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ECONOMIC CONFIDENCE AND THE YIELD CURVE AFTER THE 2008 CRISIS

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Abstract:

The purpose of this thesis is to examine the bidirectional relationship between the yield curve level, slope and curvature, and the proxies that represent the confidence of consumers, producers, and investors, using the dynamic latent factor approach. The empirical results show that the bidirectional relation between the term structure of interest rates and economic agents' confidence has shifted with the surge of the 2008 financial crisis. We find evidence that after the financial crisis, expectations have a smaller influence on the yield curve shape and, the latter has a stronger influence over the first.

Keywords: Yield Curve, Expectations, Dynamic Nelson-Siegel, State Space Models,

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(1) Introduction:

The term structure of interest rates, frequently known as the yield curve, is the relationship between short- and long-term rates that influence the decisions of economic agents when deciding, for example, to save for retirement or borrowing to buy a new house. Some might wonder why do exist different annual interest rates associated with different maturities or even look more deeply into this relationship when choosing between a fixed or variable mortgage rate. Market participants and researchers always attempt to develop functional models of the yield curve for policy evaluation or surveillance, but also to arbitrage purposes in hedge funds. It is also of pivotal interest for banks and other financial institutions, the tracking of the yield curve dynamics, for example, to interest rate forecasting or bond and options pricing.

Among specialists, it is widely accepted that bond markets quickly assimilate economic news. So expected future policy actions should affect the shape of the yield curve, as confirmed by Morales (2010). In line with this, Kurmann and Otrock (2013) demonstrate that news shocks explain more than 50% of the unpredictable movements in the slope of the yield curve in a ten years horizon forecast. From the Macroeconomic perspective, following Dong and Wang (2014) conclusion, the treasuries yield curve contains economic expectations that are helpful to predict other macroeconomic variables. Also, Estrella and Mishkin (1995), concluded that the yield curve slope is one of the most useful recession predictors. In line with this, Dueker (1997) stated that the expectations theory for the term structure of interest rates provides a theoretic substance for the predictive power of the yield curve.

The purpose of this work project is to construct a yield curve model using the same framework of the latent factor model of Diebold, Rudebush, and Aruoba (2006) (DRA (2006)), aiming equally to demonstrate that it approximates U.S. yield curve dynamics and delivers good forecasts. The Dynamic Nelson Siegel (DNS) Model, under a state-space framework, allows the inclusion of exogenous variables such as the Consumer Confidence Index (CCI), Business Confidence Index (BCI), and the VIX Index (VIX), that has been acutely studied in this work project. This last variable measures the implied volatility of the S&P500 options over the subsequent 30 days, being also known as the “investor fear gauge” or the “sentiment index”, see Tu and Chen (2018). It will be considered as the proxy of investors’ confidence, as well as CCI for consumers’ confidence and BCI for producers’ confidence. A state-space representation facilitates the integration of these exogenous variables without impacting the parsimonious estimation procedure, which also gives the possibility to test and explore the bidirectional relation to the latent factors of the yield curve.

We aim to give a possible answer to questions like: Does the uncertainty in financial markets or the lack of confidence of consumers or producers impact the shape of the yield curve? Does an increase in the level of the yield curve depreciate producers or consumer's confidence? Does a decrease in the slope increase the financial market's volatility or decrease consumers and producers’ confidence? How have those relations changed with the 2008 Financial Crisis?

Impulse Response Functions and Variance Decompositions will help to comprehend these topics. In the Literature Review and Methodology sections (2 and 3), further details will be provided. Data presentation and analysis stands in section 4, while the estimation analysis in section 5 and 6. Section 7 has the analysed Impulse Response Functions, and the respective variance decompositions are in section 8. Sections 9 presents the conclusions.

(2) Literature Review:

The first question that can surge when modelling the term structure of interest rates is: how to summarize the yields at any point in time for dozens of nominal bonds that are at the market? Assuming that only a small source of systematic risks influences the price of many assets in financial markets, it is reasonable to assume that all bond information can be summarized in a few factors or variables, see Diebold, Piazzesi, and Rudebush (2015). Accordingly, yield curve models usually employ a structure of small factor loadings that summarizes the yields of different maturities, with a parsimonious estimation procedure while showing a valuable understanding of the data.

Ang and Piazzesi (2003) were the pioneers to introduce macroeconomic variables in a yield curve model. They introduced in a no-arbitrage framework, the analysis of unidirectional relationships from macroeconomics factors to the yield curve factors. They have concluded that macro factors explain up 85% of the movements in the middle and short-term yields and 40% at the long end. Also, they showed that incorporating macro factors in a yield curve model may improve out-of-sample forecasts. Rudebusch and Williams (2009) demonstrated that the inclusion of exogenous variables could make yield curve models to have better performance in predicting recessions than professional forecasters. Following that, in the last two decades, the inclusion of macro and other external factors into yield curve models is becoming trendy. For example, Diebold, Rudebusch, and Aruoba (2006) integrated the dynamic Nelson and Siegel parametric factor model of Diebold and Li (2006) into a State-Space framework to understand the bidirectional relation between the three latent factors (level, slope, and curvature) and macroeconomic variables, finding evidence of relations in both directions.

There are many ways to incorporate macroeconomic and other external variables into the yield curve, accordingly to Dong and Wang (2014). The first one is to extract the level, slope, and curvature factors directly from the yield curve, evaluating how the external variables impact them and vice-versa. Despite the ease of method, as it is lack of economic explanation of the impact path, it is out of the scope. The second method is the addition of exogenous variables into the Nelson and Siegel (1987) latent factor model, like in the already mentioned study of Diebold, Rudebush, and Aruoba (2006). The last possible method is the inclusion of exogenous variables into no-arbitrage affine yield curve models, deriving the yields by the pricing kernel.

This approach suggests flexible linear or affine forms for the latent factors and their loadings, assuring that there are no arbitrage opportunities in the yield curve, each point in time. These models frequently fit the cross-section of yields at a particular point in time, but they do not usually succeed in fitting the dynamics of the term structure along time, see Duffee (2002). A dynamic fit is critical to our objective of analyzing the reaction of the yield curve, over time, to changes in exogenous factors.

The yield curve model, adopted in Diebold and Li (2006), has the required flexibility, being parsimonious and easy to estimate, without the need to enforce no-arbitrage restrictions, see DRA (2006). Although it might be a loss of efficiency not to impose the no-arbitrage restriction if valid, it must be weighed against the possibility of misspecification if transitory arbitrage opportunities do not disappear immediately.

Usually, in literature, studies relate the yield curve to exogenous variables, by using impulse-responses and variance-decompositions from an estimated VAR system. This estimation approach, accordingly to Ang and Piazzesi (2003), is very flexible, and the Impulse Response Functions and Variance Decompositions give prodigious insights into the relationships between macro shocks and movements in the yield curve, and vice-versa.

Whit respect to the conclusions of this kind of study in the literature, there is no consensus about what macroeconomic variables help to explain or predict better the yield curve movements. In the majority of those type of studies, the focus was on the relation between the term structure latent factors and macroeconomic variables like inflation and proxies of monetary policy or real activity, like for example, Afonso and Martins (2012), Stona and Caldeira (2019) or Cherif and Kamoun (2007). However other examples can include variables like the aggregate supply shocks from the private sector as demonstrated by Wu (2003), technology or innovation investment shocks, like Yan and Guo (2015), the rate of preferences for current consumption, Evans and Marshall, (2001) or Inflation expectations, Levant and Ma(2016).

Regarding the exogenous variable of this study, Tu and Chen (2018) defend that the Financial Stress shocks, mainly on the VIX, are found to have statistically and economically significant impacts on the yield curve when studying bond portfolios Value at Risk Nelson Siegel Models. Jubinski and Lipton (2012) have found that bond yields are statistically sensitive to changes in implied volatility and also that both short- and long-term treasury yields fall when implied volatility increases in addition to the fact that the yield curve flattens modestly. Following Jakas (2012), an increase in the Consumer Confidence Index will result in a yield curve Finally flattening. Regarding the Business Confidence Index, none relevant bibliography was found.

(2) Methodology:

To study the dynamics of the United States yield curve, we start presenting the model of Diebold and Li (2006), that interprets the model of Nelson and Siegel, as a model in which $\beta_{1,t}$,$\beta_{2,t}$,$\beta_{3,t}$ are time series that represent the Level(L_t), Slope(S_t), and Curvature(C_t) latent factors (Equation (1)). Thereby, it is possible to summarize the yields of nearly all maturities by simply estimating these factors. The terms multiplying each factor are known as “factor loadings”.

$$y(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right)\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \quad (1)$$

In equation (1), y characterizes the vector of zero-coupon bond yields, τ the maturity of each bond and λ being the decay parameter described in Nelson and Siegel (1987). However, by replicating DRA (2006), it is viable to estimate and forecast the presented Level, Slope, and Curvature using a state-space model.

$$\begin{bmatrix} L_t \\ S_t \\ C_t \end{bmatrix} = \begin{bmatrix} \mu_L \\ \mu_S \\ \mu_C \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \times \begin{bmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{bmatrix} + \begin{bmatrix} \epsilon_t(L) \\ \epsilon_t(S) \\ \epsilon_t(C) \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{bmatrix} = \begin{bmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau} - e^{-\lambda\tau_2} \\ & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} - e^{-\lambda\tau_N} \end{bmatrix} \begin{bmatrix} L_t \\ S_t \\ C_t \end{bmatrix} + \begin{bmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{bmatrix} \quad (3)$$

Equations 2 and 3 represent a state-space model, which has the convenience that methods to analyze the information available exist. It is also possible to extract the unknown latent factors and parameters applying the Kalman Filter by maximum likelihood estimation. This model is composed of a transition equation (Equation (2)), which assumes a vector autoregression of order 1, useful to make a parsimonious estimation. It follows DRA (2006), which states that "ARMA state vector dynamics of any order may be readily accommodated in the state-space form". Equation 3, known as measurement equation, follows the typical Nelson Siegel Representation, relating in this case, the known yields of different maturity's, with the unknown latent factors, accurately, the level, slope, and curvature.

Equations (2) and (3) are represented in matrix notation, respectively in equations (4) and (5):

$$\beta_t = \mu_t + A\beta_{t-1} + \epsilon_t \quad (4)$$

$$y_t = \Lambda\beta_t + \varepsilon_t \quad (5)$$

Where βt is a 3x1 vector with the latent factors, μt a 3x1 vector including the means of the factors; A, a 3x3 matrix containing the VAR (1) coefficients and ϵt is a 3x1 error term vector; yt is the Nx1 vector composed by the observed yields, Λ a Nx3 matrix containing the factor loadings, and εt being the Nx1 error vector. Likewise, in DRA (2006), it is needed to establish some assumptions:

$$\begin{pmatrix} \epsilon_t \\ \varepsilon_t \end{pmatrix} \sim WN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q_{3\times3} & 0 \\ 0 & H_{(N\times N)} \end{pmatrix}\right] \quad (7)$$

$$E(\beta_0 \epsilon_t^{'}) = 0 \quad (8)$$

$$E(\beta_0 \varepsilon_t^{'}) = 0 \quad (9)$$

One of the assumptions is that the matrix H is diagonal, meaning that the errors of different maturities are uncorrelated. However, matrix Q will be non-diagonal, letting shocks to the latent factors to be correlated. Also, the white noise errors of both equations are orthogonal to each other and the initial state, accordingly to equations (8) and (9).

The introduction of a state-space model was an improvement to the Diebold and Li (2006) model because now it is possible to compute these parameters simultaneously, applying the Kalman Filter. With this filter, it is possible to obtain the estimated factors to all periods with just one step, without the need to fix λ like in Diebold and Li. This filter is an algorithm for serially updating the one-step-ahead estimate of the state mean and variance given new information based on the observable variables. It delivers “maximum-likelihood estimates and optimal filtered and smoothed estimates of the underlying factors”, see DRA (2006). That method allows the writing down, under normality, of the likelihood function “based in the prediction error decomposition”, see Cherif and Kamoun (2007). Given the obtained likelihood function, it is possible to estimate the coefficients employing numerical optimization methods.

(4) Data:

For the Nelson Siegel State-Space Framework, it is required the input of past yield curve data. The selected sample ranges from January 1990 until April 2019, monthly data, of the constant-maturity US Treasury rates. The Yield´s data was taken from Bloomberg of the subsequent maturities: 3 Months, 6 Months, 1 Year, 2 Years, 3 Years, 5 Years, 7 Years, and 10 Years, totalizing 352 observations. Later, exogenous variables will be added to the State Space model previously presented as a way to study the relationship between the economic agent's expectations and yield curve factors. The US Business Confidence Index (BCI), collected from the OECD database, is a variable created from opinion surveys in the industry sector, representing the confidence of the production side. Also, from this database, was collected the US Consumer Confidence Index (CCI), a variable “based upon answers regarding their expected financial situation, their sentiment about the general economic situation, unemployment and capability of savings” that represent the confidence of the consumer side. The VIX, collected from Bloomberg, is a measure of the expected volatility of the U.S. stock market, derived from of S&P 500 Index, which represents the confidence of the investors reflected in the financial markets. Table 1 contains some descriptive statistics of the data presented.

	3m	6m	Y1	Y2	Y3	Y5	Y7	Y10	CCI	BCI	VIX
Mean	2.79	2.93	3.07	3.38	3.61	4.01	4.33	4.57	100.06	99.84	19.26
Median	2.64	2.78	2.84	3.30	3.59	4.01	4.32	4.50	100.39	99.92	17.28
Maximum	8.17	8.28	8.40	8.72	8.78	8.77	8.81	8.89	102.69	102.00	59.89
Minimum	0.01	0.04	0.10	0.21	0.33	0.62	0.98	1.50	96.71	96.02	9.51
Std. Dev.	2.31	2.34	2.33	2.34	2.28	2.14	2.03	1.91	1.40	1.03	7.42
Skewness	0.27	0.24	0.22	0.21	0.21	0.21	0.24	0.28	-0.38	-0.82	1.74
Kurtosis	1.80	1.78	1.78	1.81	1.86	1.94	2.01	2.11	2.58	4.34	7.67

Table 1 – Data descriptive statistics.

(5) Estimation Analysis: Yield Only Model

To follow the estimation of the Yields Only Model, like the one represented by the equations (5) and (6), we need to define the initial values for the estimation of the state space model. To find the initial guesses for the model, following DRA (2006), I first estimated the two-step approach of Diebold and Li (2006) term structure model, also getting the baseline yield curve latent factors and the possibility to previously study the dynamics of the yield curve.

Diebold and Li (2006) interpreted the Nelson Siegel Model factors, from equation (1), as “three latent factors”, also presenting empirical proxies, which I will use, to compare with my estimation. They defined the Level as $y_t(\infty)$, in our case, the yield of maximum maturity, $y_t(120M)$. The Slope factor is $y_t(120M) - y_t(3M)$. Finally, the curvature is definite as $2 \times y_t(24M) - y_t(3M) - y_t(120M)$.

Following Diebold and Li (2006), based on equation (1), I estimated the times series of $\beta_{1,t}$, $\beta_{2,t}$, $\beta_{3,t}$ by applying Ordinary Least Squares for each month of the dataset, fixing λ at 0.0609, the same used by Diebold Li (2016). However, to follow the empirical notation, from now on, the slope will be considered as the symmetric value of $\beta_{2,t}$.

Some statistics of the series of the Empirical Proxies (Emp.) and the calculated Diebold and Li (DL) Betas, are provided in table 2.

	Emp Level	DL Level	Emp Slope	DL Slope	Emp Curv	DL Curv
Mean	4.57	5.12	1.77	2.34	-0.74	-1.83
Median	4.50	5.01	1.84	2.36	-0.69	-1.63
Maximum	8.89	9.29	3.69	5.08	0.87	4.08
Minimum	1.50	1.88	-0.70	-0.85	-2.35	-6.51
Std. Dev.	1.91	1.80	1.11	1.56	0.77	2.25
Skewness	0.28	0.27	-0.13	-0.05	-0.21	-0.17
Kurtosis	2.11	2.33	2.02	1.95	2.00	2.10

Table 2 – Descriptive statistics of the estimated latent factors.

At this moment, it is already possible to estimate the state-space model, represented by equations (4) and (5), namely the transition and measurement equations as well as the Matrixes Q and H exposed in error distribution assumption (7).

Because we have yields data of 8 different maturities, there are 27 unknown parameters to estimate: 9 in matrix A, 3 in vector μ, as well as 3 unknown variances and 3 unknown covariances in the covariance matrix, all from the transition equation. From the measurement equations, there are for each maturity, one error term resulting in 8 variances to estimate, also with the decay parameter λ.

Before starting the estimation, initial values had to be defined. The coefficients got from the estimation of an unrestricted VAR(1) with the three estimated Diebold Li (2006) factors, initialize the Matrix A. The means, variances and covariances of the same factors set the initial values for the vector μ_t and the matrix Q. Matrix H was set with the variances of the yields. The estimation was reached through Kalman Filter, using the Broyden, Fletcher, Goldfarb, and Shanno (BFGS) algorithm and Dogleg step method, being possible to estimate the three latent factors. By comparing the estimated factors with the Diebold and Li (2006) method series, and the calculated empirical proxies defined in DRA (2006), it is confirmed the excellent fit of the model. The latent factors estimated by the Diebold and Li method are strongly correlated with the empirical proxies, presenting a correlation of 96,89% on Level, 99,39% on Slope, and 99,06% on the curvature factor. Comparing the estimated factors by the state-space model, with Diebold and Li (2006) method, they also present impressive values of correlation, particularly 99.09% on the level factor, 98.99% on the slope, and 97.11% on curvature factor. It is also possible to analyze the evolution of the estimated series by regarding their plots, exposed in figures 1, 2, and 3.

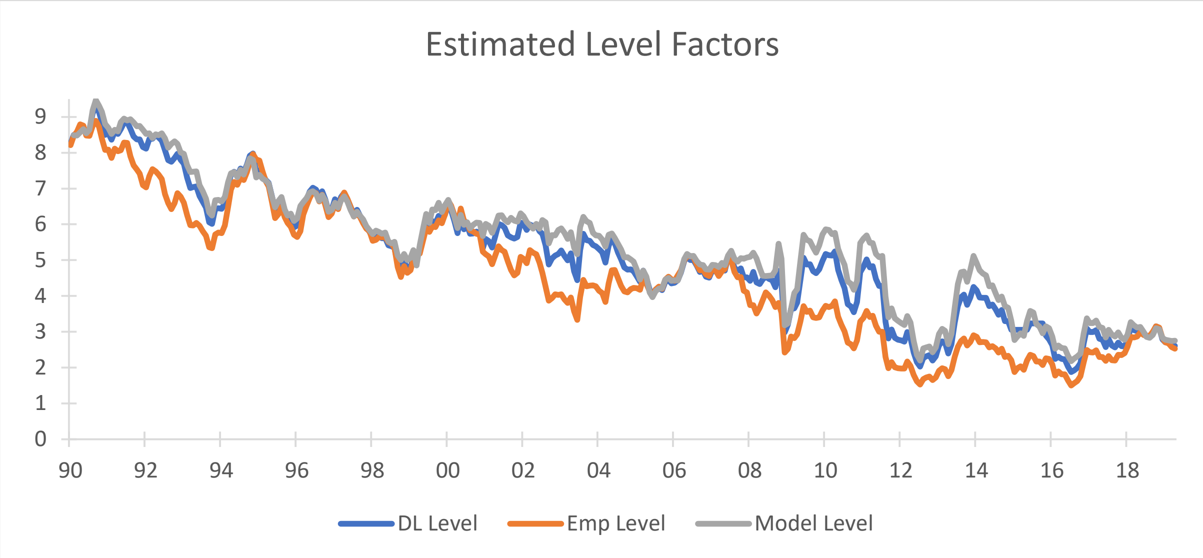

Figure 1- Estimated Level factors. DL - Diebold and Li (2006) method; Emp -Empirical Proxy (Emp); Model – State Space model estimated factor

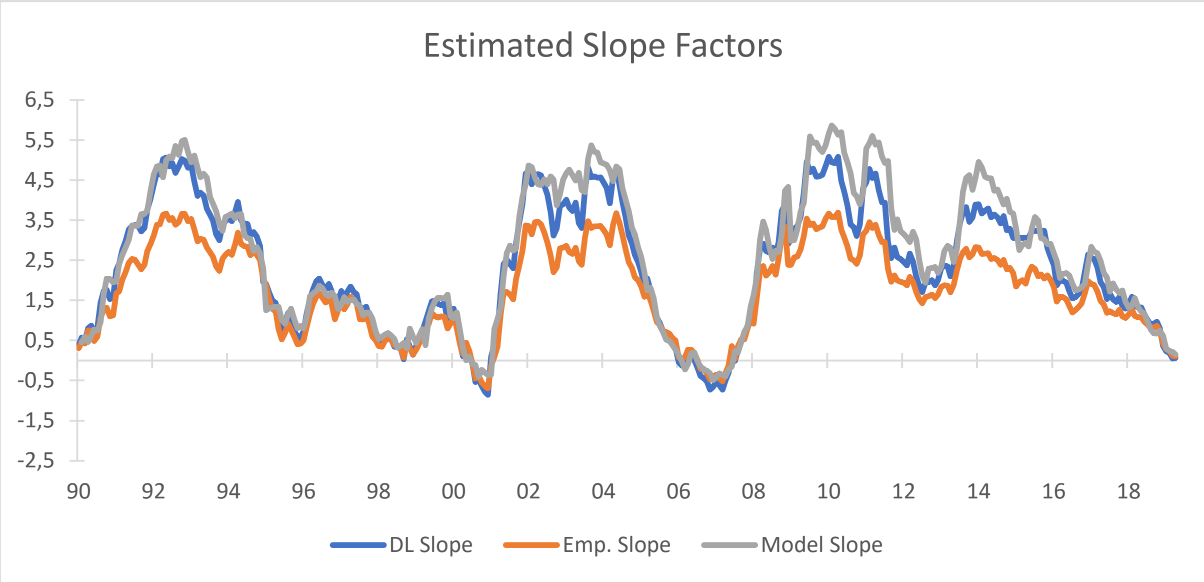

Figure 2- Estimated Slope factors. *DL - Diebold and Li (2006) method; Emp -Empirical Proxy (Emp); Model – State Space model estimated factor*

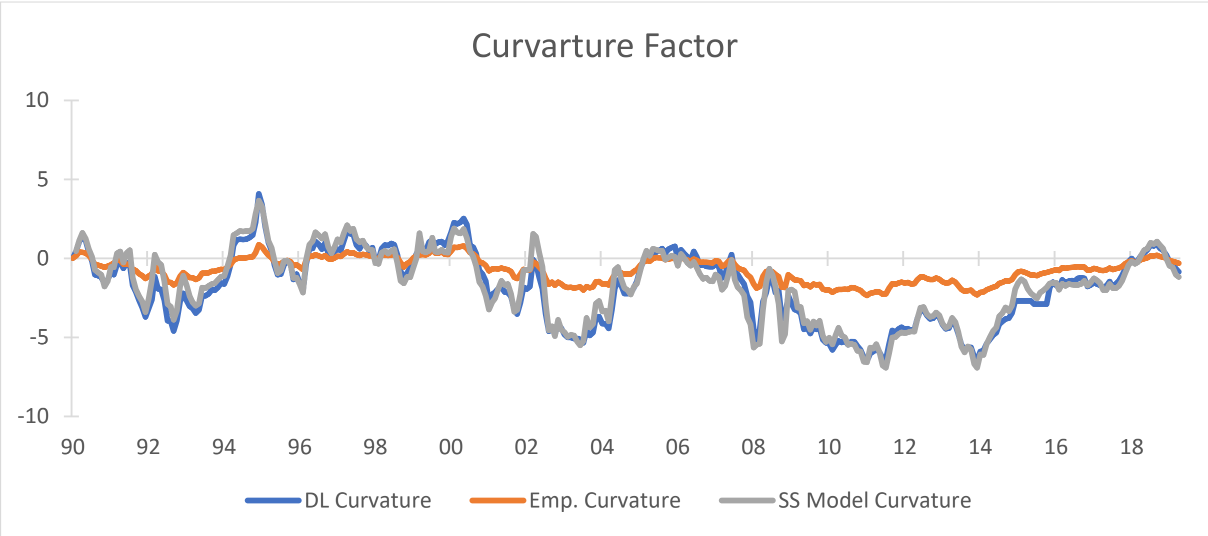

Figure 3 - Estimated Curvarure factors. *DL - Diebold and Li (2006) method; Emp -Empirical Proxy (Emp); Model – State Space model estimated factor*

Also, it is possible to assess the quality of the US yield curve fit obtained by the estimated model. Figure 4 contains the graph of the average yield curve accordingly to the estimated methods, being possible to see that all of them are very close to each other, proving the good in-sample fit.

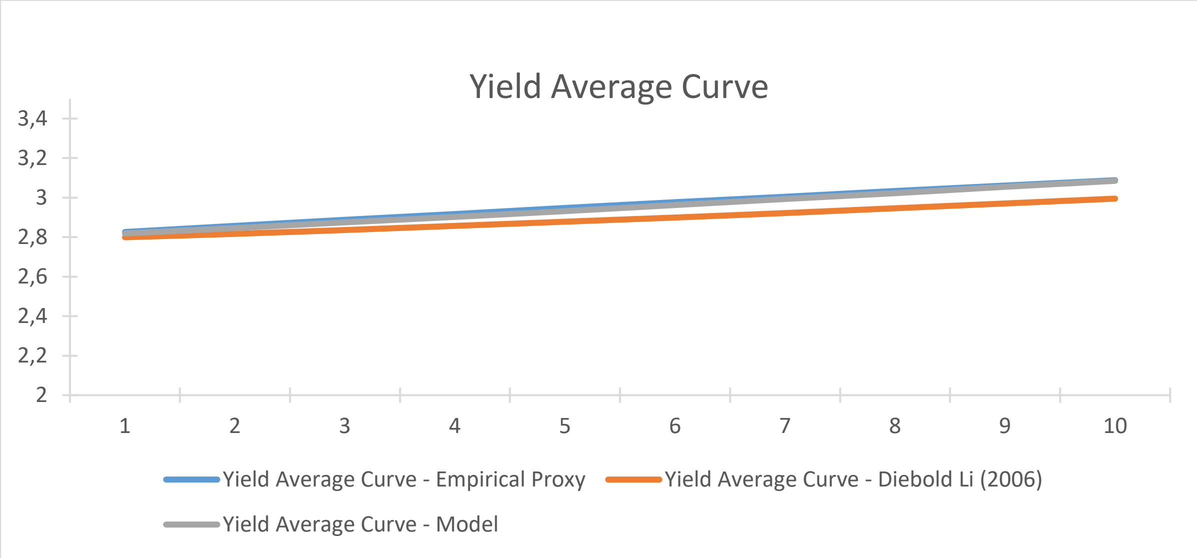

Figure 4 – Average Yield Curves

Having analyzed the fit of the yield curve in sample forecasts provided by the estimated state-space model represented by equations (4) and (5), adopting the assumptions of equations (6), (7) and (8), we can finally analyze the VAR transition equation Matrix A and the Variance Covariance Matrix Q, to access the relations between the Latent Factors, in Table 3.

	CONST	Level(-1)	Slope(-1)	Curvature(-1)
Level	0.086762151	**0.987634776**	0.010415708	0.00521764
Slope	0.11888281	<u>-0.034500163</u>	**0.938822679**	**0.052323462**
Curvature	-0.151963146	0.010318543	-0.007446193	**0.957531088**

Table 3 – Matrix A – Yield Only model - Bold Values are significant at 5% and underlined at significant at 10%.

	DL Level	DL Slope	DL Curvature
DL Level	**0.067838893**	**-0.064190386**	**-0.042743612**
DL Slope		**0.088936298**	**0.043064073**
DL Curvature			**0.424891375**

Table 4 – Matrix Q - Yield Only model - Bold Values are significant at 5% and underlined at significant at 10%.

Regarding Matrix A, it is possible to observe that the own dynamics of all Latent Factors are highly persistent. The coefficients that represent the relations between the same Latent Factors play a minor role. Only the coefficients of the Curvature lag effect to the slope and the effect of a lag on the Level factor to the slope, are significant at 5% and 10%, respectively. Regarding the constant values, they are not significant, but they present reasonable values. These results are highly similar to DRA(2006) and other works like Cherif and Kamoun (2007) that study the yield curve for Euro Area and also similar to Levant et al. (2016) study for the United Kingdom. On Matrix Q, all terms are significant at the 5 % level, likewise DRA (2006). It is also possible to verify that the transition shock volatility increases going from the level factor to the curvature factor.

(6) Estimation Analysis: Augmented DNS Model

Until now, we have denoted that the three latent factors of the Nelson Siegel Model provide an acceptable representation of the yield curve, being of interest, to study the relationship with the economic agents' expectations proxies. One difference between the Diebold and Li (2006) methodology and DRA (2006) is that, in the latter, it is possible to include exogenous variables. It is possible to extend the model presented in equations (1) and (2) in order to include the variables CCI, BCI, and the VIX. Regarding the correlation between these exogenous variables and the latent factors, we can denote an interesting correlation of the Curvature (66%) and the Slope (49%) to the Consumer Confidence Index, and a negative correlation (-18%) between Business Consumer Index and the Level factor. The Extended state-space model of the Nelson Siegel, are represented in equations (10) and (11), maintaining the assumptions related to the equations (7), (8), and (9).

$$\begin{bmatrix} L_t \\ S_t \\ C_t \\ CCI_t \\ BCI_t \\ VIX_t \end{bmatrix} = \begin{bmatrix} \mu_L \\ \mu_S \\ \mu_C \\ \mu_{CCI} \\ \mu_{BCI} \\ \mu_{VIX} \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \alpha_{15} & \alpha_{16} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \alpha_{25} & \alpha_{26} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \alpha_{35} & \alpha_{36} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \alpha_{45} & \alpha_{46} \\ \alpha_{51} & \alpha_{52} & \alpha_{53} & \alpha_{54} & \alpha_{55} & \alpha_{56} \\ \alpha_{61} & \alpha_{62} & \alpha_{63} & \alpha_{65} & \alpha_{65} & \alpha_{66} \end{bmatrix} \times \begin{bmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \\ CII_{t-1} - \mu_{CII} \\ BCI_{t-1} - \mu_{BCI} \\ VIX_{t-1} - \mu_{VIX} \end{bmatrix} + \begin{bmatrix} \epsilon_t(L) \\ \epsilon_t(S) \\ \epsilon_t(C) \\ \epsilon_t(CCI) \\ \epsilon_t(BCI) \\ \epsilon_t(VIX) \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{bmatrix} = \begin{bmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau} - e^{-\lambda\tau_2} \\ & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} - e^{-\lambda\tau_N} \end{bmatrix} \begin{bmatrix} L_t \\ S_t \\ C_t \end{bmatrix} + \begin{bmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{bmatrix} \quad (11)$$

$$\begin{pmatrix} \epsilon_t \\ \varepsilon_t \end{pmatrix} \sim WN \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q_{6\times 6} & 0 \\ 0 & H_{(8\times 8)} \end{pmatrix} \right] \quad (12)$$

$$E(\beta_0 \epsilon_t') = 0 \quad (13)$$

$$E(\beta_0 \varepsilon_t') = 0 \quad (14)$$

At this stage, there are more parameters to estimate: 36 in the transition matrix A, 6 in the mean vector μ, 21 unknown variances and covariances in matrix Q (Now this matrix has a dimension of 6×6), 8 variances on H and the decay parameter λ. The estimation method is similar to the one presented in the Yields Only section.

	CONST	Level(-1)	Slope(-1)	Curvature(-1)	CCI(-1)	BCI(-1)	VIX(-1)
Level	-0.411	0.979	-0.013	0.000	**0.015**	-0.009	0.000
Slope	-2.461	0.013	0.742	0.057	**0.070**	-0.047	0.002
Curvature	-3.966	0.078	-0.109	1.002	0.019	0.019	0.002
CCI	91.109	-0.043	0.084	0.058	**0.949**	-0.049	**0.016**
BCI	69.188	0.012	-0.017	-0.019	0.029	**0.938**	0.016
VIX	-1706.660	0.007	-0.015	-1.184	-0.090	0.141	**0.576**

Table 5 - Matrix A – Extended Model - Bold Values are significant at 5% and underlined at significant at 10%.

	Level	Slope	Curvature	CCI	BCI	VIX
Level	**0.446**	**-0.444**	**-0.921**	**-0.651**	-0.126	**-12.883**
Slope		0.573	**0.998**	-0.883	-2.010	**77.409**
Curvature			**2.355**	0.690	-0.651	**58.942**
CCI				1.820	0.271	-11.244
BCI					1.403	16.595
VIX						145.806

Table 6 - Matrix Q – Extended Model - Bold Values are significant at 5% and underlined at significant at 10%.

For interpretation purposes, we can divide the matrix A into four parts: A_1 for the relations of the yield curve factors lags on contemporaneous yield curve factors; A_2 for the dynamics of exogenous factors lag on yield curve factors; A_3 for the yield curve factors lag to exogenous factors and A_4 for the relationship between exogenous factors lag to current exogenous factors.

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$$

From A_1, although it does not have any significant coefficient as in the Yields Model only, the coefficients have not changed significantly in value. The exception is the own dynamics of the slope factor that had decreased with the inclusion of the exogenous variables. Regarding A_2, more interesting conclusions can be reached. The level and the slope are influenced positively by the Consumer Confidence Index, with a significance of 5%. In A_3, there are not any significant coefficient. However, later, it will be shown that they are jointly significant. Finally,

A_4 reveals significant and relevant own dynamics of all exogenous factors. It also indicates an influence of VIX over CCI. Matrix Q presents the existence of significant relations between the Level factor and all factors in exception to BCI. Also, it points to a deep relation of VIX with yield curve factors.

By performing Wald tests, it is possible to test the existence of bidirectional relations between the yield curve factors, and the exogenous factors. It is possible to test the unidirectional impact of the yield curve factors on the exogenous variables (A_3=0 and Q_m =0), the unidirectional impact of the external variables on the yield curve factors (A_2 = 0), or the bidirectional effects (A_3=Q_m=A_2 = 0). Q_m is the block that contains the covariance terms between exogenous and yield factors in matrix Q.

Wald Test	Restrictions	P-value
No interaction	27	0
No interaction exogenous to Yields	9	0
No Interaction Yields to Exogenous	18	0

Table 7 - Wald Tests.

All three tests for both subsamples, reject the null hypothesis of no interactions between external and term structure factors in both directions. Given that, it is possible to conclude that the economic agent's expectations impact the yield curve dynamics, as well as yield curve factors impact consumers, producers, and investors' expectations.

	Maturidade (Months)	**120**	**84**	**60**	**32**	**24**	**12**	**6**	**3**
Measurement errors (Basis Points)	**Augmented**	0.684	3.773	0.282	-1.667	0.761	-0.972	2.522	-2.244
	Yields Only	-3.988	0.142	-1.421	-0.273	3.045	-0.506	-0.200	-7.454
Correlation	**Augmened**	0.994	0.993	0.994	0.995	0.995	0.997	0.997	0.995
	Yields Only	0.992	0.994	0.994	0.995	0.995	0.996	0.997	0.997

Table 8 - Forecasted Yields descriptive statistics - Models Comparison

Finally, it is crucial to evaluate the fit of both estimated state-space models in order to understand the yield curve forecasting accuracy for the two state-space models. Regarding table 8, we can observe the quality of the fit to the historical values. They are very similar in both models, given the high correlation and the low measurement errors, being even slightly lower than those reported by DRA (2006).

(7) Impulse Response Functions

To better infer about the relations between our exogenous variables and the yield curve factors, we will follow Afonso and Martins (2012). They analyzed Impulse Response Functions from separate VAR´s based in two arguments: 1) There is no guarantee that a VAR (1) "would be the outcome of the optimal lag length analysis". 2) A state-space model would not produce estimates of the yield curve factors significantly different from those obtained from the yields only model.

In the middle of our sample, we have one of the most important events of the recent economic history, the financial crisis of 2007-08, having the United States as the epicenter of the phenomena. The existence of a structural break can indicate that the relation of the yield curve and our proxies of the expectations of investors, consumers, and producers might have changed with this event. Given the new paradigm of low-interest rates after the financial crisis, this can be an interesting analysis. To test this, we can perform a likelihood ratio test. Our goal is to test the significance of the financial crisis over the term structure bidirectional relations with our exogenous variables. To do this, we can break the sample in two: 1990 until 2008 and 2008 until 2019, estimating two VAR models for each sub-sample additionally with the entire sample model. To perform this, I computed a simple Likelihood Ratio Test.

$$LR = 2 \times (\mathcal{L}_{u1} + \mathcal{L}_{u2} - \mathcal{L}_R)$$

Following the Schwarz Information Criterion, it was computed a VAR(2) for both sub-samples and a VAR(3) for the all sample period, using the state variables estimated in the previous model and the exogenous data already presented, ensuring that all series are stationary. Examining the correlograms of the residuals, all models have a good fit for the relations between exogenous variables and yield curve factors. The Log-Likelihood value of the first sub-sample is -237.5, for the second sub-sample is -107.3928 and for the whole sample is -374.6314. The LR value is 59,48, which is higher than the Critical Value for 5% significance, $\chi(72)$ =45,245, meaning the rejection of the null hypothesis of no existing break. Given this, it is possible to conclude the existence of a structural break in the financial crisis period. Given this vital conclusion, we will present and analyze the two sub-sample Impulse Response Functions.

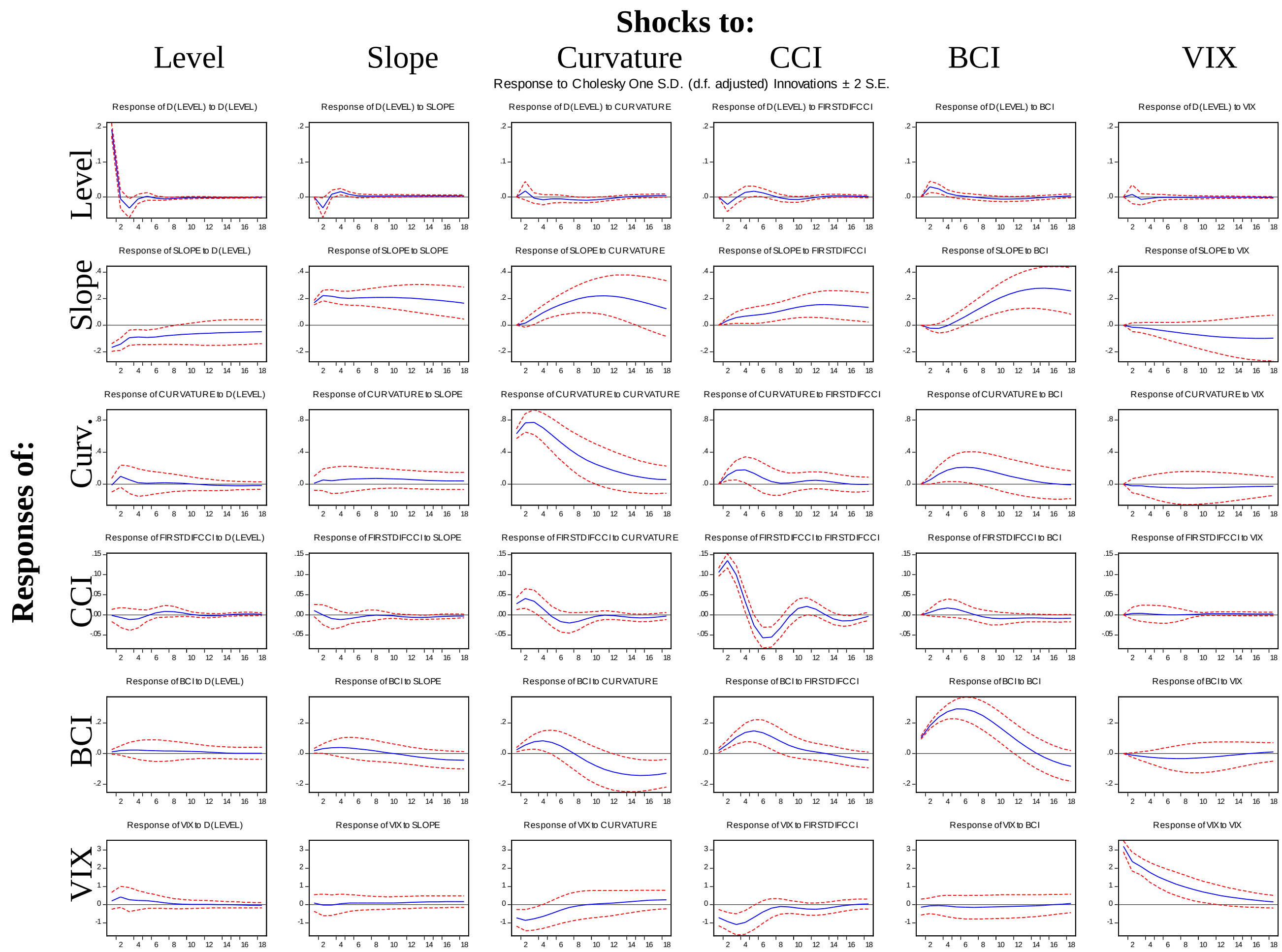

Figure 5 Impulse Response Functions: Sub-Sample 1990-2008.

In figure 5, it is possible to evaluate the Impulse Response Functions for the period before financial crises. In the lower first quarter, regarding the relations between the exogenous factors and the yield curve factors, there is not any particular significant reaction to yield curve shocks. The only slight exception is the responses of the exogenous variables to a Curvature Shock, which are positive in the first periods in the case of BCI and CCI and negative in the case of VIX.

Analyzing the responses of the yield curve factors to the exogenous variable's shocks, we can observe much more interesting interactions. To a shock over the BCI or CCI, it follows a positive response of the Slope factor. However, facing a shock to VIX, the slope factor has the opposite response. That would mean that the increase in the volatility of financial markets, or like mentioned by some authors, the fear of invests, would make the slope of the yield curve to decrease, which is expectable. Also, the curvature factor has a slightly positive response to a shock on BCI.

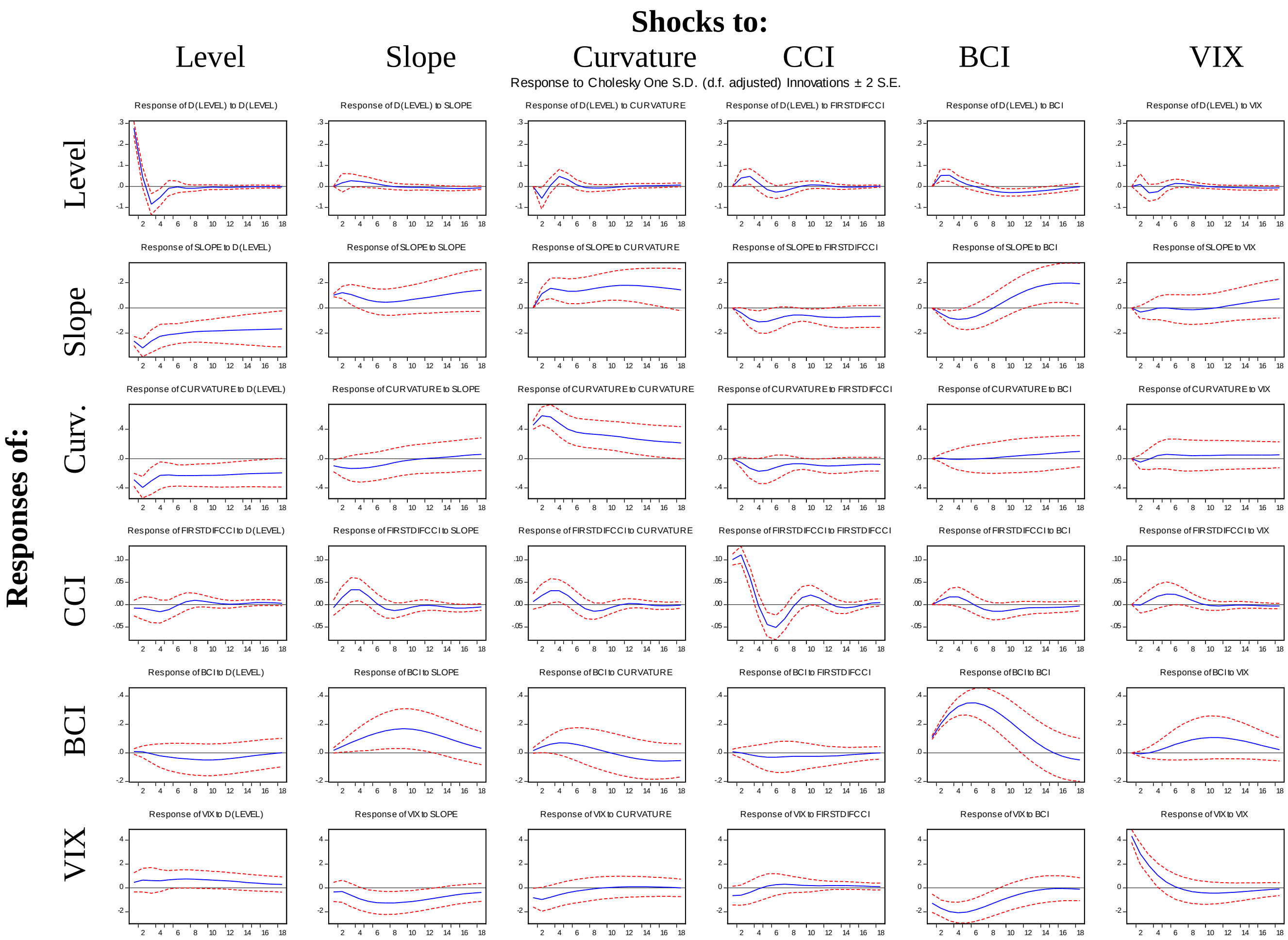

Figure 6 - Impulse Response Functions: Sub-Sample 2008-2019

We can observe some exciting features and differences between the two sub-samples. In figure 6, observing the responses of the exogenous variables to yield curve factors, after the crisis, the BCI responds positively to a shock on the Slope yield curve factor, which did not happen in the period before the Financial Crisis. After the financial crisis, although the reaction starts to be null, the Business Confidence Index increases 0.17 points to a one standard deviation shock to the slope factor, at its peak at eight months. Before the crisis, the BCI reaction was nearly insignificant. It could mean that after the crisis, business makers are more aware of the predictive power of the yield curve. Also, BCI has a slightly positive response to a shock in the Curvature, although not as sharp as before the financial crisis. Also, the VIX, in the after crisis sub-sample, seems to react further to shocks in the yield curve. It reacts positively to a shock in the level and negatively to the slope. After the crisis, the VIX index had an increasingly negative reaction until its peak (6 months), in which it reached a decrease of -1.6 index points to a one positive standard deviation shock of the slope. When a one standard deviation shock on the level triggers, the VIX response is nearly stable, reaching an increase of around 0.6 index points along 18 months. Before the financial crisis, these responses were nearly insignificant. That goes with the expectable, pointing that in recent times, financial markets discount more the famed predictive power of the yield curve. Investors fear decrease when slope increases and increase when the level increase. Also, the Consumer Confidence Index has an increasingly positive reaction to a slope increase until the fourth month, in which it reaches an increase of 0.033 index points (in first differences).

Concerning the responses of the yield curve factors to the exogenous variables, we can perceive that there are only a few differences between the two sub-samples. In the period after the financial crisis, we can verify that the slope and curvature of the yield curve react less to a shock on the BCI. In both samples, the slope starts to react slightly negatively in the first 4-8 months, having a strongly positive reaction after that. Before the crisis, the peak reached 0.28 of BCI index points increase, while after the crisis, it reached 0.20. That is a nearly 30% reaction decrease after the crisis. The responses of the Slope to VIX and CCI shocks are, after the crisis, less significant, even though having an opposite sign.

In conclusion, after the financial crisis, the exogenous variables responses to yield curve factors are more significant. We can assume, as a possible argument, that investors and producers became aware of the predictive power of the yield curve. Conversely, as the yield curve factors become less reactive to exogenous variables shocks, one possible argument is that it could happen because of the new normal of low-interest rates. After the crisis, due to quantitative easing, given the supplementary intervention of the Central Banks on the financial markets, it is possible to confirm that expectations played a minor role in shaping the Yield Curve.

(8) Variance Decompositions

In order to improve the previous analysis, it is possible to examine the variance decompositions of some selected interest rates. The analysis indicates the amount of information each of the variables of the model (and itself) contributes to explain the short-term interest rate (3 months) and the medium/long term interest rate (5 years) before and after the financial crisis.

Variance Decomposition of 3 months the Yield							
Period	**3M**	**Lev.**	**Slo.**	**Cur.**	**CCI**	**BCI**	**VIX**
3	84.36	1.01	1.61	6.98	3.73	0.97	1.34
		6%	**10%**	**45%**	**24%**	**6%**	**9%**
60	65.31	1.33	2.55	12.30	5.16	9.92	3.43
		4%	**7%**	**35%**	**15%**	**29%**	**10%**
120	65.26	1.35	2.55	12.32	5.17	9.92	3.44
		4%	**7%**	**35%**	**15%**	**29%**	**10%**

Table 9 – 3 months Yield Variance Decomposition: Sample 1990-2008

Variance Decomposition of 3 months the Yield							
Period	**3M**	**Lev.**	**Slo.**	**Cur.**	**CCI**	**BCI**	**VIX**
3	66.98	3.90	1.19	20.89	0.39	3.63	3.02
		12%	**4%**	**63%**	**1%**	**11%**	**9%**
60	59.38	3.98	2.11	19.21	1.71	9.13	4.49
		10%	**5%**	**47%**	**4%**	**22%**	**11%**
120	59.38	3.98	2.11	19.21	1.71	9.13	4.49
		10%	**5%**	**47%**	**4%**	**22%**	**11%**

Table 10 - 3 months Yield - Variance Decomposition: Sample 2008 – 2019

Variance Decomposition of 5 Years Yield							
Period	5Y	Lev.	Slo.	Cur.	CCI	BCI	VIX
3	89.57	0.13	1.20	1.69	2.21	4.91	0.27
		1%	**12%**	**16%**	**21%**	**47%**	**3%**
60	84.98	0.21	1.46	1.65	2.84	8.07	0.79
		1%	**10%**	**11%**	**19%**	**54%**	**5%**
120	84.94	0.22	1.46	1.65	2.85	8.08	0.80
		1%	**10%**	**11%**	**19%**	**54%**	**5%**

Table 11 – 5 years Yield - Variance Decomposition: Sample 1990-2008

Variance Decomposition of 5 Years Yield							
Period	5Y Yield	Lev.	Slo.	Cur.	CCI	BCI	VIX
3	80.86	7.62	0.75	0.69	1.00	7.80	1.28
		40%	**4%**	**4%**	**5%**	**41%**	**7%**
60	69.77	8.77	4.02	1.65	1.62	12.00	2.16
		29%	**13%**	**5%**	**5%**	**40%**	**7%**
120	69.77	8.77	4.02	1.65	1.62	12.00	2.16
		29%	**13%**	**5%**	**5%**	**40%**	**7%**

Table 12 - 5 years Yield - Variance Decomposition: Sample 2008-2019

Analyzing the Variance Decompositions before Financial Crisis, we can see that the variation of the 3-month yield is mostly explained by itself, either at a horizon of 3 months or 120 months. At shorter horizons, the remaining variances (grey areas) are explained by the curvature factor (45%) and CCI (24%). Nevertheless, in a ten years horizon, it is also explained by BCI (29%) and less by the CCI (15%). The five years Yields Variance decomposition presents a more intense self-explanatory behaviour. However, the remaining variance is growingly explained by the Business Confidence Index, along the horizon (about 50%). Comparing the previous results to the period after the financial crisis provides valuable insights. Both the specific interest rates have less self-explanatory variance after the financial crisis as they are, therefore, more explained by the yield curve factors. Also, the remaining variances of both interest rates are less explained by the exogenous factors. Given that the yield curve factors, respond less to the exogenous variables shocks after the crisis, as already discussed in the previous section, this was a predicted conclusion. Another valuable insight is that the VIX, explains more the individual yields after the financial crisis when the BCI and CCI have less explanatory influence.

(9) Conclusion

The objective of characterizing the yield curve using a dynamic Nelson-Siegel model was fulfilled with success, through state-space modelling. The suitableness of the Level, Slope, and Curvature obtained by both models suggested by DRA (2006), seem pretty good, being possible to find almost insignificant measurement errors.

From the addition of the Consumer Confidence Index, Business Confidence Index, and the VIX, as proxies of the expectations/fears of consumers, producers and financial markets participants to the Yields-Only model, it was possible to conclude that both yield curve factors impact the economic agents expectations and that economic agents expectations impact the yield curve shape for the whole sample period. It was also possible to conclude that this relationship had changed with the financial crisis, resulting in some enthusiastic differences, analyzed with the computed Impulse Response Functions and Variance Decompositions, for each sub-sample.

After the Financial Crisis, yield curve latent factors had a higher impact on economic agents' expectations. For example, after the financial crisis, producers tend to react positively to an increase in the slope of the yield curve, while no evidence was found before the crisis. Moreover, financial markets volatility decreased in reaction to yield curve slope increases, and increased, after a rise in the level factor. One possible argument is that with the financial crisis, producers and investors became more aware of the predictive power of the yield curve, widely spoken in recent years, see Jari Hännikäinen (2017).

On the opposite side, the confidence of the economic agents, after the financial crisis, seems to impact less the yield curve shape. Before the financial crisis, an increase in the consumers' and producers' expectations would incite by a higher magnitude the level and the slope of the yield curve. For example, the slope reaction to a BCI shock has a nearly 30% reaction decrease after the crisis. Also, the Slope and Level reaction to a shock to the VIX is, after the crisis, less significant. Variance Decompositions, besides confirming the previous findings, show that specific interest rates have less self-explanatory variance after the financial crisis.

Those findings would raise the questions: Is the yield curve shape more impacted by Federal Reverse mission of stabilizing and powering the economy after the financial crisis? This question can be an initiative for studying the monetary and fiscal policies' impact on the yield curve, before and after the Financial Crisis. As the expectations explain less the yield curve, after the crisis, has the predictability of the yield curve changed with the 2008 financial crisis?

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