Diogo Carvalho dos Santos Mota

Bachelor Degree in Engineering and Industrial Management Sciences

GameOn: A Game-Theoretic Approach to Digital Marketing and Online Lead Generation for Oligopoly Markets

Submitted to the graduate faculty Universidade Nova de Lisboa – Faculdade de Ciências e Tecnologia in partial fulfillment of the requirements for the degree of Master in Engineering and Industrial Management

Supervisor: Professor António Carlos Bárbara Grilo, Professor Auxiliar, Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa

Co-Supervisor: Professor Marta Cristina Vieira Faias Mateus, Professora Auxiliar, Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa

Examination Committee**:**

Chairperson: Professor Rogério Salema de Araújo Puga Leal
Examiner: Professor Ricardo Jardim Gonçalves
Member of the Committee: Professor António Carlos Bárbara Grilo

FACULDADE DE CIÊNCIAS E TECNOLOGIA
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First, I’d like to thank both my supervisor and my co-supervisor. To Professor António Grilo, thank you for all the support, suggestions and constructive criticisms throughout these months of research and investigation. Thank you also for the provided situations and meetings that helped to gather the necessary data to develop the best work possible. To Professor Marta Faias, thank you for all the help and the assistance especially, for the mathematical section of work. Thank you also for all the time and availability to discuss the dissertation and all the incentives transmitted.

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Abstract

The importance and role of digital marketing in today's competitive world is rapidly increasing. The surge and rapid expansion of digital technologies and especially, the Internet has propelled a shift in the consumers' habits and consequently in the strategies that firms must employ to attract the maximum number of consumers possible towards their products and/or services. Within these efforts, the online lead generation process is gaining steam and is currently an extremely important activity which most firms in most industries perform. This process has been a target of much attention very recently with several articles published in world renowned business magazines and websites, however up until now, no scientific methodology has been proposed to advise firms in the optimal quantity of online leads to be generated.

This research suggests the application of Game Theory to develop a useful model that has the potential to help digital marketing managers in their task of selecting the optimal quantity of online leads to be generated for the period in preparation in the most efficient and effective way possible in order to achieve their objectives. The proposed model, GameOn, can be used in several scenarios evidencing its wide flexibility. GameOn is developed with two different functional forms, the natural logarithm and square root, to provide digital marketing managers with the choice of which best fits their data and select it.

An application scenario is developed, as well as a case study, and their results discussed in depth. Two types of equilibrium, Nash and Stackelberg, are determined and discussed. With very satisfactory and encouraging results, the potential, efficiency and effective improvements brought on by the use of GameOn are evaluated and discussed and other possible applications scenarios of GameOn are introduced.

Keywords: Digital Marketing; Game Theory; Online Lead Generation; Efficiency; Effectiveness

Resumo

A importância e papel do marketing digital atualmente, neste mundo competitivo têm vindo a aumentar a um ritmo acelerado. O aparecimento e a rápida expansão das tecnologias digitais, especialmente da Internet, têm impulsionado uma mudança nos hábitos dos consumidores e, consequentemente nas estratégias que as empresas devem empregar de forma a atraírem o número máximo possível de consumidores para os seus produtos e/ou serviços. Dentro destes esforços, o processo de geração de *leads online* tem vindo a destacar-se e é atualmente uma atividade de extrema importância que a maioria das empresas na maioria das indústrias desempenha. Este processo tem sido alvo de bastante atenção muito recente com vários artigos publicados em revistas e *websites* de renome internacional no mundo empresarial, no entanto, até agora, nenhuma metodologia científica foi proposta para aconselhar empresas da quantidade ótima de *leads online* a ser gerada.

Este trabalho de investigação sugere a aplicação da Teoria dos Jogos para o desenvolvimento de um modelo útil com o potencial para ajudar os gestores de marketing digital na sua tarefa de selecionar a quantidade ótima de *leads online* a serem geradas para o período em preparação da forma mais eficaz e eficiente possível de forma a atingirem os seus objetivos. O modelo proposto, GameOn, pode ser utilizado em vários cenários evidenciando a sua vasta flexibilidade. GameOn é desenvolvido em duas diferentes formas funcionais, o logaritmo natural e a raiz quadrada, para fornecer aos gestores de marketing digital a escolha para utilizarem aquele que melhor se ajusta aos dados.

São desenvolvidos um cenário de aplicação e um caso de estudo, e os seus resultados discutidos em profundidade. Os tipos de equilíbrio, Nash e Stackelberg, são determinados e discutidos. Com resultados muito satisfatórios e encorajadores, o potencial, eficiência e eficácia resultantes da utilização de GameOn são avaliados e discutidos e outros cenários de aplicação possíveis de GameOn são introduzidos.

Palavras-chave: Marketing Digital; Teoria dos Jogos; Geração de *Leads Online*; Eficiência; Eficácia

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Chapter 1. Introduction

1.1 Motivation for the Research

The process of lead generation has been evolving in a rapid scale up throughout the years. In marketing, lead generation has a straightforward meaning that is universally accepted, and simply defined as the generation of consumer interest or inquiry into products or services of a business[1]. Most firms today are in the middle of a process of switching from offline generated leads, such as leads through the media of television, newspapers, and telemarketing or even door-to-door, to leads generated online, making use of the latest digital technologies such as the Internet, the smartphones, the social media and others. One of the causes for this shift is a visible increase of readily available information to consumers in the digital world.

The importance of digital marketing and online advertising has been rising for years, and the fact that there are already websites with the sole purpose of giving advice regarding of much capital should a firm invest in digital marketing is a proof of that. Additionally, many articles have been published online very recently, since this is a new and rapid growing subject with very little scientific research done to date, defending a greater share of online advertising spending when comparing to advertising in other media. In March of 2015, Tim Bourgeois wrote an article in EContent magazine in 2015, which specializes in digital and content marketing, stating that firms should be spending 90% of their advertising budget on digital media. He states, that according to Forrest Research, digital advertising accounted for 24% of total ad activity and in 2016 will overtake television as the largest media in advertising. A 2015 study by PwC and The Interactive Advertising Bureau (IAB) shows a slow and steady growth in advertising in television until 2018 while there is a much larger growth in the digital advertising, surpassing television, again in 2016. The same report from PwC and IAB (2015), is clear in the rapid growth tendency that online ad spend is incurring. The report shows that online ad spending grew by 15.6% year-over-year in 2014, from $42.8 billion to almost $49.5 billion. It is another record spending year, which is something that is becoming more and more common. These statistics show the growing importance of digital marketing and online advertising. This is a trend that the digital marketing managers from most firms need (and are) to be extremely attentive, for this is the future.

[1] http://www.zoominfo.com/business/lead-generation?utm_source=zoominfo.com&utm_medium=navigation&utm_content=resources-lead-generation&utm_campaign=navigation_links

After a meeting with one marketing manager of a top firm in its market, that deals with the process of generating online leads in his own firm on a continuously basis, other two experts within the subject area of digital marketing, including one of them that works in a firm that is responsible for the effective and efficient generation of the online leads for the previously mentioned marketing manager's firm, and after a careful investigation through scientific articles, books and online articles – since there is very little scientific work done regarding the subject of online lead generation – it couldn't be found any model/system to support the decision of the quantity of online leads to generate, and that this subject can be highly improved within not only this particular firm but in every firm that deals and carries out campaigns through digital marketing. Usually, the one who makes this decision resorts to his/her experience and/or "*gut feeling*" with the help of some historical data. Raman *et al.* (2012) reference a global survey by McKinsey & Co. where it reports "that companies tend to allocate marketing spending based on historical allocations and rules of thumb far more than quantitative measures". This situation also occurs in the online lead generation process. Raman *et al.* (2012) also state that optimal allocation can increase the profitability of a firm, sometimes by 400%. This method of decision making can be extremely ineffective and inefficient, especially when multiple firms interact and compete and have to make the same type of decisions, with different data, different results and different goals, making it a very difficult task to reach high levels of efficiency in these decisions.

Given that exists a deep interaction and competition among firms, the use of game theory makes complete sense. Dutta (1999) states that game theory is "a formal way to analyze interaction among a group of rational agents who behave strategically". In today's competitive corporate world, it is assumed that no firm makes a major decision such as the quantity of online leads to generate, which will take away a large chunk of the digital marketing department's budget, without considering the strategies employed (or an estimation of these strategies) by the rival firm(s). This is where game theory enters. Thus, it is suggested, in this dissertation, the application of game theory to online lead generation. The work here provided intends to demonstrate the applicability of game theory to such a problem and to provide marketing managers with a useful model to support their decisions. Following this line of thought, there are two research questions to be answered in this dissertation:

1. How can game theory be used to model the quantity of online leads to be generated by a firm?
2. How can game theory contribute to the effectiveness and efficiency improvement of the marketing managers' decision making process relating to online lead generation?

1.2 Objectives

In order to answer the research questions, the present dissertation has the following main objectives:

- Evaluate the viability and identify the key points relevant to the application of game theory to digital marketing, specifically to online lead generation;
- Create and develop a model based on game theory to assist digital marketing managers in their task of selecting the quantity of online leads to be generated given a specific goal to be achieved;
- Apply the proposed model to a real world situation, simulating an application in a real world context;
- Analyze the results and generalize the approach to demonstrate the potential value that a model based on game theory has on an effective and efficient management of the digital marketing department, especially when it comes to online lead generation.

1.3 Methodology

In order to answer both research questions and to complete the objectives of this dissertation, the methodology presented in Figure 1.1 was followed.

As Figure 1.1 shows, a literature review was made in the fields of marketing – specifically of digital marketing – and game theory. The marketing literature review is justified because the subject of this dissertation – online leads and its generation – falls within the scope of the firms' marketing managers and it's their decision how to go about performing this task. A review of game theory is also essential due to the fact that the work and the model developed – GameOn – were based entirely on this theory. The next step in the methodology is to establish a summary of the state of the art regarding the application of game theory to fields related to marketing such as the allocation of a firm's advertising investment. This step also comprises the identification of the gap to be solved by GameOn. Next on the methodology is the selection of the published models that will serve as basis and inspiration for the work in this dissertation. The contribution of the dissertation, i.e. GameOn, is then presented through an adaptation of the previous models selected, with additional innovations. After the model development, an application scenario and a case study are provided and some fine tuning on the model is done given the limitations and results derived from the case study and the application scenario, and finally, preliminary results are presented and discussed and recommendations for future work are proposed.

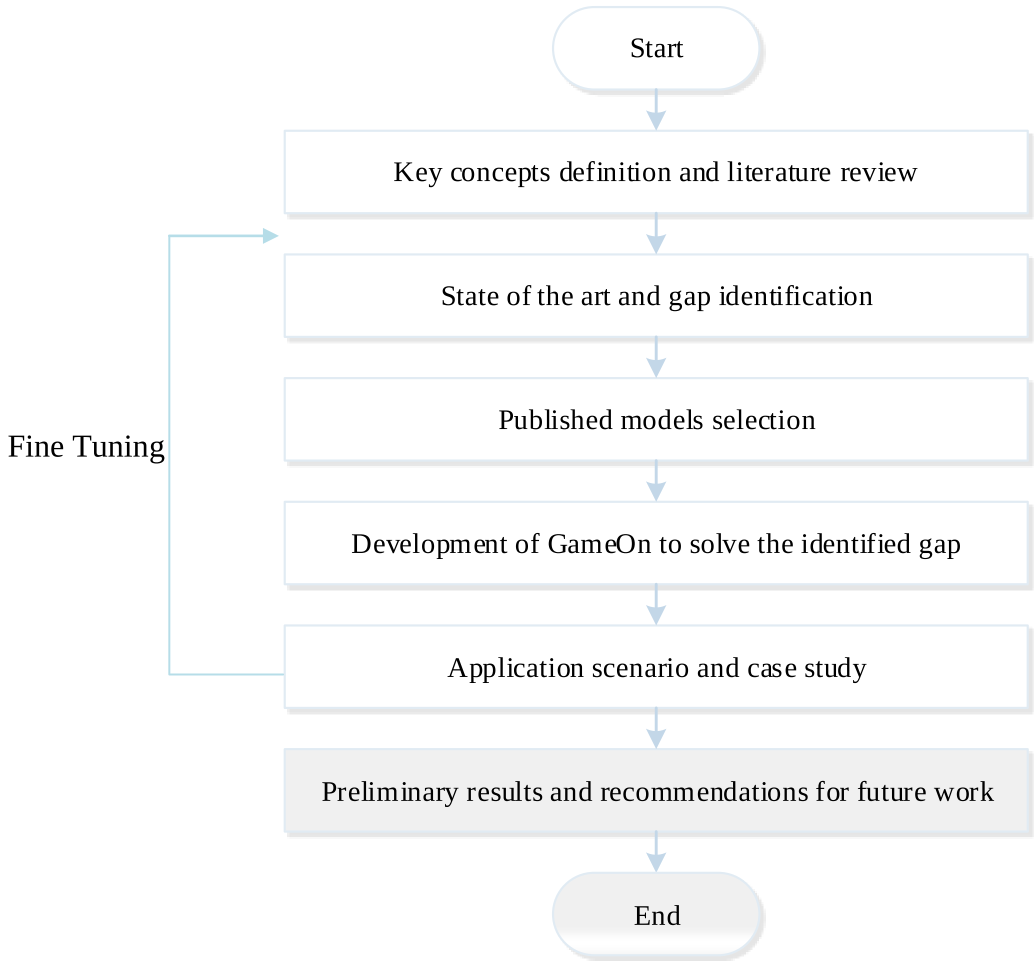

Figure 1.1 – Dissertation's Methodology

1.4 Work Structure

This dissertation is divided into six chapters. The first chapter is the present one which gives an introductory note on the work developed, including the motivation and the methodology followed. Chapter 2 provides a theoretical overview of marketing and digital marketing, evidencing general and key definitions that are important to comprehend the full intent of the work. The focus of chapter 2 will be on the (online) lead generation subject that will be one of the central themes throughout the dissertation. Chapter 3 presents a broad theoretical foundation of game theory, since it will be essential to understand the basis of this theory to comprehend both the more complex mathematical formulation developed for the model and the general idea behind the model. This chapter also provides a literature review on what some prominent authors have developed to relate game theory to marketing, especially, in the models created for the allocation of advertising investment or budget. Some of these models serve as basis and initial inspirations for GameOn. In chapter 4 the broad and specific idea and the mathematical formulation of GameOn is developed and demonstrated and generalized. In chapter 5, through some initial results, an application scenario and a case study, the model is applied and the results discussed in order to evidence the benefits and the added value that GameOn can provide to the digital marketing departments and organizations as a whole. Finally, in chapter 6 the work's conclusions, limitations and recommendations for future work are presented.

Chapter 2. Traditional and Digital Marketing

In this chapter, the marketing concept and definition are introduced based on different sources, and different periods. It is discussed the evolution of marketing as well as its "eras" or stages and role of marketing in corporations. The advertising process is explained and a literature review of digital marketing is provided. Finally, the concept, idea and importance of leads and online lead generation are introduced and explained.

2.1 Marketing Defined

The concept of marketing has been debated and evaluated regularly since marketing was recognized as a distinctive discipline and domain (Gamble *et al.*, 2011). Several definitions have been provided over the past several decades by a large number of not only authors, but also associations and institutes. Even the same authors, associations and institutes feel the need to change their own definition of marketing with the passing of the years. This is a proof that marketing is a concept that has been evolving constantly and that there is no universally accepted definition.

Philip Kotler and Sidney J. Levy tried, with their paper "Broadening the Concept of Marketing" (1969) to *"expand"* the concept of marketing because at the time it was "the author's contention that marketing is a pervasive societal activity that goes considerably beyond the selling of toothpaste, soap, and steel". Until that point marketing was seen only "as the task of finding and stimulating buyers for the firm's output". Later on, in 1984, Philip Kotler proposed the following definition of marketing:

> "Marketing is a social process by which individuals and groups obtain what they need and want through creating and exchanging products and value with others".

With this definition, Kotler exposes several core concepts portrayed in Figure 2.1:

Figure 2.1 – Marketing's core concepts
Adapted from Kotler (1984)

Forty-three years have passed since Kotler and Levy tried to expand the concept of marketing and still Kotler and Armstrong (2012) state that many people think of marketing only as selling and advertising. The authors defend that these activities are only the tip of the marketing iceberg. They define marketing as follows: “Marketing is the process by which companies create value for customers and build strong customer relationships in order to capture value from customers in return”. According to Dibb *et al.* (2008), “the basic rational of marketing is that, to succeed, a business requires satisfied and happy customers who return to the business to provide additional custom”. So it comes as no surprise, to see that the definition of marketing provided by Kotler and Armstrong mentions the word “customer(s)” three times.

In 1976, the *Chartered Institute of Marketing*, proposed the following definition of marketing: “The management process responsible for identifying, anticipating and satisfying customer requirements profitably”. However, over thirty years later, in 2007, the Institute proposed a new and lengthier (and controversial, for not all experts are convinced, according to a 2007 article in the “*The Economic Times”*) definition, because the Institute felt that the previous definition didn’t adjust to the new and contemporary reality of marketing:

> “The strategic business function that creates value by stimulating, facilitating and fulfilling customer demand. It does this by building brands, nurturing innovation, developing relationships, creating good customer service and communicating benefits. By operating customer-centrically, marketing brings positive return on investment, satisfies shareholders and stake-holders from business and the community, and contributes to positive behavioral change and a sustainable business future”.

The *American Marketing Association* (2013) approved, in July 2013, their own marketing definition: “Marketing is the activity, set of institutions, and processes for creating, communicating, delivering, and exchanging offerings, that have value for customers, clients, partners, and society at large”. As it can be verified on Figure 2.1, this recent definition touches several of the core concepts that Philip Kotler mentioned in 1984.

Finally, Lindon *et al.* (2009) provide, in their view, a broad definition of marketing: “marketing is the set of methods and means that an organization possesses to promote, to their public of interest, behaviors conducive to the realization of their own goals”. The authors argue that, by broadening its field of application, marketing is able to diversify and specialize itself.

2.2 Marketing’s Role in Organizations

Before analyzing the role of marketing in today´s organizations, it is shown the point of view of the author of Kotler (1984). The author states that not many companies have adapted the marketing concept – which he defines as the following: “the marketing concept holds that the key to achieving organizational goals consists in determining the needs and wants of target markets and delivering the desired satisfactions more effectively and efficiently than the competitors". At that time only a small number of companies stood out as "master practitioners of the marketing concept, companies such as Procter & Gamble, IBM, McDonald's" and a few others. What separated these companies was that they were not only focused on the customer but were organized to respond effectively to changing customer needs. However, other companies hadn't yet arrived at full marketing maturity. They thought they had marketing because they had a "marketing vice-president, product managers, sales force, advertising budgets, and so on". They failed to adapt to, not only changing customers' needs, but also to changing competition.

In thirty years of development some changes have occurred and today´s role of marketing is not the same as it was in the 1980's. But first, and according to Moorman and Rust (1999), marketing has been gaining prominence as an orientation that all of the people in an organization share as a whole and as a process that all of the organization's functions participate in several stages such as the deployment. According to these authors, during the 1990's, a movement occurred where marketing was thought more as a set of values and processes that all of the functions of an organization participated in implementing and less as a function itself. Webster Jr. (2005) argues that in a large number of organizations, the importance of the marketing function has been declining at an alarming rate. This decline can be measured in capital, (due to the declining budget) and in lost human resources (downsizing). Its influence and confidence has also been declining, "resulting in strategic consequences that run deeper than many seniors managers may realize". The author cites a study[2] that reveals that the chief marketing officers of 100 top-branded companies have an average tenure of less than two years. However, a study conducted by the same organization 10 years later, in 2014, determined that since 2004, "the average tenure for chief marketing officers of leading U.S. consumer brand companies" doubled, that is it increased to 48 months, which can be interpreted as sign that marketing's influence on firms has been increasing for the last 10 years.

Sheth and Sisodia (2005) address this issue by stating that marketing has been going through an ordeal due to major problems such as the lack of respect within the corporation world and a lack of confidence from the consumers. According to these authors, these two deficiencies, taken together, have placed the entire field of marketing in "society's doghouse as a shallow, wasteful, and polluting influence". Sheth and Sisodia (2005) call out for a need for a new approach to marketing across the corporation world and even across society.

[2] Study conducted by executive recruiter Spencer Stuart ("CMO Tenure: Slowing Down the Revolving Door," blue paper, G. Welch, July 2004, www.spencerstuart.com/research/articles/744/)

Wirtz *et al.* (2014) understood this decline in marketing's importance and proposed an empirical study to explore the current role that marketing indeed has in today's corporations, for very little empirical research had been done regarding this particular subject. What the authors found was that, "a strong and influential marketing department contributes positively to firm performance". A finding that supports the idea that marketing has to retrieve the importance and influence (across the corporate world) it once had, because that will be beneficial for the firms themselves.

2.3 Marketing Channels

Marketing channel, or as it is sometimes called, distribution channel, has a wide variety of definitions proposed by several authors. According to Dibb *et al.* (2008), a marketing channel is defined as "a group of individuals and organizations that direct the flow of products from producers to customers". Young and Merritt (2013) use a definition put forward by Rosenbloom in 2013, where this author defines marketing channel as "the external contractual organization that management operates to achieve its distribution objectives".

Kotler (1984) states that there are different types of channel flows, namely the physical flow – that describes the movement of physical products from raw materials to final customers; the title flow – that describes the passage of ownership from one marketing institution to another; the payment flow – where customers pay their bills to the banks or some others institutions that, in their turn pay the producer, that pays its suppliers; and the information flow – that describes "flows of influence", such as advertising, from one entity to another. It is important to mention that the same author acknowledges that there are also marketing channels in the service sector.

Marketing channels are indeed important to almost any organization in the world as they add value in several forms. Dibb *et al.* (2008) argue that marketing channels create four different types of utility: Time – by making the products available when the customers want them; Place – by making the products available in locations where the customers can access them; Possession – by giving the customer access to the products to make it possible for them to use or storage them; and creating utility by "assembling, preparing or otherwise refining the product to suit individual customer needs".

A proof that the marketing channels have become extremely important is the proliferation of scientific articles that present frameworks, models or provide new knowledge to manage them. Chen, Kou and Shang (2014) proposed a framework to evaluate multiple marketing channels, while Fleming and Hawes (2014) provided research on the topic of sales and negotiations within the marketing channels, arguing that this topic hasn't received much attention "even though that is how conflict is managed and cooperation extended among channel parties".

2.4 Marketing Communications

To quote Patrick De Pelsmacker, a Professor of the University of Antwerp, "Marketing communication is the art of seducing a consumer on his way to your competitor" (Dibb *et al,*. 2008). This quote emphasizes the importance that marketing communications has in today's competitive corporate world. A good marketing communications' strategy can be the difference between gaining a customer and losing one. Dibb *et al.* (2008) offer a more comprehensive definition: "Marketing communication is the transmission of persuasive information about a good, service or an idea, targeted at key stakeholders and consumers within the targeted market segment". The people who proposed this definition also have a term that describes those within the target market segment that are intended as the principal recipients of the message – "target audience". According to Van Raaij, Stazzieri and Woodside (2001), marketing communications don't serve only the purpose of attracting new customers, but also play a crucial role when it comes to reinforce and retain loyal customers.

2.4.1 Communication Process

Kotler (1984) makes a reference to Harold D. Lasswell[3], where this author, in 1948 stated that the process of communication or "communication model", "will answer (1) who (2) says what (3) in what channel (4) to whom (5) to what effect". Kotler (1984) and Dibb *et al.* (2008) describe, through similar figures, the flow and elements of the communication process. Kotler (1984) differentiates nine elements: **Sender** – The sender of the message; **Encoding** – Converts meaning into symbolic form; **Message** – "The set of symbols the sender transmits"; **Media** – The channel used to carry the message from the sender to the receiver; **Decoding** – "The process by which the receiver assigns meaning to the symbols transmitted by the sender"; **Receiver** – Who receives a message from another party; **Response** – "The set of reactions that the receiver has after being exposed to the message"; **Feedback** – The response of the receiver to a message; **Noise** – "Unplanned static or distortion during the communication process, resulting in the receiver's receiving a different message than the sender sent". Figure 2.2 shows the flow and the elements of the communication process.

[3] Harold D. Lasswell, Power and Personality (New York: W. W. Norton & Co., 1948), pp. 37-51, cited by Kotler (1984)

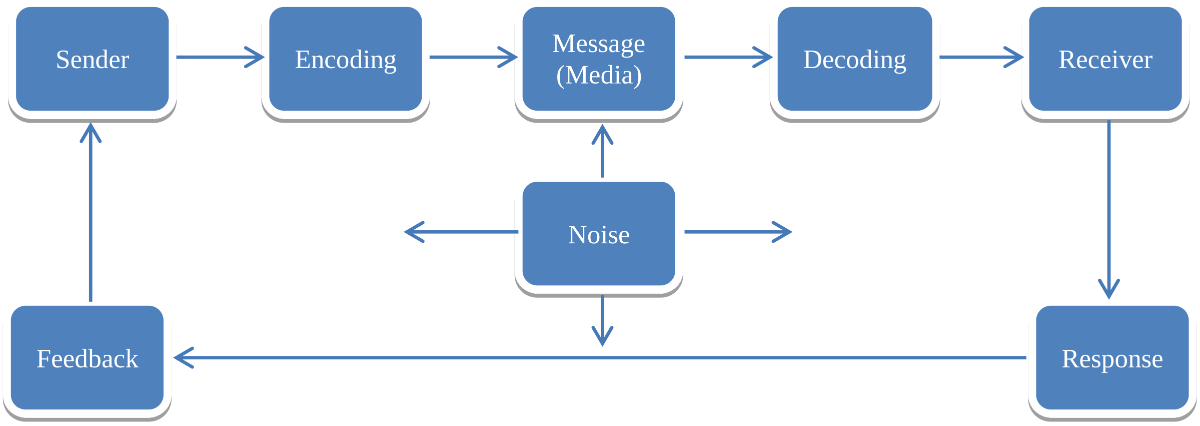

Figure 2.2 – Communication process's elements and flow
Adapted from Kotler (1984)

2.4.2 Integrated Marketing Communications

Hutton (1996) stated that the concept of integrated marketing communications is not new. "It has, in fact, been practiced by good marketing communicators for decades, if not centuries". S. Low (2000) points out something that Zinkhan and Watson (1996) have said: Integrated marketing communications is praised as the best way to take advantage of brand new technological means to communicate more directly with individual consumers. Schultz (1996) came to a conclusion, "the question of integration or not is moot". The author defends that it doesn't matter if the marketing professional integrates his/hers communications programs because the consumers integrate that communication, whether the organization has done it or not. Therefore, what marketing professionals should do is try to understand more clearly how do customers go about doing this integration and adjust and modify their own approaches to this concept.

Integrated marketing communications can be defined, according to Dibb *et al.* (2008), as the coordination and integration of all marketing communication tools and sources within a company into a seamless program that maximizes the impact on consumers and other final users, at minimal cost. It is intuitive to understand, that the main goal of this concept is to communicate at an effective and efficient level.

2.5 Advertising

The definition here proposed for advertising is a mix of the definitions provided by both Kotler and Armstrong (2012) and by Dibb *et al.* (2008) – Advertising is a paid form of nonpersonal presentation and promotion of ideas, goods or services that is transmitted through any form of mass media, such as television, direct mail, the Internet and so on, by an identified sponsor. Kotler and Armstrong (2012) state that there are records of advertising efforts that took place centuries and even millennia ago. Even the Romans "painted walls to announce gladiator fights". Today's firms understand perfectly the importance of advertising. According to a study done by the "Advertising Age" in 2014, Procter & Gamble Co., the world's leading advertiser, spent, in the year ended June 2013, an estimated $13.9 billion – about 16.5% of sales – on worldwide advertising. Due to the massive investment that firms undertake towards their advertising efforts, it is crucial to design an effective advertising strategy and to make efficient advertising decisions. Figure 2.3 structures these decisions:

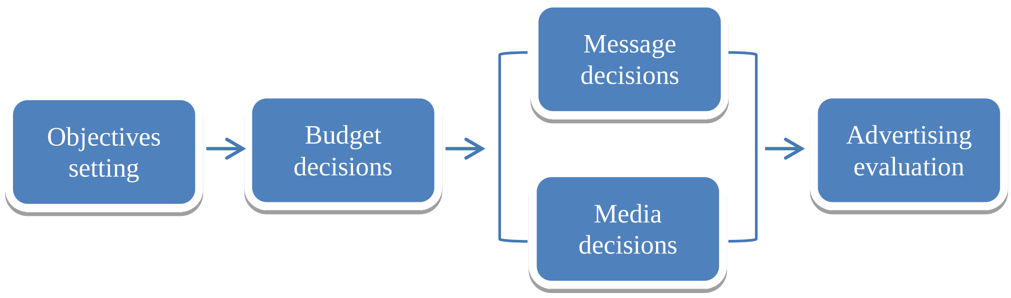

Figure 2.3 – Advertising decisions
Adapted from Kotler and Armstrong (2012)

Kotler and Armstrong (2012) state that "an advertising objective is a specific communication task to be accomplished with a specific target audience during a specific period of time". The authors classify the advertising's objectives into three different categories: Informative advertising, where the goal is to build primary demand, and that is why it is heavily used when introducing a new product; Persuasive advertising, where the company wants to build "selective demand", arguing that their product is better than the competitors' products; and Reminder advertising, that is crucial for more mature products, to ensure that customers keep those products in mind.

Budget decisions consist on determining how many dollars and other resources are allocated to the firm's advertising program. This decision depends highly on the product's stage in the product life cycle[4], where new products, typically need more resources.

The next two decisions comprise the advertising strategy. The first decision, creating advertising messages, is crucial to the entire advertising program, because if the message is not effective, then the advertisements don't gain any attention and don't have any impact. The message has to be creative, distinctive and meaningful. The next big decision regarding the advertising strategy is the media selection. The advertising media are "the vehicles through which advertising messages are delivered to their intended audiences". This decision comprises

[4] Product life cycle – "The course of a product's sales and profits over its lifetime. It involves distinct stages: product development, introduction, growth, maturity, and decline" (Kotler and Armstrong, 2012).

four self-explanatory steps: deciding on reach, frequency, and impact; choosing among major media types (television, radio, newspapers, and others); selecting specific media vehicles (for example, which television channel, or which newspaper); and deciding on media timing.

Finally comes the moment to evaluate the advertising effectiveness and the return on advertising investment[5]. According to Kotler and Armstrong (2012), “advertising accountability and return on advertising investment have become hot issues for most companies”. This is due to the concern of high ranking corporate executives that their firm might not be spending their sometimes massive advertising budget the right way. The great problem that advertisers are facing when it comes to this evaluation, is that, while they have gotten very good at measuring the communication effects of an ad campaign (whether the message was well received, what were the attitudes of the target audience regarding the campaign and how they liked it), it is much more difficult to measure the impact in terms of sales and profits, because these are affected by many other factors such as the product´s price, availability and features.

2.6 The Internet

According to Cerf (2004), although the Internet has never sustained a growth of 100% per year, it has, since 1988, maintained a steady growth of 50%-80% per year (as of 2004). According to an article in the Wall Street Journal (2015) about a report developed by Carl Benedikt and Michael Osborne, two Oxford University economists for Citi GPS, it took the telephone 75 years to reach 50 million users, 38 years for the radio to achieve the same number of users, 13 years for the TV to reach that milestone and only 4 years for the Internet to reach 50 million users. As incredible as it may seem, it only took the game *Angry Birds* 35 calendar days to reach 50 million users. This is a clear evidence of the rapid evolution and acceptance of the Internet and its functionalities by the population.

According to the Internet World Stats (2015), as of December 31st, 2014, an estimated 3,079,339,857 people use the Internet, which means that, approximately 42,4% of the world population use the Internet, a growth of 753% when compared to the data of the year 2000.

Figure 2.4 represents the evolution of adult users of the Internet in percentage relative to the entire adult population in the United States of America from the year 2000 to the year 2015.

[5] Return on advertising investment – “The net return on advertising investment divided by the cost of the advertising investment” (Kotler and Armstrong, 2012).

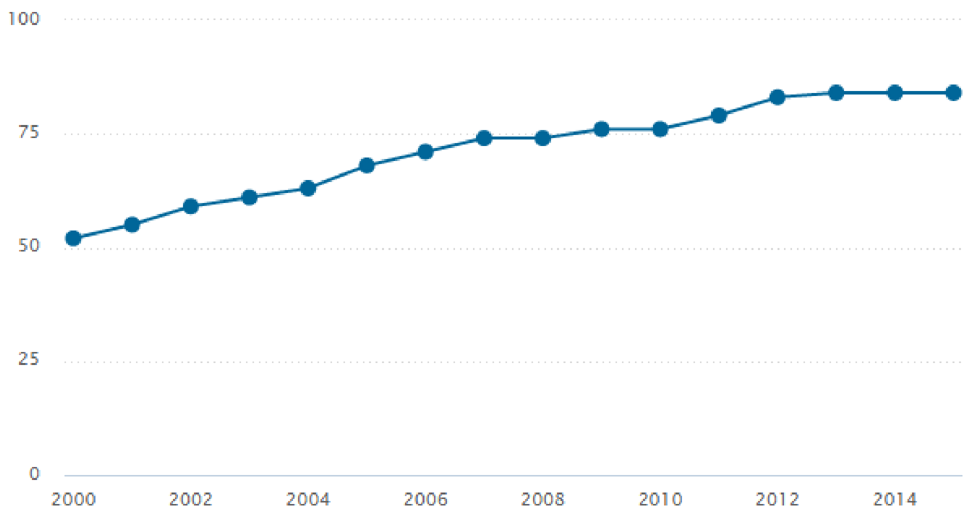

Figure 2.4 - Percentage of American Adults Use the Internet
Pew Research Center. (2015a)

As it can be observed by Figure 2.4, the percentage of American adults who use the Internet has been increasing since the year 2000 when only 52% of American adults used the Internet to 2015 where 84% of American adults use it. It is interesting to observe however, that in 2012 the percentage was 83, in 2013 it was 84, the same as 2014 and 2015 which indicates a stagnation of the growth in new American adult users.

When it comes to the use of smartphones, a growing high-tech industry extremely connected to the Internet world, a growth of 35% was verified between the year 2011 and 2015 where 65% of American adults currently own a smartphone (Pew Research Center, 2015b).

In Portugal, as of 2013 according to a study carried out by *Marktest* (2014), an estimated 5,7 million people used Internet, which is more than half of the Portuguese population. A study carried out by Pordata and INE (2015) shows that, in 2014 an estimated 65% of Portuguese people used the Internet. And according to a *Marktest* study discussed in an article in *Jornal de Notícias* (2014), 46,4% of the people that possess a mobile phone, own a smartphone, up 80% from the 2012 data.

It is very important to highlight the statistics from the smartphones users because this is a fast growing industry that, according to a study developed by *eMarketer* (2015b), the mobile ad spend will, for the first time ever, top 100 billion dollars worldwide in 2016, accounting for 51% of the digital market, which will be also the first time that the mobile ad spend will surpass the 50% of all digital ad expenditure. According to be same study, “the $101,37 billion to be spent on ads served to mobile phone and tablets worldwide next year represents a nearly 430%

increase from 2013”. It is expected that, in 2019, the spending on mobile ads will “account for 70,1% of digital ad spend as well as over one-quarter of total media ad spending globally”.

2.7 Web 2.0

According to O’Reilly (2005) the term and concept of “Web 2.0” actually began with a conference brainstorming session between one of the authors – Tim O’Reilly – and MediaLive International, specifically with Dale Dougherty. The term was first coined by Darcy DiNucci in 1999. It is composed by websites that emphasize user-generated content, usability and interoperability. Constantinides and Fountain (2008) argue that there is no universally accepted definition of the concept Web 2.0 and that it is already involved in controversy due to the fact that its “applications are by and large based on content generated by users often being anonymous and lacking qualitative credentials”. Lai and Turban (2008) state that Web 2.0 “is the popular term for advanced Internet technology” and its applications. The authors argue that “one of the most significant differences between Web 2.0 and the traditional Web is that content is user-generated, and there is greater collaboration among Internet users”. Web 2.0 is not only changing the content on the Web, but also how the Web works. Ryan and Jones (2009) begin by highlighting “what Web 2.0 is not”. The authors state that it is not a new version of Web 1.0, is not a revolution in technology. Web 2.0 is an “evolution in the way people are using technology”. It is about creating communities, sharing knowledge, ideas, about communication with other people and many other activities.

According to Ryan and Jones (2009), the consumers used to be happy just to sit in front of a broadcast media, accepting and watching whatever the editors and program schedulers choose. There is actually a certain degree of choice the user has, for example the user can change channel or read another newspaper, or listen to a different radio channel. However, ultimately it is not the user’s choice the actual content presented. With the arrival of the Web 2.0, “today’s consumers are in control as never before. They can choose the content they want, when they want it, in the way they want it. They can even create their own and share it with their friends, their peers and the world for free”.

Finally, and according to Constantinides and Fountain (2008) “the effect of a new kind of internet applications on shaping a new class of consumers increasingly integrating the web into their daily life” is the phenomenon known as Web 2.0 or Social Media. These authors state that “the terms Social Media and Web 2.0 are often used as interchangeable; however, some observers associate the term Web 2.0 mainly with online applications and the term Social Media with the social aspects of Web 2.0 applications”.

2.8 Understanding Digital Marketing

Several authors have put forward their own definitions of digital marketing. Some definitions are considerably simpler than others. Kotler and Armstrong (2012) define digital marketing (although they refer to it as "online marketing") as the efforts a firm makes to "market products and services and build customer relationships over the Internet". Dibb *et al.* (2006) differentiate and define two different aspects of digital marketing: Electronic commerce (e-commerce) which the authors define as "sharing business information, maintaining business relationships and conducting business transactions by means of telecommunications networks"; and Electronic marketing (e-marketing) which is defined as "the strategic process of creating, distributing, promoting and pricing products for targeted customers in the virtual environment of the Internet". Parsons, Zeisser and Waitman (1998) who wrote their scientific paper in the midst of the digital marketing emergence, describe and define two activities that together, make up digital marketing: "first, leveraging the unique capabilities of new interactive media to create new forms of interactions and transactions between consumers and marketers[6]; and second, integrating interactive media with the other elements of the marketing mix[7]". Finally, Chaffey *et al.* (2006) differentiate between Internet marketing and digital marketing, something that most authors don't differentiate and use interchangeably. Chaffey *et al.* (2006) provide a very simple and succinct definition of "Internet marketing": "defined as achieving marketing objectives through applying digital technologies". They offer a great reminder when they state that this "definition helps remind us that it is the results delivered by technology that should determine investment in Internet marketing, not the adoption of the technology". The authors also propose a slightly more comprehensive definition of Internet marketing by defining it as "the application of the Internet and related digital technologies in conjunction with traditional communications to achieve marketing objectives". As for digital marketing, Chaffey *et al.* (2006) states that it describes "the management and execution of marketing using electronic media such as the web, e-mail, interactive TV and wireless media in conjunction with digital data about customers' characteristics and behavior".

Zhu and Zhang (2010) state that consumers tend to seek quality information, or in other words, the information they can gather, when planning to purchase new products. For one hand, the emergence of digital marketing made the search of products or services by the consumers much easier and "with the Internet's growing popularity, online consumer reviews have become an important resource for consumers seeking to discover product quality" (Zhu and Zhang, 2010). Leeflang *et al.* (2014) state that "the Internet has become one of the most important

[6] A marketing professional.
[7] Marketing mix is "the set of controllable tactical marketing tools – product, price, place, and promotion – that the firm blends to produce the response it wants in the target market" (Kotler and Armstrong, 2012).

marketplaces for transactions of goods and services”. Thus, it is extremely important and even vital in some cases, for firms to have a noticeable presence in the digital world and to take advantage of its benefits in order to reach out to increasingly more consumers to increase sales and gain competitive advantage over its rival(s). It is, however commonly stated that the presence of a firm in the Internet is no long a nice-to-have situation, but must-have situation.

The world of digital marketing, according to Leeflang *et al.* (2014), has potential challenges for virtually every firm that makes a use of it, namely: 1) “the ability to generate and leverage deep customer insights; 2) Managing brand health and reputation in a marketing environment where social media plays an important role; and 3) Assessing the effectiveness of digital marketing”.

Ryan and Jones (2009) defend that every firm or entity involved in any sort of business need a digital marketing strategy, because without one they’ll miss opportunities and lose business. The authors even state that it doesn’t matter in what business one’s in; it’s a fairly safe bet that an increasing number of one’s target market rely on digital technology every day to research, evaluate and purchase the products and/or services they consume. Without a solid and coherent strategy, businesses are in the risk of not retaining customers, but even worst of being left behind in their respective industries while their rival(s) adapt better and faster to the changes of the corporate world by make an intelligent use of the digital technologies to improve their marketing efforts. Chaffey *et al.* (2006), to emphasize the importance of adopting the Internet technology for any firms’ marketing efforts, quote Porter (2001) when this author stated that “the key question is not whether to deploy Internet technology – companies have no choice if they want to stay competitive – but how to deploy it”. Ryan and Jones (2009) state that even while the Internet puts consumers in control as never before, it is also “important to remember that the Internet also delivers an unprecedented suite of tools, techniques and tactics that allow marketers to reach out and engage with those same consumers”.

Chaffey *et al.* (2006) state that Internet marketing strategy relates to the use of the Internet-related technologies to support and enforce the marketing efforts in that platform. It is needed to “provide consistent direction for an organization’s e-marketing activities so that they integrate with its other marketing activities and support it objectives. Ryan and Jones (2009) argue that there is no “one size fits all” strategic framework to be employed by every firm. There is no “magic recipe to ensure digital marketing success”. Every firm needs to personalize its approach and its strategy in the way they see most fit based on their needs, resources and objectives. According to Chaffey *et al.* (2006), “Internet marketing strategy has many similarities to the typical aims of traditional marketing activities”, in the sense that it will: “provide a future direction to Internet marketing activities; involve analysis of the organisation’s external environment and internal resources to inform strategy; articulate Internet marketing objectives that support marketing objectives; involve selection of strategic options to achieve Internet marketing objectives and create sustainable differential competitive advantage; specify how resources will be deployed and the organisation will be structured to achieve the strategy".

2.9 Online Marketing Domains

Kotler and Armstrong (2012) differentiate four major online marketing domains, depicted in Figure 2.5, which include B2C (business to consumer), B2B (business to business), C2C (consumer to consumer), and C2B (consumer to business).

According to Kotler and Armstrong (2012), "the popular press has paid the most attention to B2C, which the authors define as the "selling goods and services online to final consumers". The authors state that today's consumers can buy almost anything online, from kitchen appliances, clothing, car, airline tickets and much else. According to *eMarketer* (2014), worldwide business-to-consumer (B2C) e-commerce sales will increase by 20.1% from 2013 to 2014 to reach $1.500 trillion.

	Targeted to consumers	**Targeted to business**
Initiated by business	B2C (business to consumer)	B2B (business to business)
Initiated by consumer	C2C (consumer to consumer)	C2B (consumer to business)

Figure 2.5 - Online Marketing Domains
Adapted from Kotler and Armstrong (2012)

Chaffey *et al.* (2006) define B2B as "commercial transactions between an organization and other organizations (inter-organizational marketing)". The authors state the some firms might have products and/or services that appeal to both consumers and businesses, so they "will have different parts" of their online platforms to appeal to these audiences.

Consumer to consumer (C2C) online marketing is defined as "online exchanges of goods and information between final consumers" (Chaffey *et al.*, 2006). Good examples of this online marketing domain are the EBay platform and ordinary online blogs.

Finally, the last online marketing domain to be defined is C2B, which Chaffey *et al.* (2006) describe as "consumers approach the business with an offer", while Kotler and Armstrong (2012) provide a more detailed definition by stating that C2B is defined as "online exchanges in which consumers search out sellers, learn about their offers, and initiate purchases, sometimes even driving transaction terms". Examples of this type of online marketing domain are the feedback from the consumers, and bidding platforms where the consumers bid on the price of a good or service "leaving the sellers to decide whether to accept their offers" (Kotler and Armstrong, 2012).

2.10 Relevant Components of Digital Marketing

Digital marketing has great variety of components that are proposed and defined by several authors, such as Kotler and Armstrong (2012), Chaffey *et al.* (2006), Ryan and Jones (2009) and much more. However, this subchapter will only be focused on and only define (and succinctly explain) the most relevant components of digital marketing for the present dissertation, while acknowledging that a careful reading and study of the previous authors' work regarding the remaining components mentioned by them is not only interesting but highly recommended.

- *Websites*: According to Kiang, Raghu and Shang (2000), the rapid development of online computing technology makes it imperative for businesses to seriously consider the Internet to avoid losing competitive advantage. Thus, a website gives direct contact between the organization and the consumer. Kotler and Armstrong (2012) define a corporate website as "a website designed to build customer goodwill and to supplement other sales channels, rather than to sell the company's products directly". These authors state that a marketing website on the other hand has the objective of engaging the consumers in interactions that will move them closer to make a purchase.

Today, most consumers when learning about a firm, they go directly to its website in order to know and study more about it, its products and/or services. A firm's website should include its mission statement, values, vision, and every other information relative to its business in an appealing manner to bring in costumers and convince them to buy its products/services (Dionísio *et al*., 2011; Kotler and Armstrong, 2012).

- *Search Engine Marketing*: Ryan and Jones (2009) define this concept as "the process which aims to get websites listed prominently in search engine results". This is extremely important because no matter how wonderful and appealing a firm's website is, it won't get any customers or visualizations, if it's not well advertised and people don't know it. This is why "search engines are vital for generating quality visitors to a website". An intuitive example of a search engine is *Google*. It is here introduced two main search engine marketing techniques for making a firm (and its products and/or services) visible through search engines: 1) Search Engine Optimization (SEO) – This is a structured approach that is used to increase the position of a firm or its products/services in search engine results listings for selected keywords or phrases (Chaffey *et al.*, 2006). The goal of a firm is to achieve the highest position possible or ranking possible to be the first firm and/or product/service that consumers views after typing certain keywords or phrases into the search engine; 2) Pay per Click (PPC), or as it also called, sponsored/paid search – “Allows advertisers to bid for placement in the paid listings search results on terms that are relevant to their business. Advertisers pay the amount of their bid only when a consumer clicks on their listing” (Ryan and Jones, 2009).

- *Online Advertising*: “Advertising on the web takes place when an advertiser pays to place advertising content on another website” (Chaffey *et al.*, 2006). This is advertising that appears while consumers are surfing the web and can be divided into several forms, but only two forms are here defined: 1) Display Ads or Banner Ads – This form of advertising might appear anywhere on an user’s Internet screen. The most common graphical form are banners, where banner-shaped ads can appear anywhere in the screen advertising some product or service. Most display ads have, as objective, to direct the Internet users to the advertiser website (Kotler and Armstrong, 2012); Search Ads – This form on online advertising is a form “in which text-based ads and links appear alongside search engine results on websites such as Google” (Kotler and Armstrong, 2012). This form of online advertising has become extremely popular and used by most firms (Dionísio *et al.*, 2011).

- *E-mail Marketing*: Kotler and Armstrong (2012) state that “e-mail has exploded onto the scene as an important online marketing tool”. Up to 2012 the authors refer a study that found that about half of the B2B and B2C firms surveyed used e-mail marketing to reach costumers. “E-mail marketing is a fusion of marketing savvy and imaginative copy. In its simplest form, it’s an e-mail sent to a customer list that usually contains a sales pitch and a call to action” (Ryan and Jones, 2009). The call to action might be as simple as encouraging the costumer to click on a link to the firm’s website.

Must like other types of online marketing, firms need to be careful that they don’t cause any type of resentment among Internet users. Related to e-mail, the explosion of spam – unsolicited, unwanted commercial e-mail messages – has produced consumer frustration and anger. To avoid this anger, firms should always ask customer permission to pitch their messages through e-mail (Kotler and Armstrong, 2012; Chaffey *et al.*, 2006).

- *Social Media Marketing*: According to Ryan and Jones (2009), social media “is the umbrella term for web-based software and services that allow users to come together online and exchange, discuss, communicate and participate in any form of social interaction”. The users can generate content, evaluate and critic existing content and much more. This can be a great advantage for marketers because it amplifies a firm’s exposure and traffic, it makes it easier for firms to find out what people are interested in and thus develop or enhance their products/services in order to better appeal to consumers. Through social media users are in much more control of the content and have more power to "influence" firms to make more efforts to develop innovations that better appeals to them. And finally, by social media marketing, it is perceived as the use of social media to interact with the users and search in them a positive feeling towards a firm and its products/services (Ryan and Jones, 2009; Talpau and Vierasu, 2012).

2.11 Leads and Online Lead Generation

As an introductory note to this subchapter, it is important to stress that much of the information provided ahead was derived and taken from meetings and clarifying conversations with a digital marketing manager from a major firm that operates in Portugal and from a director of a firm that is specialized in online lead generation. This is the case because the type of depth of the information provided could not be found in the literature and also because the literature itself is not yet at a place that is able to provide the type of insightful information gathered from the professionals.

Ryan and Jones (2009) defined lead by stating that a lead is generated "when a visitor registers, signs up for, or downloads something on an advertiser's site. A lead might also comprise a visitor filling out a form on an advertiser's site". This is a topic (online lead generation) that has been gaining prominence and relevance in recent years. And to gain a sense of the growing importance of this subject, all the techniques mentioned in the previous subchapter exist to generate online leads for firms, and thus it is clear that firms today are involved in a great effort and are spending lots of capital and time to generate online leads. Sabnis *et al.* (2013) mention a study developed by Sirius-Decision Inc. in 2006, when the authors determined that on average, (just) "B2B firms spend 65% of their marketing budgets on activities such as trade shows, product seminars, cold-calling, data-base purchases, telemarketing" and other activities to attract new customers. This is nothing more than generating leads. These leads are considered *offline* leads but the principle and goal are the same, with firms shifting their attention to generating online leads. When anyone searches almost any word or phrase into a search engine like Google, almost certainly there will be an ad for some firm's product or service and the goal of that firm is for the consumers to feel appealed and click on the ad to perhaps make a purchase. But there is still a lack of scientific articles that not only explain and provide information regarding this subject but also that develop methodologies for better ways to generate online leads. However, articles in business magazines and websites such as Forbes[8]

[8] For example, http://www.forbes.com/sites/miketempleman/2015/07/22/how-social-media-can-be-your-best-source-for-leads/ (2015)

and *eMarketer*[9] have published very recent articles, has of 2015 evidencing the growing importance of online leads. Moreover, the definition itself of lead, or more precisely, online lead is something that not all experts can agree on. This happens because some people state that a lead is generated simply by the visit of a customer to a firm's website, and other people defend that an online lead is only generated when the customer makes a formal inquiry on a firm's products/services. For the purposes of this dissertation, online leads are only considered generated when a customer makes a formal inquiry to a firm. Many firms only wish to generate what they call "quality leads", and for this to happen, they make use of one or more of the techniques mentioned and explained in the previous subchapter, but sometimes the consumers click on the links by accident, or without any intent to truly investigate a firm's offerings, and thus the firms "complicate" the generation of the online lead by putting up forms that the consumers need to fill out sometimes with data that is not that easy to gather such as a consumer's fiscal identification number, to prevent false online leads. Logically, the goal of every firm is to develop a lead into a sale.

Today there are some offerings (websites, not scientific methodologies) when it comes to assist firms to generate better quality leads. It is important to understand that what these websites provide is a service that helps firms to better their process of online lead generation to provide more online leads in a period of time than their current result. One real world problem that still remains despite this offering is the fact that the firms still don't have an idea of how many online leads they should be generating to achieve their goals. Most firms have periodical goals, such as the goal to achieve a certain number of sold contracts in a period of time (for example a month), and for that they should generate an adequate number of online leads to help achieve this goal in the most efficient and effective way possible. There is no method, however to perform this task and currently it is left to the digital marketing managers that might not have good historical data (and currently most managers are only beginning to possess this data, and still it is not of high quality) to make a more or less accurate decision and most of these managers end up deciding be "the rule of thumb" (Raman *et al.*, 2012).

Currently most firms outsource the task of actually generating leads specialized firms. The only decision that a firm, in this case the digital marketing manager of the firm, has to make is the number of online leads that should be generated in order to achieve the established objectives. There is a true gap and lack of models and methodologies to assist the firms and their digital marketing managers in this task, and it is with the objective of solving this gap and problem that this dissertation is developed.

[9] For example, http://www.emarketer.com/Article/Lead-Generation-B2Bs-Choose-Quality-Over-Quantity/1012740 (2015a)

2.12 Summary

In this chapter, an overview of marketing was provided. With an initial focus on the traditional marketing and its concepts, processes and prominent features it was designed to show its evolution and relevance in today's corporate world. Additionally, this chapter presented the foundations of digital marketing, its importance in the current business world and its most relevant components within the scope of the dissertation. The concept of (online) leads and (online) lead generation was introduced and explained to provide an understanding on this subject. The next chapter focuses on game theory.

Chapter 3. Game Theory

This chapter presents a theoretical introduction to the theory of games and its history. It explains key concepts and definitions of this theory and provides clarifying examples. It illustrates and explains what has been done in the field of game theory applied to advertising in the last 60 years, and some of the models already developed throughout the decades by several authors. These will serve as theoretical basis for the study presented ahead.

3.1 History of Game Theory

When the birth of game theory occurred is something that most authors cannot agree on. According to Fudenberg and Tirole (1995), the first studies of games in the economic literature were the papers written by numerous authors such as Cournot (1838), Bertrand (1883), and Edgeworth (1925). These papers were related to oligopoly pricing and production, and were seen as "special models that did little to change the way economists thought about most problems". Dutta (1999) also mentions the works of Cournot (1838) – the same paper mentioned by Fudenberg and Tirole (1995) – and Edgeworth (1881). Dutta (1999) also points out that these works were special cases. Most authors agree that the early breakthrough in modern game theory came from Zermelo (1913), von Neumann (1928) and the subsequent book by von Neumann and Oskar Morgenstern titled *Theory of Games & Economic Behavior* (1944). According to Dutta (1999), Zermelo "showed that the game of chess always has a solution", meaning that from any position on the chess board, one of the players has a winning strategy – though this strategy may not be easy to figure out. Through this work, Zermelo pioneered a technique that is used to solve a particular class of games (extensive form games – which are explained ahead), known as "backward induction[10]".

In their book, von Neumann and Morgenstern made several major contributions and introduced many concepts that are today considered the basis for modern game theory. These authors introduced the foundations and ideas of the extensive form and strategic form representations of a game, proposed a base for the utility theory that explains what players get from playing a game and characterized the optimal solutions to the *zero-sum games*[11].

The next major advance in game theory came in 1950 when John Nash introduced the concept

[10] Backward induction – "The idea is to work backward to compute the optimal choice for the player before" – see Fudenberg and Tirole (1995), pages 68-69.
[11] Zero-sum games – Two player games in which one player wins if and only if the other loses (Dutta, 1999).

of equilibrium – to be known as “Nash equilibrium”, which will be discussed in much more detail ahead. This is one of the most used concepts in modern game theory. This approach, due to its great influence, advanced game theory from zero-form to nonzero-form games[12] (Dutta, 1999).

John Harsanyi also played a crucial role in advancing game theory. The author, in 1967-68, “generalized Nash’s ideas to settings in which players have incomplete information about each other’s choices or preferences” (Dutta, 1999).

Other developments were made between the 1970’s and today, but more importantly during this period were not the new concepts and theories that appeared but the extension of the fields of application of game theory. This theory has been applied to economics and financial conflicts, such as pricing of products and mergers and acquisitions, it has been applied to bankruptcy law, which specifies when, how and what the creditors can collect from a firm that has gone bankrupt, it has been applied to trench warfare in World War I, which specifies the locations of the trenches for example, it has been applied to political science (such as the voting process) and many others areas.

3.2 Game Theory’s Basic Concepts and Assumptions Defined

Myerson (1997) defined game theory “as the study of mathematical models of conflict and cooperation between intelligent rational decision-makers”. Dutta (1999) provided his own definition of game theory stating that game theory is “a formal way to analyze interaction among a group of rational agents who behave strategically”.

Just like Watson (2013) discusses, in all societies, people are in constant interaction. This interaction can be one of cooperation, such as business partners joining forces to complete a final goal, and sometimes this interaction can be one of competition, for example when two firms are in competition for additional market share or when two politicians are running for the same office. In both these situations, there is an important term that applies and is extremely important to game theory, *interdependence*. Watson (2013) describes this concept as “one person’s behavior affects another person’s well-being, either positively or negatively”. In his turn, Dutta (1999) describes interdependence as “situations in which an entire group of people is affected by the choices made by every individual within that group”. Building on this definition, Dutta stated that “if game theory were a company, its corporate slogan would be *No man is an island*”.

Building on the previous paragraph, a *game* can be simply defined as any social situation

[12] Nonzero-form games – These games are “situations in which both players could win or lose” (Dutta, 1999).

(competition or cooperation) that involves multiples entities where the action of each individual can affect the wellbeing of all entities. And the entities involved in the game are known as *players*.

Dutta (1999) states that every game is played according to a certain set of rules that have to specify four things:

1. **Who** is playing – the group of players that strategically interacts;
2. **What** are they playing with – the alternative actions or choices (the strategies), that each player has available;
3. **When** each player gets to play – the playing order, or choice selection;
4. **How much** they stand to gain, or lose, from the choices made in the game by everyone.

Watson (2013), in his turn says that the representations of games he provides have the following formal elements in common:

1. A list of players;
2. A complete description of what the players can do (their possible actions or strategies);
3. A description of what the players know when they act;
4. A specification of how the players' actions lead to outcomes;
5. A specification of the players' preferences over outcomes.

As can be seen, both authors are in agreement with most elements that have to be present in every game. Watson (2013) goes somewhat further (and is more explicit) than Dutta (1999), adding one element – the fourth one – that Dutta (1999) does not consider. Watson (2013) also states more clearly that is important to know exactly what do players know when they are making their choices (or strategies).

Before going any further and discuss the different forms of representing games, it is important to state key general assumptions within game theory (the presented group of assumptions is based on Watson (2013) and Dutta (1999), although they are generally accepted throughout the entire game theoretical community):

1. **Rationality –** Possibly the most immediate and important assumption, even present

in the game theory's definitions of both Myerson (1997) and Dutta (1999), it means, according to Watson (2013), that "each player behaves according to his preferences". More precisely, if a player's action will determine which of several outcomes will occur in a game, then this player will select the action that leads to the outcome he/she most prefers. Watson (2013) states that this assumption can be considered "weak" because it doesn't necessarily mean that each player seeks to maximize their own gains, because there could be situations in which the players are purely altruistic. According to Yu (2014), "the concept of rationalizability has been introduced into game theory independently by Bernheim (1984) and Pearce (1984)". They assert that players only select choices that are best responses (explained in detail ahead) to their forecasts and therefore some strategies in the action set (the set of strategies that can be played) will never be played.

2. It is standard to assume what is called as **common knowledge about the rules** – the "rules" are related to the rules that Dutta (1999) stated and are mentioned in the previous page. This means that "every player knows the rules of a game and that fact is commonly known". Watson (2013) explains this concept by stating that "the game is common knowledge between the players". However, this doesn't mean that the players are "equally well informed or equally influential; it simply means that they know the same rules" (Dutta, 1999).

3. There is another key issue that needs to be addressed – the issue of **realism**. According to Watson (2013), "rational decision making may require complex calculations and a sophisticated understanding of the other players' motivations. Standard game theory assumes that the players are sophisticated and that they can handle whatever difficult calculations are needed for the maximization of the gains". As is it known, instinctively, this is not always the case, and therefore it should not be expected in every situation (game) that the outcome of the theoretical models perfectly describes the complexity of the real world.

3.3 Representation of Games: The Extensive Form

Dutta (1999) defines the extensive form as "a pictorial representation of the rules. The main pictorial form is called the *game tree*, which is made up of a root and branches arranged in order". Fudenberg and Tirole (1995) argue that game theorists use this concept to model dynamic situations. The authors make this argument due to the capability that this representation form as to make "explicit the order in which players move (make decisions), and what each player knows when making each decision". Thus, the "strategies correspond to contingency plans instead of uncontingent actions". The extensive form "can be viewed as a multi-player generalization of a decision tree[13]".

It is important to state that in order for a tree to be called a *game tree* (to represent a game), it must satisfy three conditions; 1) *Single Starting Point* – the place where the game starts has to be clear; 2) *No Cycles* – important so the players don't hit an impasse while playing the game; and 3) *One Way to Proceed* – there can be no ambiguity as to how the game proceeds. (Dutta 1999)

In order to understand more clearly the explanation of how this representation form works, it is shown in Figure 3.1 a graphical example:

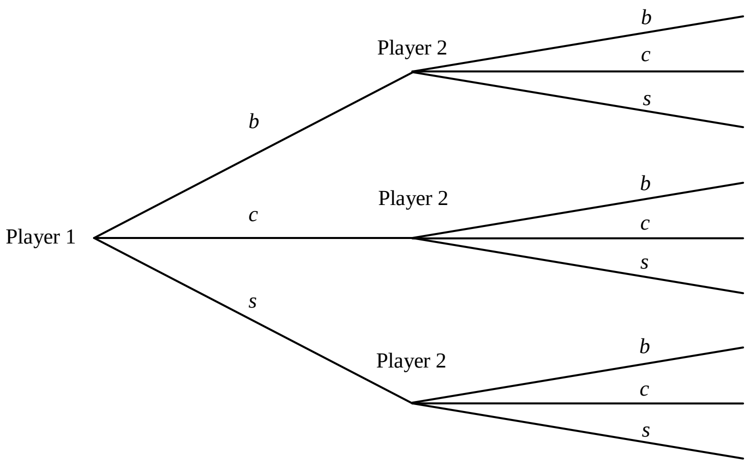

Figure 3.1 – Extensive form representation of a sequential game
Adapted from Dutta (1999)

Player 1 is at the root, that is, the starting point and has to make a choice. The player can select one of the three choices available – there are three branches emanating from the root and each branch represents a possible choice. The choices each player has are: b(us), c(ab), and s(ubway).

This extensive form as represented above permits only one player to move at a time – this represents a sequential game, meaning that when player 2 plays, he knows that player 1 already made his decision and knows what that decision was. The way to represent a simultaneous game (in the extensive form) – where each player doesn't know what decision the other player took – is represented in Figure 3.2.

[13] Decision tree – Decision trees provide a useful way of visually displaying the problem and then organizing the work developed. These trees are especially helpful when a sequence of decisions must be made. (Hillier and Lieberman, 2005)

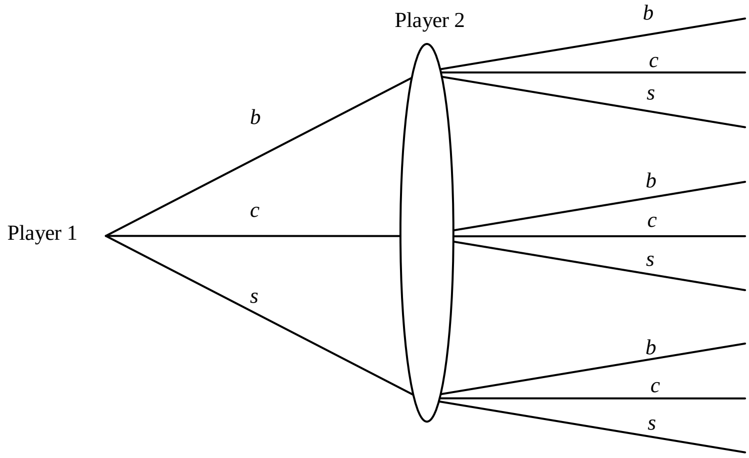

Figure 3.2 – Extensive form representation of a simultaneous game
Adapted from Dutta (1999)

From these "building blocks", more complicated game trees can be drawn, even with much more players than just the two, allowing for many choices at each decision node[14], and allowing each player to choose multiple times.

3.3.1 Information Sets and Strategies

In Figure 3.2, an oval can be seen blocking the decisions nodes of player 2. This oval is called an **information set**. An information set is "a collection of decision nodes that a player cannot distinguish between" (Dutta, 1999; Watson, 2013). According to Watson (2013), the term *information set* is commonly used to "specify the players' information at the decision nodes in the game".

According to Watson (2013) "the most important concept in the theory of games is the notion of a *strategy*". Dutta (1999) argues that "every player needs a *strategy* to play a game!". These two authors provide their own definition of strategy within game theory – which are presented in Table 3.1.

Watson (2013) acknowledges that his proposed definition might be somewhat confusing for some people due to the term "complete contingent" that he uses. However, the author states that what makes his definition "powerful" is precisely that term. By *complete contingent*, the author means a "full specification of a player's behavior, which describes the actions that the player would take at each of his possible decision points". The common ground between both definitions is found when Watson (2013) points out that "information sets represent places in the game at which players make decisions, *a player's strategy describes what he will do at each*

[14] Decision node – "Indicates that a decision needs to be made at that point in the process" (Hillier and Lieberman, 2005).

of his information sets".

Table 3.1 – Strategy definition

Definition of strategy	Author
"A strategy is a complete contingent plan for a player in the game".	Watson (2013)
"A strategy for a player specifies what to do at every information set at which the player has to make a choice".	Dutta (1999)

Dutta's definition can be argued as more "straight to the point" than Watson's. Dutta (1999) states that, succinctly, "a strategy is a blueprint for action", this because "for every decision node, it tells the player how to choose".

Returning to Figure 3.1, player 1 only has one decision node, so he has three possible strategies to choose from: *b*, *c* or *s*. Player 2 has three decision nodes: what to choose if player 1 chose *b*, what to choose if player 1 chose *c*, and what to choose if player 1 chose *s*. This means that every strategy of player 2 has three components (one for each of his decision nodes). One possible strategy of player 2 is (*s*, *s*, *b*); where the first entry is related to his choice if player 1 chooses *b* (and his choice is *s*); the second entry specifies his choice if player 1 chooses *c* (and his choice is *s*); and the last entry is conditional on player 1 choosing *s* (and his choice is *b*). In this example, it is trivial to note that players 2 has 3^3, or 27 strategies.

According to Dutta (1999) (still concerning the example illustrated in Figure 3.1) "a pair of strategies, one for player 1 and the other for player 2, determines the way in which the game actually gets played". For example, if player 1 chooses the strategy *c*, and player 2 chooses the strategy (*s*, *s*, *b*), then the game is as follows: player 1 takes a cab and player 2 takes the subway (this because, according to the strategy of player 2, if player 1 choses *c*, then player 2 chooses *s*, the second entry).

3.3.2 Utility/Payoff Function

The utility or payoff function specifies how much each player stands to gain or lose by playing the game (in the way he does). More specifically, the payoff or utility function is the "function that would specify the payoff to a player for every possible strategy combination that he – and the other player – might pick" (Dutta, 1999).

The payoffs are numbers that represent the players' motivations. The payoffs can be a number of different things, for example, when the outcome of a game is monetary, then the payoff for each player would be the amount winnings. But the payoffs are not always amounts of money. They can be prison sentences (in justice) or even amount of voters (in politics). According to

Watson (2013), the "payoff numbers describe the players' preferences over outcomes".

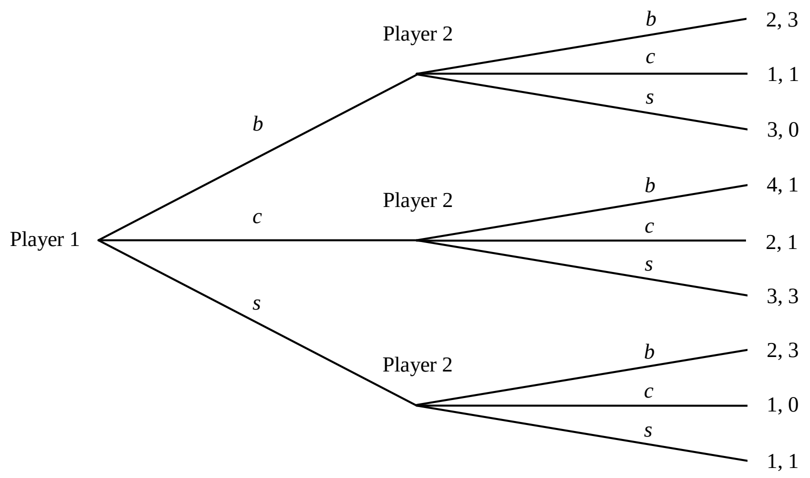

Figure 3.3 – Extensive form representation of a sequential game with payoffs

Figure 3.3 depicts a point that Dutta (1999) makes when he states that in the extensive form, the "utility numbers would get written at each one of the nodes where the game terminates". The first coordinate of each payoff pair, is related to player 1 and player 2's payoff is the second coordinate of the payoff pair.

3.3.3 Backward Induction

The way to solve games presented in the extensive form is through a process known as backward induction. This process "identifies an optimal action for each information set by working backwards in the game tree" (Watson 2013). The same author states that backward induction "is a process of analyzing a game from the end to the beginning". At each node it is eliminated the worst (dominated as it will be seen ahead) strategy(ies), "given the terminal nodes that can be reached through the play of the actions identified in the successor nodes" (Watson, 2013).

Figure 3.4 shows an example of a simple game in the extensive form to be solved as an example of how this method works.

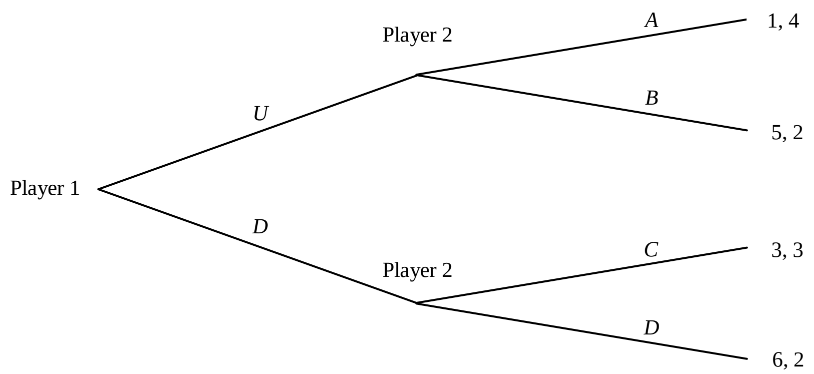

Figure 3.4 – Extensive Game Example: Backward Induction

Just like backward induction suggests, the start of the analysis should be in the end of the tree. Thus, it is obvious that player 2, for the upper decision should select A, since it yields him a payoff of 4 units, whereas if he chose B, it would've yielded him a payoff of just 2 units. For the lower decision node, player 2 is expected to select C, since it yields him a payoff of 3 units, which is larger when compared to the yielded payoff of D: 2 units. Player 1 also knows this given that he performs this analysis as well. Thus, he knows that for the upper decision node player 2 will select A and if player 1 selects U, it yields him a payoff of 1 unit, whereas if player 1 selects D, and given that he knows that player 2 will select C, this strategy yields him a payoff of 3 units. Given that 3 is larger than 1 unit, the best strategy for player 1 to select is D. Thus, there is only a single sequentially rational strategy profile: (D, C).

This is how backward induction works and it this knowledge basis will be important, especially to better understand the Stackelberg equilibrium as introduced ahead.

3.3.4 Representation of Games: The Normal Form

An alternative way of formally describing games and represent their rules, is called the **normal** or **strategic** form. Dutta (1999) defines this form as "a complete list of who the players are, what strategies are available to each of them, and how much each gets".

Myerson (1997) argues that the strategic form of representing games is much simpler than the extensive form. The author states that there is only the "need to specify the set of players in the game, the set of options available to each player, and the way that players' payoffs depend on the options that they choose". These needs that the author identifies are the exact same that Osborne and Rubinstein (1994) also defend.

According to Osborne and Rubinstein (1994), this model has a high level of abstraction, allowing it "to be applied to a wide variety of situations. A player may be an individual human being or any other decision-making entity like a government, a board of directors, or even a flower or an animal". According to the same author, the only limitation of this model is the needed requirement to "associate with each player a preference relation". This relation can be the players' feeling towards an outcome of the game or, in the cases of organisms (players) that don't act consciously, its chances of reproductive success.

In Figure 3.5 is provided an example of this representation form. First of all, it can be observed that the payoffs for each strategy of each player are represented inside the cells. This is a game between two players, player 1 and player 2. By convention, the row player is player 1 and the column player is player 2. The set of actions that player 1 has at his disposal is {T, B} (meaning **T**op and **B**ottom). The set of actions that player 2 has at his disposal is {L, R} (meaning **L**eft and **R**ight). If player 1 chooses T, and player 2 chooses L, then the row player's payoff from the outcome of (T, L) is w_1, and the column player's payoff is w_2. This logic follows for every other outcome: (T, R), (B, L) and (B, R).

Player 1 \ Player 2	L	R
T	w_1, w_2	x_1, x_2
B	y_1, y_2	z_1, z_2

Figure 3.5 – Normal form representation
Adapted from Osborne and Rubinstein (1994)

3.4 Dominance, Best Response and Nash Equilibrium and Stackelberg Model

3.4.1 Dominance

It is easy to understand that, when a game is played by more than two players and each player has more than two strategies, a matrix representation of the game can become "very cumbersome very quickly", according to Dutta (1999). This is why it helps to have a symbolic representation of the game. For the purposes of this literature review, it will be used the notions proposed by Dutta (1999).

The players that take part in the game will be labelled 1, 2, …, *N*. A representative player will be denoted the *i-th* player, with the index *i* running from 1 to *N*. S_{-i} and S_i are strategy sets (strategy space) of each of the possible strategies for all players except player i, and for player i, respectively. Player *i's* strategies will be denoted as s_i and sometimes a specific strategy will be marked s_i^* or $s_i^\#$ and so on. A strategy choice of all players other than player *i* will be denoted as

s_{-i}. Lastly, $\pi_i$will denote player *i's* payoff function. For a profile of strategies[15], $s_1^*, s_2^*, \dots, s_N^*$, one strategy for each player, player *i's* payoff will be denoted $\pi_i(s_1^*, s_2^*, \dots, s_N^*)$.

Here is introduced an important concept: *dominance*. The general idea behind this concept is that a player has a strategy that he can choose, that no matter what the other player chooses, it yields him the highest payoff. Some authors, like Dutta (1999) differentiate two types of dominance: *strong* and *weak*. According to Dutta (1999), "strategy s_i' strongly dominates all other strategies of player i if the payoff to s_i' is strictly greater than the payoff to any other strategy, regardless of which strategy is chosen by the other player(s)". Meaning:

$$\pi_i(s_i', s_{-i}) > \pi_i(s_i, s_{-i}), \text{ for all } s_i \text{ and all } s_{-i} \in S_{-i} .$$

where s_{-i} is a strategy profile of players other than i. An example is shown in Figure 3.6:

Player 2

Player 1	L	R
T	2, 3	5, 0
B	1, 0	4, 3

Figure 3.6 – Example of a game in the normal form I
Adapted from Osborne and Rubinstein (1994) and Watson (2013)

From the example in Figure 3.6, it can be seen that, for player 1, strategy T is a strictly dominant strategy (assuming the goal of the game is to maximize the payoff). This because, regardless of the strategy chosen by player 2 (either L or R), strategy T yields a higher payoff for player 1. If player 2 selects L, and player 1 selects T then player 1's payoff is 2 units, whereas if he chose B, then his payoff would be just 1 unit. And if player 2 selects strategy R, and player 1 selects strategy T, his payoff are 5 units, whereas if he chose B, his payoff would be 4 units. Thus, an intelligent and rational player, in the "shoes" of player 1, would always choose strategy T. In this case, strategy B is called a dominated strategy.

According to Dutta (1999), the definition of the second type of dominance is as follows: "a strategy s_i' (weakly) dominates another strategy, say $s_i^{\#}$, if it does at least as well as $s_i^{\#}$ against every strategy of the other players, and against some it does strictly better, i.e.,"

$$\pi_i(s_i', s_{-i}) \geq \pi_i(s_i^{\#}, s_{-i}), \text{ for all } s_{-i} \in S_{-i}.$$
$$\pi_i(s_i', \hat{s}_{-i}) > \pi_i(s_i^{\#}, \hat{s}_{-i}), \text{ for some } \hat{s}_{-i} \in S_{-i}.$$

[15] Strategy profile – "A strategy profile is a vector of strategies, one for each player. In other words, a strategy profile describes strategies for all of the players in the game" (Watson, 2013).

An example is shown in Figure 3.7:

Player 1 \ Player 2	L	R
T	1, 1	1, 1
B	0, 2	2, 0

Figure 3.7 – Example of a game in the normal form II
Adapted from Yale University open course – Game Theory (2007)

Focusing on player 2, it can be observed that if this player chooses the strategy L, and player 1 chooses the strategy T, he gets 1 unit, which is the exact same he would've gotten had he chosen the strategy R (assuming player 1 makes the same choice – T). But if player 1 chooses the strategy B, it is not same for player 2 to choose the strategy L or R, because, if he chooses strategy L, he gets a payoff of 2 units, and if he chooses the strategy R, he gets a payoff of 0 units. This means that the strategy L weakly dominates the strategy R – it's equal in some situations and better in others.

3.4.2 Best Response

The basic idea behind this concept is to select a player's best strategy (the best one player can do) given his belief about what strategy will the other player(s) select. This is an intuitive idea because rational people always "think about the actions that the other players might take; that is, people form beliefs about one another's behavior" (Watson, 2013). According to the same author, in game theory, it is crucial to "form an opinion about the other players' behavior before deciding one's own strategy".

For example, in Figure 3.6, player 2 will form an opinion regarding player 1. Player 2 is well aware of the player 1's payoff for all his strategies. Thus, being both rational and intelligent players, player 2 assumes that player 1 will choose his best strategy, and it has already been established that his best strategy is T. This means that players 2 will choose L, because it is his *best response* given what he believes the other player will do. By choosing strategy L, player 2 gets a payoff of 3 units, whereas if he chooses strategy R he gets a payoff of 0 units.

The formal definition of best response is the following: a strategy s_i^* is a best response to a strategy profile $s_{\_i}^*$ of the other players if:

$\pi_i(s_i^*, s_{\_i}^*) \geq \pi_i(s_i, s_{\_i}^*)$, for all $s_i \in S_i$.

3.4.3 Nash Equilibrium

According to Watson (2013), “the simplest notion of congruity is that the players are best responding in a setting of *strategic certainty*”. This means that the players “coordinate on a single strategy profile”. Watson (2013) states that in this case, “the player’s beliefs and behavior are consistent, with each player’s belief about another player’s strategy concentrated on the actual strategy that the other player uses. Because of these accurate beliefs, the players are best responding to each other’s strategies”. Dutta (1999) argues that in this case, the players have no reason to do anything else (select any other strategy) if they had to do it all over again (no incentive to deviate). Thus the players are in a Nash equilibrium. Equilibrium is the term that John Nash used for this concept. This concept and the “idea of mutual best response is one of the many contributions of Nobel laureate John Nash to the field of game theory”.

The formal definition of Nash equilibrium is as follows: the strategy profile $s^* = (s_1^*, s_2^*, \ldots, s_N^*)$ is a Nash equilibrium if:

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*), \text{ for all } s_i \in S_i \text{ and all } i.$$

An example is shown in Figure 3.8:

Player 2

Player 1		L	R
	T	2, 2	0, 3
	B	3, 0	1, 1

Figure 3.8 – Example of a game in the normal form III
Adapted from Watson (2013)

To find the Nash equilibrium in this case it is useful to determine the best response of each player to the strategy chosen by the other player. If player 1 chooses T, player 2’s best response is to choose R because 3 > 2 units. If player 1 chooses B, player 2’s best response is to choose R because 1 > 0 units. Looking at player 2’s strategies: if player 2 selects L, player 1 selects B because 3 > 2 units. If player 2 selects R, player 1 chooses the strategy B because 1 > 0 units. In Figure 3.8, it is possible to see, graphically, these best responses. The best responses for player 2 given the strategies chosen by player 1 are depicted in red, and the best responses for player 1 are shown in blue. The only place where they meet is in the strategy (B, R). This is the Nash equilibrium of this game. It is the only pair of strategies that are the best responses to each other.

3.4.4 Stackelberg Model

The Nash equilibrium is applied to simultaneous games, that is, games where the decisions made by the players are simultaneous in time or if not simultaneous in time, the players have no information regarding the other players' selected strategies before they select their own. But this situation is not the only possible situation that can be studied and observed in a real world context.

Osborne and Rubinstein (1994) state that a Stackelberg game is a two-player extensive game with perfect information (which will be explained ahead) in which a "leader" chooses an action from his profile of strategies and a "follower", informed of the leader's choice, chooses an action from his own profile of strategies.

When the leader selects its best strategy and the follower selects its own strategy that, given the other player's strategy, maximizes its payoff, then that is a situation of Stackelberg equilibrium. According to Dutta (1999), the leader is able to make a higher profit when compared to the solution provided by the Nash equilibrium, and follower, usually, yields a lower profit when compared to its Nash equilibrium.

The Stackelberg model and equilibrium is frequently used when two players are in the game, but it can be used when more players are involved. However, to make this game work, some assumptions need to be made, such as, when there are three players, two of the players select their strategy first and simultaneously (the leaders), and the third player, knowing the actions chosen by the other two players, chooses last.

3.5 Relevant Types of Games

In this subchapter some types of games are explained and some examples are provided. The type of games here represented will be useful to understand not only the work in this dissertation but also the models that served as foundations for it.

3.5.1 Simultaneous and Sequential (Dynamic) Games

The differences between these types of games were already lightly addressed. Dutta (1999) put it simple when he stated that simultaneous games are games where both players make their decisions (choose their strategies) simultaneously – at the exact same time. Or if they don't make their decisions at the exact same time, the players are unaware of the other players' earlier decisions.

Sequential (or dynamic) games are games where the players who make their decisions after other players have moved, have knowledge about those decisions. This knowledge might not be perfect, in the sense that they might not know every strategy that the previous player chose but they have some knowledge about those strategies.

At this point it should be easy to understand that the representation of sequential games is better using the extensive form and the simultaneous games are often depicted through the strategic form (although this is in no way a rule).

3.5.2 Perfect Information and Imperfect Information Games

According to Myerson (1997), in a game with perfect information, whenever a player moves, he knows the past moves of all other players and their chances, as well as his own past moves. Dutta (1999) provides a more formal definition by stating that a game of perfect information is "an extensive form game with the property that there is exactly one node in every information set". Dutta (1999) also mentions an important fact – in a game of perfect information, there cannot be simultaneous moves. An example of such a game is already provided in Figure 3.3.

The concept of perfect information is sometimes confused with the concept of *complete information* – that requires that every player involved in the game know all the strategies and payoffs of the other players, but doesn't necessarily knows their actions.

Games of imperfect information are logically the opposite of perfect information games, where the players don't know either the past moves of other players, or the payoffs or other information. Figure 3.9 illustrates a game of imperfect information:

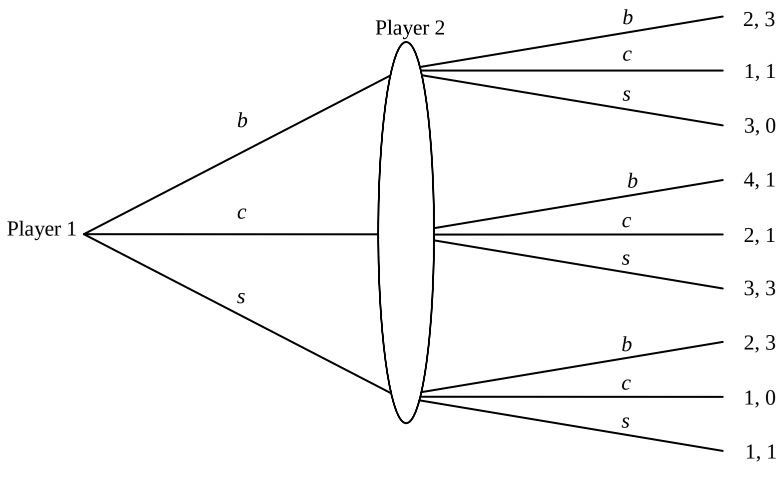

Figure 3.9 – Game of imperfect information
Adapted from Dutta (1999)

3.6 Game Theory and Advertising

Since the 1950's, there have been some prominent authors in this particular field that have created and disseminated knowledge through the work portrayed in their scientific papers. These authors and their work served as the inspiration and basis for this dissertation.

The specific problem area that is mainly discussed in this section is the allocation of advertising investment/expenditures. Friedman (1958) stated that “the allocation of advertising funds is a problem faced by most of the companies in the United States today”, and this observation was as true in the 1950’s as it was in the 1990’s when Chintagunta and Vilcassim (1992) mentioned that “the problem of determining the profit-maximizing advertising expenditure levels in a competitive setting has continued to interest marketing researchers”. This problem interests marketing researchers and firms overall due to their primary objective: maximize profits. In today’s extremely competitive world, advertising expenditures can carry a big weight on a firm’s total budget, and the ability to allocate its investment in an efficient manner, is a big and real concern of most firms in the world.

Gupta and Steenburgh (2008) claim that “the process of marketing resource allocation consists of two stages”. In the first stage, the demand is modeled and estimated. The model assesses (empirically) the impact of marketing action on the demand of a firm’s product. According to these authors, the model should also include competitive activities, although in some cases the data relative to these activities is difficult to obtain. In the second stage, the demand estimates are inputs to an optimization model that tries to “assess the economic impact of marketing actions” for the firm. Logically, this stage has to take into account the firm’s cost structure, in some models the firm’s market share, and budget.

Many authors have studied this subject in oligopolies (eg. Friedman, 1958; Erickson, 2009; Chintagunta and Vilcassim, 1995; and others), and some authors have studied it in duopolies (eg. Chintagunta and Vilcassim, 1992; Schoonbeek and Kooreman, 2007; and others) and the latter will be the focus of this dissertation. The research done in this topic also differs between static markets (eg. Friedman, 1958; Gupta and Krishnan, 1967) and dynamic market settings (eg. Chintagunta, 1993; Erickson, 2009).

Chintagunta and Vilcassim (1995) claim that a variety of factors influence advertising, sales promotion and other marketing related decisions, and among the factors is the “effectiveness of the firm’s own marketing activities, the actions taken by the competition, general economic conditions, and the dynamic nature of evolution of the firm’s sales or market share”. The model that Chintagunta and Vilcassim (1995) propose is different from the work in the present dissertation, due to the fact that their model is dynamic. These authors state that the dynamic models are capable of incorporating the “dynamics of marketing response or competitive interdependencies (Eliashberg and Chatterjee, 1985)”. They also claim that the investment decisions made by the firms are based on observed sales or market shares and changes in those are better modelled through dynamic models. In this dissertation, the model is static. But although static, it can be adapted to portrait changes in the overall marketplace (sales or relative market share), so the situations pointed out by Chintagunta and Vilcassim (1995) are not seen as weaknesses. And since the study here presented is a study made for one period, i.e. there are no carry-over effects of the actions taken (not to be confused with different data on sales or other indicators that are continually inserted in the model to continually improve it), it is appropriate to develop a static model. Moreover, with the relatively simple structure of the overall static models and of the model here presented in particular, one can obtain “unambiguous and intuitively appealing conclusions” (Schoonbeek and Kooreman, 2007). The authors also state that “many studies in marketing have found that advertising effects upon demand depreciate very rapidly”. Finally, due to the complexity of many dynamic models, sometimes palpable and explicit results are only possible in particular situations, while a relatively simpler static model often provides explicit results that are crucial for the firms (and what the firms real want and need). Schoonbeek and Kooreman (2007) defend the importance of static modelling of advertising decisions while Viscolani (2012) stresses that this type of modelling proves to be useful in order to understand “basic results when introducing model innovations”.

Most of the cited literature has one key assumption: the product demand is directly affected by the level of advertising. The way in which the authors choose to represent this relation may differ in some cases, but is it clear that advertising has positive effect on demand, although this effect exhibits diminishing returns, as it will be seen in the model proposed in the present dissertation. Viscolani (2012) points out another idea that is inherent to several models in the cited literature. It is the idea that one player’s advertising effort may hamper the competitor’s sales. The author defends that this idea is “intuitively appealing”, but as it will be shown, this idea does not come to effect in the model presented due to industry and competition specificities. This intuitive idea might not be congruent to real facts and data for a variety of reasons, like the competitive landscape of the market, for example if it is a market dominated by only two firms and the type of products commercialized, for example if the products require a prudent weighing and careful consideration (comparing products of rival firms and their features) before a customer makes a decision.

Sorato and Viscolani (2011) acknowledge a trend that has been growing throughout recent years, the use of multiple media and the synergy phenomenon. These authors proposed several static models that they hope enrich the existent knowledge.

The model developed for this dissertation was inspired by the models developed by Chintagunta and Vilcassim (1995), Chintagunta and Vilcassim (1994) and Erickson (2009). The model developed by Chintagunta and Vilcassim (1995) was studied in more detail with ideas and parts of that model being adapted since it provides a way to allocate the advertisement budget to different marketing channels. Much like the model developed by these authors, the model here proposed is based on the concept of "reaction functions". According to Chintagunta and Vilcassim (1995) "this framework allows for multiple competitive reactions, such as an increase in advertising spending by one firm may involve a price-cutting response response by another rival". The concept of reaction functions allows one to draw conclusions regarding the differences in the firms' strategies selection, "such as, classification of firms into leaders and followers and investigation of various "what if" scenarios". Their model is a dynamic one and the authors state that the "advertising dynamic result from the advertising expenditures of each the firms studied, having an expected positive impact on its own sales and a negative impact on the sales of each of the rival firms for two periods, that is, the current and next periods". The authors claim that there exists a "considerable body of empirical evidence that the carryover effects of advertising are short in during and usually last only a few periods". The sales response function in the log-log form. For firm *j* in period *t*:

$$\ln S_{jt} = \ln \alpha_j + \beta_j \ln A_{jt} + \delta_j \ln(A_{j,t-1}) - \sum_{\substack{k=1 \\ k\neq j}}^{N} \gamma_{jk} \ln(A_{kt}) - \sum_{\substack{k=1 \\ k\neq j}}^{N} \theta_{jk} \ln(A_{k,t-1})$$

$$j = 1, 2, \dots, N; t = 1, 2, \dots, T;$$

where

S_{jt} = sales (in units) of firm j in period t;

A_{jt} = advertising expenditures (measured in constant dollars) of firm j in period t;

α_j = firm-specific intercept term;

β_j, δ_j = parameters that measure the own current and lagged effects of advertising, respectively:

γ_{jk}, θ_{jk} = parameters that measure the competitive effects of firm k's current and lagged advertising, respectively, on firm j's sales; and

ln = denotes the natural logarithm.

According to Chintagunta and Vilcassim (1995) this model is modelling non-myopic firms, meaning that a myopic firm chooses its advertising spending with the goal of maximizing its profit in the current period, ignoring the impact of the current advertising on next period's sales and profits. Non-myopic firms take this impact into account. The authors state that Kadiyali (1992) proposes a similar methodology.

These author denote firms j's profit function in period $(t+1)$ as:

$$\Pi_{j,t+1} = M_j S_{j,t+1} - A_{j,t+1}, \qquad j = 1, 2, \ldots, N$$

where M_j is the unit contribution in dollars of brand j, meaning that knowledge of the firm's cost structure is imperative. The advertising expenditure, $A^*_{j,t+1}$ which maximizes the profit function, is obtained by solving:

$$\frac{\partial \Pi_{j,t+1}}{\partial A_{j,t+1}} = 0 \qquad j = 1, 2, \ldots, N$$

which yields the following relationship:

$$\ln\left(A_{j,t+1}\right) = X_j + Y_j \ln(A_{jt}) - \sum_{\substack{k=1 \\ k \neq j}}^{N} Z_{jk} \ln(A_{kt}),$$

where X_j, Y_j, Z_{jk} are known functions of the various model parameters. It is worth noting that the previous expression was derived assuming a one-period profit maximizing (i.e., myopic) behaviour of firms. Because this model is a two-period, by applying backward induction, the next step would be to choose the level of advertising for period t, taking into account the solution at $t+1$, but for the purposes of the present dissertation they have no significance (for more information, it is recommended the consultation of Chintagunta and Vilcassim (1995)).

During the literature research executed for this dissertation, mainly in this chapter, there couldn't be found any evidence of a proposed model, specifically for the problem of lead generation in digital marketing, within the theory of games. Although the process of lead generation in digital marketing is vital and an everyday task of a large portion of firms, it is being done without major scientific basis. From research and interviews, it is obvious that this process is executed on the basis on the experience of the marketing professionals alone. Marketing professionals do not know everything and they believe that models and platforms that help them perform this task would be great advantage for them and the firm overall.

One other feature of the proposed model that is innovative when comparing to the models proposed by the previous mentioned authors is the fact that in the model here presented, it is with the quantities of online generated leads and quantities of sales that the work will be based, not with the monetary amounts of investment or revenue. In the previous works, it was always used the monetary values, but due to the great difficulty in estimating the monetary investment and revenue in this particular field of the rival firm(s), and the fact that it is much easier and more reliable to only estimate the quantity of online generated leads and contracts sold in a period, while the estimation of the profit per online contract can be gathered also with ease, the work presented in this dissertation does not have the monetary values as basis for its formulation and applications.

3.7 Summary

In this chapter, a theoretical overview to game theory was provided as well as some examples to make the understanding of key concepts clearer. Relevant games and examples were presented and will be important in order to understand the contributions of the dissertation, for their concepts are used to support them. Finally, the more prominent and relevant game theoretical models applied to advertising proposed through the years, since the 1950's and by many authors, were presented and compared with each other and with the model that is explained and formulated ahead. The next chapter focuses on defining and explaining key concepts that are applied in the proposed model.

Chapter 4. Proposed Model - GameOn

In this chapter, the conceptualization and mathematical formulation of the proposed model – GameOn – is presented. Before that, it is compared and discussed several approaches to the (sales) model, and selected the best fitted one (or more). Every step of the formulation and every parameter of the model is carefully examined and explained to make the idea simple and clear for the reader.

4.1 GameOn Description

The decision of generating a certain quantity of online leads during a determined period of time is a common one for marketing managers, and with the fast growing importance of digital advertising, this activity will gain more prominence by becoming an almost continuously routine instead of a periodical task, like it was in the recent past. The budget that marketing managers possess for digital marketing has also been increasing, due to the shift from non-digital marketing channels to digital marketing channels. This means that an inefficient allocation of advertising resources, namely capital, and the inefficient generation of online leads can have a dramatic effect on the performance of not only the marketing department but the firm itself, due to repercussions caused by poor and wasteful advertising campaigns (and the consequent financial setbacks) for product or service awareness. The main problem and concern and source of capital waste, that marketing managers face when performing the task of deciding how many online leads to generate given their objective, for a determined period of time (for example, marketing managers can have an objective of a gaining a determined number of online contracts in a month, and they have to decide how many online leads they have to generate in order to gain product or service awareness from potential customers), is that they don't have a scientifically supported management model to help them reach an effective and efficient decision. Today's marketing managers make these decisions based solely on their experience and "*gut feeling*", and while many professionals are indubitably extremely competent, they often make feeble and faulty decisions that can have grave repercussions.

This dissertation suggests the application of game theory to estimate the quantity of online leads to be generated by a firm in a certain period of time, and to estimate the quantity of online contracts that set firm would gain, given the quantity of online generated leads for that period. The application of the theory of games is something that is natural, immediate and easy to understand, because in today's competitive environment no firm makes such an important decision without considering what its competitors are doing or about to do (obviously, assuming it's not a monopolistic market). This means that the firms are the players in the game, fighting for sales, market share or any other advantage they see fit. The application of this theory has other advantages in this context, such as:

i) Empirical application

Firms want, above all, models that can be easily applied to their reality and to give support or even solve their problems. Solely theoretical models do not add much value to most firms, and that is why a static game-theoretic model has the potential to be extremely useful and to fulfill the needs most firms have. It can be easily applied to real world situations.

ii) No need for exact data from competitors

It has already been addressed that game theory is based on data from the own firm and it(s) competitor(s). The level of precision of the data from the competitor(s) can be flexible, i.e. it may not be exact (unlike the data from one's own firm, which is usually exact). Obviously, the more precise all the data is, the more reliable the model and its outputs are, but giving all the uncertainty that most managers face daily, a degree of estimation is realistic and acceptable.

iii) Supports decision management and managers

The goal of a model based on game theory is to model the strategic behavior and decision making process of certain players (firms, individuals, governments, and many others). Thus, the use of this theory fits perfectly the goal of this dissertation of developing a model to support the decisions of marketing managers. The application of game theory also meets the growing need present in virtually every market (except the monopolistic markets), where the knowledge of the decision and strategies of each competitor in the market is essential to gain competitive advantage.

iv) Possibility of setting up "what-if" scenarios

Whether the competitors' data are exact or not, a model based on game theory provides the possibility of developing "what-if" scenarios. With some adjustments to the inputs and the parameters of the model, a manager is able to analyze the evolution of the estimations provided by the model and have a panoply of possible strategies to be implemented and their respective estimated effects/outputs to make a more supported and informed decision.

v) Benchmarking analysis

As already referred, game theory takes into consideration the strategy employed by a firm's competitor(s), and with the addition "what-if" scenarios, the application of this theory makes it possible to know a firm's position relative to its competitor(s) in the market. Through a benchmarking analysis, a firm can select the most effective and efficient strategy to maximize its profits.

vi) Flexibility – adapts to different situations

A model developed based on game theory will always be flexible, making possible an application in different firms, in different departments and with different goals. Varieties of the model also make possible other outputs and even the direct application of econometric models with the parameters calculated from game-theoretic based models; a small change in the inputs, not the model itself can yield different outputs with different meanings when compared to the initial model, providing a great flexibility that can be used by managers.

4.2 Initial GameOn and Market Considerations

The management model – GameOn – presented is based on game theory and its goal is to estimate the quantity of online generated leads and the estimated gained online contracts based on the competitors' strategy: the quantity of online leads the rival(s) generated during a certain period of time.

The focus of GameOn for the purposes of the present dissertation is on duopoly markets. There exists, indeed a wide variety of industries that are characterized by competition primarily among two rival firms, but there are also market situations where multiple firms interact (Chintagunta and Vilcassim, 1995). One could also adapt the situation of a duopoly market to a market situation with multiple firms, simply by gathering the market data, such as the sales, the market share, market leads, and other indicators needed to the model (obviously excluding the firm's own data) and it becomes an interaction between the own firm and the remaining market, instead of interactions between all the firms. Additionally, the data present in this dissertation is from two existing firms that operate in a duopoly, but this model can easily support the entrance of other firms with some alterations, without the market aggregation. The idea of the proposed model is indeed richer in scope, but due to the lack of quality data for other markets, the focus of the present study is on duopoly markets.

The market studied is characterized by the competition between firm A and firm B, i.e. the market – and the game – has two players. This is a duopolistic Portuguese market where firm A has a market share of 45% and firm B has a market share of 55%. This is a consumer-based market where the consumers' buying decisions are rational and pondered instead of instinctive, though sometimes there are spot peaks due to, for example, news in the media. Thus, the market is extremely volatile – to news in the media, to trends and mainly to advertising campaigns – and seasonal. This market feature leads to very interesting strategic decisions by both firms, for example, firm A might be willing or even want to spend less or just as much financial resources in publicity campaigns as firm B is willing to spend, because firm A knows that, being the consumers' buying decisions rational, they are going to research both firms' products (their advantages and disadvantages) and then make their choice, regardless on which firm first attracted them to the products. Firm A is actually contented to have a lower market share, – or doesn't have any major incentive to invest in enormous quantities to try to surpass firm B's market share – it only wants (as well as firm B) to be one of just two players in market, i.e. both firms don't want any other player to enter the market, so they are happy to divide the market share among each other. According to the marketing manager of firm A, this is a market that has been experiencing a major growth in the last years, including a projected growth of 20% during the year of 2015. Firm B has a much larger digital marketing budget when compared to the digital marketing budget of firm A, approximately three times more: 220000€ and 650000€, approximately. It is important to point out that, according to the same manager and despite the difference in the market share of both firms, the EBITDA (Earnings Before Interests Taxes, Depreciations and Amortizations) of firm A is actually higher than the EBITDA of firm B.

This market commercializes technological advanced products and services upon those products. Thus, this market is following very closely the technological advances that characterize today's world, not only to implement them in their line of products but also when it comes to marketing channels through the intelligent use of the many features of digital marketing. Through the use of GameOn it is the goal of the present dissertation to assist the marketing managers in their digital marketing's decisions, by estimating the quantity of online leads they should generate in a given period of time (in this case monthly) to achieve their goal, measured in gained online contracts, in the most effective and efficient way possible, while maximizing profits.

As it was already discussed in chapter 3, the model developed is a static one where there are no carry-over effects of the actions taken (study made for only one period). The relatively simple structure of the static models allows for intuitive conclusions, and interesting, easy to understand results. A static model also, usually provides explicit results that are crucial to most firms, as firms struggle to use and interpret purely theoretical results.

4.3 GameOn Usage – Inputs and Outputs

GameOn is very flexible and it is possible to use it to achieve different goals (different outputs) based on the users' specific objectives. There is however, a predetermined (natural and intuitive) mode of use of the presented model. The objective is to estimate the quantity of online leads that a firm has to generate, in a period of time, and to estimate (based on the estimated online leads) the quantity of online contracts that a firm sells during that period of time. It can then also estimate the profits and, given the data from the competitor it can estimate the Nash and Stackelberg equilibriums. This requires the following inputs:

1) Quantity of online generated leads from all previous periods from one's own firm and the competitors. To perform the initial estimations of the parameters and the posterior determination of one other factor (k), in the model, it is essential to have the data relative to the quantities of online generated leads from as many periods (for example, months) as possible of every firm;

2) Quantity of online sold contracts from all previous periods from one's own firm and the competitors. Again, to perform the initial estimations of the parameters, the number of online contracts sold in the previous periods by every firm is essential;

3) The unit contribution (or unit profit per online contract sold) of all firms is necessary to perform several calculation such as the reaction functions, the profits and the Nash and Stackelberg equilibriums;

4) The estimation of the competitors' quantity of online leads generated for the period in planning in order to use it in the reaction function.

The outputs that result from the standard application of GameOn are:

1) The estimated quantity of online leads to be generated for the month (or another period of time) in planning;

2) The estimation of the number of online contracts sold for that period;

3) The estimation of the profit for that period;

4) The estimated Nash equilibrium;

5) The estimated Stackelberg equilibrium.

Other applications of GameOn will be discussed in chapter 5.

Figure 4.1 depicts the basic flow of GameOn. With the basic and most important inputs illustrated (and then the additional needed inputs such as the unit contribution inserted in the model), the user can get crucial and precise information on what to expect to gain from a simple analysis of the outputs.

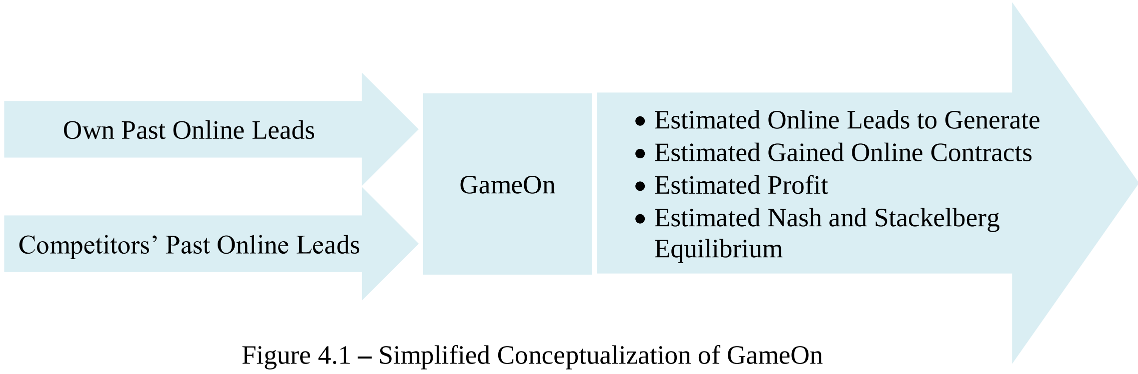

Figure 4.1 – Simplified Conceptualization of GameOn

4.4 GameOn's Sales Response Function(s)

The data available from each firm in the studied market used to develop GameOn is presented in Table 4.1. The data is from 16 months, spanning from January 2014 to April 2015; monthly quantity of sold online contracts, online leads generated by both firms and the quantity of offline leads generated by firm A. The data from firm A is exact while the data from firm B is based on estimations, and although they are a great approximation of the reality, they only represent estimates from professionals from market intelligence. In a real world context, this is the case with most firms, i.e. a firm does not always know what its competitors are doing and their strategies but they have estimates (again, in this particular case the data from firm B is a very close estimation). For the purposes of the sales response function formulation, every equation for each firm (A and B) will be presented – with the equations for firm A above the equations for firm B. The explanation of each parameter and variable will be given in the specific case of firm A; however for firm B, the explanations are analogous.

The basic idea behind a "sales response function" is to model the sales of a firm based on the data from itself and the competitors. In this specific case, it is to model the sales of a firm's online contracts based on the quantity of online leads generated by the own firm, and/or the quantity of online leads generated by the competitor(s), and/or the quantity of offline leads generated by the own firm (this last quantity is just for firm A).

As it can be seen in Table 4.1, the quantity of online leads generated by firm B is vastly superior to quantity of online leads generated by firm A, approximately 365% superior. Despite this difference, it doesn't exist such a level of disparity when it comes to the total online contracts gained by each firm, where the difference stands at, approximately 107%, again, with firm B gaining more online contracts. The difference between the quantity of online generated leads and the quantity of online contracts gained each month is explained by the "conversion rate", i.e. the rate at which the online generated leads are converted into online contracts where, and as it is obvious, firm A has a much higher total conversion rate than firm B. Additionally, the monthly quantity of offline leads generated by firm B could not be gathered.

Table 4.1 – Data Set from Each Firm

Month	Online Contracts_A	Online Contracts_B	Online Leads_A	Online Leads_B	Offline Leads_A
Jan 14	98	155	852	780	525
Feb 14	110	200	534	1950	558
Mar 14	116	231	678	1950	681
Apr 14	114	256	825	1560	582
May 14	111	267	502	5850	685
Jun 14	198	501	227	7800	907
Jul 14	275	561	703	7800	785
Aug 14	210	267	574	780	565
Sep 14	127	299	571	1950	737
Oct 14	88	301	488	3900	1081
Nov 14	39	287	118	3900	1191
Dec 14	34	54	90	780	709
Jan 15	88	110	334	780	499
Feb 15	133	210	861	1950	519
Mar 15	144	202	1176	1950	598
Apr 15	80	176	1191	1560	589
Total	**1965**	**4077**	**9724**	**45240**	**11211**

Due to the fact already addressed that the market is volatile and seasonal, the conversion rate peaks in certain months throughout the period studied, namely during the summer months (Jun 14 and Jul 14) and the fall months (Nov 14 and Dec 14), which coincides with the bulk of the yearly advertising campaigns promoted by both firms. Also, due to the lack of data (in terms of quantity) it is virtually impossible to model a seasonality aspect or even to point out periods or months where the data regarding the conversion rate differs from what one would expect and then provide an explanation, such as news in the media.

Four different sales models are formulated and a comparison is made between all of them in order to select the model(s), which best fits the data. The number of parameters in the several functions varies as well as the functional form, either natural logarithm or square root. The natural logarithm functional form was chosen because of an assumption that is made for GameOn where the effect of the own and the competitors' online contracts sales are subject to diminishing returns. Additionally, estimating a natural logarithm functional form produces estimates with constant elasticity, and according to an industry expert, this is, approximately the case for the studied market. A linear functional form was also considered but it was selected a square root functional form instead due the diminishing returns effect that comes along with it, while the linear functional form doesn't produce such effect. The decision of considering a square root in the variable "sales" (online contracts sold/gained) as it is seen in (3) and (4), was made because if that variable didn't have a square root, the reaction function that determines the optimal quantity of online leads to be generated by a firm would not depend on the strategy selected by the rival firm(s). And for the purposes of this research, it doesn't make sense that the reaction functions don't take into account the other firms' strategies, which is the basis of game theory itself. To conclude, GameOn is developed using two functional forms, natural logarithm and square root, that are formulated ahead. Every equation for each firm (A and B) will be presented, with the equations for firm A above the equations for firm B.

1. The first sales model – M1 – has the following sales response function:

$$\ln S_A = \ln \alpha_A + \beta_A \ln L_A + \gamma_{AB} \ln L_B \tag{1}$$

$$\ln S_B = \ln \alpha_B + \beta_B \ln L_B + \gamma_{BA} \ln L_A \tag{2}$$

where S_A are the sales (online contracts) of firm A in a given period of time, α_A is a firm-specific intercept term, L_A and L_B represent the quantity of online leads generated in a given period of time, by firms A and B respectively, β_A and γ_{AB} are the parameters that measure the sensitivity of firm A's sales to its own quantity of online generated leads and firm B's leads respectively, and finally, ln denotes the natural logarithm.

2. The second sales model to be studied, M2, resorts to the square root instead of the natural logarithm to model the sales response of each firm:

$$\sqrt{S_A} = \alpha_A + \beta_A \sqrt{L_A} + \gamma_{AB} \sqrt{L_B} \tag{3}$$

$$\sqrt{S_B} = \alpha_B + \beta_B \sqrt{L_B} + \gamma_{BA} \sqrt{L_A} \tag{4}$$

where the significance of each parameter is the same as it was in M1.

Each of the above sales response functions, M1 and M2, do not use the offline generated leads of each firm. To thoroughly study which model(s) and which individual and set of parameters best fit the real data with the objective of building a proper management model, one other version of the previous sales response functions was studied: the addition of the offline generated leads parameter (except for the case of firm B, given the fact that this particular data could not be gathered).

3. For the model M3, the offline leads parameter is added to the model M1, refer to (1):

$$\ln S_A = \ln \alpha_A + \beta_A \ln L_A + \gamma_{AB} \ln L_B + \rho_A \ln O_A \qquad (5)$$

where ρ_A is the parameter that measures the sensitivity of firm A's sales to its own quantity of offline generated leads, and O_A is the quantity of offline leads generated by firm A in a given period of time.

4. For the model M4, the same line of thought is followed, and its sales response function is as follows:

$$\sqrt{S_A} = \alpha_A + \beta_A \sqrt{L_A} + \gamma_{AB} \sqrt{L_B} + \rho_A \sqrt{O_A} \qquad (6)$$

Table 4.2 contains the code written in "R" a statistical programming language software, used to perform the regressions. In Table 4.3 the results of the estimation of the sales response functions of M1, M2, M3, and M4 are presented for firm A. It is presented the value of each estimated parameter, as well as their standard error and whether or not they are significant and if they are, at what level of significance. The estimations were also were computed in "R".

Table 4.2 – Regressions' "R" Code for Each Model for Firm A

Model	Regression's "R" code
M1	RegLin1 <- lm (log(OnlineContracts_A) ~ log(OnlineLeads_A) + log(OnlineLeads_B), Leads)
M2	RegLin2 <- lm (sqrt(OnlineContracts_A) ~ sqrt(OnlineLeads_A) + sqrt(OnlineLeads_B), Leads)
M3	RegLin3 <- lm (log(OnlineContracts_A) ~ log(OnlineLeads_A) + log(OnlineLeads_B) + log(OfflineLeads_A), Leads)
M4	RegLin4 <- lm (sqrt(OnlineContracts_A) ~ sqrt(OnlineLeads_A) + sqrt(OnlineLeads_B) + sqrt(OfflineLeads_A), Leads)

RegLin1,2,3, and 4 was the name given to each regression; "lm" is the "R" function that performs the regression, and its inputs are the data provided in Table 4.1; the last input, "Leads" is the name given to the entire data (Table 4.1 as a whole). The results from these regressions are provided in Table 4.3:

Table 4.3 – Parameters' Estimates, (Standard Errors) and "Significant at the Level" for Firm A

Parameter	M1	M2	M3	M4
$\boldsymbol{\alpha_A}$	0,981 (1,427) "Not Signif."	3,082 (2,676) "Not Signif."	281,181 (4,543) "Not Signif."	14,214 (6,816) "10%"
$\boldsymbol{\beta_A}$	0,447 (0,145) "1%"	0,183 (0,082) "5%"	0,2847 (0,188) "Not Signif."	0,091 (0,093) "Not Signif."
$\boldsymbol{\gamma_{AB}}$	0,252 (0,137) "10%"	0,068 (0,029) "5%"	0,446 (0,2) "5%"	0,108 (0,035) "5%"
$\boldsymbol{\rho_A}$	-	-	-0,941 (0,719) "Not Signif."	-0,416 (0,237) "Not Signif."
$\boldsymbol{R^2}$	0,406	0,312	0,436	0,407

From Table 4.3 it can be immediately observed that some parameters are not significant. For M1 and M2, where only the respective parameter α_A is not significant, the regression will be performed once again but without the parameter α_A. For M3 and M4 there is more than one parameter that is not significant. To select the one that is going to be eliminated, each parameter's p-value will be analyzed. Starting with M4, the p-value for β_A is 0,3471 and the p-value for ρ_A is 0,1049, which means that β_A, having the highest p-value should be eliminated, but, for the purposes of this study, it makes very little sense to eliminate the parameter that corresponds to the effect that firm A's own online generated leads has in its own online sales. Additionally, if the parameter ρ_A were to be excluded, then M4 (the formulation and parameters' estimates) would be the exact same as M2 which is, obviously, redundant. Thus, M4 will be eliminated. For M3, there are three non-significant parameters: α_A, with a p-value of 0,2382; β_A with a p-value of 0,1555; and ρ_A with a p-value of 0,2154. The highest p-value of these three parameters is 0,2382 that relates to α_A, and thus, this parameter will be eliminated and the regression performed again.

Since M1, M2 and M3 were modified, the new formulations for these models are presented in Table 4.4 with the non-significant parameters eliminated.

Table 4.4 – Models' New Formulation for Firm A

Model	Formulation
M1	$\ln S_A = \beta_A \ln L_A + \gamma_{AB} \ln L_B$
M2	$\sqrt{S_A} = \beta_A \sqrt{L_A} + \gamma_{AB}\sqrt{L_B}$
M3	$\ln S_A = \beta_A \ln L_A + \gamma_{AB} \ln L_B + \rho_A \ln O_A$

The regression for each model was again computed in "R", and the code used is portrayed in Table 4.5, with the values of the estimations, their standard errors and significance level presented in Table 4.6.

Table 4.5 – Updated Regressions' "R" Code for Firm A

Model	Regression's "R" code
M1	RegLin1 <- lm (log(OnlineContracts_A) ~ log(OnlineLeads_A) + log(OnlineLeads_B) - 1, Leads)
M2	RegLin2 <- lm (sqrt(OnlineContracts_A) ~ sqrt(OnlineLeads_A) + sqrt(OnlineLeads_B) - 1, Leads)
M3	RegLin3 <- lm (log(OnlineContracts_A) ~ log(OnlineLeads_A) + log(OnlineLeads_B) + log(OfflineLeads_A) - 1, Leads)

The difference between the code from Table 4.2 and Table 4.5 is the "-1" near the end of each model's code, that signifies the exclusion of the parameter α_A from each model.

Table 4.6 – Parameters' Estimates, (Standard Errors) and "Significant at the Level" for Firm A Updated

Parameter	M1	M2	M3
$\boldsymbol{\beta_A}$	0,446 (0,102) "0,1%"	0,260 (0,047) "0,1%"	0,468 (0,119) "1%"
$\boldsymbol{\gamma_{AB}}$	0,251 (0,083) "1%"	0,091 (0,022) "1%"	0,310 (0,170) "10%"
$\boldsymbol{\rho_A}$	-	-	-0,091 (0,225) "Not Signif."
$\boldsymbol{R^2}$	0,994	0,962	0,994

There is still a non-significant parameter, the parameter ρ_A of M3. There is no need to eliminate this parameter and perform the regression once more because if the parameter is removed, then

M3 would be the exact same as M1 (mathematical formulation and parameters' estimations), see Table 4.4. Therefore, the model M3 is excluded from the analysis.

The R^2 values, that give the proportion of variation in the dependent variable that is explained by the regressor variables (Griffiths, Hill and Judge, 1993), weren't discussed following Table 4.3 because there was no need for such an analysis, since the important discussion regarding Table 4.3 was on the non-significant parameters, thus rendering the analysis of the R^2 values irrelevant. It is important to point out that the R^2 values from Table 4.3 were adjusted R^2 values. This is the measure that can evaluate the fit of the models with different number of parameters and compare them with each other, while the R^2 values present in Table 4.6 are multiple R^2 values, used to evaluate the fit of the models with the same number of parameters and compare them with each other also. This is here differentiated because the R^2 values of M1 and M2 will not be directly compared with the M3's R^2 value since there is no need to do that as was discussed in the previous paragraph (although M2 could never be compared to M3 given that they have different dependent variables). From M1 and M2's R^2 value, 0,994 and 0,962 respectively, it can be concluded that both these models provide an adequate fit of the data. Both these values give enough confidence to pursue with the analysis in both cases.

In M1 and M2 the parameters β_A and γ_{AB} are significant at the 0,1 and 1 percent level respectively. These parameters also have the same signs in both models. For M1, β_A measures the sensitivity of firm A's sales (of online contracts) to its own quantity of online generated leads where if the quantity of online generated leads, by firm A increases by 1%, then the sales of online contracts of firm A would increase by 0,446%. For M2, the same cannot be said: if the quantity of online generated leads, by firm A increases by 1%, then the sales of firm A would only increase by 0,26% in a certain situation, because since the elasticity from the square root functional form is not constant, the fact portrayed is only true in a certain situation when there is a specific combination of online contracts sold by firm A and online leads generated by both firm A and B. Contrary to the natural logarithm functional form, the percentage variations are not constant. The elasticity is calculated as follows:

$$\delta_{S_A L_A} = \frac{\frac{\partial \mathrm{S}_A}{\partial \mathrm{L}_A}}{\frac{S_A}{L_A}} = \beta_A \frac{\sqrt{L_A}}{\sqrt{S_A}} \tag{7}$$

As it can be observed by (7), although β_A and γ_{AB} are already known from Table 4.6, the elasticity (the percentage variations of the online generated leads from firms A and B, from the increase in 1% of the online contracts' sales) value, is not constant, will always depend on the specific situations provided by specific values of S_A, L_A and L_B.

For the natural logarithm functional form, the elasticity is given by just the parameter "β_A". It is clear then, that the elasticity is constant, unlike the elasticity of the square root functional form.

The fact that the sales of online contracts by a firm increases when the quantity of online generated leads by the same firm increases as well, is a normal and intuitive result (derived from the fact that, in both models $\beta_A > 0$). While this observation comes as no surprise, the sign of the parameter γ_{AB}'s estimations, in both M1 and M2 might be counterintuitive, because in both models $\gamma_{AB} > 0$. Even according to the literature, especially Viscolani (2012) who states that there is a compelling idea, which relates to the particular application of the theory of games, that the advertising levels of one firm, has (negative) implications on the rival firms' sales and own advertising levels. But unlike the idea that Viscolani (2012) argues, the present study reveals something entirely different. The fact that both γ_{AB} values are positive means that an increase in the quantity of online generated leads by firm B has a positive impact on the sales of online contracts of firm A. For M1, if the quantity of online generated leads of firm B increases 1%, then the sales of online contracts of firm A will increase by 0,251%. This discrepancy from the literature - and what some might consider as common sense - and the results presented is justified due to the particular characteristics of the studied industry and the product it commercializes. Because the product that both firms commercialize is something that a consumer never buys instinctively, but ponders greatly, an increase in the advertising from a firm has a positive effect on the other firm's sales because the consumers also study their product and might want to analyze whether it has better characteristics, price or both. Additionally, the fact that this is duopolistic market, forces a consumer to only choose between two firms and that contributes to increase this effect.

At this point, the performance of the residual (which is/are the estimation(s) of the model/experimentation error(s)) analysis[16] is needed to verify the assumptions of the error normality and the variance homogeneity for both M1 and M2, in order to be sure that no false conclusions are drawn from the application(s) of these models. This is done by analyzing the residual plots that are shown ahead. The plots were computed recurring to the statistical programming language "R" and its specific software. The code written in the "R" console to compute the plots is presented in Table 4.7 and Table 4.8 for firm A's M1 and M2 respectively.

[16] For more detailed information on residual analysis, it is recommended the consultation of Pereira and Requeijo (2012)

Table 4.7 – Residual Analysis's "R" Code for Firm A's M1

Figure	Residual Analysis's "R" code
4.2	>RegLin1 <- lm(log(OnlineContracts_A)~log(OnlineLeads_A)+log(OnlineLeads_B)-1,Leads) > analysis.res = rstandard(RegLin1) > qqnorm(analysis.res, + ylab="Standardized Residuals", + xlab="Normal Scores", + main="") > qqline(analysis.res)
4.3	> plot(fitted(RegLin1), residuals(RegLin1), + ylab="Residuals", xlab="Fitted Values", + main="") > abline(0,0)

Figure 4.2 and Figure 4.3 represent the residual plots to be analyzed. Figure 4.2 relates to the plot of the residual and predicted values and it is a visual method to verify the homogeneity of the variance. An ideal plot would show no special pattern and the points should be scattered as much as possible. Figure 4.2 shows an acceptable plot despite one section of plot showing a quantity of points somewhat close. Figure 4.3 relates to the normality verification and also doesn't show an ideal plot. Although there is a gathering of a number of values around "0" and the fact that the distribution of the values follows an unusual pattern (similar to a sigmoid) it is clear that the plot is not asymmetric and it doesn't indicate a strong violation of the normality assumption. Thus, it is acceptable to conclude that the assumptions made for the model are valid.

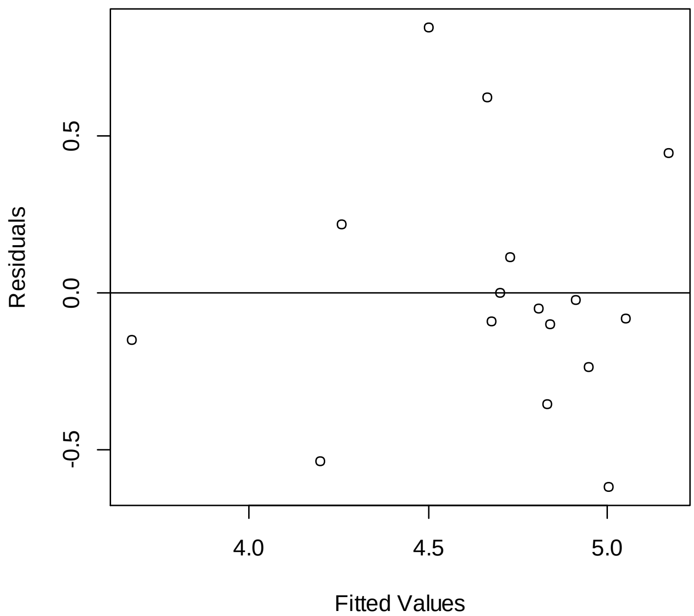

Figure 4.2 – Residuals vs Fitted Plot for Firm A's M1

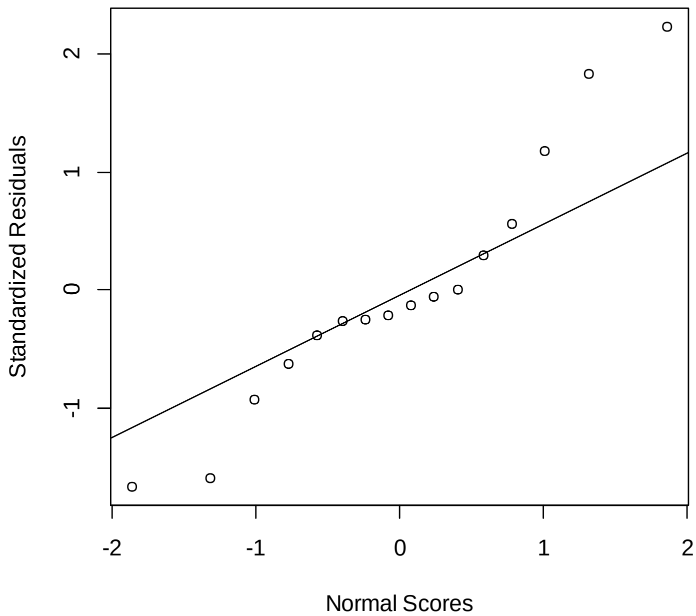

Figure 4.3 – Standardized Residuals vs Normal Scores Plot for Firm A's M1

Table 4.8 – Residual Analysis's "R" Code for Firm A's M2

Figure	Residual Analysis's "R" code
4.4	>RegLin2 <- lm(log(OnlineContracts_A)~log(OnlineLeads_A)+log(OnlineLeads_B)-1,Leads) > analysis.res = rstandard(RegLin2) > qqnorm(analysis.res, + ylab="Standardized Residuals", + xlab="Normal Scores", + main="") > qqline(analysis.res)
4.5	> plot(fitted(RegLin2), residuals(RegLin2), + ylab="Residuals", xlab="Fitted Values", + main="") > abline(0,0)

For M2 the residual analysis is similar in all aspects. For the plot portrayed in Figure 4.4, an acceptable value dispersion can be observed and thus it is adequate to state that there is no clear violation of the variance homogeneity assumption. Figure 4.5 also shows no clear violation of the normality assumption given that the values are distributed along the straight line and there is no asymmetry. Thus for this case, the assumptions made for the model are also valid.

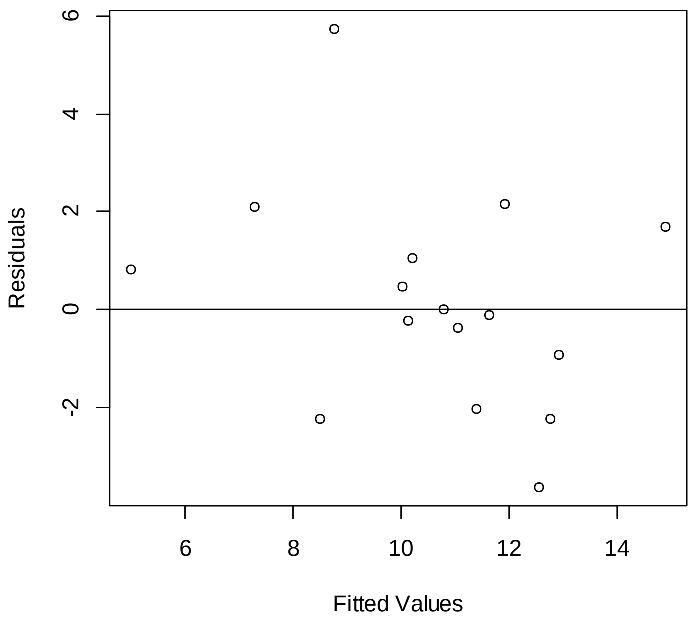

Figure 4.4 – Residuals vs Fitted Plot for Firm A's M2

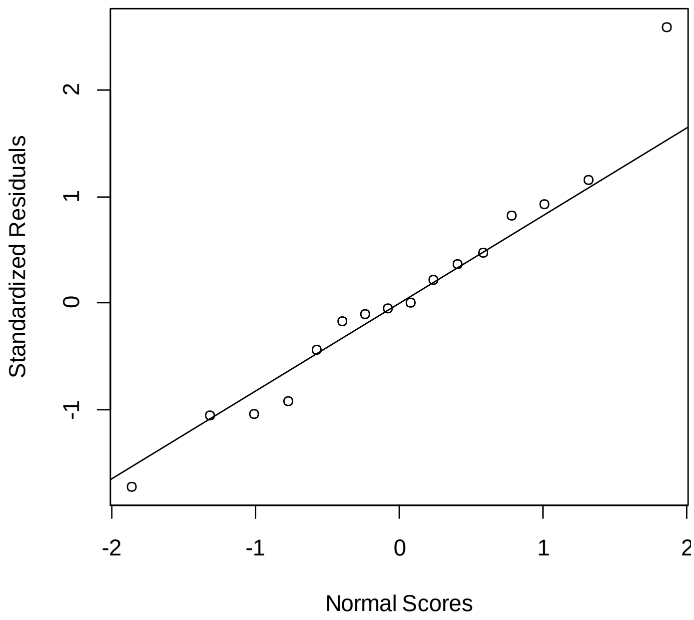

Figure 4.5 – Standardized Residuals vs Normal Scores Plot for Firm A's M2

With the residual analysis' conditions met, the bootstrap[17] technique is now utilized in order to better estimate the parameters of the regressions. The use of this technique is especially important due to the small number of observations in the data (only for 16 months or observations), and because the data, as it was already discussed in the residual analysis, are not

[17] For more information and a deeper understanding of the bootstrap technique, it is recommended the consultation of Efron (1979) and http://www.sagepub.com/sites/default/files/upm-binaries/21122_Chapter_21.pdf

extremely well behaved, especially for M1 (the bootstrap technique improves the fit when the data presented in the residual analysis' normality plot are not that well behaved). The technique will be used for M1 and M2. Just like for the regressions and the residual analysis discussed, the bootstrap's results were computed using the "R" software. The code to perform the technique for both models is presented in Table 4.9.

Table 4.9 – Bootstraps' "R" Code for Each Model for Firm A

Model	Bootstrap's "R" code
M1	> ft<-function(data, i,formula){ + d<-data[i,] + fit<- lm(formula, data=d) + return(coef(fit)) + } > results <- boot(data=Leads, statistic=ft, R=250, formula=log(OnlineContracts_A)~log(OnlineLeads_A)+log(OnlineLeads_B)-1) > results
M2	> ft<-function(data, i,formula){ + d<-data[i,] + fit<- lm(formula, data=d) + return(coef(fit)) + } > results <- boot(data=Leads, statistic=ft, R=250, formula=sqrt(OnlineContracts_A)~sqrt(OnlineLeads_A)+sqrt(OnlineLeads_B)-1) > results

What is relevant to point out from the functions on Table 4.9 that return the parameters' estimations recurring to the bootstrap technique is the "R" input for the "boot" function. In the case portrayed in Table 4.9, the "R" is equal to 250, i.e. there will be 250 replications (250 samples from the original sample data). While it is common to assume that somewhere between 1000 and 2000 bootstrap replications are sufficient because it is, generally at that point when the standard deviation of the estimations stabilize as it will be seen ahead, a studied is conducted in order to arrive at the best possible estimations for the parameters to provide confidence in the models and applications presented.

When the bootstrap is performed in the "R" software, it yields three different results: 1) the original estimation, i.e. the parameter estimation of the regression in question; 2) the bias, i.e. the difference between the estimation from the bootstrap technique and the original estimation; and 3) the standard error from the bootstrap estimation.

When the literature states that one should perform between 1000 and 2000 bootstrap replications, it is only considering the "normal" period in which the standard errors stabilize, while in this study, in order to provide more confidence to the estimations, what is ideal to stabilize are the bias values for β_A and γ_{AB}. As it can be observed from Figure 4.6, the bias values for each parameter are never stable; although the standard errors can be considered to be stabilized close to the 2000th replication (see Figure 4.7).

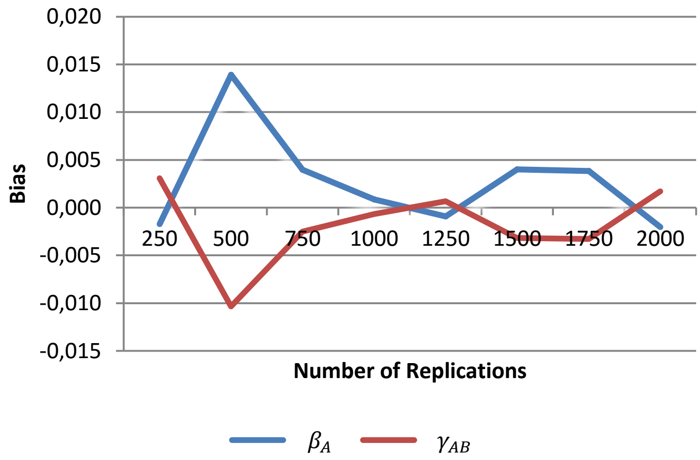

Figure 4.6 – Bootstrap’s Estimation for M1 with Replications from 250 to 2000

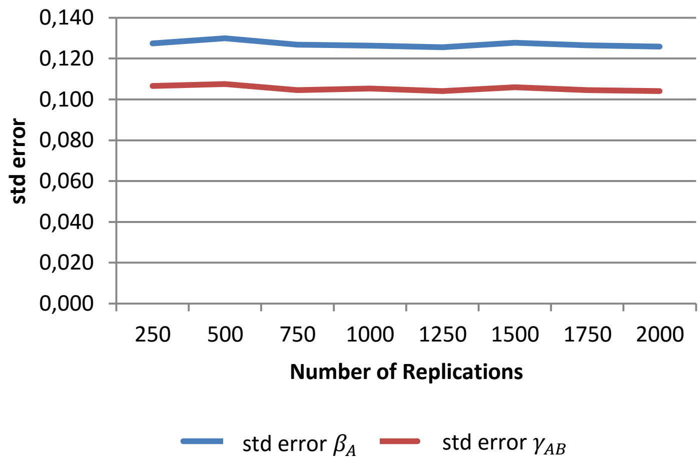

Figure 4.7 – Bootstrap's Std Error for the Estimation for M1 with Replications from 250 to 2000

Since the estimations for the parameters are far from stabilizing, according to Figure 4.6, the next step is to increase the number of bootstrap replications. Figures 4.8 and 4.9 present the evolution of the estimates for 10000 to 100000 bootstrap replications. Figure 4.9 presents a stable evolution for the standard errors, while Figure 4.8 portrays an evolution that cannot be considered stabilized yet, because there is a clear value discrepancy throughout every set of replications.

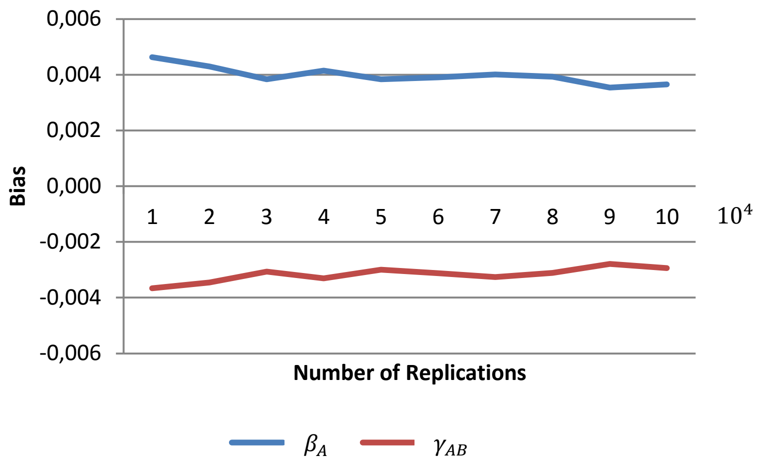

Figure 4.8 – Bootstrap's Estimation for M1 with Replications from 10000 to 100000

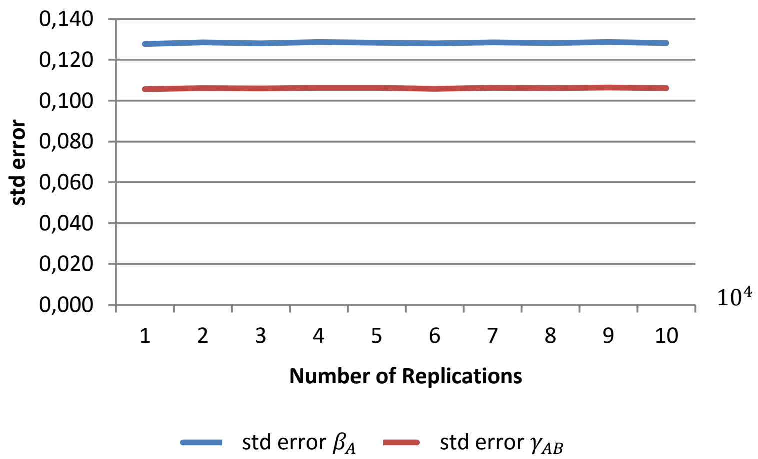

Figure 4.9 – Bootstrap's Std Error for the Estimation for M1 with Replications from 10000 to 100000

Since the values for the bootstrap's parameters estimations are not stabilized yet, the number of replications was once again increased. It was increased to a range from 100000 to 1.2 million replications. From Figure 4.10 it can be observed that the estimations have finally stabilized, and for applications purposes it will be used the last estimations computed (from the 1.2 million replications). The standard errors are still stable, see Figure 4.11. In terms of computing time, the last estimation was computed in, approximately 45 minutes.

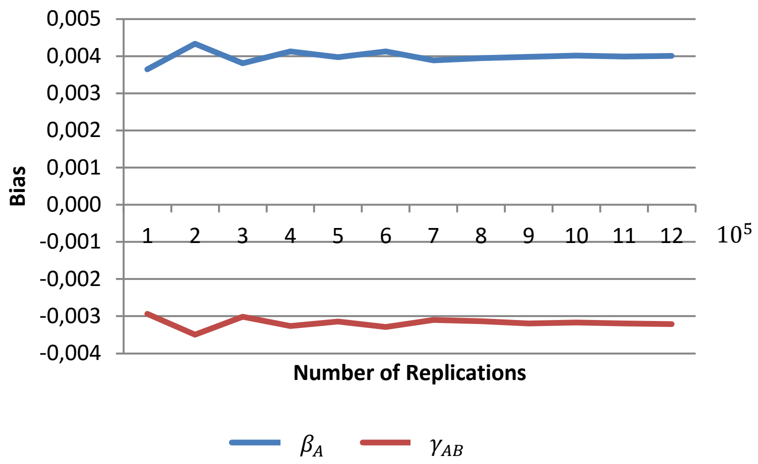

Figure 4.10 – Bootstrap's Estimation for M1 with Replications from 100000 to 1000000

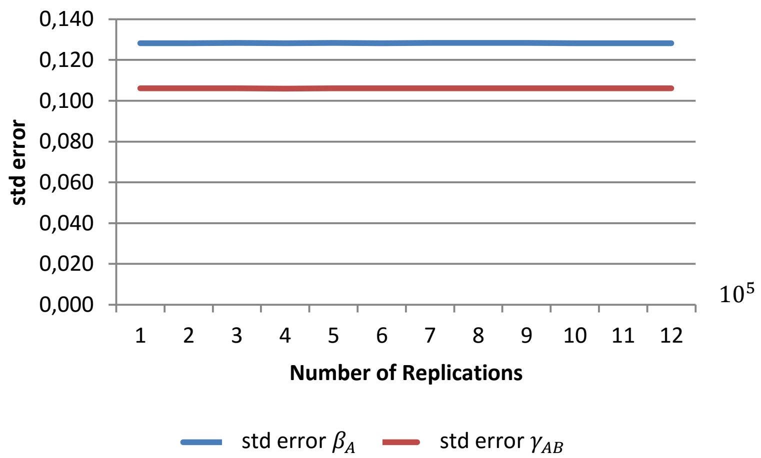

Figure 4.11 – Bootstrap's Std Error for the Estimation for M1 with Replications from 100000 to 1000000

Table 4.10 provides the final results of the parameters' estimations recurring to the bootstrap technique, and also provides the final estimations (calculated by adding the original values to the bootstrap's bias values) that will be used in the application scenario and case study.

Table 4.10 – M1's Bootstraps' Final Results and Estimations for Firm A

Parameter	Original	Bias	Final Estimation	Std error
$\boldsymbol{\beta_A}$	0,4457097	0,004009137	0,449718837	0,1282706
$\boldsymbol{\gamma_{AB}}$	0,2508342	-0,003207038	0,247627162	0,1060251

By observing the values of the final estimations of both parameters, it is concluded that the discussion from Table 4.6 regarding β_A and γ_{AB} is still valid.

From here on in, the bootstrap analysis for the remaining models will be solely focused on the stabilized figures.

To perform the bootstrap technique for M2 the same logic is applied. For this case, the stability of the parameters' estimations is reached, as it can be observed in Figure 4.12, between 110000 and 150000 replications (as well as the standard error for both parameters, although the stability for this case is reached long before this quantity of replications). As per the previous case (M1), the results to be used will be the ones from the highest number of replications, i.e. the results for 150000 replications (see Table 4.11).

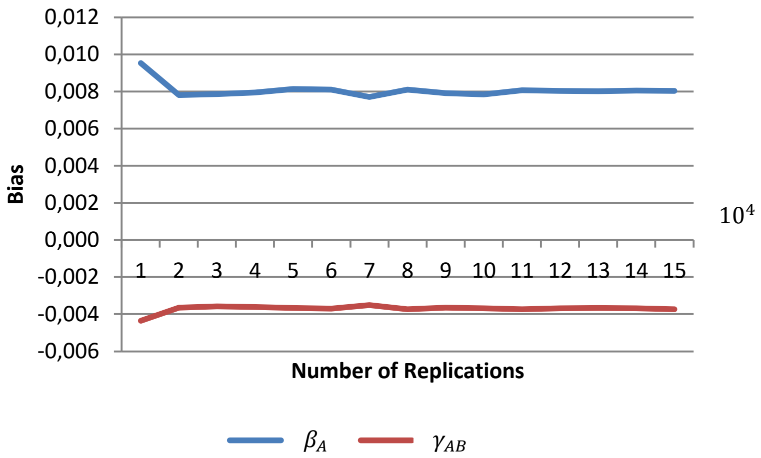

Figure 4.12 – Bootstrap's Estimation for M2 with Replications from 10000 to 150000

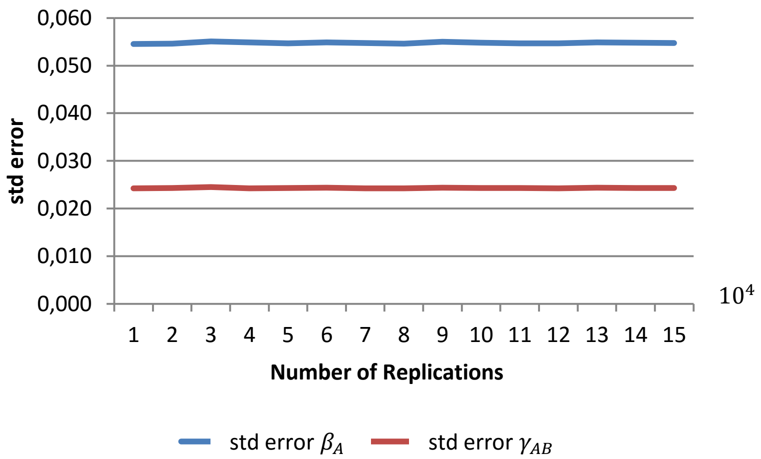

Figure 4.13 — Bootstrap's Std Error for the Estimation for M2 with Replications from 10000 to 150000

Table 4.11 provides the final results of the parameters' estimations for M2, recurring to the bootstrap technique, and also provides the final estimations that will be used in the next chapter.

Table 4.11 – M2's Bootstraps' Final Result and Estimations for Firm A

Parameter	Original	Bias	Final Estimation	Std error
$\boldsymbol{\beta_A}$	0,26019604	0,008031711	0,268227751	0,05474064
$\boldsymbol{\gamma_{AB}}$	0,09063759	-0,003739385	0,086898205	0,02428831

By observing the values of the final estimations of both parameters, it is concluded that the discussion from Table 4.6 regarding β_A and γ_{AB} is still valid.

The models relative to firm A and its parameters' estimations are concluded, addressed and discussed. The following step is to proceed to a similar discussion and evaluation for firm B. First, it is important to stress once again that since there is no data regarding the monthly quantity of offline generated leads of firm B, there are only two models to be analyzed, M1 and M2, each one with a sales response function to model firm B's online contracts sales according to certain parameters (see (3) and (4)). Much like in the case of firm A, the regressions for firm B were computed recurring to the "R" software, and its code can be seen in Table 4.12. In Table 4.13 it is presented the values for the parameters' estimations, as well as their standard errors and significance levels.

Table 4.12 – Regressions' "R" Code for Each Model for Firm B

Model	Regression R code
M1	RegLin5 <- lm (log(OnlineContracts_B) ~ log(OnlineLeads_B) + log(OnlineLeads_A), Leads)
M2	RegLin6 <- lm (sqrt(OnlineContracts_B) ~ sqrt(OnlineLeads_B) + sqrt(OnlineLeads_A), Leads)

Table 4.13 – Parameter Estimates, (Standard Errors) and "Significant at the Level" for Firm B

Parameter	M1	M2
$\boldsymbol{\alpha_B}$	0,715 (1,083) "Not Signif."	4,400 (2,551) "Not Signif."
$\boldsymbol{\beta_B}$	0,560 (0,104) "0,1%"	0,169 (0,028) "0,1%"
$\boldsymbol{\gamma_{BA}}$	0,237 (0,110) "10%"	0,118 (0,078) "Not Signif."
$\boldsymbol{R^2}$	0,713	0,738

As it can be observed in Table 4.13, the only non-significant parameter in M1 is α_B and thus this parameter will be excluded and another regression to estimate the remaining parameters will be performed. For M2, there are two non-significant parameters and to select the one to be excluded, the respective p-value is studied. For α_B the p-value is 0,109 and for γ_{BA} the p-value is 0,154. While the theory states that γ_{BA} should be excluded due to its higher p-value, it is worth noting that the difference between both p-values is not large, and for the purposes of GameOn which this dissertation proposes, it doesn't make sense to exclude the parameter γ_{BA} because it measures the sensitivity of firm B's online contracts sales to the quantity of online generated leads by firm A, thus the parameter to be excluded is α_B.

Since M1 and M2 were modified, it is presented, in Table 4.14, the new formulations of these models with the non-significant parameters excluded:

Table 4.14 – Models' New Formulation for Firm B

Model	**Formulation**
M1	$\ln S_B = \beta_B \ln L_B + \gamma_{BA} \ln L_A$
M2	$\sqrt{S_B} = \beta_B \sqrt{L_B} + \gamma_{BA}\sqrt{L_A}$

The regression for each model was again computed in "R", and the code written is presented in Table 4.15, with the values of the estimations, their standard errors and significance level shown in Table 4.16:

Table 4.15 – Updated Regressions' "R" Code for Firm B

Model	**Regression R code**
M1	RegLin1 <- lm (log(OnlineContracts_B) ~ log(OnlineLeads_B) + log(OnlineLeads_A) - 1, Leads)
M2	RegLin2 <- lm (sqrt(OnlineContracts_B) ~ sqrt(OnlineLeads_B) + sqrt(OnlineLeads_A) - 1, Leads)

The difference between the code from Table 4.13 and Table 4.16 is the "-1" near the end of each model's code, that signifies the exclusion of the parameter α_B from both models.

Table 4.16 – Parameter Estimates, (Standard Errors) and "Significant at the Level" for Firm B Updated

Parameter	**M1**	**M2**
$\boldsymbol{\beta_B}$	0,535 (0,063) "0,1%"	0,201 (0,022) "0,1%"
$\boldsymbol{\gamma_{BA}}$	0,214 (0,078) "5%"	0,228 (0,048) "0,1%"
$\boldsymbol{R^2}$	0,997	0,981

By observing the R^2 values of both M1 and M2, 0,997 and 0,981 respectively, it is fair and rational to conclude that both models provide an adequate fit of the data. Both these values give confidence to pursue with the analysis in both cases.

Regarding the concern that arose from the non-removal of the parameter that had the highest p-value of M2 in Table 4.13 (γ_{BA}), and the consequent removal of α_B, the high R^2's value and the fact that γ_{BA} is now significant at the level of 0,1% shows that the exclusion of α_B was adequate and didn't "damage" the model itself.

In M1 the parameters β_A and γ_{BA} are significant at the 0,1 and 5 percent level respectively, while in M2 the parameters β_A and γ_{BA} are both significant at the 0,1 percent level. These parameters also have the same signs in both models. The analysis of these parameters and their respective signs is analogous to the analysis presented for Table 4.6. Although the analysis is analogous, there are some interesting facts to point out, for example, for firm B's M1, the weight that its own quantity of online generated leads has on its own sales is larger than the same situation but for firm A's M1 (β_B>β_A). Interestingly, the exact opposite happens for both firms in M2.

One can argue that, due to the larger market share of firm B (and the much larger number of online leads generated, as seen in Table 4.1), it would be intuitive to think that the quantity of online generated leads by firm B would have more weight on the sales of online contracts of firm A than the quantity of online generated leads by firm A would have on the sales of online contracts of firm B ($\gamma_{AB} > \gamma_{BA}$), and that is what actually happens for M1. Once again, the exact opposite happens when it comes to the parameter's sensitivity for M2, where $\gamma_{AB} < \gamma_{BA}$.

It is interesting to verify that, for firm B's M2, $\gamma_{BA} > \beta_B$, which means that, for this model and for firm B specifically, γ_{BA}, the parameter that measures the sensitivity of firm B's sales to firm A's quantity of online generated leads, has more weight than β_B, which measures the sensitivity of firm B's sales to its own quantity of online generated leads.

Once again it is important to perform the residual analysis to verify the error normality and the variance homogeneity for both M1 and M2. This is done by analyzing the residual plots that are shown ahead. The code written in the console of the "R" software to compute the plots is presented in Table 4.17 and Table 4.18 for firm B's M1 and M2 respectively.

Table 4.17 – Residual Analysis's "R" Code for Firm B's M1

Figure	Residual Analysis's "R" code
4.14	>RegLin1 <- lm(log((OnlineContracts_B)~log(OnlineLeads_B)+log(OnlineLeads_A)-1,Leads) > analysis.res = rstandard(RegLin1) > qqnorm(analysis.res, + ylab="Standardized Residuals", + xlab="Normal Scores", + main="") > qqline(analysis.res)
4.15	> plot(fitted(RegLin1), residuals(RegLin1), + ylab="Residuals", xlab="Fitted Values", + main="") > abline(0,0)

Figure 4.14 shows a very acceptable scattered plot and it allows to state that the assumption relative to the variance homogeneity is valid. Figure 4.15 shows a distribution of values along the straight line with no great visual asymmetry, as well as no indication of any clear violation of the normality assumption. Thus, it is acceptable to conclude that the assumptions made for the model are valid.

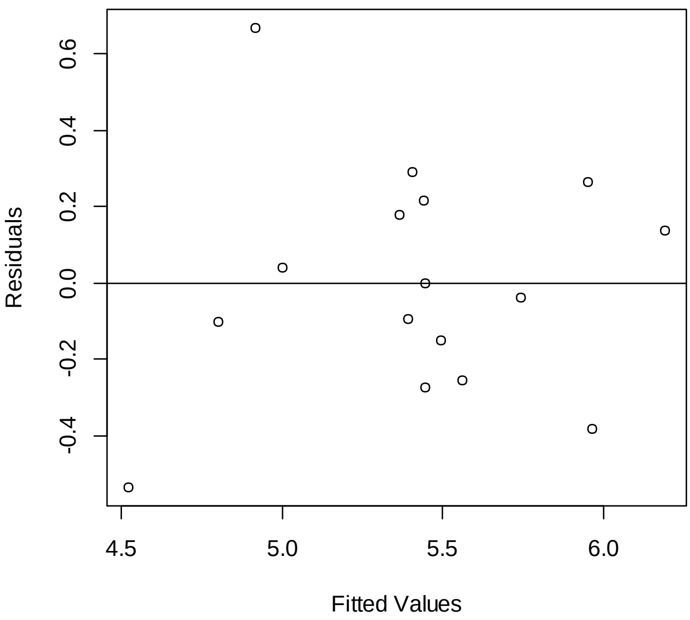

Figure 4.14 – Residuals vs Fitted Values Plot for Firm B's M1

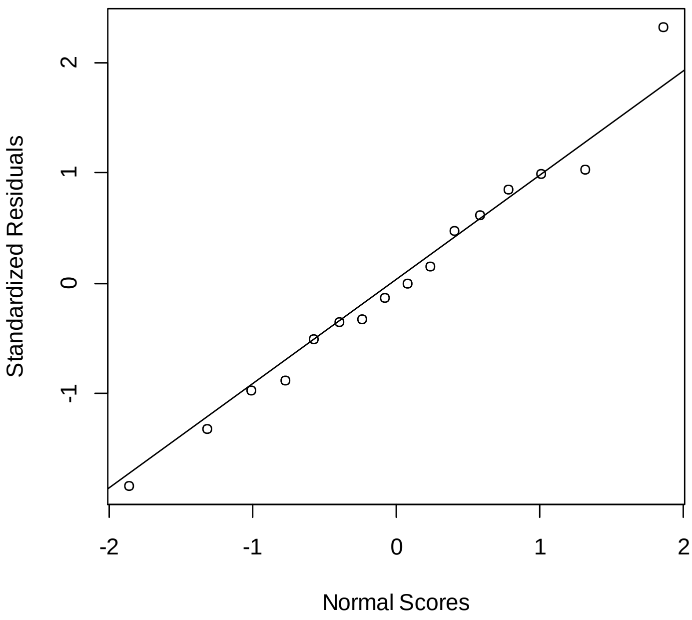

Figure 4.15 – Standardized Residuals vs Normal Scores Plot for Firm B's M1

Table 4.18 – Residual Analysis's "R" Code for Firm B's M2

Figure	Residual Analysis's "R" code
4.16	>RegLin2 <- lm(log(OnlineContracts_B)~log(OnlineLeads_B)+log(OnlineLeads_A)-1,Leads) > analysis.res = rstandard(RegLin2) > qqnorm(analysis.res, + ylab="Standardized Residuals", + xlab="Normal Scores", + main="") > qqline(analysis.res)
4.17	> plot(fitted(RegLin2), residuals(RegLin2), + ylab="Residuals", xlab="Fitted Values", + main="") > abline(0,0)

For M2 the residual analysis is similar in all aspects. For the plot portrayed in Figure 4.16, an appropriate value dispersion can be observed and thus it is adequate to state that there is no clear violation of the assumption for variance homogeneity. Figure 4.17 shows no clear violation of the normality assumption given that there is no visual asymmetry and the values are distributed along the straight line, especially towards the center of the plot, while at the corners some values are more dispersed relatively to the straight line but it isn't extremely significant. Thus, for this case, the assumptions made for the model are valid as well.

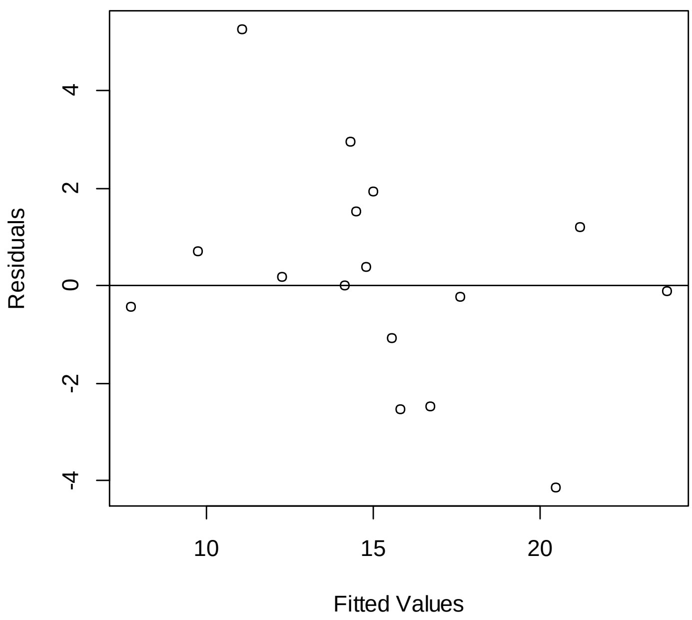

Figure 4.16 – Residuals vs Fitted Values Plot for Firm B's M2

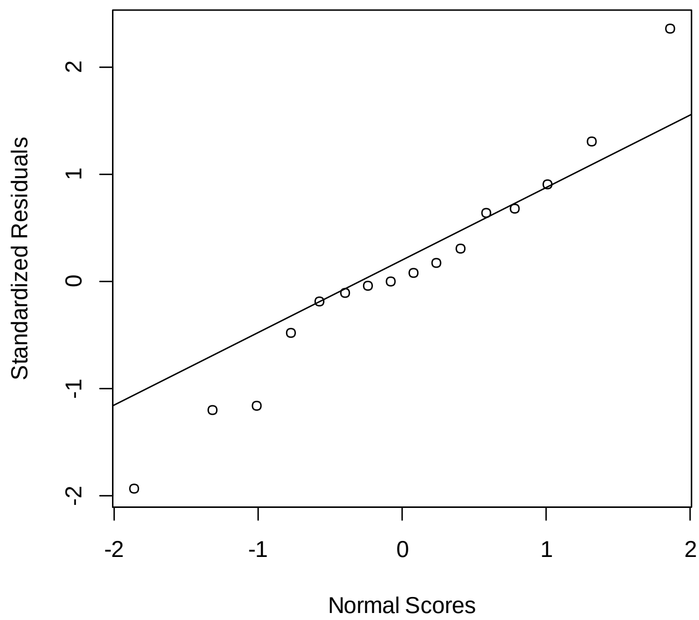

Figure 4.17 – Standardized Residuals vs Normal Scores Plot for Firm B's M2

With the residual analysis' conditions met, the bootstrap technique is now used for the same reasons as in the case for firm A. The technique will be used for M1 and M2. Just like for the regressions discussed, the bootstrap's results were computed using the R software. The code for performing the technique for both models is presented in Table 4.19.

Table 4.19 – Bootstraps' "R" Code for Each Model for Firm B

Model	Bootstrap R code
M1	> ft<-function(data, i,formula){ + d<-data[i,] + fit<- lm(formula, data=d) + return(coef(fit)) + } > results <- boot(data=Leads, statistic=ft, R=250, formula=log(OnlineContracts_B)~log(OnlineLeads_B)+log(OnlineLeads_A)-1) > results
M2	> ft<-function(data, i,formula){ + d<-data[i,] + fit<- lm(formula, data=d) + return(coef(fit)) + } > results <- boot(data=Leads, statistic=ft, R=250, formula=sqrt(OnlineContracts_B)~sqrt(OnlineLeads_B)+sqrt(OnlineLeads_A)-1) > results

For M1, the stabilized parameters' estimations and standard errors can be observed in Figures 4.18 and 4.19 respectively. This stabilization was achieved, roughly after 300000 replications, but the estimates to be used will be the ones given by the 700000 replications. Table 4.20 presents the final results of the parameters' estimations recurring to the bootstrap technique, and also provides the final estimations that will be used in the next chapter.

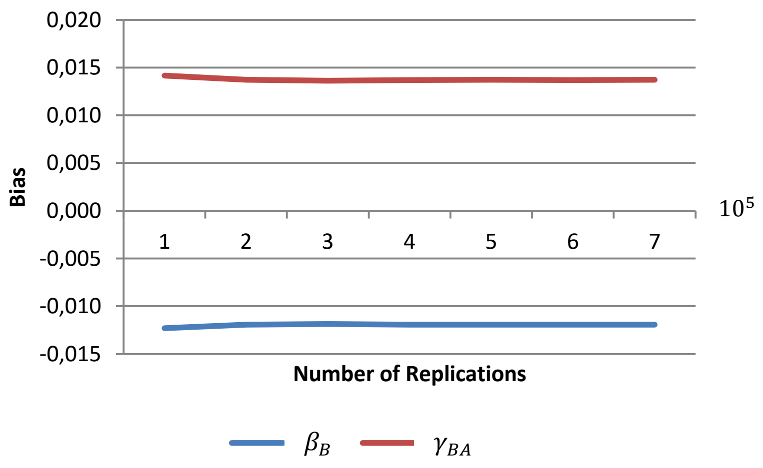

Figure 4.18 – Bootstrap's Estimation for M1 with Replications from 100000 to 700000

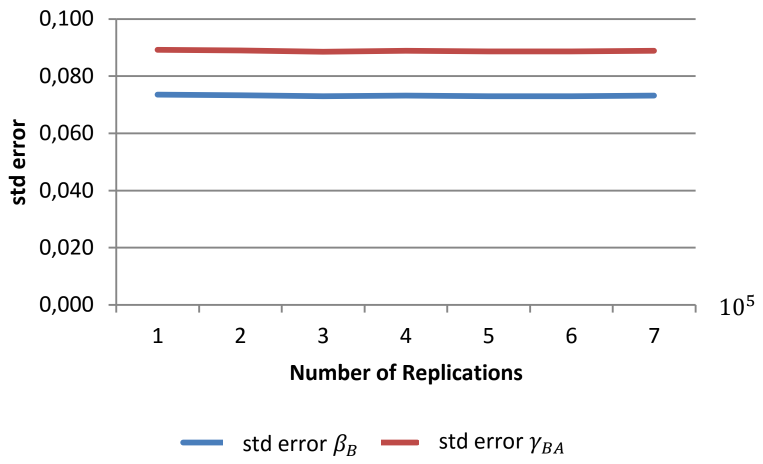

Figure 4.19 – Bootstrap's Std Error for the Estimation for M1 with Replications from 100000 to 700000

Table 4.20 – M1's Bootstrap Final Result and Estimations for Firm B

Parameter	Original	Bias	Final Estimation	Std error
$\boldsymbol{\beta_B}$	0,5348627	-0,01194434	0,52291836	0,07323479
$\boldsymbol{\gamma_{BA}}$	0,2135802	0,01374036	0,22732056	0,08892551

By observing the values of the final estimations of both parameters, it is concluded that the discussion from Table 4.16 regarding both parameters is still valid.

The code for running the bootstrap technique for M2 is presented in Table 4.19. For this case, the stability of the parameters' estimations is reached, as it can be observed in Figure 4.20, between 110000 and 150000 replications (as well as the standard error for both parameters, although the stability for this case is reached long before this quantity of replications). The results to be used will be the ones from the highest number of replications, i.e. the results for 150000 replications. Its values can be observed in Table 4.21.

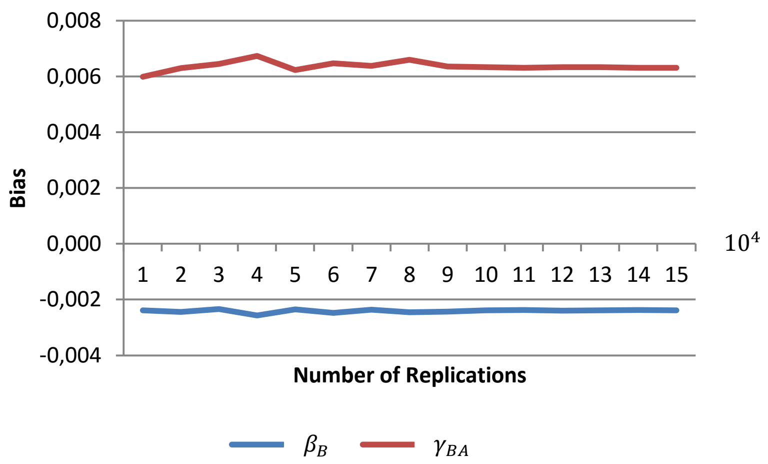

Figure 4.20 – Bootstrap's Estimation for M2 with Replications from 10000 to 150000

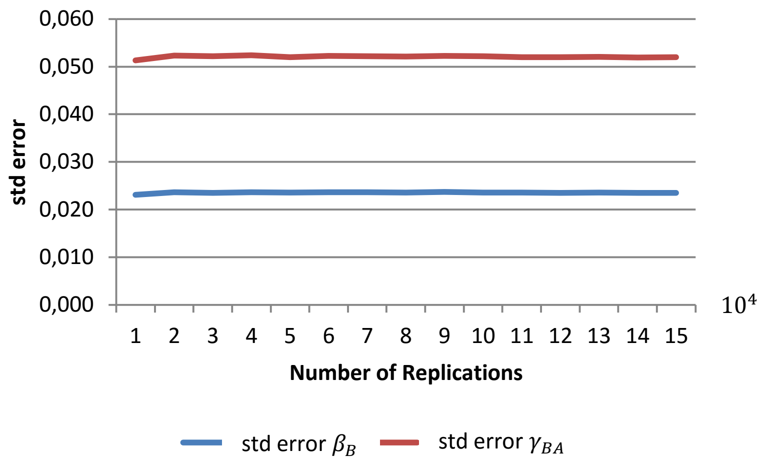

Figure 4.21 – Bootstrap's Std Error for the Estimation for M2 with Replications from 10000 to 150000

Table 4.21 – M2's Bootstrap Final Result and Estimations for Firm B

Parameter	Original	Bias	Final Estimation	Std error
β_B	0,200756	-0,002381787	0,198374213	0,02352048
γ_{BA}	0,2282532	0,00630343	0,23455663	0,0519939

Finally, by observing the values of the final estimations of both parameters, it is concluded that the discussion from Table 4.16 regarding both parameters is still valid.

Table 4.22 provides a summary of all the important parameters and their respective estimations that will be used in the following chapter when the application scenario and case study are discussed.

Table 4.22 – Summary of the Parameters' Estimations

Parameter	Firm A		Firm B	
	M1	M2	M1	M2
β_A	0,449718837	0,268227751	-	-
β_B	-	-	0,52291836	0,198374213
γ_{AB}	0,247627162	0,086898205	-	-
γ_{BA}	-	-	0,22732056	0,23455663

With the formulation and parameters' estimation for both firms' sales models' response function completed, the next step of the process to reach the goals of present dissertation is to develop the mathematical formulation of GameOn relative to both models (one with a log-log functional form and the other with a square root functional form) based on game theory that is provided and explained in the next subchapter.

4.5 GameOn's Mathematical Formulation

After the selection of two sales response functions, one based on the natural logarithm's functional form and the other based on the square root's functional form, and after the estimation of the relevant parameters, the conditions are gathered to develop the mathematical formulation of GameOn itself. First the model with the natural logarithm as functional form will be formulated and when set formulation is completed, the model based on the square root's functional form will be formulated. Again, every equation for each firm (A and B) will be presented – with the equations for firm A above the equations for firm B. Thus, for the sales response function:

$$S_A = (L_A)^{\beta_A}(L_B)^{\gamma_{AB}} \tag{8}$$

$$S_B = (L_B)^{\beta_B}(L_A)^{\gamma_{BA}} \tag{9}$$

What is presented above is the selected sales response functions from the previous subchapter, but without the natural logarithm. Now, let Π_A and Π_B denote firms A and B's profit functions respectively. Then,

$$\Pi_A(L_A, L_B) = k_A M_A S_A - L_A \tag{10}$$

$$\Pi_B(L_A, L_B) = k_B M_B S_B - L_B \tag{11}$$

where k_A and k_B are firm-specific factors that standardize the units of each profit function, and they are formulated and described in more detail ahead. M_A and M_B are called unit contribution, for firms A and B respectively, and they are calculated knowing each firm's cost structure. The unit contribution is no more than the profit per online contract of each firm. The data relative to the unit contribution of both firms was provided by the marketing manager of firm A, who has a deep knowledge, not only about his own firm but also about the market and the competitor. To know the unit contribution it is imperative to know (or have an estimation) of the cost structure, relative to the product that's the target of study, of the firm studied. In the unit contribution should not be included the investment in online lead generation, as it is clear by (10) and (11), however, due to the difficulty in gathering the necessary data relative to the investment in online leads by the rival firm (this difficulty generally exists) throughout most industries, since many firms don't know this data regarding their rival(s) the unit contribution presented is truly the profit per online contract sold including the online lead generation investment. As it will be seen in the next chapter, this has no effect on the results since this method is used in this work for both firms and since the profits calculated are actually percentage changes in profit relative to the actual profit of each firm, it is all standardized and no mathematic or logical concern should arise from the application of GameOn. The unit contribution is measured between 0 and 1; with 0 being no profit per online contract; and 1 being 100% profit per online contract.

The first step to determine the quantity of online leads to be generated in order to maximize the above functions is to substitute (8) and (9) into (10) and (11) respectively:

$$\Pi_A(L_A, L_B) = k_A M_A {L_A}^{\beta_A} {L_B}^{\gamma_{AB}} - L_A \quad (12)$$

$$\Pi_B(L_A, L_B) = k_B M_B {L_B}^{\beta_B} {L_A}^{\gamma_{BA}} - L_B \quad (13)$$

Finally, the quantity of online leads to be generated in order to maximize the profit functions is obtained by solving:

$$\frac{\partial \Pi_A}{\partial L_A} = 0 \Leftrightarrow k_A M_A \beta_A {L_A}^{\beta_A - 1} {L_B}^{\gamma_{AB}} - 1 = 0 \quad (14)$$

$$\frac{\partial \Pi_B}{\partial L_B} = 0 \Leftrightarrow k_B M_B \beta_B {L_B}^{\beta_B - 1} {L_A}^{\gamma_{BA}} - 1 = 0 \quad (15)$$

In order to guarantee that the reaction functions (explained ahead), are indeed reaction functions, meaning that those functions return the best option for a player given the other players' strategy – in order to guarantee the payoff maximization – and additionally, in order to guarantee that if the players' solutions intercept then that point is the Nash equilibrium, it is imperative that the 2nd order conditions are verified, and to perform this verification, the following derivatives are solved:

$$\frac{\partial^2 \Pi_A}{\partial {L_A}^2} = (\beta_A - 1) k_A M_A \beta_A {L_A}^{\beta_A - 2} {L_B}^{\gamma_{AB}} \quad (16)$$

$$\frac{\partial^2 \Pi_B}{\partial {L_B}^2} = (\beta_B - 1) k_B M_B \beta_B {L_B}^{\beta_B - 2} {L_A}^{\gamma_{BA}} \quad (17)$$

To guarantee all the aspects mentioned above, the signs of the derivatives above need to be negative, and since, $(\beta_A - 1)$ and $(\beta_B - 1)$ are always negative and the remaining elements of the derivatives are positive, both these derivatives will always be negative. This means that the 2nd order conditions are met and the study can proceed.

When solved, the equations (14) and (15) yield the following set of equations:

$$\begin{cases} k_A M_A \beta_A {L_B}^{\gamma_{AB}} = {L_A}^{1-\beta_A} & (18) \\ k_B M_B \beta_B {L_A}^{\gamma_{BA}} = {L_B}^{1-\beta_B} & (19) \end{cases}$$

When the above set of equations is solved, they yield the following reaction functions:

$$L_A(L_B) = (k_A M_A \beta_A L_B{}^{\gamma_{AB}})^{\frac{1}{1-\beta_A}} \tag{20}$$

$$L_B(L_A) = (k_B M_B \beta_B L_A{}^{\gamma_{BA}})^{\frac{1}{1-\beta_B}} \tag{21}$$

The idea behind reaction functions is to estimate what a firm should do given what its competitors(s) are doing. Using (20) as an example, the idea is that firm A, already has an estimation of how many online leads firm B will generate the following month thus, with this knowledge, it will insert that estimation in L_B and use its reaction function to determine how many online leads it should generate in order to maximize its profit.

To calculate k_A and k_B, (20) and (21) are solved for k_A and k_B, respectively, and:

$$k_A = \frac{\overline{L_A}^{(1-\beta_A)}\overline{L_B}^{(-\gamma_{AB})}}{\beta_A M_A} \tag{22}$$

$$k_B = \frac{\overline{L_B}^{(1-\beta_B)}\overline{L_A}^{(-\gamma_{BA})}}{\beta_B M_B} \tag{23}$$

where $\overline{L_A}$ and $\overline{L_B}$ are the arithmetic mean of the quantity of online generated leads by firm's A and B respectively, of the entire studied period (the arithmetic mean of the 16 observations for firm A and B).

Both factors, k_A and k_B, are calculated using the averages of the total online generated leads in the studied, or targeted, period. However, this is merely one of the many ways that a firm has to determine its own k factor. A firm can, for example, select a specific month, such as the one with the best conversion rate and determine the factor with that data. Or even by stating that its goal and their best strategy is when they generate three times more online leads than the rival, and determine the factor accordingly. This calculation is done only once, being k_A and k_B parameters of the model, meaning that after assigning the parameters, each one with a value, then that value is fixed and not changed in the following utilizations of the model.

Finally, to calculate the Nash equilibrium – at the equilibrium, no firm as an incentive to change the quantity of online leads to generate – the first order conditions are written as a system and a natural logarithm is applied to each (18) and (19) and the equations are rearranged:

$$\begin{cases} (1-\beta_A)\ln L_A - \gamma_{AB}\ln L_B = \ln k_A + \ln M_A + \ln \beta_A & (24) \\ (1-\beta_B)\ln L_B - \gamma_{BA}\ln L_A = \ln k_B + \ln M_B + \ln \beta_B & (25) \end{cases}$$

Let,

$A = \ln k_A + \ln M_A + \ln \beta_A \Leftrightarrow A = \ln(k_A M_A \beta_A)$

$B = \ln k_B + \ln M_B + \ln \beta_B \Leftrightarrow B = \ln(k_B M_B \beta_B)$

The system is solved by using Cramer's rule, thus:

$$\begin{bmatrix} 1-\beta_A & -\gamma_{AB} \\ -\gamma_{BA} & 1-\beta_B \end{bmatrix}\begin{bmatrix} \ln L_A \\ \ln L_B \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix} \Leftrightarrow \begin{bmatrix} \ln L_A \\ \ln L_B \end{bmatrix} = \frac{\begin{bmatrix} 1-\beta_B & \gamma_{AB} \\ \gamma_{BA} & 1-\beta_A \end{bmatrix}}{(1-\beta_A)(1-\beta_B) - \gamma_{AB}\gamma_{BA}}\begin{bmatrix} A \\ B \end{bmatrix}$$

$$\ln L_A = \frac{A(1-\beta_B) - B\gamma_{BA}}{(1-\beta_A)(1-\beta_B) - \gamma_{BA}\gamma_{AB}} \tag{26}$$

$$\ln L_B = \frac{-A\gamma_{AB} + B(1-\beta_A)}{(1-\beta_A)(1-\beta_B) - \gamma_{BA}\gamma_{AB}} \tag{27}$$

Finally, the Nash equilibrium is given by:

$$L_A = (k_A M_A \beta_A)^{\frac{1-\beta_B}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}}(k_B M_B \beta_B)^{\frac{\gamma_{AB}}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}} \tag{28}$$

$$L_B = (k_A M_A \beta_A)^{\frac{\gamma_{BA}}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}}(k_B M_B \beta_B)^{\frac{1-\beta_A}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}} \tag{29}$$

Additionally, if both firms are symmetric, i.e. $\beta_A = \beta_B = \beta, \gamma_{BA} = \gamma_{AB} = \gamma, M_A = M_B = M$, the Nash equilibrium is as follows:

$$L = (kM\beta)^{\frac{1}{(1-\beta)+\gamma}} \tag{30}$$

and then $L_A = L_B = L$.

The Nash equilibrium computed for GameOn assumes that both firms made their decisions simultaneously, perhaps not simultaneously in time but each firm didn't know what its rivals did, when selecting their strategy for the period in question. The Stackelberg equilibrium that is formulated and presented ahead, shows what would happen if a firm decides on how many online leads to generate before its competitor. In this situation, how many online leads should both firms generate? This is the question that the Stackelberg equilibrium answers. Additionally, the profits from both situations (Nash and Stackelberg equilibrium) can be compared, and interesting conclusions can arise. There are two situations to be described and formulated: 1) Firm B moves first and firm A follows; and 2) Firm A moves first and firm B follows. The first scenario might be considered more intuitive because since firm B has a larger market share and more financial power when compared to firm A, and thus it should have a bigger incentive to make its decision first, and for firm A, this situation also makes sense because, since the quantity of online generated leads from firm B has a positive influence on the sales of online contracts of firm A, this firm might have an incentive itself to wait for a decision from firm B and then make its decision afterwards.

1) For the first scenario, firm A's reaction function (see (20)) is rewritten as:

$$L_A = (k_A M_A \beta_A)^{\frac{1}{1-\beta_A}} L_B^{\frac{\gamma_{AB}}{1-\beta_A}} \tag{31}$$

Now, the quantity of online leads generated by firm A in firm B's sales function (see (9)) is substituted by firm A's rewritten reaction function (31):

$$S_B = (L_B)^{\beta_B} (k_A M_A \beta_A)^{\frac{\gamma_{BA}}{1-\beta_A}} L_B^{\frac{\gamma_{BA}\gamma_{AB}}{1-\beta_A}} \tag{32}$$

The sales function from firm B in its profit function (11) is substituted by (32). The resulting equation is rewritten and it yields the following relationship:

$$\Pi_B(L_A, L_B) = \ k_B M_B (L_B)^{\beta_B} (k_A M_A \beta_A)^{\frac{\gamma_{BA}}{1-\beta_A}} L_B^{\frac{\gamma_{BA}\gamma_{AB}}{1-\beta_A}} - \ L_B$$

$$\Leftrightarrow \Pi_B(L_A, L_B) = \ k_B M_B (L_B)^{\beta_B + \frac{\gamma_{BA}\gamma_{AB}}{1-\beta_A}} (k_A M_A \beta_A)^{\frac{\gamma_{BA}}{1-\beta_A}} - L_B \tag{33}$$

Firm B's quantity of online leads to be generated in order to maximize its profit function is obtained by solving:

$$\frac{\partial \Pi_B}{\partial L_B} = 0 \Leftrightarrow \left(\beta_B + \frac{\gamma_{BA}\gamma_{AB}}{1-\beta_A}\right) k_B M_B (k_A M_A \beta_A)^{\frac{\gamma_{BA}}{1-\beta_A}} L_B^{\beta_B + \frac{\gamma_{BA}\gamma_{AB}}{1-\beta_A} - 1} = 1 \tag{34}$$

Thus, the Stackelberg equilibrium, for firm B is given by:

$$L_B = \left[\left(\beta_B + \frac{\gamma_{BA}\gamma_{AB}}{1-\beta_A}\right) k_B M_B (k_A M_A \beta_A)^{\frac{\gamma_{BA}}{1-\beta_A}}\right]^{\frac{1-\beta_A}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}} \tag{35}$$

The Stackelberg equilibrium for firm A is given by substituting (35) into firm A's reaction function (31). Thus, the rewritten Stackelberg equilibrium for firm A is given by:

$$L_A = (k_A M_A \beta_A)^{\frac{1}{1-\beta_A}} \left[\left(\beta_B + \frac{\gamma_{BA}\gamma_{AB}}{1-\beta_A}\right) k_B M_B (k_A M_A \beta_A)^{\frac{\gamma_{BA}}{1-\beta_A}}\right]^{\frac{\gamma_{AB}}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}} \tag{36}$$

2) The second scenario is analogous. So, firm B's reaction function (see (21)) is

rewritten as:

$$L_B = (k_B M_B \beta_B)^{\frac{1}{1-\beta_B}} L_A^{\frac{\gamma_{BA}}{1-\beta_B}} \tag{37}$$

Now, the quantity of online leads generated by firm B in firm A's sales function (see (8)) is substituted by firm B's rewritten reaction function (37):

$$S_A = (L_A)^{\beta_A} (k_B M_B \beta_B)^{\frac{\gamma_{AB}}{1-\beta_B}} L_A^{\frac{\gamma_{AB}\gamma_{BA}}{1-\beta_B}} \tag{38}$$

The sales function from firm A in its profit function (10) is substituted by (38). The resulting equation is rewritten and it yields the following relationship:

$$\Pi_A(L_A, L_B) = k_A M_A (L_A)^{\beta_A} (k_B M_B \beta_B)^{\frac{\gamma_{AB}}{1-\beta_B}} L_A^{\frac{\gamma_{AB}\gamma_{BA}}{1-\beta_B}} - L_A$$

$$\Leftrightarrow \Pi_A(L_A, L_B) = k_A M_A (L_A)^{\beta_A + \frac{\gamma_{AB}\gamma_{BA}}{1-\beta_B}} (k_B M_B \beta_B)^{\frac{\gamma_{AB}}{1-\beta_B}} - L_A \tag{39}$$

Firm A's quantity of online leads to be generated in order to maximize its profit function is obtained by solving:

$$\frac{\partial \Pi_A}{\partial L_A} = 0 \Leftrightarrow \left(\beta_A + \frac{\gamma_{AB}\gamma_{BA}}{1-\beta_B}\right) k_A M_A (k_B M_B \beta_B)^{\frac{\gamma_{AB}}{1-\beta_B}} L_A^{\beta_A + \frac{\gamma_{AB}\gamma_{BA}}{1-\beta_B} - 1} = 1 \tag{40}$$

Thus, the Stackelberg equilibrium, for firm A is given by:

$$L_A = \left[\left(\beta_A + \frac{\gamma_{AB}\gamma_{BA}}{1-\beta_B}\right) k_A M_A (k_B M_B \beta_B)^{\frac{\gamma_{AB}}{1-\beta_B}}\right]^{\frac{1-\beta_B}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}} \tag{41}$$

The Stackelberg equilibrium for firm B is given by substituting (41) into firm B's reaction function (37). Thus, the rewritten Stackelberg equilibrium for firm B is given by:

$$L_B = (k_B M_B \beta_B)^{\frac{1}{1-\beta_B}} \left[\left(\beta_A + \frac{\gamma_{AB}\gamma_{BA}}{1-\beta_B}\right) k_A M_A (k_B M_B \beta_B)^{\frac{\gamma_{AB}}{1-\beta_B}}\right]^{\frac{\gamma_{BA}}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}} \tag{42}$$

Table 4.23 presents a summary of the most important equations for the natural logarithm functional form of GameOn, which will be used in the next chapter when the application scenario and case study are discussed.

Table 4.23 – Summary of the Functions for the Model's "ln" Functional Form

Equation	Firm A	Firm B
Sales	$S_A = (L_A)^{\beta_A}(L_B)^{\gamma_{AB}}$	$S_B = (L_B)^{\beta_B}(L_A)^{\gamma_{BA}}$
Profit	$\Pi_A(L_A, L_B) = k_A M_A S_A - L_A$	$\Pi_B(L_A, L_B) = k_B M_B S_B - L_B$
Reaction Function	$L_A(L_B) = (k_A M_A \beta_A L_B{}^{\gamma_{AB}})^{\frac{1}{1-\beta_A}}$	$L_B(L_A) = (k_B M_B \beta_B L_A{}^{\gamma_{BA}})^{\frac{1}{1-\beta_B}}$
Nash Equilibrium	$L_A = (k_A M_A \beta_A)^{\frac{1-\beta_B}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}} (k_B M_B \beta_B)^{\frac{\gamma_{AB}}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}}$	$L_B = (k_A M_A \beta_A)^{\frac{\gamma_{BA}}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}} (k_B M_B \beta_B)^{\frac{1-\beta_A}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}}$
Stackelberg Equilibrium 1)	$L_A = (k_A M_A \beta_A)^{\frac{1}{1-\beta_A}} \left[\left(\beta_B + \frac{\gamma_{BA}\gamma_{AB}}{1-\beta_A}\right) k_B M_B (k_A M_A \beta_A)^{\frac{\gamma_{BA}}{1-\beta_A}}\right]^{\frac{\gamma_{AB}}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}}$	$L_B = \left[\left(\beta_B + \frac{\gamma_{BA}\gamma_{AB}}{1-\beta_A}\right) k_B M_B (k_A M_A \beta_A)^{\frac{\gamma_{BA}}{1-\beta_A}}\right]^{\frac{1-\beta_A}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}}$
Stackelberg Equilibrium 2)	$L_A = \left[\left(\beta_A + \frac{\gamma_{AB}\gamma_{BA}}{1-\beta_B}\right) k_A M_A (k_B M_B \beta_B)^{\frac{\gamma_{AB}}{1-\beta_B}}\right]^{\frac{1-\beta_B}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}}$	$L_B = (k_B M_B \beta_B)^{\frac{1}{1-\beta_B}} \left[\left(\beta_A + \frac{\gamma_{AB}\gamma_{BA}}{1-\beta_B}\right) k_A M_A (k_B M_B \beta_B)^{\frac{\gamma_{AB}}{1-\beta_B}}\right]^{\frac{\gamma_{BA}}{(1-\beta_A)(1-\beta_B)-\gamma_{BA}\gamma_{AB}}}$

In an analogous way it is provided the mathematical formulation for GameOn based on the square root functional form. Since the formulation is similar to the one already presented (for the natural logarithm functional form) and the explanations for each step are also identical, it will be only provided deeper explanations when the formulation process differs from what was presented previously. Once more, every equation for each firm (A and B) will be presented, with the equations for firm A above the equations for firm B. Thus, for the sales response function:

$$S_A = \left(\beta_A\sqrt{L_A} + \gamma_{AB}\sqrt{L_B}\right)^2 \tag{43}$$

$$S_B = \left(\beta_B\sqrt{L_B} + \gamma_{BA}\sqrt{L_A}\right)^2 \tag{44}$$

For the profits functions of firms A and B, see (10) and (11) respectively. And to determine quantity of online leads to be generated in order to maximize each profit function, (43) and (44) are substituted into (10) and (11) respectively:

$$\Pi_A = k_A M_A (\beta_A\sqrt{L_A} + \gamma_{AB}\sqrt{L_B})^2 - L_A \tag{45}$$

$$\Pi_B = k_B M_B (\beta_B\sqrt{L_B} + \gamma_{BA}\sqrt{L_A})^2 - L_B \tag{46}$$

The quantity of online leads to be generated in order to maximize the profit functions is obtained by solving:

$$\frac{\partial \Pi_A}{\partial L_A} = 0 \iff k_A M_A\left(\beta_A\sqrt{L_A} + \gamma_{AB}\sqrt{L_B}\right)\frac{\beta_A}{\sqrt{L_A}} = 1 \tag{47}$$

$$\frac{\partial \Pi_B}{\partial L_B} = 0 \iff k_B M_B\left(\beta_B\sqrt{L_B} + \gamma_{BA}\sqrt{L_A}\right)\frac{\beta_B}{\sqrt{L_B}} = 1 \tag{48}$$

To guarantee the 2nd order conditions, the following derivatives need to be negative:

$$\frac{\partial^2 \Pi_A}{\partial {L_A}^2} = -\frac{k_A M_A \beta_A \gamma_{AB}\sqrt{L_B}}{2{L_A}^{3/2}} \tag{49}$$

$$\frac{\partial^2 \Pi_B}{\partial {L_B}^2} = -\frac{k_B M_B \beta_B \gamma_{BA}\sqrt{L_A}}{2{L_B}^{3/2}} \tag{50}$$

The 2nd order conditions are only guaranteed if γ_{AB} and γ_{BA} are positive (or if only one of those parameters is positive than the reaction function relative to that parameter, and only that, does maximize the firm's profit). Although in the present study both parameters are indeed positive (see Table 4.22 for both M2 cases), it is not guaranteed that they will be positive for every case, even so due to what the literature states regarding this subject by pointing out that the intuitive result for this situation is that the values of both parameters should be negative. So, when using the present model with the square root as the functional form, one needs to be careful regarding this subject.

In the cases where the 2nd order conditions are met (because when they are not met, then there are no reaction functions), then the reaction functions for firms A and B are determined by solving (47) and (48) respectively, and:

$$L_A(L_B) = \left(\frac{k_A M_A \beta_A \gamma_{AB} \sqrt{L_B}}{1 - k_A M_A {\beta_A}^2}\right)^2 \tag{51}$$

$$L_B(L_A) = \left(\frac{k_B M_B \beta_B \gamma_{BA} \sqrt{L_A}}{1 - k_B M_B {\beta_B}^2}\right)^2 \tag{52}$$

To calculate k_A and k_B, (51) and (52) are solved for k_A and k_B, respectively, and:

$$k_A = \frac{\sqrt{\overline{L_A}}}{\beta_A M_A \left(\beta_A \sqrt{\overline{L_A}} + \gamma_{AB} \sqrt{\overline{L_B}}\right)} \tag{53}$$

$$k_B = \frac{\sqrt{\overline{L_B}}}{\beta_B M_B \left(\beta_B \sqrt{\overline{L_B}} + \gamma_{BA} \sqrt{\overline{L_A}}\right)} \tag{54}$$

Given that the square root is a function that passes through the origin when drawn on an x-y axis, when the firm-specific intercept term (in this case both α_A and α_B) is eliminated from the function, then what happens for Nash equilibrium is that the only solution for the intersection of the reaction functions of both (or more) firms is the origin itself (0,0). This solution has no interest for purposes of this study. Thus, the Nash equilibrium, as well as the Stackelberg equilibrium will not be formulated nor calculated.

Table 4.24 presents a summary of the most important equations for the square root functional form of GameOn, which will be used in the next chapter when the application scenarios are discussed.

Table 4.24 – Summary of the Functions for the Model's "Square Root" Functional Form

Equation	Firm A	Firm B
Sales	$S_A = (\beta_A\sqrt{L_A} + \gamma_{AB}\sqrt{L_B})^2$	$S_B = (\beta_B\sqrt{L_B} + \gamma_{BA}\sqrt{L_A})^2$
Profit	$\Pi_A(L_A, L_B) = k_A M_A S_A - L_A$	$\Pi_A(L_A, L_B) = k_B M_B S_B - L_B$
Reaction Function	$L_A(L_B) = \left(\frac{k_A M_A \beta_A \gamma_{AB}\sqrt{L_B}}{1 - k_A M_A {\beta_A}^2}\right)^2$	$L_B(L_A) = \left(\frac{k_B M_B \beta_B \gamma_{BA}\sqrt{L_A}}{1 - k_B M_B {\beta_B}^2}\right)^2$
Nash Equilibrium	-	-
Stackelberg Equilibrium 1)	-	-
Stackelberg Equilibrium 2)	-	-

4.6 GameOn Generalization

As it was previously stated, GameOn is richer in scope than the above presented mathematical formulation implies, given that it was developed for a duopoly due to the limitations already addressed. In this subchapter it is shown the generalization of GameOn demonstrating its richness and potential. With the following mathematical formulation, the model can be used by any firm in any industry with no restrictions to the number of competitors. For this generalization the same parameters are used as the previous formulation plus the addition of a firm-specific intercept term and without the quantity of offline generated leads by any firm. Firstly, the model with the natural logarithm as the functional form is formulated and more ahead the model with the square root as the functional form is formulated.

Assuming that there are N firms in the market, indexed $i = 1, 2, \dots, N$, then for firm i, the sales response function is formulated as follows:

$$S_i = \alpha_i (L_i)^{\beta_i} \prod_{\substack{j=1 \\ j \neq i}}^{N} (L_j)^{\gamma_{ij}}, \qquad i = 1, 2, \dots, N \tag{55}$$

where S_i are the sales (online contracts) of firm i in a given period of time, α_i is a firm-specific intercept term, L_i and L_j represent the quantity of online leads generated by firms i and j respectively, a given period of time, β_i and γ_{ij} are the parameters that measure the sensitivity of firm i's sales to its own quantity of online generated leads and firm j's leads respectively, and finally, ln denotes the natural logarithm.

Now, let Π_i denote firm i's profit function. Then,

$$\Pi_i(L_1, \cdots, L_i, \cdots, L_N) = k_i M_i S_i - L_i, \qquad i = 1, 2, \dots, N \tag{56}$$

where k_i is a firm-specific factor that standardizes the units of the profit function. M_i is the unit contribution, for firm i. The quantity of online generated leads that maximizes the above function is obtained by solving:

$$\frac{\partial \Pi_i}{\partial L_i} = 0, \qquad i = 1, 2, \dots, N \tag{57}$$

To verify the existence of the reaction function, the 2nd order conditions must be verified, i.e. the resulting derivatives must be negative, because if not, then there are no reaction functions and GameOn cannot be used. If the conditions are verified, then when (57) is solved, it yields the following reaction function:

$$L_i(L_j) = exp\left(\frac{-\ln(\alpha_i k_i M_i \beta_i) - \sum_{\substack{j=1 \\ j \neq i}}^{N} \gamma_{ij} \ln L_j}{\beta_i - 1}\right), \qquad i = 1, 2, \dots, N \tag{58}$$

The factor k_i is calculated by solving the above function for k_i, yielding the following relationship:

$$k_i = \frac{\overline{L_i}^{1-\beta_i} \prod_{\substack{j=1 \\ j \neq i}}^{N} \overline{L_j}^{-\gamma_{ij}}}{\beta_i M_i}, \qquad i = 1, 2, \dots, N \tag{59}$$

where $\overline{L_i}$ and $\overline{L_j}$ are the arithmetic average for the entire studied period for the online generated leads of firms i and j, respectively. It was already mentioned earlier that this factor can be determined without recurring to the average number of online generated leads, but also with any other method or rate – or proportion – that each firm intends to be best.

The Nash equilibrium is determined by solving the following system:

$$L = Q^{-1} P \tag{60}$$

where, $\begin{bmatrix} L_1 \\ \vdots \\ L_j \\ \vdots \\ L_N \end{bmatrix}$, $Q = [Q_{ij}]_{i,j=1,\dots,N}$ where $Q_{ij} = \begin{cases} \beta_i - 1, & if\ i = j \\ \gamma_{ij}, & if\ i \neq j \end{cases}$

$$P = \begin{bmatrix} -\ln(\alpha_1 k_1 M_1 \beta_1) \\ \vdots \\ -\ln(\alpha_j k_j M_j \beta_j) \\ \vdots \\ -\ln(\alpha_N k_N M_N \beta_N) \end{bmatrix}$$

The Stackelberg equilibrium is not generalized because it doesn't make sense to perform this formulation given that this equilibrium offers a solution when there are two firms and one of them selects its strategy before the other makes its selection. When the market has more than two firms, some assumptions need to be made, for example, when three firms compete in the same market, to compute the Stackelberg equilibrium, an order of strategy selection needs to be made, where one assumption can be that firm A and firm B select their strategy first and firm C selects its own secondly (and such as this, several other assumptions can be inferred). For the purposes of the work here presented, this oligopoly situation irrelevant for the Stackelberg equilibrium. When in a duopoly situation, it is recommended an analysis of (35), (36), (41) and (42), and the process to determined set equations.

Below, GameOn is generalized for the square root functional form. All the parameters and variables have the same meaning as the previous case (with the natural logarithm as functional form), and thus only the equations are presented (without long and repeated explanations). For firm i, the sales response function is formulated as follows:

$$S_i = \left(\alpha_i + \beta_i \sqrt{L_i} + \sum_{\substack{j=1 \\ j \neq i}}^{N} \gamma_{ij} \sqrt{L_j} \right)^2, \qquad i = 1, 2, \dots, N \tag{61}$$

Now, let Π_i denote firm i's profit function. Then,

$$\Pi_i(L_i, \cdots, L_j, \cdots, L_N) = k_i M_i S_i - L_i, \qquad i = 1, 2, \dots, N \tag{62}$$

The quantity of online generated leads that maximizes the above function is obtained by solving:

$$\frac{\partial \Pi_i}{\partial L_i} = 0, \qquad i = 1, 2, \dots, N \tag{63}$$

If the 2nd order conditions are verified, then when (63) is solved, it yields the following reaction function:

$$L_i(L_j) = \left(\frac{\alpha_i k_i M_i \beta_i + \sum_{\substack{j=1 \\ j \neq i}}^{N} k_i M_i \beta_i \gamma_{ij} \sqrt{L_j}}{1 - k_i M_i (\beta_i)^2} \right)^2 , \qquad i = 1, 2, \dots, N \tag{64}$$

The factor k_i is calculated by solving the above function for k_i, yielding the following relationship:

$$k_i = \frac{\sqrt{L_i}}{\beta_i M_i \left(\alpha_i + \beta_i \sqrt{L_i} + \sum_{\substack{j=1 \\ j \neq i}}^{N} \gamma_{ij} \sqrt{L_j} \right)}, \qquad i = 1, 2, \dots, N \tag{65}$$

The Nash equilibrium is determined by solving the following system:

$$L = Q^{-1} P \tag{66}$$

where, $\begin{bmatrix} L_1 \\ \vdots \\ L_j \\ \vdots \\ L_N \end{bmatrix}$, $Q = \left[Q_{ij} \right]_{i,j=1,\dots,N}$ where $Q_{ij} = \begin{cases} k_i M_i (\beta_i)^2 - 1, & if\ i = j \\ k_i M_i \beta_i \gamma_{ij}, & if\ i \neq j \end{cases}$

$$P = \begin{bmatrix} -\alpha_1 k_1 M_1 \beta_1 \\ \vdots \\ -\alpha_j k_j M_j \beta_j \\ \vdots \\ -\alpha_N k_N M_N \beta_N \end{bmatrix}$$

The Stackelberg equilibrium is not generalized once again for the same reason as previously mentioned. For the analysis of a duopolistic market, it is recommended an analysis of (35), (36), (41) and (42), and the process to determined set equations (although the equations mentioned are taken from the natural logarithm functional form's formulation, the process is analogous for the square root functional form).

4.7 Summary

In this chapter GameOn was introduced as a management model that is a game-theoretic approach to online lead generation and the goal is to estimate the quantity of online generated leads and the gained online contracts based on the competitors' strategy – the quantity of online leads the rival(s) generate in a certain period of time. Two sales response functions were selected, one with the natural logarithm as the functional form and the other with the square root as the functional form, as well as the significant parameters for each sales response function. GameOn was then mathematically formulated in much detail for the situation portrayed in the present dissertation, which is a duopolistic market situation. And finally the generalization of

GameOn, with both functional forms, was provided and explained, making it possible to apply GameOn to every firm and market situation possible, except for monopolistic markets. In the next chapter, an application scenario and a case study are provided and their results discussed.

Chapter 5. Application Scenario and Case Study - GameOn

In this chapter, some initial GameOn considerations are made and initial results are shown and discussed. An application scenario and a case study are evaluated, each one with its own objectives to demonstrate the potential, utility and validity of GameOn.

5.1 Methodology

This chapter encompasses an application scenario, a case study, and sets up the application of GameOn to an even a larger number of scenarios. Initially it is presented and discussed a set of interesting results. To make the analysis clearer for the reader, and since GameOn can be used with two different functional forms as it was concluded in the previous chapter, special notation will be used to differentiate more intuitively which functional form is being analyzed and for the comparison of the results from both functional forms: 1) For the application of GameOn with the natural logarithm functional form, GameOn_L will be used; and 2) For the application of GameOn with the square root as functional form, GameOn_SR will be used. The model is, conceptually the same – GameOn – only the functional form and the value of the parameters change (as discussed in chapter 4).

GameOn was tested in a duopoly market, where firm A and B interact and dominate. Firm A showed to be extremely interested in this study and in GameOn since it believes it can be a very useful tool that can help them make more effective and efficient decisions. As most markets, they are competing in one that requires a shift towards the digital world and its marketing department needs to make more and more decisions relative to this digital feature and it struggles, most of the time, to reach solid decisions that don't compromise their (and the firm's) budget while gaining awareness and contracts from consumers.

With the available data, it was possible to determine several results, from the estimation of the quantity of online leads to generate for the targeted period, to the Nash and Stackelberg equilibriums, while determining the percentage change in multiple situations. The results were then compared (comparing also GameOn_L with GameOn_SR) and discussed.

5.2 GameOn Data

The data was supplied by the marketing manager of firm A and by experts in market intelligence from one other firm that is not related to the present study. The data used for the GameOn_L and GameOn_SR's inputs are presented in Tables 4.1 and 4.22 from the previous chapter. Table 4.1 represents the data gathered from firm's A marketing manager and from the experts in market intelligent while Table 4.22 represents the results from the empirical study discussed in chapter 4 that provides the values for the parameters that will be used in this chapter. Table 5.1 below provides the same data as Table 4.22 but with the addition of the factors k_A, k_B, M_A and M_B for a more detailed view. Thus, Table 5.1 has the data for all the calculated factors and parameters that will be used in this chapter. In Tables 4.23 and 4.24 are present the functions used to compute the relevant results for the application scenario and case study for GameOn_L and GameOn_SR respectively.

Table 5.1 – Summary of the Parameters' Estimations and of the Factors' Calculations

Parameter	Firm A		Firm B	
	GameOn_L	GameOn_SR	GameOn_L	GameOn_SR
β_A	0,449718837	0,268227751	-	-
β_B	-	-	0,52291836	0,198374213
γ_{AB}	0,247627162	0,086898205	-	-
γ_{BA}	-	-	0,22732056	0,23455663
k_A	52,86878	40,90941	-	-
k_B	-	-	197,4099	164,1375
M_A	0,2	0,2	-	-
M_B	-	-	0,1	0,1

From Table 5.1 it's relevant to state that, for GameOn_L the equations used for the calculations of k_A and k_B are (22) and (23) respectively and for GameOn_SR the same factors are calculated recurring to (53) and (54). M_A and M_B are firm-specific factors, where for firm A it means that the profit per online contract is 20% and for firm B the profit is 10%.

Finally, for each of the following subchapters and when there are calculations involved, a brief explanation of the calculation is provided.

5.3 GameOn Initial Results

Before delving into the application scenario and case study, it is shown and discussed some interesting initial results. The aim of these results is to observe the evolution of the online generated leads of both firms, compared to each other in both situations, GameOn_L and GameOn_SR. Firstly, Figure 5.1 presents the evolution of the real data, i.e. the real quantity of online generated leads as shown in Table 4.1, with the data from firm B rearranged from the least quantity (monthly) to the highest, where each number represents a certain period (month). These values are then matched with the values from firm A.

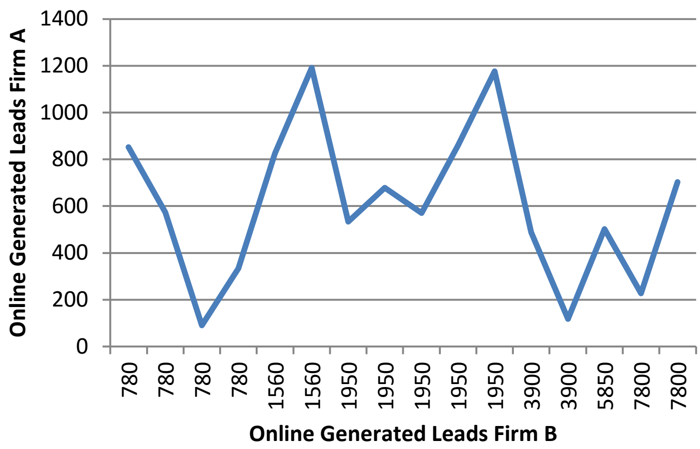

Figure 5.1 – Online Generated Leads Firm A vs Online Generated Leads Firm B

From Figure 5.1 it can be easily observed that there is a great variability among both firms when it comes to the quantity of online leads generated for the same period. The firms appear to not follow in any way what the rival is doing; they do not appear to take into consideration the quantity of online leads the rival firm generates. Even when firm B generates the same amount of online leads in different period, firm A never really stabilizes. It can be seen that firm B generates 780 online leads in four separate months, and the variability of online leads generated by firm A in those months is great, it ranges from 852 online generated leads to 90 online generated leads. And with the increase of the online generated leads by firm B, firm A does not have a clear tendency. It is, however, somewhat stable when firm B generates 1950 online leads, except for the last value where the difference spikes. One can make the argument that, only based on the observation of Figure 5.1, firm A does not take into considerations the strategy of firm B.

Figure 5.2 presents the first results of the application of GameOn. The aim is to show the evolution of the quantity of online generated leads of firm A when the online leads of firm B vary from 500 to 8000, in increments of 500 units. Both variances of GameOn are present in the figure: GameOn_L and GameOn_SR. For the calculations of the results presented in this subchapter for firm A, it was used (20) and (21) for GameOn_L and (51) and (52) for GameOn_SR.

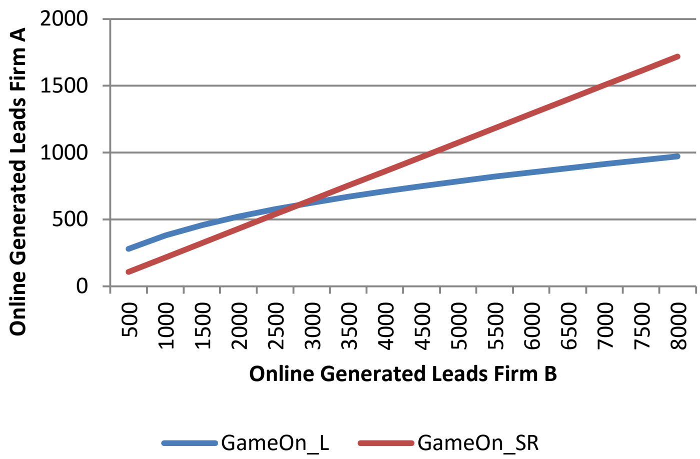

Figure 5.2 – Online Generated Leads Firm A vs Online Generated Leads Firm B for GameOn_L and GameOn_SR

A multitude of interesting facts arise from the observation of Figure 5.2. First of all, for both GameOn_L and GameOn_SR, a clear tendency can be observed as the quantity of online generated leads of firm B is increased. This figure shows a market where both firms clearly take into account the strategy employed by their respective rival and act accordingly. For GameOn_L, effect of the natural logarithm is very evident where it can be argued that when firm B has generated up to around 3500 online, then firm A needs to increase at a higher pace its quantity of online generated leads, whereas after the mark of 3500 online generated leads from firm B, firm A has little to gain by increasing at the same rhythm as the previous case, reaching a maximum value of, approximately, 971 online generated leads when firm B generates 8000 online leads. This value of online leads from firm A is actually less than the two highest actual values: 1176 and 1191, from Mar 15 an Apr 15 respectively. Thus, according to GameOn_L, firm A should never have generated such quantities of online leads (due to the fact that the highest actual values for firm B are 7800 online generated leads in Jun 14 and Jul 14).

For GameOn_SR a different analysis is conducted due to the distinct behavior of the evolution of the estimated quantity of online generated leads of GameOn_L and GameOn_SR portrayed in Figure 5.2. As it can be seen in that figure, the evolution of the estimated quantity of online generated leads for GameOn_SR has a linear tendency with the increasing quantity of online generated leads of firm B. It starts at a lower quantity than GameOn_L and then both models intercept each other at, approximately at the 608 online leads of firm A and 2827 online leads of firm B. The explanation for this intersection is the factor k (k_A and k_B). Both factors k_A and k_B were calculated based on the average quantity of online generated leads by both firm A and B, and their respective average is, approximately, 608 and 2827 online leads. After this intersection, the estimated quantity of online generated leads by firm A continues its linear tendency, increasing at a much faster pace when compared to the pace of GameOn_L. It is relevant to point out that there is a budget to be respected and it would be unsustainable for firm A to continually generate a very high quantity of online leads such as GameOn_SR might suggest, for example each time that firm B generates more than 4653 online leads, firm A would have to generate more than 1000 online leads, which after some time would be unsustainable.

Figure 5.3 presents the evolution of the real data, i.e. the real quantity of online generated leads as shown in Table 4.1, with the data from firm A rearranged from the least quantity (monthly) to the highest, where each number represents a certain period (month). These values are then matched with the values from firm B.

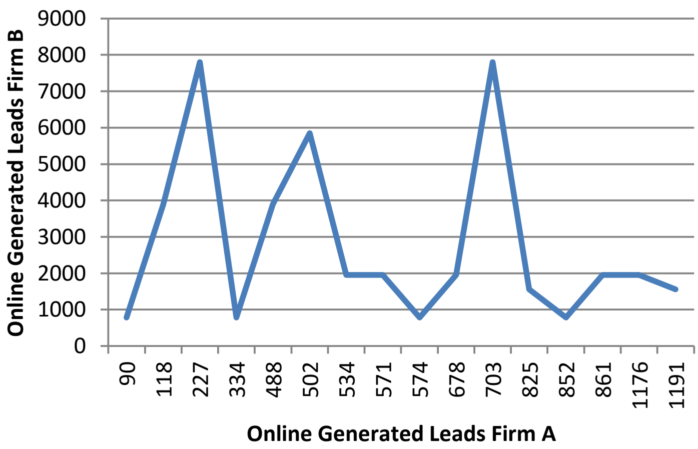

Figure 5.3 – Online Generated Leads Firm B vs Online Generated Leads Firm A

Figure 5.3 also shows a significant variability among both firms when it comes to the quantity of online leads generated for the same period. There are three great spikes throughout the 16 months. It is once again observed that, when the quantity of online generated leads increases, this time by firm A, there is no clear pattern for the evolution of firm B's online generated leads. It is fair to conclude then, that firm B also selects its strategy disregarding firm A's strategy. Giving that firm A also acted this way, the decisions taken by both firms might, and probably are, a source of ineffectiveness and inefficiency where both firms could be performing much better, at higher levels of efficiency.

Figure 5.4 shows the evolution of the quantity of the estimated quantity of online generated leads of firm B when the online leads of firm A vary from 75 to 1200, in increments of 75 units. Both variances of GameOn are present in the figure: GameOn_L and GameOn_SR.

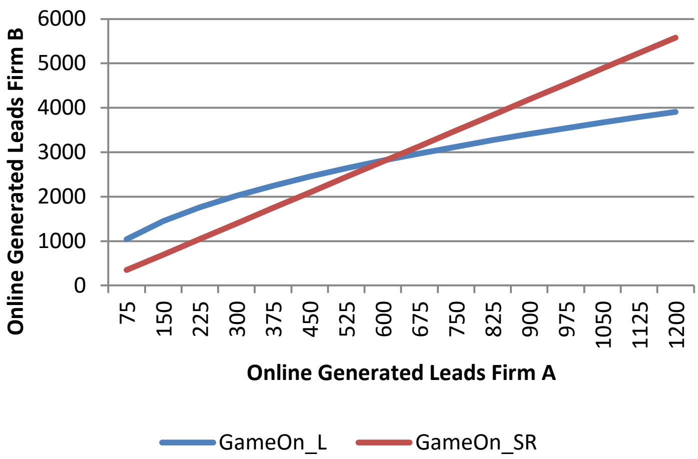

Figure 5.4 – Online Generated Leads Firm B vs Online Generated Leads Firm A for GameOn_L and GameOn_SR

The analysis of Figure 5.4 is in all aspects identical to the analysis made for Figure 5.2. There are, however, two interesting points worth mentioning and discussing. In both situations, GameOn_L and GameOn_SR, the maximum estimated quantity of online generated leads of firm B does not equal nor surpass the actual maximum value of 7800 online leads (in Jun 14 and Jul 14). For GameOn_SR, the maximum estimated quantity of online generated leads is, approximately 5583 and this number is reached when firm A generates 1200 online leads which actually never happens, given that the maximum for firm A is 1191 online leads in Apr 15. The number 5583 is actually inferior to another quantity of online generated leads by firm B, 5850 in May 14. For GameOn_L, the maximum estimated quantity of online generated leads is even less when compared to the previous case, only reaching, approximately 3910 online leads. When observing the range of values for firm B, it is also interesting to see that the range is much narrower than the actual range of values, where for GameOn_L the maximum and minimum quantities of online generated leads are 3910 and 1043, respectively, for GameOn_SR – although the range is wider when comparing to GameOn_L, is still narrower when compared to the range of the actual values – the maximum and minimum quantities of online generated leads are 5583 and 349, respectively, whereas in the actual situation, the maximum and minimum quantities of online generated leads are 7800 and 780, respectively.

5.4 GameOn Application Scenario

5.4.1 GameOn Application Scenario Contextualization

The application scenario, hereinafter "AS1", actually serves two purposes:

1) It's a method that firms can use when they want to estimate the online contracts they can, potentially, sell in a given period of time, knowing (or estimating) a quantity of

online leads to be generated by the rival(s) and their own quantity of online leads to be generated. Through this use of GameOn it is possible to perform what-if analysis to better understand the implications of some decisions and to arrive at a more efficient and effective strategy;

2) It serves as a preliminary validation because AS1 uses the data relative to a firm's own online leads and the competitor's online leads to estimate the contracts that a firm would gain in that month and then it crosses with the actual data of online sold contracts by each firm and calculates the error of the estimations.

AS1 requires the quantity of online leads to be generated by the own firm and the quantity (or estimation) of the online leads to be generated by the rival firm (in the duopoly case that this chapter relates to: firms A and B) that are presented in Table 4.1, and it calculates the estimated online sold contracts using the data from Table 5.1 and (8) and (9) (for firm A and B respectively) for GameOn_L, and (43) and (44) (for firm A and B respectively) for GameOn_SR.

The error percentage is calculated as follows:

$$Error\ (\%) = \frac{|Estimated\ Value - Exact\ Value|}{Exact\ Value} . 100 \qquad (66)$$

5.4.2 GameOn Application Scenario Results and Discussion

For AS1 the goal is to test both GameOn_L and GameOn_SR in order to verify their forecast precision. Table 5.2 presents the application of GameOn to estimate the contracts sold by firm A since the beginning to the end of the studied period, from Jan 14 to Apr 15. These monthly estimations are then compared to the actual values of the contracts sold seen in Table 5.2 in the column marked as "Online Contracts_A" with the computation of the percentage error.

According to the estimations in Table 5.2, firm A would sell 130 less online contracts with the application of GameOn_L and 73 less online contracts with the application of GameOn_SR when compared to the actual values. This difference can be explained by the fact that some of the data used in the parameters' estimations is not completely exact, the fact that 16 is not a high number of observations that can inspire a model with absolute confidence, although that is one of the reasons why the bootstrap technique was used. Still it is important to state that the more observations one can gather, the better and more trustworthy the model will be, and also the fact that there is no clear pattern when observing the evolution of the monthly quantity of online leads that each firm generates and the quantity of monthly online contracts sold.

Table 5.2 – Firm A's Estimated Online Contracts: GameOn_L and GameOn_SR

Month	Online Contracts_A	GameOn_L		GameOn_SR	
		Estimated Online Contracts_A	Error (%)	Estimated Online Contracts_A	Error (%)
Jan 14	98	108	10,2	105	7,1
Feb 14	110	110	0,0	101	8,2
Mar 14	116	122	5,2	117	0,9
Apr 14	114	127	11,4	124	8,8
May 14	111	140	26,1	160	44,1
Jun 14	198	106	46,5	137	30,8
Jul 14	275	175	36,4	219	20,4
Aug 14	210	91	56,7	78	62,9
Sep 14	127	113	11,0	105	17,3
Oct 14	88	125	42,0	129	46,6
Nov 14	39	66	69,2	70	79,5
Dec 14	34	39	14,7	25	26,5
Jan 15	88	71	19,3	54	38,6
Feb 15	133	136	2,3	137	3,0
Mar 15	144	157	9,0	170	18,1
Apr 15	80	149	86,3	161	101,3
Total	**1965**	**1835**	**27,9**	**1892**	**32,1**

Since the average monthly error throughout the entire 16 months related to GameOn_L is inferior to the average monthly error related to GameOn_SR (27,9% and 32,1% respectively), it is normal that in some months, the error of the estimations from GameOn_SR is higher than the estimations from GameOn_L and for the case of GameOn_SR there are more months with elevated percentage errors, thus contributing for the higher average monthly error of GameOn_SR.

The months of Aug 14, Oct 14, Nov 14, and especially Apr 15, for both GameOn_L and GameOn_SR have very high percentage error relative to their monthly averages and they can be explained by some unforeseen events such as trends and news in the media, and advertising campaigns for example. Other months, such as Jun 14 and Jul 14 have high percentage errors relative to GameOn_L, making this version of GameOn more susceptible to seasonality (and interestingly when most of the advertising campaigns are carried out by both firms) than

GameOn_SR. GameOn_L also has high values on Aug 14, another summer month, Oct 14, Nov 14 and Apr 15. These errors are deeply related to the conversion rate already explained that changes month to month and is very sensitive to exterior events, like news or publicity campaigns again. For GameOn_SR, there are two other months that have much high error percentages when compared to its average: May 14 and Jan 15.

To correct these discrepancies it is recommended the addition of a multiplying factor, to be called τ, to (8) and (9) in the case of GameOn_L, and to (43) and (44) in the case of GameOn_SR. The value to be assigned to the factor τ will be determined by the knowledge and experience of the digital marketing manager of the firm that is using GameOn. This factor, which varies from 1 to a non-fixed maximum, represents a percentage increase that relates to the expected change in the conversion rate. For example, for a month in which the conversion rate is stable and in, approximately, accordance to the standard GameOn's estimations, τ will be 1, but if an advertising campaign is incurred and the digital marketing manager forecasts an increase in the conversion rate of 50%, then the value for the factor to be used is 1,5. With the addition of this factor, it is expected that the average monthly percentage error for the estimations to decrease substantially, providing more confidence to GameOn.

Another possibility to correct the verified discrepancies is to consider a separated model for the months where the conversion rate differs largely from the annual average, such as the summer months. However, due to the low number of observations, even because there is only one observation for each of the summer months, this proposed solution cannot be developed as of now.

Table 5.3 presents GameOn's monthly contracts estimations and the respective percentage error when compared to the actual values, for firm B.

According to the estimations in Table 5.3, firm B would sell 181 less online contracts with the application of GameOn_L and 65 less online contracts with the application of GameOn_SR when compared to the actual values. The explanation for these discrepancies can be found in the exact same manner as it was explained for the case of Table 5.2. For both Table 5.2 and Table 5.3, total percentage errors relative to these discrepancies will be analyzed in more detail ahead.

Following the same analysis as for the previous table, the months of May 14, Aug 14, Mar 15, and, again especially Apr 15, for both GameOn_L and GameOn_SR have very high percentage errors relative to their monthly averages (22,9% and 20,3% respectively) and they can be explained by some unforeseen events such trends such as news in the media, and advertising campaigns for example. One other month, Dec 14 has an extremely high percentage error relative to GameOn_L, whereas the same month but for GameOn_SR has a very acceptable percentage error of 11,1. For GameOn_SR, there is one other month that has a much high error percentage when compared to its annual average: Sep 14.

Table 5.3 – Firm B's Estimated Online Contracts: GameOn_L and GameOn_SR

Month	**Online Contracts_B**	**GameOn_L**		**GameOn_SR**	
		Estimated Online Contracts_B	Error (%)	Estimated Online Contracts_B	Error (%)
Jan 14	155	151	2,6	153	1,3
Feb 14	200	219	9,5	201	0,5
Mar 14	231	231	0,0	221	4,3
Apr 14	256	215	16,0	212	17,2
May 14	267	384	43,8	417	56,2
Jun 14	501	372	25,7	443	11,6
Jul 14	561	481	14,3	564	0,5
Aug 14	267	138	48,3	125	53,2
Sep 14	299	222	25,8	206	31,1
Oct 14	301	308	2,3	309	2,7
Nov 14	287	223	22,3	223	22,3
Dec 14	54	90	66,7	60	11,1
Jan 15	110	122	10,9	97	11,8
Feb 15	210	244	16,2	245	16,7
Mar 15	202	262	29,7	282	39,6
Apr 15	176	234	33,0	254	44,3
Total	**4077**	**3896**	**22,9**	**4012**	**20,3**

These errors are deeply related to the conversion rate already explained that changes month to month and is very sensitive to exterior events, like news or publicity campaigns.

Again, to correct these discrepancies it is recommended the addition of a multiplying factor, to be called τ, to (8) and (9) in the case of GameOn_L, and to (43) and (44) in the case of GameOn_SR. The other possibility is still relevant and applicable, that is to consider a separated model for the months where the conversion rate differs largely from the annual average. However, due to the low number of observations, this proposed solution still cannot be developed as of now.

In Table 5.4 a summary of the more important and interesting results from AS1 is presented, namely the average monthly error already presented in Tables 5.2 and 5.3, and the percentage error of the estimation of the total quantity of online leads generated by firm A and B, through the application of GameOn_L and GameOn_SR.

Table 5.4 – Final Results Summary for Online Contracts Estimation for Firm A vs Firm B: GameOn_L and GameOn_SR

	Firm A		Firm B	
	GameOn_L	GameOn_SR	GameOn_L	GameOn_SR
Average Monthly Error (%)	27,9	32,1	22,9	20,3
Total	1835	1892	3896	4012
Total Error (%)	6,6	3,7	4,4	1,6

As it can be easily observed in Table 5.4, the estimations' average monthly error for firm B is lower, when compared to firm A for both cases GameOn_L and GameOn_SR. These results are interesting due to the fact that the data from firm B is not exact because it was gathered with the assistance of experts in the studied market but not from the actual marketing manager of firm B, and the data from firm A is exact. As already discussed for firm A, there is a number of months, in both GameOn_L and GameOn_SR that have an extremely high percentage error, while in the case of firm B, there is never an error so high as some errors in firm A, and additionally, the high errors for firm B occur less frequently. It is crucial to point out that the parameters' estimations and, consequently the results for both firms and both versions of GameOn, will undoubtedly get better with more observations with a better (or, ideally exact) precision. Another opposite result that can be observed in the above table is that for firm A, the better version of GameOn, i.e. the version that provides estimations whose average monthly percentage error is inferior, is GameOn_L, while for firm B, the best version is GameOn_SR. Additionally, for both firms, the percentage errors for GameOn_L and GameOn_SR, do not differ much, making any argument where one prefers one version of GameOn over the other, very difficult to sustain and justify.

Finally, it is important to discuss the difference in the actual total quantity of online contracts sold during the studied period and the estimated total quantity of online contracts sold for both firm A and B with the application of GameOn_L and GameOn_SR. As it can be seen in Table 5.4, all of the total quantities estimated are inferior to the actual total quantities of online contracts sold. And again, firm B has the better error percentages when compared to firm A. In both firms, the percentage error of GameOn_SR is better than the percentage error of GameOn_L, with the difference between both versions of GameOn, for both firms, standing on, approximately 3%. The total error percentage of GameOn_L for firm A can be somewhat troubling, since it is the highest error percentage of all cases and with a significant difference when compared to the other errors percentage.

After these discussions, the analysis of the following case study will be done for both GameOn_L and GameOn_SR since there is no definite proof that supports an elimination of either version of GameOn, and additionaly, it is interesting to still compare both GameOn_L and GameOn_SR for both firms in the case study.

5.5 GameOn Case Study

5.5.1 GameOn Case Study Contextualization

The case study, hereinafter "CS1", can be considered as the "standard" way to use GameOn. The idea behind CS1 is to first estimate the quantity of online leads a firm should generate for the period in study, and then GameOn delivers an estimation of the online contracts gained for that period. Thus, there are two equations to be used in CS1, one to calculate the estimation of the quantity of online leads a firm should generate, using as input the estimation of the quantity of online leads the rival firm will generate, and the second equation provides the estimation of the quantity of online contracts gained in that period, using as inputs the rival's online leads already used in the previous equation and the output of the previous equation: the estimation of the quantity of online leads the firm should generate.

Given that the explanation for the calculations presented in the next subchapter can be confusing when written all together in same paragraph, the respective explanation is provided before each case portrayed ahead.

5.5.2 GameOn Case Study Results and Discussion

For CS1, the goal is to simulate the standard use of GameOn. Table 5.5 presents the estimated quantity of online leads to be generated and the estimated online contracts to be sold in the period in question for firm A. It also presents the respective deltas so that a comparison between the actual values and the estimations can be made.

Table 5.5 – Firm A's Estimated Online Contracts: GameOn_L and GameOn_SR

Month	GameOn_L				GameOn_SR			
	Estimated Online Leads_A	Delta Online Leads_A	Estimated Online Contracts_A	Delta Online Contracts_A	Estimated Online Leads_A	Delta Online Leads_A	Estimated Online Contracts_A	Delta Online Contracts_A
Jan 14	340	-512	72	-26	168	-684	35	-63
Feb 14	514	-20	108	-2	419	-115	87	-23
Mar 14	514	-164	108	-8	419	-259	87	-29
Apr 14	465	-360	98	-16	335	-490	70	-44
May 14	843	341	177	66	1257	755	261	150
Jun 14	959	732	202	4	1677	1450	348	150
Jul 14	959	256	202	-73	1677	974	348	73
Aug 14	340	-234	72	-138	168	-406	35	-175
Sep 14	514	-57	108	-19	419	-152	87	-40
Oct 14	702	214	148	60	838	350	174	86
Nov 14	702	584	148	109	838	720	174	135
Dec 14	340	250	72	38	168	78	35	1
Jan 15	340	6	72	-16	168	-166	35	-53
Feb 15	514	-347	108	-25	419	-442	87	-46
Mar 15	514	-662	108	-36	419	-757	87	-57
Apr 15	465	-726	98	18	335	-856	70	-10
Total	**9025**	**-699**	**1901**	**-64**	**9724**	**0**	**2020**	**55**

By observing the Table 5.5, the only months where the difference between the actual quantity of online generated leads and its estimation for GameOn_L, are somewhat similar are Feb 14, Sep 14 and Jan 15, although the difference in Sep 14 can be viewed as a little high. During these months the difference in the quantity of online sold contracts varies, where in Feb 14 it is estimated that firm A would have sold 2 online contracts less, in Sep 14 19 less and in Jan 15 16 less. There are some months with results that may seem counterintuitive because, for example the estimation might suggest an inferior quantity of online leads to be generated and then the estimated quantity of online sold contracts would be higher (or the inverse), such is the case of Jul 14, Jan 15 and Apr 15. This might happen due to the fact that in these months the conversion rate from online lead to online contract follows an extremely unusual pattern when compared to the generality of the other months, and because there is a high degree of uncertainty when it comes to this rate. It is important to point out that throughout the 16 months, according to GameOn_L it is estimated that firm A would have generated 699 less online leads than it actually did and it would have gained less 64 online contracts than it actually did. The potential difference in profits will be addressed ahead.

For GameOn_SR the same analysis can be made but the more interesting fact to refer is that the counterintuitive argument presented earlier does not apply in this case. There is not one month in which the estimated quantity of online generated leads increases or decreases and the quantity of online sold contracts decreases or increases respectively. This difference between both versions of GameOn can be a sign that GameOn_SR is more adaptable to abrupt changes in the expected monthly conversion rate. It is curious to see that the total quantity of online generated leads throughout the 16 months is the exact same as the actual total quantity. However, the total estimated quantity of online sold contracts throughout that period increases, indicating an increase in the total estimated profit when compared to the actual value, concluding that GameOn_SR as the potential increase the efficiency and better the effectiveness at which these decisions are currently made, by redistributing the monthly quantities of online generated leads.

Table 5.6 presents the estimated quantity of online leads to be generated and the estimated quantity of online contracts to be sold in the period in question for firm B. It also presents the respective deltas so that a comparison between the actual values and the estimations can be made.

The analysis for firm B is in all ways similar to the one presented for firm A. For GameOn_L, although there are no months in which the estimated quantity of online generated leads is similar the actual values, there are a couple of months that have very interesting results, namely Aug 14 and Sep 14. For Aug 14, although the difference between estimated online generated leads and its actual value is 1972 more estimated online generated leads, the difference in the quantity of online sold contracts is zero. This can be, again counterintuitive because with such an increase in the estimated quantity of online generated leads, it would be expected an increase in the quantity of online gained contracts, but this doesn’t happen. And in Sep 14, an increase in the estimated quantity of online generated leads actually leads to a decrease in the estimated quantity of online sold contracts. This situation can also be explained by the uncertainty of the monthly conversion rate as it was in the situation regarding firm A. For GameOn_L the estimated total quantity of online generated leads is much less, 1920 less to be precise, than the actual value, while the estimated total quantity of online sold contracts is higher, by 120 units than the actual value. This means that with GameOn_L, for firm B, these decisions can be made in a much more efficient and effective manner to provide, as it will be seen ahead, added profit.

Table 5.6 – Firm B's Estimated Online Contracts: GameOn_L and GameOn_SR

Month	GameOn_L				GameOn_SR			
	Estimated Online Leads_B	Delta Online Leads_B	Estimated Online Contracts_B	Delta Online Contracts_B	Estimated Online Leads_B	Delta Online Leads_B	Estimated Online Contracts_B	Delta Online Contracts_B
Jan 14	3321	2541	322	167	3964	3184	374	219
Feb 14	2658	708	258	58	2484	534	234	34
Mar 14	2979	1029	289	58	3154	1204	298	67
Apr 14	3271	1711	317	61	3838	2278	362	106
May 14	2581	-3269	250	-17	2336	-3514	220	-47
Jun 14	1769	-6031	171	-330	1056	-6744	100	-401
Jul 14	3031	-4769	294	-267	3271	-4529	308	-253
Aug 14	2752	1972	267	0	2670	1890	252	-15
Sep 14	2745	795	266	-33	2657	707	251	-48
Oct 14	2547	-1353	247	-54	2270	-1630	214	-87
Nov14	1295	-2605	125	-162	549	-3351	52	-235
Dec 14	1138	358	110	56	419	-361	39	-15
Jan 15	2126	1346	206	96	1554	774	147	37
Feb 15	3338	1388	323	113	4006	2056	378	168
Mar 15	3873	1923	375	173	5471	3521	516	314
Apr 15	3896	2336	377	201	5541	3981	523	347
Total	**43320**	**-1920**	**4197**	**120**	**45240**	**0**	**4268**	**191**

For GameOn_SR, the same months as GameOn_L, Aug 14 and Sep 14, still have the same counterintuitive problem. In both months the estimated quantity of online generated leads is larger than the actual values, but the estimated quantity of online sold contracts is inferior to the actual quantity.

Once again, just like in the case of GameOn_SR for firm A, the case of GameOn_SR for firm B shows that the total estimated quantity of online generated leads is the exact same as the actual value. However, and just like in the previous case, the total estimated quantity of online sold contracts throughout that period increases, indicating an increase in the total estimated profit when compared to the actual value, concluding that GameOn_SR has the potential increase the efficiency and better the effectiveness at which these decisions are currently made.

Table 5.7 shows some interesting values from which pertinent and essential conclusions can be derived. In Table 5.7 the profits of CS1 are compared to the actual profits. This comparison is made not with monetary values, but through percentages. This is done because it was impossible to gather precise values related to the costs per online generated lead by the rival. Additionally, the profit per online contract of firm B is not an exact value and thus, other approximations would decrease the confidence from the use of GameOn. Thus, the profit from the actual values, the quantity of online generated leads and online sold contracts, is considered to be 0%, because it is the change in percentage of profit that is computed in CS1.

Table 5.7 – Final Results Summary for Online Contracts Estimation for Firm A vs Firm B: GameOn_L and GameOn_SR

	Firm		**Delta Profit**	
	A	B	Firm A	Firm B
Actual				
Online Leads	**9724**	**45240**	0%	0%
Online Contracts	**1965**	**4077**		
GameOn_L				
Online Leads	**9025**	**43320**	-0,04%	+12,14%
Online Contracts	**1901**	**4197**		
GameOn_SR				
Online Leads	**9724**	**45240**	+6,95%	+14,40%
Online Contracts	**2020**	**4268**		

As it was earlier discussed, for firm A and in the case of GameOn_L, it is estimated that firm A would have generated less online leads during the 16 studied months and it would have sold less online contracts. These two quantities almost offset each other and the profit would have been almost the same, with a decrease of 0,04%. It is understandable that firm A actually generated much more online leads, gaining more online contracts because it was able to reach its objective, if the actual number of online contracts gained was the objective (or close to it), and firm A was able to gain more customer awareness due to these larger quantities. For the same case, GameOn_L but for firm B the percentage change in profit is quite different. In this situation, there is actually a decrease in the estimated quantity of online generated leads and an increase in the estimated quantity of online contracts sold for an increase in profit of 12,14%, which means that a decrease and a redistribution of the monthly online generated leads can lead to a more efficient and effective outcome; the increase in profits.

For GameOn_SR, as it was mentioned earlier, it is interesting to observe that the estimated quantity of online generated leads by both firms is the exact same as the actual quantity generated by both firms. But while the quantity of online leads generated stays the same in the estimation, the quantity of estimated online sold contracts increases for both firms. This will, obviously lead to an increase in the estimated profits, where for firm A the increase is 6,95%, while for firm B, the profit is increased by 14,4%.

It is interesting to observe that firm B as a larger increase in the estimated profits in both situations, GameOn_L and GameOn_SR when compared to the estimated profits of firm A. This result is indeed interesting given the fact that, as it was already seen in Table 5.4, firm B also performed better. It can also be observed that the estimated percentage change in profits from GameOn_SR is higher than the percentage change in profits of GameOn_L for both firms.

5.5.3 Nash and Stackelberg Equilibrium

Table 5.8 presents the Nash equilibrium for firms A and B using (28) and (29) respectively. As it was already discussed (in chapter 4), the Nash equilibrium is only calculated for GameOn_L, since that equilibrium for GameOn_SR is the actual origin of the x-y axis, (0,0) and that result holds no interest for the present study.

At the Nash equilibrium no firm as an incentive to change the quantity of online leads generated, this is to say, that at the equilibrium each firm is choosing the best strategy for itself given the other strategies prevailing in the market (the strategy of the rival in this case). As it can be observed in Table 5.8 given the quantities of online leads to generate in the Nash equilibrium – 608 online leads for firm A and 2827 online leads for firm B – firm A would get 128 monthly online contracts and firm B would get 274 monthly online contracts.

Table 5.8 – Nash Equilibrium

Nash Equilibrium	**Firm**	
	A	B
Online Leads	608	2827
Online Contracts	128	274

It should be clear by now that the Nash equilibrium computed for both firms is the exact same as the average actual quantities of both the monthly online generated leads. This happens because of how the factors k_A and k_B were calculated. When the same numbers are used to calculate both factors, in this case the monthly average of online generated leads, then what that is really doing, is fixating the Nash equilibrium and pre determining it, given that the Nash equilibrium is the intersection between the reaction functions. But as it was already stated, there are innumerous forms of calculating these factors. For example, it is assumed that firm A calculates the factor k_A with the average quantities of online generated leads by both firms during the total period studied, thus, the final value of k_A is the same as showed in Table 5.1, and firm B determines its own factor by selecting the month in which its conversion rate was the highest – Aug 14 – and it will use the quantity of online generated leads by firm A and its own quantity to compute the factor k_B, 108,1867188. The remaining process is identical to what it was already described. The results from this example of the Nash equilibrium are showed in Table 5.9.

Table 5.9 - Nash Equilibrium - Example

Nash Equilibrium	**Firm**	
	A	B
Online Leads	295	568
Online Contracts	62	100

As it can be immediately seen by in Table 5.9, the values from this example of the Nash equilibrium differ greatly from the values initially determined, which goes to show the importance that the factors k_A and k_B have.

Returning to the values present in Table 5.8, the monthly quantity of online sold contracts in the Nash equilibrium is higher in both firms compared to the actual average monthly quantities of online sold contracts (123 and 255 for firm A and B respectively). This means that with the same total quantity of online generated leads, with a monthly quantity redistribution of the quantity of online leads to be generated in each month (generating each month the Nash equilibrium's quantities), a firm can sell more online contracts.

When observing Tables 5.8 and Table 4.1, it is possible to see that in no month did firm A and B behaved according to their Nash equilibrium, thus making a comparison impossible. However, when analyzing the values presented in the tables relative to CS1, Tables 5.5 and 5.6, it is possible to observe some similarities to the Nash equilibrium's values. For Firm A there is no month where the monthly quantity of online generated leads is similar to the quantity of the Nash equilibrium but for firm B this doesn't happen, there are indeed several months in which the quantities of online generated leads are very similar and almost identical to the Nash equilibrium's quantities. It needs to be noted that only GameOn_L is being analyzed since, as it was already discussed, there is no Nash equilibrium for GameOn_SR. In Table 5.6 the months of Feb 14, Mar 14, Jul 14, Aug 14 and Sep 14 have similar quantities to the Nash equilibrium although for firm A, in these months, the closest to the values of the Nash equilibrium are Aug 14 and Sep 14 (see Table 4.1). When comparing the quantities of estimated online sold contracts of firm B it is easily observed that those quantities, 267 and 266 respectively, are very similar to the quantities of the Nash equilibrium, although the quantities of the estimated online sold contracts of firm B are lower since the actual quantities of online generated leads by firm A during those months is lower than its Nash equilibrium, as well as the estimated quantity of online generated leads by firm B (lower than its own Nash equilibrium).

In Table 5.10 the results from the Stackelberg equilibrium are presented. The Nash equilibrium results are for simultaneous games, i.e. where the decisions made by, in this situation, both firms, A and B, are made simultaneously or without knowledge of each other's strategy. In version "1)" of the Stackelberg equilibrium, firm B moves first, i.e. makes its decision first by choosing the strategy that maximizes its profit function, taking into account that firm A moves afterward according with its reaction function, and accordingly firm A makes its decision based on firm B's strategy, i.e. it uses its reaction function with firm B's best strategy to determine what it should do (whereas firm B never uses its reaction function, it just chooses the best strategy by substituting the rival's reaction function into its profit function and selects the strategy that maximizes the profit). In version "2)", the only difference is that the first mover is firm A and firm B follows. As discussed previously (chapter 4) the Stackelberg equilibrium is only calculated for GameOn_L.

Table 5.10 - Stackelberg Equilibrium

Stackelberg Equilibrium		**Firm** A	**Firm** B
1)	Online Leads	753	4554
	Online Contracts	158	369
2)	Online Leads	1042	3655
	Online Contracts	174	354

Some immediate conclusions can be derived by observing Table 5.10, such as the fact that, in both versions of the Stackelberg equilibrium, the estimated quantity of online generated leads by both firms are much higher than the actual average quantity (and Nash equilibrium), 608 and 2827 for firm A and B respectively. The estimated quantity of online sold contracts also follows the same trend, where in both versions of the Stackelberg equilibrium, the estimated quantity of online sold contracts by both firms are much higher than the actual average, and the Nash equilibriums' quantity of online sold contracts. It will be seen and analyzed ahead the interesting results and conclusions that arise from the fact that the higher quantity of online generated leads and online sold contracts, and posterior higher profit when comparing the Stackelberg and Nash equilibrium, has very interesting theoretical and practical impacts. But before delving in that discussion, it is important to point out that both firms, as every firm and every department in every industry, is subjected to budget limits that has to comply with. The ability and feasibility of generating the large quantities of online leads expressed in both versions of the Stackelberg equilibrium in both firms, is dependent on the budget that each firm and each marketing department has to perform this task. It is entirely possible that one or even both firms cannot generate such an amount of online leads for too many months, even though in terms of profits it could be favorable.

As it can be observed in Table 4.1, there is no month in which both firms seem to be in a situation of Stackelberg equilibrium, and thus no discussion can be derived from this observation.

When looking at Table 5.5 it can be observed that the only months in which the estimated quantity of online generated leads is somewhat similar to the first version of the Stackelberg equilibrium are Oct 14 and Nov 14 (for firm A). And in these months the actual quantity of online leads generated by firm B is lower than the Stackelberg equilibrium, however it is a number that is sufficiently high that can provide some important information, such as the fact that the estimated quantity of online contracts gained by firm A is lower than the Stackelberg equilibrium, and that is to be expected since the estimated quantity of online generated leads by firm A and the actual quantity of online generated leads by firm B are lower than the Stackelberg equilibrium. Concerning firm B and looking at Table 5.6 there is no month in which the estimated quantity of online generated leads is similar to the first version of the Stackelberg equilibrium.

Looking at the second version of the Stackelberg equilibrium, where firm A makes its decision first and firm B follows, and at Table 5.5, only two months have an estimated quantity of online generated leads that is similar to firm A's Staceklberg equilibrium, Jun 14 and Jul 14. During these months, firm B actually generated a far superior quantity of online leads when compared to its Stackelberg equilibrium. Thus, it is natural to expect the estimated quantity of online contracts gained by firm A during these months to be greater than its Stackelberg equilibrium, given the fact that although the quantity of the online generated leads in the Stackelberg equilibrium for firm A is superior to the estimated quantity in CS1, the Stackelberg quantity of online generated leads by firm B is vastly inferior to the actual quantity (seen in Table 4.1).

According to Table 5.6 there are only two months in which the estimated quantity of online generated leads by firm B resembles the second version of the Stackelberg equilibrium, Mar 15 and Apr 15. Interestingly enough, when observing Table 4.1, those two months have values relative to the quantity of online generated leads by firm A that are somewhat similar to the Stackelberg equilibrium, although somewhat higher, and thus, the estimated quantity of online sold contracts by firm B does not differ much from the second version of its Stackelberg equilibrium. However the estimated quantities of the online sold contracts for both months are higher than the Stackelberg equilibrium which is to be expected since the estimated quantity of online generated leads by firm B and the actual quantities of online generated leads by firm A are both higher than the values of the second version of the Stackelberg equilibrium.

It is easily observed that when a firm moves first, it will generate more online leads and gain more contracts that it does when moving second. This is true for both firms A and B. However, this observation, as it is seen ahead, might not translate in terms of profit. Table 5.11 presents the actual profits, and the profit estimations for the Nash equilibrium and for both versions of the Stackelberg equilibrium.

The goal by showing and analyzing Table 5.11, is to demonstrate the difference, in this case the percentage difference, between the actual case, that is the actual situation of the 16 studied months, the Nash equilibrium and both versions of the Stackelberg equilibrium. For the actual case the profit change is, obviously 0% for both firms since this is the value that it is to be compared against. The percentage change in profits with the application of the Nash equilibrium is +7,64% and +17,11% for firm A and B respectively. As it was already explained earlier, these values are intrinsic to the way in which the factors k_A and k_B were determined. If those factors were determined using other quantities as opposed to the average quantity of online generated leads in at least one firm, or both firms as long as the values used weren't the same for both, then the results from the Nash equilibrium and consequently, the percentage change would've something different.

Table 5.11 - Profit Comparison Actual vs Nash vs Stackelberg Equilibrium

		Firm		**Delta Profit**	
		A	B	Firm A	Firm B
	Actual				
	Online Leads	608	2827	**0%**	**0%**
	Online Contracts	123	255		
	Nash Equilibrium				
	Online Leads	608	2827	**+7,64%**	**+17,11%**
	Online Contracts	128	274		
	Stackelberg Equilibrium				
1)	Online Leads	753	4554	**+33,41%**	**+23,95%**
	Online Contracts	158	369		
2)	Online Leads	1042	3655	**+14,84%**	**+51,4%**
	Online Contracts	174	354		

As it can be easily seen simply by observing Table 5.11, the monthly quantities relative to the online generated leads and online gained contracts by firm A and B for both versions of the Stackelberg equilibrium, are all higher when compared to the same quantities in the actual situation and the Nash equilibrium. It is interesting to observe that the Nash equilibrium is not the most efficient equilibrium by far, due to the lower profits when compared to the Stackelberg equilibrium. Sure that one needs to be very careful when pointing out this fact because the Nash equilibrium was "forced" or fixed previously by the firms, when determining k_A and k_B.

When analyzing both versions of the Stackelberg equilibrium, it is observed that the difference in the percentage change in profit is very large when compared to the actual situation and the

Nash equilibrium, pointing out that the firms should be behaving according to one of the versions of Stackelberg equilibrium in order to obtain a larger profit. It needs to be said something that was already stated earlier and that is the budget restrictions that both firms need to respect, and thus the continued application of the Stackelberg equilibrium as the firms' strategy might not be sustainable after a certain period of time.

Some extremely interesting and innovative conclusions arise when analyzing both versions of the Stackelberg equilibrium, such as the fact that for both versions, the firm that selects its strategy secondly, that is the follower – in the case of the first version, firm A is the follower and in the case of the second version, firm B is the follower – yields a larger profit percentage change when compared to the profit percentage change of the leader. This fact happens in both versions and it is extremely interesting given that the literature suggests that the Stackelberg equilibrium yields better results for the leader when compared to the results of the follower – when the Stackelberg equilibrium is compared to the Nash equilibrium this situation also occurs – and it is observed in both versions of the Stackelberg equilibrium that the opposite happens. An explanation for this situation can be found in the values of γ_{AB} and γ_{BA}. Since both these parameters are positive – meaning that the quantity of online generated leads by a rival firm will have a positive impact on the sales of the own firm – when a firm generates high quantity of online leads, the firm that selects secondly, will insert that quantity in its reaction function and the resulting quantity of online leads to be generated will be inflated when determining the Stackelberg equilibrium (when compared to the actual situation and the Nash equilibrium), and thusly so will the estimated quantity of contracts to be sold for that period. For this situation, in this particular market and competitive environment, it can be said that the best strategy that each firm can employ is to be follower in the Stackelberg equilibrium.

Figure 5.5 shows a graphic depicting the reaction functions of both firms, A and B and the respective places where the Nash equilibrium and both versions of the Stackelberg equilibrium are situated.

As it is clear by observing Figure 5.5, and as it was already mentioned, the Nash equilibrium is the point of intersection of both reaction functions, approximately 608 and 2827 online leads for firm A and B respectively, and it is situated within the green circle. The other circles symbolize the Stackelberg equilibrium. Within the black circle, is the first version of the Stackelberg equilibrium. This is to be expected because the Stackelberg equilibrium falls in the reaction function's curve of the follower, and in this case, the follower is firm A. The same goes for the second version of the Stackelberg equilibrium that is represented with an orange circle and it is situated on the reaction function's curve of the follower, and this is case the follower is firm B.

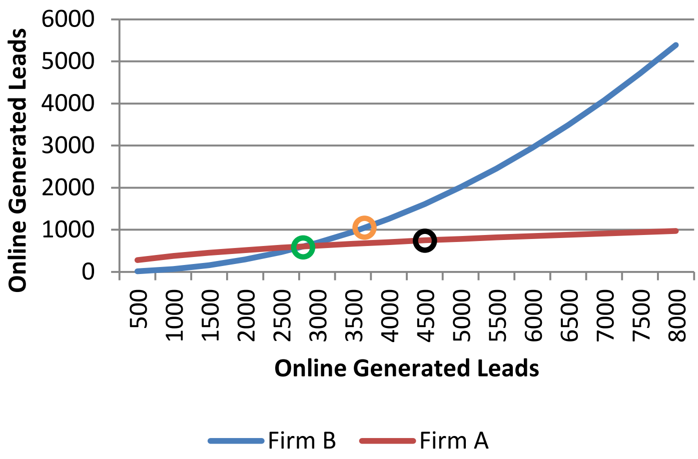

Figure 5.5 – Reaction Functions Firm A vs Firm B

5.6 GameOn Final Considerations

An application scenario and a case study were presented and discussed in this chapter and some interesting conclusions were derived from the results. As mentioned before, GameOn is very flexible and it can be used in different scenarios to achieve different objectives or when a firm has/wants different inputs from the ones that AS1 or CS1 requires. Besides the various what-if scenarios that a firm can perform in order to have to better perception of what it needs to do, or which strategy it is better to select given its belief of what the rival(s) will do, it is proposed three situations to use GameOn:

1. Use GameOn in similar way as CS1, where the objective of a firm is to estimate the quantity of online leads it should generate and the quantity of online contracts it should gain. As inputs, it is needed the quantity of online leads the rival(s) will generate for the period in question, and if necessary the factor τ.

2. To estimate the number online contracts to be sold in the period that is being prepared, and when the firm that is using GameOn has at its disposal the quantities of online leads to be generated (or an estimation) in that period for its rival(s), it can use them as inputs, as well as the quantity of online leads it will generate and, if necessary the factor τ, and as output the firm will have an estimate of how many online contracts it will sell in that period. A firm can use this method repeatedly as several what-ifs scenarios to choose the best strategy in their view.

3. The last GameOn use to be proposed is the one that, most likely, most firms would find more useful. Most firms have predetermined objectives to reach at the end of each period. For example, a firm might have as a goal to sell 150 online contracts in a given month. Thus, as inputs a firm inserts in GameOn the quantity of online

contracts it has to sell (its selling objective) for the period that is being prepared, the quantity of online leads the rival(s) will generated for the same period and, if necessary, the factor τ. As output, the firm will have an estimate of how many online leads it should generate in order to achieve its goal.

5.7 Summary

In this chapter an initial analysis and discussion was provided for some preliminary results as well as for an application scenario and a case study. For the initial results shown it was discussed the evolution of the estimated quantity of online generated leads by firm A with the application of GameOn_L and GameOn_SR and compared with the evolution of firm B's estimation of online generated leads with the application of both GameOn_L and GameOn_SR.

The results for AS1 were discussed as one way of using GameOn and its potential was evaluated as a means of forecasting the estimated quantity of online contracts to be sold in the studied period. The use of this scenario as a source for what-if analysis was also lightly mentioned. Finally, for AS1, the differences between the actual values and the estimated values were compared.

A case study, CS1 was also introduced and its results discussed. CS1 took into consideration the estimation of the quantity of online leads generated by the rival and estimated the quantity of online leads the own firm should generate and with those estimations, it would estimate the quantity of online contracts it would sell in the studied period. From these results, it was possible to compute the difference in profits from the real situation and from CS1 that was also discussed in detail. The Nash and Stackelberg equilibriums were determined and the results discussed, culminating in extremely interesting theoretic and practical observations.

Finally, it was presented some possible real world applications of GameOn that firms can use to support their decisions in the context they feel GameOn to be more appropriate. The next and final chapter, deals with the conclusions from the study presented, the limitations inherent to the work and some recommendations for future work.

Chapter 6. Conclusions, Limitations and Recommendations for Future Work

6.1 Conclusions

The present research work comes from the real world need for a management model that supports the decision-making process of digital marketing managers when it comes to know how many online leads should be generated for a certain period in order to achieve their objectives measured in, for example online contracts gained, in the most efficient and effective way possible. Thus, two research questions were formulated to approach this problem:

1. How can game theory be used to model the quantity of online leads to be generated by a firm?
2. How can game theory contribute to the effectiveness and efficiency improvement of the digital marketing managers' decision making process relating to the online leads generation?

To address both questions, four objectives were identified and achieved throughout the dissertation. The first one was related to the identification and evaluation of the key relevant points to the application of game theory to online lead generation. A careful review of the state of the art in the literature available was made to evaluate the work that has already been done, in this field of study, and while there have been developed some approaches regarding game theory applied to advertising, there hasn't been conducted any research specifically relating game theory to online lead generation. Some models were identified, especially Chintagunta and Vilcassim's (1995), who proposed model regarding the allocation of advertisement investments, and adapted to the studied situation. Game theory's specific feature that it takes into account the strategies employed by the rival firm(s) proved very useful and vital in the development of the GameOn.

To achieve the next objective, related to the actual development of the proposed management model – GameOn – a study was conducted in order to clarify what parameters are significant for the development and application of GameOn. After the parameters were identified, the mathematical foundation and formulation were developed to support the model. The significant parameters selected might differ from each studied situation, due to specificities of the studied market. For the situation studied a very interesting result arose: the fact that both parameters γ_{AB} and γ_{BA} are positive. This implies that the quantity of online generated leads by each firm has a positive impact on the sales of its rival, and not a negative impact as it would be, perhaps, intuitive to assume. GameOn, being itself based on game theory, was able to determine two types of equilibrium that can be very useful to every firm that resorts to the model, because for one hand it is possible to calculate the Nash equilibrium where if each firm selects each strategy simultaneously or not knowing what strategy its rival(s) selected, it yields the best outcome for both firms, and the Stackelberg equilibrium where there is one leader that selects its strategy first and the follower selects its own strategy later. These situations can provide the firms with knowledge that they currently do not possess and can be a source of competitive advantage that every firm should strive for. Additionally, it was formulated the generalization of GameOn and its variations, GameOn_L and GameOn_SR, to demonstrate the wide potential it has. With the generalization of GameOn, any firm, in any industry and in any competitive environment can use it as a support tool to achieve its objectives more effective and efficiently.

The third objective was to apply GameOn and its variations, GameOn_L and GameOn_SR, to situations identical to the ones they could be used in a real world context. GameOn was used in a context of a very specific Portuguese duopoly market with peculiar characteristics, and the application of GameOn provided very compelling results showing that GameOn has the potential to improve the digital marketing department's performance within a firm. GameOn can also be used in a variety of industries, be it duopolies or oligopolies, with the exception of monopolies. It can also be used in firms that operate in the food industry, clothing industry, and any other industry that requires online advertising, namely the investment in online leads, which in today's competitive world is the case for most industries.

The fourth objective was achieved through the development and discussion of one application scenario and one case study, where it is was demonstrated and discussed through several indicators such as the percentage errors from the estimated values and the percentage change in the estimated profits, that GameOn can be very useful and accurate as a support tool to assist the digital marketing manager in his/hers decision of how many online leads to generate to achieve a certain goal. The results relative to the percentage errors from the estimated values for both firms studied and with both functional forms were very interesting and encouraging and proof of the potential of GameOn. For the case study, the computed results were also encouraging, despite the application of GameOn_L to firm A returned an estimated -0,04% profit, while all the other scenarios return a positive change in estimated profits and in the case of firm B more than 12% change for GameOn_L and more than 14% change for GameOn_SR. One of the great capabilities of GameOn is the possibility of computing the Nash and Stackelberg equilibriums and thus the firm has a better understanding of which strategy it should employ in order to maximize its profit. Through the way in which certain factors, namely k_A and k_B, were determined, the Nash equilibrium was already predetermined and thus a discussion revolving this subject will always have to be very carefully done due to this issue. However, very interesting results arose, specifically, relative to the Stackelberg equilibrium where it was concluded that both versions of this equilibrium, one version where one firm was the leader, and the other version where the rival firm was the leader and selected its strategy first, are more efficient than the Nash equilibrium, in the situation computed, and more interestingly, that the best strategy for a firm to select when it comes to the Stackelberg equilibrium is to be the follower, that is, to select its strategy after the rival firm selected its own strategy. This situation happens due to the specificities of the studied market and the competitive environment where the quantity of online generated leads by a firm has a positive effect on the online contract sales of its rival. When both firms follow the Stackelberg equilibrium, and whether a firm is the leader or the follower, their estimated profits are much higher than their actual profits, varying from 14,84% to 33,41% higher in the case of firm A and 23,95% and 51,4% higher for firm B.

In general, game theory is very useful for virtually any firm, with the obvious exception of a monopoly market, since it takes into account the strategies that its rival(s) is/are employing. It can be an excellent source of benchmarking and competitive advantage, by increasing a firm's efficiency and effectiveness in its performance.

Overall both research questions were successfully answered and it was demonstrated the great potential that, not only game theory has in assisting the digital marketing managers in their decisions, but specifically, the potential that GameOn has to help a firm gain competitive advantage by helping it operate at a higher level of efficiency.

6.2 Limitations

The presented work is not without some important limitations. These limitations arise from the characteristics of game theory itself that may hamper the great potential that GameOn has to maximize a firm's profits, to data quality and quantity limitations.

A very important limitation not only for the work here presented but for the application of GameOn, is a limitation that is inherent to the application of any game theoretic model, and that is the knowledge about the necessary data from the rival firm(s). Sometimes, that knowledge is not perfect or maybe there is no knowledge at all, which makes the application of GameOn very difficult; although it is possible to, with a number of estimations, develop a variety of what-if scenarios that can be analyzed.

There is the obvious limitation relative to the quantity and quality of the data available. The total number of observations from which the entire study was derived is 16, meaning the 16 months of data here studied. This is clearly not a high number of observations from which one could infer solid and trustworthy mathematic and statistical conclusions, and that is why, in order to surpass this limitation, the bootstrap technique was used. The quality of data is also something that needs some improvement for future applications for more accurate results and conclusions. Again, the data presented in the study was gathered from a marketing manager of one of firms studied and one other specialist, and there is nothing wrong in using less than accurate data because of the flexibility that GameOn provides, but if one wants to get extremely accurate and precise results then accurate, precise, and extensive data is required.

The cost per lead is also one other information that is difficult to obtain when it comes to the rival firm(s). It is actually for this reason that GameOn was designed so that it wasn't necessary to figure out or to estimate this value.

One more limitation to be mentioned is the fact that it was impossible to gather information regarding the monthly quantity of offline generated leads for firm B that rendered the analysis of whether a parameter regarding the offline generated leads by that firm would be significant or not, and if it was significant, it could have change the mathematical formulation, the application scenario and the case study for both GameOn_L and GameOn_SR.

6.3 Recommendations for Future Work

First of all, it is recommended an application of both GameOn_L and GameOn_SR in a real world context to definitely validate GameOn. Several approaches are possible with GameOn as it was explained in chapter 5, and thus it is possible for any firm to apply GameOn to its own situation and to benefit from it. And to definitely prove the potential and the benefits of GameOn, a wide application in a real world context is highly recommended. Ideally, GameOn would be applied in different contexts, different industries and in different competitive environments to demonstrate its wide potential. It would be very interesting to analyze the results from the application of GameOn to other realities in order to figure out if the same results occur or if different, perhaps more natural and intuitive results arise.

Logically it is recommended the application of GameOn with better and more data. Since this was precisely one of the limitations of the work presented, it is important to mention that, for future works, the data from more periods than just 16 would be very useful and exact data from the competitor(s) would also help to increase the confidence in the application of GameOn.

It is also recommended, as long as there is sufficient data for such, to develop a separate model for the months with the most unusual conversion rates, which can coincide with the periods in which most firms in the studied market carried out publicity campaigns. Concerning the conversion rate, it is recommended the addition of the factor τ in the sales equations of the studied market to better model the changing monthly conversion rate, which can add confidence and reliability to the model.

It would be very interesting to adapt GameOn so that instead of working with quantities of online generated leads and online sold contracts, the user would be working with monetary values, such as Euros or American dollars. The disadvantage of building GameOn that way is that the information relative to the competitor(s)' cost per online lead is extremely difficult to gather, and consequently it would lose reliability. Additionally, it is recommended to model the factors k_A and k_A as random generated numbers through a probabilistic distribution.

Lastly, GameOn is a static model, and it is recommended an adaptation of GameOn to transform it in a dynamic model which in certain situations can perform better, specially, in highly volatile markets where the market situation is constantly changing and the decisions are not punctual, but constant. Thusly, it is recommended an adaptation to a dynamic model, and perhaps an adaptation to a 2-period model to take into account carry-over effects of the actions taken.

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