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Two dimensional upper and lower-bound numerical analysis of the basal stability of deep excavations in clay

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ABSTRACT: The basal stability of deep excavations in clay is often analysed using the classical approaches of Terzaghi (1943) and Bjerrum & Eide (1956). Terzaghi's approach seems to be appropriate to excavation with small depth to width ratios, h/B , whereas Bjerrum & Eide give better results for greater h/B ratios. Not many attempts have been made to unify the two situations. In the present paper, the problem is analysed using upper and lower-bound considering two two-dimensional cases: a basic case, where the rigid stratum was considered deep, not affecting the stability of the excavation, and a case considering a rigid stratum at a certain depth. The embedded length of the wall was assumed null. A proposal for the stability numbers at failure is presented, for a wide range of h/B ratios.

1 INTRODUCTION

The problem of the basal stability of deep excavations in clayey soils under undrained conditions has been addressed by several authors. For a case such as the one shown in Figure 1a, considering an excavation of depth h and width B performed under undrained conditions in a homogeneous clayey soil with unit weight γ and undrained shear strength c_u , basal instability can occur if the total stresses imposed by the supported soil, above the base of the excavation, cause the failure of the soil below, due to insufficient strength. The classical approaches to this problem are those from Terzaghi (1943) and from Bjerrum & Eide (1956). These methods are based on limit equilibrium principles and are developed from a basic case of no wall embedded length and considering a deep stratum, which means that (Figure 1a) d is large and does not affect the mechanism.

Collapse occurs, according to Terzaghi's method, when the stability number $\gamma h/c_u$ is:

$$\frac{\gamma h}{c_u} = 5.7 + \sqrt{2} \frac{h}{B} \quad (1)$$

using Terzaghi's method, and:

$$\frac{\gamma h}{c_u} = (2 + \pi) \left(1 + 0.34 \arctan \frac{h}{B} \right) \quad (2)$$

using Bjerrum and Eide's method, which is based on the depth correction for shallow foundations proposed by Skempton (1951).

Figure 1. Geometry of the problem (a); cases analysed: Case 1 (b) and Case 2 (c).

In both methods the failure mechanism is affected by the rigid stratum when the depth d (Figure 1a) is:

$$d < \frac{\sqrt{2}}{2} B \quad (3)$$

and in this case equations (1) and (2) remain valid by replacing B by $\sqrt{2}d$:

$$\frac{\gamma h}{c_u} = 5.7 + \frac{h}{d} \quad (4)$$

in Terzaghi's method and:

$$\frac{\gamma h}{c_u} = (2 + \pi) \left(1 + 0.34 \arctan \frac{h}{\sqrt{2}d} \right) \quad (5)$$

in Bjerrum and Eide's method.

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These methods have also been extended to the cases of wall embedded length, soil-to-wall adhesion, soil anisotropy, etc, but these cases are beyond the scope of the present paper.

Numerical limit analysis has been applied by Ukritchon et al. (2003) to this problem, using both upper (UB) and lower-bound (LB) methods. Santos Josefino et al. (2010) have also applied to the same problem the upper-bound method using a numerical implementation and results have significantly improved the previous upper-bound ones.

In the present paper, the calculations presented by Santos Josefino et al. (2010) are extended to other geometries and are complemented by lower-bound numerical calculations using the finite element program *mechpy*, developed within the authors' research team.

2 THE FINITE ELEMENT PROGRAM

The finite element limit analysis code is implemented in *mechpy*, a platform written in Python language for the development of non-conventional finite element formulations (Deusdado 2017). The limit analysis module of this software is an evolution of the *sublim3D* program, also developed within the authors' research team (Vicente da Silva and Antão 2008, Vicente da Silva 2009, Antão et al. 2012). As is well-known, the determination of plastic collapse loads can be done based on the Limit Analysis theorems. From a numerical viewpoint, this approach leads to a non-linear optimization problem. The optimization technique used in *mechpy* is the Alternating Direction Method of Multipliers (ADMM). Its iterative solution scheme is based on an operator splitting algorithm, which is suitable to efficiently solve large-scale variational problems with parallel processing. It has been applied successfully to several geotechnical problems, including earth pressures and deep excavations (Antão et al. 2008, Antão et al. 2011, Deusdado et al. 2015). This software is capable of producing either lower or upper bounds depending on the type of the finite element used. In this work, 6-noded triangles (with the usual 3 corner nodes and 3 midside nodes) are used to obtain strict upper bounds. In these elements, the velocity field approximation is assumed to be quadratic and the approximation of both the strain rate and stress fields to be linear. Whereas to obtain strict lower bounds, a more peculiar 7-noded triangular element is used (with 2 nodes associated to each edge of the element, plus an additional node located at the baricenter). In this element the strain rate and stress fields approximations are assumed to be linear. Conversely, the velocity field is incoherent and discontinuous. It is worth mentioning that in the current formulation

the velocity, strain rate and stresses fields are independently approximated.

3 NUMERICAL ANALYSIS

The geometry of the problem analysed is presented in Figure 1. Two cases were considered: Case 1, considering a smooth interface and not modelling the wall itself, and Case 2, which considers the wall and a rough soil-to-wall interface.

A set of calculations was initially performed considering Case 1 and Case 2 assuming depth d to be large enough to have no influence on the mechanism and on the results. Different h/B ratios were assumed (between 0.2 and 3). For Case 2 several e/h ratios were considered (in the range 0.005 to 0.1).

Another set of calculations was performed, using Case 1, considering different d/h ratios (from 1/3 to 3).

4 RESULTS OBTAINED FROM CASE 1 FOR DEEP RIGID STRUTUM

Results obtained from Case 1 for deep rigid stratum are included in Table 1 (column 2) and are represented in Figure 2. This figure also represents other available solutions, such as those from Terzaghi (1943) and from Bjerrum & Eide (1956) and the numerical limit analysis results from Ukritchon et al. (2003).

Both the table and the figure allow the following remarks: i) the values obtained for the stability numbers are between Terzaghi's and Bjerrum and Eide's solutions; ii) the results obtained are slightly closer to Terzaghi's solution for small values of h/B and to Bjerrum and Eide's for greater values of h/B ; iii) upper and lower-bound solutions significantly improve the ones from Ukritchon et al. (2003); iv) upper and lower-bound results are quite close, with

Figure 2. Results of the stability numbers obtained from Case 1. Comparison with other solutions.

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Table 1. Results of the stability number $\gamma h/c_u$ obtained from the numerical upper and lower-bound calculations for Cases 1 and 2.

h/B	Case 1		Case 2			
			e/h			
			0*	0.005	0.01	0.02
Lower-bound (LB)						
0.2	5.733	5.739	5.736	5.739	5.744	5.744
0.5	6.186	6.196	6.199	6.213	6.240	6.372
1	6.579	6.580	6.597	6.621	6.671	7.005
1.5	6.908	6.911	6.936	6.968	7.023	7.462
2	7.168	7.173	7.201	7.240	7.304	7.815
2.5	7.393	7.399	7.430	7.474	7.545	8.118
3	7.589	7.596	7.630	7.678	7.756	8.382
Upper-bound (UB)						
0.2	5.745	5.740	5.748	5.750	5.756	5.842
0.5	6.222	6.232	6.236	6.248	6.272	6.402
1	6.642	6.645	6.666	6.688	6.735	7.076
1.5	6.944	6.949	6.974	7.004	7.066	7.506
2	7.214	7.222	7.250	7.286	7.360	7.874
2.5	7.449	7.458	7.489	7.530	7.613	8.190
3	7.659	7.667	7.698	7.747	7.839	8.470

*obtained by linear regression (Figure 5)

the maximum error margin being around 1%. This error was evaluated dividing the difference between the upper and the lower-bound stability numbers by the lower-bound ones.

Results from Case 1 should, therefore, be almost exact. The following proposed equation fit the results:

$$\frac{\gamma h}{c_u} = (2 + \pi) \left\{ 1 + 0.815 \arctan \left[0.359 \left(\frac{h}{B} \right)^{0.55} \right] \right\} \quad (6)$$

This equation was obtained using the lower-bound values, for an estimate on a (slightly) conservative side. This equation is also shown in Figure 2 and, as it can be seen, it represents an accurate approximation to the obtained numerical results.

5 RESULTS OBTAINED FROM CASE 2 FOR DEEP RIGID STRATUM. INFLUENCE OF THE WALL THICKNESS

Results obtained from Case 2 for deep rigid stratum, for different values of the wall thickness ratio e/h , are shown in Table 1 (columns 4 to 7). The evolution of collapse values of the stability number with h/B for two e/h ratios are represented in Figure 3 (only two ratios were represented in

Figure 3. Results of the stability numbers obtained from Case 1 and from Case 2 as a function of h/B for two e/h ratios.

the figure to keep it legible). It can be seen from the table and the figure that a greater thickness of the wall leads to greater values of the stability number. This is expected, as a greater wall thickness means that a greater base of the wall is in contact with the soil below, which results in a more restricted problem and therefore in greater collapse loads.

Figure 4 shows the yield condition zones (for LB calculations) and plastic deformation zones (for UB calculations) for the different e/h ratios analysed for Case 2 and also for Case 1, for $h/B = 2$. From these figures mechanisms corresponding to the collapse situations can be inferred. It can be seen that greater wall thickness involve larger volumes of soil in the mechanisms and for the lesser thickness the mechanisms obtained are practically the same and quite close to the mechanism for Case 1. The mechanisms also show that the wall has no other effect besides the one at the base, as the soil behind it always moves as a rigid block.

Similar mechanisms between Case 1 and Case 2 for the lesser e/h ratio results in similar values of the stability number. This can be seen in Table 1, comparing values in columns 2 and 4. Also, the results from this table are represented in Figure 5 as a function of e/h , for different h/B ratios and a linear relation is found between e/h and $\gamma h/c_u$. For each one of the h/B ratio (and for both LB and UB results) a linear regression was performed and the obtained straight lines are represented in the figure. Results from Case 1 are also plotted and it can be seen that they match the y-intercept of the linear regression lines. This can also be seen in Table 1, comparing columns 2 and 3.

Case 1, therefore, represents a limit situation for Case 2 when e/h tends to zero. For this limit situation the most conservative results are obtained and in the next calculations only this case will be considered.

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Figure 4. Yield condition zones (LB) and plastic deformation zones (UP) for $h/B = 2$ obtained for Case 1 and for Case 2, for different e/h ratios.

Figure 5. Results of the stability numbers obtained from Case 2 as a function of e/h for different h/B ratios. Lines are the result of linear regression and results from Case 1 are shown for $e/h = 0$.

6 INFLUENCE OF THE DEPTH OF THE RIGID STRATUM

Considering Case 1, a series of numerical LB and UB calculations were performed, considering different d/h ratios. Results are presented in Table 2 and in Figure 6. The last column in Table 2 shows

the results for deepstratum (not affecting the results) and are the same shown in Table 1 for Case 1.

It can be seen, again, that results from LB calculations are quite close to the ones from UB calculations. The finite element meshes do not always have the same refinement; larger problems, involving greater soil volumes, were modelled using slightly larger finite elements. This could be the reason for some apparent very small inconsistencies on the results presented in table 2, usually on the third decimal place. It can also be seen that for a given d/h ratio values of the stability number are constant with h/B up to a certain h/B ratio. Although with significant differences in the values, the same type of results are obtained from Terzaghi's and Bjerrum and Eide's methods. This is caused by the fact that for larger excavations (lesser h/B ratios), mechanisms are controlled by the d/h ratio and are therefore independent of the excavation width. For narrower excavations their width becomes relevant. This is also true for Terzaghi's and Bjerrum and Eide's solutions, as also represented in Figure 6 and can be observed in the mechanisms inferred from the plastic deformation zones obtained from the UB calculations represented in Figure 7. In this Figure only the UB results are shown, but similar results from representing the yield condition zones are obtained from LB calculations.

Results obtained from the numerical calculations, as can be seen in Figure 6, are therefore represented by a horizontal line for a given d/h ratio, but this line merges smoothly in the curve

Table 2. Results of the stability numbers γ/c_u obtained from the numerical upper and lower-bound calculations for Case 1 for different d/h ratios.

h/B	d/h				
	1/3	1/2	1	3	deep
Lower-bound (LB)					
0.2	7.679	7.153	6.512	5.886	5.733
0.5	7.679	7.153	6.512	6.196	6.186
1	7.679	7.153	6.616	6.595	6.579
1.5	7.679	7.153	6.908	6.908	6.908
2	7.678	7.262	7.168	7.168	7.168
2.5	7.700	7.431	7.393	7.393	7.393
3	7.785	7.605	7.599	7.627	7.589
Upper-bound (UB)					
0.2	7.760	7.205	6.537	5.894	5.745
0.5	7.760	7.205	6.537	6.210	6.222
1	7.760	7.205	6.643	6.620	6.642
1.5	7.760	7.201	6.944	6.944	6.944
2	7.751	7.317	7.214	7.214	7.214
2.5	7.777	7.493	7.449	7.467	7.449
3	7.870	7.676	7.658	7.681	7.659

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for deep stratum represented in Figure 2. This smooth transition represents a situation where both the excavation width and the depth of the rigid stratum are affecting the results. If this smooth transition is not considered, the problem can be described by equation 6 for deep stratum and by the different values of the stability number that depend on d/h . Values of the stability numbers corresponding to different d/h ratios are represented in Figure 8. For this figure, additional calculations were performed for d/h equal to 1.5, 2, 2.5 and 3 using a small value of the h/B ratio of 0.1, corresponding to a very large excavation, to ensure that the excavation width does not affect the results.

Results represented in Figure 8 can be obtained with the following proposed equation (obtained using the lower-bound results):

$$\frac{\gamma h}{c_u} = 1 + \pi + \exp \left[-0.409 + 1.275 \left(\frac{d}{h} \right)^{-0.246} \right] \quad (7)$$

Figure 6. Influence of d/h ratio. Results of the stability numbers obtained in the calculations compared with other solutions.

7 PROPOSED EQUATION FOR BASAL INSTABILITY IN DEEP EXCAVATIONS IN CLAYEY SOILS

The quality of the numerical results obtained, that can be shown by the very small gap between the LB and the UB calculations, justifies a proposal for the basal instability of deep excavations in clayey soils, for the case with no wall embedded length and possible influence of rigid stratum below the excavation, based on equations 6 and 7.

Such proposal allows the determination for the collapse situation of the stability number, given the depth ratio, d/h , of the rigid stratum and the h/B ratio, using:

$$\left[\frac{\gamma h}{c_u} \right] \left(\frac{h}{B}, \frac{d}{h} \right) = \max \left\{ \left[\frac{\gamma h}{c_u} \right]_{eq. 6}; \left[\frac{\gamma h}{c_u} \right]_{eq. 7} \right\} \quad (8)$$

Such equation results in the values represented in Figure 9, where the proposal is superposed with the results from the numerical calculations that allowed the proposal.

Figure 8. Values of $\gamma h / c_u$ for the cases where d/h influences the results (horizontal lines in Figure 6).

Figure 7. Plastic deformation zones obtained from the UB calculations for different d/h and h/B ratios.

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Figure 9. Proposal for the calculation of basal instability in clayey soils superposed with the numerical calculations that lead to such proposal.

8 CONCLUSIONS

The problem of the two-dimensional basal stability of deep excavations in clayey soils under undrained conditions was addressed with upper and lower-bound limit analysis using the finite element program *mechpy*, developed within the authors' research team.

The wall was assumed with no embedded length and a rigid stratum was considered at a variable depth. Two cases were initially analysed: one where the wall was not modelled (Case 1) and one other where the wall was modelled and different wall thicknesses were considered (Case 2). Case 1 was found to be a limit situation of Case 2 when the wall thickness tends to zero and to give more conservative results. The main study was therefore conducted using Case 1 only.

Results for deep rigid stratum (not affecting the stability numbers) and for different depths of the rigid stratum were obtained using UB and LB calculations. The two approaches—LB and UB—were found to give very close results, usually with a margin of less than 1%.

An equation for the calculation of the stability numbers for the case analyses was proposed, derived from the numerical results.

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