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QUANTITATIVE EASING IN THE SHADOWS: MODELLING THE TRANSMISSION OF ASSET PURCHASES THROUGH SHADOW RATE DYNAMICS

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Abstract

This paper investigates the relationship between the Federal Reserve's unconventional monetary policies and estimates of the shadow interest rate from 2003 to 2024. Using estimates of Wu and Xia (2016) and Krippner (2015) for the shadow rate, the effects of asset purchases are quantified across several quantitative easing episodes, including the Great Financial Crisis and the COVID-19 pandemic. Findings reveal that asset purchases significantly reduced the shadow rate, but there is considerable heterogeneity in their effectiveness across programs, with forward guidance also playing an important role.

Keywords: Balance sheet policy; Shadow rate; Effective Lower Bound; Monetary transmission mechanism

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1 Introduction

For decades, central banks around the globe pursued monetary policy mandates almost exclusively by influencing short-term interest rates. However, this approach reached its limits in the aftermath of the 2007-2008 great financial crisis (GFC), when central banks effectively lowered nominal short-term interest rates to levels close to their effective lower bound (ELB). In an attempt to stimulate economic growth, they consequently turned to new tools, commonly referred to as unconventional monetary policies (UMP). These include a multitude of mechanisms, such as quantitative easing (QE), forward guidance (FG), and seldomly also negative interest rate policies (Sims and Wu 2019). While all play an important role from a monetary policy (MP) perspective, quantitative easing (QE) swiftly emerged as the tool employed the most frequent and to the largest extent after the GFC (Gagnon et al. 2011; Gambacorta, Hofmann, and Peersman 2014). QE refers to central banks purchasing large amounts of public and private assets to reduce the outstanding bond supply for investors, increase asset prices and in turn apply downward pressure on long-term interest rates, thereby incentivizing consumption and investment (Engen, Laubach, and Reifschneider 2015; Chung et al. 2011).[1] The Federal Reserve Bank of the U.S. (Fed), for instance, grew its balance sheet to $4.5 trillion by 2014, compared with only $900 billion prior to the GFC (Gulati and Smith 2022) (Figure 1). Immediate market effects indicated that QE was effective in easing financial conditions during the GFC as the first round of QE in the United States (U.S.) lowered 10-year treasury notes and corporate bonds yields by about half a percentage point (Chung et al., 2011).

Under such an ELB scenario, official policy rates lose not only their place as a policy tool but also much of their predictive power since they no longer reflect the full range of measures adopted by central banks. While this amplifies the difficulty of assessing the MP stance over

[1] The quantitative easing definition is adopted from Sims and Wu (2020) whose model characterizes both public and private bonds as large-scale asset purchases.

the long term, a consistent indicator is still widely viewed as a prerequisite for quantifying its effects (Hamilton and Wu 2012). One key empirical concept suggested for this scenario that has gained new relevance in recent years is the shadow rate. Originally introduced by Black (1995) as a means to model interest rates as financial options, the shadow rate constructs a hypothetical short-term interest rate that measures the nominal interest rate if not constrained by the ELB (Bauer and Rudebusch 2016). This concept allows market operations conducted by central banks to be evaluated in terms of the interest rate, even when markets operate at the ELB. This paper follows the definition by Black (1995):

$$r_t = max(0, s_t) \tag{1}$$

where r_t refers to the nominal interest rate, and s_t denotes the shadow rate. While shadow rates have been widely used as policy evaluation tools for ELB periods, the understanding of their behaviour in the context of expansive asset purchasing programs is surprisingly limited. This becomes especially relevant given that shadow rates have strong correlation with financial indices, which in turn closely track the central bank's balance sheet in the U.S. Wu and Zhang (2019), for instance, report a correlation coefficient of -0.94 between the Wu and Xia (2016) shadow rate and the Fed's balance sheet during QE phases, hinting at a significant link between QE and shadow rate fluctuations. To the best of the author's knowledge, Sims and Wu (2019) provide the only estimate in mapping QE with a shadow rate and find that the first three rounds of QE accounted for two thirds in the variation of the Wu and Xia (2016) shadow rate. While considering the variation of QE compared to other UMP and channels, they do not quantify the effect of any one period and solely evaluated QE until 2014. Sims and Wu (2020) expand this estimate by constructing a model shadow rate using the Fed's balance sheet from November 2008 to October 2014 and find that it closely resembles the Wu and Xia (2016) shadow rate (Figures 2 and 3). However, they also remain silent about the size of the relationship between this measure and the QE-driven shadow rate during various purchasing rounds at the ELB.

Drawing on these promising findings, a research gap emerges from the magnitude and persistence of the effect of QE on the shadow rate. Specifically, this paper aims to establish a relationship between the scale of asset purchases and variation in the shadow rate by unveiling the exact periods where QE effectively exerted influence. The analysis starts mapping several QE rounds for the Wu-Xia shadow rate in the U.S. and later extends the model with forward guidance events and the shadow rate by Krippner (2015) for robustness, thereby comparing effects between arguably the two most important UMP tools. Accordingly, this paper contributes to the existing literature by providing a measure to quantify the short-run relationship between UMP and the shadow rate across several ELB periods, including both the GFC and COVID-19 pandemic, to improve the understanding of monetary transmission mechanisms of QE under different MP environments. Main findings suggest that QE played a vital role in reducing the shadow rate in the immediate aftermath of the GFC and during the COVID-19 pandemic, while FG was relatively more important during the second and third purchasing round between 2011 and 2014. The paper is structed as follows: Section 2 provides a summary of the existing literature, followed by a description of the methodological approach in section 3. Section 4 introduces the data set and section 5 presents and discussed the results for the baseline models and extensions for robustness. Section 6 presents alternative models results. In section 7, limitations and idea for further research are discussed. Section 8 concludes.

2 Literature Review

For the shadow rate to be a decent proxy for the official policy rate, it needs to reflect the overall MP stance well and remain consistent under both conventional and unconventional MP regimes (Hirata et al. 2024). For the U.S., Claus, Kim, and Kato (2014) show that the shadow rate tracks the MP stance during ELB periods just as well as the policy rate during non-ELB periods. Kuusela and Hännikäinen (2017) add that shadow rates are well-suited to predict U.S. inflation for both non-ELB and ELB periods, regardless of model specification or forecast horizon. Bernanke, Kiley, and Roberts (2019) evaluate the effectiveness of MP at the ELB for a total of ten MP tools in the U.S. and find that shadow rates which take into account the weighted sum of inflation and output gaps provide better economic results during the ELB than interest rates implied by any of the remaining tools, such as Taylor rules. The dynamics between the shadow rate and the ELB is investigated by Jones, Kulish and Morley (2022) who explicitly account for the ELB. Their model finds that a policy response constrained by the ELB effectively contracts and increases the shadow rate, but that UMP such as forward guidance are expansionary and thereby decrease the shadow rate, mainly through extending the expected duration of ELB periods. Rezende and Ristiniemi (2022) update the model to the 'new normal' times where UMP has been established as a standard policy tool. They estimate shadow rates without a ELB constraint for several countries, including the U.S. and the Euro area, and find that expansionary MP that includes QE but also FG, effectively lowers the shadow rate. Wu and Zhang (2019) extend this by showing that the macroeconomic effects of UMP, including QE, are the same as those from negative shadow rates for the U.S. (Hirata et al.2024). Consequently, shadow rates are generally considered a very accurate measure of MP under both ELB and non-ELB regimes.

In the literature, most shadow rates estimated are constructed using one of three models: A three-term Gaussian affine shadow rate term structure model (SRTSM) (e.g., Wu and Xia 2016), a two-term affine SRTSM (such as Krippner 2015) or a shadow rate dynamic factor model (SRDFM) (e.g., Lombardi and Zhu 2018). SRTSMs utilize the term structure of interest rates, where short-term interest rates, commonly proxied by three-month treasury bills, are driven by the long end of the yield curve, which makes them responsive to financial variables other than MP (Lombardi and Zhu 2018). This implies that SRTSMs can have shadow rates different from official policy rates (Jones, Kulish and Morley 2022). Shadow rate DFMs, on the other hand, not only incorporate asset purchases but rather all UMP instruments (Anderl and Caporale

2022). A comprehensive overview of the applicability of shadow rates in both countries with a permanent inflation target (e.g., Canada or the United Kingdom) and those that pursued an inflation mandate infrequently (e.g., U.S. or the Euro area) is provided by Anderl and Caporale (2022). They compare shadow rates to official rates and three Taylor rules and find that shadow rate models that account for UMP like QE better capture MP shocks during crisis periods.

Given the vast structural variety in shadow rate models and their forecasting accuracy for macroeconomic variables, it is important to motivate the choice of a shadow rate for empirical work. Here, the SRTSMs of Wu and Xia (2016) and Krippner (2015) have produced consistent estimates for most monetary regions and are accordingly utilized often. Hohberger, Gerke, and Scheufele (2023), for instance, base a large-scale three-region dynamic stochastic general equilibrium model spanning twenty years and including ELB periods on the shadow rate of Wu and Xia (2016). Kuusela and Hännikäinen (2017) compare the two-factor TSM from Wu and Xia (2016) to Krippner's three-factor TSM (2015) and find that the former is superior in inflation forecasting accuracy since it more closely fits the yield curve. Krippner (2015) provides an extension on the robustness of several shadow rate models and reports slightly better performances of two-factor- over three-factor TSMs. The latest vast use for Wu and Xia (2016) and Krippner (2015) is provided by Hirata et al. (2024) who report the model strength using Kendall's rank correlation coefficient and find that both shadow rates are indeed well suited for UMP analysis (Table 5).

3 Methodology

To investigate a potential relationship between the two policy variables in a time-series analysis and accordingly quantify the magnitude of QE in driving the shadow rate, the paper starts by using a simple linear regression model that allows to model different rounds of QE in a coherent way. The model employs an ordinary least squares (OLS) estimator which can handle occasionally binding constraints like the ELB much better than, for instance, vector autoregressive (VAR) models which are informative about the interplay between two variables but often aggregate effects which would exacerbate the isolation of QE periods (Gerke, Kienzler, and Schee 2021). Arguably the most important reason in favour of this simple linear regression is that it provides period-specific coefficients, thus ensuring clear interpretability of how asset holdings affect the shadow rate in different policy contexts. The OLS estimator then provides the vector of coefficient estimates that minimizes the sum of squared residuals.

To ensure the soundness of the econometric specification, all variables in the model must be stationary. A time series is considered stationary when it does not follow a trend in the long run and its statistical properties, such as mean, variance and autocovariance, are constant over time (Wooldridge 2016). To induce stationarity, the time series is differenced p times such that the process becomes integrated of order p. Non-stationary time series drift over time while also responding to permanent shocks, which can cause serious econometric issues, such as spurious regressions, where even significantly estimated coefficients are meaningless. Stationarity of each variable is tested for with the Augmented Dickey-Fuller (ADF) test (Dickey and Fuller 1979). Under the null, the time series is deemed non-stationary, implying that the data would exhibit a trend, and must thus be differenced. Based on the ADF results, the shadow rate and the control variables are stationary at a 10% significance level while Fed assets are highly non-stationary (Table 6). Accordingly, all variables are differenced once to ensure compatibility in the OLS model, which now predicts the short-run effect of the first-differenced Fed assets on the shadow rate in first differences. The model follows as:

$$\begin{aligned} \Delta S_t = \alpha &+ \beta_1 \Delta FedAssets_t QE_1 + \beta_2 \Delta FedAssets_t QE_2 + \beta_3 \Delta FedAssets_t QE_3 \\ &+ \beta_4 \Delta FedAssets_t QE_4 + \beta_5 \Delta FedAssets_t NonQE \\ &+ \beta_6 \Delta FederalFundsRate_t NonQE + \varepsilon_t \end{aligned} \tag{2}$$

where S_t represents the Wu and Xia (2016) U.S. shadow rate, $FedAssets_t$ are the main regressing variables of interest in period t with coefficients β_t, α is the intercept and ε_t the error term. The regressors are interaction terms for distinct QE periods during ELB regimes and asset holdings outside QE periods which are complemented by an interaction term to account for the federal funds rate as the main policy tool under a non-ELB regime. The regressors for each period are constructed as the central bank's balance sheet interacting with a dummy variable taking the value of 1 during the respective period and 0 otherwise. Incorporating QE in such a way allows to maintain a baseline balance sheet effect during non-ELB periods and strictly compare QE period coefficients during ELB periods. This is relevant to cover the pre-GFC period where QE was not yet used but also in line with asset holdings potentially influencing economic variables, such as real interest rates or yields (Gagnon et al. 2011). Although it is important to control for macroeconomic conditions, this paper refrains from the inclusion of variables such as inflation, unemployment or GDP growth, as this would potentially violate the exogeneity assumption. Given that QE is designed to target inflation (and unemployment in the case of the Fed), the model would likely suffer from reverse causality, leading to biased OLS estimators. This methodological approach is in line with Bernanke (2020) who stresses that variables subject to influence from regressors should not be controlled for. Accordingly, the model only accounts for overall financial conditions through the volatility index of the Chicago Board Options Exchange (VIX), and for forward guidance through FG announcement date dummies. The OLS hence enables the consistent estimation of marginal QE effects to enable a comparison of policy decisions across regimes. To interpret the shadow rate in level terms, first differences in t can be added to the value at time $t-1$ such that:

$$S_t = S_{t-1} + \Delta S_t \tag{3}$$

3.1 OLS assumptions

To produce unbiased and consistent estimators, the OLS model needs to fulfil several assumptions covered by the Gauss-Markov Theorem (Wooldridge 2016). For the model to be the correct functional specification for the dataset, it has to be linear in its parameters, implying that changes in the dependent variable must be related to the regressors a linear fashion. Note that while parameters must be strictly linear, the OLS is sufficiently flexible in the construction of its variable such that creating interaction terms or applying first differences does not violate this assumption. The linearity assumption is commonly evaluated through residual analysis, scatter plots, and specification tests (Wooldridge 2016). If the actual form of the data were non-linear, the model would produce biased estimates, leading to false conclusions at extreme values. The Ramsey RESET test shows roughly a linear distribution, validating that the OLS is indeed appropriate as an estimation technique (Table 7). In addition to the form of the data, the independent variables must also not be endogenously correlated with each other in a perfect way. In time series, significant correlations can arise due to trends or persistent patterns, especially in non-stationary data. This assumption ensures that the model matrix is invertible and has full rank such that the OLS estimator is reliable and has constant variance. If multicollinearity was present, a unique identification of the individual model parameters would not be possible, and the individual effects of QE compared to conventional MP measures would be rendered unreliable. In this model, it would occur if in addition to the QE interaction terms that use Fed's assets the model also included the asset holdings in level terms or failed to use a non-QE period for reference. A variance inflation factor (VIF) larger than 10 provides strong evidence of multicollinearity in the data (Wooldridge 2016). The results of the VIF show that this condition is not present for regressors in any of the models (Table 8). Next, autocorrelation can often occur in time series that exhibit persistent shock. If this assumption is violated, the error term at time t is correlated with the previous error at $t-1$, which is known as first-order autocorrelation and can be represented by:

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t \tag{4}$$

where ε_t and ε_{t-1} are the error terms at t and $t-1$, ρ represents the autocorrelation coefficient and u_t is a white noise process. While coefficients will not be biased, the standard errors would be wrongly estimated, potentially causing spurious regressions that produce significant but meaningless estimates. The error terms must therefore not be correlated. This issue might arise in this model if the actual effect of asset purchases or their expectation persist beyond one period. Given that the real effect of QE stems from both the signaling channel of announcements and purchasing amounts, balance sheet policies could affect the shadow rate over multiple months. The Breusch-Godfrey test evaluates the pattern of residuals in the model (Wooldridge 2016). Assuming the presence of autocorrelation under the null, autocorrelation is likely present in the models (Table 7). Accordingly, the model adopts heteroskedasticity and autocorrelation consistent (HAC) Newey-West robust standard errors to reliably obtain consistent estimators.

The OLS model also needs to follow the homoskedasticity assumption, which states that the variance of the error term must be constant across all observations. This condition is especially relevant in macroeconomic models where volatility might not be randomly distributed across observations because of financial turmoil or policy changes. Formally:

$$Var(\varepsilon_t \mid X_t) = \sigma^2 \tag{5}$$

where σ^2 is a constant parameter. In this specification, heteroskedasticity might arise if the volatility of MP effects differs systematically between conventional MP and UMP regimes, for instance, if supply shocks are larger in the former. Furthermore, residuals might be more dispersed during the first round of QE due to the scale of the recession. The Breusch-Pagan test assumes homoscedastic errors under the null and tests for the presence of heteroskedasticity, which would increase the likelihood of Type I errors (Wooldridge 2016). Its results indicate that errors are homoscedastic (Table 7), but Newey-West standard errors are still employed.

Next, the error term must also follow the zero conditional mean assumption, which rules out correlation of the error term with any regressors to ensure they are fully exogenous. Formally:

$$E[\varepsilon_t \mid X_t] = 0 \tag{6}$$

Arguably the most relevant condition, a violation would lead OLS estimators to become both biased and inconsistent. This often occurs due to unobserved factors hidden in the error term but also because of omitted variable bias or reverse causality. Reverse causality is unlikely since the interaction terms are constructed exogenously and the shadow rate acts a descriptive tool rather than being set explicitly during ELB periods. If the shadow rate correlates with unobserved variables which the central bank also considers when deciding on the size of QE, the interaction terms might not be completely exogenous. As introduced above, the model was carefully specified to ensure exogeneity to the largest extent possible. Finally, residuals need to be normally distributed, with a zero mean and constant variance:

$$\varepsilon_t \sim N(0, \sigma^2) \tag{7}$$

While not directly related to the unbiasedness or consistency of OLS estimators, this assumption has vast implications for confidence intervals. The QE periods are characterized by increased volatility and financial market turmoil, and consequently the data might exhibit unusual skewness or kurtosis into the residuals, especially in months with distinct shifts in the shadow rate. Testing for normality of the residuals can be done by the Jarque-Bera Test or visually examining the distribution of residuals, which indicates that errors are indeed not normally distributed in most models (Figures 5, 7, 9 and 11). However, for sufficiently large samples as in this dataset, the Central Limit Theorem (CLT) states that distribution of OLS estimators will approximate normality regardless of the error distribution (Wooldridge 2016). In addition, the use of HAC robust standard errors further ensures consistency for hypothesis testing. As such, the fulfilment of all Gauss-Markov conditions leads to best linear unbiased estimators (BLUE) that are the basis for sound economic interpretation of model coefficients.

4 Data description

The dataset contains monthly starting values for the Wu and Xia (WX) (2016) shadow rate and monthly averages for the Krippner (KR) (2015) shadow rate. Observations begin in January 2003 and extend to June 2023 for WX (n = 246) and to June 2024 for KR (n = 258) for all variables in the respective models to account for both sufficient ELB and non-ELB periods.[2] Total assets are reported by the Fed using end-of-month values.[3] Following Sims and Wu (2020), the central bank's balance sheet is transformed into logarithmic form for stable variance and ensured interpretability. The federal funds overnight rate (FFR) was used as the main policy rate for the US set by the FED. The VIX data was aggregated from daily into monthly estimates.[4] Forward guidance classifications are adopted from Sutherland (2022) to distinguish whether a FG event occurred in the respective period in two ways: first as normal binary indicators (1 if a FG event occurred during period t, 0 elsewise), and then using qualitative measures that account for the nature of the FG event in t (-1 = dovish, 0 = neutral, +1 = hawkish). The classification of QE periods listed in Table 9 is adopted from the Fed New York (2024) and Labonte (2022). Concretely, the data set encompasses three QE purchasing rounds from December 2008 to March 2010 (QE1), November 2010 to June 2011 (QE2) and September 2012 to October 2014 (QE3), as well as the normalization period from 2015 until 2019 and the response to the COVID-19 pandemic from March 2020 to February 2022 (QE4).

[2] The shadow rate of Wu and Xia (2016) is from the personal website of Wu. The measure of Krippner (2015) is also from the author's personal website.

[3] Fed assets include security holdings, US treasuries, US T-bills and US T-bonds and are from the FRED dataset of the St. Louis Fed.

[4] The VIX and federal funds rate are also taken from the FRED dataset of the St. Louis Fed.

5 Results

The linear model aims to compare the relative effect of QE on shadow rate variation between purchasing rounds by constructing QE-specific periods as regressors. The following section summarizes the results that contain the relative magnitude and significance of the key coefficients for different model variations using the Wu and Wia (2015) shadow rate, including versions with FG and the Krippner (2016) shadow rate. Note that since all time series are integrated of order one and asset holdings are taken in logarithmic form, each QE-period coefficient can be interpreted as the relative basis point change in the shadow rate given a one percent increase in Fed assets.

5.1 Model considering only asset purchases

The baseline model includes the change in Fed assets in the four different purchasing rounds considered and outside of these periods. Table 1 presents the results including different controls.

Table 1: OLS Results for the Wu and Xia (2016) shadow rate

Variable	**Model 1 (QE)**	**Model 2 (QE + VIX)**	**Model 3 (QE+ FFR)**	**Model 4 (QE + FFR+ VIX)**
Constant	0.04 (0.028)	0.16*** (0.049)	0.04 (0.024)	0.10* (0.049)
Fed Assets QE1	-1.66** (0.789)	-1.60*** (0.610)	-1.64** (0.774)	-1.61** (0.666)
Fed Assets QE2	-1.50 (1.051)	-1.57 (1.021)	-1.29 (0.944)	-1.37 (0.916)
Fed Assets QE3	-4.32** (2.146)	-5.52*** (2.082)	-4.06** (2.046)	-4.69** (1.985)
Fed Assets QE4	-2.09*** (0.427)	-1.27** (0.618)	-2.05*** (0.419)	-1.62*** (0.611)
Fed Assets non-QE	-0.54*** (0.146)	-0.96 (0.999)	0.51* (0.278)	0.67* (0.369)
FFR non-QE	–	–	0.70*** (0.101)	0.63*** (0.118)
VIX	–	-0.01* (0.003)	–	-0.01 (0.003)
Adj. R^2	**0.033**	**0.060**	**0.127**	**0.131**
Observations	**246**	**246**	**246**	**244**

Note: Standard errors are in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

In the first specification without controls, the coefficients during each period indeed indicate an inverse relationship between expansionary MP through QE and changes in the shadow rate. Concretely, a one percent increase in Fed asset holdings is associated with a 1.66 basis point (bp) decrease in the shadow rate during QE1, a 4.32 basis point decrease during QE3 and a 2.09 basis point decrease during QE4, with the 1.50 basis point decrease in QE2 not being statistically significant (Model 1). Outside of QE periods, asset holdings also exhibit a significant, but lower effect on the WX shadow rate at 0.54 basis points (bps). Results are almost unaffected by controlling for financial market conditions as proxied by the VIX. In the third and fourth specifications the FFR is included. During normal times, the variance in the WX shadow rate is accounted for more through the FFR (0.70), while Fed assets become positively correlated with the shadow rate (0.51), pointing towards fundamentally different transmission mechanisms between conventional and unconventional MP regimes. In all QE periods, the coefficients of Fed assets are rather robust to the inclusion of the FFR and VIX (Model 4). Specifically, the effects of QE1 and QE4 converge to virtually the same magnitude, with coefficients at -1.61 and -1.62 respectively, while QE2 remains insignificant and QE3 exhibits a substantially larger effect, where a one percent increase in Fed assets corresponded to a 4.69 bp drop in the WX shadow rate. The main adjustment occurs during non-ELB periods where Fed holdings move positively with the WX shadow rate. The overall model fit as measured by the adjusted R^2 lies at around 3.3% for the base model but improves to 13.1% when including the FFR and VIX. While appearing modest, this explanatory power is reasonable for a model in first differences (Rogers, Scotti, and Wright 2014).

5.2 Model considering asset purchases and forward guidance

Table 2 extends the analysis by incorporating forward guidance (FG) as a UMP measure during all periods in the normal and qualitative manner as introduced, controlling for the FFR and VIX.

As before, each specification is estimated in base terms and with the FFR during non-ELB times. The model estimated is thus:

$$\Delta S_t = \alpha + \beta_1 \Delta FedAssets_t QE_1 + \beta_2 \Delta FedAssets_t QE_2 + \beta_3 \Delta FedAssets_t QE_3 + \beta_4 \Delta FedAssets_t QE_4 + \beta_5 \Delta FedAssets_t NonQE + \beta_6 \Delta FederalFundsrate_t NonQE + \gamma_1 FG_t QE_1 + \gamma_2 FG_t QE_2 + \gamma_3 FG_t QE_3 + \gamma_4 FG_t QE_4 + \gamma_5 FG_t NonQE + \varepsilon_t \quad (8)$$

Table 2: OLS Results for the Wu and Xia (2016) shadow rate - with Forward Guidance

Variable	**Model 5 (QE + Normal FG + FFR)**	**Model 6 (QE + Normal FG + FFR + VIX)**	**Model 7 (QE + Qualitative FG + FFR)**	**Model 8 (QE + Qualitative FG + FFR + VIX)**
Constant	0.10 (0.07)	0.25 (0.08) ***	0.03 (0.02)	0.17 (0.08) *
Fed Assets QE1	-1.66 (0.81) **	-1.69 (0.72) **	-1.54 (0.76) **	-1.64 (0.72) *
Fed Assets QE2	4.45 (1.03) ***	4.69 (1.21) ***	4.45 (1.03) ***	4.59 (1.15) **
Fed Assets QE3	-0.66 (1.90)	-0.75 (1.85)	-1.28 (1.84)	-2.12 (1.79)
Fed Assets QE4	-0.91 (0.51) *	-0.25 (0.62)	-1.14 (0.34) ***	-0.87 (0.65)
Fed Assets Non-QE	0.57 (0.29) *	-0.04 (0.42)	0.65 (0.26) **	-0.17 (0.44)
FFR Non-QE	0.73 (0.10) ***	0.13 (0.09)	0.76 (0.12) ***	0.12 (0.09)
FG QE1	-0.17 (0.08) *	-0.12 (0.07) ***	0.10 (0.06)	0.24 (0.05) **
FG QE2	-0.23 (0.07) ***	-0.27 (0.06) ***	0.17 (0.05) ***	0.13 (0.05) *
FG QE3	-0.15 (0.08) *	-0.28 (0.11) **	0.06 (0.06)	0.33 (0.10) **
FG QE4	-0.29 (0.11) **	-0.04 (0.05)	0.29 (0.09) ***	0.09 (0.04) **
FG Non-QE	-0.08 (0.07)	-0.01 (0.00)	0.02 (0.02)	-0.00 (0.01)
VIX	–	-0.01 (0.00) *	–	-0.04 (0.02) **
Adjusted R^2	**0.280**	**0.127**	**0.278**	**0.154**
Observations	**240**	**240**	**240**	**240**

FG events have little but mostly significant effects during non-QE periods. Concretely, binary FG is consistently associated with a decrease in the WX shadow rate of between 15 basis points (QE3) and 28 basis points (QE4) (Model 4), while qualitative FG can be linked to a shadow rate *increase* between 10 basis points (QE1) and 29 basis points (QE4) (Model 7), underlining the importance of accounting for the nature of FG. Accounting for both the FFR and the VIX, the effects are largest for QE1 (24 bp increase) and QE3 (33 bp increase) and all significant with qualitative FG, while similar in revered size with binary FG. For asset purchases, the QE1 coefficient remains stable across specifications, such that the true effect of a one percent increase in Fed assets corresponded to a drop of about 1.64 bps in the WX shadow (Model 8). The inclusion of FG substantially does, however, alter the effect of asset purchases during QE2, which now reports a highly significant *increase* of 4.45 bps in the WX shadow rate, starkly contradicting previous models. Consistent across all specifications, this result may reflect anomalies of the economic characteristics of this period (Fawley and Neely 2013). The impact during QE3 diminishes drastically (from -4.69 to -2.16) and becomes insignificant in both FG specifications, underlining a potential upward bias of the coefficient due to capturing concurrent FG effects. Asset purchases in the Fed's latest purchasing round during COVID-19 report a slightly lower magnitude of a 1.48 bps decline in the WX shadow rate (Model 7) which turns insignificant in Model 8. The model fit increases drastically from Model 4 (0.131) to Models 5 (0.280) and 6 (0.278), underlining that the efficacy of UMP measures should ideally be assessed not in isolation but as part of the total MP toolbox (Rogers, Scotti, and Wright 2014).

5.3 Discussion

This section will consider the presented OLS results against both the wider balance sheet policy context in the respective period and similar QE effects on yields. While the shadow rate constructs a short-term interest rate and is thus not directly comparable with longer-term yields, the distinct effect mapping on both shadow rates and yields provides an insight into the potential size of the role of asset purchase across periods. Based on the Fed's balance sheet expansion of approximately 150% during the first QE round from December 2008 to March 2010 (Gagnon et al. 2011), the estimated reduction of 1.64 basis points implies a cumulative shadow rate reduction of 241 basis points. Gagnon et al. (2011) estimated QE1 to have reduced ten-year yields by 100 basis points at most, such that both measures indicate a largely significant effect of QE right after the GFC.

For QE2, the statistically insignificant coefficient in the baseline models may reflect not only the smaller balance sheet expansion of 29% during the eight month period from November 2010 to June 2011, but also a lower effect from treasury-targeted purchases (Krishnamurthy and Vissing-Jorgensen 2011). It also aligns with a lower effect on long-term rates at roughly 10 to 30 basis points. Unlike QE1, the policy announcements before purchases in the second round of QE potentially resulted in market anticipation effects which likely reduced QE efficacy (Bauer and Rudebusch 2014).

The effects during QE3 were more heterogeneous across specifications, but following the drop of 4.69 bps, a cumulative decline of approximately 375 basis points in the shadow rate could be attributable to this QE round, given that the Fed’s balance sheet expansion by 80% from September 2012 to October 2014 (Ihrig et al. 2018). These results compare with an effect of around 100 basis points on ten-year yields (Bhattarai and Neely 2022). Indeed, QE3 had several anomalies, such as being more open-ended in nature and specifically addressing labor market outcomes (Bhattarai and Neely 2022, Fawley and Neely 2013). In light of the design of purchasing programs substantially altering purchase efficacy (Wright 2012), these might explain the vastly superior coefficient of the third purchasing round.

The final period during COVID-19 (QE4) is of distinct interest as it reflects QE under a MP stance of a completely different nature. Between March 2020 and December 2021, the estimation reveals highly significant 1.61 basis point decrease in the shadow rate in the baseline model. Since the Fed engaged in asset purchases of unprecedented scope after the onset of the pandemic, accordingly increasing its asset holdings by 201% from $4.3 trillion to $8.9 trillion (Logan and Kent 2022), this finding would correspond to a 323 bps decline in the shadow rate. In this context, Vissing-Jorgensen (2021) found ten-year yields to reduce by 200 bps. Aside from the sheer implementation velocity (more than $1 trillion worth of assets purchased in the first month), the altered effectiveness is potentially driven by the Fed's commitment to engage in unlimited purchases and a more coordinated approach with fiscal policy this time (Clarida, Duygan-Bump and Scotti 2022). The varying FG coefficients across periods also hold key policy insights. As introduced, FG events classified only binary decreased the shadow rate by around 25 basis points during QE2 and QE4 while qualitative FG increased it by about 17 bps in QE2 and 29 bps during the pandemic. This apparent reversal of effects might appear through the likely nature of FG at the ELB. If the majority of FG announcement events were dovish [-1] (Fed maintaining a low interest regime, coinciding with QE periods), then the binary [1] and qualitative dummies should be inversely correlated. With qualitative FG likely capturing such policy commitments more explicitly, the results provide support for the 'fed information effect', whereby central bank announcements heavily influence economic assessments and expected policy decisions, underlining the relevance of FG nature (Nakamura and Steinsson 2018).

5.4 Robustness: Wu and Xia (2016) compared with Krippner (2015) shadow rate

To validate the robustness of findings and explore potential structural differences in shadow rate measures, the analysis extends by incorporating the measure of Krippner (2015) from a two-term SRTSM. Table 12 and 15 report both models from above using the KR shadow rate measure. In these specifications, a one percent increase in Fed asset holdings corresponds to a consistent 1.50 basis point decrease in the KR shadow rate during QE1 both with and without FG, which resembles the WX findings (-1.64) almost perfectly. Fed assets during QE2 (2.41) and QE3 (2.66) turn highly insignificant and are positive in baseline models but revert to a negative (QE2: -1.69) and significant (QE3: -7.86) effect in qualitative FG models. This result is especially surprising for QE3 and contrasts with WX results where an increase in assets during QE2 corresponded to a significant increase of 4.45 bps and varied less across specifications. The pandemic-era purchases show a significant -2.42 bp effect (-2.79 bp in the FG specification), considerably exceeding WX findings of -1.62 bps with higher magnitude and thereby hinting that QE had greater efficacy for the KR measure. During non-ELB periods, asset holdings had virtually the same effect on the KR shadow rate as the FFR at a 0.72 bps, in line with WX model results. The model fit of KR models is consistently higher than the respective WX model, both for baseline QE models (adj. R^2: 0.318 vs. 0.131) and the extension with FG events (adj. R^2: 0.351 vs. 0.278). The stark contrast in explanatory power across most model specifications may hint at the dynamic relationship between UMP tools and the shadow rate being differently across measures. Evaluating these patterns, the analysis finds consistent estimates during QE1, but a considerably stronger effect in QE3 ls using the WX measure. The KR measure may react more sensitively to QE purchases with leading communications, such as in periods with distinct policy transitions (QE2) or crisis-response programs (QE4). Outside of the ELB, both measures indicate a positive relationship once the FFR is included, such that maintaining an elevated level of assets without new purchases may not be sufficient to depress the shadow rate and the size of the balance sheet can only complement rather than substitute conventional MP. Conclusively, these findings challenge the ‘stock effect’ of QE theorized by Bernanke (2020) and instead align with Greenwood, Hanson, and Stein (2016) who find balance sheet effects to vanish as markets swiftly adjust to a new supply equilibrium.

6 Alternative Models

While the linear regression model provided clear distinction of effects across periods to answer the research question, the definition of purchase periods was enforced upon the model. However, the data set structure also allows for endogenous estimation techniques. The paper thus extends the OLS results by two different model techniques, with the first identifying QE-effective periods endogenously and the latter estimating QE magnitude depending on the prevailing policy stance.

6.1 Markov Regime Switching Model

Each MP regime can reflect a distinct economic environment for highly effective QE periods. The first extension thus applies a Markov Regime-Switching (MRS) model discriminating between a baseline regime ('normal') and a regime which establishes a clear link between asset purchases and the shadow rate ('effective') (Krolzig 1997). In line with Hamilton (1989), the intercept and slope coefficients switch endogenously such that shifts in monetary transmission mechanisms can be uncovered, thereby unveiling how the responsiveness of the shadow rate to balance sheet expansions evolves across latent MP states. The MRS model follows as:

$$S_t = X_t\beta_s + \varepsilon_t \tag{9}$$

where S_t is the first differenced WX shadow rate in the U.S., X_t is the vector of similar explanatory variables as in the baseline OLS models in first differences, β_s are the regime-specific coefficients and ε_t the error term. The latent regimes indicate a period's effectiveness in explaining the shadow rate with QE and evolve via a first-order Markov process:

$$P(s_t = j \mid s_{t-1} = i) = p_{ij} \tag{10}$$

where p_{ij} are the transition probabilities across regimes (Krolzig 1997). The model is run in two specifications, once only with Fed assets where $\beta_{1,s}$ captures the effect of changes in asset holdings that vary by regime and again including the FFR such that $\beta_{2,s}$ similarly reflects the effect of regime-dependent changes in the FFR. Table 3 reports the parameters results distinguishing between both regimes and the according transition probabilities.

Table 3: MRS results for the Wu and Xia (2016) shadow rate

Regime	**Parameter**	**Model 1: Fed Assets**	**Model 2: Fed Assets + FFR**
Regime 0 (Normal)	Constant	0.01 (0.068)	0.05 (0.054)
	Fed Assets	-0.46 (0.195) **	0.84 (0.970)
	Fed Funds Rate	–	0.98 (0.320) ***
	Variance (σ^2)	0.01 (0.003) ***	0.14 (0.031) ***
Regime 1 (QE-effective)	Constant	0.10 (0.068)	-0.01 (0.011)
	Fed Assets	-4.24 (2.071) **	-0.19 (0.353)
	Fed Funds Rate	–	0.61 (0.107) ***
	Variance (σ^2)	0.19 (0.047) ***	0.01 (0.002) ***
Transition Probabilities	**P (0→0)**	**0.93 (0.078) *** **	**0.93 (0.029) *** **
	P (1→0)	**0.21 (0.027) *** **	**0.18 (0.098) * **

In the normal regime, the WX shadow rate decreases by 46 bps per 100% balance sheet expansion, while this effect amplifies to a 424 bp decline for effective QE periods. The transition probabilities document high persistence for both regimes, indicating that only unusual conditions force a switch to regime 1 where QE has a strong effect, which also has a high chance of prevailing at 79%. Once the FFR is accounted for, this probability remains almost unchanged, however, the QE coefficient declines and is no longer significant. Instead, the FFR plays the main role under both regimes, such that relationship between the WX shadow rate and the FFR is virtually uniform at of 0.98%.While the MRS model may thus challenge QE effectiveness across regimes, the results foremost point to the limitations of the endogenous approach to efficiently distinguish regimes solely based on asset purchases without further details on composition or financial context.

6.2 Threshold model

In testing another data-driven approach to capture state-dependent effects for the studied relationship, a smooth threshold regression (STR) model is employed. In this specification, the effect of regressors is estimated based on a transition variable crossing a critical value. The STR model uses a continuous transition function (here: logistic) to split the data set into high- and low-regime states based on the transition variable, allowing slope coefficients to change gradually across regimes rather than abruptly (van Dijk, Teräsvirta, and Franses 2002). It thus retains the ability to model regime-dependent QE effects and can be classified between the MRS and linear regression model. The STR model is defined as:

$$S_t = \beta_0 + \beta_1 x_t + \beta_2 x_t \cdot G(q_t; \gamma, c) + \varepsilon_t \tag{11}$$

where S_t is again the WX shadow rate and x_t the Fed's balance sheet, both in first differences. $G(q_t; \gamma, c)$ is the logistic transition function bound in the interval $[0,1]$ that is estimated using the threshold variable q_t, a steepness parameter γ and c_t as the threshold parameter. β_1 reflects the base effect of QE under the low regime ($G = 0$) whereas β_2 measures the change in slope for the higher regime. ε_t is the error term assumed to be independent and identically distributed with zero mean and constant variance. Table 4 reports estimations using the FFR as the threshold variable of the transition function, with and without it as a control.

Table 4: STR results for Wu and Xia (2016) shadow rate [5]

Parameter	**Model 1: Fed Assets**	**Model 2: Fed Assets + FFR**
Intercept (β_0)	0.04** (0.017)	0.01 (0.015)
Base QE Effect (β_1)	-2.15** (0.733)	-0.10 (0.675)
Effect Shift (β_2)	1.57 (1.081)	0.77 (0.961)
Effect FFR	-	0.80 (0.098)
Threshold (c)	0.80 (0.6427)	0.70 (1.950)

[5] The steepness parameter γ is 19.62 for model 1 and 20.00 for model 2. The Root Mean Square Error is 0.2494 for model 1 and 0.2516 for model 2. The adjusted R^2 is 0.0545 for model 1 and 0.0378 for model 2.

The results estimate a threshold of between 0.7% and 0.8% of the FFR, which is broadly consistent with splitting the sample between QE periods (when FFR is below this level) and non-QE periods (when FFR is above). The steepness parameter coefficient of 19.61 reveals a relatively sharp regime switch but with a continuous transition. The effect of a 1% increase in Fed asset holdings is estimated to decrease the WX shadow rate by around 2.15 bps when the FFR is below the threshold, which comes very close to the linear regression coefficients that varied between -1.61 (QE1) and -4.69 (QE3). During non-ELB regimes (here: FFR > 0.8%), the effect shift by 1.57 bps to -0.58 bps is not significant, such that the finding from the linear regression where the relationship between QE and the WX shadow rate turned positive and significant is not challenged. When including the FFR, the base effect occurs at a level of 0.7%, and asset purchases are not significant once more. In fact, none of the estimated coefficients, including the estimated threshold, are statistically significant in the second model specification. As such, the findings from the STR model align with those from the MRS as both find a significant QE effect across thresholds (STR) respectively regimes (MRS) but cast severe doubt on the role of QE when including the FFR.

7 Limitations and Further Research

While the empirical approach of this paper is considered the most fitting to fully answer the research question, several limitations in the data set should be noted. Specifically, first-differencing was required to ensure stationarity of all time series, however, this omits a conclusion about the long-term dynamics in the tested relationship, especially for a potential persistent effect of cumulative asset holdings. Furthermore, the structure of the Fed's balance sheet in this model ignores its composition, therefore not being able to capture, e.g., the maturity extension program from September 2011 through 2012 where the Fed lengthened the term structure of its assets (Fed New York, 2024). This aspect also applies to communication events beyond forward guidance, such as anticipation effects during lower purchasing rounds. Accordingly, the coefficients in some periods might underestimate the actual effect of QE purchases (Krishnamurthy and Vissing-Jorgensen 2011).

Furthermore, the conducted analysis can be extended in significant ways, such as methodologically through a shadow rate VAR approach to model interactions between Fed assets, shadow rates, and further macroeconomic variables to investigate feedback effects of the monetary transmission mechanism. The differential results for both shadow rate measures also point at structural variation in construction, which should be further taken into account. While Sims and Wu (2020) implicitly assumed an ELB duration ex ante (nine quarters), an interesting research gap also concerns the endogenous determination the ELB duration to validate this assumption. Finally, future research could also explore the composition of assets by bond term structure and extend the analysis to other monetary policy regions to identify institutional factors that influence the effectiveness of QE under a different monetary policy context.

8 Conclusion

This paper empirically investigated the relationship between different quantitative easing programs and shadow rates in the U.S over a twenty year period including both the GFC and the COVID-19 pandemic. Drawing on first-difference regression models with the Wu and Xia (2016) and Krippner (2015) shadow rates as the dependent variable, the results provide evidence for significant effects of asset purchases conducted by the Fed of varying robustness. The most robust findings for the first QE program and the pandemic purchases indicate that a one percent increase in Fed asset holdings is associated with a drop in the shadow rate of around 1.5 basis points. The effectiveness of asset purchases depends highly on the specific QE program features in the context of wider financial market conditions, such as a weaker impact from treasury-targeted purchases in QE2 and a stronger effect if accompanied by forward guidance in QE3. In assessing the sensitivity of results to shadow rate specifications, the effects during the first and fourth QE period proved highly consistent for both shadow rate measures while differing by quite a margin in the remaining purchasing rounds. While the models using the Krippner (2015) shadow rate displayed a consistently higher fit, both measures provided similar estimates across periods. This robustness hence allows for a coherent qualitative conclusion of QE effects on the shadow rate, which are compatible with those on long-term Treasury yields (Gagnon et al. 2011; Bhattarai and Neely 2022). To uncover the distinct monetary transmission mechanisms across regimes, the results were complemented by a Markov regime switching model, which suggested a high persistence of both conventional and QE-effective regimes once entered, but only considered asset purchases significant absent the inclusion of the FFR. In evaluating the overall efficacy of QE, a smooth transition model provided similar findings as the MRS, but revealed an overall decrease of 2.14 basis in the shadow rate which might occur if the FFR is close to but still above the ELB. Conclusively, the results from the Fed's asset purchasing programs underline the importance of considering QE as part of the wider unconventional monetary policy toolbox of central banks that also contains forward guidance as well as considering the methodological construction of shadow rate measures to soundly evaluate monetary policy effectiveness.

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Appendix

Figure 1: Monthly Fed Assets – Total and by asset type

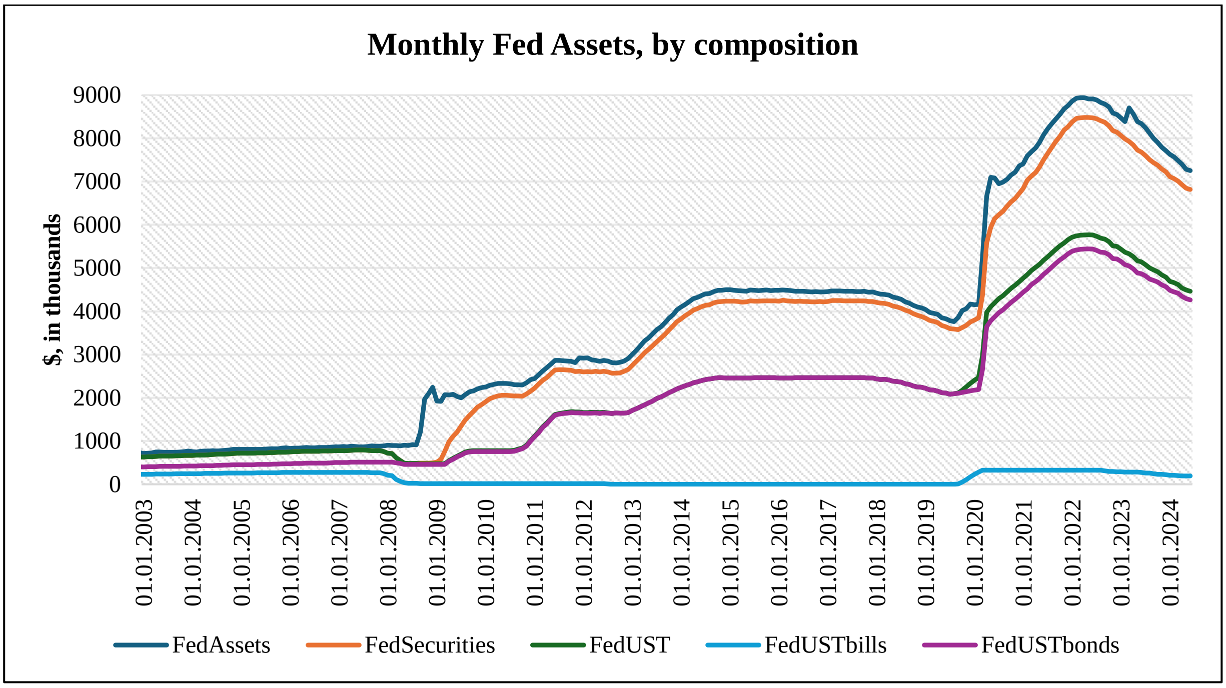

Figure 2: WX shadow rate and the Fed's balance sheet (adopted from Sims and Wu 2020)

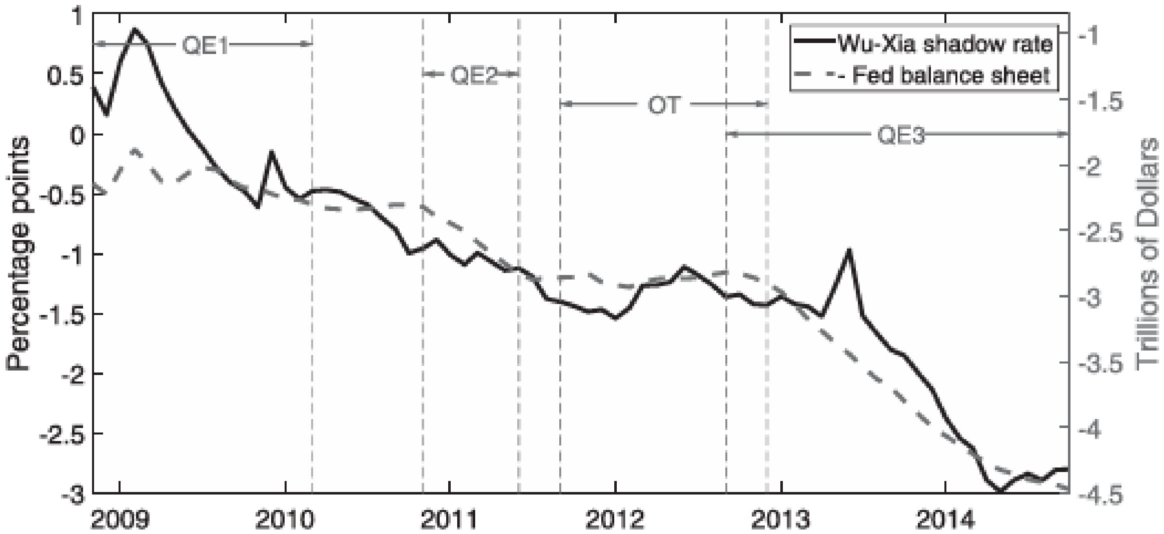

Figure 3: WX shadow rate and model-implied shadow rate (adopted from Sims and Wu 2020)

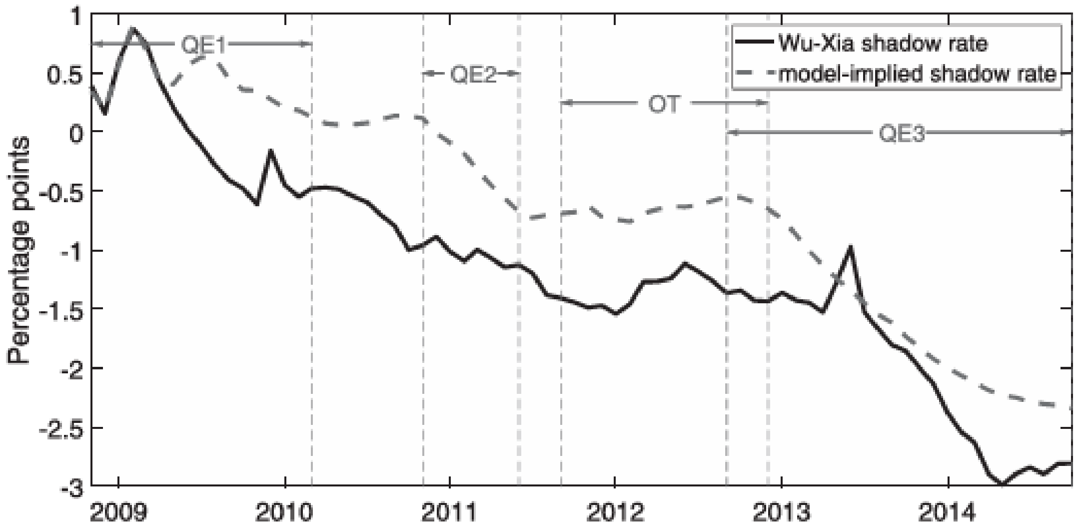

Table 5: Kendall’s Rank Correlation Coefficients (adopted from Hirata et al. 2024)

Monetary policy phase	Uncollateralized O/N call rate	IN	WX	Krippner
Conventional	0.68***	0.68***	0.68***	0.68***
Unconventional	0.09	0.21**	0.23***	0.24***

Note: p-values denote the statistical significance for changes in monetary policy stance over 1995-2016.

Table 6: Augmented Dickey-Fuller test results

Variable	Level Test Statistic	p-value	FD Test Statistic	p-value
Fed Assets	-1.17	0.687	-5.999	1.68e-07
Krippner Shadow Rate	-2.69	0.076	-7.06	5.38e-10
Wu-Xia Shadow Rate	-2.68	0.078	-4.88	3.73e-05
VIX	-4.54	0.00017	-11.17	2.62e-20
Fed Funds Rate (FFR)	-4.21	0.00064	-3.90	0.00239

Table 7: OLS Robustness Tests for baseline WX models in Table 1 (p-values)

Test	**Model 1 (QE)**	**Model 2 (QE+ FFR)**	**Model 3 (QE + VIX)**	**Model 4 (QE + FFR+ VIX)**
Breusch-Pagan (Heteroskedasticity)	p = 0.951	p = 0.929	p = 0.195	p = 0.278
Breusch-Godfrey (Autocorrelation)	p = 0.000	p = 0.000	p = 0.000	p = 0.000
Ramsey RESET (Functional Form)	p = 0.376	p = 0.802	p = 0.238	p = 0.649
Jarque-Bera (Normality of Residuals)	p = 0.000	p = 0.000	p = 0.000	p = 0.000

Table 8: Variance Inflation Factors for baseline OLS WX models in Table 1

Variable	**Model 1 (QE)**	**Model 2 (QE+ FFR)**	**Model 3 (QE + VIX)**	**Model 4 (QE + FFR+ VIX)**
Constant	1.1709	1.1811	8.2536	9.1632
Fed Assets QE1	1.0002	1.0002	1.0005	1.0005
Fed Assets QE2	1.0041	1.0045	1.0044	1.0048
Fed Assets QE3	1.0061	1.0069	1.0364	1.0416
Fed Assets QE4	1.0029	1.0032	1.1611	1.1826
Fed Assets Non-QE	1.0024	1.2451	1.1543	1.3133
FFR Non-QE	—	1.2420	—	1.3680
VIX	—	—	1.3464	1.4991

Table 9: Quantitative Easing periods and characteristics in the US (adopted from the Fed New York 2024 and Labonte 2022)

QE period	**Start**	**End**	**Scale**	**Asset Type**
1	December 2008	March 2010	$1,750 billion	Mortgage-backed securities (MBS) ($1,250 billion) and longer-term Treasury securities ($300 billion)
2	November 2010	June 2011	$600 billion	Longer-term Treasury securities
3	September 2012	October 2014	$1,600 billion	Treasury securities ($790 billion) and MBS ($823 billion)
4	March 2020	February 2022	$4,600 billion	Treasury securities ($2.5 trillion) and MBS ($2.1 trillion)

Table 10: OLS Robustness Tests for WX models with FG in Table 2 (p-values)

Test	**Model 5 (QE + Normal FG + FFR)**	**Model 6 (QE + Normal FG + FFR + VIX)**	**Model 7 (QE + Qualitative FG + FFR)**	**Model 8 (QE + Qualitative FG + FFR + VIX)**
Breusch-Pagan	p = 0.270	p = 0.372	p = 0.515	p = 0.371
Breusch-Godfrey	p = 0.000	p = 0.000	p = 0.006	p = 0.000
Ramsey RESET	p = 0.178	p = 0.0269	p = 0.886	p = 0.023
Jarque-Bera	p = 0.000	p = 0.000	p = 0.000	p = 0.000

Table 11: Variance Inflation Factors for WX models with FG in Table 2

Variable	**Model 5 (QE + Normal FG + FFR)**	**Model 6 (QE + Normal FG + FFR + VIX)**	**Model 7 (QE + Qualitative FG + FFR)**	**Model 8 (QE + Qualitative FG + FFR + VIX)**
const	3.59	14.57	1.67	12.91
Fed Assets QE1	1.00	1.00	1.00	1.00
Fed Assets QE2	12.20	12.20	12.20	12.20
Fed Assets QE3	3.69	3.68	3.93	3.94
Fed Assets QE4	1.27	1.41	1.11	1.31
Fed Assets Non-QE	1.01	1.24	1.19	1.23
FFR Non-QE	1.44	1.26	1.27	1.28
FG QE1	1.14	12.29	1.03	12.24
FG QE2	12.28	4.02	12.22	4.05
FG QE3	3.92	1.42	3.69	1.14
FG QE4	1.41	1.82	1.12	1.41
FG Non-QE	1.76	1.79	1.13	1.93
VIX	—	1.48	—	1.37

Table 12: OLS Results for the Krippner (2015) shadow rate

Variable	**Model 9 (QE)**	**Model 10 (QE + VIX)**	**Model 11 (QE+ FFR)**	**Model 12 (QE + FFR+ VIX)**
Constant	0.03 (0.026)	0.22*** (0.049)	0.02 (0.021)	0.15 *** (0.041)
Fed Assets QE1	-1.65** (0.522)	-1.53 *** (0.357)	-1.63*** (0.498)	-1.50*** (0.322)
Fed Assets QE2	1.76 (3.351)	1.57 (3.387)	2.01 (3.319)	2.41 (3.351)
Fed Assets QE3	3.10 (3.432)	1.27 (3.322)	3.41 (3.394)	2.66 (3.350)
Fed Assets QE4	-3.64*** (0.506)	-2.27*** (0.661)	-3.59*** (0.510)	-2.42*** (0.574)
Fed Assets non-QE	-1.10*** (0.120)	-0.30 (0.322)	-0.19 (0.226)	0.73 *** (0.246)
FFR non-QE	–		0.85*** (0.107)	0.73*** (0.092)
VIX	–	-0.01 *** (0.003)	–	-0.1*** (0.002)
Adj. R^2	**0.101**	**0.174**	**0.230**	**0.318**
Observations	**257**	**258**	**257**	**258**

Table 13: OLS Robustness Tests for baseline Krippner models in Table 12 (p-values)

Test	**Model 9 (QE)**	**Model 10 (QE + VIX)**	**Model 11 (QE+ FFR)**	**Model 12 (QE + FFR+ VIX)**
Breusch-Pagan (Heteroskedasticity)	p = 0.2816	p = 0.0661	p = 0.0001	p = 0.0000
Breusch-Godfrey (Autocorrelation)	p = 0.0000	p = 0.0000	p = 0.0000	p = 0.0000
Ramsey RESET (Functional Form)	p = 0.0435	p = 0.2176	p = 0.2021	p = 0.2021
Jarque-Bera (Normality of Residuals)	p = 0.0000	p = 0.0000	p = 0.0000	p = 0.0000

Table 14: Variance Inflation Factors for baseline OLS Krippner models in Table 12

Variable	**Model 9 (QE)**	**Model 10 (QE + VIX)**	**Model 11 (QE+ FFR)**	**Model 12 (QE + FFR+ VIX)**
Constant	1.1558	1.1616	8.3756	8.7949
Fed Assets QE1	1.0001	1.0001	1.0005	1.0005
Fed Assets QE2	1.0031	1.0033	1.0031	1.0046
Fed Assets QE3	1.0052	1.0056	1.0033	1.0041
Fed Assets QE4	1.0022	1.2331	1.1596	1.1605
Fed Assets Non-QE	1.0022	1.2303	1.1634	1.2996
FFR Non-QE	—	1.2303	—	1.2138
VIX	—	—	1.3536	1.3861

Table 15: OLS Results for the Krippner (2015) shadow rate – with Forward Guidance

Variable	Model 13 (QE + Normal FG + VIX)	Model 14 (QE + Normal FG + FFR + VIX)	Model 15 (QE + Qualitative FG + VIX)	Model 16 (QE + Qualitative FG + FFR + VIX)
Constant	0.53*** (0.082)	0.39*** (0.087)	0.16*** (0.058)	0.12*** (0.045)
Fed Assets QE1	-1.89*** (0.488)	-1.77*** (0.398)	-1.51*** (0.282)	-1.50*** (0.264)
Fed Assets QE2	-1.46 (6.488)	-1.55 (6.573)	-1.66 (6.675)	-1.69 (6.706)
Fed Assets QE3	-0.27 (6.047)	-0.28 (6.099)	-8.02** (3.488)	-7.86** (3.435)
Fed Assets QE4	-1.85*** (0.686)	-2.10*** (0.688)	-2.70*** (0.704)	-2.79*** (0.596)
Fed Assets Non-QE	0.05 (0.323)	0.84*** (0.319)	-0.34 (0.388)	0.72*** (0.247)
FG QE1	-0.09 (0.109)	-0.05 (0.076)	-0.07 (0.082)	-0.07 (0.058)
FG QE2	-0.12 (0.155)	-0.04 (0.152)	-0.13 (0.147)	-0.14 (0.147)
FG QE3	-0.19 (0.147)	-0.10 (0.145)	-0.29*** (0.049)	-0.30*** (0.000)
FG QE4	-0.19 (0.138)	-0.14 (0.132)	-0.14 (0.087)	-0.15* (0.084)
FG Non-QE	-0.26*** (0.074)	0.19*** (0.060)	0.08** (0.028)	0.04 (0.029)
VIX	-0.02*** (0.003)	-0.01*** (0.003)	-0.01** (0.004)	-0.01*** (0.003)
FFR Non-QE		0.63*** (0.102)		0.72*** (0.101)
Adjusted R^2	**0.267**	**0.367**	**0.214**	**0.351**
Observations	**240**	**240**	**240**	**240**

Table 16: Robustness Tests for OLS KR with FG models in Table 15 (p-values)

Test	**Model 13 (QE + Normal FG + VIX)**	**Model 14 (QE + Normal FG + FFR + VIX)**	**Model 15 (QE + Qualitative FG + VIX)**	**Model 16 (QE + Qualitative FG + FFR + VIX)**
Breusch-Pagan (heteroskedasticity)	p = 0.0001	p = 0.0001	p = 0.0001	p = 0.0004
Breusch-Godfrey (autocorrelation)	p = 0.0000	p = 0.0001	p = 0.0000	p = 0.0001
Ramsey RESET (functional form)	p = 0.6504	p = 0.7622	p = 0.1944	p = 0.1944
Jarque-Bera (normality of residuals)	p = 0.0000	p = 0.0000	p = 0.0000	p = 0.0000

Table 17: Variance Inflation Factors for OLS KR models with FG in Table 15

Variable	**Model 13 (QE + Normal FG + VIX)**	**Model 14 (QE + Normal FG + FFR + VIX)**	**Model 15 (QE + Qualitative FG + VIX)**	**Model 16 (QE + Qualitative FG + FFR + VIX)**
const	21.07	23.79	12.44	12.44
Fed Assets QE1	1.00	1.00	1.00	1.00
Fed Assets QE2	10.80	10.80	10.80	10.80
Fed Assets QE3	3.88	3.88	4.25	4.25
Fed Assets QE4	1.41	1.41	1.31	1.31
Fed Assets Non-QE	1.22	1.34	1.37	1.37
FFR Non-QE	—	1.31	—	1.27
FG QE1	1.41	1.42	1.20	1.20
FG QE2	11.00	11.03	10.82	10.82
FG QE3	4.63	4.74	4.34	4.34
FG QE4	1.60	1.62	1.33	1.33
FG Non-QE	2.33	2.50	1.31	1.31

Figure 4: Model fit of Model 1 (Wu and Xia 2016)

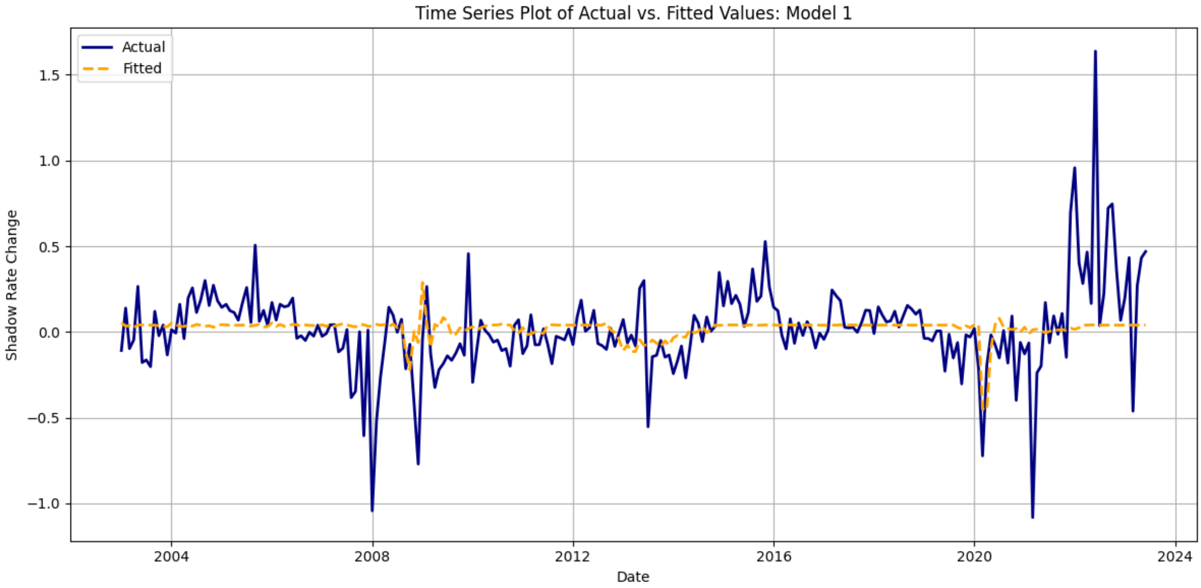

Figure 5: Residuals vs. Fitted Values for Model 1 (Wu and Xia 2016)

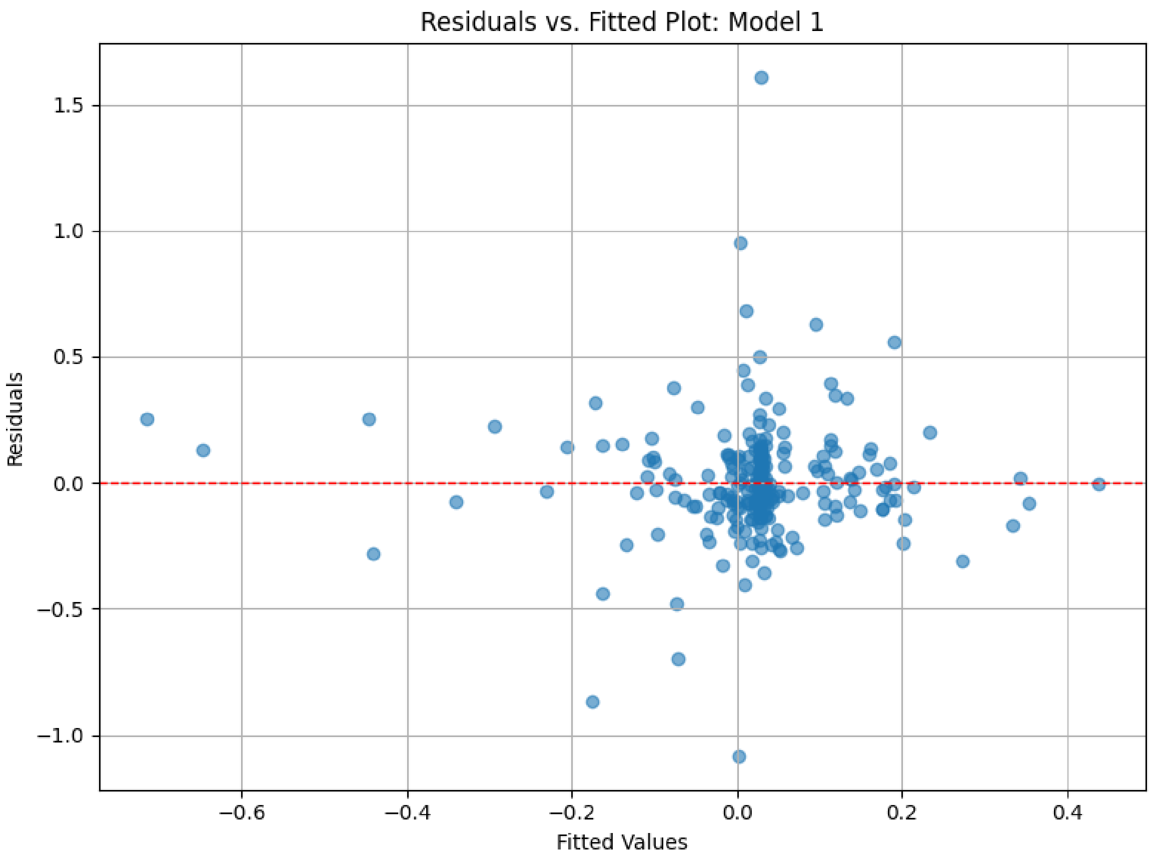

Figure 6: Model fit of Model 2 (Wu and Xia 2016)

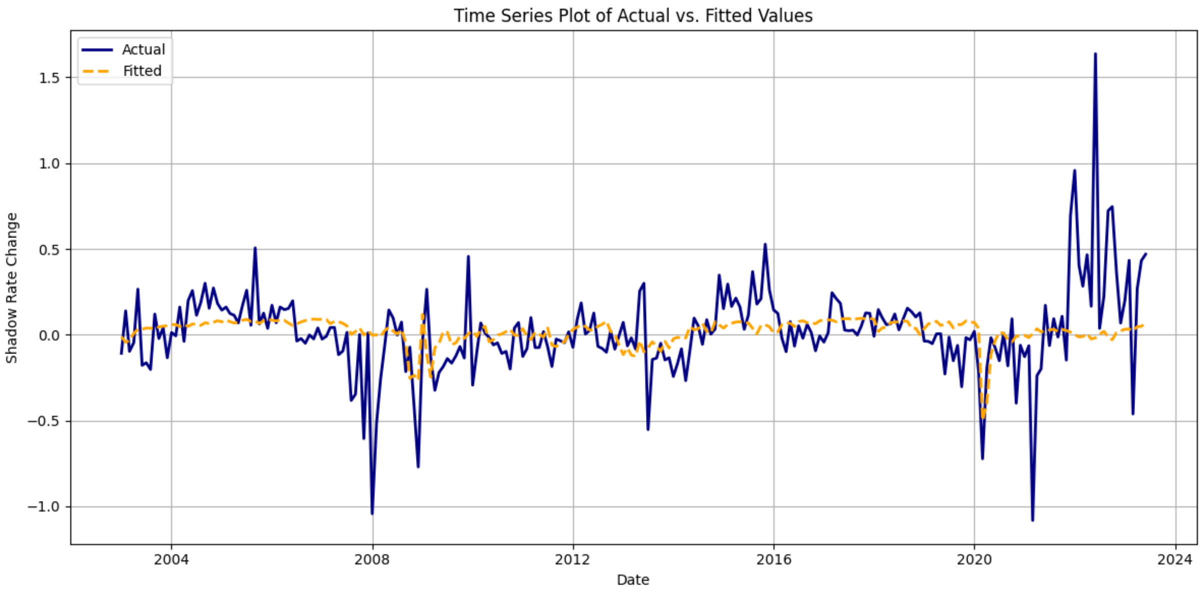

Figure 7: Residuals vs. Fitted Values for Model 2 (Wu and Xia 2016)

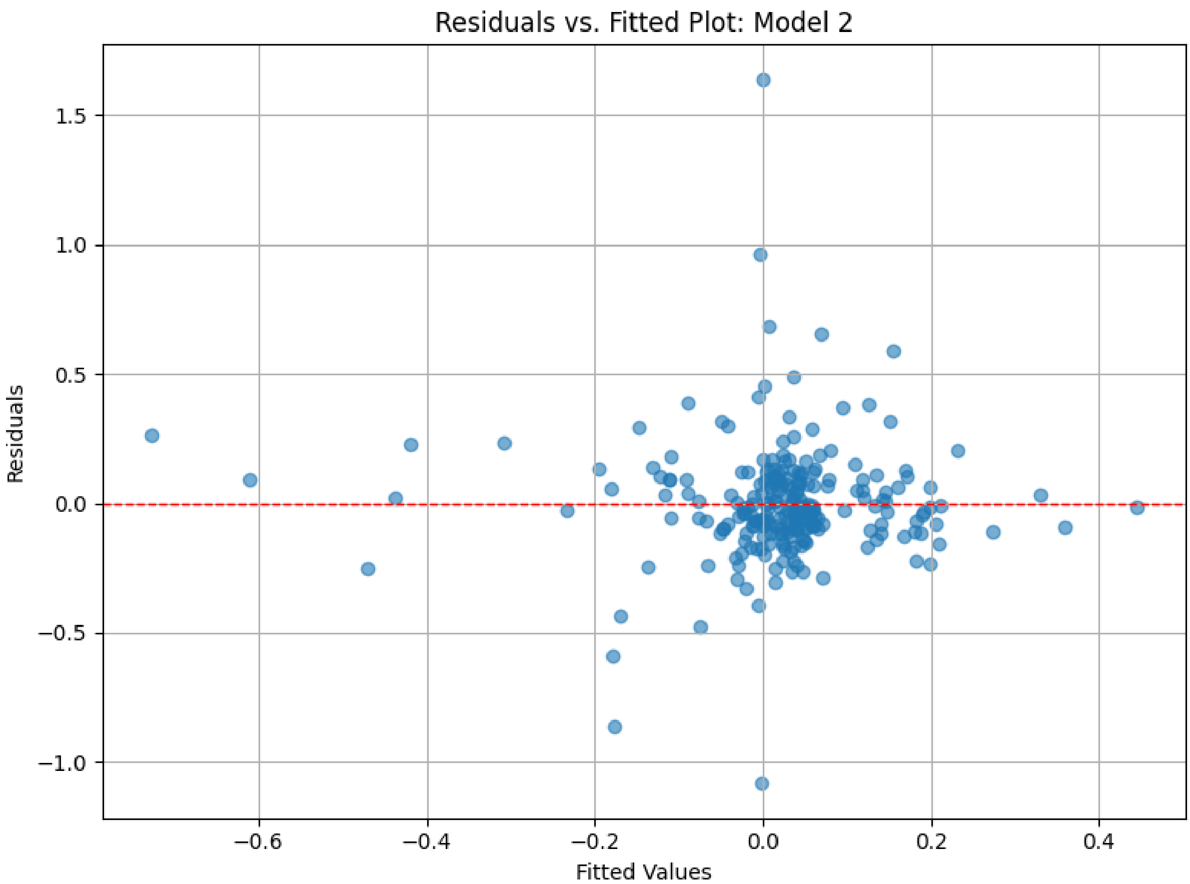

Figure 8: Model fit of Model 3 (Wu and Xia 2016)

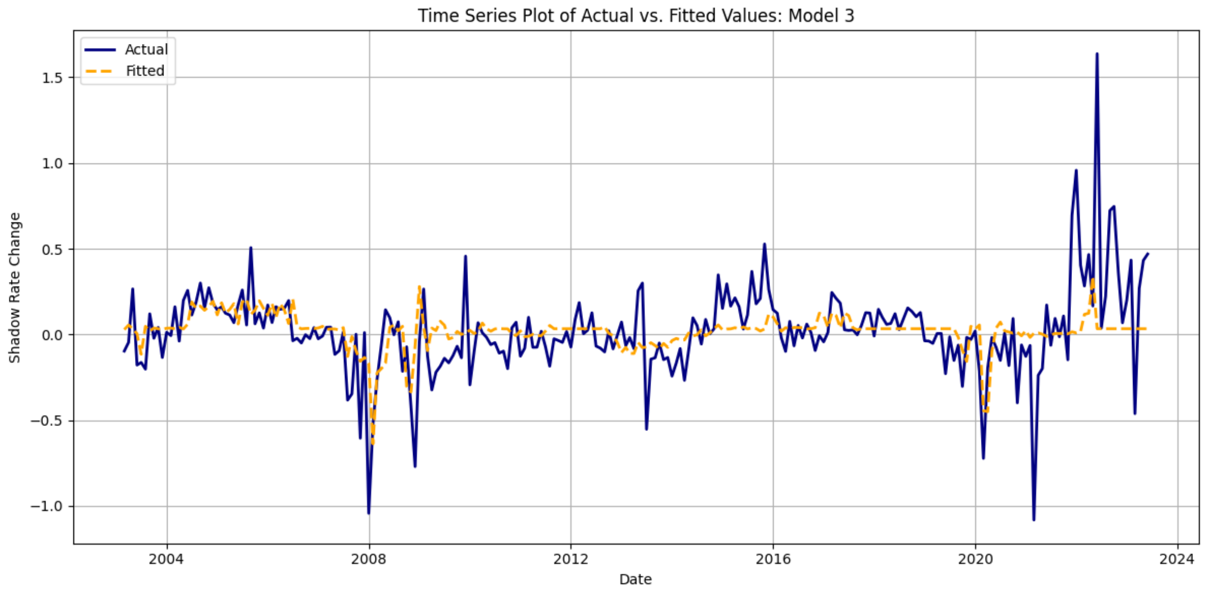

Figure 9: Residuals vs. Fitted Values for Model 3 (Wu and Xia 2016)

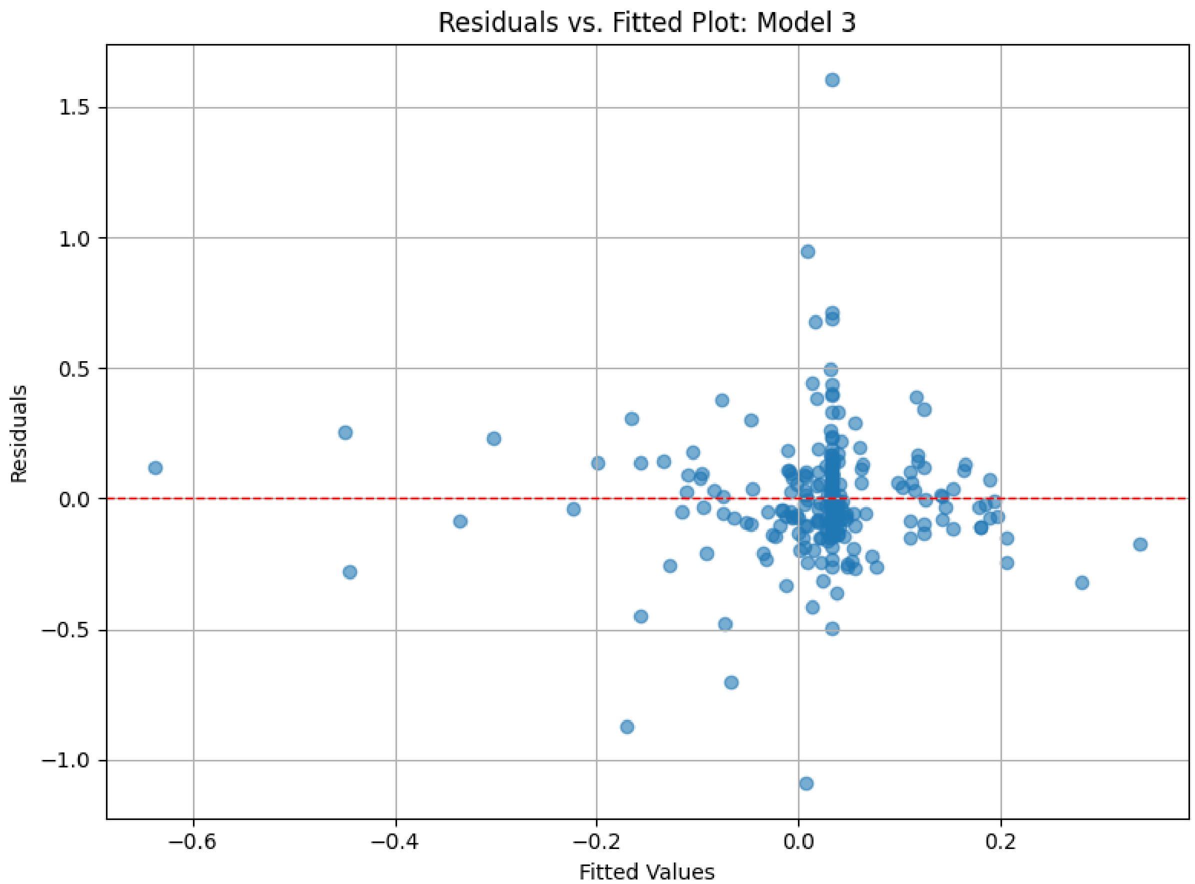

Figure 10: Model fit of Model 4 (Wu and Xia 2016)

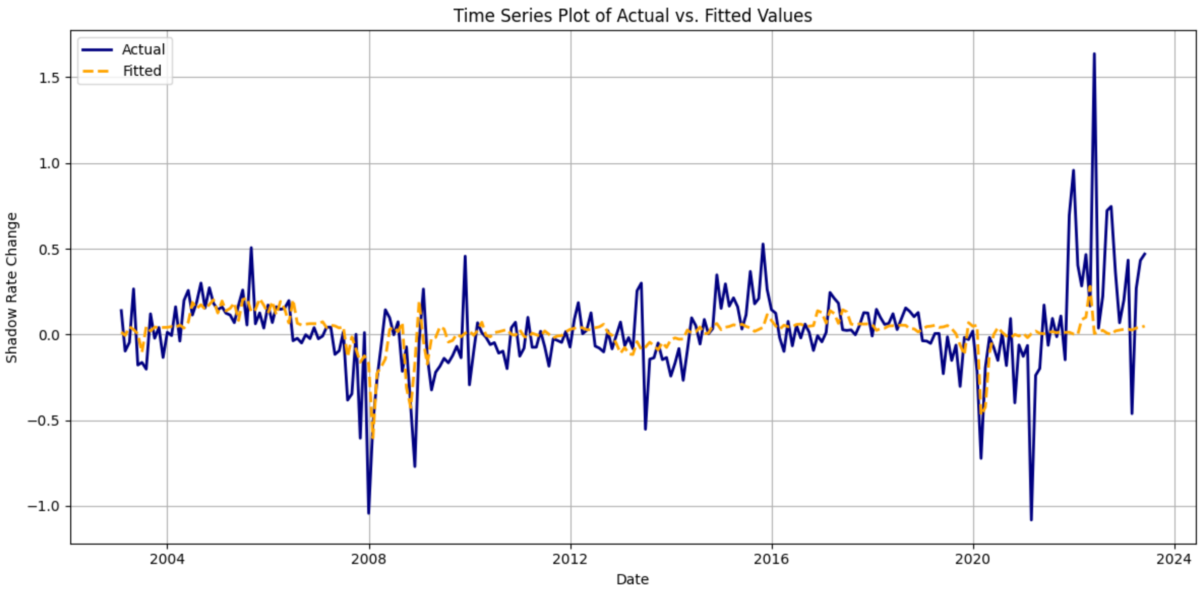

Figure 11: Residuals vs. Fitted Values for Model 4 (Wu and Xia 2016)

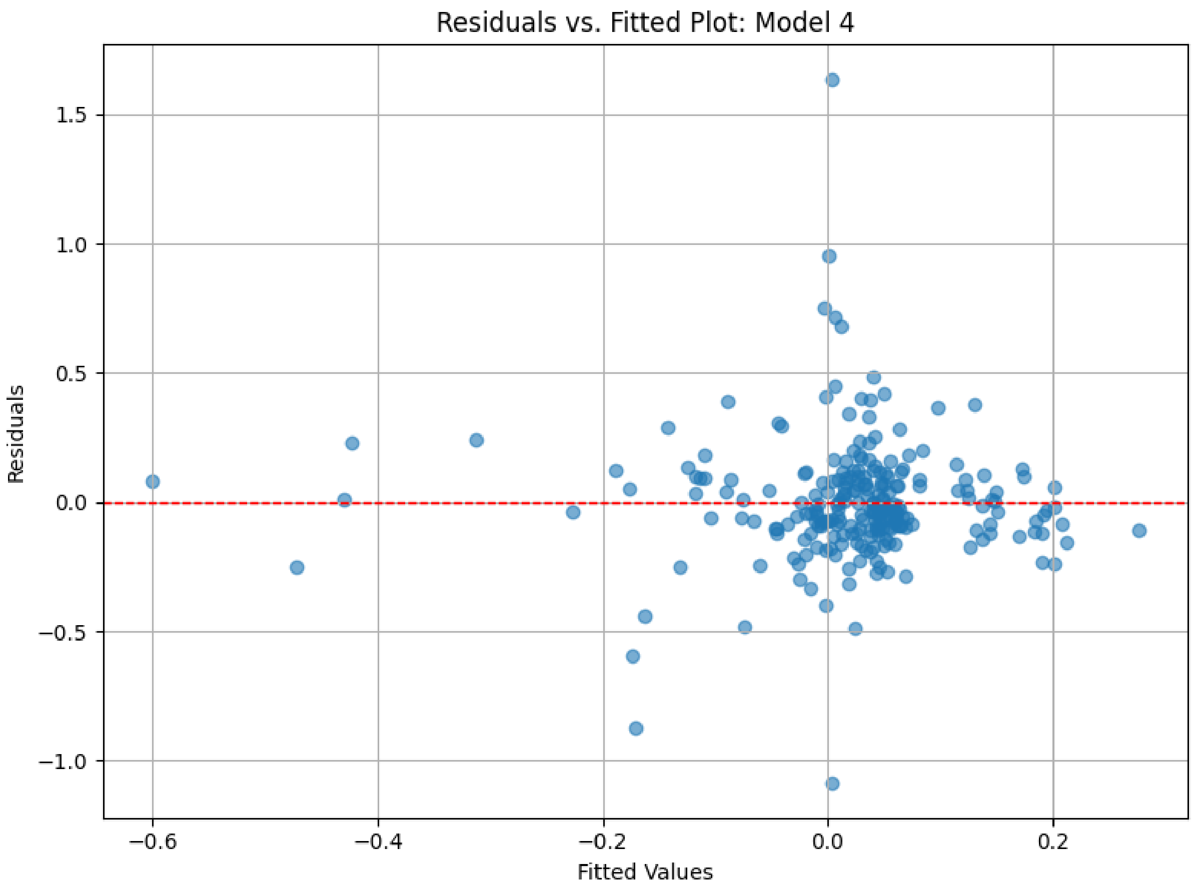

Figure 12: Model fit of Model 5 (Wu and Xia 2016)

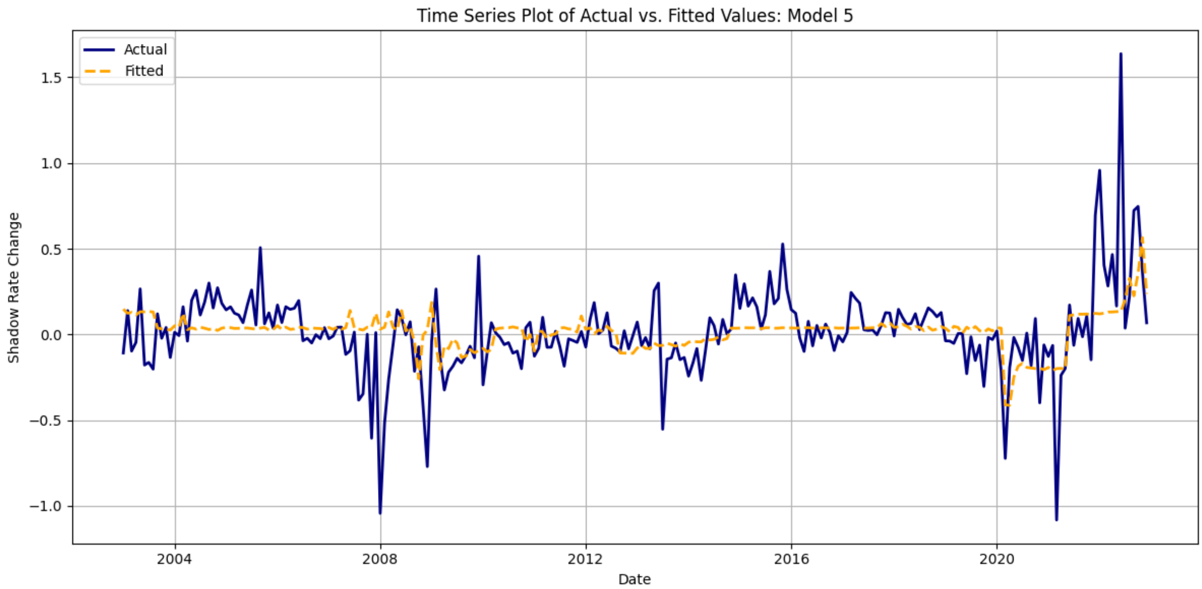

Figure 13: Residuals vs. Fitted Values for Model 5 (Wu and Xia 2016)

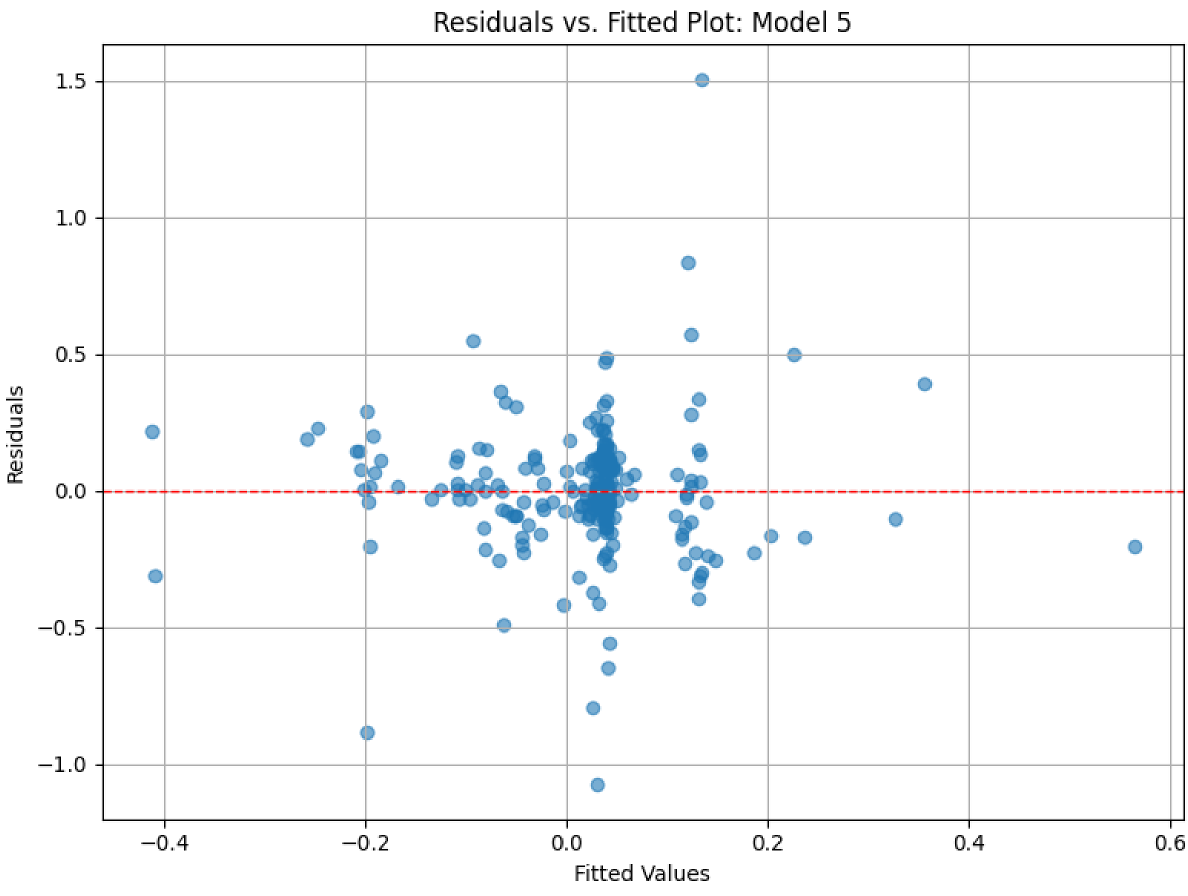

Figure 14: Model fit of Model 6 (Wu and Xia 2016)

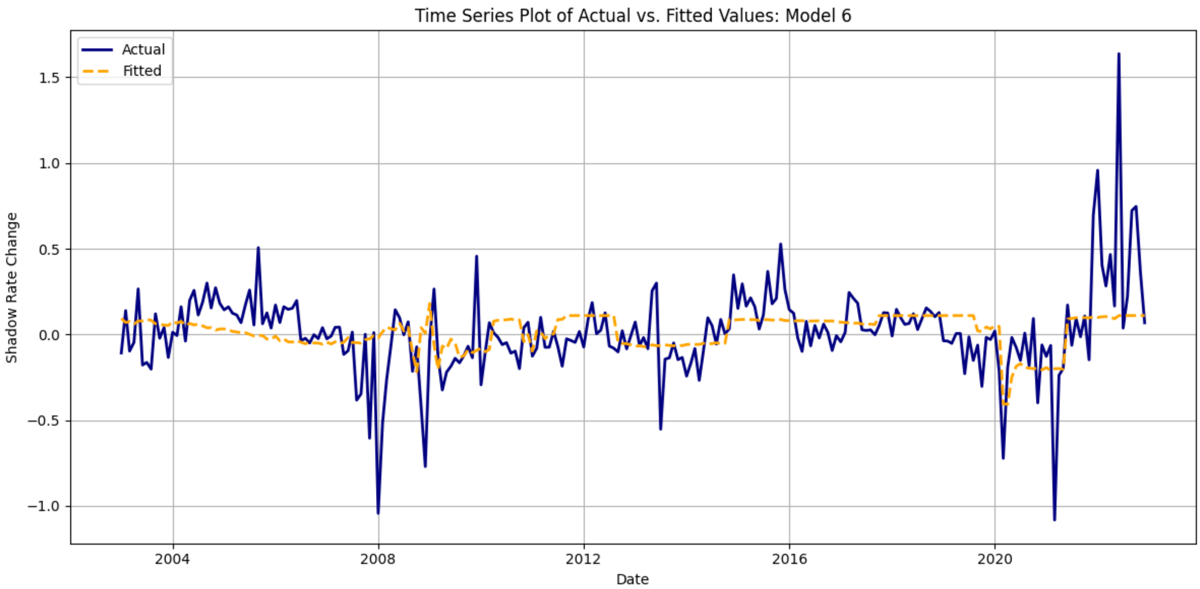

Figure 15: Residuals vs. Fitted Values for Model 6 (Wu and Xia 2016)

Figure 16: Model fit of Model 7 (Wu and Xia 2016)

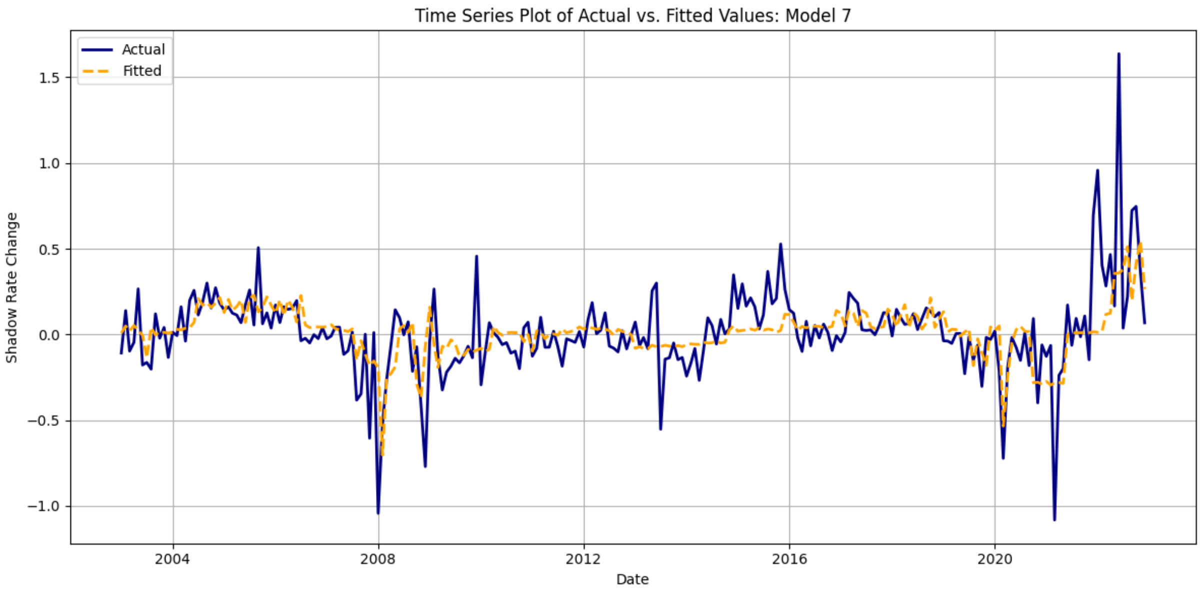

Figure 17: Residuals vs. Fitted Values for Model 7 (Wu and Xia 2016)

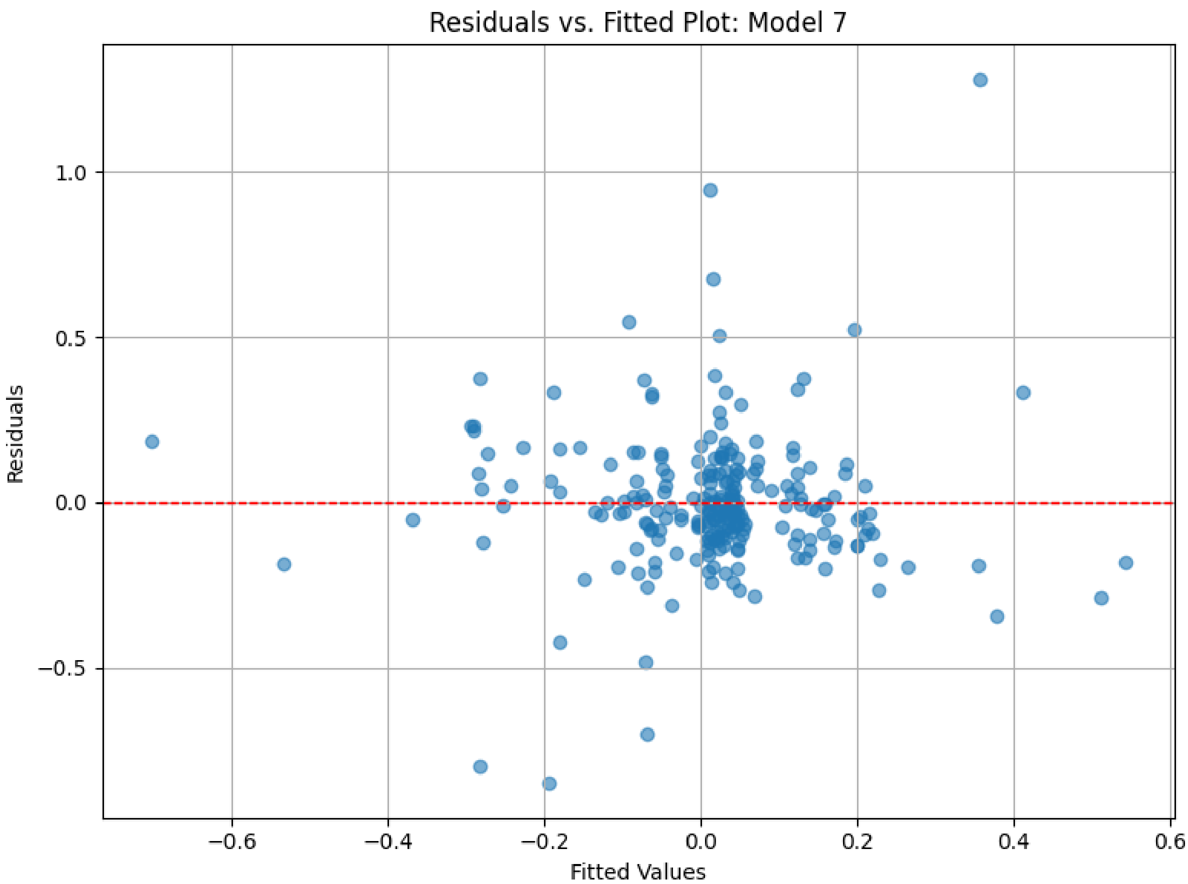

Figure 18: Model fit of Model 8 (Wu and Xia 2016)

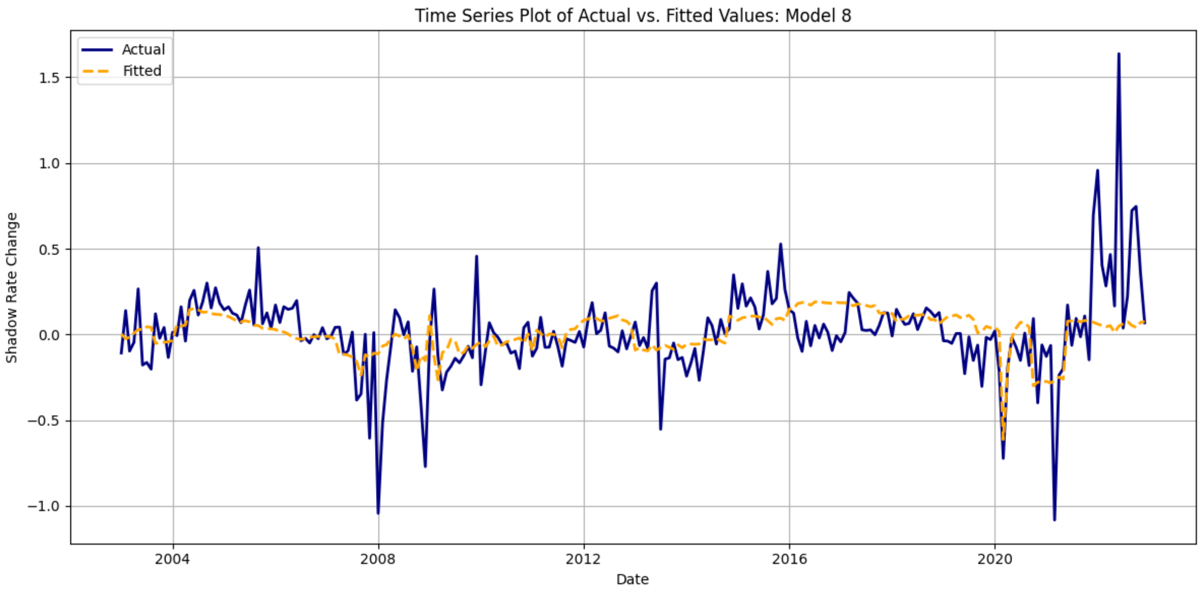

Figure 19: Residuals vs. Fitted Values for Model 8 (Wu and Xia 2016)

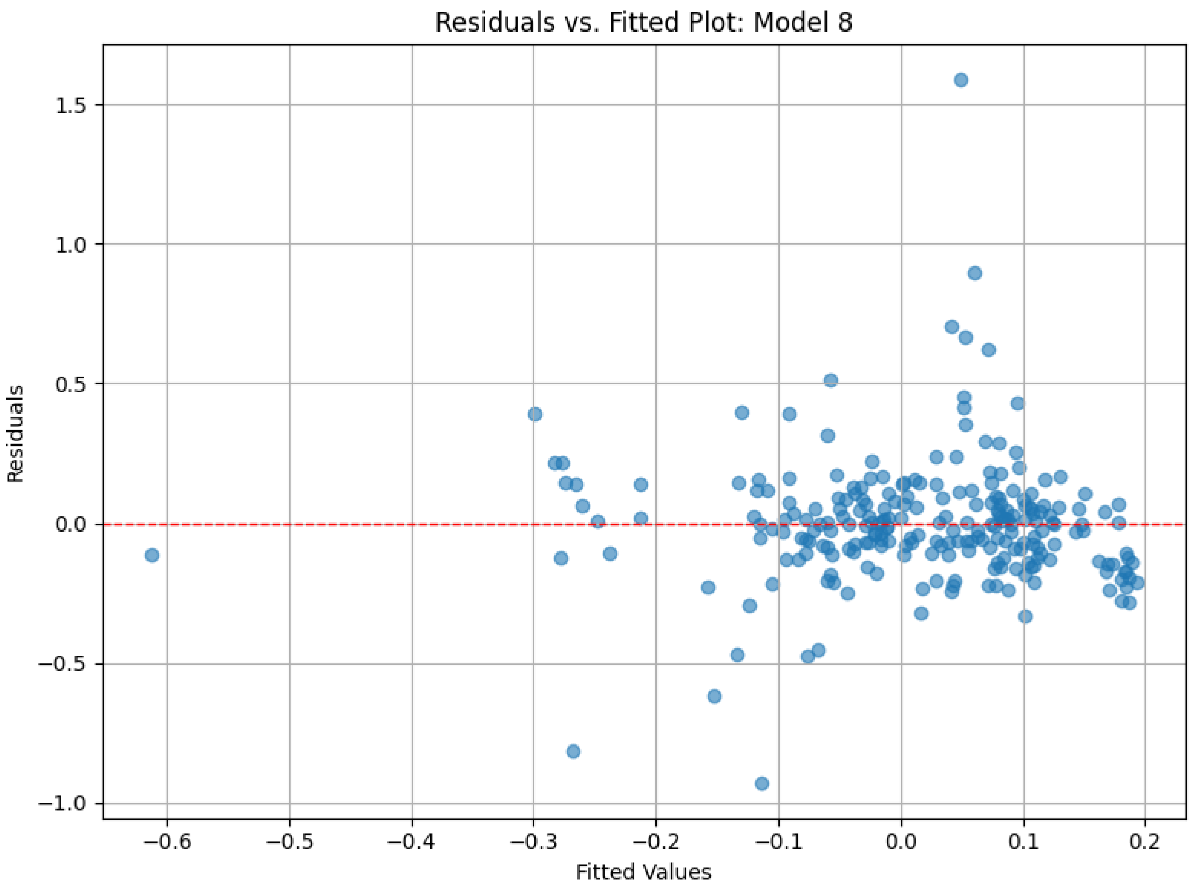

Figure 20: MRS model specification 1 (only using Fed assets) - QE-effective regime probability across QE periods

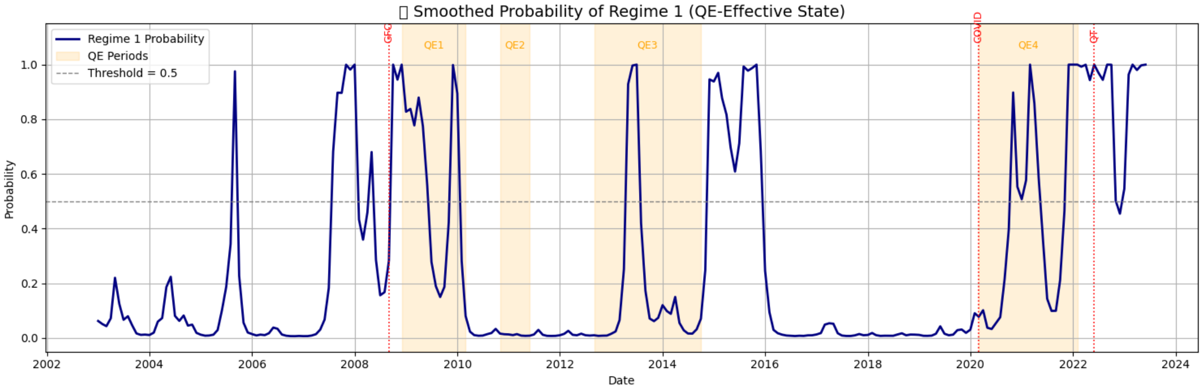

Figure 21: MRS model specification 2 (using Fed assets and Federal funds rate) – QE-effective regime probability across QE periods

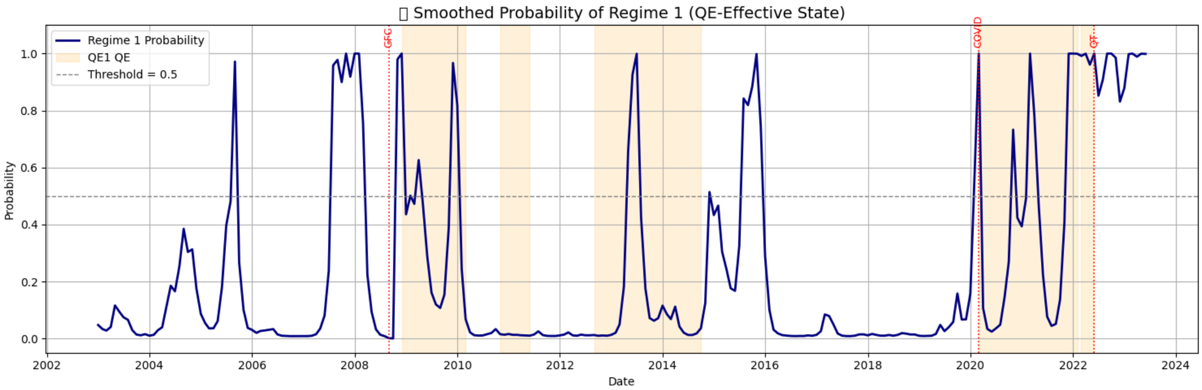

Figure 22: MRS model specification 1 (only using Fed assets) – QE-effective regime probability compared with Federal Funds Rate and Key events

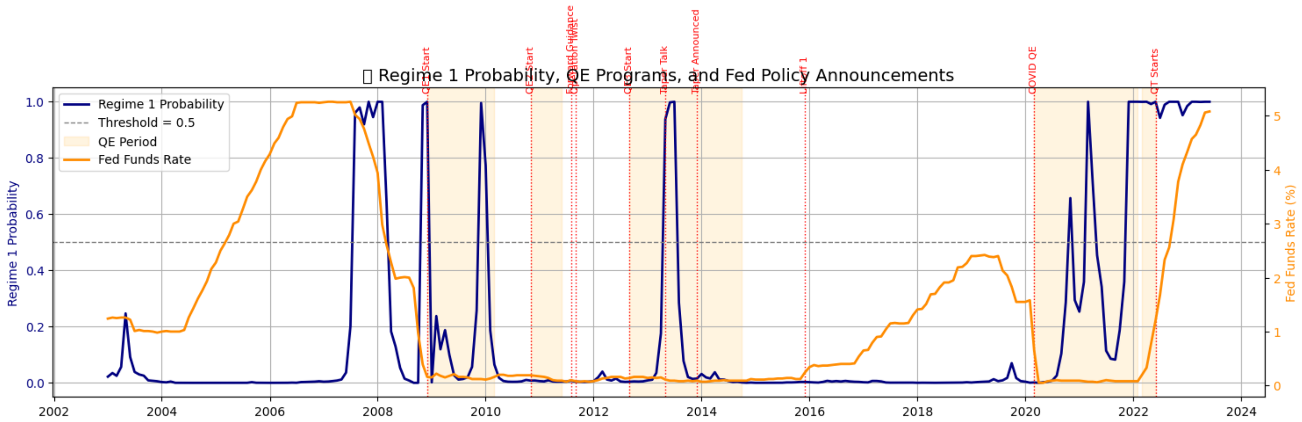

Figure 23: MRS model specification 2 (using Fed assets and Federal funds rate) – QE-effective regime probability compared with Federal Funds Rate and Key events

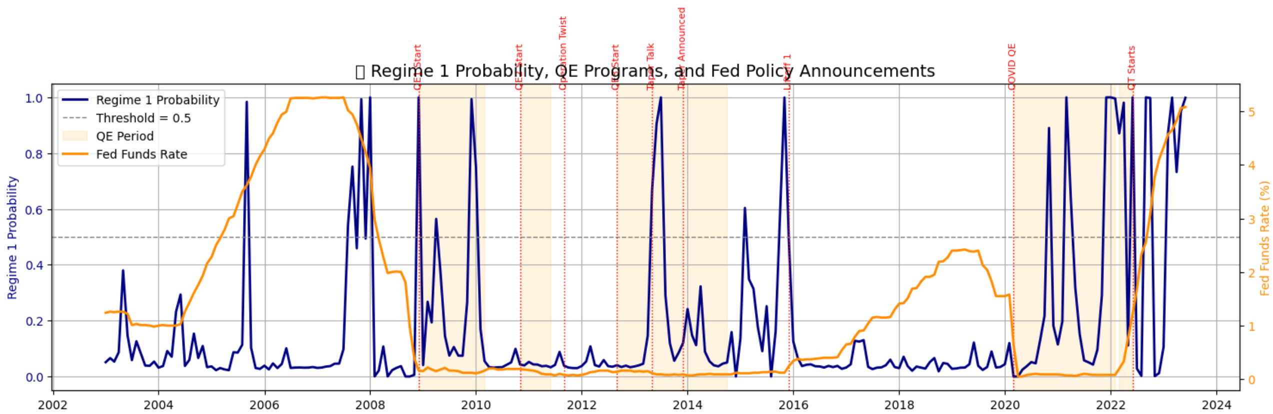

Figure 24: MRS model specification 1 (only using Fed assets) – QE-effective regime probability compared with first differenced WX shadow rate and Fed assets

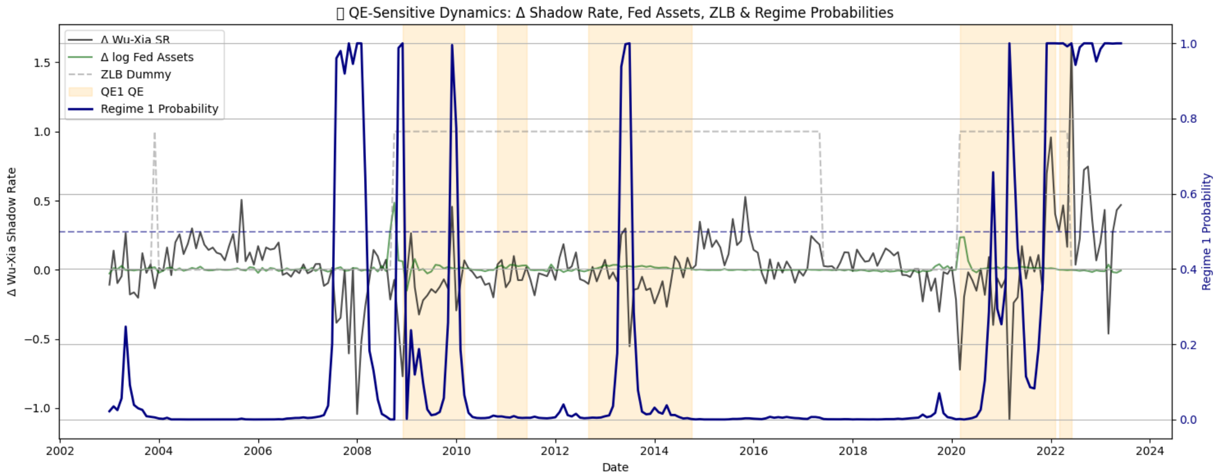

Figure 25: MRS model specification 2 (using Fed assets and Federal funds rate) – QE-effective regime probability compared with first differenced WX shadow rate and Fed assets

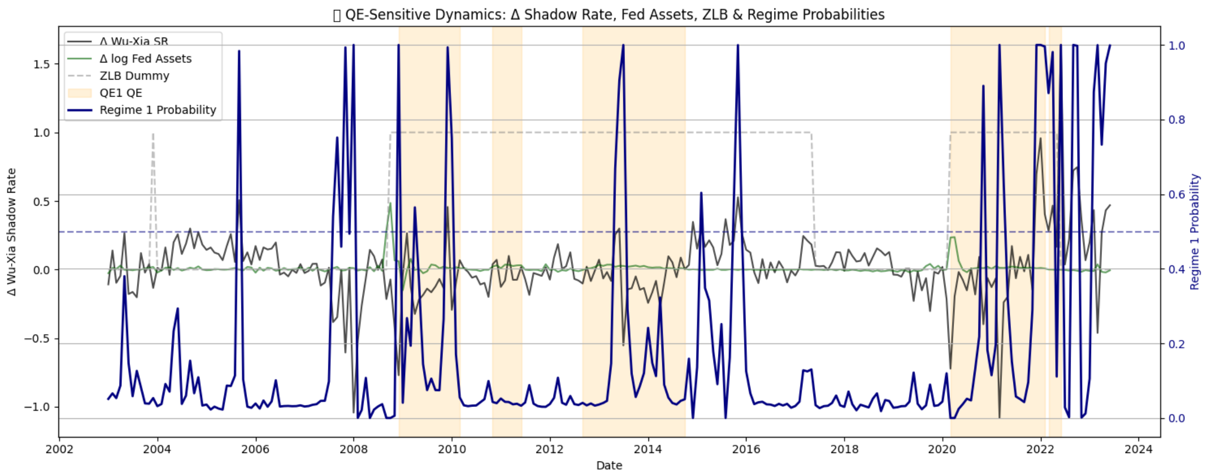

Figure 26: MRS model specification 1 (only using Fed assets) - QE-effective regime probability compared with total Fed assets

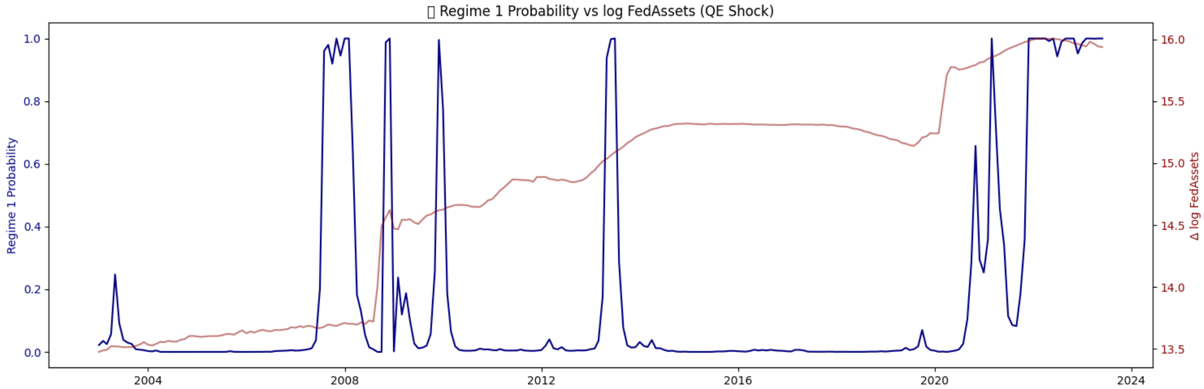

Figure 27: MRS model specification 2 (using Fed assets and Federal funds rate) - QE-effective regime probability compared with total Fed assets

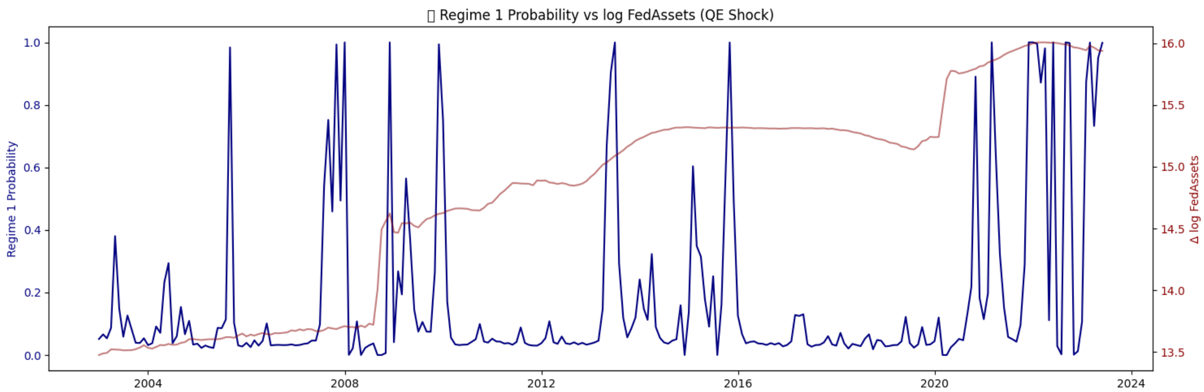

Figure 28: MRS model specification 1 (only using Fed assets) - QE-effective regime probability compared with first differenced Fed assets

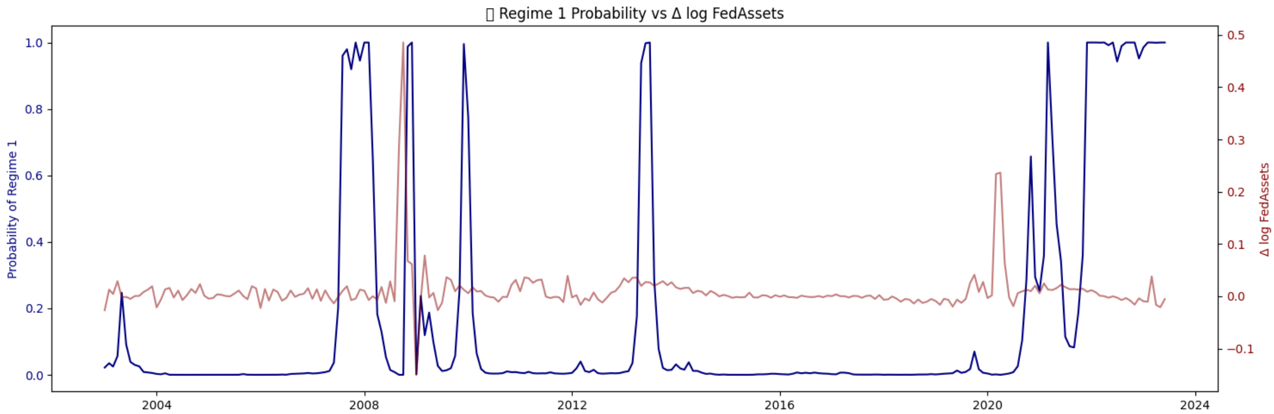

Figure 29: MRS model specification 2 (using Fed assets and Federal funds rate) - QE-effective regime probability compared with first differenced Fed assets

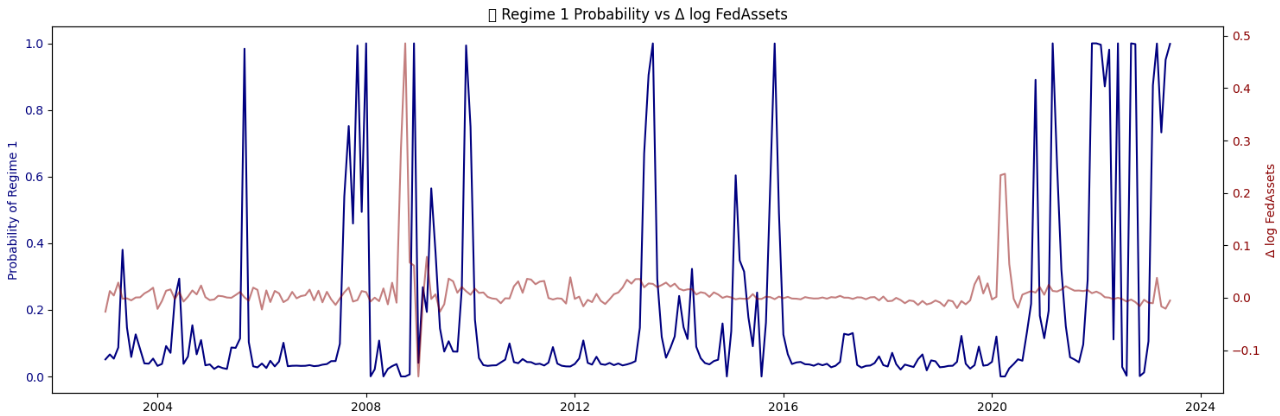

Figure 30: MRS model specification 1 (only using Fed assets) - QE-effective regime probability compared with WX shadow rate

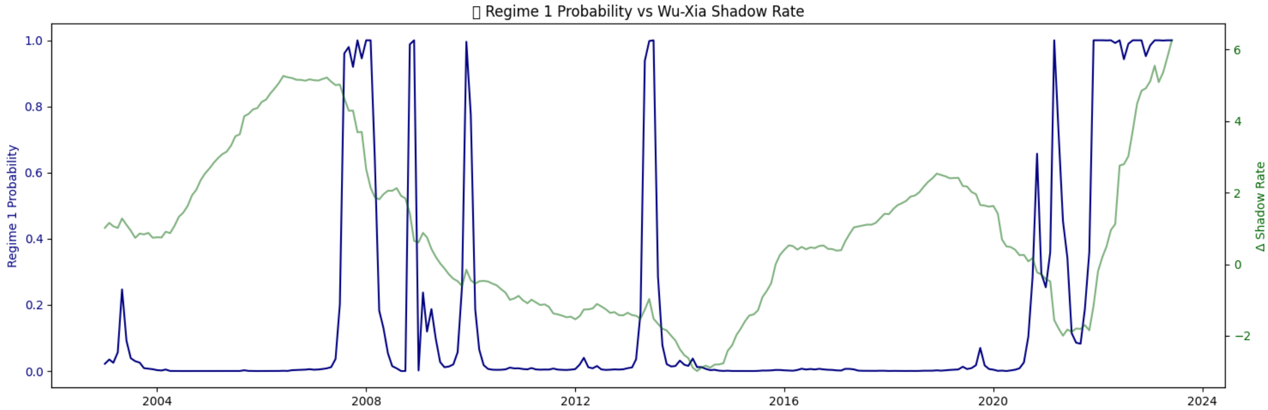

Figure 31: MRS model specification 2 (using Fed assets and Federal funds rate) - QE-effective regime probability compared with WX shadow rate

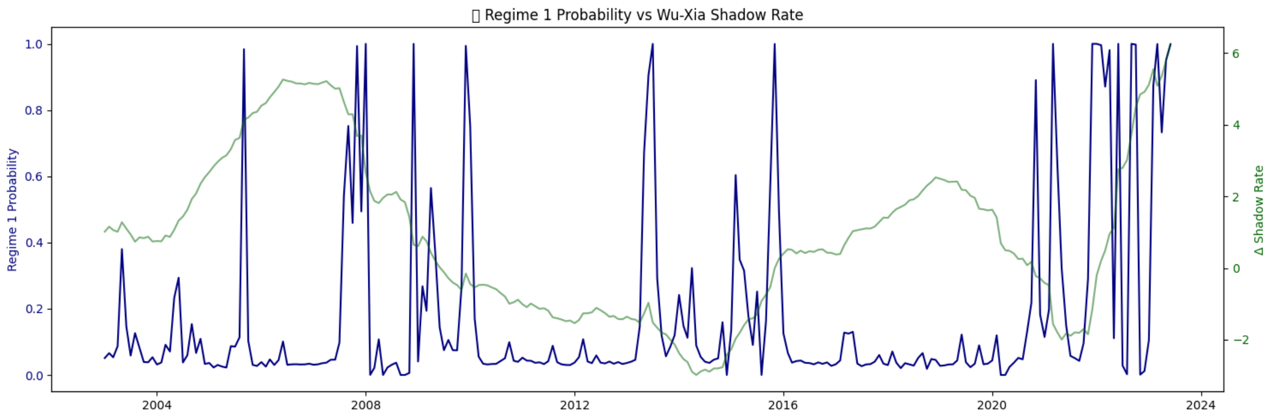

Figure 32: MRS model specification 1 (only using Fed assets) – QE-effective regime probability compared with first differenced WX shadow rate

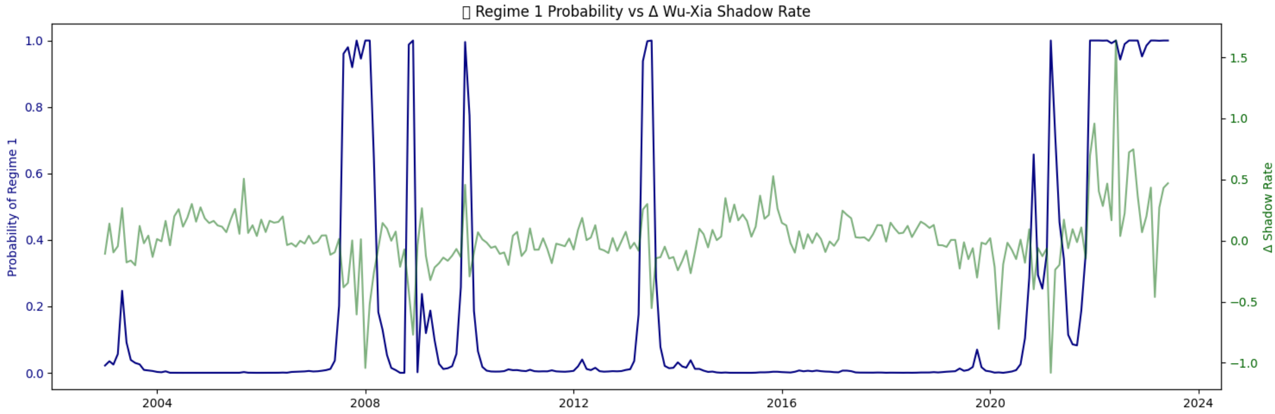

Figure 33: MRS model specification 2 (using Fed assets and Federal funds rate) - QE-effective regime probability compared with first differenced WX shadow rate

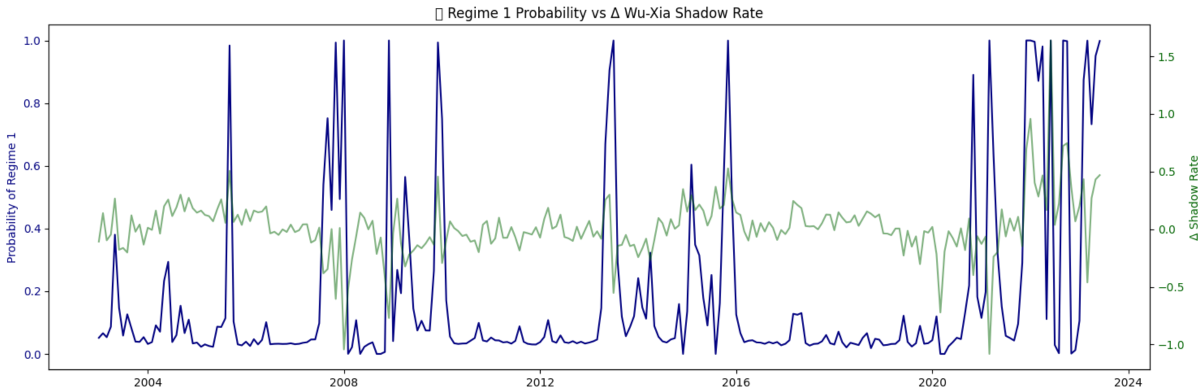

Figure 34: MRS model specification 1 (only using Fed assets) – QE-effective regime probability as binary (threshold: 0.5)

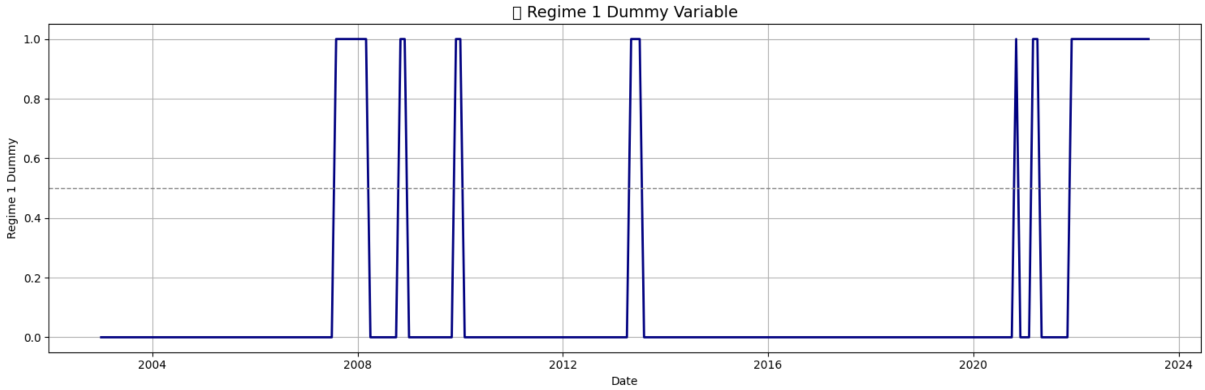

Figure 35: MRS model specification 2 (using Fed assets and Federal funds rate) - QE-effective regime probability as binary (threshold: 0.5)