



JOÃO MIGUEL ALPOIM PINHEIRO DOS SANTOS
BSc in Computer Science

**AUTOMATIC WILDFIRE CLASSIFICATION
WITH FIRE DANGER IF-THEN RULES
EXTRACTION**

MASTER IN STUDY PROGRAM NAME

NOVA University Lisbon
September, 2023



NOVA

NOVA SCHOOL OF
SCIENCE & TECHNOLOGY

DEPARTMENT OF
COMPUTER SCIENCE

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JOÃO MIGUEL ALPOIM PINHEIRO DOS SANTOS

BSc in Computer Science

Adviser: Susana Nascimento

Assistant Professor, NOVA University Lisbon

Co-advisers: Carlos Viegas Damásio

Associate Professor, NOVA University Lisbon

Maria Lourdes Bugalho

Meteorologist, Instituto Português do Mar e da Atmosfera

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*Para a minha Mãe, Pai e Irmão,
pelo vosso apoio constante.*

ABSTRACT

This dissertation aims to continue existing studies related to the definition of fire danger scales, applying data mining and rule extraction tools to wildfire data. Here, twelve wildfire datasets from mainland Portugal and its five geographic regions were studied, with wildfires from 2001 to 2020 and with a burned area larger than 100 hectares.

The studied approach follows up on the work done in [35] by applying Archetypal Analysis and Fuzzy c-Means for grouping wildfires and deriving fire danger classification.

Using the classified wildfires, we applied Decision Trees and RIPPER classifiers to extract IF-THEN rules of fire danger classes.

The analysis of the experimental results suggested that Archetypal Analysis is a viable if not better alternative to Fuzzy c-Means for this problem, further highlighting the various fire danger classes for each of the studied regions and resulting in the better rules, while the RIPPER classifier still obtained better results confirming previous studies.

Keywords: Archetypal Analysis, Fuzzy c-Means, Decision Trees, RIPPER, Fire Risk Danger, Canadian Forest Fire Weather Index (FWI), Continuous Haines Index (CHI),

RESUMO

Esta dissertação procura continuar estudos atuais ligados à definição de intervalos para escalas de risco de incêndios florestais, utilizando ferramentas de prospeção de dados e extração de regras a dados de incêndios florestais. Aqui, foram estudados doze conjuntos de dados relativos a Portugal continental e as suas cinco regiões geográficas, com dados de incêndios de 2001 a 2020 e com área ardida superior a 100 hectares.

A metodologia estudada segue o trabalho realizado em [35], aplicando Análise de Arquétipos e Fuzzy c-Means para agrupamento de incêndios florestais e atribuição de classe de risco de severidade.

Usando os incêndios florestais classificados, foram aplicados os classificadores de Árvores de Decisão e RIPPER numa medida de extração de regras SE-ENTÃO sobre classes de risco de incêndios.

A análise dos resultados experimentais sugere que a Análise de Arquétipos é uma alternativa tão viável ou até melhor a Fuzzy c-Means para este problema, destacando as várias classes de risco de incêndio para cada uma das regiões estudadas e resultando em melhores regras de classificação, enquanto que o classificador RIPPER gerou igualmente resultados positivos, confirmando estudos anteriores.

Palavras-chave: Análise de Arquétipos, Fuzzy c-Means, Árvores de Decisão, RIPPER, Risco de Severidade de Incêndios, Canadian Forest Fire Weather Index (FWI), Continuous Haines Index (CHI),

CONTENTS

1	Introduction	1
1.1	Motivation	1
1.2	Objectives	2
1.3	Organization of the Document	2
2	The Study of Wildfires	3
2.1	Wildfire Indexes	3
2.1.1	Continuous Haines Index	3
2.1.2	The Canadian Forest Fire Weather Index System	5
2.1.3	Vegetation Indexes	7
2.1.4	Dangerousness Index (ICNF)	8
2.2	The Fire Danger Rating Scales	8
3	Background Knowledge	11
3.1	The Archetypal Analysis Algorithm	11
3.1.1	The Model	11
3.1.2	The Algorithm	12
3.1.3	N-Partition Validation	13
3.2	The Fuzzy c-Means Algorithm	13
3.2.1	The Algorithm	13
3.2.2	c-Partition Validation	14
3.3	Tree-Based Classification with Rule Extraction	14
3.3.1	Decision Trees	15
3.3.2	RIPPER	16
3.3.3	Rule Evaluation	17
4	Data Collection and Description	19
4.1	Data Collection	19
4.2	Data Preprocessing	21

5	Experimental Protocol	23
5.1	General Workflow	23
5.2	Fire Danger Classification through Clustering: Archetypal Analysis vs Fuzzy c-Means	25
5.2.1	Wildfire Clustering using Archetypal Analysis and Fuzzy c-Means	25
5.2.2	Archetypal Analysis and Fuzzy C-Mean Partition Validation and Selection	25
5.2.3	Fire Danger Archetype and Prototype Labelling	27
5.2.4	Wildfire Classification using Archetypes and Prototypes	28
5.3	IF-THEN Rules Extraction for Fire Danger Classification	30
5.3.1	Decision Tree and RIPPER Models Construction	30
5.3.2	Decision Tree and RIPPER Models Selection	31
5.3.3	Decision Tree and RIPPER Ruleset Evaluation	32
5.3.4	IF-THEN Fire Danger Rules	32
5.4	Summary	33
6	Experimental Study	34
6.1	Main Objectives	34
6.2	Wildfire Clustering and Classification	35
6.2.1	Mainland Portugal	35
6.2.2	Center East	38
6.2.3	Center West	41
6.2.4	North East	44
6.2.5	North West	47
6.2.6	South	50
6.3	Decision Tree and RIPPER: Model Construction and Evaluation	53
6.3.1	Mainland Portugal	53
6.3.2	Center East	54
6.3.3	Center West	55
6.3.4	North East	56
6.3.5	North West	57
6.3.6	South	58
6.4	Fire Danger Rulesets: Comparison and Evaluation	59
6.4.1	Best Fire Danger Rule Sets Selection	59
6.4.2	Fire Danger IF-THEN Rules	62
6.5	Results Summary	74
6.6	Burned Area Study for Archetypal Analysis and Fuzzy c-Means Classifications	76
7	Conclusion and Future Work	78
	Bibliography	80

INTRODUCTION

1.1 Motivation

Wildfires are one of the most common naturally occurring disasters in the world. Due to their great impact on wildlife, natural vegetation and our own socioeconomic well-being, an even greater effort is put on both researching and responding to these disasters.

Over the years, researchers have developed a vast array of tools and metrics to help them analyse and classify these wildfire occurrences. Many of these metrics are indexes, used to describe and quantify certain aspects of both wildfires and the region where they occur, often using data regarding the location, temperature, relative humidity, wind speed and precipitation. Among them, the Canadian Fire Weather Index (FWI) is one of the most used indexes for monitoring wildfire danger. The FWI is normally accompanied with an associated Fire Danger Scale, which describes different levels of fire danger for different types of wildfires. However, as different regions are characterized by different geographical and climatological factors, this scale and its thresholds need to be adapted for each region.

The current dissertation aims to continue studies related to the definition of these fire danger scale thresholds, adapted for the case of mainland Portugal and its five geographic regions: North West and East, Center West and East and South. In particular, this project follows up on the work done in [35], where Fuzzy c-Means was used as the clustering algorithm from which different rule extraction models (Decision Trees, RIPPER, SIRUS) were applied, based on different combinations of indexes/sub-indexes. These rules were then used to propose new thresholds for the wildfire danger scale. This work concluded with positive results for the usage of RIPPER and by proposing a comparative study between Fuzzy c-Means and another clustering algorithm -like Archetypal Analysis- in the context of this approach.

This dissertation is to starts with a comparative study of the Archetypal Analysis and Fuzzy c-Means algorithms in the context of wildfire classification, using the same twelve datasets collected during [35] for mainland Portugal and its different geographic regions. Following this, IF-THEN rules are to be extracted using Decision Tree and RIPPER

algorithms and the classified wildfires. These are then compared and evaluated using interpretability metrics and the analysis of an expert in the field.

1.2 Objectives

Following the work done in [35], the main objective of this dissertation is to make a comparative study of Archetypal Analysis as an alternative to Fuzzy c-Means in a wildfire severity classification approach and as a basis for rule extraction using the Decision Tree and RIPPER algorithms. This study aims to help field experts tune and adapt commonly used fire danger rating scales to mainland Portugal and its five geographical regions.

The main contributions of this study are:

- The application and concluded viability of the Archetypal Analysis algorithm in the domain of wildfire studies.
- A comparative study between the Archetypal Analysis and Fuzzy c-Means clustering algorithms for wildfire clustering and classification, using different combinations of features and samples from across different geographic regions and occurring between 2001 and 2020.
- Extraction of new wildfire classification rules using Decision Tree and RIPPER algorithms and further evaluation using interpretability rules and analysis from an expert in the field, for mainland Portugal and each of its geographic regions.

1.3 Organization of the Document

This document is divided into nine chapters, including this one, which serves as an Introduction where the main motivation and objectives are presented.

The second Chapter is about the study of Wildfires. It includes a description of many commonly used wildfire indexes and scales.

The third and fourth Chapters are about the Archetypal Analysis and Fuzzy c-Means algorithms respectively. They include a description of the algorithms and their expected validation methods.

The fifth Chapter is a description of the rule extraction algorithms used: Decision Trees and RIPPER.

The sixth Chapter is a description of the wildfire datasets used in this study.

The seventh and eighth Chapters showcase the developed experimental protocol and the results of the study, respectively.

The ninth and final Chapter presents the conclusions of this dissertation and possible future works.

THE STUDY OF WILDFIRES

As an impactful natural disaster, wildfires are a highly researched subject. Over the years, many studies about this topic have been done worldwide and many techniques have been analysed and set up in order to better prevent and manage wildfire occurrences. In this Chapter, some of the most commonly used indexes for describing aspects of wildfires are presented and some state of the art studies in this field are explored.

2.1 Wildfire Indexes

Wildfire risk levels can be related to different atmospheric, meteorologic and geographical. As such, the study of factors like the wind, precipitation and dryness of the lands can help specialists in the field to predict higher or lower wildfire risk levels. Here follow some of the existing metrics used in order to best monitor and predict this.

2.1.1 Continuous Haines Index

One of the indexes that we will be using in our studies is the Continuous Haines Index (CHI). In order to understand the Continuous Haines Index and why it is used, we must first understand the base Haines Index.

The Haines Index (HI) was first proposed in [19] under the name Lower Atmosphere Severity Index (LASI), aiming to provide a new index that would describe the stability and moisture deficit of the air, which has a great impact on wildfire size and behaviour. The higher the value of HI is, the higher the potential is for large and erratic wildfires to occur.

HI is divided into two components: the Lapse Rate and the Moisture Deficit of the air. The Lapse Rate is calculated by the difference of temperature T at two different atmospheric layers P_1 and P_2 . This component represents the stability S of the air: a bigger difference of temperatures indicates a higher air instability.

$$S = T_{P_1} - T_{P_2} \quad (2.1)$$

Elevation	Stability (A) component		Humidity (B) component	
	Calculation	Categories	Calculation	Categories
Low	950-hPa temperature – 850-hPa temperature	A = 1 if <4°C A = 2 if 4°–7°C A = 3 if ≥8°C	850-hPa temperature – 850-hPa dewpoint	B = 1 if <6°C B = 2 if 6°–9°C B = 3 if ≥10°C
Mid	850-hPa temperature – 700-hPa temperature	A = 1 if <6°C A = 2 if 6°–10°C A = 3 if ≥11°C	850-hPa temperature – 850-hPa dewpoint	B = 1 if <6°C B = 2 if 6°–12°C B = 3 if ≥13°C
High	700-hPa temperature – 500-hPa temperature	A = 1 if <18°C A = 2 if 18°–21°C A = 3 if ≥22°C	700-hPa temperature – 700-hPa dewpoint	B = 1 if <15°C B = 2 if 15°–20°C B = 3 if ≥21°C

Figure 2.1: Calculation Table for Haines Index [19][31]

The Moisture term is calculated by the difference of the temperature T_{P3} and the dew-point temperature T_{dP3} at the same atmospheric level P3. This component represents the Moisture Deficit M of the air: a bigger difference of temperatures indicates a higher air dryness.

$$M = T_{P3} - T_{dP3} \quad (2.2)$$

Using the table offered in Figure 2.1 for the appropriate elevation and atmospheric levels, stability S and moisture deficit M can be calculated and translated into A and B respectively, which take values 1, 2 or 3. To calculate the final HI value, A and B are added together, meaning that HI takes whole values from 2 to 6.

$$HI = A + B \quad (2.3)$$

Haines based his index on the topographical analysis of the United States of America, which limited its use on regions outside of the country, leading to the creation of the Continuous Haines Index.

The Continuous Haines Index is an extension of the Haines Index, proposed in [26], by researchers who did not find the base Haines Index suitable for identifying the extreme conditions in places like Australia, due to the differences in the climate, and hoping to generalize this metric.

Similarly to HI, CHI also has two components representative of the air stability CA and moisture deficit CB:

$$CA = (T_{850} - T_{700})/2 - 2 \quad (2.4)$$

$$CB = (T_{850} - T_{d850})/3 - 1 \quad (2.5)$$

$$CHI = CA + CB \quad (2.6)$$

where T_{850} and T_{700} are the temperatures at atmospheres of 850hPa and 700hPa respectively and T_{d850} is the dew-point temperature at an atmosphere of 850hPa.

The following conditions must also be met, as occasional dew-point depressions may lead to disproportionately large CB values:

CHI	Likely fire behaviour and fire prediction reliability
<4	Fires are easily controlled. Modelling is highly likely to overpredict fire travel.
"4 - 8"	Fires may be difficult to control and fire behaviour may be erratic. This is the transition phase of fire behaviour. Modelling is likely to be close to actual fire behaviour.
"8 - 10"	Fires will be difficult to control and fire behaviour will be erratic. Modelling is likely to under-predict fire behaviour.
>10	Fires will be uncontrollable and extremely difficult to extinguish. Modelling is highly likely to dramatically under-predict fire behaviour.

Figure 2.2: Expected Wildfire Behaviour according to CHI [28]

1. IF $CB > 9$, THEN $CB = 9$;
2. IF $CB > 5$, THEN $CB = 5 + (CB-5)/2$.

where both conditions are checked sequentially, which limits the maximum value of CB to 7.

The Continuous Haines Index can take any value, usually between 0 and 14, where the higher the value is, the higher the potential is for large and erratic wildfires to occur, just like the base Haines Index. Figure 2.2 describes said wildfire behaviour for the different values of CHI.

2.1.2 The Canadian Forest Fire Weather Index System

The Canadian Forest Fire Weather Index System (CFFWIS) is composed of different metrics describing the forest floor fuel moisture and the relative wildfire behaviour. The metrics are calculated daily at noon, using local temperature, relative humidity, wind speed and the last 24h precipitation measures.

CFFWIS has six connected components, highlighted in Figure 2.3. The first three are the primary components, called fuel moisture codes, which describe the fuel moisture at three different levels of depth. They are as follow [10]:

- Fine Fuel Moisture Code - FFMC

The FFMC represents the moisture of the fine fuel (like grasses, leaves and mosses) found in the first layer of depth and measured at 1 to 2 centimeters underground, using the temperature, relative humidity, wind speed and 24h precipitation measures. As fires tend to start and spread in this layer, FFMC can be an indicator for ease of fire ignition. FFMC values range from 0 to 99, where fire usually starts at around 70 and the likelihood of it increases exponentially the higher this value is.

- Duff Moisture Code - DMC

The DMC represents the moisture of the duff layer fuels and measured at 5 to 10 centimeters underground, using the temperature, relative humidity and 24h

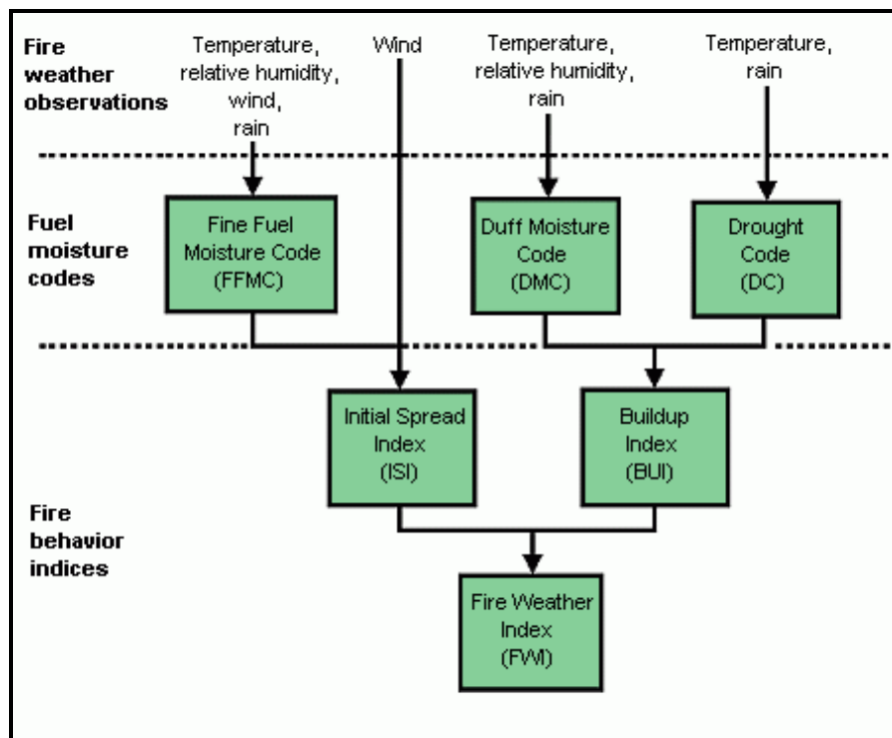


Figure 2.3: CFFWIS Component Hierarchy [4]

precipitation measures (it is too far underground for wind speed to be considered). As fires that start in this layer are usually due to lightning strikes, DMC can be an indicator of lightning fire ignition probability. DMC values are open-ended but usually fall in the range of 0 to 150.

- Drought Code - DC

The DC represents the moisture of the deep layer fuels and measured at 10 to 20 centimeters underground, using the temperature and 24h precipitation measures (it is too far underground for wind speed and relative humidity to be considered). DC values are open-ended but usually fall in the range of 0 to 800.

These components are then used to calculate the following intermediate components, which are connected to the wildfire behaviour:

- Initial Spread Index - ISI

The ISI represents the expected fire spread rate. It is calculated using FFMC, which represents the ease of fire ignition, and the wind speed measures, which influences the speed at which the fire might spread.

- Buildup Index - BUI

The BUI represents the total amount of fuel available for combustion. It is calculated using DMC and DC, the moisture codes for depths of 5 to 20 centimeters underground, where DMC has a higher influence than DC on the final value.

Finally these components are used to calculate the final risk index:

- Fire Weather Index - FWI

The FWI represents the expected fire intensity and danger. It is calculated using ISI, which represents the fire spread rate, and BUI, which represents the amount of available fuel for combustion.

CFFWIS also provides a seventh index, derived from FWI. The Daily Severity Rating (DSR) represents the expected efforts for fire suppression more accurately. Unlike the FWI, the DSR can also be averaged through time and/or space, allowing for more behavioural analysis.

$$DSR = 0.0272(FWI)^{1.77} \quad (2.7)$$

2.1.3 Vegetation Indexes

Vegetation Indexes represent different features of the forest vegetation, like health and moisture, which may have an impact on wildfire ignition. Many of these indexes are calculated using data from satellites or drones. Here are just a few of these indexes:

- Smoothed Normalized Difference Vegetation Index - SMN

SMN is an indicator of vegetation denseness and health, as it is calculated using the reflectance difference between visible and near infrared light, with noise removal. Values range between -1 and 1.

- Smoothed Brightness Temperature - SMT

SMT represents the infrared reflection of vegetation at a band of 10.3 to 11.3 μm , with noise removal.

- Temperature Condition Index - TCI

TCI is calculated using SMT, carrying more insight on the thermal condition of the vegetation. Values range between 0 and 100.

- Vegetation Condition Index - VCI

VCI is calculated using SMN, carrying more insight on the moisture of the vegetation. Values range between 0 and 100.

- Vegetation Health Index - VHI

VHI is a general vegetation health indicator, calculated using both VCI and TCI. Values range between 0 and 100.

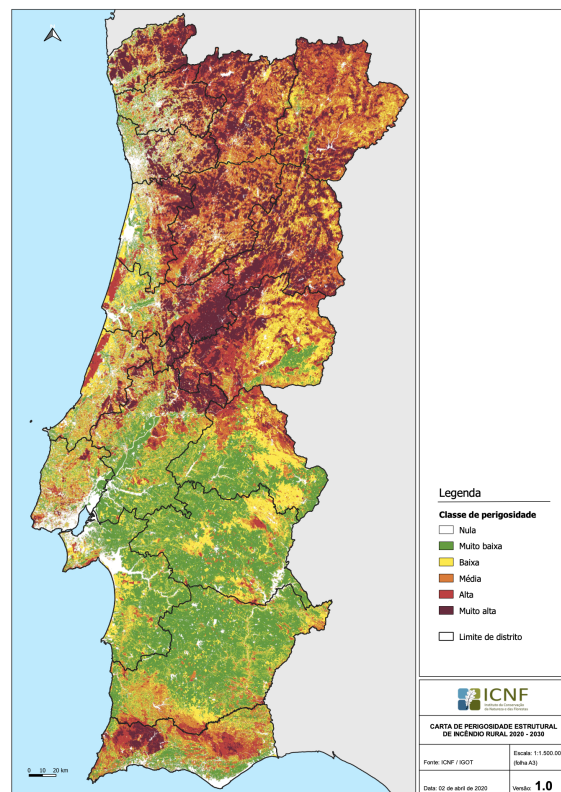


Figure 2.4: Structural Dangerousness Chart for decade of 2020-2030 [5]

2.1.4 Dangerousness Index (ICNF)

The Dangerousness Index (or PERIG) is a metric made available by the Instituto da Conservação da Natureza e das Florestas (ICNF) and represents the likelihood of fire occurrences. The PERIG is calculated using the slope, elevation and land cover, for each ground unit ($25 m^2$), which is then classified into one of five classes: Very Low, Low, Moderate, High, Very High. Every year and every decade, the ICNF makes available a Structural Danger Chart for Portugal for every year and decade respectively (example in Figure 2.4).

2.2 The Fire Danger Rating Scales

Using the different metric systems detailed in the previous section, researchers have over the years implemented different danger scales, as ways of classifying the danger level of wildfire occurrences.

The CFFWIS is the Canadian system and developed its own scale adapted to their country. One of the original danger scales is presented in Figure 2.5 and shows five danger levels: Very Low, Low, Moderate, High, Extreme, associated with different FWI ranges.

The increase in popularity of the FWI in wildfire classification lead to further studies and research on its application to other countries [20] [14] [13] [41]. In 2007, the European Forest Fire Information System (EFFIS) adapted the CFFWIS scale to the European Union

2.2. THE FIRE DANGER RATING SCALES

Fire Danger Classes	FWI
Very Low	0 - 1
Low	2 - 5
Moderate	6 - 12
High	13 - 24
Extreme	25 < FWI

Figure 2.5: CFFWIS scale, from [38]

Fire Danger Classes	FWI	FFMC	DMC	DC	ISI	BUI
Very Low	FWI < 5.2	FFMC < 82.7	DMC < 15.7	DC < 256.1	ISI < 3.2	BUI < 24.2
Low	5.2 - 11.2	82.7 - 86.1	15.7 - 27.9	256.1 - 334.1	3.2 - 5.0	24.2 - 40.7
Moderate	11.2 - 21.3	86.1 - 89.2	27.9 - 53.1	334.1 - 450.6	5.0 - 7.5	40.7 - 73.3
High	21.3 - 38.0	89.2 - 93.0	53.1 - 140.7	450.6 - 749.4	7.5 - 13.4	73.3 - 178.1
Very High	38.0 - 50	93.0 < FFMC	140.7 < DMC	749.4 < DC	13.4 < ISI	178.1 < BUI
Extreme	50 < FWI					

Figure 2.6: EFFIS Fire Danger Scale [16]

Risk	FWI	ISI	DC	FFMC	BUI	DMC
Very Low	0 - 8.5	0 - 2	0 - 250	0 - 75	0 - 50	0 - 50
Low	8.5 - 17.2	2 - 6	250 - 400	75 - 85	50 - 100	50 - 100
Moderate	17.2 - 24.6	6 - 12	400 - 600	85 - 90	100 - 150	100 - 150
High	24.6 - 38.3	12 - 17	600 - 750	90 - 93	150 - 200	150 - 200
Very High	38.3 - 50	17 - 23	750 - 900	93 - 95	200 - 250	200 - 250
Maximum	50 - 64	23 - 30	900 - 1100	95 - 97	250 - 325	250 - 325
Extreme	64 < FWI	30 < ISI	1100 < DC	97 < FFMC	325 < BUI	325 < DMC

Figure 2.7: IPMA FWI Danger Scale [17]

[16]. This EFFIS scale, as seen in Figure 2.6, defines the usual range of values for each index and sub-index, for each of the following six danger levels: Very Low, Low, Moderate, High, Very High and Extreme. In June 2023, another wildfire danger level was added, Very Extreme, following the need to further discriminate wildfire danger classification for Mediterranean countries during Summer months.

In the case of Portugal, IPMA has also been working with FWI since 1994 [39] [32] [40]. Although Portugal is a part of Europe, IPMA adopts a specialized scale for wildfire classification in the country, similar to but different than the EFFIS scale. The range of values for each index and sub-index are adapted to the region's particular ecological and geographical conditions. The IPMA scale details seven danger levels: Very Low, Low, Moderate, High, Very High, Extreme and Maximum, the latter being added after the great wildfires of 2017 [17]. This scale can be found in Figure 2.7.

For the present work, a combination of both CHI and IPMA FWI scales will be used. This scale, presented on Figure 5.3, has adapted both of the original scales to include the following fire danger levels: Very Low and Low, Moderate, High, Very High and Maximum.

Risk	CHI	FWI	ISI	DC	FFMC	BUI	DMC
Low/VLow	CHI < 4	FWI < 17.2	ISI < 6	DC < 400	FFMC < 85	BUI < 100	DMC < 100
Moderate	4 - 6	17.2 - 24.6	6 - 12	400 - 600	85 - 90	100 - 150	100 - 150
High	6 - 8	24.6 - 38.3	12 - 17	600 - 750	90 - 93	150 - 200	150 - 200
VHigh	8 - 10	38.3 - 50.1	17 - 23	750 - 900	93 - 95	200 - 250	200 - 250
Maximum	10 < CHI	50.1 < FWI	23 < ISI	900 < DC	95 < FFMC	250 < BUI	250 < DMC

Figure 2.8: Combination of CHI and IPMA FWI scales [17]

BACKGROUND KNOWLEDGE

3.1 The Archetypal Analysis Algorithm

Archetypal Analysis (AA) is a statistical method introduced by Cutler and Breiman [9] that treats each data point from a data set as a convex linear combination of the so-called "pure types" or "archetypes". Said "archetypes" are also treated as a convex linear combination of all data points and lie on the edge of the data space or convex hull.

These archetypes can be viewed as extreme potential instances of the data, which might generally represent different important aspects of the whole data set. This is useful when applying an AA clustering algorithm, as the archetypes share some of the same qualities of a prototype.

Due to its profile as a clustering algorithm capable of characterizing data through the definition of extremes, Archetypal Analysis can be useful in a myriad of areas of applications, such as Talent Analysis [15], Computer Vision [37][44], Behaviour Profiling [22], Medical Studies [1], Banking [43][27] and Benchmarking [29][30], among others.

3.1.1 The Model

Given a set of x_i ($i=1\dots N$), where x_i represents a data entity with D attributes and N is the number of entities, Archetypal Analysis seeks to find K archetypes, such that each archetype a_k is the convex combination of every data entity and each data entity x_i is the convex combination of every archetype. This is achieved by minimizing the residual sum of squares RSS

$$RSS = \min_{\delta, \beta} \sum_{i=1}^N \|x_i - \sum_{k=1}^K \delta_{ik} a_k\|^2 = \min_{\delta, \beta} \sum_{i=1}^N \|x_i - \sum_{k=1}^K \sum_{j=1}^N \delta_{ik} \delta_{kj} x_j\|^2 \quad (3.1)$$

and where:

$$\sum_{k=1}^K \delta_{ik} = 1, \text{ with } \delta_{ik} \geq 0 \text{ and } i = 1, \dots, N; \quad (3.2)$$

$$\sum_{i=1}^N \beta_{ki} = 1, \text{ with } \beta_{ki} \geq 0 \text{ and } k = 1, \dots, K; \quad (3.3)$$

Due to these constraints, the following is also ensured:

$$x_i = \sum_{k=1}^K \delta_{ik} a_k; \quad (3.4)$$

$$a_k = \sum_{i=1}^N \beta_{ki} x_i; \quad (3.5)$$

These constraints ensure that the archetypes will lie on the convex hull [9], except for the trivial case where number of archetypes is 1, where the archetype will be the sample mean of the data.

3.1.2 The Algorithm

In their introductory work [9], Cutler and Breiman also present an algorithm to solve the minimization problem of the Residual Sum of Squares criterion. This algorithm alternates between two convex least square problems, reducing the RSS over time, until convergence to local minima. The problem is framed as such:

Given a set of x_i ($i = 1, \dots, N$), where x_i represents a data entity and N is the number of entities, and a set of a_k ($k=1\dots q$), where a_k represents an archetype, we seek to find a matrix of coefficients β such that we minimize

$$\|X - \sum_{k=1}^q \beta_k a_k\|^2 \quad (3.6)$$

where $\beta_k \geq 0$ for $k = 1, \dots, q$ and $\sum_{k=1}^q \beta_k = 1$.

1. The values of β are randomly initialized while respecting the constraints and a set of archetypes a_k is calculated accordingly.
2. Using the obtained a_k values, we solve the first set of convex least square problems:

$$\|x_i - \sum_{k=1}^K \delta_{ik} a_k\|^2 \quad (3.7)$$

3. Using the obtained δ values, we solve the second set of convex least square problems, minimizing β :

$$\|\bar{v} - \sum_{i=1}^N \beta_{lj} x_j\|^2, \text{ where } \bar{v} = \frac{\sum_{i=1}^n \delta_{il}^2 \frac{(x_i - \sum_{k \neq l}^p \delta_{ik} a_k)}{\delta_{il}}}{\sum_{i=1}^n \delta_{il}^2} \quad (3.8)$$

4. Repeat from step (1), while using the newly obtained β values instead of initializing.

3.1.3 N-Partition Validation

As a clustering algorithm, the study of partition quality is essential in Archetypal Analysis. When the clustering process is done with a value of clusters closer to the actual unrevealed cluster structure of the data, said clusters will be of higher quality, being more representative of said data.

In their work [9], Cutler and Breiman proposed the usage of the minimized RSS (Equation 3.1) value and the "elbow method" in order to evaluate the best number of archetypes for a particular problem. As a score already indicative of partition quality, the RSS lends itself to this method. It consists in running the algorithm for different incremental numbers of archetypes and selecting the one value where the RSS curve appears to flatten.

This method has since been the most widely used for establishing an optimal number of archetypes for Archetypal Analysis.

3.2 The Fuzzy c-Means Algorithm

Fuzzy c-Means is a soft clustering algorithm originally introduced by J.C. Dunn in [12] and improved by J.C. Bezdek in [3], where each data point does not belong to a single cluster, but has a degree of membership for each of the clusters which represents how similar it is to each of said clusters.

This sort of fuzzy clustering is very useful when trying to view data based on a fuzzy variable, since the degree of membership allows for a more gradual transition between states, instead of a more binary analysis.

3.2.1 The Algorithm

Given a set of x_i ($i = 1, \dots, N$), where x_i represents a data entity and N is the number of entities, and a number of clusters K , we seek a matrix $U = [\mu_{i,k}]$, where μ is the degree of membership between 0 and 1 of x_i to cluster k , such that we minimize

$$J(X, U, V) = \sum_{k=1}^K \sum_{i=1}^N (\mu_{i,k})^m \|x_i - v_k\|_A^2 \quad (3.9)$$

where m is a fuzziness degree (generally $m=2.0$) and A is a distance matrix (generally $A=I$)

- The values of μ are randomly initialized with values between 0 and 1.
- Calculate new centroids for each cluster:

$$v_k = \frac{\sum_{i=1}^N (\mu_{i,k}^{(l-1)})^m x_i}{\sum_{i=1}^N (\mu_{i,k}^{(l-1)})^m} \quad (3.10)$$

where l is the iteration number and $1 \leq k \leq K$.

- Calculate the distance between each data point and each new centroid:

$$D_{i,kA}^2 = (x_i - v_k)^T A (x_i - v_k) \quad (3.11)$$

where $1 \leq i \leq N$ and $1 \leq k \leq K$.

- Update U such as:

$$\mu_{i,k} = \frac{1}{\sum_{j=1}^K \left(\frac{D_{i,kA}^2}{D_{i,jA}^2} \right)^{\frac{2}{m-1}}} \quad (3.12)$$

- Repeat from step (2) while the error difference between iterations is lower than ϵ .

3.2.2 c-Partition Validation

As a clustering algorithm, the study of partition quality is necessary when searching for the best number of clusters for a given dataset. For this, we can try a range of number of cluster values and compare their respective results. This quality can be estimated using internal validity indices.

Over the years, many internal validity indices have been presented [2][21][33]. As each validity index measures different aspects of the resulting clusters, there is no real consensus on a singular index being preferable over the others and being capable of estimating partition quality on their own. Any conclusion is then based on multiple tests being made, using different internal validity indices.

These internal validity indices estimate clustering results quality by measuring intra-cluster proximity and inter-cluster separation, meaning that better cluster quality is achieved when clusters maximize proximity of data points that belong to the same cluster and maximize separation between data points that belong to different clusters.

One of these indices is the Xie-Beni Index[42]. XB represents the ratio between the total inter-cluster variation and the separation of the clusters. We seek to minimize

$$XB(c) = \frac{\sum_{i=1}^c \sum_{j=1}^n u_{ij}^2 \|x_j - v_i\|^2}{n \left(\min_{1 \leq i, k \leq c, i \neq k} \|v_i - v_k\|^2 \right)} \quad (3.13)$$

Its implementation is available for fuzzy clustering algorithms in [6].

3.3 Tree-Based Classification with Rule Extraction

Rule extraction can be used to generate an easily understandable representation of data classification from a model, while maintaining its predictive capabilities. The idea is to translate the model into a set of rules which the data adheres by.

These IF-THEN Rules generally follow the same format: an IF statement, with a set number of conditions that must be met in order to activate said rule, and a THEN statement, that specifies some information about the samples where all the conditions are met, like a data label for example.

Here are presented two tree-based classification algorithms that can be used to extract rules from data.

3.3.1 Decision Trees

Due to their tree-based structure, Decision Trees can be used for rule extraction. A Decision Tree can be seen as a set of inter-connected nodes, where each node branches into other nodes until getting to the so-called leaf-nodes, as seen in Figure 3.1.

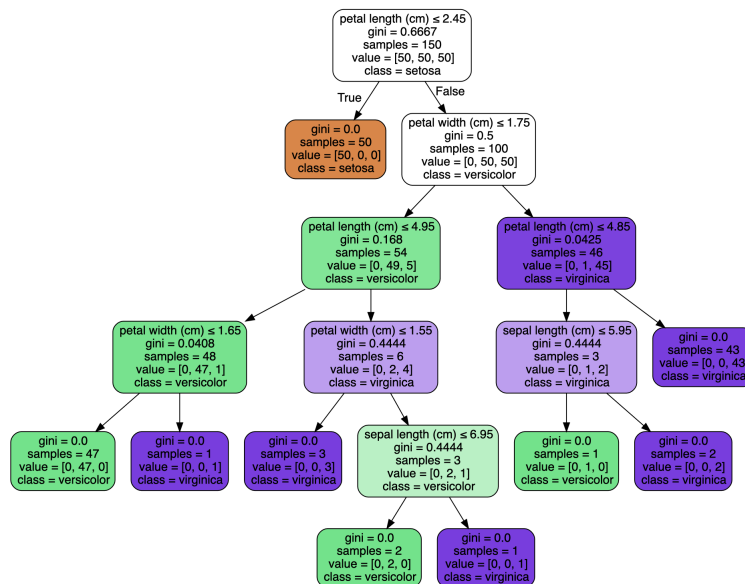


Figure 3.1: Decision Tree example, applied to the Iris Dataset [11]

Decision Trees models keep dividing the data into different disjointed subsets, according to different parameters of the data and until the data is sufficiently divided. These divisions are represented by the multiple branching paths from a particular node, where one can "make a decision" on which path to follow based on the condition of a certain parameter. Finally, each leaf-node details some concluding information regarding the nodes on the path to get there, like a data label or feature value. These can then be translated into rules, where each path is a rule where the internal nodes are IF conditions and the leaf-node serves as a THEN statement.

Finally, there are two types of Decision Trees. Classification Trees are Decision Trees where the deciding feature in the leaf-node is categorical, like sex, age or class. On the other hand, Regression Trees are Decision Trees where the deciding feature in the leaf-node is continuous, like weight, distances or percentages.

In order to divide the data in the best way possible, the Decision Tree algorithm measures the quality of a split. Two of the most used criterion are the Gini Impurity and the Entropy.

The Gini Impurity measures the probability of randomly labelled samples from the set being incorrectly labelled, following class distribution. It can be calculated as such:

$$I_G(p) = \sum_{i=1}^J \left(p_i \sum_{k \neq i} p_k \right) = \sum_{i=1}^J p_i (1 - p_i) = \sum_{i=1}^J (p_i - p_i^2) = \sum_{i=1}^J p_i - \sum_{i=1}^J p_i^2 = 1 - \sum_{i=1}^J p_i^2 \quad (3.14)$$

where J is the number of classes and p_i is the probability of a sample being from class i .

The Entropy criterion is used based on the concepts of Information Gain from Information Theory. Entropy represents the level of uncertainty of a variable, due to its possible values. It can be calculated as such:

$$H(T) = I_E(p_1, p_2, \dots, p_J) = - \sum_{i=1}^J p_i \log_2 p_i \quad (3.15)$$

where J is the number of classes and p_i is the percentage of class i present in child nodes, after the split.

Given the Information Gain Formula 3.16, the best data split occurs when we maximize the difference between node entropy and the sum of entropy of its children.

$$IG(T, a) = H(T) - H(T|a) \quad (3.16)$$

where $H(T)$ is the entropy of a node and $H(T|a)$ is the sum entropy of the children nodes.

3.3.2 RIPPER

The Repeated Incremental Pruning to Produce Error Reduction algorithm [8] or RIPPER is a rule-based classification algorithm. RIPPER works by dividing the data into two sets: a growing set, used to generate rules, and a pruning set, used to remove conditions that reduce the precision of the rules.

To generate a single rule for a given class, RIPPER starts by adding conditions from the growing set to an empty rule as long as they improve the information gain and stopping when the rule starts covering samples that are labelled with another, different class.

After generating the rule, the pruning process begins, where different combinations of conditions are removed from the rule if that increases the metric presented in Equation 3.17.

$$m = \frac{p - n}{p + n} \quad (3.17)$$

where p (positive examples) is the number of samples from the pruning set that validate the rule for the correct class and n (negative examples) is the number of samples from the pruning set that validate the rule for other classes.

After both growing and pruning, a new rule is generated, the positive and negative examples activated by that rule are removed and this process repeats.

The main difference between rules generated with Decision Trees and RIPPER is that the RIPPER ruleset as sequential rules. Instead of having each rule on its own, RIPPER rules must be analysed in context with all the rules that came before, as in a IF-THEN-ELSE structure. As seen in example Figure 3.2, the second rule (seen as C4 AND C5) implies the negation of the first rule (seen as C1 and C2 and C3). The same can be said for the third rule and the negation of both previous rules.

```

IF (C1 and C2 and C3):
  THEN Class1
ELSE IF (C4 and C5):
  THEN Class2
ELSE:
  Class3

```

Figure 3.2: RIPPER Ruleset example

3.3.3 Rule Evaluation

In order to analyse and compare extracted rules, we use the Interpretability Score, as presented in [25]. This rule score consists on the averaging of three secondary scores: the Predictivity, Simplicity and Stability scores.

3.3.3.1 Predictivity Score

The Predictivity Score represents the model ability to accurately predict the value of a random variable. It is calculated as:

$$\mathcal{P}_n(g_n, h_n) = 1 - \frac{\mathcal{L}_n(g_n)}{\mathcal{L}_n(h_n)} \quad (3.18)$$

where \mathcal{L}_n is the empirical risk, g_n is our model and h_n is a naive baseline predictor chosen by the analyst which we are comparing g_n to.

Since this is a fraction of the empirical risk of a model over the empirical risk of a naive predictor, we expect the model to always have better, lower risk. As such, the Predictivity Score ranges from 0 to 1, where higher Scores indicate a model with lower empirical risk and thus better precision.

3.3.3.2 Simplicity Score

The Simplicity Score represents how simple and easily verifiable are the rules from a model in relation to another model, by comparing the size and number of the rules or the interpretability index of a ruleset [24] (note: this index is unrelated to the Interpretability Score discussed in this section).

This interpretability index is calculated as such:

$$Int(g_n) = \sum_{r \in R_n} length(r) \quad (3.19)$$

where $length(r)$ is the length of a rule from ruleset R_n

In order to compare ruleset simplicity, we calculate the Simplicity Score of each algorithm A_i in \mathcal{A} as:

$$S_n(A_i, \mathcal{A}) = \frac{\min\{Int(g_n^A) : A \in \mathcal{A}\}}{Int(g_n^{A_i})} \quad (3.20)$$

Since this is a fraction of the lowest found interpretability index in all studied algorithms over the interpretability index in a studied algorithm, the Simplicity Score ranges from 0 to 1, where higher Scores indicate an algorithm simpler than the others.

3.3.3.3 Stability Score

The (Q-)Stability Score represents how stable a model is, based on how similar two independent ruleset estimations of two disjoint subsets of the original data are.

In order to compare rulesets that involve continuous values, a series of equally distributed range intervals are generated for each feature and their value is replaced in the rules by an according discrete label. This is done since otherwise it would be highly improbable to find two rules with the same exact continuous value.

The Stability Score is calculated as such:

$$S_n^q(\mathcal{A}) = \frac{2|Q_q(R_n) \cap Q_q(R'_n)|}{|Q_q(R_n)| + |Q_q(R'_n)|} \quad (3.21)$$

where $Q_q(R_n)$ is the function that replaces continuous values with discrete labels in each rule from R_n and R_n and R'_n are two independently estimated rulesets.

The Stability Score ranges from 0 to 1, as the higher the intersection between both rulesets is, the higher their similarity and thus the stability of the model.

DATA COLLECTION AND DESCRIPTION

4.1 Data Collection

As this work is a continuation of [35], the same datasets will be used, which were assembled during said work.

The datasets contain a collection of attributes from different sources, representing wildfires from mainland Portugal between 2001 and 2020. They are organised into six sets, each dedicated to a particular region: mainland Portugal, North West, North East, Center West, Center East, South. This division allows for a more specific view of the country's very diverse regions, while also having a more general view of the whole data. Figure 4.1 shows the different regions.

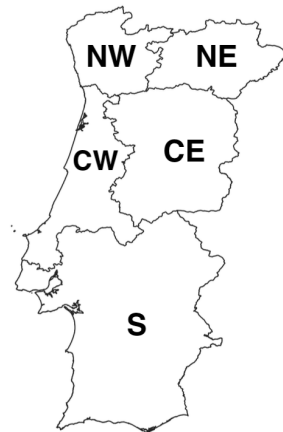


Figure 4.1: Portuguese Region Partitions: North West (NW), North East (NE), Center West (CW), Center East (CE) and South (S).

Here follow the attributes:

- date: the date of wildfire start, in format YYYY-MM-DD
- area_ha: the burned area, in hectares
- source: the date and burned area data source (can be Maryland or ICNF)

- CHI: the Continuous Haines Index
- FWI: the Fire Weather Index
- ISI: the Initial Spread Index
- DC: the Drought Code
- FFMC: the Fine Fuel Moisture Code
- BUI: the Buildup Index
- DMC: the Duff Moisture Code
- DSR: the Daily Severity Rating
- PERIG: the Dangerousness Index
- SMN: the Smoothed Normalized Difference Vegetation Index
- SMT: the Smoothed Brightness Temperature
- TCI: the Temperature Condition Index
- VCI: the Vegetation Condition Index
- VHI: the Vegetation Health Index
- region: the region of wildfire occurrence

Date, burned area and region information were made available by the ICNF and complemented with data from a repository from the University of Maryland, obtained using daily recordings of the MODIS satellite [18]. ICNF had data from wildfires from 2012 to 2019 (excluding 2017) and the University of Maryland from 2001 to 2020 (excluding 2019). The source attribute represents the origin of this data.

The risk indexes CHI, FWI, ISI, DC, FFMC, BUI, DMC and DSR were made available by IPMA and were calculated using measurements from European Center for Medium-Range Weather Forecasts (ECMWF). These indexes were calculated daily at noon, using local temperature, relative humidity, wind speed and the last 24h precipitation measures, at regular distances on a geographic mesh of mainland Portugal (see Figure 4.2). Their description can be found in Section 2.1.

The Dangerousness Index PERIG was obtained via the Structural Dangerousness Charts from ICNF. Its description can be found in Section 2.1.

The vegetation indexes SMN, SMT, TCI, VCI and VHI were made available from data obtained from the MODIS satellite and through the National Oceanic and Atmospheric Administration repository. Their description can be found in Section 2.1.

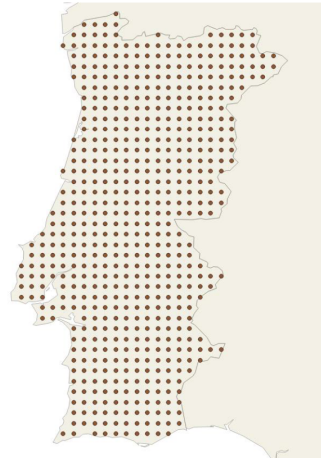


Figure 4.2: Mainland Portugal Geographic Mesh, points at $0,125^\circ$ Latitude and $0,125^\circ$ Longitude intervals. Image from [35].

4.2 Data Preprocessing

For this dissertation and since we are searching for more extreme cases of forest fires, the data was filtered to include wildfires with burned area greater than 100ha and that occurred between May and October of each year. The number of samples for each region can be found in Table 4.1.

Region	2001-2018	2019-2020	Total
North East	238	16	254
North West	482	22	504
Center East	596	35	631
Center West	181	9	190
South	203	16	219
Mainland Portugal	1700	98	1798

Table 4.1: Number of Wildfires per Region

Following the conclusions from [35], a clustering approach using the Dangerousness Index PERIG and the Vegetation Indexes SMN, SMT, TCI, VCI and VHI did not lend themselves to satisfying results. As such and in order to study and compare results to current fire danger rating scales, only the CHI, FWI, ISI, DC, FFMC, BUI and DMC indexes and sub-indexes were taken into account. As a way to explore the impact of different combinations of features, the data for each region was divided into two collections with the help of an expert in the field: one using only five of the features - CHI, FWI, ISI, DC, FFMC - and one using the seven - CHI, FWI, ISI, DC, FFMC, BUI and DMC.

Finally, for Training and Testing phases of algorithms, the datasets were also divided in two, where the first half contained samples from 2001 to 2018 and the second half samples from 2019 and 2020. On top of the necessity to train and test the algorithms, this division also allows for a better understanding on how well do these approaches react to

more recent data.

Table 4.2 shows the final distribution of samples per dataset and their dimensions.

Region	Features	Years	Dataset	Dimensions
North East	5	2001-2018	NE5-Train	238x5
		2019-2020	NE5-Test	16x5
	7	2001-2018	NE7-Train	238x7
		2019-2020	NE7-Test	16x7
North West	5	2001-2018	NW5-Train	482x5
		2019-2020	NW5-Test	22x5
	7	2001-2018	NW7-Train	482x7
		2019-2020	NW7-Test	22x7
Center East	5	2001-2018	CE5-Train	596x5
		2019-2020	CE5-Test	35x5
	7	2001-2018	CE7-Train	596x7
		2019-2020	CE7-Test	35x7
Center West	5	2001-2018	CW5-Train	181x5
		2019-2020	CW5-Test	9x5
	7	2001-2018	CW7-Train	181x7
		2019-2020	CW7-Test	9x7
South	5	2001-2018	S5-Train	203x5
		2019-2020	S5-Test	16x5
	7	2001-2018	S7-Train	203x7
		2019-2020	S7-Test	16x7
Mainland Portugal	5	2001-2018	PT5-Train	1700x5
		2019-2020	PT5-Test	98x5
	7	2001-2018	PT7-Train	1700x7
		2019-2020	PT7-Test	98x7

Table 4.2: Datasets Dimensions and Characteristics. Five Features include CHI, FWI, ISI, DC, FFMC, BUI and DMC. Seven Features include CHI, FWI, ISI, DC, FFMC, BUI and DMC

EXPERIMENTAL PROTOCOL

This chapter describes the Experimental Protocol developed for this dissertation. It starts with the protocol Workflow, followed by the description of the main stages, illustrated with examples.

5.1 General Workflow

The developed protocol is divided into two phases: a Wildfire Clustering and Classification Phase and a Risk-Based Rule Extraction Phase. The Workflow is presented in Figure 5.1.

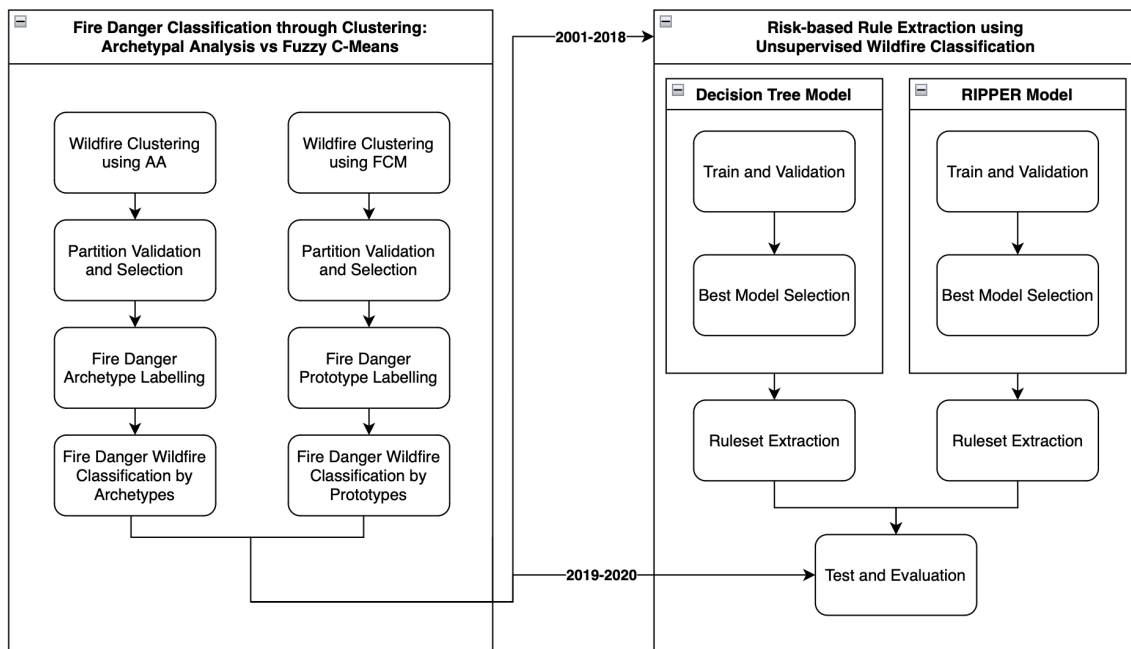


Figure 5.1: Project Workflow

This protocol is applied for each of the six datasets - Mainland Portugal (PT), Center East (CE), Center West (CW), North East (NE), North West (NW), and South (S) -, for both 5- and 7-Feature groups - CHI,FWI,ISI,DC and FFMC and CHI,FWI,ISI,DC,FFMC,BUI and DMC.

Phase 1: Fire Danger Classification through Clustering: Archetypal Analysis vs Fuzzy c-Means

1- Wildfire Clustering using Archetypal Analysis and Fuzzy c-Means

- Both algorithms are run several times, for tuning of the number of clusters hyperparameter.

2- Archetypal Analysis and Fuzzy C-Mean Partition Validation and Selection

- For each algorithm, partitions of 5 to 9 clusters are tested.
- The RSS is used to determine the best number of clusters for Archetypal Analysis.
- The Xie-Beni Index is used to determine the best number of clusters for Fuzzy c-Means.
- For each algorithm, the best partition for both 5 and 7 Features is selected by comparing and taking the best results in these metrics.

3- Fire Danger Archetype and Prototype Labelling

- The resulting Archetypes and Prototypes are classified using the fire danger rating scales and with the help of a specialist in the field.

4- Fire Danger Wildfire Classification by Archetypes and Prototypes

- The same risk level label of an Archetype/Prototype is attributed to each Wildfire in its cluster.
- For each classification set, a precision and error score are calculated.

Phase 2: IF-THEN Rules Extraction for Fire Danger Classification

1- Decision Tree and RIPPER Models Construction

- Model Training and Validation is done using training data from 2001-2018.

2- Decision Tree and RIPPER Models Selection

- Testing is done using the testing data from 2019-2020.
- For each classifier, the best model is selected using the F1 measure.

3- Decision Tree and RIPPER Ruleset Evaluation

- Ruleset Evaluation is done using the F1 measure and the Interpretability score (Predictivity, Stability and Simplicity scores).

4- IF-THEN Fire Danger Rules

- Rulesets are compared to the reference Fire Danger Rating Scales.

5.2 Fire Danger Classification through Clustering: Archetypal Analysis vs Fuzzy c-Means

The goal of this phase is to cluster and derive the fire danger classification of wildfires from the two data collections (5- and 7- Features). In order to do this, we are using two clustering algorithms to group wildfires by their similarities, which allows us to classify them according to the group they are attributed to. These clustering algorithms are the Archetypal Analysis and the Fuzzy c-Means algorithms, described in Sections 3.1 and 3.2.

5.2.1 Wildfire Clustering using Archetypal Analysis and Fuzzy c-Means

Clustering algorithms are used to group samples according to how similar they are to each other. Both Archetypal Analysis and Fuzzy c-Means generate N- or c-partitions where N or c is hyperparameter denoting the number of clusters. Each of these clusters has a representative point belonging to the domain space of the entities, that is characterized by the same features as the data points.

In the case of Archetypal Analysis, these representative points are the archetypes, which represent extreme wildfire instances with extreme feature values, as they lie at the edge of the data space. On the other hand, Fuzzy c-Means generates prototypes which are the weighted average of the fire occurrences belonging to that cluster. An example of found Archetypes can be found in Table 5.1.

CE5	CHI	FWI	ISI	DC	FFMC
Arq1	1.59	-3.55	-1.0	617.8	14.19
Arq2	-1.7	12.47	3.16	328.03	81.1
Arq3	11.57	27.19	7.24	419.12	92.22
Arq4	5.96	39.74	10.01	1385.15	93.5
Arq5	9.23	69.68	25.08	623.64	97.14

Table 5.1: Example of Archetypes found in Center East, for 5 Features and 5 Clusters.

In order to have a better indicator of the algorithm performance, they are both run several times and their performance metrics averaged. This will result in more stable metrics for us to use during the following stages.

5.2.2 Archetypal Analysis and Fuzzy C-Mean Partition Validation and Selection

When working with partitional clustering algorithms, one of the most important steps is the fine tuning of the hyperparameters and the main hyperparameter that needs adjustments is the number of clusters.

For this study, we want to see how well adjusted fire danger rating scales with 5, 6 and 7 risk levels are to the classification of wildfires. As such, we can associate the number

of risk levels we expect to see in the data to the number of clusters for the algorithms. The number of clusters tested ranged between 5 and 9, allowing for a wider range of information gathering and analysis.

The remaining hyperparameters for Fuzzy c-Means are defined as follows: stopping criterion of 0.001, maximum number of iterations of 1000 and the degree of fuzziness $m = 2.0$.

For each algorithm, we validate and compare the distinct N- and c-partitions using state of the art validity measures. Specifically, for Archetypal Analysis, we use the Residual Sum of Squared Error (RSS) and the "elbow method" in order to choose the best number of archetypes, as described in Section 3.1.3. The RSS for a partition of 4 clusters is calculated just so that this method can be applied to the results of 5 clusters. For Fuzzy c-Means, we choose the c-partition with the lowest Xie-Beni Index internal validity index, which represents a ratio between cluster compactness and cluster separation and is widely used to evaluate Fuzzy c-Means partitions, as described in Section 3.2.2. In order to compare results between partitions, a normalized Xie-Beni Index is also calculated, using Range Normalization. Like any other validity index, Xie-Beni should not be analysed on its own. For each algorithm and number of features, the best partition is selected using these approaches.

Figure 6.3 illustrates the RSS metric across different number of clusters, for Archetypal Analysis, where the elbow method is applied.

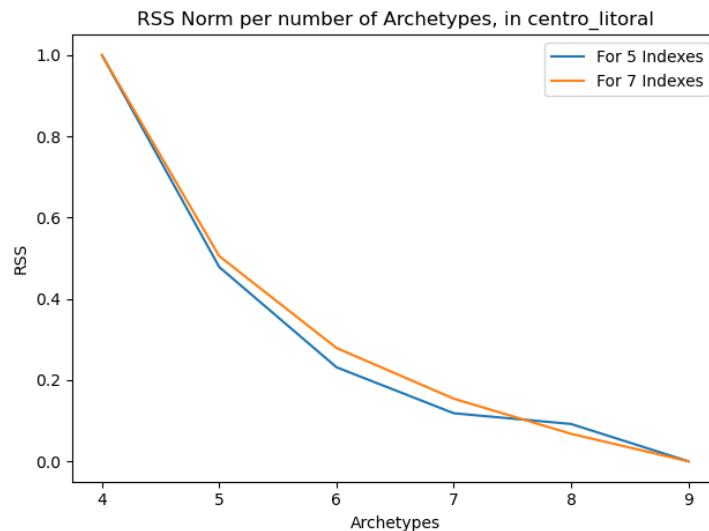


Figure 5.2: RSS for Center West, for 5 and 7 Features. In this case, 5 Archetypes appears to be the beginning of the flattening of the curve for both 5 and 7 Features, so that would be the chosen partition.

5.2. FIRE DANGER CLASSIFICATION THROUGH CLUSTERING: ARCHETYPAL ANALYSIS VS FUZZY C-MEANS

Risk	CHI	FWI	ISI	DC	FFMC	BUI	DMC
Low/VLow	CHI < 4	FWI < 17.2	ISI < 6	DC < 400	FFMC < 85	BUI < 100	DMC < 100
Moderate	4 - 6	17.2 - 24.6	6 - 12	400 - 600	85 - 90	100 - 150	100 - 150
High	6 - 8	24.6 - 38.3	12 - 17	600 - 750	90 - 93	150 - 200	150 - 200
VHigh	8 - 10	38.3 - 50.1	17 - 23	750 - 900	93 - 95	200 - 250	200 - 250
Maximum	10 < CHI	50.1 < FWI	23 < ISI	900 < DC	95 < FFMC	250 < BUI	250 < DMC

Figure 5.3: Combination of CHI and IPMA FWI scales [17]

5.2.3 Fire Danger Archetype and Prototype Labelling

After selecting the best Archetypal Analysis N-partitions and Fuzzy c-Means c-partitions, we can label their cluster representatives - archetypes and prototypes -, before using them to help classify the wildfire occurrences.

Firstly, each of the archetype and prototype's features (i.e. the fire indexes/sub-indexes) are compared to their respective fire danger rating scales, presented in Section 2.2 and seen in Table 5.3, and attributed a label according to their risk (see example on Table 5.2).

CE5	CHI	FWI	ISI	DC	FFMC	Risk Level
Arq1	1.59	-3.55	-1.0	617.8	14.19	???
Arq2	-1.7	12.47	3.16	328.03	81.1	???
Arq3	11.57	27.19	7.24	419.12	92.22	???
Arq4	5.96	39.74	10.01	1385.15	93.5	???
Arq5	9.23	69.68	25.08	623.64	97.14	???

Table 5.2: Example of Archetype Features Classification. For the colors, Green represents a "Low" classification, Yellow represents a "Moderate", Orange represents a "High", Light Red represents a "Very High" and Dark Red represents a "Maximum".

Then, having the different label for each index, the archetype/prototype is classified taking into account the average risk levels from these indexes. For example, in Arq4 from Table 5.2, CHI and ISI are "Moderate", FWI and FFMC are "Very High" and DC is "Maximum". As such, Arq4 could be labelled as "Very High". This approach does have some limitations. In real cases, specialists in the field will give some indexes more importance when choosing a Risk Level. For example, if we had a CHI and ISI "Maximum" and the remaining indexes as "Moderate", should we average to "High", "Very High" or should we give importance to some of the indexes and keep it at "Maximum" or "Moderate"?

To decide about this, a manual revision with the aid of a specialist in the field follows this process. Here, more importance is given to CHI, FWI and ISI indexes, which allows for some corrections in the final labelling. This is especially the case for "Very Low/Low" and "Maximum" Risk Levels, due to their extreme nature and the softening of said extremes during averaging.

Having finished the labelling process, the individual indexes label are compared to the final archetype or prototype label as a way to analyse how well does said archetype or prototype match the fire danger rating scales. Each index is color-coded in order to

represent how well it fits with the expected range of values for that index, for that risk level. Green represents an index value within the specified range for that label. Yellow represents an index value one risk level above or below the specified range for that label. Red represents an index value with two or more risk levels above or below the specified range for that label. An example of this can be found in Table 5.3.

CE5	CHI	FWI	ISI	DC	FFMC	Risk Level
Arq1	1.59	-3.55	-1.0	617.8	14.19	Low
Arq2	-1.7	12.47	3.16	328.03	81.1	Low
Arq3	11.57	27.19	7.24	419.12	92.22	High
Arq4	5.96	39.74	10.01	1385.15	93.5	Very High
Arq5	9.23	69.68	25.08	623.64	97.14	Maximum

Table 5.3: Example of Archetype Classification based on the Risk Level of each feature.

5.2.4 Wildfire Classification using Archetypes and Prototypes

After labelling the Archetypes and Prototypes, we can use their label to classify the wildfires. Each wildfire can then be analysed and classified according to how similar it is to each of the Archetypes/Prototypes.

In the case of Archetypal Analysis, since each wildfire can be seen as a convex linear combination of every extreme archetype, we calculate the classification based on this combination and the different coefficients for each archetype. As such, the Risk Level of a wildfire is based on the weighted combination of the Risk Levels of each of the Archetypes.

For Fuzzy c-Means, each wildfire does not belong to a single cluster due to fuzziness and has instead a membership degree to each of the clusters that represents its similarity to the prototype. We calculated the classification based on this membership degree to each of the prototypes. As such, similarly to Archetypal Analysis, the Risk Level of a wildfire is based on the weighted combination of the Risk Levels of each of the Prototypes.

For example, let us consider a wildfire w such that we want to find its classification wc , where w is a convex linear combination of five archetypes a and c_j is the coefficient of similarity of w to a_j . We start by attributing a numeric value to each archetype a_j , based on their risk level label ac_j : "1" for "Very Low/Low", "2" for "Moderate", "3" for "High", "4" for "Very High" and "5" for "Maximum". Table 5.4 shows the similarity coefficients c_j for w and each archetype.

	Arq1	Arq2	Arq3	Arq4	Arq5
w	0,05	0,05	0,2	0,4	0,3

Table 5.4: Similarity Coefficients for each Archetype, for example w

Since, w is a weighted combination of all archetypes, we can calculate the final classification as:

5.2. FIRE DANGER CLASSIFICATION THROUGH CLUSTERING: ARCHETYPAL ANALYSIS VS FUZZY C-MEANS

$$\begin{aligned}
 wc &= ac_1 * c_1 + ac_2 * c_2 + ac_3 * c_3 + ac_4 * c_4 + ac_5 * c_5 \\
 wc &= "VL/Low" * 0,05 + "Mod" * 0,05 + "High" * 0,2 + "VHigh" * 0,4 + "Max" * 0,3 \\
 wc &= 1 * 0,05 + 2 * 0,05 + 3 * 0,2 + 4 * 0,4 + 5 * 0,3 \\
 wc &= 3,85
 \end{aligned}$$

Rounding to the nearest integer, w has a classification of 4, or "Very High". This method is also applied to Fuzzy c-Means, where c_j is not the coefficient of similarity of w to the archetype a_j , but the degree of membership from w to the cluster j .

At this point, for each geographic region and algorithm, we have two collections of data sets - each collection corresponding to 5 and 7 Features, respectively - with an additional feature: the attribute class "Risk Level", corresponding to the fire danger derived from the labeling of the corresponding archetypes or prototypes.

The resulting classifications are then analysed using confusion tables, comparing wildfires classified using our approach and wildfires classified using the current fire danger rating scales. An example of these confusion tables can be found in Table 5.5.

CE	VL+Low	Moderate	High	Very High	Maximum	Total
VL+Low	19	16	0	0	0	35
Moderate	0	100	41	0	0	141
High	0	2	306	56	0	364
Very High	0	0	4	82	4	90
Maximum	0	0	0	0	1	1
Error	0	2	4	0	0	631 6 0.95%
Precision	19	100	306	82	1	508 80.51%

Table 5.5: Example of Confusion Table for Archetype Classification in Center East, for 5 Features and 5 Clusters.

To evaluate the matching between the domain based classification (in rows) and the one derived from archetypes or prototypes (in columns), we calculate the Precision and a Misclassification Error. Here, Precision is the standard precision used for Multi-Class Classification, as seen in Equation 5.1, where TP_c and FP_c are the number of True and Negative Positives for class c , respectively. In our case, True Positives are wildfire occurrences that were classified as having the same risk by both our approach and the scales. In the confusion tables, those are all the occurrences that in the diagonal.

$$Prec = \frac{\sum_{c=1}^C TP_c}{\sum_{c=1}^C TP_c + FP_c} \quad (5.1)$$

As we are dealing with a real-life risk, where classifying wildfires above their actual risk level more acceptable than classifying below its real risk, a pessimistic classification is preferable over an optimistic one. As such, the Misclassification Error only takes into account wildfire occurrences that were classified with a lower risk using our approach than using the scales. In the confusion tables, those are all the occurrences that happen below the diagonal. For example, in Table 5.5, there were two wildfire occurrences that were classified as Moderate (column) using the Archetypes, that would have been classified as High (row) using the fire danger rating scales. Since these wildfires were classified lower than they should, these are counted as a misclassification errors.

Finally, for each region, the best 5- and 7-Feature partitions are analysed and compared and the partition with the best results is then selected for that region and algorithm. This means that for the next Phase, only one partition with either 5 or 7 Features, per Algorithm and per Region, is taken into consideration.

5.3 IF-THEN Rules Extraction for Fire Danger Classification

The second stage of this study focuses on the definition of IF-THEN rules, upon which the risk level of any new wildfire can be correctly attributed. For this, we use Decision Tree and RIPPER rule-based classifiers to generate rulesets that are tested and compared to the fire danger rating scales in use.

For this stage, we are dividing the newly classified data into a training and testing subsets, where training contains wildfire samples from the years 2001 to 2018 and testing contains wildfire samples from the years 2019 and 2020. The goal is to analyse how the models behave with the extension of more recent samples.

5.3.1 Decision Tree and RIPPER Models Construction

Decision Trees are one of the classifiers that we are using to extract IF-THEN rules from the data. A Decision Tree can be seen as a series of interconnected nodes as seen in Section 3.3.1, where each downwards path is viewed as a sequence of True and False conditions that lead to a final classification.

In this study, the following hyperparameters are used to build the Decision Tree models:

Split Criterion: it is the function that measures the quality of a node split. The tested functions are the "Gini Impurity" and "Entropy".

Maximum Depth: limits how deep can a tree be. Deeper trees may result in more accurate classifications but more complex trees. The tested depths range from 2 to 9.

Minimum Number of Samples per Split: limits the number of samples required to split a node. The tested values range from 2 to 40.

Minimum Number of Samples per Leaf: limits the number of samples required to be at a leaf node. The tested values range from 1 to 20.

During the training and validation phase, we are also using a cross-validation technique called Stratified Shufflesplit. Cross-validation allows for a more accurate understanding of the model performance by dividing the training data into a train and validation subsets multiple times and measuring their average performance. On top of this, in Stratified Shufflesplit, the subsets are randomly split while preserving the original data class distribution for each subset.

Decision Trees, the performance metrics and the Stratified Shufflesplit are implemented using the packages available in the scikit-learn python library [11][7][36].

Similarly to Decision Trees, we are also using RIPPER as a rule-extractor. As seen in Section 3.3.2, RIPPER extracts IF-THEN-ELSE rules from our data. The biggest difference between Decision Trees and RIPPER rules is that RIPPER rules are sequenced one after the other, meaning that a rule cannot be examined on its own but by also taking into account the negation of the previous rules.

RIPPER is implemented using the python-weka-wrapper3 python library [34].

5.3.2 Decision Tree and RIPPER Models Selection

For Decision Trees, each combination of hyperparameters is tested for each decision tree depth and the selected combination for that depth maximizes the value of the average F1 measure across multiple runs. This leaves us with eight Decision Tree Models, one for each depth ranging from 2 to 9.

In order to select the best Decision Tree Model, we compare not only the F1 measure of each one, which we want to maximize, but we also take into account the number of nodes and number of leaves. Not only is the accuracy important, but the readability as well. Generally, as the Decision Tree depth increases, so do the number of nodes and leaves. The number of leaves represent the number of rules that can be generated and the number of nodes is the number of conditions. Taking an example from Table 5.6, Decision Trees with depth of 5 and above have very good F1 scores, all above 0.9, but they might be hard to interpret, with 28 or more rules and 55 or more conditions. As such, the Decision Tree of depth 4 is be a better choice, having a still-good, slightly lower F1 score and much better readability.

The best Decision Tree Model is then selected, in conjunction with the RIPPER Model.

PT	Model	Criterion	ML	MS	NN	NL	Train/Val			Test		
							Precis	Rec	F1	Precis	Rec	F1
AA 5F5C	DT2	Gini	17	32	7	4	0.82	0.82	0.82	0.88	0.88	0.88
	DT3	Gini	15	9	15	8	0.86	0.86	0.86	0.88	0.88	0.88
	DT4	Entropy	18	24	29	15	0.89	0.89	0.89	0.87	0.87	0.87
	DT5	Entropy	3	2	55	28	0.9	0.9	0.9	0.9	0.9	0.9
	DT6	Entropy	2	4	85	43	0.91	0.91	0.91	0.91	0.91	0.91
	DT7	Entropy	2	14	97	49	0.89	0.89	0.89	0.92	0.92	0.92
	DT8	Gini	3	9	123	62	0.87	0.87	0.87	0.91	0.91	0.91
	DT9	Gini	5	6	123	62	0.91	0.91	0.91	0.92	0.92	0.92

Table 5.6: Results for the best partitions using Archetypal Analysis in Mainland Portugal, with respective Precision, Recall and F1 scores for both Training/Validation and Testing phases and best hyperparameters. DT{X} refers to the Decision Tree Model of depth X. ML, MS, NN and NL are the Minimum Number of Samples per Leaf, Minimum Number of Samples per Split, Number of Nodes for that Model and Number of Leaves for that Model, respectively.

5.3.3 Decision Tree and RIPPER Ruleset Evaluation

Point out the role of the test set to select the best DT/RIPPER classifier models using as evaluation measures:

- F1-measure - interpretability score

In order to analyse and compare rulesets from the different models, the testing dataset is applied to these models and both the F1 Measure and an Interpretability Score were extracted from the results. The Interpretability Score was implemented based on the works detailed in Section 3.3.3. This Score is calculated by averaging the following scores:

Predictivity Score: represents how precise a model is, by comparing the empirical risk of the model with a baseline predictor (Equation 3.18).

Simplicity Score: represents how simple or complex a ruleset is in comparison to other rulesets, by taking into account the total number of conditions and rules in each one (Equation 3.20).

Stability Score: represents how stable a model is, by comparing how similar rulesets derived from the same model using different subsets of the data are (Equation 3.21).

5.3.4 IF-THEN Fire Danger Rules

Finally, IF-THEN rules extracted from the best DT/RIPPER models for each Region are compared to the current fire danger rating scales, with the aid of a specialist in the field, where most attention is given to how the different index thresholds compare to the scales, for each Risk Level. This analysis can then be used to further explore the definition of new thresholds.

5.4 Summary

Our experimental protocol is thus divided into two phases, those being the Fire Danger Classification through Clustering Phase and the IF-THEN Rule Extraction Phase. This protocol is applied to the 5- and 7-Features collections of each geographic region, as presented in Table 4.2 of Section 4.2.

In the Fire Danger Classification through Clustering Phase, both clustering algorithms - the Archetypal Analysis and Fuzzy c-Means algorithms - are used to group wildfire data for each dataset into partitions of 5 to 9 clusters. The best partitions for each collection are selected and compared using the respective metrics - RSS or Xie-Beni Index. For each of these partitions, the resulting Archetypes and Prototypes are labelled using the Fire Danger Rating Scales and specialist knowledge in the field. Consequently, all wildfires are classified according to their similarity to each labelled Archetype and Prototype. The best partitions for each algorithm and region are selected for the next Phase, according to their precision and error in determining accurate wildfire classification.

In the Risk-Based Rule Extraction Phase, different rulesets are extracted from different classifiers - Decision Trees and RIPPER -, trained, validated and tested using the resulting classifications of the best partitions from last Phase. These rulesets are then compared and the best selected, using the precision, recall and F1 scores, as well as the rule interpretability score (with its predictivity, stability and simplicity scores). The resulting best rulesets for each region are then analysed and compared to the currently used Fire Danger Rating Scales.

EXPERIMENTAL STUDY

The following Chapter presents the results from each of the different stages of the experimental protocol, applied to the 5- and 7-Feature collections of each region.

6.1 Main Objectives

We recall here the objectives of this study:

- Apply the Archetypal Analysis algorithm in the domain of wildfire clustering. Analysis of wildfire archetypes and definition of their risk levels according to their extreme indexes and specialist knowledge in the field. Using the different wildfire archetypes as a representation of different types of wildfires, with different risk levels, classify all wildfires according to their similarity to these archetypes.
- Highlight the value of Archetypal Analysis for clustering and classification in this domain.
- Extract rules from the classified datasets using Decision Tree and RIPPER algorithms.
- Evaluate the extracted rules with the aid of rule interpretability scores and a specialist in the field.
- Compare the application of Archetypal Analysis and Fuzzy c-Means as clustering algorithms during all stages of this study, in order to highlight their resulting differences when applied to this domain.
- Highlight the difference between the six studied regions, as seen in their respective extracted risk-based rules.
- Highlight the difference between the use of different combinations of sub-indexes.

6.2 Wildfire Clustering and Classification

The following section contains the results of the first phase of the study, concerning the clustering and classification of wildfire occurrences, using both Archetypal Analysis and Fuzzy c-Means algorithms, presented stage by stage according to the experimental protocol described in Section 5.1.

For each 5- and 7-Feature collection, each partition was compared to one another using different internal and external validity indexes. Archetypal Analysis uses the RSS (see Section 3.1.3), while Fuzzy c-Means uses the Xie-Beni internal validity index (see Section 3.2.2). The best partition for each collection was then selected according to these metrics.

After labelling the archetypes and prototypes, each was compared to the fire danger rating scales in order to evaluate how well they match. Archetype and Prototype Tables in this section are colored following the process described in Section 5.2.4 and Table 5.3.

Finally, based on the classification of wildfires using archetypes and prototypes, the Precision and Misclassification Error were calculated, as described in Section 5.2.4. Using these metrics, the best 5- or 7-Feature partition was selected for each region and algorithm.

It is worth noting that for this last step, all Confusion Tables have been studied, however, in order to present these results for all partitions and algorithms, a Summary Table is instead presented for each case, including both Precision and Error metrics (as well the RSS/XB indexes from before).

6.2.1 Mainland Portugal

PT	AA				FCM			
		RSS	Err%(↓)	Precis%(↑)	XB(↓)	nXB(↓)	Err%(↓)	Precis%(↑)
PT5	5c	12.02*	13.3	79.52*	31507.46	1.0	0.95*	53.81
	6c	6.02	18.2	70.06	13804.33	0.382	5.9	67.28
	7c	3.98	2.11*	53.42	14533.31	0.408	6.07	71.34
	8c	2.67	5.06	63.11	10907.11*	0.281	5.45	71.34
	9c	1.92	12.69	76.85	13936.72	0.387	11.13	71.56*
PT7	5c	29.94*	1.22*	65.66	3266.59	0.014	5.18	55.65*
	6c	16.95	13.52	71.29*	4648.73	0.063	1.17	39.4
	7c	11.47	2.39	45.24	4290.28	0.05	0.83*	37.34
	8c	9.64	7.96	63.11	2852.61*	0.0	9.24	52.25
	9c	6.46	3.12	42.68	5556.68	0.094	2.78	49.58

Table 6.1: Error, Precision, RSS and XB Index results for Archetypal Analysis and Fuzzy c-Means clustering with 5 and 7 features, for Mainland Portugal

For Archetypal Analysis

- According to Figure 6.1 and using the elbow method for RSS, the best number of archetypes is 5 for both five and seven features, so these partitions are selected for those collections.
- When comparing the wildfire archetypes seen in Tables 6.2 and 6.3 to the fire danger rating scales, the partition of five features contains archetypes more in line with the scales.
- As such, the best partition for Archetypal Analysis in Mainland Portugal is with the 5-Features collection and for 5 archetypes.
- In Table 6.1, we can see that for five features and 5 archetypes, this partition has a low error of 13,3% and high precision of 79,52%. For seven features and 5 archetypes, the error is also very low at 1,22% but the precision is worse at 65,66%.

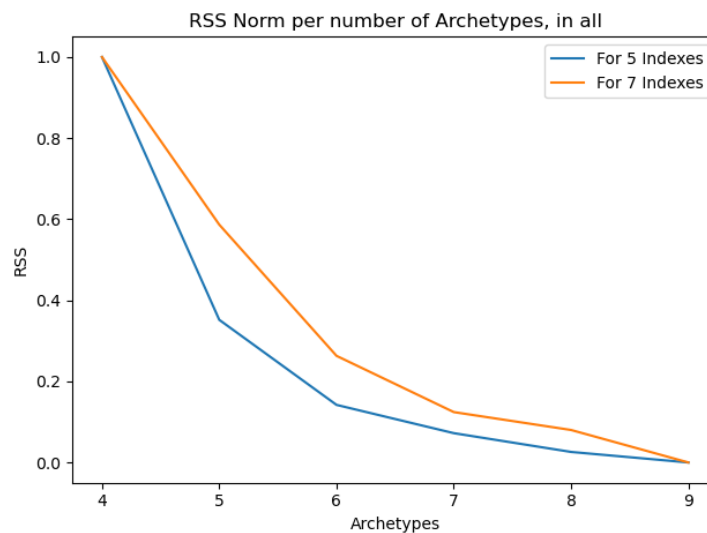


Figure 6.1: RSS for Mainland Portugal with 5 and 7 features

PT5	CHI	FWI	ISI	DC	FFMC	Risk Level
Arq1	2.83	-3.4	-1.0	604.08	17.96	Low
Arq2	-1.16	7.67	2.85	109.09	80.72	Low
Arq3	12.27	26.59	6.41	575.69	92.43	High
Arq4	1.97	27.45	5.48	1357.76	90.12	Moderate
Arq5	10.99	72.79	26.28	625.54	97.5	Maximum

Table 6.2: Archetypes found in Mainland Portugal, for 5 features

For Fuzzy c-Means

PT7	CHI	FWI	ISI	DC	FFMC	BUI	DMC	Risk Level
Arq1	1.86	-2.44	-0.63	688.24	40.05	16.98	5.36	Low
Arq2	-2.83	17.87	6.55	243.89	85.62	54.2	36.36	Low
Arq3	12.09	22.02	6.24	506.96	91.94	92.32	59.47	High
Arq4	3.76	29.5	6.23	1362.12	91.0	495.89	463.74	Very High
Arq5	11.37	72.06	26.19	725.79	98.43	220.64	176.93	Maximum

Table 6.3: Archetypes found in Mainland Portugal, for 7 features

- According to Table 6.1 and the Xie-Beni Index, the best number of clusters is 8 for both five and seven features, so these partitions are selected for those collections.
- When comparing the prototypes with the fire danger rating scales, partitions indexes are very in line with the scales. However, a prototype of risk level "Maximum" is not present in the partition for the 7-Feature collection, as seen in Table 6.5, which we want to be represented when considering wildfires across Portugal.
- As such, the selected partition for Fuzzy c-Means in Mainland Portugal is with the 5-Features collection and for 8 clusters.
- In Table 6.1, we can see that both partitions have very low error at 5,45% and 9,24% respectively. However, the partition of five features appears to have a much better precision than the one with seven: 71,34% to 52,25%.

PT5	CHI	FWI	ISI	DC	FFMC	Risk Level
Prot1	2.55	2.9	0.78	573.25	51.4	Low
Prot2	2.48	21.73	5.45	560.86	86.94	Moderate
Prot3	6.1	24.35	6.4	572.71	89.29	Moderate
Prot4	8.43	30.7	8.49	521.93	91.6	High
Prot5	8.19	33.79	8.76	723.41	91.85	High
Prot6	4.5	34.19	8.79	728.86	90.61	High
Prot7	8.58	42.68	12.2	678.97	93.54	Very High
Prot8	9.45	56.6	18.91	631.58	94.96	Maximum

Table 6.4: Prototypes found in Mainland Portugal, for 5 features

PT7	CHI	FWI	ISI	DC	FFMC	BUI	DMC	Risk Level
Prot1	2.61	3.56	1.0	558.57	53.88	69.97	44.39	Low
Prot2	3.05	21.43	5.55	540.22	87.07	117.59	84.7	Moderate
Prot3	6.77	24.84	6.92	551.68	89.64	114.28	80.9	High
Prot4	8.28	32.08	8.89	569.44	91.65	140.54	106.17	High
Prot5	4.84	32.4	8.22	690.54	90.14	181.79	141.61	High
Prot6	6.93	38.71	9.95	824.95	92.41	274.88	242.37	Very High
Prot7	8.42	39.03	10.66	672.04	92.82	188.44	150.56	High
Prot8	9.12	53.4	17.3	627.9	94.44	191.21	160.26	Very High

Table 6.5: Prototypes found in Mainland Portugal, for 7 features

6.2.2 Center East

CE		AA			FCM			
		RSS	Err%(↓)	Precis%(↑)	XB(↓)	nXB(↓)	Err%(↓)	Precis%(↑)
CE5	5c	4.67*	1.11*	80.48	5118.37*	0.365	10.16	62.7
	6c	2.86	3.81	82.54*	8962.0	0.733	3.17*	56.35
	7c	1.69	3.49	68.89	5900.95	0.439	6.67	64.92
	8c	1.11	6.03	78.57	11664.51	0.991	13.33	64.76
	9c	1.09	4.76	73.81	8385.8	0.677	9.84	66.67*
CE7	5c	12.48*	2.38*	54.13	5158.58	0.368	1.27*	43.65
	6c	6.47	7.78	68.41	2459.92	0.11	13.65	63.02*
	7c	4.9	5.56	52.7	1311.07*	0.0	9.84	57.46
	8c	3.68	9.68	64.76	4165.35	0.273	10.79	56.03
	9c	3.08	10.32	72.06*	4302.63	0.286	12.54	52.54

Table 6.6: Error, Precision, RSS and XB Index results for Archetypal Analysis and Fuzzy c-Means clustering with 5 and 7 features, for Center East

For Archetypal Analysis

- According to Figure 6.2 and using the elbow method for RSS, the best number of archetypes is 5 for both five and seven features, so these partitions are selected for those collections.
- When comparing the wildfire archetypes seen in Tables 6.7 and 6.8 to the fire danger rating scales, the partition of five features contains archetypes more in tune with the scales.
- As such, the selected partition for Archetypal Analysis in Center East is with the 5-Features collection and for 5 archetypes.
- In Table 6.6, we can see that for five features and 5 archetypes, this partition has a very low error of 1,11% and high precision of 80,48%. For seven features

and 5 archetypes, the error is also very low at 2,38% but the precision is worse at 54,13%.

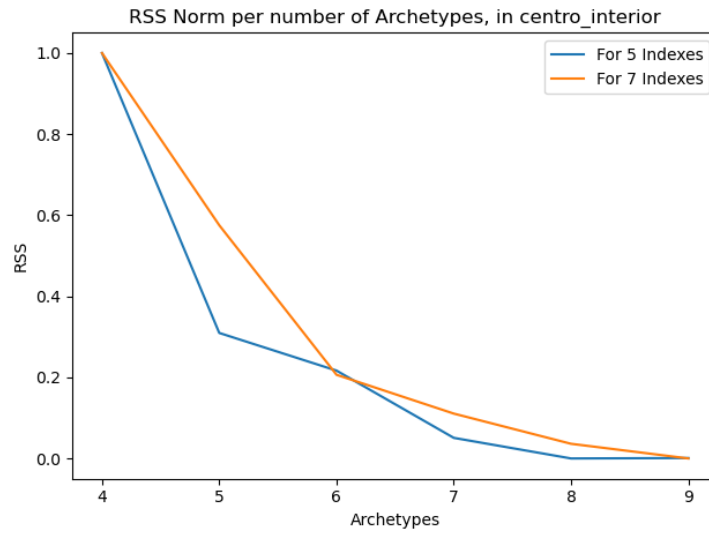


Figure 6.2: RSS for Center East with 5 and 7 features

CE5	CHI	FWI	ISI	DC	FFMC	Risk Level
Arq1	1.59	-3.55	-1.0	617.8	14.19	Low
Arq2	-1.7	12.47	3.16	328.03	81.1	Low
Arq3	11.57	27.19	7.24	419.12	92.22	High
Arq4	5.96	39.74	10.01	1385.15	93.5	Very High
Arq5	9.23	69.68	25.08	623.64	97.14	Maximum

Table 6.7: Archetypes found in Center East, for 5 features

CE7	CHI	FWI	ISI	DC	FFMC	BUI	DMC	Risk Level
Arq1	1.05	-3.25	-0.91	663.16	17.12	48.49	24.55	Low
Arq2	-1.59	12.35	3.32	393.13	81.2	78.23	54.58	Low
Arq3	11.67	27.09	7.17	527.33	92.1	116.75	81.05	High
Arq4	6.19	46.31	12.57	1364.91	92.62	455.02	411.44	Very High
Arq5	8.51	68.66	24.76	641.55	96.89	197.92	163.0	Maximum

Table 6.8: Archetypes found in Center East, for 7 features

For Fuzzy c-Means

- According to table 6.6 and the Xie-Beni Index, the best number of clusters is 5 for five features and 7 for seven features, so these partitions are selected for those collections.

CE5	CHI	FWI	ISI	DC	FFMC	Risk Level
Prot1	2.35	6.94	1.51	651.84	57.84	Low
Prot2	3.41	28.97	7.23	641.13	88.55	High
Prot3	6.16	32.38	8.28	733.73	90.94	High
Prot4	8.46	33.86	9.13	571.16	92.43	High
Prot5	7.72	44.81	13.01	735.56	93.7	Very High

Table 6.9: Prototypes found in Center East, for 5 features

- Also in Table 6.6, we can see that both partitions have low error at 10,16% and 9,84% respectively. However, both partition also have a lower precision at 62,7% and 57,46% respectively. If we take into consideration the partition of 6 clusters instead of 5 for seven features, the partition with the second best Xie-Beni Index, we have a slightly higher precision (63,02%), although with a slightly higher but still acceptable error as well (13,65%).
- When looking at and comparing them to the fire danger rating scales, the prototypes from both partitions appear to be in line with the fire danger rating scales, as seen in Tables 6.9 and 6.10, where the 7-Feature Partition has better representation of the risk levels, as it includes a "Moderate" prototype.
- As such, the selected partition for Fuzzy c-Means in Center East is with the 7-Features collection and for 6 clusters.

CE7	CHI	FWI	ISI	DC	FFMC	BUI	DMC	Risk Level
Prot1	2.39	7.03	1.59	642.51	58.74	106.67	70.15	Low
Prot2	4.09	27.11	6.96	598.44	88.27	143.2	107.0	Moderate
Prot3	7.6	31.11	8.39	580.13	91.15	143.2	108.35	High
Prot4	4.86	36.0	9.18	758.54	90.67	228.02	191.87	High
Prot5	8.27	38.07	10.63	622.31	92.76	170.5	135.15	High
Prot6	7.57	42.46	11.7	790.7	93.09	235.56	196.57	Very High

Table 6.10: Prototypes found in Center East, for 7 features

6.2.3 Center West

CW		AA			FCM			
		RSS	Err%(↓)	Precis%(↑)	XB(↓)	nXB(↓)	Err%(↓)	Precis%(↑)
CW5	5c	1.77*	17.46	68.78*	2847.38	1.0	14.29	67.2
	6c	1.05	12.17	55.56	2054.23	0.688	6.35*	67.72*
	7c	0.71	62.96	29.63	2130.37	0.718	9.52	65.08
	8c	0.63	22.22	62.96	1604.37	0.511	9.52	62.96
	9c	0.36	10.58*	62.43	1409.76*	0.434	20.11	60.32
CW7	5c	5.06*	3.17	60.32	344.7	0.015	0.53	59.79*
	6c	3.49	0.53*	66.67	333.62	0.011	0.0*	43.39
	7c	2.63	6.88	65.08	317.35	0.005	0.0*	53.97
	8c	2.07	2.65	71.96*	305.49*	0.0	0.0*	52.91
	9c	1.57	5.82	60.32	934.9	0.248	0.0*	53.44

Table 6.11: Error, Precision, RSS and XB Index results for Archetypal Analysis and Fuzzy c-Means clustering with 5 and 7 features, for Center West

For Archetypal Analysis

- According to Figure 6.3 and using the elbow method for RSS, the best number of archetypes is 5 for both five and seven features, so these partitions are selected for those collections.
- In Table 6.11, we can see that for five features and 5 archetypes, this partition has an error of 17,46% and precision of 68,78%. For seven features and 5 archetypes, the error is very low at 3,17% but the precision is worse at 60,32%. If we take into consideration the partition of 6 archetypes instead of 5 for seven features, we have a higher precision (66,67%) and while still not higher than the precision for the partition of five features, it also has a much lower error than that partition (0,53%).
- When comparing the wildfire archetypes seen in Tables 6.12 and 6.13 to the fire danger rating scales, both partitions are in line with the scales and the 7-Feature Partition has better representation of the risk levels, as it includes a "Very High" archetype.
- As such, the selected partition for Archetypal Analysis in Center West is for seven features and 6 archetypes.

For Fuzzy c-Means

- According to Table 6.11 and the Xie-Beni Index, the best number of clusters is 9 for five features and 8 for seven features, so these partitions are selected for those collections.

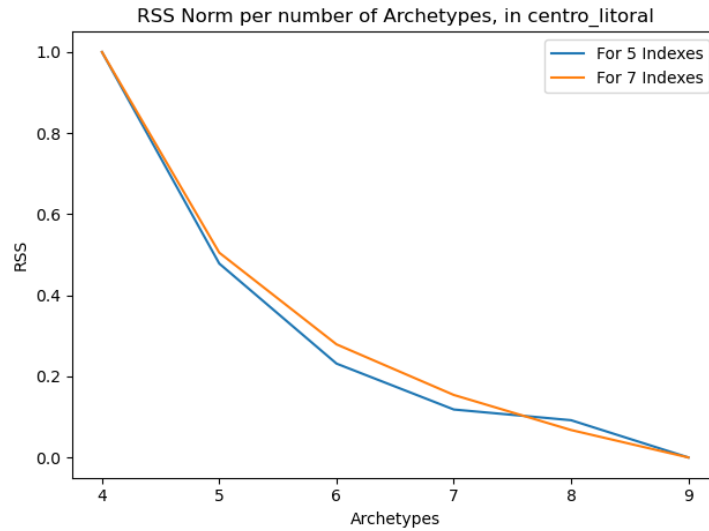


Figure 6.3: RSS for Center West with 5 and 7 features

CW5	CHI	FWI	ISI	DC	FFMC	Risk Level
Arq1	2.22	-2.97	-1.06	624.45	12.16	Low
Arq2	7.86	21.19	4.03	1144.01	83.73	Moderate
Arq3	-1.9	23.52	6.25	469.95	82.18	Moderate
Arq4	9.46	26.69	8.39	174.58	91.36	High
Arq5	10.8	68.27	24.26	725.06	98.86	Maximum

Table 6.12: Archetypes found in Center West, for 5 features

CW7	CHI	FWI	ISI	DC	FFMC	BUI	DMC	Risk Level
Arq1	2.48	-1.76	-0.76	713.62	33.42	44.93	21.62	Low
Arq2	8.82	18.26	7.61	48.89	86.82	23.44	30.28	Moderate
Arq3	-2.58	24.06	7.7	319.75	81.92	83.56	65.09	Moderate
Arq4	8.84	27.43	7.98	990.29	84.63	88.59	46.76	High
Arq5	8.85	37.36	8.93	996.61	93.63	354.73	309.44	Very High
Arq6	10.6	68.08	24.22	713.97	98.16	222.67	175.06	Maximum

Table 6.13: Archetypes found in Center West, for 7 features

- Also in Table 6.11, we can see that the partition of five features has a high error of 20,11% and a precision of 60,32%. The partition of seven features has a much lower error (0%) but a lower precision as well (52,91%). However, if we take into consideration the partition of 5 clusters instead of 8 for seven features, we have a higher precision (59,79%) while keeping a lower error as well (0,53%).
- When looking at and comparing them to the fire danger rating scales, the prototypes from the 5-Feature collection are more in tune with the fire danger rating scales, as seen in Tables 6.14 and 6.15.
- As such, the selected partition for Fuzzy c-Means in Center West is for seven

features and 5 clusters.

CW5	CHI	FWI	ISI	DC	FFMC	Risk Level
Prot1	1.83	18.24	4.65	614.64	76.25	Low
Prot2	7.08	24.72	5.86	871.56	84.68	Moderate
Prot3	6.91	25.33	6.62	629.44	83.79	Moderate
Prot4	6.79	26.53	7.68	463.56	85.29	High
Prot5	8.76	33.4	8.93	624.98	88.7	High
Prot6	7.82	39.96	11.18	764.72	90.19	High
Prot7	5.28	42.34	12.15	668.12	89.02	Very High
Prot8	9.69	44.57	13.07	634.25	92.34	Very High
Prot9	8.33	54.0	18.63	595.95	92.53	Maximum

Table 6.14: Prototypes found in Center West, for 5 features

CW7	CHI	FWI	ISI	DC	FFMC	BUI	DMC	Risk Level
Prot1	6.04	21.89	6.75	568.81	83.23	76.99	50.54	Moderate
Prot2	7.49	32.79	8.59	662.85	88.28	168.22	127.32	High
Prot3	7.35	33.29	9.43	651.96	87.93	138.53	99.92	High
Prot4	7.84	38.96	10.2	760.29	91.12	249.48	208.86	Very High
Prot5	8.32	48.63	15.24	647.4	93.23	180.73	142.21	Very High

Table 6.15: Prototypes found in Center West, for 7 features

6.2.4 North East

NE		AA			FCM			
		RSS	Err%(↓)	Precis%(↑)	XB(↓)	nXB(↓)	Err%(↓)	Precis%(↑)
NE5	5c	1.67*	1.19*	69.57	3448.01	0.856	4.35*	56.52
	6c	1.02	1.19*	73.52*	3309.64	0.818	5.14	68.77*
	7c	0.92	3.16	62.06	2105.07*	0.487	5.14	65.61
	8c	0.55	28.06	68.77	2653.34	0.638	12.25	64.03
	9c	0.44	19.76	68.38	3971.26	1.0	6.32	66.01
NE7	5c	5.96	0.4*	73.52	358.93	0.008	1.19*	39.13
	6c	2.72*	0.79	73.12	1052.08	0.198	1.98	42.69
	7c	2.19	2.37	72.73	345.04	0.004	6.32	49.41
	8c	1.72	2.37	75.1*	341.32	0.003	5.14	70.75*
	9c	1.21	8.7	72.73	330.73*	0.0	5.53	67.98

Table 6.16: Error, Precision, RSS and XB Index results for Archetypal Analysis and Fuzzy c-Means clustering with 5 and 7 features, for North East

For Archetypal Analysis

- According to figure 6.4 and using the elbow method for RSS, the best number of archetypes is 5 for five features and 6 for seven features, so these partitions are selected for those collections.
- When comparing the wildfire archetypes seen in Tables 6.17 and 6.18 to the fire danger rating scales, both partitions are very in line with the scales. The 7-Feature Partition does include another "Low" and a "Very High" archetypes instead of the "Moderate" archetype from the 5-Feature Partition.
- As such, the selected partition for Archetypal Analysis in North East is for seven features and 6 archetypes.
- In table 6.16, we can see that for five features and 5 archetypes, this partition has a very low error of 1,19% and high precision of 69,57%. For seven features and 6 archetypes, the error is also very low at 0,79% and has a better precision at 73,12%.

NE5	CHI	FWI	ISI	DC	FFMC	Risk Level
Arq1	1.35	-2.98	-0.8	499.45	27.55	Low
Arq2	0.92	20.33	5.55	224.3	85.19	Moderate
Arq3	12.08	26.98	6.41	606.19	91.3	High
Arq4	1.31	28.76	6.11	1345.69	87.82	High
Arq5	8.48	64.96	23.05	620.23	96.39	Maximum

Table 6.17: Archetypes found in North East, for 5 features

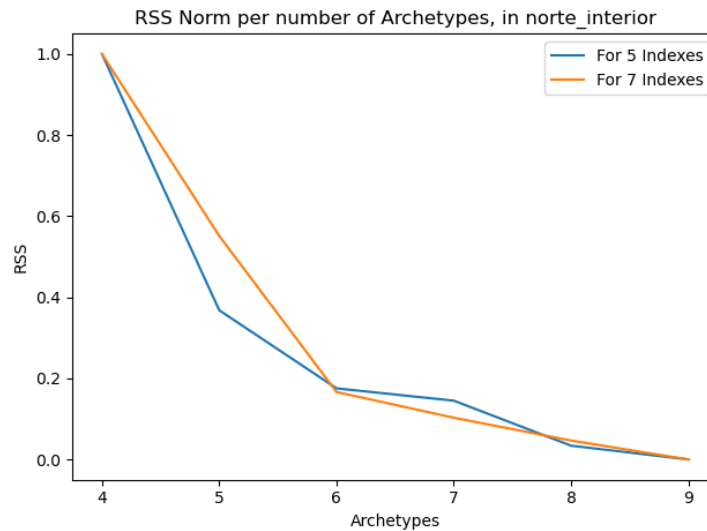


Figure 6.4: RSS for North East with 5 and 7 features

NE7	CHI	FWI	ISI	DC	FFMC	BUI	DMC	Risk Level
Arq1	1.32	-3.38	-0.87	481.0	23.99	56.72	33.43	Low
Arq2	0.86	15.49	5.78	229.32	82.36	34.32	32.93	Low
Arq3	11.84	25.95	6.44	634.03	90.71	119.7	79.16	High
Arq4	1.28	28.64	6.08	1343.16	87.48	242.68	154.94	High
Arq5	4.56	38.02	9.26	847.95	90.77	447.39	442.36	Very High
Arq6	8.19	64.51	23.15	582.27	95.87	173.85	135.86	Maximum

Table 6.18: Archetypes found in North East, for 7 features

For Fuzzy c-Means

- According to Table 6.16 and the Xie-Beni Index, the best number of clusters is 7 for five features and 9 for seven features, so these partitions are selected for those collections.
- Also in Table 6.16, we can see that both partitions have very low error at 5,14% and 5,53% respectively. If we take into consideration the partition of 8 clusters instead of 9 for seven features, the partition with the second best Xie-Beni Index, we have a higher precision (70,75%) and lower error as well (5,14%).
- When looking at and comparing them to the fire danger rating scales, the prototypes from both partitions appear to be in line with the scales, as seen in Tables 6.19 and 6.20.
- As such, the selected partition for Fuzzy c-Means in North East is for seven features and 8 clusters.

NE5	CHI	FWI	ISI	DC	FFMC	Risk Level
Prot1	2.07	19.5	4.87	479.09	83.33	Moderate
Prot2	3.75	21.37	5.15	597.03	85.09	Moderate
Prot3	6.55	25.29	6.19	648.7	87.71	High
Prot4	9.08	29.12	7.29	643.85	89.62	High
Prot5	3.39	33.62	8.32	784.79	88.6	High
Prot6	5.72	33.97	8.94	670.41	89.41	High
Prot7	8.53	42.71	12.32	685.0	91.78	Very High

Table 6.19: Prototypes found in North East, for 5 features

NE7	CHI	FWI	ISI	DC	FFMC	BUI	DMC	Risk Level
Prot1	2.44	19.23	4.95	484.02	83.19	103.15	73.98	Moderate
Prot2	3.67	22.44	5.46	587.95	85.06	126.45	89.55	Moderate
Prot3	6.57	24.36	6.32	613.47	87.21	115.3	78.5	Moderate
Prot4	9.11	28.87	7.42	624.87	89.24	139.11	99.56	High
Prot5	5.71	30.97	7.46	760.6	88.1	200.85	153.87	High
Prot6	3.9	31.21	7.76	684.26	87.62	177.39	134.51	High
Prot7	5.59	36.42	9.02	792.17	89.99	282.94	249.59	Very High
Prot8	8.23	39.91	11.21	659.91	90.95	175.96	133.84	Very High

Table 6.20: Prototypes found in North East, for 7 features

6.2.5 North West

NW		AA			FCM			
		RSS	Err%(↓)	Precis%(↑)	XB(↓)	nXB(↓)	Err%(↓)	Precis%(↑)
NW5	5c	3.34*	5.57	77.34*	5421.48	0.97	23.06	68.99
	6c	2.04	10.54	68.19	3284.69*	0.544	2.98	63.22
	7c	1.38	2.58*	64.21	4312.04	0.749	1.59*	58.85
	8c	0.98	11.33	65.81	5571.7	1.0	3.78	72.17*
	9c	0.74	5.57	68.19	5291.1	0.944	1.59*	65.41
NW7	5c	8.87*	0.0*	34.19	554.28*	0.0	0.4*	28.83
	6c	5.64	0.2	57.26	797.7	0.049	0.6	49.9
	7c	4.51	0.0*	36.78	783.72	0.046	1.39	60.64*
	8c	2.82	1.19	71.97*	871.58	0.063	1.19	51.49
	9c	2.38	3.78	64.41	841.52	0.057	0.6	38.77

Table 6.21: Error, Precision, RSS and XB Index results for Archetypal Analysis and Fuzzy c-Means clustering with 5 and 7 features, for North West

For Archetypal Analysis

- According to figure 6.5 and using the elbow method for RSS, the best number of archetypes is 5 for both five and seven features, so these partitions are selected for those collections.
- When comparing the wildfire archetypes seen in Tables 6.22 and 6.23 to the fire danger rating scales, both partitions are in line with the scales.
- As such, the selected partition for Archetypal Analysis in North West is for five features and 5 archetypes.
- In Table 6.21, we can see that both partitions have very low error at 5,57% and 0% respectively. However, the partition of five features has a much higher precision at 77,34%, as opposed to 34,19%.

NW5	CHI	FWI	ISI	DC	FFMC	Risk Level
Arq1	2.24	-3.25	-0.95	551.35	10.97	Low
Arq2	6.17	-0.85	1.48	-32.08	80.4	Low
Arq3	-1.55	21.07	4.15	617.15	85.71	Moderate
Arq4	10.44	33.54	7.92	757.85	93.43	High
Arq5	11.03	72.32	27.16	474.75	96.41	Maximum

Table 6.22: Archetypes found in North West, for 5 features

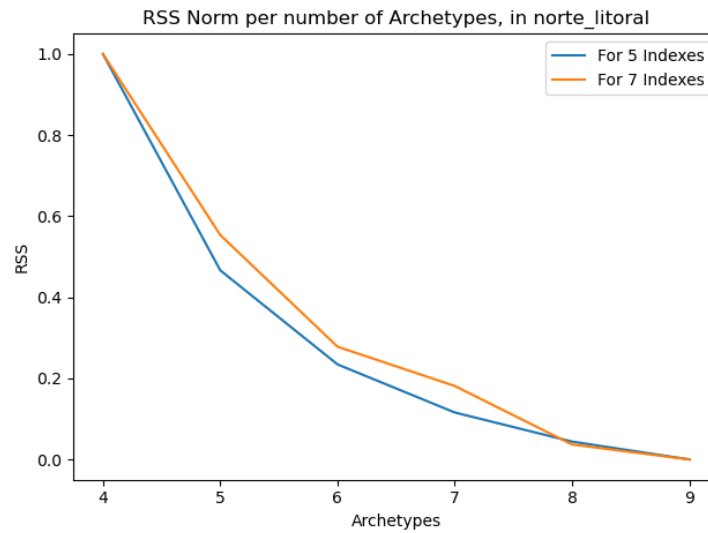


Figure 6.5: RSS for North West with 5 and 7 features

NW7	CHI	FWI	ISI	DC	FFMC	BUI	DMC	Risk Level
Arq1	2.86	-2.6	0.56	-46.66	73.11	-6.89	-3.56	Low
Arq2	-0.83	18.95	4.71	648.09	83.8	109.0	68.74	Moderate
Arq3	12.04	21.85	6.65	592.28	90.65	85.84	51.35	High
Arq4	8.3	33.69	7.85	749.13	90.06	278.07	258.1	Very High
Arq5	10.94	70.11	25.96	508.75	96.26	180.64	164.68	Maximum

Table 6.23: Archetypes found in North West, for 7 features

For Fuzzy c-Means

- According to table 6.21 and the Xie-Beni Index, the best number of clusters is 6 for five features and 5 for seven features, so these partitions are selected for those collections.
- When looking at and comparing them to the fire danger rating scales, the prototypes from the 5-Feature collection appear to more be in line with the scales and all of the risk levels are represented there, as seen in Tables 6.24 and 6.25.
- As such, the selected partition for Fuzzy c-Means in North West is for five features and 6 clusters.
- In Table 6.21, we can also see that both partitions have very low error at 2,98% and 0,4% respectively. However, the partition of five features has a much higher precision at 63,22%, as opposed to 28,83%.

NW5	CHI	FWI	ISI	DC	FFMC	Risk Level
Prot1	2.87	12.16	3.85	350.66	79.6	Low
Prot2	2.89	22.17	5.81	528.33	86.45	Moderate
Prot3	6.36	24.33	6.73	511.97	88.8	Moderate
Prot4	8.65	27.85	8.28	439.03	90.5	High
Prot5	8.2	36.14	10.29	585.71	91.45	Very High
Prot6	9.72	56.09	18.88	519.25	93.61	Maximum

Table 6.24: Prototypes found in North West, for 5 features

NW7	CHI	FWI	ISI	DC	FFMC	BUI	DMC	Risk Level
Prot1	3.78	12.51	4.3	369.42	80.56	60.6	41.15	Moderate
Prot2	4.13	23.28	6.23	508.86	87.44	113.94	81.59	High
Prot3	7.94	25.93	7.99	462.41	89.81	96.95	68.07	High
Prot4	7.73	33.92	9.08	583.38	91.2	154.77	118.88	High
Prot5	9.29	51.99	16.82	538.3	93.21	172.56	148.97	Very High

Table 6.25: Prototypes found in North West, for 7 features

6.2.6 South

S		AA			FCM			
		RSS	Err%(↓)	Precis%(↑)	XB(↓)	nXB(↓)	Err%(↓)	Precis%(↑)
S5	5c	2.75*	8.72*	77.52	1903.25	0.457	2.75*	47.71
	6c	1.68	8.72*	86.7*	3686.31	1.0	9.17	63.3
	7c	1.15	13.76	73.85	2655.09	0.686	18.35	68.35
	8c	0.75	19.27	70.18	1524.17	0.341	12.39	66.06
	9c	0.55	10.09	60.55	1218.76*	0.248	15.14	72.48*
S7	5c	6.67*	30.73	61.93	1257.27	0.26	5.5*	60.09
	6c	4.06	36.24	57.34	1209.15	0.245	9.63	71.56*
	7c	3.21	27.06	57.34	3405.06	0.914	6.88	66.06
	8c	1.95	8.72*	65.14*	1836.21	0.436	6.42	63.76
	9c	1.43	27.98	60.55	988.75*	0.178	11.93	62.84

Table 6.26: Error, Precision, RSS and XB Index results for Archetypal Analysis and Fuzzy c-Means clustering with 5 and 7 features, for South

For Archetypal Analysis

- According to figure 6.6 and using the elbow method for RSS, the best number of archetypes is 5 for both five and seven features, so these partitions are selected for those collections.
- When comparing the wildfire archetypes seen in Tables 6.27 and 6.28 to the fire danger rating scales, both partitions are very in line with the scales and represent very similar archetypes.
- As such, the selected partition for Archetypal Analysis in South is for five features and 5 archetypes.
- In table 6.26, we can see that for five features and 5 archetypes, this partition has a low error of 8,72% and high precision of 77,52%. For seven features and 5 archetypes, the error is much higher at 30,73%, with a lower precision at 61,93%.

S5	CHI	FWI	ISI	DC	FFMC	Risk Level
Arq1	2.66	-4.12	-1.18	802.98	2.63	Low
Arq2	1.19	2.7	1.88	-30.09	70.21	Low
Arq3	0.3	22.4	3.99	1379.31	86.62	Moderate
Arq4	11.86	37.32	9.24	898.41	97.69	Very High
Arq5	11.05	72.57	26.32	613.14	100.87	Maximum

Table 6.27: Archetypes found in South, for 5 features

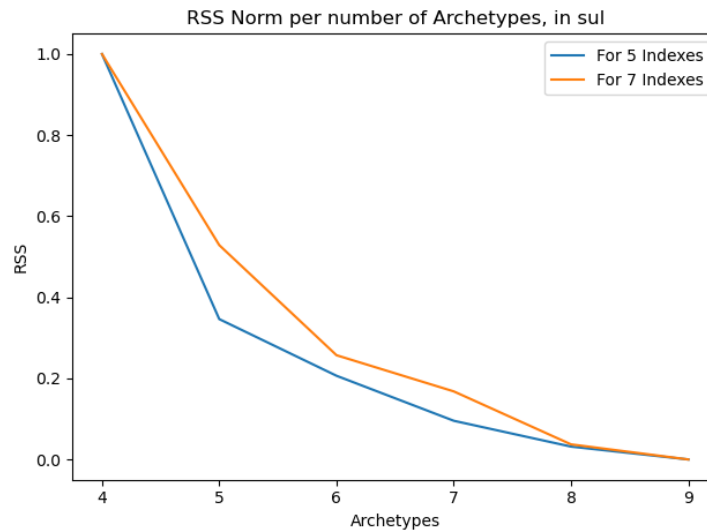


Figure 6.6: RSS for South with 5 and 7 features

S7	CHI	FWI	ISI	DC	FFMC	BUI	DMC	Risk Level
Arq1	2.76	-3.5	-0.94	824.97	17.71	43.68	16.07	Low
Arq2	0.42	1.97	1.7	108.38	73.76	1.99	-3.05	Low
Arq3	1.33	23.64	4.36	1373.59	88.61	520.28	499.08	Moderate
Arq4	12.39	40.99	10.74	829.61	98.69	267.52	224.53	Very High
Arq5	11.37	72.42	26.36	690.76	101.35	230.01	193.83	Maximum

Table 6.28: Archetypes found in South, for 7 features

For Fuzzy c-Means

- According to Table 6.26 and the Xie-Beni Index, the best number of clusters is 9 for both five and seven features, so these partitions are selected for those collections.
- Also in Table 6.26, we can see that the partition of five features has an error of 15,14% and a high precision of 72,48%. The partition of seven features has a lower error (11,93%) but a lower precision as well (62,84%). However, if we take into consideration the partition of 6 clusters instead of 9 for seven features, the partition with the second best Xie-Beni Index, we have a much higher precision (71,56%) and lower error as well (9,63%).
- When looking at and comparing them to the fire danger rating scales, the prototypes from both partitions appear to be in line with the fire danger rating scales, as seen in Tables 6.29 and 6.30.
- As such, the selected partition for Fuzzy c-Means in South is for seven features and 6 clusters.

S5	CHI	FWI	ISI	DC	FFMC	Risk Level
Prot1	2.8	-0.19	0.12	411.12	31.93	Low
Prot2	2.0	4.45	0.96	773.62	56.66	Low
Prot3	3.55	23.81	4.79	1207.8	85.95	Moderate
Prot4	3.6	25.57	6.02	578.83	85.31	Moderate
Prot5	7.62	33.05	7.98	971.47	90.13	High
Prot6	8.81	36.03	9.15	679.44	91.71	High
Prot7	5.71	46.78	13.68	673.87	92.63	Very High
Prot8	10.64	47.43	13.76	713.24	97.23	Very High
Prot9	10.54	62.48	21.39	707.18	98.19	Maximum

Table 6.29: Prototypes found in South, for 5 features

S7	CHI	FWI	ISI	DC	FFMC	BUI	DMC	Risk Level
Prot1	3.02	0.9	0.42	494.17	34.91	37.91	21.73	Low
Prot2	2.48	9.71	2.36	709.64	69.58	92.96	60.17	Low
Prot3	4.02	25.84	5.36	1227.13	88.39	450.94	424.46	High
Prot4	6.25	34.61	9.16	661.45	90.89	185.74	152.56	High
Prot5	8.68	39.21	10.22	810.1	93.74	273.99	243.25	Very High
Prot6	10.09	55.52	17.81	703.89	98.1	239.76	211.06	Maximum

Table 6.30: Prototypes found in South, for 7 features

6.3 Decision Tree and RIPPER: Model Construction and Evaluation

Following the selection of the different best partitions for each dataset and algorithm that result in the best wildfire classifications, each wildfire now has a new Risk Level label that will be used as the class variable for our rule extraction models.

This next section presents the results of the training, validation and testing stages for the creation of the Decision Trees and RIPPER models, using the selected best configurations of features and partitions for each of the studied regions.

6.3.1 Mainland Portugal

PT	Model	Criterion					Train/Val			Test		
			ML	MS	NN	NL	Precis	Rec	F1	Precis	Rec	F1
AA PT5 5C	DT2	Gini	17	32	7	4	0.82	0.82	0.82	0.88	0.88	0.88
	DT3	Gini	15	9	15	8	0.86	0.86	0.86	0.88	0.88	0.88
	DT4	Entropy	18	24	29	15	0.89	0.89	0.89	0.87	0.87	0.87
	DT5	Entropy	3	2	55	28	0.9	0.9	0.9	0.9	0.9	0.9
	DT6	Entropy	2	4	85	43	0.91	0.91	0.91	0.91	0.91	0.91
	DT7	Entropy	2	14	97	49	0.89	0.89	0.89	0.92	0.92	0.92
	DT8	Gini	3	9	123	62	0.87	0.87	0.87	0.91	0.91	0.91
	DT9	Gini	5	6	123	62	0.91	0.91	0.91	0.92	0.92	0.92
	RIPPER	—	—	—	—	—	—	—	—	0.93	0.93	0.93
FCM PT5 8C	DT2	Gini	18	2	7	4	0.83	0.83	0.83	0.81	0.81	0.81
	DT3	Gini	13	31	15	8	0.86	0.86	0.86	0.87	0.87	0.87
	DT4	Gini	5	10	29	15	0.87	0.87	0.87	0.88	0.88	0.88
	DT5	Gini	8	10	45	23	0.86	0.86	0.86	0.9	0.9	0.9
	DT6	Gini	8	10	61	31	0.89	0.89	0.89	0.9	0.9	0.9
	DT7	Gini	1	16	79	40	0.89	0.89	0.89	0.91	0.91	0.91
	DT8	Entropy	7	22	83	42	0.88	0.88	0.88	0.91	0.91	0.91
	DT9	Entropy	2	9	111	56	0.89	0.89	0.89	0.92	0.92	0.92
	RIPPER	—	—	—	—	—	—	—	—	0.91	0.91	0.91

Table 6.31: Results for the best partitions using Archetypal Analysis and Fuzzy c-Means in Mainland Portugal, with respective Precision, Recall and F1 scores for both Training/Validation and Testing phases and best hyperparameters. DT{X} refers to the Decision Tree Model of depth X. ML, MS, NN and NL are the Minimum Number of Samples per Leaf, Minimum Number of Samples per Split, Number of Nodes for that Model and Number of Leaves for that Model, respectively.

As seen in Table 6.31, for Archetypal Analysis, the Decision Trees of depths 7 and 9 appear to have the best F1 score at 0,92. However they have a high number of nodes (90+) which might result in rules that are harder to interpret. For a decision tree depth of 3, we

still have a very high F1 at 0,88, but with a tree with a good balance between readability and complexity at 15 nodes.

For Fuzzy c-Means, the Decision Tree of depth 9 appears to have the best F1 score at 0,92. However it also has a high number of nodes (111) which might result in rules that are harder to interpret. For a decision tree depth of 4, we still have a very high F1 at 0,88, but with a tree with better readability at 29 nodes.

6.3.2 Center East

CE	Model	Criterion	ML	MS	NN	NL	Train/Val			Test		
							Precis	Rec	F1	Precis	Rec	F1
AA CE5 5C	DT2	Gini	1	6	7	4	0.72	0.72	0.72	0.74	0.74	0.74
	DT3	Gini	6	20	15	8	0.72	0.72	0.72	0.86	0.86	0.86
	DT4	Gini	4	5	29	15	0.75	0.75	0.75	0.83	0.83	0.83
	DT5	Entropy	7	26	39	20	0.76	0.76	0.76	0.66	0.66	0.66
	DT6	Gini	1	2	77	39	0.81	0.81	0.81	0.77	0.77	0.77
	DT7	Entropy	3	2	71	36	0.81	0.81	0.81	0.89	0.89	0.89
	DT8	Entropy	4	12	65	33	0.77	0.77	0.77	0.83	0.83	0.83
	DT9	Entropy	2	2	87	44	0.77	0.77	0.77	0.86	0.86	0.86
	RIPPER	—	—	—	—	—	—	—	—	0.91	0.91	0.91
FCM CE7 6C	DT2	Gini	11	24	7	4	0.88	0.88	0.88	0.8	0.8	0.8
	DT3	Gini	6	12	15	8	0.86	0.86	0.86	0.8	0.8	0.8
	DT4	Gini	2	10	27	14	0.91	0.91	0.91	0.71	0.71	0.71
	DT5	Gini	8	16	33	17	0.9	0.9	0.9	0.77	0.77	0.77
	DT6	Entropy	4	16	35	18	0.91	0.91	0.91	0.77	0.77	0.77
	DT7	Gini	5	8	43	22	0.92	0.92	0.92	0.77	0.77	0.77
	DT8	Entropy	10	15	33	17	0.91	0.91	0.91	0.77	0.77	0.77
	DT9	Gini	2	13	51	26	0.9	0.9	0.9	0.8	0.8	0.8
	RIPPER	—	—	—	—	—	—	—	—	0.8	0.8	0.8

Table 6.32: Results for the best partitions using Archetypal Analysis and Fuzzy c-Means in Center East, with respective Precision, Recall and F1 scores for both Training/Validation and Testing phases and best hyperparameters. DT{X} refers to the Decision Tree Model of depth X. ML, MS, NN and NL are the Minimum Number of Samples per Leaf, Minimum Number of Samples per Split, Number of Nodes for that Model and Number of Leaves for that Model, respectively.

As seen in Table 6.32, for Archetypal Analysis, the Decision Tree of depth 7 appears to have the best F1 score at 0,89. However it has a high number of nodes (71) which might result in rules that are harder to interpret. For a decision tree depth of 3, we still have a very high F1 at 0,86, but with a tree with better readability at 15 nodes.

For Fuzzy c-Means, the Decision Trees of depths 2,3 and 9 appear to have the best F1 score at 0,8. Among these, the Decision Tree of Depth 3 has a good balance between readability and complexity at 15 nodes.

6.3. DECISION TREE AND RIPPER: MODEL CONSTRUCTION AND EVALUATION

6.3.3 Center West

CW	Model	Criterion	ML	MS	NN	NL	Train/Val			Test		
							Precis	Rec	F1	Precis	Rec	F1
AA CW7 6C	DT2	Gini	20	14	7	4	0.73	0.73	0.73	0.44	0.44	0.44
	DT3	Entropy	16	2	13	7	0.82	0.82	0.82	0.89	0.89	0.89
	DT4	Gini	3	9	21	11	0.8	0.8	0.8	0.67	0.67	0.67
	DT5	Gini	19	31	11	6	0.79	0.79	0.79	0.44	0.44	0.44
	DT6	Entropy	20	14	11	6	0.8	0.8	0.8	0.78	0.78	0.78
	DT7	Entropy	19	31	11	6	0.77	0.77	0.77	0.78	0.78	0.78
	DT8	Entropy	16	15	13	7	0.76	0.76	0.76	0.89	0.89	0.89
	DT9	Gini	5	30	21	11	0.73	0.73	0.73	0.89	0.89	0.89
	RIPPER	—	—	—	—	—	—	—	—	0.56	0.56	0.56
FCM 7F5C	DT2	Entropy	12	5	7	4	0.81	0.81	0.81	0.67	0.67	0.67
	DT3	Gini	5	12	13	7	0.8	0.8	0.8	0.67	0.67	0.67
	DT4	Gini	9	15	15	8	0.81	0.81	0.81	0.78	0.78	0.78
	DT5	Entropy	3	2	25	13	0.81	0.81	0.81	0.67	0.67	0.67
	DT6	Gini	4	38	13	7	0.81	0.81	0.81	0.67	0.67	0.67
	DT7	Gini	2	8	23	12	0.78	0.78	0.78	0.78	0.78	0.78
	DT8	Entropy	1	6	27	14	0.81	0.81	0.81	0.78	0.78	0.78
	DT9	Entropy	10	16	15	8	0.81	0.81	0.81	0.78	0.78	0.78
	RIPPER	—	—	—	—	—	—	—	—	0.67	0.67	0.67

Table 6.33: Results for the best partitions using Archetypal Analysis and Fuzzy c-Means in Center West, with respective Precision, Recall and F1 scores for both Training/Validation and Testing phases and best hyperparameters. DT{X} refers to the Decision Tree Model of depth X. ML, MS, NN and NL are the Minimum Number of Samples per Leaf, Minimum Number of Samples per Split, Number of Nodes for that Model and Number of Leaves for that Model, respectively.

As seen in Table 6.33, for Archetypal Analysis, the Decision Trees of depths 3,8 and 9 appear to have the best F1 score at 0,89. Among these, the Decision Tree of Depth 3 has a good balance between readability and complexity at 13 nodes.

For Fuzzy c-Means, the Decision Trees of depths 4,7,8 and 9 appear to have the best F1 score at 0,78. Among these, the Decision Tree of Depth 4 has better readability at 13 nodes.

6.3.4 North East

NE	Model	Criterion	ML	MS	NN	NL	Train/Val			Test		
							Precis	Rec	F1	Precis	Rec	F1
AA NE7 6C	DT2	Entropy	10	15	7	4	0.73	0.73	0.73	0.62	0.62	0.62
	DT3	Gini	8	9	13	7	0.77	0.77	0.77	0.62	0.62	0.62
	DT4	Entropy	2	14	23	12	0.76	0.76	0.76	0.62	0.62	0.62
	DT5	Entropy	3	4	35	18	0.82	0.82	0.82	0.5	0.5	0.5
	DT6	Entropy	2	2	45	23	0.77	0.77	0.77	0.56	0.56	0.56
	DT7	Entropy	2	7	37	19	0.79	0.79	0.79	0.56	0.56	0.56
	DT8	Entropy	3	4	39	20	0.77	0.77	0.77	0.56	0.56	0.56
	DT9	Entropy	1	8	37	19	0.74	0.74	0.74	0.56	0.56	0.56
	RIPPER	—	—	—	—	—	—	—	—	0.81	0.81	0.81
FCM NE7 8C	DT2	Gini	7	35	7	4	0.87	0.87	0.87	0.94	0.94	0.94
	DT3	Gini	13	10	11	6	0.8	0.8	0.8	0.94	0.94	0.94
	DT4	Entropy	2	40	17	9	0.83	0.83	0.83	0.88	0.88	0.88
	DT5	Entropy	4	39	21	11	0.84	0.84	0.84	0.88	0.88	0.88
	DT6	Entropy	6	25	25	13	0.84	0.84	0.84	0.88	0.88	0.88
	DT7	Entropy	4	27	25	13	0.81	0.81	0.81	0.88	0.88	0.88
	DT8	Gini	1	35	23	12	0.82	0.82	0.82	0.94	0.94	0.94
	DT9	Entropy	1	7	33	17	0.83	0.83	0.83	0.88	0.88	0.88
	RIPPER	—	—	—	—	—	—	—	—	0.88	0.88	0.88

Table 6.34: Results for the best partitions using Archetypal Analysis and Fuzzy c-Means in North East, with respective Precision, Recall and F1 scores for both Training/Validation and Testing phases and best hyperparameters. DT{X} refers to the Decision Tree Model of depth X. ML, MS, NN and NL are the Minimum Number of Samples per Leaf, Minimum Number of Samples per Split, Number of Nodes for that Model and Number of Leaves for that Model, respectively.

As seen in Table 6.34, for Archetypal Analysis, the Decision Trees of depths 2,3 and 4 appear to have the best F1 score at 0,62. Among these, the Decision Tree of Depth 3 has a good balance between readability and complexity at 13 nodes.

For Fuzzy c-Means, the Decision Trees of depths 2,3 and 8 appear to have the best F1 score at 0,94. Among these, the Decision Tree of Depth 3 has better readability at 11 nodes.

It also is worth pointing out that for this region, F1 scores for Decision Trees are considerably worse for Archetypal Analysis than for Fuzzy c-Means.

6.3. DECISION TREE AND RIPPER: MODEL CONSTRUCTION AND EVALUATION

6.3.5 North West

NW	Model	Criterion					Train/Val			Test		
			ML	MS	NN	NL	Precis	Rec	F1	Precis	Rec	F1
AA 5F5C	DT2	Gini	4	38	7	4	0.87	0.87	0.87	0.86	0.86	0.86
	DT3	Gini	4	3	13	7	0.87	0.87	0.87	0.95	0.95	0.95
	DT4	Entropy	17	14	19	10	0.91	0.91	0.91	0.86	0.86	0.86
	DT5	Gini	2	16	23	12	0.89	0.89	0.89	0.86	0.86	0.86
	DT6	Entropy	3	13	39	20	0.91	0.91	0.91	0.95	0.95	0.95
	DT7	Gini	1	13	33	17	0.91	0.91	0.91	0.86	0.86	0.86
	DT8	Gini	2	16	33	17	0.9	0.9	0.9	0.86	0.86	0.86
	DT9	Entropy	5	2	45	23	0.9	0.9	0.9	0.91	0.91	0.91
	RIPPER	—	—	—	—	—	—	—	—	0.95	0.95	0.95
FCM 5F6C	DT2	Gini	8	32	7	4	0.76	0.76	0.76	0.86	0.86	0.86
	DT3	Gini	2	22	15	8	0.76	0.76	0.76	0.91	0.91	0.91
	DT4	Entropy	7	14	31	16	0.77	0.77	0.77	0.77	0.77	0.77
	DT5	Entropy	4	16	45	23	0.81	0.81	0.81	0.77	0.77	0.77
	DT6	Entropy	8	22	47	24	0.79	0.79	0.79	0.77	0.77	0.77
	DT7	Entropy	2	8	73	37	0.81	0.81	0.81	0.86	0.86	0.86
	DT8	Entropy	3	10	69	35	0.8	0.8	0.8	0.86	0.86	0.86
	DT9	Entropy	2	3	87	44	0.81	0.81	0.81	0.86	0.86	0.86
	RIPPER	—	—	—	—	—	—	—	—	0.91	0.91	0.91

Table 6.35: Results for the best partitions using Archetypal Analysis and Fuzzy c-Means in North West, with respective Precision, Recall and F1 scores for both Training/Validation and Testing phases and best hyperparameters. DT{X} refers to the Decision Tree Model of depth X. ML, MS, NN and NL are the Minimum Number of Samples per Leaf, Minimum Number of Samples per Split, Number of Nodes for that Model and Number of Leaves for that Model, respectively.

As seen in Table 6.35, for Archetypal Analysis, the Decision Trees of depths 3 and 6 appear to have the best F1 score at 0,95. Among these, the Decision Tree of Depth 3 has a good balance between readability and complexity at 13 nodes.

For Fuzzy c-Means, the Decision Tree of depth 3 appears to have the best F1 score at 0,91, with a good balance between readability and complexity at 15 nodes.

6.3.6 South

S	Model	Criterion					Train/Val			Test		
			ML	MS	NN	NL	Precis	Rec	F1	Precis	Rec	F1
AA S5 5C	DT2	Entropy	7	23	7	4	0.68	0.68	0.68	0.5	0.5	0.5
	DT3	Entropy	7	32	13	7	0.71	0.71	0.71	0.62	0.62	0.62
	DT4	Entropy	5	10	25	13	0.73	0.73	0.73	0.88	0.88	0.88
	DT5	Entropy	4	4	31	16	0.72	0.72	0.72	0.88	0.88	0.88
	DT6	Entropy	3	4	35	18	0.74	0.74	0.74	0.81	0.81	0.81
	DT7	Entropy	3	8	33	17	0.69	0.69	0.69	0.88	0.88	0.88
	DT8	Entropy	4	10	29	15	0.66	0.66	0.66	0.88	0.88	0.88
	DT9	Entropy	4	10	29	15	0.72	0.72	0.72	0.88	0.88	0.88
	RIPPER	—	—	—	—	—	—	—	—	0.75	0.75	0.75
FCM S7 6C	DT2	Entropy	4	15	7	4	0.75	0.75	0.75	0.56	0.56	0.56
	DT3	Entropy	8	19	13	7	0.72	0.72	0.72	0.62	0.62	0.62
	DT4	Entropy	3	19	23	12	0.76	0.76	0.76	0.75	0.75	0.75
	DT5	Entropy	5	6	33	17	0.7	0.7	0.7	0.75	0.75	0.75
	DT6	Gini	1	3	47	24	0.75	0.75	0.75	0.88	0.88	0.88
	DT7	Entropy	1	4	45	23	0.75	0.75	0.75	0.69	0.69	0.69
	DT8	Gini	1	4	45	23	0.69	0.69	0.69	0.88	0.88	0.88
	DT9	Gini	1	9	39	20	0.7	0.7	0.7	0.88	0.88	0.88
	RIPPER	—	—	—	—	—	—	—	—	0.69	0.69	0.69

Table 6.36: Results for the best partitions using Archetypal Analysis and Fuzzy c-Means in South, with respective Precision, Recall and F1 scores for both Training/Validation and Testing phases and best hyperparameters. DT{X} refers to the Decision Tree Model of depth X. ML, MS, NN and NL are the Minimum Number of Samples per Leaf, Minimum Number of Samples per Split, Number of Nodes for that Model and Number of Leaves for that Model, respectively.

As seen in Table 6.36, for Archetypal Analysis, the Decision Trees of depths 4,5,7,8 and 9 appear to have the best F1 score at 0,88. Among these, the Decision Tree of Depth 4 has better readability at 25 nodes.

For Fuzzy c-Means, the Decision Trees of depths 6,8 and 9 appear to have the best F1 score at 0,88. However they have a high number of nodes (39+) which might result in rules that are harder to interpret. For a decision tree depth of 4, we still have a high F1 at 0,75, but with a tree with better readability at 23 nodes.

6.4 Fire Danger Rulesets: Comparison and Evaluation

6.4.1 Best Fire Danger Rule Sets Selection

In order to select the best rules for each region, the rulesets from both models and clustering algorithms are compared using their F1 and rule Interpretability scores. In the case of Decision Trees, the F1 score and tree readability was used to select the model with best depth.

6.4.1.1 Mainland Portugal

PT	Model	Precision	Recall	F1	Pred	Simp	Stab	Score
AA	DT3	0.88	0.88	0.88	0.74	1.0	0.25	0.66
5F5C	RIPPER	0.93	0.93	0.93	0.85	0.55	0.21	0.53
FCM	DT4	0.88	0.88	0.88	0.67	1.0	0.0	0.56
5F8C	RIPPER	0.91	0.91	0.91	0.77	0.47	0.12	0.45

Table 6.37: Rule Evaluation Scores for the best models with both Archetypal Analysis and Fuzzy c-Means, in Mainland Portugal

As seen in Table 6.37, for both Archetypal Analysis and Fuzzy c-Means, both the Decision Tree and RIPPER show very high F1 scores, with RIPPER being slightly better in both cases.

While the Simplicity score is generally in favour of the Decision Trees, RIPPER has generally higher Predictivity Scores with the Stability score being similar between the two.

As such, the best ruleset for Mainland Portugal is the one from RIPPER using Archetypal Analysis.

6.4.1.2 Center East

CE	Model	Precision	Recall	F1	Pred	Simp	Stab	Score
AA	DT3	0.86	0.86	0.86	0.71	1.0	0.0	0.57
5F5C	RIPPER	0.91	0.91	0.91	0.82	0.71	0.32	0.62
FCM	DT3	0.8	0.8	0.8	0	0.71	0.0	0.24
7F6C	RIPPER	0.8	0.8	0.8	0	1.0	0.25	0.42

Table 6.38: Rule Evaluation Scores for the best models with both Archetypal Analysis and Fuzzy c-Means, in Center East

As seen in Table 6.38, RIPPER generally has similar or better F1 scores and better Interpretability score than the Decision Trees, with the best results for each score being found for RIPPER when using Archetypal Analysis: F1 at 0,91 and Interpretability at 0,62.

As such, the best ruleset for Center East is the one from RIPPER using Archetypal Analysis.

6.4.1.3 Center West

CW	Model	Precision	Recall	F1	Pred	Simp	Stab	Score
AA	DT3	0.89	0.89	0.89	0.72	0.4	0.0	0.37
7F6C	RIPPER	0.56	0.56	0.56	0	1.0	0.15	0.38
FCM	DT4	0.78	0.78	0.78	0.37	0.45	0.0	0.27
7F5C	RIPPER	0.67	0.67	0.67	0.37	1.0	0.25	0.54

Table 6.39: Rule Evaluation Scores for the best models with both Archetypal Analysis and Fuzzy c-Means, in Center West

As seen in Table 6.39, for both Archetypal Analysis and Fuzzy c-Means, the Decision Trees show much higher F1 scores than RIPPER.

While the Simplicity and Stability score are generally in favour of RIPPER, the results of the Decision Tree using Archetypal Analysis show a very high Predictivity score of 0,72 on top of the previously mentioned very high F1 score of 0,89.

As such, the best ruleset for Center West is the one from the Decision Tree of depth 3 using Archetypal Analysis.

6.4.1.4 North East

NE	Model	Precision	Recall	F1	Pred	Simp	Stab	Score
AA	DT3	0.62	0.62	0.62	0.6	0.55	0.0	0.38
7F6C	RIPPER	0.81	0.81	0.81	0.8	1.0	0.2	0.67
FCM	DT3	0.94	0.94	0.94	0.65	0.81	0.0	0.49
7F8C	RIPPER	0.88	0.88	0.88	0.3	1.0	0.4	0.57

Table 6.40: Rule Evaluation Scores for the best models with both Archetypal Analysis and Fuzzy c-Means, in North East

As seen in Table 6.40, RIPPER has very high F1 scores, using both Archetypal Analysis and Fuzzy c-Means, while the Decision Tree using Fuzzy c-Means has the highest F1 score at 0,94.

While having a very high F1 score at 0,81, the results of RIPPER using Archetypal Analysis also show the highest rule Interpretability score at 0,67, with a great Predictivity score of 0,8.

As such, the best ruleset for North East is the one from RIPPER using Archetypal Analysis.

6.4.1.5 North West

As seen in Table 6.41, for both Archetypal Analysis and Fuzzy c-Means, both the Decision Tree and RIPPER show very high F1 scores, with Archetypal Analysis being slightly better in both cases.

NW	Model	Precision	Recall	F1	Pred	Simp	Stab	Score
AA	DT3	0.95	0.95	0.95	0.85	0.75	0.0	0.53
5F5C	RIPPER	0.95	0.95	0.95	0.85	1.0	0.15	0.67
FCM	DT3	0.91	0.91	0.91	0.8	1.0	0.0	0.6
5F6C	RIPPER	0.91	0.91	0.91	0.8	0.83	0.0	0.54

Table 6.41: Rule Evaluation Scores for the best models with both Archetypal Analysis and Fuzzy c-Means, in North West

Although all have very similar Interpretability scores, the highest score belongs to RIPPER using Archetypal Analysis at 0,67, with a very high Predictivity score as well (0,85).

As such, the best ruleset for North West is the one from RIPPER using Archetypal Analysis.

6.4.1.6 South

S	Model	Precision	Recall	F1	Pred	Simp	Stab	Score
AA	DT4	0.88	0.88	0.88	0.88	0.31	0.0	0.4
5F5C	RIPPER	0.75	0.75	0.75	0.77	1.0	0.31	0.69
FCM	DT4	0.75	0.75	0.75	0.77	0.38	0.0	0.38
7F6C	RIPPER	0.69	0.69	0.69	0.71	1.0	0.31	0.67

Table 6.42: Rule Evaluation Scores for the best models with both Archetypal Analysis and Fuzzy c-Means, in South

As seen in Table 6.42, for both Archetypal Analysis and Fuzzy c-Means, both the Decision Tree and RIPPER show high to very high F1 scores, with Decision Trees being slightly better in both cases.

While the Predictivity scores are similarly high (with Decision Tree using Archetypal Analysis being slightly better), the Interpretability scores are much more in favour of RIPPER than Decision Trees.

As such, the best ruleset for South is the one from RIPPER using Archetypal Analysis.

6.4.2 Fire Danger IF-THEN Rules

In this section, we are evaluating the selected rulesets based on how they compare to the fire danger rating scales. For the RIPPER rulesets, we take into consideration the first rules with the highest support for each Risk Level, in descending order. Support is the number of cases where the rule is true over the number of cases where it is not.

6.4.2.1 Mainland Portugal

RiskLvl	Rules	Support
Maximum	ISI \geq 22.68 If Not Then:	21/4
Low/VLow	FWI \leq 13.02 and DC \leq 585.36 and FFMC \leq 79.27	56/0
	FWI \leq 14.86 and ISI \leq 0.57 and DC \leq 863.41	15/0
	DC \leq 320.68 and FWI \leq 14.86 and CHI \leq 6.52	14/0
	FFMC \leq 83.71 and DC \leq 618.03 and CHI \geq 1.59 and CHI \leq 2.35 If Not Then:	4/0
VeryHigh	FWI \geq 50.72	104/2
	FWI \geq 47.42 and DC \geq 800.43	13/1
	FWI \geq 43.07 and DC \geq 1058.6	7/0
	FWI \geq 48.5 and DC \geq 668.52 and CHI \leq 9.35 If Not Then:	9/2
High	FWI \geq 35.23	474/7
	FWI \geq 30.08 and DC \geq 700.63	99/10
	FWI \geq 32.14 and ISI \geq 8.78 and DC \geq 506.58	37/2
	FWI \geq 28.89 and ISI \geq 10.24	28/4
	DC \geq 646 and FWI \geq 28.5 and CHI \leq 5.62 and ISI \geq 7.66	10/0
	DC \geq 919.03 and FWI \geq 22.83	30/4
	CHI \leq 6.88 and FWI \geq 32.98	10/2
	FWI \geq 28.98 and DC \geq 674.33 and CHI \leq 2.9	6/0
	DC \geq 610.1 and FWI \geq 33.96 If Not Then:	6/0
Moderate		757/16

Table 6.43: Rules extracted with RIPPER using Archetypal Analysis, in Mainland Portugal

By analysing the rules in Table 6.43, we can observe that:

Maximum :

1. ISI appears as the sole necessary factor in classifying a Mainland Portugal wildfire with Maximum Risk Level. Its value is in the expected range for this Risk Level, very close to the current ISI Maximum Risk Level threshold (where ISI is higher than 23).

Low/VLow :

1. Both FWI and FFMC are in the expected range of values for this Risk Level, slightly lower than their current threshold. DC is Moderate, higher than expected for this Risk Level.
2. Both FWI and ISI are in the expected range of values for this Risk Level, slightly lower than their current threshold. DC is Very High, much higher than expected for this Risk Level, but possibly balanced by the other indexes.
3. Both FWI and DC are in the expected range of values for this Risk Level, slightly lower than their current threshold. CHI is High, higher than expected for this Risk Level.

Very High :

1. FWI is much higher than the range of values for this Risk Level. In fact, it is more representative of the Maximum Risk Level, as this Risk Level is expected to have FWI values greater than 50,1.
2. FWI is in the expected range of values, although much higher than the current inferior FWI threshold for this Risk Level. DC is also in the expected range of values for this Risk Level, higher than the current inferior threshold.

High :

1. FWI is in the expected range of values for this Risk Level, higher than its current threshold.
2. Both FWI and DC are in the expected range of values for this Risk Level, higher than their current threshold.
3. FWI is in the expected range of values for this Risk Level, higher than its current threshold. Both DC and ISI are Moderate, lower than expected for this Risk Level.
4. FWI is in the expected range of values for this Risk Level, higher than its current threshold. ISI is Moderate, lower than expected for this Risk Level.

6.4.2.2 Center East

RiskLvl	Rules	Support
Maximum	FWI \geq 60.99 If Not Then:	6/1
Low/VLow	FWI \leq 0.9 CHI \leq -1.52 ISI \leq 1.79 and DC \leq 499.01 If Not Then:	13/0 3/0 3/0
Moderate	FWI \leq 24.68 and CHI \leq 6.17 and FFMC \leq 89.24 FFMC \leq 90.62 and DC \leq 638.94 and CHI \leq 5.92 FWI \leq 29.58 and DC \leq 523.6 and CHI \leq 6.72 ISI \leq 6.8 and CHI \leq 3.05 FWI \leq 21.06 and DC \leq 555.52 If Not Then:	55/0 28/2 9/0 12/3 7/2
VeryHigh	FWI \geq 39.05 and CHI \geq 5.71 and DC \geq 658.01 FWI \geq 40.93 and FFMC \geq 93.91 FWI \geq 42.24 and DC \geq 889.22 FWI \geq 35.08 and DC \geq 796.06 and CHI \geq 6.84 FWI \geq 42.98 and CHI \geq 6.92 and FFMC \geq 92.19 DC \geq 931.66 and FFMC \geq 92.11 If Not Then:	77/3 21/2 6/0 11/1 8/0 7/2
High		330/12

Table 6.44: Rules extracted with RIPPER using Archetypal Analysis, in Center East

By analysing the rules in Table 6.44, we can observe that:

Maximum :

1. FWI is in the expected range of values for this Risk Level, although much higher than its current threshold.

Low/VLow :

1. FWI is in the expected range of values for this Risk Level, although much lower than its current threshold, it is near zero.

Moderate :

1. FWI is in the expected range of values for this Risk Level, interchangeable with the current superior FWI Moderate Risk Level threshold (where FWI is no higher than 24,6). Both CHI and FFMC are also very close to their respective superior Moderate Risk Level threshold, which stands at 6 for CHI and 90 for FFMC.

2. FFMC, DC and CHI are very close to their respective superior Moderate Risk Level threshold, which stands at 90 for FFMC, 600 for DC and 6 for CHI.

Very High :

1. FWI is in the expected range of values for this Risk Level, very close to the current inferior FWI Very High Risk Level threshold (where FWI is no lower than 38,3). Both CHI and DC are Moderate and High respectively, lower than expected for this Risk Level.
2. Both FWI and FFMC are in the expected range of values for this Risk Level, very close to their respective inferior Very High Risk Level threshold.

6.4.2.3 Center West

RiskLvl	Rules
Low/VLow	$\text{FWI} \leq 37.40$ and $\text{FWI} \leq 25.02$ and $\text{CHI} \leq 6.49$
Moderate	$\text{FWI} \leq 37.40$ and $\text{FWI} \leq 25.02$ and $\text{CHI} > 6.49$ $\text{FWI} \leq 37.40$ and $\text{FWI} > 25.02$ and $\text{DC} \leq 567.48$ $\text{FWI} \leq 37.40$ and $\text{FWI} > 25.02$ and $\text{DC} > 567.48$ $\text{FWI} > 37.40$ and $\text{FWI} \leq 48.10$ and $\text{DMC} \leq 154.37$
High	$\text{FWI} > 37.40$ and $\text{FWI} \leq 48.10$ and $\text{DMC} > 154.37$ $\text{FWI} > 37.40$ and $\text{FWI} > 48.10$

Table 6.45: Rules extracted with Decision Tree of depth 3 using Archetypal Analysis, in Center West

By analysing the rules in Table 6.45, which is the only selected ruleset from Decision Trees, we can observe that:

Low/VLow :

1. Both FWI and CHI are much higher than expected for this Risk Level, both being very close to their respective superior Moderate Risk Level threshold, which stands at 24,6 for FWI and 6 for CHI.

Moderate :

1. FWI is in the expected range of values for this Risk Level, very close to the current superior FWI Moderate Risk Level threshold (where FWI is no higher than 24,6). However, CHI is higher than expected for this Risk Level, opening up to higher CHI values while already being close to the superior CHI Moderate Risk Level threshold.
2. The presented range of FWI values seem higher than expected for this range. In fact, both thresholds are very close to the thresholds representative of the High Risk Level. DC is in the expected range of values for this Risk Level.
3. Can be simplified using the previous rule. The same information regarding FWI still applies here.
4. An even higher FWI range of values, now closer to the range of values expected from a Very High Risk Level. However, DMC is in the expected range of values for this Risk Level, very close to the current superior DMC Moderate Risk Level threshold.

High :

1. As for the previous rule, the presented FWI range of values are closer to what is expected of a Very High Risk Level. However, DMC is in the expected range

6.4. FIRE DANGER RULESETS: COMPARISON AND EVALUATION

of values for this Risk Level, very close to the current inferior DMC High Risk Level threshold.

2. As for the previous rules, FWI is higher than expected, being closer to the inferior threshold of a Very High Risk Level.

6.4.2.4 North East

RiskLvl	Rules	Support
Maximum	FWI \geq 59.43 If Not Then:	2/0
Low/VLow	FWI \leq 12.72 and BUI \leq 112.35 If Not Then:	7/0
VeryHigh	FWI \geq 39.62 BUI \geq 289.21 If Not Then:	30/5 3/1
Moderate	FWI \leq 26.71 and DC \leq 668.71 CHI \leq 4.83 and FWI \leq 27.63 and BUI \leq 178.28 If Not Then:	61/2 14/0
High		121/13

Table 6.46: Rules extracted with RIPPER using Archetypal Analysis, in North East

By analysing the rules in Table 6.46, we can observe that:

Maximum :

1. FWI is in the expected range of values for this Risk Level, although much higher than its current threshold.

Low/VLow :

1. Both FWI and BUI are very close to their respective superior Low Risk Level threshold, which stands at 17,2 for FWI and 100 for BUI.

Very High :

1. FWI is in the expected range of values for this Risk Level, very close to and slightly higher than the current inferior FWI Very High Risk Level threshold (where FWI is no lower than 38,3).

Moderate :

1. FWI is in the expected range of values for this Risk Level, very close to and slightly higher than the current superior FWI Moderate Risk Level threshold (where FWI is no higher than 24,6). DC is High, but can also be viewed as being close to the current superior DC Moderate Risk Level threshold (where DC is no higher than 600).
2. Both FWI and BUI are slightly higher than expected for this Risk Level, being very close to their respective superior Moderate Risk Level threshold, which stands at 24,6 for FWI and 150 for BUI. CHI is in the expected range of values

6.4. FIRE DANGER RULESETS: COMPARISON AND EVALUATION

for this Risk Level, while being considerably lower than the current superior CHI Moderate Risk Level threshold (where CHI is no higher than 6).

6.4.2.5 North West

RiskLvl	Rules	Support
Maximum	FWI \geq 66.17 If Not Then:	4/0
Low/VLow	FWI \leq 2.59	19/0
	FWI \leq 7.79 and DC \leq 350.94 If Not Then:	3/0
VeryHigh	FWI \geq 46.77 If Not Then:	42/0
Moderate	FWI \leq 24.06	152/5
	FWI \leq 27.58 and CHI \leq 7.81 and DC \leq 526.04	29/0
	CHI \leq 5.74 and ISI \leq 7.58	17/0
	FFMC \leq 90.19 and ISI \leq 8.24 and ISI \geq 8.1 If Not Then:	4/0
High		212/6

Table 6.47: Rules extracted with RIPPER using Archetypal Analysis, in North West

By analysing the rules in Table 6.47, we can observe that:

Maximum :

1. FWI is in the expected range of values for this Risk Level, although much higher than its current threshold.

Low/VLow :

1. FWI is in the expected range of values for this Risk Level, although much lower than its current threshold, it is near zero.

Very High :

1. FWI is in the expected range of values, although much higher than the current inferior FWI threshold for this Risk Level.

Moderate :

1. FWI is in the expected range of values for this Risk Level, interchangeable with the current superior FWI Moderate Risk Level threshold (where FWI is no higher than 24,6).
2. FWI is slightly higher than expected for this Risk Level, but is still comparable to the current superior FWI Moderate Risk Level threshold (where FWI is no higher than 24,6). CHI is considerably higher than expected and can even be compared to the the current superior CHI High Risk Level threshold (where

6.4. FIRE DANGER RULESETS: COMPARISON AND EVALUATION

CHI is no higher than 8). DC is in the expected range of values for this Risk Level.

3. Both CHI and ISI are in the expected range of values for this Risk Level, slightly lower than their current threshold.

6.4.2.6 South

RiskLvl	Rules	Support
Maximum	FWI \geq 57.63 and CHI \geq 10.44 If Not Then:	12/0
Low/VLow	ISI \leq 0.76	19/1
	FFMC \leq 81.9 and DC \leq 700.06	11/1
	FFMC \leq 67.1 and CHI \leq 1.21 If Not Then:	4/0
Moderate	FFMC \leq 89.15 and CHI \leq 5.89	28/3
	CHI \leq 3.07 and FWI \leq 35.88 If Not Then:	5/0
VeryHigh	FWI \geq 42.41	47/4
	CHI \geq 9.2 and ISI \geq 9.23 If Not Then:	15/2
High		62/2

Table 6.48: Rules extracted with RIPPER using Archetypal Analysis, in South

By analysing the rules in Table 6.48, we can observe that:

Maximum :

1. FWI is in the expected range of values for this Risk Level, although much higher than its current threshold. CHI is also in the expected range of values and very close to the current threshold.

Low/VLow :

1. ISI is in the expected range of values for this Risk Level, although much lower than its current threshold, it is near zero.
2. FFMC is in the expected range of values for this Risk Level, slightly lower than the current threshold. DC is High, much higher than expected for this Risk Level.

Moderate :

1. Both CHI and FFMC are in the expected range of values for this Risk Level, very close to their respective superior Moderate Risk Level threshold.

Very High :

1. FWI is in the expected range of values for this Risk Level, slightly higher than the current inferior FWI Very High Risk Level threshold (where FWI is no lower than 38,3).

6.4. FIRE DANGER RULESETS: COMPARISON AND EVALUATION

2. CHI is in the expected range of values for this Risk Level, slightly higher than the current inferior threshold. ISI is Moderate, much lower threshold than expected for this Risk Level.

6.5 Results Summary

Comparing the selected partitions from Archetypal Analysis and Fuzzy c-Means, we can observe that:

- For four out of six regions, both Archetypal Analysis and Fuzzy c-Means preferred the same number of features: 5 features for Mainland Portugal and North West and 7 features for Center West and North East.
- On top of that, when their preference did differ (Center East and South), Archetypal Analysis preferred 5 features and Fuzzy c-Means preferred 7 features.
- Archetypal Analysis generally preferred partitions with fewer clusters than Fuzzy c-Means, the exception being in Center West.

Regarding the rules-extraction models, we can see that:

- Models using Archetypal Analysis were always selected instead of those using Fuzzy c-Means for each region, due to a slightly better balance between the F1 and Interpretability scores. Archetypal Analysis had an average F1 of 0,833 and Interpretability score of 0,539. Fuzzy c-Means had an average F1 of 0,826 and Interpretability score of 0,478.
- RIPPER generally resulted in better results than Decision Tree as well, the latter only being selected for one of the regions (Center West). RIPPER had an average F1 of 0,814 and Interpretability score of 0,563. Decision Trees had an average F1 of 0,845 and Interpretability score of 0,454.
- Mainland Portugal had a better F1 score than the average of the other regions for the testing data: 0,93 to 0,86.

Finally, about the extracted rules, we can notice that:

- Mainland Portugal has the only RIPPER ruleset that does not start with a Maximum Risk Level rule based on FWI, but instead with ISI. This highlight might present the fact that the Wind conditions are more impactful when studying wildfire classifications in a broader scale, more than the remaining indexes which are more regional.
- For each sub-region with a RIPPER ruleset, FWI was the sole necessary factor in defining the first Maximum Risk Level Rule, except for South, where CHI is also included. Similarly to the first point, this seems to highlight the impact of the different local conditions for wildfire classification, which FWI sums up.

- For the two regions where all seven features were used (Center West and North East), BUI was not a manifested in the ruleset of the first and DMC of the second. This might be related to the fact that BUI and DMC are closely related and very numerically similar, thus not being necessary to include both in the rules.
- Further highlighting this index, FWI is present in 48 out of 61 rules, or 79%. In decreasing order, the other indexes are: DC (44%), CHI (39%), ISI (18%) and FFMC (16%).
- However, in South, CHI has a higher presence and impact than FWI and DC, highlighting the difference in how this region should be analysed in comparison to the others. This also makes sense in context with the type of climate and wildfires we expect from that region.
- Of all the sub-regions, Center East is the most complex, having a higher number of rules and conditions. This might imply that Center Eastern wildfires are more varied in their main characteristics.

6.6 Burned Area Study for Archetypal Analysis and Fuzzy c-Means Classifications

On top of the previously discussed results, this section presents the results of the study made on the relation between archetype and prototype-based classification and the burned area of the wildfires. As Burned Area is often an indicator as to the consequences of a wildfire, where larger and more dangerous wildfires cause more burned areas, in this experiment we analysed how the fire danger classifications derived from Archetypal Analysis and Fuzzy c-Means clustering (by archetypes and prototypes) compare to fire danger classification based on the wildfires burned areas.

For this study, the average Burned Area for each Risk Level is calculated, given each of the three wildfire classification methods. The Burned Area-based classification is done using the scale presented in Table 6.49, where each wildfire is classified according only to their burned area. The Archetype and Prototype-based classification are done as described in Chapter 5, where the Burned Area attribute is not taken into account during clustering and labelling.

Risk	Burned Area (ha)
Low/Very Low	BA < 150
Moderate	150 - 250
High	250 - 500
Very High	500 - 1000
Maximum	1000 < BA

Table 6.49: Burned Area Scale, provided by IPMA

The results of this study are presented in the graphic of Figure 6.7 for Mainland Portugal, for the data set described by five attributes and using the partitions with five clusters for both clustering approaches. This graphic shows the average and standard deviation of burned area for the group of wildfires classified under each Risk Level, according to each classification method.

As expected, the results show a gradual average area increase according to the Risk Level, in the case of the Burned Area-based classification.

From Risk Levels Low/Very Low to Very High, wildfires clustered using Archetypes and Prototypes show similar burned area averages, with Archetype-based classification showing slightly higher averages for Moderate, High and Very High Risk Levels.

For the Maximum Risk Level, both of these classification averages diverge considerably. The burned area average in the Archetype-based classification increases and seems to more closely resemble the drastic increase of the average in the Burned Area-based classification for this Level. The burned area average in the Prototype-based classification decreases from the Very High Level.

These results highlight the similarity between Archetypal Analysis and Fuzzy c-Means

6.6. BURNED AREA STUDY FOR ARCHETYPAL ANALYSIS AND FUZZY C-MEANS CLASSIFICATIONS

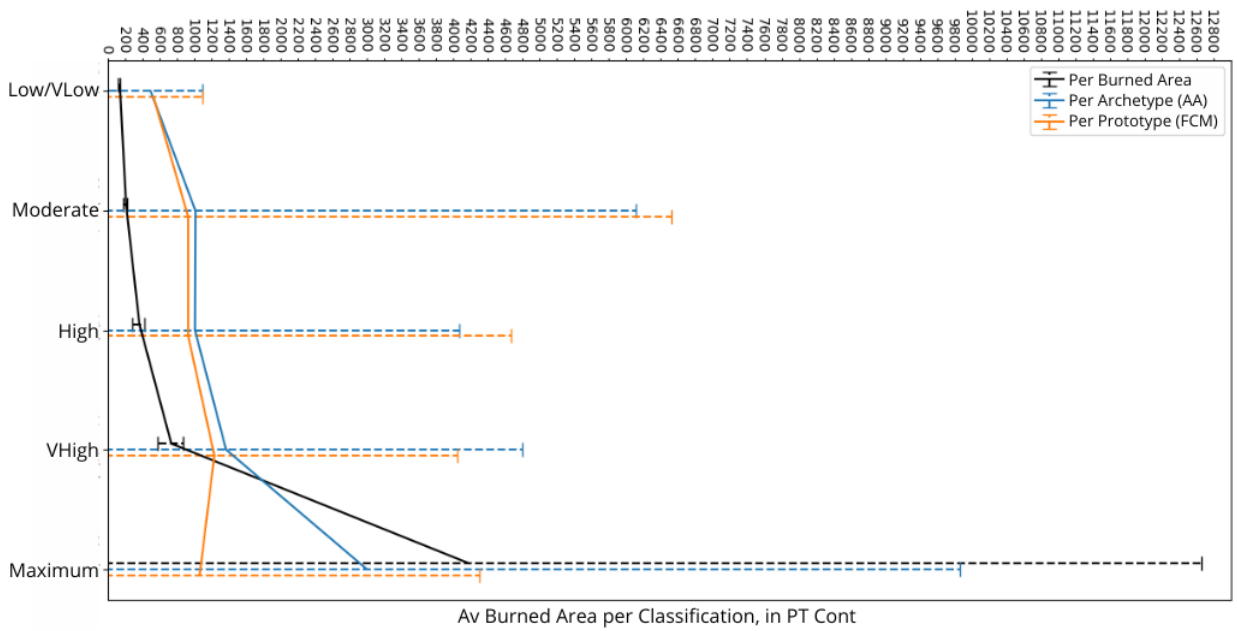


Figure 6.7: Average Burned Area per Classification Method, per Risk Level, for Mainland Portugal (and corresponding standard deviations)

classifications for the more moderate Risk Levels, while also highlighting the ability for Archetypal Analysis to classify extreme wildfire cases, specifically the Maximum Risk Level classification.

CONCLUSION AND FUTURE WORK

This dissertation attempted to explore existing fire danger rating scales for Portuguese wildfires, by studying different wildfire clustering approaches and risk-based rule extraction using wildfire data from across two decades.

Wildfires were clustered and classified with the help of two clustering algorithms - Archetypal Analysis and Fuzzy c-Means -, with results being validated using the RSS and Xie-Beni Index respectively. The application of the elbow method to RSS proved to be adequate for determining the number of archetypes for this study. Results showed that Archetypal Analysis generally preferred partitions of 5 or 6 clusters, while Fuzzy c-Means fluctuating more between 6 and 8 clusters. Due to its nature in highlighting more extreme cases, Archetypal Analysis was always able to generate "Maximum" risk level archetypes, which was not always the case for Fuzzy c-Means prototypes.

These wildfires were then used for IF-THEN rule extraction using the Decision Tree and RIPPER algorithms, which were themselves evaluated using rule interpretability scores and metrics. Across the different regions, rules extracted using wildfires classified with Archetypal Analysis led to slightly better results than those coming from Fuzzy c-Means, with an average F1 of 0,833 and Interpretability score of 0,539 versus an average F1 of 0,826 and Interpretability score of 0,478.

Regarding the rule extraction algorithms, RIPPER generally had better results than Decision Tree with Archetypal Analysis, except for one region (Center West). RIPPER and Decision Trees had similar F1 scores at 0,814 and 0,845 respectively, with RIPPER having an advantage in the Interpretability score: 0,563 to 0,454.

When comparing the extracted rules to the currently used fire danger rating scales, we can generally see the scales reflected on the rules. Sometimes, probably due to the extreme nature of archetypes, where highs are very high and lows are very low, we can note that thresholds are extremely close to what was expected for Moderate to Very High classification and remarkably higher or lower for Maximum and Low/Very Low classifications respectively.

Finally, both the extracted rules and the preferred number of features continue to highlight the difference and importance in studying these scales on a regional basis.

Regions like Center West and North East had better results when using all seven features, while the rest - including mainland Portugal as a whole - preferred five features.

As future work, some aspects can be further explored:

- The application of the Archetypal Analysis algorithm to wildfire data from other regions;
- The exploration of this approach using different wildfire index combinations, including the vegetation indexes and dangerousness index;
- As the Xie-Beni Index was the only internal validity index used for Fuzzy c-Means, a study using multiple validity indexes can be explored;

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