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Diversification and Screening

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Diversification and Screening

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Abstract

I study two-way effects between financial markets and contractual agreements with a risk sharing component, such as compensation packages within a firm, or mortgages and loans. I construct a model with many Units, in each of which one of the contracting individuals, the Agent, has private information, while the uninformed individual, the Principal, has the opportunity to trade with the Principals in other Units. I give general conditions under which financial markets induce a transfer of risk from Agents to Principals. I also show how asymmetric information interacts with financial markets through two channels. First, the distortion of the allocation of the high risk Agents, feeds back in the market portfolio increasing risk on markets, and in the contracts of the low risk Agents. Secondly, markets change the Principals' screening problem preventing low risk Agents from enjoying an information rent. The model results can explain empirical evidence from the subprime mortgage market during the securitization boom leading to the 2008 financial crisis and suggest further implications for other markets segment.

1 Introduction

Risk is transferred and pooled among individuals in different ways. For example, by trading assets on financial markets, or by contractual arrangements among individuals. I study how these two modes of risk sharing interact in presence of asymmetric information, and how their interaction affects the distribution of risk and welfare.

These interactions are relevant in a modern economy. For example, most forms of bonus compensation necessarily involve some form of risk sharing, because of the difficulty in obtaining

a precise performance measure, or because of profit sharing. Labor compensation is typically the most relevant expense for a corporation. In the US, for example, more than 60% of the payment to factors group in the 2009 GDP in US was to labor.¹ Since the stock of a company is a claim to its profits, the firm's decisions on workers' compensation affect the returns of its stock. In the aggregate this affects financial markets. On the other hand, diversification opportunities offered by markets influence the design of compensation packages.

Another interesting case of the interaction of these two modes of risk sharing is the securitization of individual contracts, like insurance policies and mortgage loans. In the past decades, these contracts have been increasingly often packaged into traded assets. The recent financial crisis is a prime example that this evolution of financial markets can and will affect non-market agreements, such as mortgages, and that these changes can at times feedback into financial markets with spectacular consequences.

In both cases, it is important for the firm, the insurer or the loan issuer, to assess certain qualities of the counterparty, in order to design the contract maximizing their profits. In some cases these can be directly observed (freely or at some cost), in other cases these qualities are private information, and they can be screened by offering the informed party a menu of different contracts to choose from. I show channels through which diversification opportunities interact with asymmetric information and how these interactions induce transfers of risk and welfare across different types of individuals in the economy. After providing conditions under which risk shifts to individuals accessing markets, I show two novel effects, specific to the case of asymmetric information. First, there is a transfer of welfare from high quality individuals to their counterparties with access to markets. This effect follows from the way in which diversification changes the tradeoffs involved in the design of optimal screening contracts. Secondly, I show how informational distortions reverberate across the economy putting more risk in the hands of high quality agents and of individuals accessing markets: asymmetric information shifts risk from low quality agents to the equilibrium portfolios of trading agents, and from there on to the contracts offered to high types.

To study the interactions between contracting and financial markets, I construct a model with private information in which all individuals are paired into Units generating returns, which are shared by means of a contract. In each Unit, one of the contracting individuals, the Agent, has private information, while the uninformed individual, the Principal, has the opportunity to trade with the Principals in the other Units. In the context of firms, Principals and Agents can be seen as Investors and Employees; in the context of securitization, Agents are individuals

¹ Economic Report of the President, 2010

taking out a loan or an insurance contract, whereas Principals are the financial institutions securitizing. This generates two-way effects from markets to Units, and vice versa.

I use this model to analyze both directions of interaction.

1. How does the existence of asset markets affect contracts within individual Units?
2. How does asymmetric information in the Units affect asset markets?

First, since financial markets allow individuals to diversify risk, one would expect that they would make Principals act as if they were less risk averse. This changes the terms of risk sharing and make contracts less risky for Agents, the “Insurance Effect”. Several examples show that this intuitive property need not hold (even in the simple case of symmetric information, in which Agents’ types are public at the contracting stage). I give sufficient conditions which rule out these unexpected effects of markets: a large number of Units with a limited level of correlation across them. These are fairly general assumptions, which arguably correspond to features of real financial markets.

Second, Principals solve the screening problem they face inside Units by designing contracts offering different levels of risk to different types of Agents. The need to screen distorts risk sharing within Units. In particular, it leads to an inefficiently high level of risk held by Principals, which they consequently trade on asset markets. Excess market risk is hence a byproduct of asymmetric information in agency relationships within Units.

However the lower risk aversion metaphor cannot explain the third result: the excess of aggregate risk in the market portfolio will partially trickle down to the contracts held by low risk agents; an effect of asymmetric information which is due to the market portfolio transferring risk across firms.

Finally, there are welfare implications. A large market, offering enough diversification possibilities, reduces informational distortions within Units. However, and perhaps more interestingly, some of the individuals who do not access markets will bear a cost. *All* Agents will see their utility pushed to the reservation level, even if they enjoyed some information rent in absence of markets (or in presence of a smaller market). In particular, those who will lose will be the low risk Agents, even if they do not directly access markets. Going back to the applications discussed earlier, this last result means that more developed financial markets can harm good workers, or safe loan applicants, if they cannot obtain access to these markets. This is because trading opportunities naturally increase the welfare of Principals’, not only by allowing them to adjust their portfolios, but also by improving their ability to extract welfare from Agents.

While these are new results, the study of financial markets as means to share risk in relation to other types of risk sharing, and their effects on each other have already been touched upon by the economic and financial literature.

I do not study the effects of asymmetric information *in* markets, but rather the effects that asymmetric information resolved elsewhere has *on* markets, and vice versa. This marks the first difference from the General Equilibrium works on insurance markets, starting from the seminal paper of Rothschild and Stiglitz (1976). Another important difference is that, in those papers, the fact that some individuals are risk neutral is usually an assumption (An exception is Dubey and Geneakoplos (2002), in which individuals endogenously pool to share risk). In this paper there are many risk averse investors, who access financial markets to trade away part of the risk they are exposed to. Traditionally, Principals are modeled as risk neutral expected utility maximizers because of their access to diversification opportunities. While I give conditions under which markets will induce a Principal to offer less risk to Agents, than they would without markets, I also show that even in this case, risk aversion plays a role as markets induce transfers of risk across types.

Within the more recent General Equilibrium literature, this paper fits best in spirit with other works, with more recent papers linking markets and non-markets transactions. For example, Legros and Newman (1996) analyze the internal organization of firms, in relation to the distribution of wealth in the economy and the imperfections of credit markets. Legros and Newman (2002) use markets to show how shocks to individual firms can cause restructuring in a sector. Similarly, in this paper the internal agreements in a firm impact those in other firms, but the channel is here that of financial markets. Gibbons, Holden, and Powell (2012) take firms formation as exogenous, as in the present work, but their focus is the effect of information sources on the allocation of control within the firm. Some papers in the General Equilibrium literature focus instead on the problem of hidden action, such as the works by Magill and Quinzii (2005), Parlour and Walden (2011), which bear similarities to this one. They also take firm formation as an exogenous process, abstracting from labor market considerations, and they allow for contracts inside firms and financial markets across firms. Magill and Quinzii show how available securities affect economic incentives, whereas Parlour and Walden consider effort in the moral hazard problem a human capital investment, and use their model to derive testable implications (for example on size effect) on the cross section of stock returns.

The work is closer in spirit to papers on financial intermediation such as Diamond (1984), Ramakrishnan and Thakor (1984) or Colla (2011). However these consider the information revelation process as a function of the effort of the intermediary. I consider a specific information

process, with novel welfare implications.

A strand of the finance literature looks at asset pricing in the presence of delegated portfolio management (for a survey, see Stracca (2003), for a more recent example including review of more recent literature see Ou-Yang (2011)). These studies look at the effect on prices and returns of the classical informational asymmetry problems. In this literature a representative principal delegates his investing decisions to an agent. Inefficiencies take the form of deviations from the non-delegated case equilibrium which take the form of changes in asset prices and optimal portfolio composition. Besides the different object of interest, the perspective in these works is in a sense opposite of the one taken here. There we have informed parties trading, whereas in the present work it is the uninformed parties who access markets.

Several findings in the empirical literature provide evidence compatible with the theoretical results in this paper. The recent subprime crisis in the United States naturally provided motivation for a wave of work enquiring the relation between mortgages and financial markets. I will mention a few here, all of which show how screening attitudes, changed before the subprime crisis, as securitization was booming. In general, these papers provide evidence of less screening and of an expansion of the mortgage pool in the direction of riskier loans. Their findings are consistent with the model's prediction that when more mortgages are securitized, then contracts of different types become closer, and that inclusion can only go in the direction of subprime applicants. Mian and Sufi (2009) and Purnandam (2011) show how institutions granting mortgage loans relaxed their screening practices, a change accelerated between 2005 and 2007, when securitization activity intensified. Dell'Ariccia et al. (2008) identify several channels affecting screening standards, including securitization practices (even though their goal is discussing in general the relation between credit growth and lending standards). Finally, Keys et al. (2010) show how the specific "rule of thumb" used by institutions has been relaxed and they relate the change to the increasing prevalence of securitization of loans. Agarwal et al. (2012) show a similar pattern, but further break it down to loans issued by a single institution, distinguishing between those which were sold off and those retained on the balance sheet. The present work suggests further venues for empirical analysis. Namely, analyzing the impact of securitization on "prime" market segments.

The paper is structured as follows. Section 2 presents a description of the model including primitives. Section 3 discusses the equilibrium concept and its existence. Section 4 presents the main results in the simpler case of independent returns. In section 5 I present an extension in which the pool of contracted Agents is endogenous. In Section 6 I discuss systemic risk and give sufficient conditions for results to hold in presence of systemic risk. Section 7 concludes.

The Appendix contains all proofs and two examples of economies which do not satisfy the assumptions and conditions discussed in Section 5.

2 The Model

To capture the feedback between contracting and trading on financial markets, I construct a model of an economy in which risk averse individuals can share risk through one-to-one contracting and some of them can diversify their risk by exchanging assets on a competitive market. Results are driven by the fact that those who access markets are in a sense solving two problems, with the same control variables: they have to design a contract *and* a security.

2.1 Primitives

There are $2N$ individuals with identical expected utility preferences $U(X) = E[u(X)]$, with quadratic utility function $u(x) = x - \frac{b}{2}x^2$.²

N individuals are Principals, and N are Agents. Principals are all identical. An Agent can be of type $t \in \{L, H\}$, which is his private information and influences the Agent's performance. Agent's types are independently drawn from a commonly known distribution f on $\{L, H\}$ such that $Pr(t(n) = L) = q$ and $Pr(t(n) = H) = 1 - q$.

There are N Units, each unit n is formed by a Principal P^n and an Agent A^n , whose type is $t(n)$. Unit n generates random returns $X(t(n))$, whose distribution is common knowledge and depends on the Agent's type. Any pattern of correlation between Units' is admissible.

Since preferences can be expressed as functions of mean or variance a single coefficient $\rho_{s,t}^{m,n} \in [-1, 1]$ describes all payoff relevant covariation between Units m and n with agents of type $t(m) = s$ and $t(n) = t$.

Principals are entitled to the returns generated by the unit. Agents obtain a reservation utility of \bar{u} if they do not participate in production, while Principals obtain zero.

2.2 Contracting

Principals and Agents within each Unit, share surplus through linear contracts. A contract $C^n = (\alpha^n, \beta^n)$ specifying the fixed amount where α^n the share of output β^n which the Principal can claim.³

² where b is small enough that utility is increasing.

³This notation is opposite to what one typically finds in a contract theory paper. However, this choice is convenient as most of the complexity here is in the Principal's objective function

After a contract is signed, Principal P^n holds the random variable $\alpha^n + \beta^n X(t(n))$, and Agent A^n holding $-\alpha^n + (1 - \beta^n) X(t(n))$.

Each Principal P^n offers to agent A^n a menu of contracts $M^n = [C_L^n, C_H^n] = [(\alpha_L^n, \beta_L^n), (\alpha_H^n, \beta_H^n)]$. A^n picks his preferred contract, or decides to not participate and obtain his reservation utility \bar{u} . Without loss of generality, we can restrict attention to Incentive Compatible (menus such that Agents of type t would pick C_t would pick over any other contract) and Individually Rational menus (menus such that agents of type t would pick C_t rather than his outside option).⁴

$$E[u(-\alpha_t + (1 - \beta_t) X_t)] \geq E[u(-\alpha_s + (1 - \beta_s) X_t)], \forall s, t \in \{L, H\}$$

$$E[u(-\alpha_t + (1 - \beta_t) X_t)] \geq \bar{u}$$

2.3 Market

In the final stage, each Principals is endowed with her claims to profits $\alpha_{t(n)}^n + \beta_{t(n)}^n X(t(n))$, which they will be able to trade at price q^n , together with a riskless asset available in zero net supply at price q^0 . A portfolio of financial assets is denoted by $\theta = (\theta_0 | \theta_1, \dots, \theta_N)$. θ_0 is the position an investor holds in the riskless asset. $\theta_1, \dots, \theta_N$ are the holdings of securities arising from contracting within each of the N Units. The solution concept used here is competitive equilibrium. All Principals choose a portfolio θ^n they can afford to maximize their expected utility, taking prices as given. Equilibrium prices will be such that all markets clear.

2.4 Timeline

The model plays out as follows

- **Time 0** Nature randomly draws the types of each Agent.
 - A type realization $\mathbf{t} = t(1) \dots t(N)$ is drawn from distribution $\mathbf{F} = X_{n=1}^N f$.
- **Time 1** Each Principal makes a take-it-or-leave-it offer to Agent he is matched with in the form of an Incentive Compatible menu of contracts
 - $\mathbf{M} = [M^1, \dots, M^N]$ is the vector of menus offered in each of the N Units.
- **Time 2** Each Agent A^n chooses a contract from the menu he is offered, or he can choose to not participate and obtain his reservation utility.

⁴ The fact that Principals interact with each other only on the asset market, makes this a straightforward application of the Revelation Principle.

– $\mathbf{c} = [c^1(\cdot) \dots c^N(\cdot)]$ the vector of Agents' choices, we have that $\mathbf{c}(\mathbf{M})$ is the vector of contracts chosen by agents facing menus $\mathbf{M} = [c^1(M^1), \dots, c^N(M^N)]$.

• **Time 3** Principals trade on the asset market.

– Since securities payoffs are determined by contracts $\mathbf{C} = [C^n]_{n=1}^N$ and by agents' skills \mathbf{t} , portfolios θ^n and prices $q = [q^1, \dots, q^N]$ will be functions $(\theta, q)(\mathbf{C}, \mathbf{t})$.

Finally, uncertainty is realized and contracts and assets pay off.

When Principals design contracts at time 1, they are also designing their endowment in the market at time 3, so that available securities are determined by risk sharing within units. This link creates the feedback between markets and contracts in equilibrium.

3 Equilibrium

Given the sequential nature of the model, I will describe payoffs within each pair and define the equilibrium concept by backward induction.

(and then the equilibrium concept) working backwards from the final stage.

Asset Market

Consider a Principal's holding portfolio θ , given a type realization \mathbf{t} , and contracts \mathbf{C} . Her utility will be:

$$U_{P^n}^3(\theta, \mathbf{C}, \mathbf{t}) = E \left[u \left(\theta_0 + \sum_{m=1}^N \theta_m [\alpha^m + \beta^m X(t(m))] \right) \right]$$

Principals will choose a portfolio to maximize their expected utility given prices.

Contracting, Agents' turn

When Agent A^n chooses contract C^n out he gets a payoff off

$$U_{A^n}^2(C^n, t(n)) = E[u(-\alpha^n + (1 - \beta^n) X(t(n)))]$$

Contracting, Principals' turn

At time 1 the *expected* utility of a Principal holding portfolio θ , when menus are \mathbf{M} , Agents choose contracts $\mathbf{c}(\mathbf{M})$ is

$$U_{P^n}^1(\mathbf{c}(\mathbf{M}), \theta) = E_t [U_{P^n}^3(\theta, \mathbf{c}(\mathbf{M}), \mathbf{t})]$$

Each principal offers a menu, without knowing the Agents' types. However, they correctly foresee the strategies of each agent, and the outcome of asset markets. In other words, they can forecast the equilibrium path for all possible offered menus \mathbf{M} , realizations of types \mathbf{t} , and chosen contracts $\mathbf{c}(\mathbf{M})$. Principals at this stage are playing a game against each other. Their strategies are menus, with Principal P^n offering menu M^n

Based on this timeline we can write the utility in the first stage in this form:

$$V^n(M^n | \mathbf{M}^{-n}) = U_{P^n}^1(\mathbf{c}(M^n | \mathbf{M}^{-n}, \tilde{\mathbf{t}}), \theta(\mathbf{c}(M^n | \mathbf{M}^{-n}, \tilde{\mathbf{t}}))) \quad (1)$$

3.1 Definition

An Equilibrium consists of

- Portfolios $\theta_*^n(\mathbf{C}, \mathbf{t})$ for each Principal P_n and prices $q_*(\mathbf{C}, \mathbf{t}) \in \mathbb{R}^{N+1}$ such that $[\theta_*, q_*](\mathbf{C}, \mathbf{t})$ is a competitive Equilibrium for the asset market taking place after contracting. Each principal is endowed with one unit of her asset so that the endowment of principal n is $w^n = [0, 0, \dots, 1, \dots, 0, 0]$ with 1 being in the n th position, and its value q_*^n .

$$\theta_*^n(\mathbf{C}, \mathbf{t}) \in \arg \max_{\theta \in \mathbb{R}_+^{N+1}} U_{P^n}^3(\theta, \mathbf{C}, \mathbf{t})$$

s.t.

$$q_*(\mathbf{C}, \mathbf{t}) \cdot \theta(\mathbf{C}, \mathbf{t}) \leq q_*^n(\mathbf{C}, \mathbf{t})$$

$$\sum_{n \in N} \theta_{*0}^n = 0$$

$$\sum_{n \in N} \theta_{*m}^n = 1, \forall m = 1, \dots, N$$

- For each agent A^n an optimal choice of contract out of the menu he is offered, conditional on hist type realization. $c_*^n(M, t)$ such that

$$c_*^n(M^n, t(n)) \in \arg \max_{C \in \mathcal{C}^n} U_{A^n}^2(C, t(n))$$

- For each principal P^n , a lottery \tilde{M}_*^n of menus such that

$$\text{supp} \left[\tilde{M}_*^n \right] \subseteq \text{arg} \max_{M^n \in \mathcal{M}} V^n(M^n | \mathbf{M}_*^{-n})$$

Before stating the existence result and sketch its proof, it is important to discuss some features of this model. First, Principal and Agents are matched randomly. There is hence no competition between Principals to obtain a better Agent. Second, Firms act as price takers at the trading stage, but they act strategically when designing contracts. This is similar to Spence (1977), and other models, in which firms choose capacity strategically but behave as price takers in the following market stage. An alternative would be to have an oligopolistic market at the second stage, as in Dixit (1980) and others, but this does not seem to reflect the size of financial markets. Thirdly, the use of IARA⁵ utility function is dictated by the necessity of having an expression for the asset market equilibrium simple enough to make the screening problem tractable.⁶ This choice is also non standard but, as I will discuss late in section , using these preferences actually stacks the card *against* the main results.

3.2 Existence

Theorem 1. *There is an equilibrium as defined in Section 3.1 .*

The proof, in the appendix, follows the traditional pattern of using the maximum theorem to guarantee that individuals' best responses are well behaved enough to apply a fixed point theorem. However, some extra care is needed to insure the continuity necessary to apply the maximum theorem. For Principals' payoffs to be continuous in their own strategies (menus), it has then to be the case that (i) the contracts chosen by Agents change continuously with the menus offered ($c^n(\mathbf{M}, \mathbf{t})$ is continuous in \mathbf{M}^N and (ii) the Equilibrium correspondence of the asset market stage is a continuous function of the prevailing contracts. The first is achieved by noting that the economy has the same equilibria if we restrict Principals to offering Incentive Compatible Menus. The second point is trickier, as the Walrasian Equilibrium correspondence is in general neither continuous, nor single-valued.

However, the assumptions on preferences and the existence of a riskless asset assure that,

⁵ Increasing Absolute Risk Aversion

⁶ The classical CARA-Gaussian framework could not be used in this context, because the Principal's objective function at the contracting stage would have been a mixture of normals making the standard "Certainty Equivalent" procedure impossible. This would have precluded a tractable solution of the screening problem (as Principals are risk averse we cannot proceed like in Jullien e al. (2007)). On the other hand Markowitz preferences on a more general domain would have posed problems for existence and consistency, as they are not expected utility preferences. See, for example, Dekel (1986) or Machina (1989).

for a given distribution of returns, the asset market equilibrium exhibits the properties of a Capital Asset Pricing Model (CAPM) equilibrium, which are

- **Portfolio Separation.** Every individual in the economy holds a fraction of the same portfolio of risky assets, the “Market Portfolio”. In this case, all traders have the same preferences, and will hence be holding the *same* fraction of the Market Portfolio. Differences in initial wealth are accounted for by a short or long position in the riskless asset.
- **“Beta” Pricing.** The price of a financial asset is a linear function of its expected payoff and the sum of its variance plus its correlation with the market portfolio, with the former becoming negligible, when many securities are available for trade.

Because of these two properties, all final holdings are uniquely determined as a continuous function of endowments. As endowments are determined by contracts, and final holdings determine utilities, the maximum theorem can be applied.

4 Markets and Contracts with Diversifiable Risk

Having established existence of equilibrium in the model, we can now turn to the relationship between financial markets and contracting.

As in the previous sections, I will here focus on the case in which both agents guarantee the same expected returns ($\mu_L = \mu_H$), but different variance ($\sigma_L^2 < \sigma_H^2$). This assumption is generalized in Section 9.4 to include familiar cases such as type L guaranteeing a higher mean, or type H having a higher probability of default).

In this section I will assume Units’ returns to be uncorrelated, which implies that, with a large number of Units in the

economy, risk can be entirely diversified away. In Section 6 I will discuss the implications of systemic risk.

Assumption 1. $\forall m, n \leq N$, $X(t(n))$ and $X(t(m))$ are independently distributed.

4.1 Insurance

When Principals can diversify some of their risk through the asset markets, they will be facing a lower cost of taking more risk at the contracting stage. In this model, this implies Principals retain a larger share of the Unit’s return β . Conversely, Agents end up with less risk, when

markets are present, even though they have no direct access to diversification by means of trading.

Let $M^n(N)$ be the equilibrium menu in unit n , in an economy with N Units. $M(1)$ is hence the optimal menu of contracts for an economy with a single Unit.⁷

$$\mathbf{M}(N) = [M^n(N)]_{n=1}^N = [C_G^n(N) \ C_H^n(N)]_{n=1}^N = [(\alpha_L^n(N), \beta_L^n(N)) \ (\alpha_H^n(N), \beta_H^n(N))]_{n=1}^N$$

Lemma 1. *Under Assumptions 1, and 3,*

- $\beta_L^n(N) < 1, \beta_H^n(N) \leq 1$
- $\lim_{N \rightarrow \infty} \beta_L^n(N) = 1, \forall n, \forall t$
- $\exists \bar{N} s.t. \beta_H^n(N) = 1, \forall N \geq \bar{N}$

Lemma 1 establishes the convergence of any economy to full insurance for both types. As a consequence we can conclude that adding enough Units, and hence securities, creates diversification opportunities for Principals to “insure” Agents. Note that the contract for H types reaches full insurance in a finite market, whereas the L type contracts never fully insure the agent.

Theorem 2. *Under Assumptions 1 and 3, increasing the number of Units in an economy, eventually induces Principals to retain more risk at the contracting stage.*

$$\forall N, \exists \bar{N} : \beta_t^n(N') \geq \beta_t^n(N), \forall n, \forall t \in \{L, H\}, \forall N' \geq \bar{N}$$

This seemingly natural “Insurance Effect” is not a foregone conclusion. The previous theorem shows that Agents are always “insured” by markets in economies with a large number of units, with independent returns.

$$\lim_{N \rightarrow \infty} V^n \left(M^n | \tilde{\mathbf{M}}^{-n} \right) \rightarrow E_t [M^n]$$

The proof of Lemma 1 shows that the problem of a Principal exhibits enough regularity⁸ to ensure that the straightforward convergence of the utility function leads to convergence of the solution. it never actually reaching its limit at a finite size.

⁷ This is also the optimal menu in any economy when Principals do not trade.

⁸The proof in the appendix uses techniques, similar to those of in Maskin and Riley (1984) to show that if type Second Order Stochastic Dominance between types reduce the set of relevant constraints.

4.2 Risk and Welfare

The previous result highlights how markets change equilibrium contracts. Here I discuss the implications for the allocation of welfare and risk.

To do so, it is first necessary to describe the effect of markets in the symmetric information case. Similar conclusions to those of Theorem 2 apply to the first best contracts $\hat{C} = (\hat{\alpha}, \hat{\beta})$.

Lemma 2.

- $\hat{\beta}_t^n(N) < 1$
- $\lim_{N \rightarrow \infty} \hat{\beta}_t^n(N) = 1, \forall n, \forall t$

Theorem 3. *Increasing the number of Units in an economy, eventually induces Principals to retain more risk at the contracting stage.*

$$\forall N, \exists \bar{N} : \hat{\beta}_t^n(N') \geq \hat{\beta}_t^n(N), \forall n, \forall t \in \{G, B\}, \forall N' \geq \bar{N}$$

Both symmetric and asymmetric information economies converge to an identical equilibrium, in which both types are offered the same full insurance contract $(\bar{\alpha}, 1)$ with $\bar{\alpha}$ such that $u(-\bar{\alpha}) = \bar{u}$. However, there is an important difference. Under asymmetric information, the contract of type H is the full insurance contract already for a finite N .

Type L values a marginal increase of his fraction of the Unit $(1 - \beta)$, more than type H does, and Agents' obtain the same utility only for full insurance contracts, consisting only of a cash transfer $((\alpha, 1))$. For any given contract with $\beta < 1$, L obtains a strictly larger utility. C_H guarantees H a utility of \bar{u} . The positive difference $\Delta u = E[u(-\alpha_H + (1 - \beta_H) X_L)] - \bar{u}$ is the "information rent" of type L . C_L has to guarantee type L at least $\bar{u} + \Delta u$, otherwise it will not be chosen over C_H . This contract leaves the principal worse off than the first best contract \hat{C}_L , which would have pushed L to his reservation utility.

To avoid giving up too much utility when the Agent is of type L , the Principal distorts C_H to make it less desirable for type L , while yielding the same utility to type H . Distortion takes the form of overinsuring type H , by giving him more cash and less of the Unit's return. However, distortion also reduces Principal's utility.

The optimal contract strikes the right balance between two extremes. On one hand, a "Rent Menu" of contracts, with minimum distortion for Agent H and maximum rent for Agent L ; On the other hand a "Distortion' Menu" under which H is fully insured, and L is pushed to his reservation utility. Because markets change the cost of bearing risk for the principal, they also make "Distortion" less costly than "Rent" moving the equilibrium contract closer to the

former, ultimately nullifying type L information rent.

As the discussion above suggests, equilibrium contracts are in general different from the symmetric information ones, and have different welfare properties.

To distort the allocation of type H , who will be insured more than he would be at the optimum, Principals take on more risk than they would if they did not have to take into account incentive compatibility. Because this risk is then poured onto the market, we can conclude that asymmetric information within units generates excess aggregate risk on the security market.

Proposition 1. *In a large market, ex ante Aggregate Risk is higher than in the symmetric information economy.*⁹

$$E \left(\sum_{i=1}^N (\beta_{t(n)}^n)^2 \sigma_{t(n)}^2 \right) \geq E \left(\sum_{i=1}^N (\hat{\beta}_{t(n)}^n)^2 \sigma_{t(n)}^2 \right)$$

To achieve this result it is also necessary to prove the following Lemma¹⁰

Lemma 3. *In a large market, asymmetric information induces H (L) agents to carry less (more) risk.*

$$\beta_H^i > \hat{\beta}_H^i$$

$$\beta_L^i < \hat{\beta}_L^i$$

The first part follows from the need to screen H types by overinsuring them. The second is due to the fact that part of the informational distortion on the type H contracts reverberates to the type L contracts through the market portfolio every Principal holds in equilibrium, because Principals are risk averse.

Having established the direction of all transfers of risk, I can now assess the effects of the size of markets on this problem. Lemma 1 (and 5 for the general case) show that the information structure change the dynamics of convergence for the contract of type H . β_H reaches 1 (full insurance) at a finite number of units \bar{N} , whereas in the first best economy none of the two types is ever fully insured.

Using the distortion with respect to the symmetric information contracts $\Delta_t(N) = |\beta_t(N) - \hat{\beta}_t(N)|$ as a measure of the effects of asymmetric information, the following result shows that financial markets mitigate the effects of information asymmetries within Units.

⁹ The result follows from Theorem 4.3 (which is stated in the Appendix for the more general case).

¹⁰ Corresponding to Theorem 4.1 and 4.2, also in the appendix.

Proposition 2. *Increasing the number of units in an economy, makes contracts closer to the symmetric information ones.*

$$\forall N, \exists \bar{N} : \forall N' \geq \bar{N}, \Delta_t(N') < \Delta_t(N)$$

This result relies on the convergence of contracts to full insurance for all agents.

Diversification opportunities make Principals' behave as if they were less risk averse. As a result they can better exploit Agents' own risk aversion to screen them. It is hence possible for a Principal to achieve a more advantageous outcome, by offering a menu closer to the first best. This need not coincide with a Pareto Improvement. In fact, while Principals are naturally made better off by the extra opportunities provided by markets, and Agents of type H see no change in their utility level (\bar{u}), and Agents' of type L can be made worse off, as the next result shows.

We have seen that L Agents enjoy a non negative rent Δu . In fact there can be no rents in a large enough market.

Corollary 1. *With large enough markets, no agent enjoys an information rent.*

$$\exists \bar{N} : \forall N \geq \bar{N}, U(C_t(N), t) = \bar{u}, \forall t$$

To optimally screen types, a Principal faces a trade off between the loss of utility due to the distortion of the contract of type H , and the loss of utility in favor of type L (the information rent). In a large market where also first best contracts get approximately close to $(\bar{\alpha}, 1)$, the cost of distorting is very small, and the Principal will optimally offer a full insurance contract to type H ($\beta_H = 1$). This contract leaves both types at their reservation utility, so that the Incentive Compatibility constraint of type L is binding exactly where his Individual Rationality constraint binds, and no rents are possible in equilibrium.

Markets often provide an efficient mechanism for resource allocation. Even though informational asymmetries change this, markets usually provide (weak) Pareto improvements over the lack of a mechanism. This is not the case here, as access to markets is not universal, and access to trading opportunities affects other relationships in ways which are detrimental to some individuals' welfare. Besides the theoretical point, this result hints at a new direction for empirical research on securitization and risk sharing across banks.

5 An Extension: Pool Selection

Suppose now that Principals can employ their technology in R , a project which does not require an Agent and gives a certain net profit of \bar{R} . If a Principal is matched with an Agent who will not accept any of the contracts he is offered, she can invest in R instead. In this case a Principal is facing two further alternatives besides offering the same menu of contracts as described in the previous section: she can decide simply to invest in R , or she can decide to offer a contract that will be accepted only by the low risk type L and invest in R if matched with a high risk type. In this last case the optimal contract to offer is the first best contract giving L his reservation utility.

Note that this might be the Principal's optimal choice even if the expected payoff of R is lower than the payoff from the second best contract for type H . It is sufficient that this difference is small enough to be offset by the loss coming from the information distortions (variables with one star * correspond to first best contracts and those with two ** correspond to second best contracts):

$$pEU(\alpha_L^* + \beta_L^* X_L) + (1-p)EU(\bar{R}) > pEU(\alpha_L^{**} + \beta_L^{**} X_L) + (1-p)EU(\alpha_H^{**} + \beta_H^{**} X_H) \quad (2)$$

If this is the case, high risk agents H will not be contracted. However, markets change the picture because diversification will make the risky project operated by agent H more attractive without affecting preferences for R . In fact, if the payoff from R is smaller than the equilibrium price of the H project in a large market, a Principal in a large market will want to offer the screening menu and hire both types.¹¹

$$\bar{R} < \bar{\alpha} + \mu \quad (3)$$

It is then the case that markets can change the pool of projects undertaken in the economy

Proposition 3. *When R satisfies both condition 2 and 3,¹² Principals in isolation will only contract L agents, whereas for a sufficiently large number of units, all agents will be contracted in equilibrium.*

Proof. To prove Proposition 3 we need to show that as markets get larger, H types will be

¹¹ $\bar{\alpha}$ is defined in Section 4.2. $-\bar{\alpha}$ is the transfer giving an Agent his reservation utility when the contract does not include any profit sharing ($1 - \beta = 0$).

¹² Given the model's assumptions the two conditions are never exclusive.

eventually contracted. We need to show that

$$\exists N : \forall M \geq N, EU \left(-\alpha_H(M) + (1 - \beta_H(M)) \mu_H - \frac{b}{M} (1 - \beta_H(M))^2 \sigma_H^2 \right) \geq \bar{u}$$

By Assumption 3 it has to be that

$$V(\bar{\alpha}_H) > V(\bar{R})$$

But by continuity there must be an N such that for all M greater than N .

$$V \left(\alpha_H(M) + \beta_H(M) \mu_H - \frac{b}{M} (\beta_H(M))^2 \sigma_H^2 | M \right) > V(\bar{R} | M) \quad (4)$$

As we know from Proposition 2, in large markets the first and second best contracts converge for both types, and eventually coincide for type L (by Corollary 2. This allows us to conclude that the following inequality holding for some large enough M , so that the Principal will also offer a contract accepted by type H agents.

$$pV \left(\alpha_L^{**}(M) + \beta_L^{**}(M) \mu_L - \frac{b}{M} (\beta_L^{**}(M))^2 \sigma_L^2 | M \right) + (1 - p) V \left(\alpha_H^{**}(M) + \beta_H^{**}(M) \mu_H - \frac{b}{M} (\beta_H^{**}(M))^2 \sigma_H^2 | M \right) > \\ pV \left(\alpha_L^*(M) + \beta_L^* \mu_L - \frac{b}{M} (\beta_L^*(M))^2 \sigma_L^2 | M \right) + (1 - p) V(\bar{R} | M)$$

□

This extension of the model suggests an explanation of the evidence showing that the boom of securitization contributed to expanding the pool of mortgages to include riskier ones.

Under the same conditions, all results from previous sections apply. Their interpretation is however slightly different, because the non-market equilibrium does not include the H types, and does not exhibit. Introducing a large market in this case does not change the welfare of the L Agents, because here they obtain their reservation utility also in absence of markets, because Principals do not need to screen them from the H types. Examples can be generated showing that the welfare of L Agents is first increasing and then decreasing in market size. In smaller finite markets, H types will be contracted, but will not be fully insured. In these economies L types obtain an information rent as soon as diversification makes H types desirable, which

eventually vanishes, as in Corollary 2.

6 Markets without Diversification

One of the key motives for trading on financial markets is the pooling and diversification of risk. This is not necessarily the case if risks are highly or perfectly perfectly correlated. It should not come as a surprise that a certain degree of independence and a certain number of Units are needed for Principals to be able to diversify away enough risk to act as if they were less risk averse. What is less expected is that enough positive correlation can generate the “opposite” effect to that of Lemma 1, as the following example shows.

Considering an economy with two Units, and the following parameters.

(μ_L, σ_L^2)	(μ_H, σ_H^2)	r	\bar{u}	ρ
$(3, 0.1)$	$(3, 1)$	0.1	0.3	0.9

The equilibrium contracts without markets will be

$$-\alpha_L + (1 - \beta_L) X = -1.4068 + 0.5711X$$

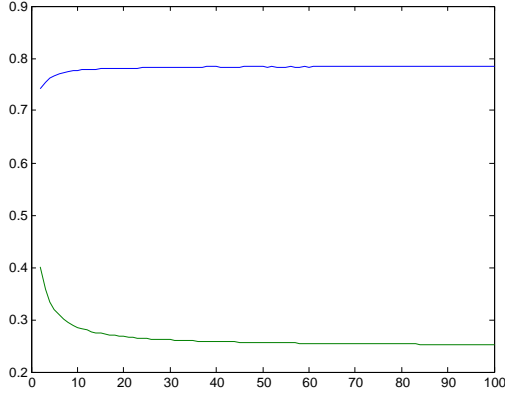
$$-\alpha_H + (1 - \beta_H) X = -1.3866 + 0.5693X$$

Those with markets will be

$$-\alpha_L^M + (1 - \beta_L^M) X = -1.4451 + 0.5838X$$

$$-\alpha_H^M + (1 - \beta_H^M) X = -1.1569 + 0.4913X$$

This economy is compatible with the conclusion of Lemma 1, which is a statement about large and finite economies. One would expect that adding enough units is enough to restore the expected insurance effect, by bringing about more diversification. However, it need not be the case. The graph shows in blue (below) $1 - \beta_H$, and in red (above) $1 - \beta_L$ as functions of N . The sensitivity to returns and hence the variance of the contract of Agent L actually increases with the size of markets.



Why does increasing markets make the variance of contract L higher? The existence of non diversifiable risk, modeled here as positive correlation ρ between Units returns, creates a strong incentive to issue securities which get as close as possible to hedging returns when the economy does bad.

To understand the mechanism at work, it is useful to take the “hedging” idea to the extreme and think in terms of underlying states. Units’ returns are binary $X_L \in \{\underline{x}_L, \bar{x}_L\}$ and $X_H \in \{\underline{x}_H, \bar{x}_H\}$, with $\mu_L = \mu_H$ and $\sigma_L^2 < \sigma_H^2$, so that $\underline{x}_H < \underline{x}_L < \bar{x}_L < \bar{x}_H$. Suppose ρ is equal to one, so that a space $S = \{\underline{s}, \bar{s}\}$ with 2 states is sufficient to represent the economy.

$$X_t(\underline{s}) = \underline{x}_t$$

$$X_t(\bar{s}) = \bar{x}_t$$

Whenever an H unit returns \underline{x}_H all other H units will do the same, and L units will return \underline{x}_L .

In absence of markets both Principals and Agents have the same risk attitude and the optimal contracts will strike some balance of risk sharing. Now suppose trading is possible. When Principals designs C_L she knows, that when she is matched with L she will then trade with some H and some L units, and end up holding a portfolio of these. She also knows, as a L unit, she has an advantage in providing returns in state \underline{s} , since $\underline{x}_H < \underline{x}_L$. Units L can hedge returns in \underline{s} and the markets provides them, through pricing, with the incentives to do so, by

making returns in that state particularly valuable. The cost she bears by giving less returns to Agent L in \underline{s} , in the form of a higher variable compensation $(1 - \beta_L)$ and a lower transfer $-\alpha_L$ is however not changed by markets. This is because Agents do not access markets, and hence their risk preferences are unaffected by trading opportunities. The optimal contract will reflect this and give less return to L in state \underline{s} and more in state \bar{s} than it would be the case in absence of trading opportunities. This translates in more risk in the hands of type L Agents.

The extent of this effect hence depends on the difference between $\underline{x}_H < \underline{x}_L$ and of course on the level of correlation ρ .

In the language of the previous sections, we can observe that the mechanism described above transfers risk from type H to type L (on top of the -smaller- transfer caused by distortion described in Lemma 3), while correlation reduces diversification opportunities to the point of undoing the insurance effect of Theorem 2. It is worth noting that in this numerical example, the low risk types are once again worse off in a large market than they are in isolation, but for a different reason than in Corollary 2 of Section 4.2. Here the cost of distortion is not lowered by markets as in the case of independent returns, but the first best contracts coefficients β_L and β_H are more distant in a large market than in isolated units, requiring less (or no) distortion to make C_H less desirable to type L . This in turn reduces (or nullifies) the need for giving him an information rent in the optimal screening contracts. In fact, examples can be constructed in which the first best contracts will coincide with the screening contracts for a finite number of units N .

7 Generalizing Results

The results from the previous two sections are obtained in the context of large markets and under more stringent assumptions than necessary, namely:

1. There is no systemic risk as all Units return are independently distributed.
2. Types L and H differ only by the variance of the profits they generate.

Under the assumption of independence (as in Lemma 1) Principals act as if they were less risk averse as N got large. In general this need not be true, because correlation across Units generates non diversifiable risk and, as shown in the previous example, a high level of correlation can change the effects of markets on contracts, undoing some of the results. However we can conclude that for sufficiently small levels of correlation the insurance effect still holds.

We can also relax the second assumption by allowing for a more general pair of types. In

fact the results of Section 4 will also apply when pairs are ranked by Second Order Stochastic Dominance. That is, as long as any expected utility maximizer prefers holding some claim on type L than some claim on type H :

Assumption 2. L Second Order Stochastically Dominates H if and only if

$$E(u(\alpha + \beta X_L)) \geq E(u(\alpha + \beta X_H))$$

$$\forall \alpha \in \mathbf{R}, \beta \in [0, 1], u(\cdot)$$

where $u(\cdot)$ is strictly increasing and concave

This intuitive condition includes the case of different variance already discussed, the case in which both types generate the same variance of returns, but L provides higher mean returns $\mu_L > \mu_H$, and the case in which they generate the same two outcomes, but L has a higher probability of the best outcome.

We can generalize all results of Section 4.1 and 4.2¹³ to economies where the following conditions hold:

1. There is an appropriately *bounded* amount of systemic risk.
2. Types L Second Order Stochastically Dominates H .

The following result is the equivalent of Lemma 1:

Proposition 4. *Suppose Assumption 2 holds, then there is a $\bar{\rho} > 0$, such that, if $\rho_{st}^{mn} < \bar{\rho}$ for all m, n, s, t , increasing the number of Units in an economy, eventually induces Principals to retain more risk at the contracting stage.*

$$\forall N, \exists \bar{N} : \beta_t^n(N') \geq \beta_t^n(N), \forall n, \forall t \in \{G, B\}, \forall N' \geq \bar{N}$$

Furthermore, $\exists \bar{N}$ s.t. $\beta_H^n(N) = 1, \forall N \geq \bar{N}$

Even though the contract of L does not converge to full insurance, all results from Section 4.2 also apply. In particular, the fact that the contract H will give him full insurance (Proposition 4) implies that no Agent will obtain more than their reservation utility (Corollary 2), as was the case without aggregate risk.

¹³Note the result on the pool extension from Section 5 may not carry over to the more general assumption on types because the outside option and type H cannot be ranked according to SSD. They do apply however in the case most relevant to securitization, that of different default probabilities. Furthermore I conjecture that allowing for a more complex outside option can replicate the result for any pair of types satisfying SSD.

8 Conclusions

To analyze how financial markets interacts with contracting in presence of asymmetric information which is dealt with outside markets. To do so I construct a model integrating Principal-Agent interaction with hidden type in an asset markets. A Unit is formed by a Principal and an Agent. Each pair produces random returns, whose distribution is known only to the agent at the contracting stage. Unlike in a standard contracting model is that Principals have access to an asset market on which they trade their shares of returns, and a riskless asset.

I present a general framework and define a notion of equilibrium, for which I prove existence. With the added structure of a low correlation and an ordering on types, I study the interactions of financial markets and contracts. The existence of markets, can have an *Insurance Effect*, inducing less risky compensation for agents, and having important welfare implications.

Asymmetric information within Units induces excessive aggregate risk in markets. The amount of extra risk is small when markets are large. But, while large markets increase the utility of Principals who access them, they will push all Agents to their reservation utility, including those who enjoyed an information rent when Units operated “in isolation” or in small markets. Introducing markets in these economies does not result in a Pareto Improvement, and it is the “better” Agents who will be worse off. This result suggests new testable implications on the effects of securitization.

Although this is not the focus on the paper, I also show how an extension of the model accomodates the empirical evidence on the expansion of the mortgage pool caused by the increase of securitization.

The seemingly natural “insurance effect” need not obtain, and how a high level of non diversifiable risk, might induce the opposite effect for some types of Agents. For contracts to exhibit less variance, three ingredients must be present in the economy. A large number of traders, a limited amount of systemic risk, and types ordered according to Second Order Stochastic Dominance. As soon as one of these assumptions is dropped, counterexamples can be constructed. It is a relevant empirical question whether systemic risk can be large enough to upset the “insurance effect”, and a theoretical question to study the implications besides the example provided in the paper.

It is worth highlighting two special features of this model to discuss its limitations.

First is the choice of a quadratic utility function, which exhibits Increasing Absolute Risk Aversion utility. This functional form was chosen mainly for two reasons. It yields a CAPM equilibrium and makes the screening problem somewhat tractable. Most importantly Increasing

Risk Aversion stacks the card *against* the results: Markets allow diversification, but they also increase the utility level of principals, making them *more* risk averse. I show that this countervailing effect is eventually overcome by a large enough market. Using a DARA utility specification would make the results even stronger.

Secondly, the matching between Principals and Agents is modeled as an exogenous process. Endogenizing the formation of Principal-Agent pairs seems an interesting venue for future research, given the welfare effects asset markets have in this model suggests the matching and financial sides might exhibit interactions.

9 Appendix

In this appendix the reader can look the hood of the model, and see the details of how financial markets change the Principal's problem. After having established the necessary notation, I will prove existence of the model, and finally generalize the Insurance effect of Sections 4 and 6 to a more general type space.

9.1 Mean-Variance and CAPM

As previously noted, the assumptions on preferences and availability of a riskless asset, pay off in terms of tractability. Let us first note that quadratic expected utility preferences on random variables can be equivalently represented as preferences over mean and variance of random variables.

$$\begin{aligned} E[u(X)] &= \\ E[X] - \frac{b}{2}E[X^2] &= \\ E[X] - \frac{b}{2}E[X]^2 - \frac{b}{2}Var[X] \end{aligned}$$

I will call the Mean-Variance representation $F(\mu_X, \sigma_X^2)$, where the first argument is the mean of X and the second argument its variance.

Linear contracts also allow a handy representation in terms of Mean and Variance.

$$\begin{aligned}\mu_{\alpha+\beta X} &= \alpha + \beta\mu_X \\ \sigma_{\alpha+\beta X}^2 &= \beta^2\sigma_X^2\end{aligned}$$

These assumptions also imply the existence of a CAPM equilibrium in the asset market.¹⁴, so that the final holdings in equilibrium can be expressed analytically.

Every individual will hold the same risky portfolio, an equal fraction $\frac{1}{N}$ of the aggregate endowment, and will spend the rest on the riskless asset (or short it if their remaining endowment is negative). With this in mind the mean and variance of the portfolio held by the agent is readily computed as a function of contracts. For a general principal n we have that¹⁵

- The holding of riskless asset is $q^n - \frac{1}{N} \sum_{m=1}^N q^m$
- The mean of the risky portfolio is $\frac{1}{N} \sum_{m=1}^N (\alpha^m + \beta^m \mu^m)$
- The variance of the risky portfolio is $\frac{1}{N^2} \sum_{m=1}^N \left(\beta_m^2 \sigma^{m2} + \sum_{k \neq m} \rho^{mk} \beta^m \beta^k \sigma^m \sigma^k \right)$

Since $q^n = \alpha^n + \beta^n \mu^n - \frac{b}{N} \beta^{n2} \sigma^{n2} \sum_{m \neq n} \rho^{mn} \beta^m \beta^n \sigma^m \sigma^n$, we have that the mean of P^n 's holdings simplifies to

$$\alpha^n + \beta^n \mu^n - \frac{b}{N} \left(\beta^{n2} \sigma^{n2} + \sum_{m \neq n} \rho^{mn} \beta^n \beta^m \sigma^n \sigma^m \right) + \frac{b}{N^2} \sum_{m=1}^N \left(\beta^{m2} \sigma^{m2} + \sum_{k \neq m} \rho^{mk} \beta^m \beta^k \sigma^m \sigma^k \right)$$

and the variance is of course the variance of the risky part $\frac{1}{N^2} \sum_{m=1}^N \left(\beta_m^2 \sigma^{m2} + \sum_{k \neq m} \rho^{mk} \beta^m \beta^k \sigma^m \sigma^k \right)$

9.2 The Objective Function

If $V(\alpha^n, \beta^n) = F(\alpha^n + \beta^n \mu^n, \beta^n \sigma^{n2})$ is the utility a Principal obtains from contract α, β when no markets are available, markets will change this into

¹⁴See the proof of Lemma 4

¹⁵Note that the covariance between two assets is

$$Cov(X_s^m, X_t^n) = \rho_{s,t}^{m,n} \sigma_s^m \sigma_t^n$$

$$\begin{aligned}
V_M(\alpha^n, \beta^n) &= \\
&F^M(\alpha^n + \beta^n \mu^n, \beta^{n2} \sigma^{n2}) = \\
F\left(\alpha^n + \beta^n \mu^n - \frac{b}{N} \left(\beta^{n2} \sigma^{n2} + \sum_{m \neq n} \rho^{nm} \beta^n \beta^m \sigma^n \sigma^m\right) + \frac{b}{N^2} \sum_{m \in N} \left(\beta^{m2} \sigma^{m2} + \sum_{k \neq m} \rho^{mk} \beta^m \beta^k \sigma^m \sigma^k\right) \right. \\
&\quad \left. , \frac{1}{N^2} \sum_{m \in N} \left(\beta^{m2} \sigma^{m2} + \sum_{k \neq m} \rho^{mk} \beta^m \beta^k \sigma^m \sigma^k\right) \right)
\end{aligned}$$

It is useful for the following proof to explicitly write the partial derivatives of V_M with respect

$$\begin{aligned}
\frac{\partial V_M}{\partial \alpha^n}(\alpha^n, \beta^n) &= F_{\mu}(\cdot, \cdot) \\
\frac{\partial V_M}{\partial \beta^n}(\alpha^n, \beta^n) &= F_{\mu^n}(\cdot, \cdot) \left[\mu^n - \frac{b}{N} \left(2\sigma^{n2} \beta^n + \sum_{m \neq n} \rho^{nm} \sigma^n \sigma^m \beta^m \right) + \frac{b}{N^2} \left(2\sigma^{n2} \beta^n + 2 \sum_{m \neq n} \rho^{nm} \sigma^n \sigma^m \beta^m \right) \right] \\
&\quad + F_{\sigma^2}(\cdot, \cdot) \left[\frac{1}{N^2} \left(2\sigma^{n2} \beta^n + 2 \sum_{m \neq n} \rho^{nm} \sigma^n \sigma^m \beta^m \right) \right]
\end{aligned}$$

When principals are allowed to trade their claims, they will act at the first stage, as if their utility functions were

$$\begin{aligned}
E_t V_M(\alpha_t^n, \beta_t^n) &= \\
E_t F^M(\alpha_t^n + \beta_t^n \mu_t^n, \beta_t^{n2} \sigma_t^{n2}) &= \\
E_t F\left(\alpha_t^n + \beta_t^n \mu_t^n - \frac{b}{N} \left(\beta_t^{n2} \sigma_t^{n2} + \sum_{m \neq n} \rho_{t(m)}^{nm} \beta_t^n \beta_t^m \sigma_t^n \sigma_t^m\right) \right. \\
&\quad \left. + \frac{b}{N^2} \sum_{m \in N} \left(\beta_{t(m)}^{m2} \sigma_{t(m)}^{m2} + \sum_{k \neq m} \rho_{t(m)(k)}^{mk} \beta_{t(m)}^m \beta_{t(k)}^k \sigma_{t(m)}^m \sigma_{t(k)}^k\right) \right. \\
&\quad \left. , \frac{1}{N^2} \sum_{m \in N} \left(\beta_{t(m)}^{m2} \sigma_{t(m)}^{m2} + \sum_{k \neq m} \rho_{t(m)(k)}^{mk} \beta_{t(m)}^m \beta_{t(k)}^k \sigma_{t(m)}^m \sigma_{t(k)}^k\right) \right)
\end{aligned}$$

9.3 Proof of Theorem 1

Proof. I am going to use a well known fixed point result by Glicksberg (1952) to show that there is an equilibrium in the first stage of the game, given that the asset market develops as predicted by the CAPM model.

I need to show that

1. The strategy space of each Principal $\Delta(\mathcal{M}^n)$ is a convex, compact subset of a locally convex Hausdorff space.
2. The best response correspondence of all principals is upper hemi-continuous, convex valued, and nonempty.

For the first part note that the space of Incentive Compatible menus \mathcal{M}^n is a subset of a Euclidean space. It is closed because it is defined by a finite number of weak inequalities, and it is bounded because it is included in the larger set of feasible contracts. To make it compact, we need to show it is bounded. β is bounded between 0 and 1. α is bounded above by $\bar{\alpha}$ such that $EU(-\bar{\alpha} + X_L)$. It is also bounded below by $\underline{\alpha}$ such that $EU(-\underline{\alpha}) = \bar{u}$. Since it will be the case that $\beta_H \geq \beta_L$ and this implies that $\alpha_H \leq \alpha_L$. Since α_L has to be larger than $\underline{\alpha}$, these bounds will be satisfied.

The space of lotteries (identified with Borel probability measures) over these Menus is of course convex. It is also compact with respect to the weak* topology. This space of probabilities is a subset of the space of continuous functions $\mathcal{C}(\mathcal{M}^n)$, which is locally convex (and Hausdorff) with respect to the weak* topology.¹⁶

For the second part, convexity of the best response correspondence follows from preferences on random variables being represented by expected utility. I will use Berge's Maximum theorem to show that it is non empty, compact-valued and upper hemi-continuous.

To apply the maximum theorem to individuals' best response, it has to be that constraints vary continuously with other principals' strategies, and that the payoff function is continuous in one's own actions.

First note how the constraints correspondence is constant with respect to other principals strategies, and is therefore continuous. Also note how the constraints correspondence maps to the space of Borel probability measures on menus, which is a Hausdorff space as noted above.

We also need to make sure that the payoff function of a principal is continuous in menus.

To do this we need to show that

¹⁶For a treatment of these and other results on weak topologies, and also to see the theorems of Berge and Glicksberg, see Aliprantis, Border (2005)

1. Payoffs at the market stage are a continuous function of the contracts chosen by agents.
2. The contracts chosen by agents are a continuous function of the menus offered.

Claim 1 By Lemma 4, if the preferences are monotonic for (μ, Ω) , they are going to be monotonic for the asset markets resulting from all possible contracts C . Under the present assumptions a CAPM equilibrium exists once contracts are chosen.¹⁷ Because in equilibrium the price of a security can be expressed as $q^n = \alpha^n + \beta^n \mu^n - \frac{b}{N} (\beta^{n2} \sigma^{n2} + \sum_{m \neq n} \rho^{mn} \beta^m \beta^n \sigma^m \sigma^n)$ The indirect utility from a contract profile in the CAPM function is continuous in contracts.

Claim 2 Without loss of generality, we can restrict attention to Incentive Compatible menus, from the set \mathcal{M}_{IC}^n . If a principal makes a small change to the menu he offers while remaining in this set, every type of agent t , will still find it optimal to pick the contract intended for him, C_t . Hence any small change, will correspond to a small change in the contract picked by each type of agent.

We can conclude that the indirect utility for a principal facing type t is a continuous function of the menus offered.

Taking expectation with respect to \mathbf{F} over these indirect utilities yields a continuous functional on the domain of lotteries on IC menus $\Delta(\mathcal{M}_{IC}^n)$.

By the maximum theorem the best response correspondence of each player is now UHC and compact valued, which implies that the game best response is as well.

By Glicksberg's theorem there is a fixed point, which is an equilibrium by construction. \square

9.4 The Type Space

Moving to Section 4 we saw that the results from the leading example extend to a more general setting, in which type can differ by more than the variance of the returns they generate. As long as the project operated by type L second order stochastically dominates H , all results go through. This is because Second Order Stochastic Dominance implies the following adaptation of Single Crossing Property, which is used in the proofs to reduce the set of relevant constraints.

Definition 1. *We say Types L and H satisfy condition R if and only if the following holds:*¹⁸

¹⁷In the literature briefly reviewed by Nielsen (1990), one can find many sufficient conditions for the existence of a CAPM equilibrium, most of them deal with the possibility of satiation of preferences. Things are particularly simple when returns are bounded (which includes this model): monotonicity and local non satiation are guaranteed by a low enough risk aversion.

¹⁸The condition amounts to Single Crossing Property in the space of the two components of contracts. Restricting attention to quadratic preferences yields a simpler definition and the same results.

$$\frac{\partial E[u(\alpha + \beta X_L)]}{\partial \beta} > \frac{\partial E[u(\alpha + \beta X_H)]}{\partial \beta}, \forall \alpha, \beta \quad (\text{R})$$

Before discussing the implications of this property, I will prove it is implied by Second Order Dominance.

Proposition 5. *Second Order Stochastic Dominance of X_L over X_H implies Condition R.*

Proof. Saying that X_L SSD X_H amounts to saying that X_L is preferred by any risk averse decision maker: $E[u(X_L)] \geq E[u(X_H)]$ for any concave u .

Since rescaling and shifting preserve convexity/concavity, this implies that $E[u(\alpha + \beta X_L)] \geq E[u(\alpha + \beta X_H)]$ for any concave u , any α and any β between 0 and 1.

In other words if, X_L SSD X_H implies $\alpha + \beta X_L$ SSD $\alpha + \beta X_H$

To reach the desired conclusion, I will now focus on the economies at hand, which exhibit quadratic preferences, strictly monotonic on the possible returns from projects.

In this context second order stochastic dominance of $\alpha + \beta X_L$ over $\alpha + \beta X_H$ translates to:

$$\alpha + \beta \mu_L - \frac{b}{2} (\alpha + \beta \mu_L)^2 - \frac{b}{2} \sigma_L^2 \geq \alpha + \beta \mu_H - \frac{b}{2} (\alpha + \beta \mu_H)^2 - \frac{b}{2} \sigma_H^2$$

Whereas condition R translates to

$$\mu_L (1 - b\alpha - b\beta \mu_L) - b\beta \sigma_L^2 \geq \mu_H (1 - b\alpha - b\beta \mu_H) - b\beta \sigma_H^2$$

First rearrange:

$$b \left\{ \alpha (\mu_L - \mu_H) + \beta \left[(\mu_L^2 - \mu_H^2) + (\sigma_L^2 - \sigma_H^2) \right] \right\} \leq \mu_L - \mu_H$$

Then divide¹⁹ by $\mu_L - \mu_H$ to obtain:

$$b \left\{ \alpha + \beta \left[\mu_L + \mu_H + \frac{\sigma_L^2 - \sigma_H^2}{\mu_L - \mu_H} \right] \right\} \leq 1$$

Note that since SSD implies $\sigma_L^2 \leq \sigma_H^2$, this is implied by:

¹⁹The difference cannot be negative, because of SSD, and when $\mu_L = \mu_H$, the claim is trivially satisfied.

$$b\{\alpha + \beta[\mu_L + \mu_H]\} \leq 1$$

Since α is bounded above and below, the condition is satisfied by the monotonicity assumption (??) allowing us to conclude that in these economies Second Order Stochastic Dominance is a sufficient condition “rank” types by Single Crossing Property. \square

Condition R simply requires that for any given contract (α, β) , a marginal change in share of profit yields more utility to type L than it does for type H .

Fact 1 substantiates the claim that R is a reasonable notion of L being better than H , as it encompasses a few interesting cases.

9.4.1 Some implications of Single Crossing Property

Fact 1. *With quadratic utility the following cases imply SCP:*

1. *Both types generate the same mean returns but L does so with lower variance. $\sigma_L^2 < \sigma_H^2$*
2. *Both types generate the same variance of returns, but L provides higher mean returns. $\mu_L > \mu_H$*
3. *Both types can generate the same two outcomes, but L has a higher probability of the best outcome.*

Fact 2. *In the case of quadratic utility, SCP amounts to*

$$\mu_L(1 - b(-\alpha) - b\mu_L(1 - \beta)) - b\sigma_L^2(1 - \beta) > \mu_H(1 - b(-\alpha) - b\mu_H(1 - \beta)) - b\sigma_H^2(1 - \beta)$$

The following two implications of SCP will be useful in the proof Theorem 2

Fact 3. $\forall (\alpha, \beta), U(-\alpha, 1 - \beta|G) > U(-\alpha, 1 - \beta|B)$

Fact 4. $\forall (\alpha, \beta), (\alpha', \beta') : \beta \leq \beta'$

$$U(-\alpha, (1 - \beta)|G) - U(-\alpha', (1 - \beta')|G) > U(-\alpha, (1 - \beta)|B) - U(-\alpha', (1 - \beta')|B)$$

Fact 5. *SCP implies that $\mu_L \geq \mu_B$*

Fact 6. *SCP implies that $\sigma_L \leq \sigma_B$*

The proof of Fact 1 follows.²⁰

Proof. 1. $\mu_L = \mu_H, \sigma_L^2 < \sigma_H^2$. As all identical parts cancel from the condition above, we are left with $-\sigma_L^2 > -\sigma_H^2$. Multiplying both sides by -1 concludes the proof of 1.

2. $\mu_L > \mu_H, \sigma_L^2 = \sigma_H^2$.

U_2 , the derivative with respect to $(1 - \beta)$ is given by

$$\mu + b\alpha\mu - b(1 - \beta)\mu^2 - b(1 - \beta)\sigma^2$$

we want to show that $U_2(\cdot|G) > U_2(\cdot|B)$, that is

$$\mu_L + b\alpha\mu_L - b(1 - \beta)\mu_L^2 - b(1 - \beta)\sigma^2 > \mu_B + b\alpha\mu_B - b(1 - \beta)\mu_B^2 - b(1 - \beta)\sigma^2$$

$$\mu_L(1 + b\alpha - b(1 - \beta)\mu_L) > \mu_B(1 + b\alpha - b(1 - \beta)\mu_B)$$

Since $b(1 - \beta)$ is smaller than 1, the claim is satisfied whenever $\mu_L > \mu_H$.

3. Without loss of generality suppose that the outcome can be either 0 or 1, and let p_L be the probability of success of an agent of type t with, and consider $p_L > p_H$. The returns from employing agent t , will have mean p_L and variance $p_L(1 - p_L)$. Again, let's start by proving that $U_2(\cdot|G) > U_2(\cdot|B)$. That is

$$p_L + b\alpha p_L - b(1 - \beta)p_L^2 - b(1 - \beta)p_L(1 - p_L) > p_B + b\alpha p_B - b(1 - \beta)p_B^2 - b(1 - \beta)p_H(1 - p_H)$$

which readily simplifies to

$$p_L + b\alpha p_L - b(1 - \beta)p_L > p_B + b\alpha p_B - b(1 - \beta)p_H$$

$$\text{and finally } (p_L - p_B)(1 + b\alpha - b(1 - \beta)) > 0.$$

This has to be true, as $p_L > p_B$ by assumption and the second term in brackets cannot be negative, otherwise individual utility $x - \frac{b}{2}x^2$ would be decreasing for $x = 1$, which violates the assumption of monotonicity of preferences.

□

For the following set of results this condition will be needed

Assumption 3. *Condition R is satisfied.*

9.5 Insurance

Lemma 4. *Under Assumption 3, there is a $\bar{\rho}$, such that, if $\rho < \bar{\rho}$*

- $\beta_L^n(N) < 1, \beta_H^n(N) \leq 1$
- $\lim_{N \rightarrow \infty} \beta_t^n(N) = 1, \forall n, \forall t$

²⁰ The other proofs are a matter of algebra and available upon request

- $\exists \bar{N}$ s.t. $\beta_H^n(N) = 1, \forall N \geq \bar{N}$

Proof. A generic Principal is solving the problem

$$\begin{aligned} & \max_{(\alpha_L, \beta_L), (\alpha_H, \beta_H)} E_t V^M(\alpha_t, \beta_t | t) \\ & \text{s.t. } U(-\alpha_t, 1 - \beta_t | t) \geq \bar{u}, \quad \forall t \in \{G, B\} \quad (\text{IR } t) \\ & \quad U(-\alpha_t, 1 - \beta_t | t) \geq U(-\alpha_{t'}, 1 - \beta_{t'} | t), \quad \forall t, t' \in \{G, B\} \quad (\text{IC } t \ t') \end{aligned}$$

SCP and IC constraints imply by usual arguments that yield that

$$(1 - \beta_L^*) \geq (1 - \beta_H^*) \quad (5)$$

Of course this implies that $\beta_L \leq \beta_H$

To attain the desired result, we have to solve for the contract of type G. To do this I show that the IC_{GB} constraint will always be binding, and that this is enough

The first thing to do is to reduce the set of relevant constraints.

Fact 3 implies that

$$U(-\alpha_L, 1 - \beta_L | G) \geq U(-\alpha_L, 1 - \beta_L | B)$$

This together with IR holding for type H and IC holding for type L implies that IR holds for type L .

In other words, Fact 3 together with

$$\begin{aligned} & U(-\alpha_H, 1 - \beta_H | B) \geq \bar{u} \\ & U(-\alpha_L, 1 - \beta_L | G) \geq U(-\alpha_H, 1 - \beta_H | G) \end{aligned}$$

implies that

$$U(-\alpha_L, 1 - \beta_L | G) \geq \bar{u}$$

We can hence solve the problem without worrying about any of the IR constraints except that of type H .

We can also infer that $IC_{G,B}$ will be binding at an optimum. Suppose that it were not binding,

$$U(-\alpha_L, 1 - \beta_L|G) > U(-\alpha_H, 1 - \beta_H|G)$$

Consider an alternative incentive scheme $\{\alpha'_L, \beta'_L\}_{t \in \{G,B\}}$, which gives a smaller fixed payment less transfer $-\alpha'_L < \alpha_L$ to type L . Because the IC constrain $IC_{G,B}$ is not binding, we are increasing the maximand while remaining in the admissible set of contracts, which contradicts the original scheme $\{\alpha_t, \beta_t\}_{t \in \{G,B\}}$ being an optimum.

This and SCP imply that $IC_{B,G}$ will not be binding.

This means that the only relevant constraint for determining the optimal β_L is in the form

$$U(-\alpha_L, 1 - \beta_L|G) = U(-\alpha_H, 1 - \beta_H|G)$$

I now have to solve a simpler problem

$$\begin{aligned} \max_{(\alpha_L, \beta_L), (\alpha_H, \beta_H)} E_{\mathbf{t}} V^M(\alpha_t, \beta_t|t) \\ s.t. \quad U(-\alpha_H, 1 - \beta_H|B) = \bar{u} \quad (\text{IR B}) \\ U(-\alpha_L, 1 - \beta_L|G) = U(-\alpha_G, 1 - \beta_G|G) \quad (\text{IC G, B}) \end{aligned}$$

The only first order conditions involving α_L and β_L are given by

$$\begin{aligned} \mathbf{F}(\mathbf{t}|t(n) = G) V_{\alpha_L} - \lambda_L U_{-\alpha_L} &= 0 \\ \mathbf{F}(\mathbf{t}|t(n) = G) V_{\beta_L} - \lambda_L U_{(1-\beta_L)} &= 0 \end{aligned}$$

Dividing both sides of each equation by $\mathbf{F}(\mathbf{t}|t(n) = G)$, and solving for $\frac{\lambda_L}{\mathbf{F}(\mathbf{t}|t(n) = G)}$ (where λ_L is the Lagrange Multiplier associated with $IC_{G,B}$ yields

$$\frac{V_{\alpha_L}}{U_{-\alpha_L}} = \frac{V_{\beta_L}}{U_{(1-\beta_L)}}$$

$$\frac{E_{\mathbf{t}|t(n)=G} \{F_{\mu}^M(\alpha_L + \beta_L \mu_L, \beta_L^2 \sigma_L^2)\}}{F_{\mu}(-\alpha_L + (1-\beta_L)\mu_L, (1-\beta_L)^2 \sigma_L^2)} = \frac{E_{\mathbf{t}|t(n)=G} \{(\mu_L - bR(N)) F_{\mu}^M(\alpha_L + \beta_L \mu_L, \beta_L^2 \sigma_L^2) + bS(N)\}}{\mu_L F_{\mu}(-\alpha_L + (1-\beta_L)\mu_L, (1-\beta_L)^2 \sigma_L^2) - b(1-\beta_L)\sigma_L^2}$$

where

$$R(N) = \frac{1}{N} \left(2\sigma^{n^2} \beta^n + \sum_{m \neq n} \rho^{nm} \sigma^n \sigma^m \beta^m \right) - \frac{1}{N^2} \left(2\sigma^{n^2} \beta^n + 2 \sum_{m \neq n} \rho^{nm} \sigma^n \sigma^m \beta^m \right)$$

$$S(N) = \frac{1}{N^2} \left(2\sigma^{n^2} \beta^n + 2 \sum_{m \neq n} \rho^{nm} \sigma^n \sigma^m \beta^m \right)$$

Note that

$$\lim_{N \rightarrow \infty} R(N) = [q\beta_L \sigma_L + (1-q)\beta_H \sigma_H] \sigma_t$$

$$\lim_{N \rightarrow \infty} S(N) = 0$$

Solving above for β_L we have

$$\beta_L(N) = \left[2\sigma_L^2 F_{\sigma^2} F_{\mu}^M - F_{\mu}(-\alpha_L + (1-\beta_L)\mu_L, (1-\beta_L)^2 \sigma_L^2) bR(N) + F_{\mu}(-\alpha_L + (1-\beta_L)\mu_L, (1-\beta_L)^2 \sigma_L^2) F_{\mu}^M(\alpha_L + \beta_L \mu_L, \beta_L^2 \sigma_L^2) bS(N) \right] / 2\sigma_L^2 F_{\sigma^2} F_{\mu}^M(\alpha_L + \beta_L \mu_L, \beta_L^2 \sigma_L^2)$$

Since $F_{\sigma^2} = -\frac{b}{2}$, this can be written as

$$\beta_L(N) = \left[\sigma_L^2 F_\mu^M - F_\mu(-\alpha_L + (1 - \beta_L)\mu_L, (1 - \beta_L)^2 \sigma_L^2) R(N) + \right. \quad (6)$$

$$\left. - F_\mu(-\alpha_L + (1 - \beta_L)\mu_L, (1 - \beta_L)^2 \sigma_L^2) F_\mu^M(\alpha_L + \beta_L\mu_L, \beta_L^2 \sigma_L^2) S(N) \right] / \quad (7)$$

$$\sigma_L^2 F_\mu^M(\alpha_L + \beta_L\mu_L, \beta_L^2 \sigma_L^2) \quad (8)$$

As N gets large, this converges to

$$1 - \frac{\lim_{N \rightarrow \infty} R(N) F_\mu(-\alpha_L + (1 - \beta_L)\mu_L, (1 - \beta_L)^2 \sigma_L^2)}{\sigma_L^2 F_\mu^M(\alpha_L + \beta_L\mu_L, \beta_L^2 \sigma_L^2)}$$

Letting $\rho_L \equiv \max_{m,n,s,t} \rho_{st}^{mn}$, and observing that F_μ is always strictly smaller than one, we can solve the following inequality to find a bound for ρ_L .

$$\beta_L^{NOMKT} < 1 - \frac{\rho_L \sigma_L \lim_{\sum m \neq G} \frac{\sigma_m}{N} F_\mu(-\alpha_L + (1 - \beta_L)\mu_L, (1 - \beta_L)^2 \sigma_L^2)}{\sigma_L^2 F_\mu^M(\alpha_L + \beta_L\mu_L, \beta_L^2 \sigma_L^2)}$$

Which is satisfied whenever $\rho_L < \frac{1 - \beta_L^{NOMKT}}{\sigma_L \lim_{\sum m \neq G} \frac{\sigma_m}{N}} = \bar{\rho}_L$. Whenever all correlations are bounded above, we have the desired result for type G.

To conclude that *every* contract is less risky, I need to rule out the case in which β_H^M goes from 1 to some value in $(\beta_L^M, 1)$, and then converges to 1, without hitting 1 in a finite time).

Consider the simplified optimization problem with markets described above. Consider the contract of type H . Let λ_L be the Lagrange multiplier of the Incentive Compatibility constraint of type L , $IC_{G,B}$ (type L prefers the contract designed for him over that designed for type H). The first order conditions determining the contract of type H are

$$\begin{aligned} & E_{\mathbf{t}|t(n)=B} \left\{ f(H) F_\mu^M(\alpha_H + \beta_H\mu_H, \beta_H^2 \sigma_H^2) \right\} + \\ & - \lambda_H F_\mu(-\alpha_H + (1 - \beta_H)\mu_H, (1 - \beta_H)^2 \sigma_H^2) + \\ & + \lambda_G F_\mu(-\alpha_H + (1 - \beta_H)\mu_L, (1 - \beta_H)^2 \sigma_G^2) = 0 \\ & E_{\mathbf{t}|t(n)=B} \left\{ f(H) \left[(\mu_H - R(N)) F_\mu^M(\alpha_H + \beta_H\mu_H, \beta_H^2 \sigma_H^2) + S(N) F_{\sigma^2}^M \right] \right\} \\ & - \lambda_H \left[\mu_H F_\mu(-\alpha_H + (1 - \beta_H)\mu_H, (1 - \beta_H)^2 \sigma_H^2) - b(1 - \beta_H) \sigma_H^2 \right] \\ & + \lambda_G \left[\mu_G F_\mu(-\alpha_H + (1 - \beta_H)\mu_L, (1 - \beta_H)^2 \sigma_G^2) - b(1 - \beta_H) \sigma_G^2 \right] = 0 \end{aligned}$$

Solving for λ_H we obtain

$$\begin{aligned}
& E_{\mathbf{t}|t(n)=B} \left\{ f(H) F_\mu^M (\alpha_H + \beta_H \mu_H, \beta_H^2 \sigma_H^2) \right\} \\
& + \lambda_G F_\mu (-\alpha_H + (1 - \beta_H) \mu_L, (1 - \beta_H)^2 \sigma_G^2) / \\
& F_\mu (-\alpha_H + (1 - \beta_H) \mu_H, (1 - \beta_H)^2 \sigma_H^2) = \\
& E_{\mathbf{t}|t(n)=B} \left\{ f(H) \left[(\mu_H - bR(N)) F_\mu^M (\alpha_H + \beta_H \mu_H, \beta_H^2 \sigma_H^2) + S(N) F_{\sigma^2}^M \right] \right\} \\
& + \lambda_G \left[\mu_G F_\mu (-\alpha_H + (1 - \beta_H) \mu_L, (1 - \beta_H)^2 \sigma_G^2) - b(1 - \beta_H) \sigma_G^2 \right] / \\
& \mu_H F_\mu (-\alpha_H + (1 - \beta_H) \mu_H, (1 - \beta_H)^2 \sigma_H^2) - b(1 - \beta_H) \sigma_H^2
\end{aligned}$$

which we can rewrite as

$$\begin{aligned}
& \left[\lambda_G F_\mu (-\alpha_H + (1 - \beta_H) \mu_H, (1 - \beta_H)^2 \sigma_H^2) F_\mu (-\alpha_H + (1 - \beta_H) \mu_G, (1 - \beta_H)^2 \sigma_G^2) (\mu_G - \mu_H) \right. \\
& \left. F_\mu (-\alpha_H + (1 - \beta_H) \mu_H, (1 - \beta_H)^2 \sigma_H^2) E_{\mathbf{t}|t(n)=B} \left\{ f(H) bR(N) F_\mu^M M (\alpha_H + \beta_H \mu_H, \beta_H^2 \sigma_H^2) \mu_H + f(H) S(N) F_{\sigma^2}^M \right\} \right] / (DEN)
\end{aligned}$$

where

$$\begin{aligned}
& DEN \equiv \\
& b \left[E_{\mathbf{t}|t(n)=B} \left\{ (1 - q) F_\mu^M (\alpha_H + \beta_H \mu_H, \beta_H^2 \sigma_H^2) \sigma_H^2 \right\} + \right. \\
& \left. \lambda_G (F_\mu (-\alpha_H + (1 - \beta_H) \mu_L, (1 - \beta_H)^2 \sigma_G^2) \sigma_H^2 - F_\mu (-\alpha_H + (1 - \beta_H) \mu_H, (1 - \beta_H)^2 \sigma_H^2) \sigma_G^2) \right] \geq 0
\end{aligned}$$

By Fact 5 The first addend in the numerator is positive. Since $F_{\sigma^2}^M$ is negative, the second addend is also positive, but it will go to zero in a large enough market, and β_H will converge to

$$\begin{aligned}
& 1 + \left[\lambda_G F_\mu (-\alpha_H + (1 - \beta_H) \mu_H, (1 - \beta_H)^2 \sigma_H^2) F_\mu (-\alpha_H + (1 - \beta_H) \mu_G, (1 - \beta_H)^2 \sigma_G^2) (\mu_G - \mu_H) \right. \\
& \left. - E_{\mathbf{t}|t(n)=B} \left\{ f(H) \mu_H R(N) F_\mu (-\alpha_H + (1 - \beta_H) \mu_H, (1 - \beta_H)^2 \sigma_H^2) F_\mu^M (\alpha_H + \beta_H \mu_H, \beta_H^2 \sigma_H^2) \right\} \right] / DEN
\end{aligned} \tag{9}$$

By putting an appropriate bound on correlation ρ_{st}^{nm} we can constrain $\lim_{N \rightarrow \infty} R(N)$, to the

point in which this solution is greater than 1, which cannot be. It has to be that $\beta_H(N) = 1$. Hence, there is a \bar{N} such that $\beta_H(N)$ is equal to 1, $\forall N \geq \bar{N}$ and type H will receive a contract with zero variance. If correlation is bounded above by this value (let it be called $\bar{\rho}$) all conclusions hold. This concludes the proof. □

The other results follow from the construction of the optimal contracts in the previous proof.

9.6 Risk and Welfare

First we need to establish a few facts about the symmetric information economy. The symmetric information equilibrium menus in the more general economy are

$$\hat{M}^n(N) = \left[\hat{C}_t^n(N) \right]_{t=1}^T = \left[\left(\hat{\alpha}_t(N), \hat{\beta}_t(N) \right) \right]_{t=1}^T$$

Lemma 5. *Under Assumption 3, there is a $\bar{\rho}$, such that, if $\rho_{st}^{mn} < \bar{\rho}$ for all m, n, s, t*

- $\hat{\beta}_t^n(N) < 1$
- $\lim_{N \rightarrow \infty} \hat{\beta}_t^n(N) = 1, \forall n, \forall t$

As noted the screening problem distorts the allocations of Units with type H Agents. Before drawing any conclusions I need to describe how this effect impacts the design of L contracts through the market portfolio, which each Principal holds in Equilibrium.

9.6.1 Best Response

The following technical Lemma is needed to prove results on aggregate risk. I need to verify that the overall effect of riskier “type H ” securities is not offset by the adjustments made in response (which will go in the direction of less risky securities).

First I need to establish a few derivatives

Fact 7.

$$\begin{aligned}
F_\mu^M &= 1 - b \left[(\alpha_i + \beta_i \mu) - \frac{b}{N} \left(\beta_i^2 \sigma_i^2 + \sum_{m \neq i} \rho \beta_i \beta_m \sigma_i \sigma_m \right) + \frac{b}{N^2} \sum_{m=1}^N \left(\beta_m^2 \sigma_m^2 + \sum_{n \neq m} \rho \beta_m \beta_n \sigma_m \sigma_n \right) \right] \\
\frac{\partial F_\mu^M}{\partial \beta_j} &= b^2 \frac{1}{N} \rho \beta_i \sigma_j \sigma_i - \frac{b^2}{N^2} \left(\beta_j \sigma_j^2 + \sum_{k \neq j} \rho \beta_k \sigma_j \sigma_k \right) \\
\frac{\partial R(N)}{\partial \beta_j} &= \frac{1}{N} \rho \sigma_i \sigma_j - \frac{1}{N^2} 2 \rho \sigma_i \sigma_j \\
\frac{\partial S(N)}{\partial \beta_j} &= \frac{1}{N^2} 2 \rho \sigma_i \sigma_j
\end{aligned}$$

Lemma 6. 1. *There is a bound $\bar{\rho}$ such that if all $\rho_{st}^{ij} < \bar{\rho}, \forall i, j, s, t$ then $\left| \frac{\partial \beta_i}{\partial \beta_j} \right| < \frac{1}{N}$*

2.

$$\forall i, j, \frac{\partial \beta_i}{\partial \beta_j} < 0$$

Proof. 1. I am going to use Equation 6 to put a bound on $\left| \frac{\partial \beta_i}{\partial \beta_j} \right|$. Note that Equation 6 is still an implicit function as β_i appears on the right hand side too, so that differentiating both sides we obtain

$$\frac{\partial \beta_i}{\partial \beta_j} = \frac{d\beta_i}{d\beta_j} + \frac{d\beta_i}{d\beta_i} \frac{\partial \beta_i}{\partial \beta_j}$$

However, rearranging shows that

$$\frac{\partial \beta_i}{\partial \beta_j} = \frac{\frac{d\beta_i}{d\beta_j}}{1 - \frac{d\beta_i}{d\beta_i}}$$

Because $\frac{d\beta_i}{d\beta_i}$ can only be negative, it is then sufficient to put a bound on $\frac{d\beta_i}{d\beta_j}$ to prove the desired result.

note we can rewrite 6 as

$$\beta_L = 1 - \frac{F_\mu E [R(N) + F_\mu^M S(N)]}{\sigma_L^2 E (F_\mu^M)} = 1 - \frac{F_\mu}{\sigma_L^2} \left[\frac{E (R(N))}{E (F_\mu^M)} - \frac{E [F_\mu^M S(N)]}{E (F_\mu^M)} \right]$$

Let $D_1 = \frac{E(R(N))}{E(F_\mu^M)}$ and $D_2 = \frac{E[F_\mu^M S(N)]}{E(F_\mu^M)}$.

We have that

$$\frac{d\beta_i}{d\beta_j} = -\frac{F_\mu}{\sigma_L^2} \left[\frac{dD_1}{d\beta_j} - \frac{dD_2}{d\beta_j} \right]$$

and

$$\begin{aligned} \frac{dD_1}{d\beta_j} &= \frac{E\left(\frac{\partial R(N)}{\partial \beta_j}\right) E(F_\mu^M) - E(R(N)) E(F_{\mu\beta_j}^M)}{[E(F_\mu^M)]^2} \\ \frac{dD_2}{d\beta_j} &= \frac{E\left[F_{\mu\beta_j}^M S(N) + F_\mu^M \frac{\partial S(N)}{\partial \beta_j}\right] E(F_\mu^M) - E(F_\mu^M) E(S(N)) E(F_{\mu\beta_j}^M)}{[E(F_\mu^M)]^2} \end{aligned}$$

Since the derivatives in Fact 7 all go to zero when N gets large, it must be the case that

$\left|\frac{d\beta_i}{d\beta_j}\right|$ also goes to zero, which in turn implies the same for $\left|\frac{\partial \beta_i}{\partial \beta_j}\right|$.

In the case in which all ρ 's are equal to zero, the claim follows trivially satisfied since $\frac{dD_2}{d\beta_j}$ is equal to zero, and $\frac{dD_1}{d\beta_j}$ is of the order of $\frac{1}{N^2}$

In the case in which ρ 's are greater than zero, I will need to put an upper bound to make sure that correlation does not affect the utility function in similar fashion to Section 6.

All of the addends in $\frac{dD_2}{d\beta_j}$ are of the order of $\frac{1}{N^2}$, so that we can focus on D_1 . Since both addends in $\frac{dD_1}{d\beta_j}$ are continuous, increasing (in absolute value) in ρ , and of order $\frac{1}{N^2}$, when ρ is equal to zero, we can conclude that there is a bound $\bar{\rho}$ below which $\frac{dD_1}{d\beta_j}$ will be smaller than $\frac{1}{N}$. This bound is given by the minimum between 1 and $\frac{E[F_\mu^M(\bar{\alpha}, 1)]}{F_\mu(-\bar{\alpha}, 0)}$

2. We have left to show that the sign of the derivative is negative. Since the derivative of D_2 is of the order of $\frac{1}{N^2}$ we only need to focus on 1, and show its derivative is positive. The first addend always is, the second depends on the sign of $F_{\mu\beta_j}^M$. Inspection shows that when N is large enough, this depends on the difference between σ_L and σ_H . Since SCP implies $\sigma_L < \sigma_H$ we can conclude that $F_{\mu\beta_j}^M$ is smaller than zero, and so is $\frac{\partial \beta_i}{\partial \beta_j}$.

□

9.6.2 Risk Transfer

Theorem 4. *For a large enough N private information implies*

1. *Higher insurance for bad types. (higher β_H than at first best)*
2. *Lower insurance for good agents (lower β_L)*

3. More aggregate risk on markets (higher variance of the market portfolio) (higher $E\left(\sum_{i=1}^N \beta_i \sigma_i\right)$).

Proof. The strategy of the proof is straightforward. All types H get full insurance (which is a distortion) after some finite N . This puts extra risk on the market. Principals take this into account when designing contracts. Corollary 2, H contracts are going to offer full insurance, even with changes in L contracts. The distortion will be compensated by the design of L contracts, which will put less risk on the market and more on the good agents.

1. We know from Corollary 2 that there is a finite N such that β_L is equal to 1, which is higher than what it would be at first best.

2. I now have to show the effects of distortion on other contracts. I will use for this the results from Lemma 6. The idea is that the change in H contracts generates a *smaller* change in L contracts.²¹ In general I will need to show that

$$\left| \sum \beta_i (\beta'_{-i}) - \beta_i (\beta_{-i}) \right| < \left| \sum \beta'_i - \beta_i \right|$$

and that the sequence of best responses starting from the first best contracts and converging to the second best contracts, generates a converging series such that the initial variation $(1 - q)(1 - \beta_H^*)N$ is only partially compensated by a smaller variation of opposite sign $q(\beta_L - \beta_L^*)N$, so that the total aggregate risk varies by a positive quantity $(1 - q)(1 - \beta_H^*)\sigma_H - q(\beta_L^* - \beta_L)\sigma_L$.

To show this we will exploit the structure of the economy. The symmetry of Principals, implies that there are only two optimal contracts C_L and C_H . Since we know that in a large market under asymmetric information β_H is equal to 1 regardless of the value of other contracts, we only need to worry about how the initial “informational shock” reverberates through β_L .

Let $s_H = (1 - \beta_H^*)$, the informational shock type L contracts, and $S_H = (1 - q)N(1 - \beta_H^*)$, the aggregate informational shock. Let β_L^0 be the “share” part of the equilibrium contract for type H under symmetric information. Let $C_L^{k+1} = (\alpha_L^{k+1}, \beta_L^{k+1})$ be the variable part of the optimal H contract for a principal when other Principals are offering $C_H = (\bar{\alpha}, 1)$ and $C_L = (\alpha_L^k, \beta_L^k)$.

Let $s_0 = \beta_L^1 - \beta_L^0$ and $s_{k+1} = \beta_L^{k+1} - \beta_L^k$, with their aggregate equivalents $S_0 = qN(\beta_L^1 - \beta_L^0)$

²¹ This is akin to the best response correspondence being a contraction, which suggests the possibility of a constructive proof of existence and uniqueness whenever the assumption of large numbers, correlation, and type ranking are satisfied.

and $S_k = qN (\beta_L^{k+1} - \beta_L^k)$

We can compute each step using the derivatives from Lemma 6 and the mean value theorem (β_I^i are the intermediate points given by the mean value theorem).

$$\begin{aligned}
s_0 &\approx (1-q) N \frac{\partial \beta_i}{\partial \beta_j} (\beta_I^0) s_H \\
s_1 &\approx qN \frac{\partial \beta_i}{\partial \beta_j} (\beta_I^1) s_0 \\
s_k &\approx qN \frac{\partial \beta_i}{\partial \beta_j} (\beta_I^k) s_{k-1} = \\
& q^k N^k \prod_{m=1}^k \frac{\partial \beta_i}{\partial \beta_j} (\beta_I^m) s_0 \\
& q^k (1-q) N^k \prod_{m=0}^k \frac{\partial \beta_i}{\partial \beta_j} (\beta_I^m) s_H
\end{aligned}$$

Visual inspection shows that s_k goes to zero as k gets larger. This means that, if the sum of these terms converges, the corresponding contracts $C = \lim C^k$, are the type H equilibrium contracts with asymmetric information.

To see that the sum converges note that

$$\sum_{k=0}^{\infty} s_k = \sum_{k=0}^{\infty} s_{2k} + s_{2k+1} = \tag{10}$$

$$\sum_{k=0}^{\infty} s_{2k} \left(1 + qN \frac{\partial \beta_i}{\partial \beta_j} (\beta_I^{2k+1}) \right) = \tag{11}$$

$$(1-q) s_H \sum_{k=0}^{\infty} \left(1 + qN \frac{\partial \beta_i}{\partial \beta_j} (\beta_I^{2k+1}) \right) q^{2k} \left[\prod_{m=0}^{2k} N \frac{\partial \beta_i}{\partial \beta_j} (\beta_I^m) \right] \tag{12}$$

Each term of the above sum is negative (because of the product term multiplying $2k+1$ negative numbers) and bounded above by q^{2k} . This means that the sum converges to some negative number s_L .

3. ($|s_L| < |s_H|$). I need to show that $q|s_L| < (1-q)|s_H|$. By Lemma 6 $|s_0| < (1-q)|s_H|$ which implies that $q|s_0| < (1-q)|s_H|$. Showing that $|s_L| < |s_0|$ will conclude the proof. Since both sides of the inequality are negative, this amounts to

$$s_L > s_0$$

Recalling how s_L is derived we can subtract s_0 on both sides and to obtain

$$\sum_{k=1}^{\infty} s_k > 0$$

The sum can be written as

$$\begin{aligned} \sum_{k=1}^{\infty} s_k &= \sum_{k=0}^{\infty} s_{2k+1} + s_{2k+2} = \\ \sum_{k=1}^{\infty} s_{2k+1} \left(1 + qN \frac{\partial \beta_i}{\partial \beta_j} \left(\beta_I^{2k+2} \right) \right) &> 0 \end{aligned}$$

s_{2k+1} is positive for all k and the term in parentheses is positive by Lemma6, which mean the sum is positive (convergence follows from part 2.)

□

9.6.3 Results

Theorem 4 immediately implies all the results, as stated in the main text of the paper.

Theorem 5. *Increasing the number of Units in an economy, eventually induces Principals to retain more risk at the contracting stage.*

$$\forall N, \exists \bar{N} : \hat{\beta}_t^n(N') \geq \hat{\beta}_t^n(N), \forall n, \forall t \in \{G, B\}, \forall N' \geq \bar{N}$$

Proposition 6. *In a large market, Aggregate Risk is higher than under symmetric information.*

$$E \left(\sum_{1=1}^N (\beta_{t(n)}^n)^2 \sigma_{t(n)}^2 \right) \geq E \left(\sum_{1=1}^N (\hat{\beta}_{t(n)}^n)^2 \sigma_{t(n)}^2 \right),$$

Proposition 7. *Increasing the number of Units in an economy, eventually makes contracts closer to the symmetric information ones. Letting $\Delta_t^n(N) = |\beta_t^n(N) - \hat{\beta}_t^n(N)|$.*

$$\forall n, \forall N, \exists \bar{N} : \forall N' \geq \bar{N}, \Delta_t^n(N') < \Delta_t^n(N)$$

Proof. The proof follows from the fact that contracts are arbitrarily close to 1 for large enough markets, regardless of the information structure and Theorem 4. □

Corollary 2. *With large enough markets, no agent enjoys an information rent.*

$$\exists \bar{N} : \forall N \geq \bar{N}, U(C_t(N), t) = \bar{u}, \forall t$$

Proof. For a given economy consider an N' which makes $\beta_H^n(N) = 1$, in every Unit, as per Equation 9. The contract will be in the form $C_H^n = (\bar{\alpha}, 1)$ where $\bar{\alpha}$ is such that $-\bar{\alpha} - \frac{b}{2}(-\bar{\alpha})^2 = \bar{u}$. This contract leaves both types at their participation constraint. This implies that the optimal contract for type L will also give him only his reservation utility, because that is the most he can obtain by mimicking type H . \square

9.7 Counterexamples

These two examples, combined with the correlation example in Section 6 show that the assumptions (independence of returns and ordering of types), and conclusions of Lemma 1 are “tight”.

To conclude the proof we have to show that.

9.7.1 A Type Space violating SCP

Consider the following set of parameters

(μ_L, σ_L^2)	(μ_H, σ_H^2)	b	\bar{u}
(2.1, 1)	(2, 0.1)	0.1	0.3

$$\begin{aligned} -\alpha_L + (1 - \beta_L) X &= -0.9301 + 0.6163X \\ -\alpha_H + (1 - \beta_H) X &= 0.3122 \end{aligned}$$

Those with markets will be

$$\begin{aligned} -\alpha_L^M + (1 - \beta_L^M) X &= 0.3083 + 0.0018X \\ -\alpha_H^M + (1 - \beta_H^M) X &= -0.9301 + 0.4913X \end{aligned}$$

We can see that the contract of type H becomes riskier as markets are introduced. This is because in this example SCP fails, so that there is not a “better” type. In this example, which type is “better” now depends on the diversification possibilities available to Principals.

9.7.2 Small Market

The insurance results are formulated as convergence results rather than Comparative Statics result. The following example shows that a stronger comparative statics would not be true. Adding a Unit does not necessarily induce insurance for Agents.

Consider the set of parameters.

(μ_L, σ_L^2)	(μ_H, σ_H^2)	r	\bar{u}
(3, 0.3)	(2, 1)	.01	.5

The equilibrium contracts without markets will be

$$-\alpha_L + (1 - \beta_L) X = -0.2483 + 0.2499X$$

$$-\alpha_H + (1 - \beta_H) X = -0.5368 + 0.5197X$$

Those with markets will be

$$-\alpha_L^M + (1 - \beta_L^M) X = -0.7713 + 0.4243X$$

$$-\alpha_H^M + (1 - \beta_H^M) X = -0.3529 + 0.4275X$$

In this economy, the contract of type H is now riskier, when markets are present.

To have an intuition for what is going on, compare the parameters with those from the previous example and note that now

- *Risk Aversion is very low.* Markets do not change much the risk taking attitude, as Principals are close to being risk neutral.
- *The distributions of returns are very different.* Entering the market amounts to selling half of a Unit to buy a fraction of the other, plus/minus a transfer of riskless asset. This introduce a big change in the risk held by a principal.

In this example, the Principal owning the safer Unit ends up holding more risk than her agent does, when the possibility of trading is introduced. She is of course compensated with a transfer of riskless asset, but at the contracting stage she acts as if her risk attitude has increased

Consider what happens to the contracts when markets are present. If we consider a replica economy with 10 Units, we will have that

$$-\alpha_L^M + (1 - \beta_L^M) X = 0.0279 + 0.1578X$$

$$-\alpha_H^M + (1 - \beta_H^M) X = 0.1835 + 0.1589X$$

Lemma 1 starts biting and both contracts are again less risky than they would have been without markets. This is because the large number of Units makes diversification and the ensuing change in risk aversion enough to countervail the effect of selling part of the less risky endowment.

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