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A Comparative Review of Risk Based Portfolio Allocations:

An Empirical Study throughout Rising Yields

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Abstract

Since the financial crisis, risk based portfolio allocations have gained a great deal in popularity. This increase in popularity is primarily due to the fact that they make no assumptions as to the expected return of the assets in the portfolio. These portfolios implicitly put risk management at the heart of asset allocation and thus their recent appeal. This paper will serve as a comparison of four well-known risk based portfolio allocation methods; minimum variance, maximum diversification, inverse volatility and equally weighted risk contribution. Empirical backtests will be performed throughout rising interest rate periods from 1953 to 2015. Additionally, I will compare these portfolios to more simple allocation methods, such as equally weighted and a 60/40 asset-allocation mix. This paper will help to answer the question if these portfolios can survive in a rising interest rate environment.

Key words: asset allocation, low volatility anomaly, portfolio optimization, rising interest rates

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Introduction

Traditionally, many asset managers have allocated their capital based on some fixed percentage amount such as an equally weighted (1/N) or a 60% allocation to equity and a 40% allocation to fixed income (60/40). However, these portfolio allocation methods have major drawbacks, namely that the majority of the total risk of the portfolio comes from the equity portion Quin (2005). Fixed income which has a relatively low volatility is given a smaller or equal weight resulting in an even lower contribution to the total risk of the portfolio. Additionally, in recent years these fixed weight portfolios have exhibited large drawdowns (MDD) and reduction of capital.

Since the financial crisis, portfolio managers have turned their interests to a more risk managed focus to asset management Martel (2014). With good reason, risk parity among other risk based allocation methods, have gained popularity in recent years. The majority of risk based portfolio allocation strategies, unlike the aforementioned method, are not reliant on historical returns and for this reason they are perceived to be more robust.

This paper will empirically review the performance of four well known long only risk based portfolio allocations. The first which is a subset of the mean-variance portfolio and the most well-known risk based portfolio allocation method is the global minimum variance portfolio (GMV) which seeks to create a portfolio with minimum risk. The second and precursor to risk parity is the inverse volatility (IV) portfolio. The objective of this portfolio is to equalize the standard deviation of the assets within the portfolio. The inverse volatility is the easiest of the four to construct making it appealing to less sophisticated investors. The third is the Risk Parity (RP) portfolio developed by Maillard et al. (2010) which is similar to inverse volatility but also takes the correlations of each asset into account. This is important because assets with high volatility but low correlation to the overall portfolio may be needlessly penalized with a lower weight than may be necessary as they would in inverse volatility. Finally the most diversified portfolio (MD) Choueifaty, Coignard (2008) which is actually the most similar in construction to the minimum variance portfolio. For comparison purposes I will also construct an equally weighted, and what I will call maximum volatility (MV), which is a portfolio whose only asset is the most volatile out of all assets in the sample. This maximum volatility portfolio will serve as an extreme opposite to all of the other risk based portfolios being that it is the most volatile and least diversified.

Being that the threat of rising interest rates is a contemporaneous concern; the primary objective of this paper is to study the performance of risk based portfolio allocations in rising interest rate environments. This is important to fully evaluate the nature of risk based portfolio allocations. This paper will add to the literature a collection of backtests of portfolios of different sizes and sample start dates of varying lengths. The majority of the literature takes a look at these portfolios through the past couple of decades. However, there have only been a small amount of short-lived interest rate increases in that time. For a full analysis it is essential to see the performance in a wide range of periods of interest rate rise.

Proponents of the risk parity strategy argue that the strategy only outperforms in bond bull markets because a high weight is given to low volatility assets such as fixed income. This paper will shed light on this argument. Additionally, throughout the paper I will make comparisons between the risk parity and inverse volatility methods to determine if risk parity can adapt quick enough in times of rapidly changing correlations. I will test all portfolios in a realistic and consistent manner.

Literature Review

The first known documentation of risk parity came from Ray Dalio, the founder of Bridgewater Associates, in a paper called Engineering Targeted Returns & Risks in 2004. However, he mentions that "All Weather principles for asset allocation" dates back further and was established in 1996. This paper lays the framework for all discussions on risk parity to come, starting from the ground up, Dalio gives an overview of his engineering process. In short, the idea behind this process is that almost all asset classes can be leveraged up to target a higher risk and return profile. For an example, if an investor wants to target say, returns of 30%, traditionally there are not many options to choose from. Investors might then be tempted to be overweight in a small number of high return assets and underweight lower return assets. To solve this problem low return assets can be leveraged to a level that would produce similar returns. Now the investor that would like to target higher returns has many more options to choose from. The results are a portfolio with the same return and less risk. The overarching goal of the All Weather asset allocation mix is to perform well throughout any economic cycle.

The foundation for the theoretical framework risk parity portfolio came from the work of Maillard, Roncalli and Teiletche's paper "On the properties of equally-weighted risk contributions portfolios." Without touching any of the nuances of risk parity as Dalio, the authors of this paper formulate the equation to solve the risk parity problem which also takes the correlation matrix into account. This paper draws the interesting comparison between the equally weighted and minimum variance portfolios. The authors describe the equally weighted risk contribution portfolio to be a compromise between the equally weighed and minimum variance portfolios. Additionally, they show a very important "natural order" of volatility between the three portfolios. That being the equally weighted portfolio is the most volatile, equally weighted risk contributions at a middle ground and minimum variance portfolio the almost obvious least volatile shown below. I will verify my results with this equation and check to see where the inverse volatility and most diversified portfolio stand in the data.

$$\sigma_{MV} \leq \sigma_{RP} \leq \sigma_{EQ}$$

In order to calculate the most diversified portfolio you first need to calculate the diversification ratio also defined by Choueifaty (2008). This ratio is essentially the weighted average of volatilities of all assets in the portfolio divided by the total portfolio volatility. They explain that by maximizing the diversification ratio you are indeed maximizing the diversification of the portfolio.

Others such as Asness, Frazzini and Pedersen 2012 try to explain the success of risk parity due to investor leverage aversion or simply being unable to do so. They explain that safer assets have to offer higher risk-adjusted returns than that of more risky assets. To take advantage of the higher risk adjusted returns of low risk assets one must use leverage. This is exactly what risk parity in practical terms is doing.

Bruder, Roncalli (2012) study a more flexible version of risk parity known as risk budgeting. This is a case of risk parity where contributions to risk do not necessarily need to be equal. This paper is useful for investors, for example, institutional investors who may not want to put an equal risk contribution in all assets. With this method an alternative asset such as real estate, can be budgeted to a lower more ideal level rather than having the same contributing risk as equity and bonds. Roncalli (2013) further expands on this risk budgeting technique to also incorporate expected returns. To do this they develop a generalized standard deviation-based risk measure, which encompasses the Gaussian value-at-risk and expected shortfall risk measures.

Standard deviation is not the only measure of risk used to calculate a risk parity portfolio. Alankar, DePalma, Scholes (2012) use “implied expected tail loss” which is a measure extracted from options-market information. The goal of this portfolio is to equalize the expected tail loss of each asset in the portfolio. This portfolio is described as being similar to risk parity when returns are normally distributed. The authors show that with this method it is possible to reduce large drawdowns cheaper than outright buying insurance while maintaining high returns over full market cycles. Another measure suggested by Martellini and Milhau (2013) describes using duration volatility measure in the context of rising interest rates. They suggest that this method can be used to address the issue of bond overweighting in a low-interest rate context.

Under realistic assumptions, Anderson, Bianchi, Goldberg (2012) confirm many of the results of Frazzini et al. and show that the differences in the two sets of research is in how the levered risk parity portfolio is scaled. Anderson et al. make some interesting conclusions on their empirical backtest on an 85 year horizon. The first of which is that the start and end dates have a large impact on the overall results of the backtest. The second is that transaction costs can wipe away outperformance.

Data

It is true that yields have been on a downtrend since the 1980s. However, there have been a substantial number of yield shocks to formulate a backtest. The first Panel (Panel 1) will test the four strategies throughout yield shocks between 1986 and 2015. First, I will start with a similar set of indexes as the global diversified portfolio used in Maillard et al. (2008) shown in Table 1. The only difference being a few less assets for the sake of a longer sample from 1986 to 2015 compared to 1995 to 2008. Periods of interest rate rise will be defined as any period of rising rates lasting longer than one year after a period of decline longer than one year. The starting and ending dates of this sample will include an additional two months of data beyond the start and end points of interest rate rise (4/1989-12/1992, 12/2000-6/2004, and 8/2006-4/2009).

Table 1: Descriptive statistics of the returns of Panel 1 (1986-2015)

	Return	Volatility	Correlation matrix (%)									
JGAGGUSD	5.7%	5.3%	100	-5.7	2.7	-15.4	-3.4	-11.4	-8.0	1.1	4.3	0.9
SPXT	8.8%	17.2%		100	13.2	49.7	11.2	48.0	84.0	53.1	38.5	17.5
SPGSCI	2.6%	20.8%			100	16.6	7.3	19.8	16.8	22.7	20.9	12.0
SX5E	5.8%	20.9%				100	26.7	81.8	49.4	49.5	53.0	37.5
TPX	-0.3%	20.4%					100	27.3	11.6	19.7	45.4	50.8
UKX	4.5%	17.2%						100	45.5	48.7	52.2	37.0
RTY	7.9%	20.1%							100	52.7	40.7	20.8
MXLA	10.4%	27.1%								100	76.1	33.3
MXEF	7.4%	18.1%									100	78.8
MXASJ	5.7%	19.8%										100

Names of the indexes are as follows: JPM Global Aggregate Bond, S&P 500 Total Return, S&P GSCI, Euro Stoxx 50, TOPIX, FTSE 100, RUSSELL 2000, MSCI EM LATIN AMERICA, MSCI EM, MSCI AC ASIA x JAPAN

Next I will use an even smaller universe of assets (Panel 2); Because of data limitations, I will recreate the price of the US 10-year treasury using 10 year constant maturity rate data taken from the U.S. Department of The Treasury website. Using simple bond math I will calculate the 10-year bond price with a constant 10 years to maturity making the only assumption that the coupon payment remains at 5% throughout the sample. Finally, I will calculate the price of the bond on the following month using all the same inputs as before except for the rate and settlement date. To estimate the 10 year minus one month yield on the bond I will use linear interpolation between the 10 year and 5 year rates. The settlement date input on the 10 year minus one month will simply be 10 years minus one month. This method will simulate an investor buying a 10-year treasury with 10 years to maturity every month and then selling it the following month only to buy another with 10 years to maturity.

Table 2: Descriptive statistics of the returns of Panel 2 (1972-2015)

	Return	Volatility	Correlation matrix (%)				
10 Year Treasury	2.2%	8.1%	100	7.7	4.6	-10.2	3.8
SPXT	8.0%	15.3%		100	57.6	10.7	63.8
FNERTR	12.9%	17.1%			100	6.8	46.5
SPGSCI	4.8%	20.4%				100	16.7
NDDUEAFE	9.9%	17.2%					100

Names of the indexes are as follows: S&P 500 Total Return, FTSE All Equity REIT, S&P GSCI, MSCI EAFE

In the third Panel (Panel 3) I will sacrifice more assets to use a longer time frame from 1952 to 2015. This Panel will examine the longstanding rising interest rate environment in the 50's and 60's. Again in panel 3 I will use the bond price creation used in Panel 2 coupled with SPXT. This will represent the most basic domestic portfolio with no international diversification.

Table 3: Descriptive statistics of the returns of panel 3 (1952-2015)

	Return	Volatility	Correlation matrix (%)	
10 Year Treasury	1.0%	6.8%	100	-7.9
SPXT	6.9%	14.6%		100

Theoretical Framework

Log returns were used for all calculations. All portfolio statistics will be shown after adjusting for the risk free rate, which in this case will be the federal funds rate. All portfolios were calculated and rebalanced at the close of each day unless otherwise stated. The portfolio's return and standard deviation are calculated in the usual way, here using r_i and x_i to denote the return and weight respectively of each individual asset. Panel 1 uses daily data whereas Panels 2 and 3 use monthly. The covariance between assets i and j are written as σ_{ij} and Ω to denote the covariance matrix. For our tests a fifty-day rolling window will be used to estimate the price volatility and covariance matrix.

$$r_p = \sum_{i=1}^{N} x_i \cdot r_i \qquad (1)$$

$$\sigma_p = \sqrt{x' \, \Omega \, x} \tag{2}$$

As I mentioned before, the diversification ratio shown below is essential to calculate the most diversified portfolio. Here I am using σ to denote a vector of individual asset volatilities.

$$\frac{x' \cdot \sigma}{\sigma_p} \tag{3}$$

Two risk measures that are essential to any discussion on risk based portfolio allocations are marginal risk contribution (MRC) and total risk contribution (TRC.) MRC is simply the covariance of the asset with the portfolio, which can also be looked at as the impact of a very small increase in an asset's weight on the risk of the total portfolio. TRC is simply the MRC multiplied by the assets weight, which tells you the total risk the asset has on the portfolio.

$$MRC_i = \frac{\partial \, \sigma_p}{\partial \, x_i} = \sum_{j=1}^{N} x_j \cdot \sigma_{ij} = cov(r_i, r_p) \tag{4}$$

$$TRC_i = x_i \cdot \frac{\partial \, \sigma_p}{\partial \, x_i} = \sum_{j=1}^{N} x_i \cdot x_j \cdot \sigma_{ij} = x_i \cdot cov(r_i, r_p) \tag{5}$$

Notice from the following equation that each asset's TRC can be viewed as separate components and the sum of those components will equal the total risk of the portfolio.

$$\sum_{i=1}^{N} TRC_i = \sum_{j=1}^{N} x_i \cdot cov(r_i, r_p) = \sigma_p^2 \tag{6}$$

Notice below that the similarities between the construction of the minimum variance portfolio and the risk parity and most diversified portfolios. The sole difference between the minimum variance portfolio and the risk parity portfolio is the inclusion of the assets' weights. Similarly the difference between the minimum variance and the most diversified portfolio is that the most diversified is scaled by the inverse of the assets volatility.

Table 4: Theoretical definitions

Portfolio Name	Objective	Strategy definition
Equal Weighted	Equalizes weights	$x_i = x_j$
Inverse Volatility	Equalizes volatility	$x_i \sigma_i^{-1} = x_j \sigma_j^{-1}$
Minimum Variance	Equalizes MRC	$\frac{\partial \sigma_p}{\partial x_i} = \frac{\partial \sigma_p}{\partial x_j}$
Most Diversified	Equalizes volatility scaled MRC	$\sigma_i^{-1} \frac{\partial \sigma_p}{\partial x_i} = \sigma_j^{-1} \frac{\partial \sigma_p}{\partial x_j}$
Risk Parity	Equalizes TRC	$x_i \cdot \frac{\partial \sigma_p}{\partial x_i} = x_j \cdot \frac{\partial \sigma_p}{\partial x_j}$

It is worth noting here that if all assets have identical pairwise correlation, risk parity will yield the same results as the inverse volatility method. In a two asset-universe, the calculation for inverse volatility will yield full risk parity. The inverse volatility portfolio is relatively simple to calculate while the full risk parity portfolio is computationally more difficult. This difficulty is due to the need of estimating a covariance matrix at every rebalancing frequency. To solve the risk parity problem, Maillard, Roncalli, and Teiletche (2010) propose minimizing the squared difference of all TRCs between all assets. This results in a portfolio whose asset's TRC are as close to identical as possible.

Inverse volatility or "naïve" risk parity is a strategy in which each assets weight is set proportional to its volatility. Said differently, x_i is the inverse volatility of asset *i* divided by the sum of all of the other assets inverse volatility.

$$x_i = \frac{\sigma_i^{-1}}{\sum_{j=1}^{N} \sigma_j^{-1}} \tag{7}$$

All Calculations were performed in the latest version of MATLAB (R2014B). Minimum variance most diversified and risk parity all need to be solved with numerical optimization with MATLAB's FMINCON and QUADPROG optimization which I will also summarize below.

Each program uses a covariance matrix in three dimensions and a loop to calculate the weights throughout time. One major difference in the calculation of risk parity versus minimum variance and most diversified is being that TRC takes into account the assets weights and MRC does not. To solve for risk parity weights, there needs to be an initial guess as to what the weights actually are. As suggested by Chaves et al. (2012) I have used the inverse volatility weights as an initial starting point. Because the weights of inverse volatility and risk parity are generally similar, FMINCON considers whatever input weights already as optimum. Scaling up the function output by a large number (1e10) solves this problem.

Table 5: Optimization set up

Portfolio Name	Objective	Equation
Minimum Variance	$x^* = min\, f\ (x)$	$f(x) = \frac{1}{2} x'\Omega x$
Most Diversified	$x^* = max\, f\ (x)$	$f(x) = \frac{x'\sigma}{\sigma_p}$
Risk Parity	$x^* = min\, f\ (x)$	$f(x) = \sum_{i=1}^{N} \sum_{j=1}^{N} (TRC_i - TRC_j)^2$

For the purpose of this thesis, long only portfolios will be examined. Each portfolio will be subject to the same constraints. That is, they will both have weights between zero and one that also sum to one.

$$x = \varepsilon\ [1,0] \quad (8)$$

And

$$\sum_{i=1}^{N} x_i = 1 \quad (9)$$

Results

Panel 1

In this section I will begin with a long sample set as previously mentioned to get a baseline for the various portfolio performances over time shown in Table 6. Using a longer time sample and slightly smaller universe of assets used in Roncalli et al. (2009) I find different results; namely that the risk parity portfolio is slightly worse than the minimum variance portfolio in this case rather than slightly better. This further strengthens the argument that Goldberg et al. (2012) pointed out that the starting and ending points of the backtest have a large effect on the results.

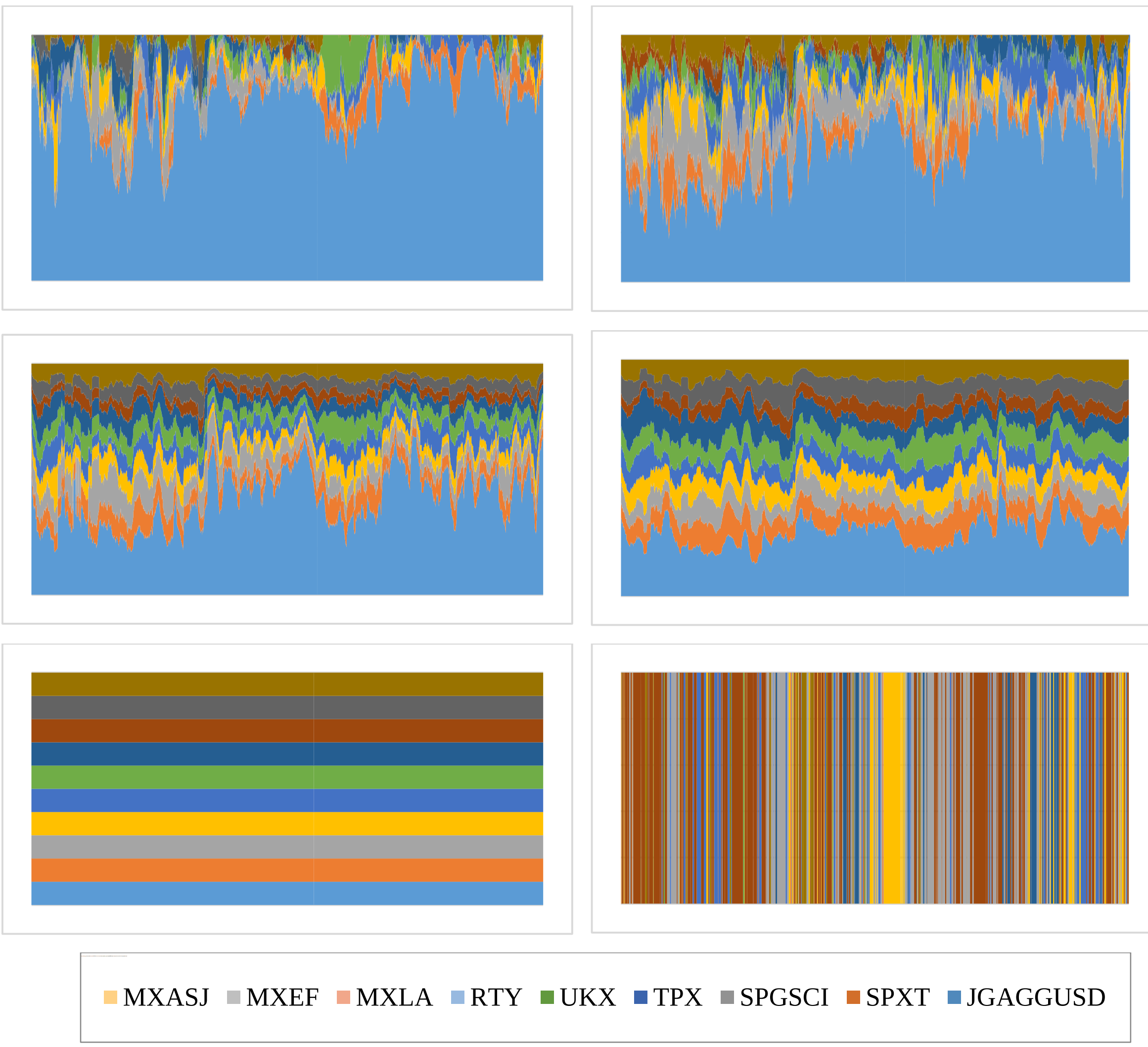
Table 6: Panel 1 Portfolio statistics total sample (1986-2015)

	GMV	MD	RP	IV	EQ	MV
Return	3.1%	2.7%	3.0%	3.0%	3.1%	9.3%
Volatility	4.5%	5.5%	6.7%	8.8%	12.2%	29.2%
Sharpe	0.69	0.49	0.44	0.34	0.25	0.32
Skew	-0.10	-0.35	-0.42	-0.51	-0.55	-0.36
Kurtosis	4.83	3.54	4.23	5.32	9.41	8.24
MDD	-14.9%	-19.9%	-28.6%	-43.6%	-55.9%	-77.8%

However, there is a pattern of volatility consistent with the literature, that being the minimum variance is the least volatile, most diversified and risk parity are somewhere in the middle and equally weighted is the most volatile out of the four. Here we can see that these results still hold out of sample. We can also see another pattern from this dataset that the least volatile portfolios have the highest Sharpe ratios. Before taking into account asset turnover and transaction costs, we can see that the minimum variance, most diversified and risk parity portfolios are all substantially better in terms of Sharpe ratio, and even maximum drawdown, than the inverse volatility portfolio.

I will now turn the focus to a more risk-managed point of view and compare the MRC and TCR of the various portfolios which can be viewed throughout time. Displayed below are the weights throughout time of the various portfolios.

Figure 1: Panel 1 weights, 1986-2015



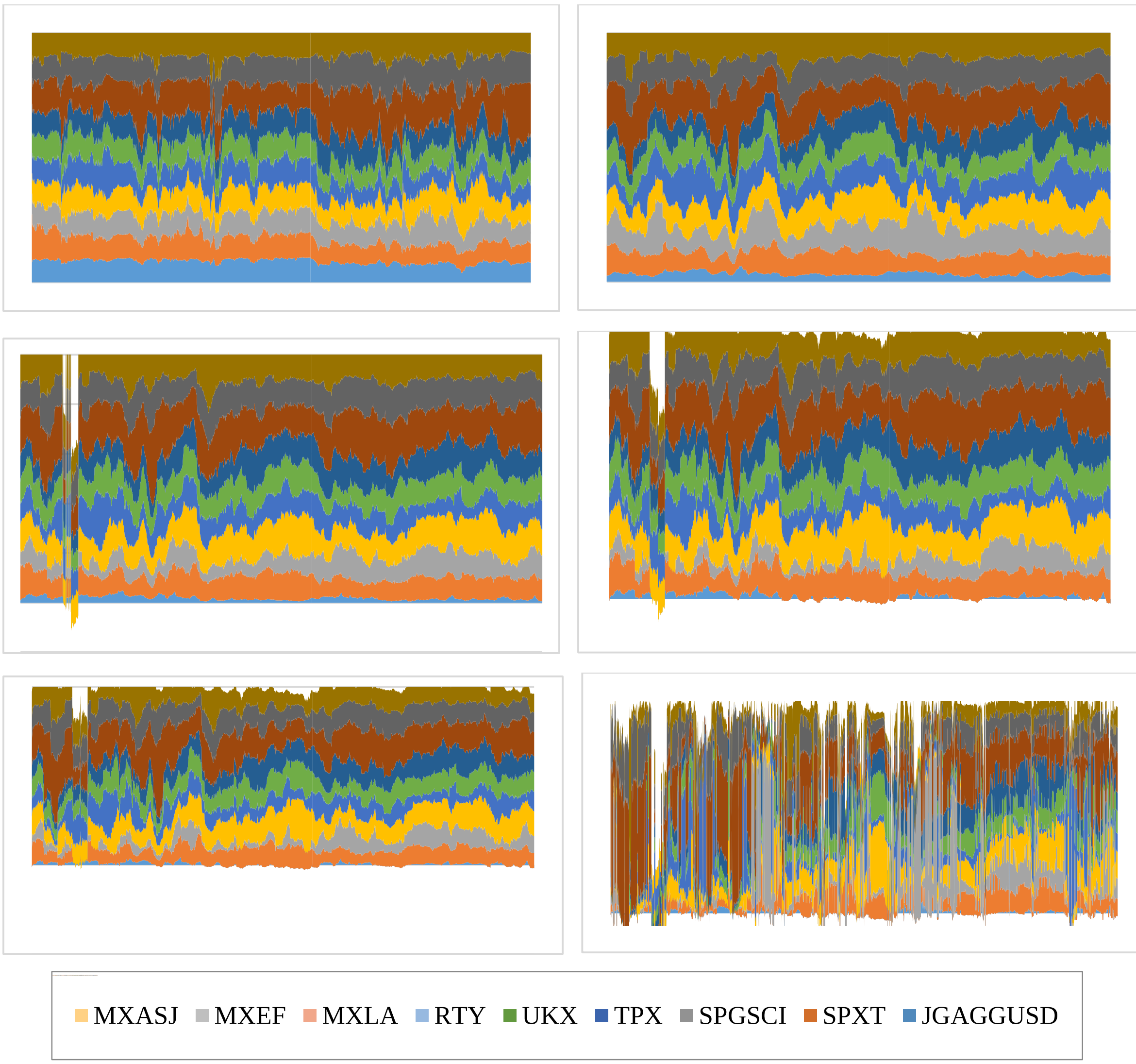
Note: Upper left (Minimum Variance), upper right (Maximum Diversification), middle left (risk parity), middle right (Inverse Volatility), lower left (Equally Weighted), lower right (Maximum Volatility)

Not surprisingly, we can see from the graphs that the lesser volatile portfolios are more heavily concentrated in fixed income. The minimum variance and most diversified portfolios are almost entirely comprised of fixed income. Furthermore, the minimum variance portfolio is also dominated by just a few assets at times. Visually, it is easy to see that the risk-based portfolio with the most even distribution in terms of asset weight is the inverse volatility portfolio. Risk parity has a similar weight distribution as inverse volatility but with a much larger weight given to fixed income.

MRC

Recall that I previously stated that the minimum variance portfolio's objective is to equalize the MRC of all assets in the portfolio. It is apparent that this is not the case with this situation because this is a constrained problem, which means making the MRC of all assets equal may not be possible. Only assets that are included in the minimum variance portfolio have an equal MRC. All assets that were given a weight of zero have an MRC unequal to the assets included in the portfolio. In an unconstrained minimum variance portfolio, all MRC would be equal. Here again we can see similarity between the minimum variance and most diversified portfolios in terms of distribution of MRC. Furthermore, we can see that MRC even becomes negative in some situations where the portfolio has a relatively lower weight in fixed income.

Figure 2: Panel 1 MRC, 1986-2015

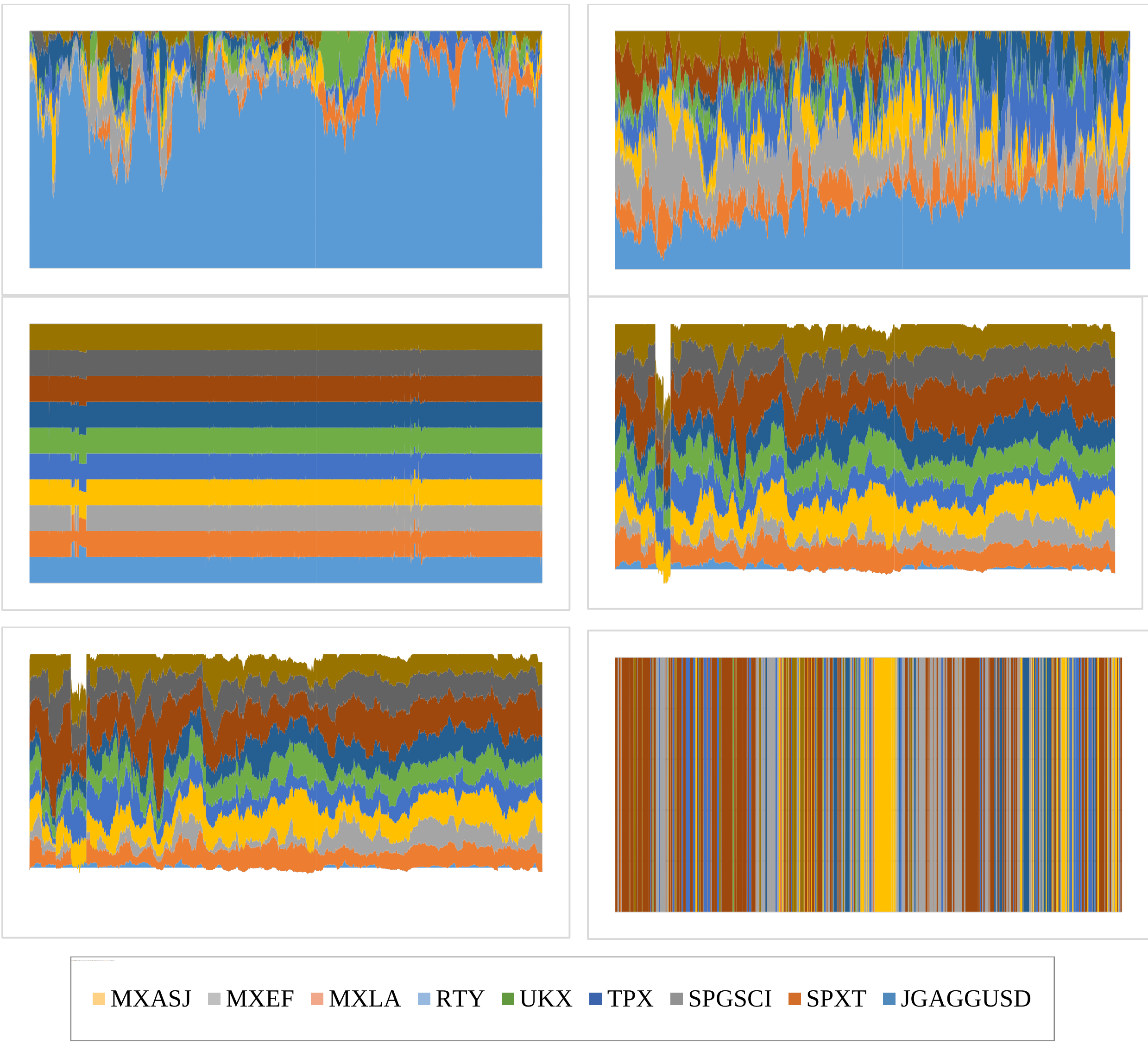


Note: Upper left (Minimum Variance), upper right (Maximum Diversification), middle left (risk parity), middle right (Inverse Volatility), lower left (Equally Weighted), lower right (Maximum Volatility)

TRC

Below we can see that the minimum variance portfolio is not only heavily concentrated in particular assets by weight but also by TRC. For illustrative purposes notice that the most volatile portfolio's TRC graph is the same as the graph for its respective weights being that that TRC is MRC multiplied by the weight of the asset and this portfolio only invests in one asset at a time. Interestingly, we can see that the equally weighted portfolio is more evenly distributed in terms of TRC than both the minimum variance and most diversified portfolios.

Figure 3: Panel 1 TRC, 1986-2015



Note: Upper left (Minimum Variance), upper right (Maximum Diversification), middle left (risk parity), middle right (Inverse Volatility), lower left (Equally Weighted), lower right (Maximum Volatility)

In periods of rising interest rates from 1986-2015 the risk parity portfolio outperforms all other risk-based portfolios except for the inverse volatility portfolio. These results show that risk based portfolios can survive short-term periods of rising rates shown in Table 7. Interestingly in this sample, the portfolios actually performed better in times of rising rates than they did in times of falling rates.

Table 7: Panel 1 Portfolio Statistics Periods of Rising Rates (1986-2015)

	GMV	MD	RP	IV	EQ	MV
Return	3.5%	5.9%	7.6%	9.7%	12.7%	15.4%
Volatility	4.3%	5.0%	5.6%	6.6%	8.1%	23.8%
Sharpe	0.81	1.18	1.34	1.47	1.57	0.65
Skew	-0.14	-0.22	-0.44	-0.48	-0.47	0.10
Kurtosis	1.02	1.18	2.36	2.46	2.63	3.57
MDD	-11.2%	-13.9%	-14.3%	-13.9%	-15.7%	-34.9%

Panel 2

This section is dedicated to the sample set from 1972-2015 using a five-asset portfolio of domestic equity, foreign equity, fixed income, real estate, and commodities. The sample set has been split into three sections; a long sample from 1972 to see baseline results, 1972-1986 to evaluate a period of rising rates and 1986-2015 to compare the results to the previous dataset.

Table 8: Panel 2 Portfolio Statistics Periods of Rising Rates (1972-1986)

	GMV	MD	RP	IV	EQ	MV
Return	0.5%	1.4%	3.8%	5.8%	7.6%	6.5%
Volatility	7.9%	8.5%	8.4%	8.7%	9.5%	19.5%
Sharpe	0.07	0.17	0.45	0.67	0.80	0.34
Skew	0.16	0.01	0.09	-0.44	-0.44	0.59
Kurt	1.35	0.65	0.69	0.89	0.65	3.65
Max DD	-30.4%	-20.1%	-20.5%	-20.9%	-19.6%	-47.3%

We can see from the results here that the risk parity portfolio outperforms the minimum variance and most diversified portfolios in all three time periods.

However, the equally weighted portfolio performs better than the risk parity portfolio in the period of rising rates in terms of Sharpe ratio and return. The risk parity and inverse volatility most often have similar Sharpe ratios but the risk parity portfolio has less drawdown than the inverse volatility.

Table 9: Panel 1 Portfolio Statistics Periods of Rising Rates (1986-2015)

	GMV	MD	RP	IV	EQ	MV
Return	1.3%	2.7%	5.7%	6.7%	10.7%	31.1%
Volatility	7.1%	7.3%	7.1%	7.0%	7.6%	16.1%
Sharpe	0.19	0.38	0.80	0.96	1.40	1.93
Skew	-0.44	-0.53	-0.64	-0.63	-0.43	-0.44
Kurt	1.06	0.48	0.57	0.28	-0.14	0.85
Max DD	-17.1%	-15.1%	-8.6%	-7.5%	-6.8%	-11.4%

Panel 3

This part of the paper is dedicated to the long sample from 1953 to 2015 examining a two-asset portfolio of stock and bond. For comparison with the other two datasets I have split the data into four different time periods; 1953-2015, 1953-1970, 1972-1986, 1986-2015. The components of the portfolio are represented by the S&P 500 index and a 10-year treasury bond recreated from 10 year constant maturity yields as mentioned previously. This section is studying the effects of rising interest over a long-term horizon on the various risk-based portfolios over the equally weighted portfolio. In Panel 3 I will also review the performance of the 60/40 portfolio as an additional benchmark. In general, risk based portfolios will have the greatest allocation to fixed income, trailed by equally weighted leaving 60/40 to have the lowest allocation to fixed income making it an appropriate comparison here.

Similar to the results from Panel 2, in the sample from 1953 to 2015 the risk parity performs better than both the minimum variance and most diversified portfolios in terms of both Sharpe ratio and drawdown. However, the equally weighted portfolio outperforms the risk parity portfolio in terms of Sharpe ratio and return.

Table 10: Panel 3 Portfolio Statistics Periods of Rising Rates (1953-1970)

	GMV	MD	RP	EQ	60/40	MV
Return	-0.9%	-1.6%	-0.6%	2.0%	2.9%	6.2%
Volatility	4.1%	3.8%	4.0%	6.4%	7.6%	12.6%
Sharpe	-0.23	-0.42	-0.15	0.32	0.38	0.49
Skew	-0.43	-0.16	-0.35	-0.58	-0.56	-0.51
Kurtosis	4.11	4.58	3.63	0.61	0.31	-0.08
MDD	-25.7%	-32.8%	-23.7%	-22.2%	-24.7%	-34.0%

Consistent with these results, we can see that in periods of rising interest rates, 1953 to 1970 and 1972-1986, these portfolios perform in a worse but similar manner. In the rising interest rate periods the risk parity portfolio performs better than the other risk-based portfolios but still worse than the equally weighted portfolio in terms of Sharpe ratio and return. It is not until the last time period from 1986-2015 that the risk parity portfolio undoubtedly outperform all other portfolios in terms of Sharpe ratio and drawdown. These results strengthen the case that portfolios that are typically heavily concentrated in low volatility assets will perform worse than their naively diversified counterparts in times of rising rates.

Table 11: Panel 3 Portfolio Statistics Periods of Rising Rates (1972-1986)

	GMV	MD	RP	EQ	60/40	MV
Return	2.0%	1.2%	2.3%	3.1%	3.5%	2.7%
Volatility	8.0%	8.3%	8.0%	9.0%	9.8%	14.2%
Sharpe	0.24	0.15	0.29	0.34	0.36	0.19
Skew	0.75	0.88	0.66	0.52	0.42	0.18
Kurtosis	1.39	1.85	1.30	0.80	0.83	1.33
MDD	-27.2%	-30.3%	-26.7%	-30.5%	-34.3%	-47.8%

Table 12: Panel 3 Portfolio Statistics Periods of Rising Rates (1986-2015)

	GMV	MD	RP	EQ	40/60	MV
Return	-0.3%	-1.3%	0.0%	1.9%	2.9%	7.0%
Volatility	5.4%	5.5%	5.3%	6.0%	6.6%	10.2%
Sharpe	-0.05	-0.24	0.00	0.31	0.44	0.69
Skew	-0.19	-0.01	-0.33	-0.22	-0.15	0.04
Kurtosis	-0.38	-0.42	-0.20	-0.37	-0.56	-0.83
Max DD	-17.4%	-22.9%	-15.6%	-11.2%	-9.0%	-7.2%

Conclusion

This paper is a demonstration of the importance of reviewing the performance of risk parity and other risk based portfolio allocations before the bull run in bonds lasting decades. If one was to look at the performance of these portfolios since the early 2000's, the conclusion would most likely be that risk based portfolios are superior to fixed weighted strategies due to the dramatic reduction in drawdown in the financial crisis. However a different picture would be painted if you look at the performance since the 1950's. It is of the utmost importance to scrutinize strategies through different regimes to evaluate their true performance.

The results of this paper show that risk based portfolio allocations, especially inverse volatility, have a positive performance throughout shorter periods of rising yields. The performance was the worst in the period between 50s and early 70s which was overall negative. Surprisingly, in Panel 1 during the short term interest rate hikes between 1986 and 2015, the risk based portfolios had a better performance than they did in the total sample between 1986 and 2015. In general, the risk based portfolios that had some degree of international diversification and commodities performed well in this time period.

From my results, two things can be said about risk parity and inverse volatility in times of rising interest rates. The first is that each portfolio does better when it has a more diverse group of assets. We can see that the first Panel with ten assets does better than the second Panel with five assets and that does better than Panel three with two assets. When there are more assets, there is a greater chance of one or more of them providing a cushion when fixed income falls. The second is that in comparison between risk parity and inverse volatility, inverse volatility performs better in times of interest rate rise. This is because inverse volatility has a closer resemblance to an equally weighted portfolio in terms of weights than that of risk parity. Inverse volatility typically weights fixed income lower than full risk parity due to the low correlation fixed income has with most assets. Therefore, in times of heightened uncertainty it is better to not make assumptions in terms of correlation.

Though there are times when the inverse volatility portfolio has a slightly higher Sharpe ratio than the risk parity portfolio but more often than not the risk parity portfolio is better in terms of drawdown. The risk parity portfolio has a major appeal of being a portfolio that can perform well in any environment. However, as the results show, this is highly dependent on which assets are selected into the portfolio. An advantage of minimum variance and most diversified is that they can weed assets out of the portfolio whereas risk parity and inverse volatility cannot. This makes asset allocation essential to the success of risk parity and inverse volatility portfolios.

In times of rising interest rates, the equally weighted portfolio is typically better in terms of Sharpe ratio than all of the risk based portfolios including the risk parity portfolio. The same cannot be said for drawdown, which is important in today’s risk sensitive world. Even though the equally-weighted portfolio typically has a better Sharpe ratio than the risk parity portfolio, the risk parity portfolio has a similar if not better maximum drawdown.

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