A Work Project, presented as part of the requirements for the Award of a Master Degree in Finance from the Nova – School of Business and Economics.

Stochastic Volatility: Volatility Models and Systematic Options Trading

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A Project carried out on the Master in Finance Program under the supervision of:

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May 2018

Abstract

This study presents an empirical analysis on the impact of stochastic volatility on options pricing and its effect on systematic option portfolio management. Through the modelling of univariate GARCH (Generalized Autoregressive Conditional Heteroskedasticity) processes and for the period between 1990 and 2017 for the S&P 500, Russell 2000 and FTSE 100, it is possible to observe deviations from the formula presented by Black and Scholes (1973) and Merton (1973). In this sense, we try to understand how stochastic volatility affects deviations from that pricing identity and the effects on speculation portfolio management policies related to this type of derivatives contracts.

Keywords: Stochastic Volatility, GARCH, Derivatives, Portfolio Management.

I. Introduction

Option trading strategies have been used even back before actual pricing formulas become available in the literature. With Black and Scholes (1973) and Merton (1973), a widely used plain-vanilla option (Call and Put) pricing formula became widely available which allowed for a spread in the trading of options. The Black-Scholes formula aims at pricing plain-vanilla calls and puts based on the option characteristics.

A crucial parameter in the model is the volatility of the underlying asset. This volatility, however, is a non-observable parameter. In this sense, an accurate estimation of volatility is crucial to an accurate depiction of an option fair price. Many authors in the literature, such as Black and Scholes (1973) have considered historical volatility as an accurate estimator of future volatility, while some practioneers in financial markets use an estimation of the previous period implied volatility.

Nevertheless, the Black-Scholes model fails to consider volatility as a stochastic process and assumes constant volatility. Therefore, many authors in the literature modeled option pricing with different volatility assumptions. The frameworks considered can vary in their GARCH (Generalized Autoregressive Conditional Heteroskedasticity) typology, but all aim at creating volatility forecasts as a stochastic process in order to fairly price options.

Many of the studies focus on the S&P 500 index options and create different and more accurate pricing models. This paper is not primarily focused on an accurate pricing of plain-vanilla options but rather on its use based on volatility modeling.

The use of this derivative product can have two main motives: i) hedging of current positions; and ii) speculation. While the former aims at hedging risk away through the creation of positions that limit the downside risk of a portfolio, the latter tries to obtain abnormal returns through the trading of options.

Therefore, the paper focuses on evaluating speculative option trading strategies for the creation of abnormal portfolio returns. To do so, it will study the practical application of GARCH models for the trading of plain-vanilla Calls and Puts in order to exploit mispricing and arbitrage opportunities in option prices for multiple indices. The core focus of the paper will be on univariate GARCH models and its implications to option portfolio strategies, with volatility forecasts being built individually for each stock index. Furthermore, the option portfolio will consist of Calls and Puts for the S&P 500, Russell 2000 and FTSE 100. Through this, it will be possible to monetize potential option mispricing, as well as to exploit diversification benefits. This modelling allows for a definition of a trading strategy based on plain vanilla options for the beforementioned indices.

The goal of this paper is then to make use of the literature with a practical application to trading strategies on synthetic (theoretical) path-dependent plain-vanilla derivative contracts on stock indices. It makes use of the literature in the sense that it exploits the pricing results derived from stochastic volatility models on option pricing. However, it aims at going beyond it as it tries to exploit these results by finding arbitrage opportunities and back test strategies that trade on that. These trading strategies can, therefore, be extended to other portfolios including options on stocks, indices and exotic options.

II. Literature Review

Black and Scholes (1973) and Merton (1973) developed an option pricing formula that ignited extensive research and led to an increase in the trading of this type of derivatives contract. In these models, one of the crucial inputs is the volatility that, since it is an unobservable parameter, must be estimated. While in the Black-Scholes model there is the assumption of constant volatility, and the use of historical estimates, many researchers have developed new pricing formulas to account for stochastic volatility. Practioneers in the financial markets, on the other hand, use the implied volatility of the previous period to price options based on the Black-Scholes model. However, extensive research developed new models that incorporate stochastic volatility (Heston (1993)). These models vary mainly between continuous-time stochastic volatility and discrete-time GARCH models. While the former is usually developed through simulation, the latter is less computational intensive, thus more efficient in empirical studies, with this paper focusing on the latter.

The concept of time-varying dynamic variances has been developed by Engle (1982), with the ARCH (Autoregressive Conditional Heteroskedasticity) models and later expanded in Bollersev (1986) with the proposal of the GARCH model. These models accommodated for the persistence of volatility in the sense that heteroscedasticity may be present, with the research aiming at modelling the heteroskedastic behavior in data. Furthermore, financial data tends to typically exhibit heteroskedastic behavior – volatility is persistent (volatility clustering) – and so these models gained the traction and have been featured prominently in financial analysis and used by financial practioneers as well as researchers. Moreover, and due to the univariate character of these volatility models, the concept of multivariate volatility models (MGARCH) came to overcome the spillover effects between asset returns by including correlations between asset returns. These multivariate GARCH models have emerged as an extension of the univariate GARCH models and the most popular are the Constant Conditional Correlation model by Bollersev (1990) and Jeantheau (1998); the BEKK model proposed by Engle and Kroner (1995) and the Dynamic Conditional Correlations model by Tse and Tsui (2002) and Engle (2002). These models, although accounting for a crucial factor in asset pricing which is the time-varying cross-correlations between assets, increase model complexity due to the cross-section dimensionality. That is, as the number of assets increases, the number of parameters expands in a way that can make estimation cumbersome. As mentioned in Francq and Zakoïan (2016), this "dimensionality curse", although characteristic of multivariate time series analysis, is particularly visible within volatility modelling frameworks. An alternative proposed in the paper is to perform an estimation of univariate models in a first stage and then proceed to dynamic correlation modeling with standardized residuals. In this sense, it is important to understand that multivariate volatility models can be crucial for the development of investment strategies on option portfolios of different assets to account for spillover effects across assets.

Given this and taking into account the importance of volatility estimation in option pricing, extensive research has been drawn towards alternative pricing formulas to overcome the biases in the framework proposed by Black and Scholes (1973) and Merton (1973). The methods developed aimed at including time-varying volatility effects (stochastic volatility) into the pricing of derivatives contracts. Duan (1995) introduced the GARCH option pricing model. This model aimed at reducing the pricing bias in the Black-Scholes formula, caused by the volatility smile (effect that results from the fact that an option implied volatility is dependent on its moneyness level). Hsieh and Ritchken (2000) compared the performance of different GARCH option pricing models and Heston and Nandi (2000) developed the same empirical research by analysing call and put options in a different way. Several studies have analyzed this issue. The model developed by Duan (1995) supported the theory that options can be priced when the evolution of the asset return follows a GARCH process. Heynen *et al* (1994), Duan (1996), Heston and Nandi (2000) and Hsieh and Ritchken (2000), among others, proved that GARCH effects can be used in the pricing of exchange-traded derivatives such as options.

These GARCH option pricing models, as exposed in Heston and Nandi (1997) are computed through simulation and do not have a closed-form solution – Duan (1995). In this sense, Heston and Nandi (1997) developed a closed-form pricing option solution for GARCH models. The model is then empirically tested on the S&P 500 index options traded at the CBOE (Chicago Board Options Exchange), showing substantially lower pricing errors when compared to the Black-Scholes model. The model developed uses S&P 500 options intraday data with the specific focus on first order GARCH modes with Maximum Likelihood Estimation as proposed by Bollersev (1986). Moreover, in Fiszeder (2007) the use of transactional data on the WIG 20 to test the GARCH option pricing models concluded that all models were able to beat the Black-Scholes; the models included in the study were GARCH and MGARCH models of various types: GARCH, MGARCH, GJR and GJR with Student-t innovations. This study shows that option valuation through volatility modes is able to explain most of the pricing biases in the Black-Scholes model: volatility smile, negative correlation between present return and future volatility and positive risk premiums (the volatility smirk results from the fact that the implied volatility on assets' returns is higher for lower levels of moneyness; this is a result of investors' aversion to crashes, relying on options for insurance purposes). Capmani and Fucci (2017) consider continuous-time stochastic volatility models. The main focus is on the continuous-time GARCH model presented in Hull (2012) aiming at forecasting the term-structure of implied volatility in the Brazilian option market, concluding that the model is able to accurately forecast shocks in volatility term-structure (term structure of implied volatilities).

As can be seen, GARCH option pricing models, both univariate and multivariate, exhibit a higher pricing accuracy compared to the Black-Scholes model. Thus, it is possible to conclude that stochastic volatility in asset returns is crucial for option valuation as shown by the extensive research. Moreover, most of the empirical research has been developed on the S&P 500 index options given their highly liquid nature. In this paper, the study will therefore be focused on taking advantage of the pricing accuracy created by stochastic volatility modeling. In this sense, instead of studying option valuation, the paper will provide an investment strategy analysis based on the modeling of volatility for the trading of options. Through this, it will be possible to monetize the mispricing of exchange-traded options as well as capturing the benefits of diversification.

III. Methodology

This paper will cover the empirical study of investment strategies on options trading through univariate GARCH models. In this sense, the aim of the paper is to provide and empirical estimation of the volatility model described. The models developed will then be used to extract volatility out-of-sample (OOS) forecasts that will be used for the creation of trading signals that will then be applied to the OOS data on option prices. Through this, it will be possible to evaluate the efficacy of the models to trade options on the indices.

In this sense, the models presented will be estimated from 05/01/1990 to 30/12/2005 (estimation sample) using weekly data on the log returns of the indices. By doing so, and through the coefficients provided, it will be possible to extract the OOS volatility forecasts for the period between 06/01/2006 to 29/12/2017 (testing sample). Furthermore, those forecasts will give insights to systematically develop trading signals across the latter period.

i. Volatility Models

The models underlying the paper are based on univariate GARCH processes following Francq and Zakoian (2016) with a focus on the GARCH model by Bollersev (1986). The BEKK MGARCH model by Engle and Kroner (1995) is important due to the time-varying covariances between asset returns that has a significant impact on volatility forecasting. Nevertheless, the computational intensity of such models increases with the number of underlying assets. Therefore, for simplicity, the paper will have an extensive focus on univariate GARCH processes applied to a multi-asset portfolio.

The GARCH model aims at estimating volatility clustering effects. This model appears as an extension of the ARCH model introduced by Engle (1983). This model characterizes autoregressive squared returns where the next period's volatility is conditional to the information from the previous period. However, in the ARCH model, volatility does not have a strong persistence, and so, the model is extended to the GARCH model by Bollersev (1986). In this model, volatility is conditional, not only on previous volatility but also on the previous squared return. As presented in the following equation, the return on each index (i) will have a mean equation (Equation 1) and a volatility equation (Equation 2) with the volatility following a GARCH (p,q) process, i.e.:

$$r_t^i = \mu_t^i + \sigma_t^i \varepsilon_t^i \ , \ \varepsilon_t^i \ i.i.d \sim N(0,1) \ , \ \eta_t^i = \sigma_t^i \varepsilon_t^i \tag{1}$$

$$\sigma^{2}{}_t^i = \alpha_0^i + \sum_{\gamma=1}^{p} \alpha_\gamma^i \, (\eta_{t-\gamma}^i)^2 + \sum_{\theta=1}^{q} \beta_\theta^i \, (\sigma_{t-\gamma}^i)^2 \, , \qquad \alpha_0^i > 0, \alpha_\gamma^i > 0, \ \beta_\theta^i > 0. \tag{2}$$

For simplification purposes, and since the model could be further extended to a GARCH (p,q) process, we consider a GARCH (1,1) process. Also, the mean equation is assumed to follow an ARMA (j,k) process. In this sense, the investment strategies will be based on the following equations (Equations 3 and 4):

$$r_t^i = \mu + \sum_{\tau=1}^{j} \delta_\tau^i \, r_{t-\tau}^i + \sum_{\vartheta=1}^{k} \rho_\tau^i \, \epsilon_{t-\vartheta}^i + \epsilon_t^i \,, \qquad \epsilon_t^i \; i.i.d \sim N\left(0, \sigma_{\epsilon^i}^2\right) \tag{3}$$

$$\sigma^{2}{}_t^i = \alpha_0^i + \alpha_1^i \left(\epsilon_{t-1}^i\right)^2 + \beta_1^i \left(\sigma_{t-1}^i\right)^2 , \qquad \alpha_0^i > 0, \alpha_1^i > 0, \; \beta_1^i > 0 \,, \alpha_1^i + \beta_1^i < 1. \tag{4}$$

Through this estimation of the univariate volatility model for each index, it is therefore possible to extract the volatility forecasts. After the estimation of the GARCH (1,1) processes for the three equity indices in the estimation sample, it is possible to extract the OOS forecasts for the period between 06/01/2006 and 29/12/2017. This is done through a rolling GARCH (1,1) model with a rolling moving window for the period between 05/01/1990 and 29/12/2017 with one-step ahead OOS forecasts starting in 06/01/2006. By doing so, it is possible to obtain OOS volatility forecasts for the testing sample and to have those forecasts including all the available information at the moment of the trading decision.

Furthermore, this volatility forecasts will then be used for an analysis of the expected volatility on the given indices, to be able to derive investment decisions on the synthetic derivatives for the monetization of the investment strategy derived based on the GARCH model.

ii. Data

Weekly data on the S&P 500, Russell 2000 and FTSE 100 is used to test the models. This data is extracted from Bloomberg and corresponds to the Historical End of Day data for the period between 05/01/1990 and 29/12/2017. Through the inclusion of three equity indices it is possible to ensure diversification benefits not only within a geographic region but also across geographic locations.

Firstly, the closing price of all the indices was taken for the period in question, in order to compute both the arithmetic return and the log return on the indices. All prices were obtained in US Dollars in order to exclude foreign exchange effects from the data, assuming that the portfolio will be held for by an US investor.

In order to have a comprehensive database for the pricing of options, the following datasets were used: Historical Call Implied Volatility, Historical Put Implied Volatility, Dividend Yield and Risk-Free Rates (Fed Funds Rate and LIBOR 6M for the US and UK respectively). Based on this dataset it is then possible, through reverse engineering[1], to extract the option prices for the period in question through the Black-Scholes formula assuming a maturity of three months and 100% moneyness as seen below (Equations 5, 6, 7 and 8). By doing so, it is possible to have an extensive dataset on option prices (both call and put options with the described maturity and moneyness). To correct for pricing errors from this assumption, a rebalancing period[2] of 1 to 4 weeks is assumed in order to allow for the same option to be fixed over that period of time. By doing this analysis and guaranteeing that the option to be traded is held across the rebalancing period, returns on the options are drawn for the period ranging from 06/01/2006 to 29/12/2017. This was done to overcome the obstacle of the absence of a solid database on option prices that allows for the analysis of the evolution of a given option on an index over time and until maturity.

[1] Bloomberg terminal assumes the Price of an option follows the Black and Scholes formula with T=3 and K=S to compute the Implied Volatilities (IV). Therefore, through reverse engineering, given the IV, it is possible to calculate the underlying call and put option price.
[2] The rebalancing period ensures that over 1 to 4 weeks, the option to be traded and for which the trading signals are created is the same.

$$Call\ Price_t = S_{t-1} * e^{-q_t \tau} N(d_{1_t}) - K e^{-r_t \tau} N(d_{2_t}) \qquad (5)$$

$$Put\ Price_t = -S_{t-1} * e^{-q_t \tau} N(-d_{1_t}) + K e^{-r_t \tau} N(-d_{2_t}) \qquad (6)$$

$$d^i{}_{1_t} = \left(\frac{ln\left(\frac{S^i_{t-1}}{K}\right) + \tau(r_t - q^i_t + \frac{(\sigma^i_t)^2}{2}}{\sigma^i_t \sqrt{\tau}} \right) \qquad (7)$$

$$d^i{}_{2_t} = d^i{}_{1_t} - \sigma^i_t \sqrt{\tau} \qquad (8)$$

$$q = dividend\ yield;\ \tau = time - to - maturity;\ r = risk - free\ rate;$$

$$S = underlying\ asset\ price$$

$$K = Strike\ Price\ ;\ \sigma = implied\ volatility$$

By applying the Black-Scholes formula to the S&P 500, Russell 2000 and FTSE 100, it is possible to create a comprehensive data set of option (plain-vanilla call and put options) prices for the period. Through this data set, it is possible to ensure an estimation sample that is large enough to avoid model misspecification as well as to include the dotcom crisis. Moreover, it allows for a testing sample that includes both the subprime crisis and the sovereign debt crisis, allowing for a pre and post crisis portfolio testing, and analyze its influence in the portfolio returns.

iii. Application

The volatility models developed on this section are then estimated for the S&P 500, Russell 2000 and FTSE 100 from 05/01/1990 to 30/12/2005. To do so, a test for data stationarity is performed to ensure that the models are not to be corrected for nonstationary data. Secondly, the estimation of the ARMA models is computed for each index and the order of the mean equation is derived. Moreover, the Ljung-Box test (Appendix 2) is applied on both the residuals and the squared residuals of the ARMA models to test for autocorrelation in the error term and for conditional heteroskedastic (GARCH) effects. After this analysis is made, the univariate volatility models are estimated. In this sense, and by having the estimated parameters for both models referring to all the indices it is possible to extract the volatility estimates. Moreover, and in order to ensure that the models use all available past information to forecast the volatility from 06/01/2006 to 29/12/2017, dynamic predictions are computed.

In this sense, as mentioned before, with regards to the univariate GARCH (1,1) estimation, the forecasts are computed using a rolling moving window GARCH model for the period between 05/01/1990 and 29/12/2017. Therefore, the one-week ahead forecasts can be derived from this rolling model, ensuring sufficient dynamism so as to include all past available information at the trading decision point in time.

Through this analysis, it is possible to ensure that the volatility forecasts use all available past information to be able to produce more accurate trading signals. These signals are computed based on the Implied Volatility for both Call and Put options on the indices (Equations 9 and 10). Therefore, since option prices react positively to volatility shocks, a forecasted volatility higher than the implied volatility for the Call/Put on the index should lead a rational investor to take a long position on that derivative contract, *ceteris paribus*. Thus, the trading signals are build based on that reasoning, with long positions being taken on the derivatives contracts when the options' Implied Volatility is lower than the forecasted volatility for the subsequent period, and short positions vice-versa as follows:

$$Call\ Position^i = \begin{cases} Long \quad if\ IV_{t,C}^i < \hat{\sigma}_{t+1}^i \\ Short \quad if\ IV_{t,C}^i > \hat{\sigma}_{t+1}^i \end{cases} \qquad (9)$$

$$Put\ Position^i = \begin{cases} Long \quad if\ IV_{t,P}^i < \hat{\sigma}_{t+1}^i \\ Short \quad if\ IV_{t,P}^i > \hat{\sigma}_{t+1}^i \end{cases} \qquad (10)$$

By doing so, it is possible to derive extensive trading signals across the testing sample in order to perform a back testing on the investment strategy based on all the past available information. In addition to this, a delta-hedged portfolio will be considered so as to isolate from the portfolio returns the returns on the stock index itself as presented below. Through Equations 11 and 12, it is possible to obtain the Delta on each option (that measures the sensibility of the option through variations in the underlying asset's spot price). Based on that, a delta-hedged portfolio can be derived (Equation 13) which is not sensitive to the underlying asset return.

$$\Delta C_t^i = \ Delta\ Call_t^i = \ e^{-q_t^i \tau} N\left(\frac{ln\left(\frac{S_{t-1}^i}{K}\right) + \tau(r_t - q_t^i + \frac{(\sigma_t^i)^2}{2}}{\sigma_t^i \sqrt{\tau}}\right) \tag{11}$$

$$\Delta P_t^i = \ Delta\ Put_t^i = \ -e^{-q_t^i \tau} N\left(-\frac{ln\left(\frac{S_{t-1}^i}{K}\right) + \tau(r_t - q_t^i + \frac{(\sigma_t^i)^2}{2}}{\sigma_t^i \sqrt{\tau}}\right) \tag{12}$$

$$q = dividend\ yield;\ \tau = time - to - maturity;\ r = \ risk - free\ rate;$$

$$S = underlying\ asset\ price$$

$$K = Strike\ Price\ ;\ \sigma = implied\ volatility$$

$$Investment\ Index_t^i = \ -(\Delta P_t^i * \ weight\ put_t^i + \ \Delta P_t^i * \ weight\ Call_t^i) \tag{13}$$

Moreover, two types of portfolios are to be considered that are: a long short portfolio characterized by being a zero-investment portfolio; and a long short portfolio with a long bias of up to 25% - which allows the portfolio to take a net long exposure of up to 25%. These portfolios are equally weighted in both their long and short leg.

The portfolios will be analyzed based on their average annualized return, standard deviation and Information Sharpe Ratio (Equation 14). This ratio measures the return per unit of risk, and so it is a measure of risk adjusted return which does not take into account the risk-free rate (and so is not the excess return). This measure provides accuracy when evaluating portfolios that are zero-investment or close to it, such as pure long short portfolios or portfolios with a reduced net exposure.

$$Information\ Sharpe\ Ratio = \frac{Portfolio's\ Average\ Annualized\ Return}{Portfolio's\ Annualized\ Standard\ Deviation} \quad (14)$$

Given this, the volatility forecasts computed by both the GARCH (1,1) models will be monetized giving further importance to the close relationship between volatility models and option pricing as well as the impact of stochastic volatility on deviations from the Black-Scholes formula.

IV. Empirical Evidence on Volatility Modeling

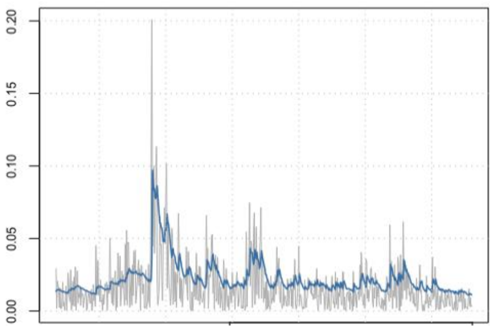
In order to derive a comprehensive investment strategy on the options portfolio based on the volatility estimation, the indices were studied for the estimation period, in order to assess their volatility behaviour. Therefore, for the three indices – S&P 500, Russell 2000 and FTSE 100 – log-returns were analyzed in terms of the GARCH effects present. To do so, firstly nonstationarity was tested through the Augmented Dickey-Fuller test against the alternative hypothesis of stationarity. Moreover, and using the Akaike Information Criteria and the Bayesian Information Criteria the ARMA structure for the indices is derived. Lastly, these ARMA models are tested with the Ljung-Box test for autocorrelation in the error term to ensure accuracy of fit of the ARMA model and in the squared error term to analyze conditional heteroskedastic effects. After that, for the estimation sample, the GARCH (1,1) univariate volatility model is estimated for all the indices with the correspondent ARMA fit for the mean equation.

The first stage of the study is characterized by an analysis of the three indices' behaviour in terms of conditional heteroskedasticity. In this sense, and after analyzing the plots presented below and performing the Augmented Dickey-Fuller test, it is possible to conclude that, for the estimation sample, all variables considered are stationary at even a 5% and 2,5% significance level. In this sense, since the p-values for all the variables is close to 1%, the null hypothesis of non-stationary data is rejected at the significance levels of 5% and 2,5%. This is fundamental to ensure that the models do not have to be corrected for non-stationarity.

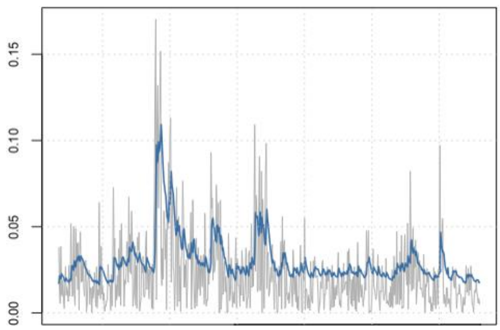
Moreover, by using the beforementioned statistics, the order of the ARMA models is computed for all the indices. In this sense, it is possible to conclude that all the indices follow an ARMA structure. These models are then estimated using Maximum Likelihood Estimation (*Appendix 5*). Furthermore, to assess the existence of GARCH effects in the residuals, the same test is performed on the squared residuals evaluating autocorrelation in the squared residuals (and so heteroskedastic behavior). By performing this test, it is possible to conclude that at all significance levels, for the estimation sample, all index log-returns exhibit autocorrelation in the squared residuals with p-values lower than 1%, and so, conditional heteroskedastic behavior.

Given this, it is possible to conclude that, in fact, given the analysis on the index log-returns, all indices exhibit heteroskedasticity. Therefore, the GARCH models are estimated for all the indices. Firstly, with regards to the S&P 500, a variance equation following a GARCH (1,1) process is estimated with a mean equation following an ARMA model of order (2,2). The estimated model for the period between 05/01/1990 and 30/12/2005 allows us to obtain the GARCH (1,1) parameters as follows: $\widehat{\alpha_0}$ = 2,758e^{-06}, $\widehat{\alpha_1}$ = 0,070 and $\widehat{\beta_1}$ = 0,925 (with the sum of the latest being 0,995 < 1). The presented rationale is also used for the estimation of the GARCH (1,1) process with an ARMA (2,2) for the mean equation related to the Russell 2000 log-return series. By doing so, the estimated parameters for the variance equation are, respectively, 1,388e^{-05}, 0,155 and 0,836 (with the sum of the latest two being 0.991 < 1). Finally, and estimating a GARCH (1,1) model for the FTSE 100 log-return series for the estimation sample with a mean equation following an ARMA (1,1) process, it is possible to obtain the estimated parameters as follows: $\widehat{\alpha_0}$ = 3,544e^{-06}, $\widehat{\alpha_1}$ = 0,037 and $\widehat{\beta_1}$ = 0,956 (with the sum of the latest being 0,993 < 1).

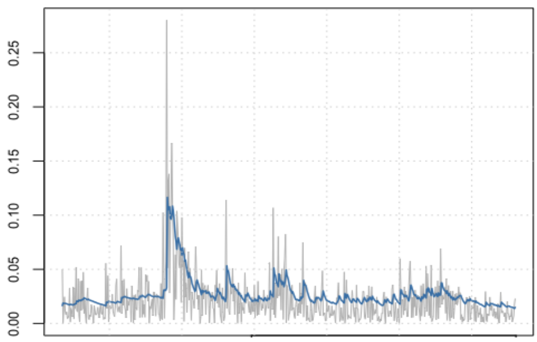
In this sense, the estimation of the model for the estimation sample is defined and the GARCH (1,1) processes are well specified as the conditional heteroskedastic behavior is removed from the ARMA series for the indices (*Appendix 6*). Therefore, the rolling GARCH (1,1) processes with the referred mean equations were estimated for the entire sample. This was performed through rolling GARCH models with a moving window to obtain the one-week ahead volatility forecasts. These volatility forecasts were generated for the period ranging from 06/01/2006 to 29/12/2017 for the S&P 500, Russell 2000 and FTSE 100 stock indices, which are presented below (**Figure 1**) and compared to the actual volatility levels verified in the period in question.



S&P 500



Russell 2000



FTSE 100

Figure 1 *– Volatility 1-week ahead Forecasts from GARCH (1,1) processes for the equity index options for the period from 06/01/2006 to 29/12/2017*

The volatility forecasts presented will then be applied to the creation of trading signals as derived before. These signals can be used for the creation of the beforementioned portfolios that can then be back tested for the testing sample. This back test of investment strategies on index options will be further developed in the next section.

V. Results

In order to accurately monetize the generated volatility forecasts through the volatility models, a back testing on the investment strategy was developed for the period between 06/01/2006 and 29/12/2017. This synthetic portfolio includes long and short positions on call and put options that trade according to the Black-Scholes formula for the three indices mentioned: S&P 500, Russell 2000 and FTSE 100. For the sample period, a pre and post subprime crisis analysis will be included. The pre-crisis period considered is from 06/01/2006 to 01/01/2008 while the post crisis period is from 01/01/2010 to 29/12/2017. In addition to this, in order to avoid portfolio returns through stock index returns, a delta-hedged portfolio will be considered to test the robustness of results. Moreover, the portfolios to be considered are a long short portfolio that is zero investment and a long short portfolio with a long bias of 25%. The following table (***Table 1***) summarizes the results for the portfolios considering the entire sample and the two subsamples.

Portfolio Statistics				
	Overall (06/01/2006 to 29/12/2017)			
	Porfolio		**Delta-Hedged Portfolio**	
	Long Short	Long Bias (25%)	Long Short	Long Bias (25%)
Average Annualized Returns	110%	62%	106%	58%
Standard Deviation	69%	66%	77%	76%
Info Sharpe	1,5940	0,9327	1,3723	0,7592
Maximum	44%	41%	49%	46%
3Q	6%	4%	6%	4%
Median	0%	0%	0%	0%
1Q	-1%	-3%	-3%	-4%
Min	-28%	-28%	-29%	-32%
Positive Weeks	40%	35%	51%	47%
Skewness	0,6347	0,7896	0,8575	0,8418
Kurtosis	2,2154	2,5498	2,3347	2,3831
	Pre Crisis (06/01/2006 to 01/01/2008)			
	Porfolio		**Delta-Hedged Portfolio**	
	Long Short	Long Bias (25%)	Long Short	Long Bias (25%)
Average Annualized Returns	91%	62%	47%	42%
Standard Deviation	87%	66%	92%	102%
Info Sharpe	1,0465	0,9327	0,5085	0,4109
Maximum	40%	41%	41%	46%
3Q	7%	4%	6%	6%
Median	0%	0%	0%	0%
1Q	-3%	-3%	-4%	-5%
Min	-26%	-28%	-28%	-32%
Positive Weeks	36%	35%	34%	50%
Skewness	0,5181	0,7896	0,3981	0,4578
Kurtosis	1,0910	2,5498	1,0433	1,1762
	Post Crisis (01/01/2010 to 29/12/2017)			
	Porfolio		**Delta-Hedged Portfolio**	
	Long Short	Long Bias (25%)	Long Short	Long Bias (25%)
Average Annualized Returns	87%	41%	79%	35%
Standard Deviation	61%	54%	64%	60%
Info Sharpe	1,4231	0,7712	1,2309	0,5719
Maximum	36%	30%	38%	33%
3Q	5%	3%	5%	3%
Median	0%	0%	0%	0%
1Q	-1%	-2%	-3%	-3%
Min	-28%	-25%	-29%	-26%
Positive Weeks	40%	35%	51%	45%
Skewness	0,3218	0,7532	0,6961	0,8927
Kurtosis	2,0192	2,2084	2,2625	2,3055

***Table 1** – Portfolio Performance Statistics*

As can be seen, for the overall testing sample, the synthetic option portfolio yields positive results. For the long short portfolio, the average annualized return is 110% with a standard deviation of 69%. This yields an Information Sharpe Ratio of 1,5940 which indicates a good investment, meaning that an investor through this strategy can attain 1,5940 of return per unit of risk in his portfolio. In addition to this, the portfolio yields positive returns in 40% of the weeks and is positively skewed (0,6247) and has a positive excess Kurtosis (2,2154). In this sense, the distribution of returns has a long right tail (positive Skew) and so extreme negative returns are not likely; the high level of excess Kurtosis points to the fact that the probability of attaining extreme results is higher than that for a Normal distribution. This also points to the facts seen in the Quartile Distribution of the portfolio returns with an extremely high maximum of 44% and extremely low (but less extreme) minimum of -28%. As can be seen from the cumulative return graph (***Figure 2***) presented below, the portfolio is highly volatile with periods of exponentially high peaks and of sudden crashes.

Figure 2 *– Cumulative Returns of the Long Short Portfolio (unhedged) for 06/01/2006 to 29/12/2017*

Moreover, the Delta-Hedged Portfolio for the long short portfolio presents similar results with a high average annualized return of 106% associated with a volatility of 77%. These results yield an Information Sharpe Ratio of 1,3723. In addition to this, the portfolio has a positive skewness of 0,8575 and an excess Kurtosis of 2,3831. The Delta-Hedged portfolio has a higher percentage of positive weeks when compared to the long short unhedged portfolio (51% compared to the beforementioned 40% for the unhedged portfolio). These portfolio statistics allied with the chart presented below (***Figure 3***) also sustain the hypothesis of volatile return distribution allied with a certain degree of crash risk present in such portfolios.

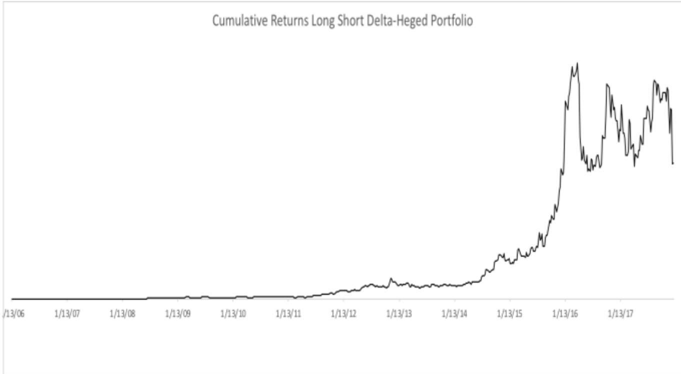


Figure 3 *– Cumulative Returns of the Long Short Delta-Hedged Portfolio for 06/01/2006 to 29/12/2017*

Furthermore, when analyzing both portfolios (unhedged and delta-hedged) for the overall testing sample regarding the long short portfolio with a long bias of 25% (that allows for a net exposure to the puts and calls of up to 25%, we can see a significant decrease in the Information Sharpe Ratio to 0,9327 and 0,7592, respectively for the unhedged portfolio and for the Delta-Hedged Portfolio. This decrease occurs mainly due to a considerably lower average annualized return (from 110% to 62% and from 106% to 68%, respectively for the two mentioned portfolios). This suggests that the short leg of the portfolios plays an important role in generating returns to investors. Moreover, this decrease is not proportional to the change in standard deviation from the portfolio (decreases from 69% to 66% and 77% to 76% respectively for the unhedged portfolio and Delta-Hedged portfolio).

Overall, and as presented in the charts below (***Figure 4*** and ***Figure 6***), the portfolios present a high crash risk, with a highly volatile return distribution – which can also be seen from both their standard deviation and their quartile distribution with both extremely high maximums and extremely low minimums.

Figure 4 *– Cumulative Returns of the Long Short with a Long Bias of 25% (unhedged) Portfolio for 06/01/2006 to 29/12/2017*

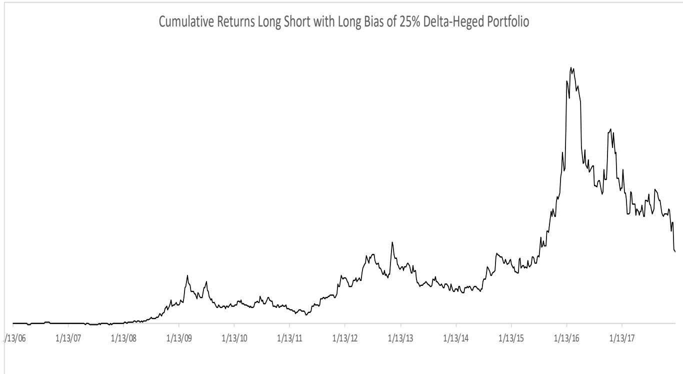


Figure 5 *– Cumulative Returns of the Long Short with a Long Bias of 25% Delta-Hedged Portfolio for 06/01/2006 to 29/12/2017*

After performing the analysis on the portfolios' behavior for the overall sample, it is also important to study the pre and post-crisis period.

As presented in ***Table 1*** the portfolio performance decreases for both portfolios (hedged and unhedged) during this period. In this sense, the pre-crisis period is characterized by a highly volatile nature with portfolios' standard deviation ranging from 66% (Long Biased Portfolio unhedged) to 102% Long Bias Delta-Hedged Portfolio. Moreover, Long Short portfolios verified a volatility in this period of 87% for the unhedged portfolio and 92% for the Delta-Hedged Portfolio. In this sense, the period between 06/01/2006 to 01/01/2008 is characterized by an highly volatile portfolio behavior regardless of exposure and hedge type which shows the nature of the period itself before the market crash with regards to derivatives.

The post-crisis period, on the other hand, is characterized by a less volatile nature with volatility ranging from 54% to 64% for the unhedged Long Bias portfolio and Long Short Portfolio that is Delta-Hedged. Overall, during this post-crisis period, and as presented in ***Table 1***, the portfolios exhibited Information Sharpe Ratios over 0,5 with the Long Short with zero net exposure having a ratio of 1,4231 and 1,2309, respectively for the unhedged and Delta-Hedged Portfolio.

Given this, it is possible to see that, the short leg of the portfolio, independently of the period and of hedging, plays a crucial role in generating portfolio performance. Therefore, the portfolios with no net exposure exhibit higher risk-return ratios and a better overall performance. This points to the fact that, in general, during this period, the underwriting of options, both call and put options, tended to exhibit superior performance and generated abnormal return compared to the buy-side.

All in all, all the portfolios, for the overall sample generate superior performance, with high risk-return tradeoff ratios. In this sense, to are mean-variance optimizer investors, these synthetic portfolios should lead to positive investment decisions.

VI. Conclusion

This study presents an empirical analysis of the effects of stochastic volatility and options portfolio management. According to Black and Scholes (1973) and Merton (1973) the valuation of options is dependent on the assumption that volatility is a constant parameter. However, Duan (1995) presents and alternative option pricing formula with non-constant stochastic volatility. Further studies have accounted for the higher pricing accuracy of GARCH valuation formulas compared to the Black-Scholes formula using, for instance, the MSE for comparison purposes. The study presented shows a different approach to this analysis. Therefore, through the analysis it is possible to show how the monetization of the Black-Scholes formula through stochastic volatility modelling affects the formula accuracy. In this sense, through the creation of synthetic weekly option prices that trade according to the formula proposed by Black and Scholes (1973), it is possible to show that the modelling of GARCH processes can generate superior portfolio performance that should be exploited by mean-variance optimizer investors.

In this sense, through the modelling of volatility according to GARCH (1,1) processes on equity index returns (S&P 500, Russell 2000 and FTSE 100) it is possible to obtain accurate volatility OOS forecasts. Moreover, by applying those OOS forecasts to the generation of systematic trading signals, the generated portfolio on those synthetic options generates superior performance that have high Information Sharpe Ratios and positive Skewness and Kurtosis and high cumulative returns over the period between 06/01/2006 and 29/12/2017, as well as during the pre and post subprime crisis periods. This portfolio, as a portfolio that is heavily reliant on a long and a short leg and given its risk-return profile, presents a valuable addition to other portfolios. Therefore, the high risk and high return profile of the portfolio, give the indication that its addition to a well-diversified investor's portfolio provides asset diversification, as well as a robust profile. In this sense, the robustness of results show that, if plain-vanilla European Call and Put options are to trade according to the Black-Scholes formula, then superior performance can be generated through stochastic volatility modelling.

This study did not use transactional data, which can be an option to exploit in the future. The use of such type of data can provide a practical application to systematic options trading, as well as complement the efforts and test further the existence of arbitrage opportunities related to volatility modelling in the options market.

Moreover, we were not able to test the impact of multivariate volatility models such as the BEKK and DCC MGARCH models that account for dynamic-covariance estimation. This can be particularly important as the number of underlying assets increases to account for spillover effects in asset returns. As presented in Fiszeder (2007), both univariate and multivariate GARCH models have pricing implications on Call and Put options. Therefore, the modelling of MGARCH models can improve portfolio performance.

All in all, the empirical analysis performed shows the clear impact of volatility modelling on the pricing of options. If options are actually to trade according to the Black-Scholes formula, arbitrage opportunities in the options market will persist due to stochastic volatility.

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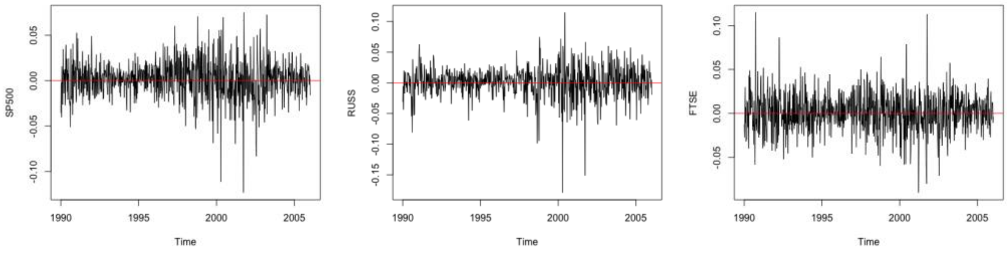
VIII. Appendix

Appendix 1 – Index Log-Returns (S&P 500, Russell 2000 and FTSE 100)



Appendix 2 – Augmented Dickey-Fuller Test (SP500, Russell 2000 and FTSE 100 log-returns)

```
        Augmented Dickey-Fuller Test

data:  SP500
Dickey-Fuller = -9.4476, Lag order = 9, p-value = 0.01
alternative hypothesis: stationary


        Augmented Dickey-Fuller Test

data:  RUSS
Dickey-Fuller = -8.4865, Lag order = 9, p-value = 0.01
alternative hypothesis: stationary

        Augmented Dickey-Fuller Test

data:  FTSE
Dickey-Fuller = -9.5718, Lag order = 9, p-value = 0.01
alternative hypothesis: stationary
```

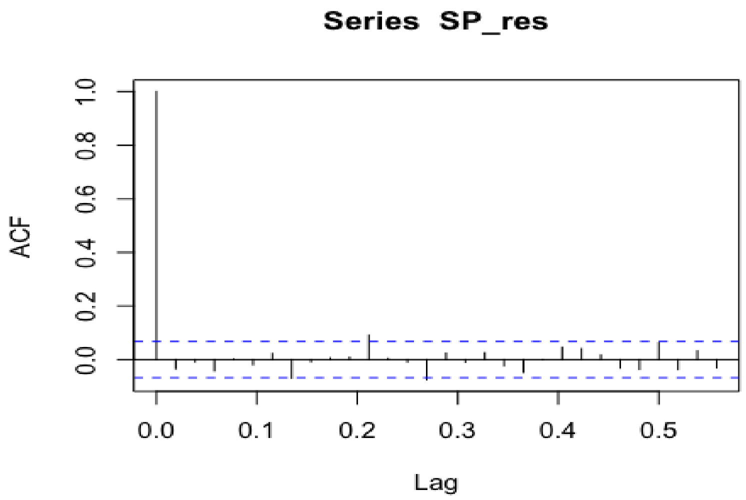
Appendix 5 – ARMA Models using AIC, AICC and BIC

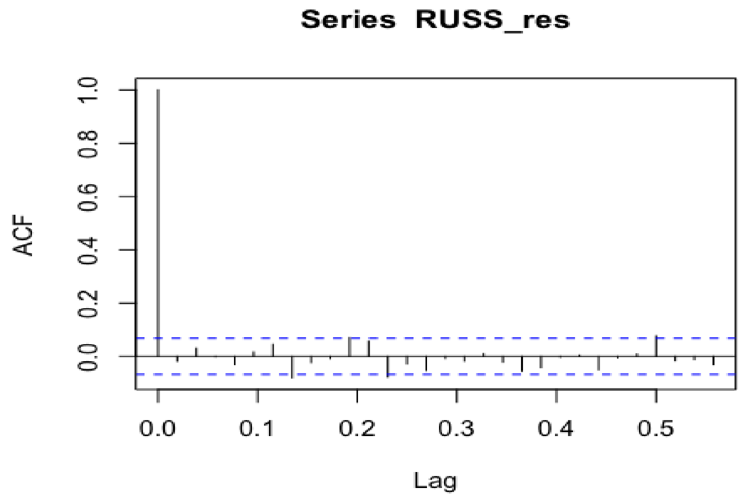
```
Series: SP500
ARIMA(2,0,2)(1,0,0)[52] with non-zero mean

Coefficients:
          ar1      ar2     ma1     ma2    sar1    mean
      -0.0836  -0.7612  0.0322  0.8389  0.0098  0.0015
s.e.   0.0901   0.1015  0.0726  0.0894  0.0354  0.0007

sigma^2 estimated as 0.0004382:  log likelihood=2044.15
AIC=-4074.31   AICc=-4074.17   BIC=-4041.23
```

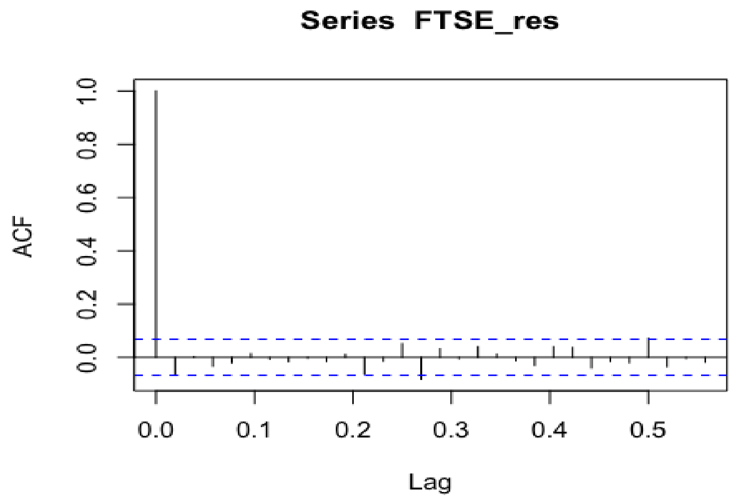
```
Series: RUSS
ARIMA(2,0,2)(1,0,0)[52] with non-zero mean

Coefficients:
         ar1      ar2      ma1     ma2    sar1    mean
      1.4923  -0.6173  -1.4485  0.5922  0.0231  0.0016
s.e.     NaN      NaN      NaN     NaN  0.0355  0.0010

sigma^2 estimated as 0.0006139:  log likelihood=1903.58
AIC=-3793.16   AICc=-3793.02   BIC=-3760.07
```

```
Series: FTSE
ARIMA(0,0,0)(2,0,0)[52] with non-zero mean

Coefficients:
        sar1     sar2    mean
      0.0223  -0.0072  0.0011
s.e.  0.0358   0.0366  0.0008

sigma^2 estimated as 0.0004954:  log likelihood=1991.52
AIC=-3975.04   AICc=-3975   BIC=-3956.14
```

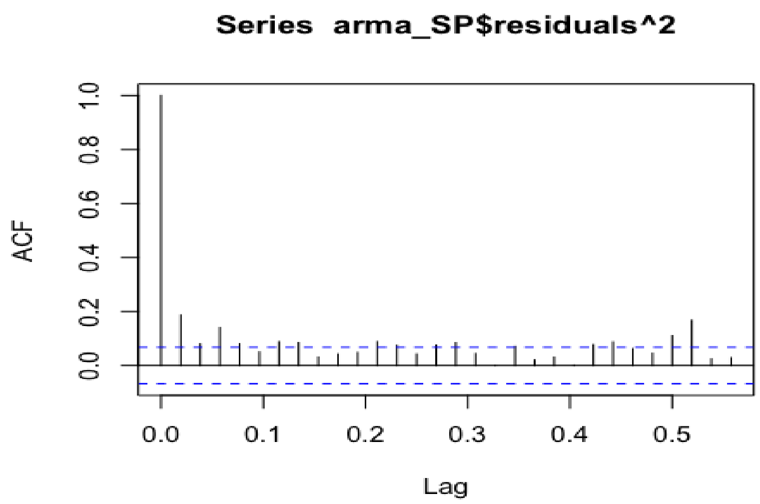
Appendix 3 – ACF for the ARMA residuals

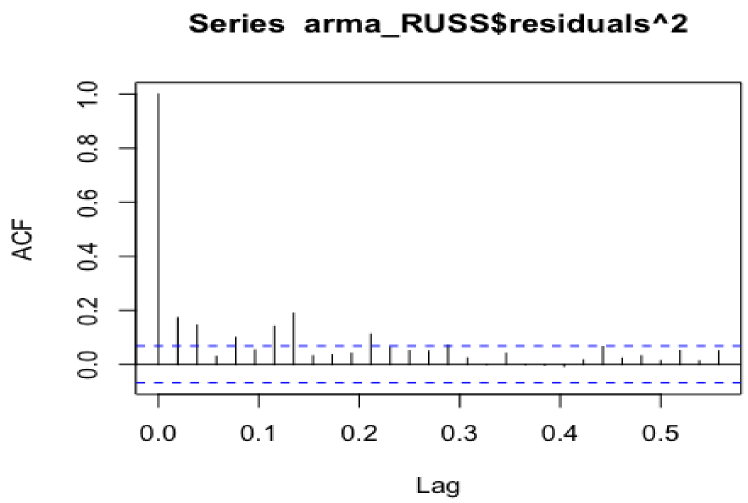


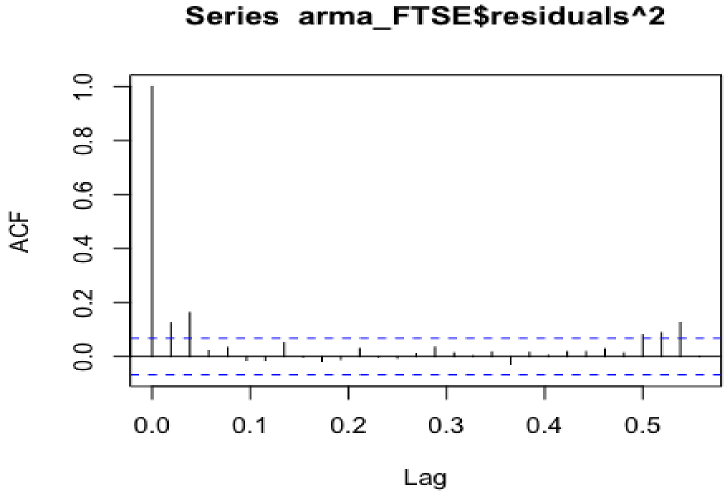




Appendix 4 – ACF for the ARMA squared residuals







Appendix 5 – Ljung-Box Test for GARCH effects

```
Box-Ljung test

data:  arma_SP$residuals^2
X-squared = 75.405, df = 10, p-value = 3.968e-12
```

```
Box-Ljung test

data:  arma_RUSS$residuals^2
X-squared = 103.71, df = 10, p-value < 2.2e-16
```

```
Box-Ljung test

data:  arma_FTSE$residuals^2
X-squared = 39.166, df = 10, p-value = 2.374e-05
```

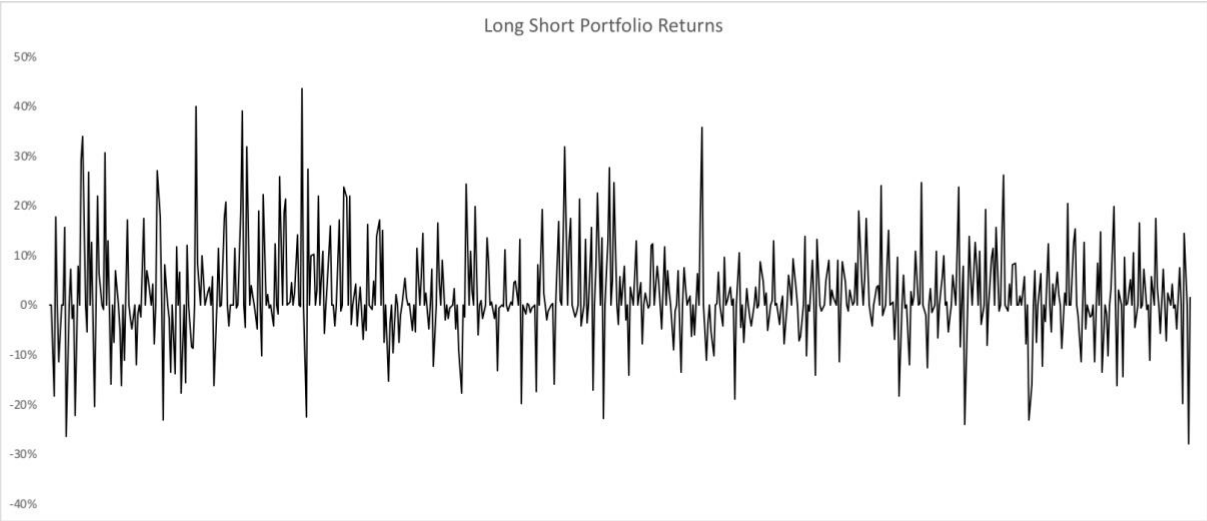
Appendix 6 – Ljung-Box Test for Squared Residuals after GARCH estimation

```
Box-Ljung test

data:  residuals(garch_SP, standardize = TRUE)^2
X-squared = 18.468, df = 20, p-value = 0.5566
```

```
Box-Ljung test

data:  residuals(garch_RUSS, standardize = TRUE)^2
X-squared = 16.371, df = 20, p-value = 0.6933
```

```
Box-Ljung test

data:  residuals(garch_FTSE, standardize = TRUE)^2
X-squared = 28.77, df = 20, p-value = 0.09239
```

Appendix 7 – Long Short Portfolio Returns for 06/01/2006 to 29/12/2017



Appendix 8 – Long Short (Long Bias – 25%) Portfolio Returns for 06/01/2006 to 29/12/2017