A Work Project, presented as part of the requirements for the Award of a Masters Degree in Economics from the NOVA – School of Business and Economics.

Hospital Efficiency

Directed Internship: Deloitte & NOVA SBE

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June 2012

1. Abstract

In a context of economic recession, where resource scarcity follows every manager, efficiency is the only way out. Health Care is no exception. Continuous budgetary cuts from central authorities led to increasing pressures in hospital managers to achieve efficient results. In light of being able to quantify efficiency, this work project has the aim of identifying the best of two frontier based analysis (*Stochastic Frontier Analysis and Data Envelopment Analysis*) by performing efficiency estimations for a single year using variables from the Portuguese reality, allowing the identification of inefficiency sources.

Efficiency scores will be obtained to compare hospitals for efficiency ranks and several efficiency-seeking suggestions will be stated in the end.

The scores obtained from the estimations show that some hospitals still have a rough path to endure if they are to achieve economic efficiency. From the analysis we can see that efficient hospitals vary for each model. (8 in SFA [hospitals with a score higher than 0.95] and 17 in DEA). Estimation outputs suggest that changes in hospital size or services provided should occur in order to achieve higher efficiency, which is in light with the presently taken health policies.

Keywords: Hospitals ; Stochastic Frontier Analysis ; Data Envelopment Analysis ; Technical Efficiency ; Allocative Efficiency

2. Introduction

In harsh times where budget cuts are the main topic in the Portuguese economy, producing more with less has been the golden goal for a country that has been struggling with low productivity. Efficiency is the only alternative to austerity.

The Portuguese economy is being largely affected by the public debt crisis. Years of careless spending with low interest rates and cheap money from the European Union, together with market speculation have led Portugal to an aggravating financial situation ending with the country calling for help to the world's financial institutions.

The coming of the so called *Troika* (European Union, European Central Bank and IMF), has been imposing strict restrictions to the Portuguese government budget.

This austerity has already reached the *Serviço Nacional de Saúde* (SNS) who has seen its annual budget cut in more than €1bn (cuts of 30% in 2012 and 20% in 2013) and several other efficiency seeking measures, like price revision and staff cuts, imposed by the *memorandum.*[1]

Although in Portugal hospitals are considered to be a public service with no profit goal, they still try to minimize costs by changing input allocation so to maximize production and comply with the given budget. (Conrad and Strauss, 1983; Scuffham et al., 1996; Carreira, 1999 and Azevedo, 2011).

The purpose of this work project is so, to study methods that will allow a more accurate and actual analysis of Hospitals in Portugal, understand the causes of inefficiency for Portuguese hospitals and recommend some solutions.

The tools used to study efficiency are based in microeconomic theory, recurring to frontier analysis, using both *Data Envelopment* (DEA) and *Stochastic Frontier* (SFA) analysis. I will be analyzing hospital efficiency using a quadratic cost function. This

[1] "Memorando de Entendimento Sobre as Condicionalidades De Política Económica"

way I am able to compute efficiency based not only in a multi-input environment but also take into account economies of scale and so use a more reality-oriented view.

The work project is divided in five parts. The first, Introduction, gives a starting insight on the theme, as well as the core objectives and personal choice. The second part reports to the Literature Overview explaining core principals of efficiency and measurement. The third and fourth sections present the econometric analysis, explaining the methodology used and the results. The fifth part will present the main conclusions and recommendations, where a more managerial view of the health care service will explain the main problems and post solutions.

Health care efficiency is for these reasons an utmost important topic, considering it tries to preserve human dignity and health standards in an economical context where austerity is the main driver. The efficiency goal is nowadays essential.

3. *Literature Review*

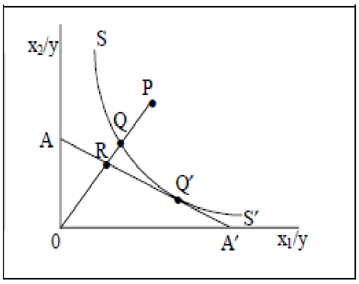
3.1 Efficiency: a definition

Efficiency is the art of attaining the highest achievement with the available resources or, on the other hand, the art of minimizing the resources used to achieve a given goal.

Farrell (1957) defines firm efficiency to be constructed after to components: *technical efficiency* and *allocative efficiency*. The first reports to the ability of the firm to produce an optimal output given the resources' (inputs) quantities available (managerial skills), the second reflects to the use of inputs in optimal quantities given their prices.

Farrell's original idea is based in input-oriented measures, where a fully efficient firm exists, its production structure is defined and so we know the isoquant associated with the firm's production. Firm's use a set of inputs to produce a unit of output.

Figure 1: Efficiency Analysis



Source: Coelli 1996

Efficiency measures are computed using distance measures from the firm set to the frontier, comparing the distance from the firm actual set (P in the graph on the left) with the distance from the optimal point to the same axis. A particular example of distance measurement is the line OP.

Regarding a particular case, let us imagine a firm that uses the combination of inputs P, to produce an output quantity defined by the isoquant S. Technical inefficiency is represented by the distance QP, which is the possible reduction in inputs for the firm maintaining the same output, and thus be more efficient (producing the same with less cost). Technical Efficiency is usually represented by the ratio $\frac{QP}{OP}$, representing the percentage by which all inputs should be reduced (Coelli, 1996). If we invert the measure Technical efficiency can be measured by $\frac{OP}{QP}$, which corresponds to $\frac{QP}{OP} - 1$, this is done to define an easier efficiency measure, taking values between 0 and 1, indicating full efficiency if one (Coelli, 1996).

From basic microeconomic consumer theory it is known that price ratios represent the slope of the budget line. In the producer side, the same applies, and so the input price ratio gives the producer its budget line, see line AA' in Figure 1.

The other component of *Total Efficiency*, *Allocative (Price) Efficiency*, is computed through the ratio $\frac{OR}{OQ}$, where RQ is the reduction in costs if production was to take place at the optimal point Q' (Coelli, 1996). Generally, it shows the efficiency attained by producing an output at observed factor prices relative to the minimum costs of producing in technical efficiency. The objective is to allocate input quantities, given factor prices, so that cost minimization is achieved (Q' in the graph).[2]

The idea is the same applied to every firm. A firm transforms input(s) into output(s) through the production process. Hospitals use its available resources (medical and non-medical staff, equipment, clinical instruments, and others) to treat patients. *"These inputs can be combined in various ways to obtain a certain level of activity, or what is known in economic theory by output".*[3] Firms have a cost associated with this output called *Production Cost*, which results from transforming inputs into outputs given input prices. Firms can reach efficiency by targeting the lowest cost possible for each output level, by choosing inputs and use them in the optimal quantities reaching cost efficiency.

3.2 Theoretical Model

In order to study efficiency the formalization of a production function is needed. The generalized production function can be written as:

$$F(Y, X) = 0 \qquad (1)$$

Where Y represents the maximum levels of production attainable with X input factors. Considering duality theory, where, if the information available allows and (1) has the necessary characteristics.[4] We can obtain the *Dual Cost Function*, the firms' costs given production levels assuming that firms minimize costs.

[2] Forsund et al; (2007)
[3] Harfouche; (2008)
[4] Continuity and almost concavity (Diewert, 1982)

$C = C(Y,W)$[5] (2)

Where W is the vector of input prices and C the production costs.

Hospital efficiency is quite a bit different though. Inputs such as medical instruments are not as controlled or as linearly used as in firms in other economic sectors. Dealing with human life is different, in patient treatment doctors need to expect the unexpected, in some cases the expenditure is larger than what it was thought and vice-versa. *"Feasible Technological combinations are not determined by the economic analyst […] however its characterization is fundamental for an efficiency analysis"*[6].

For so, instead of looking at maximizing production, we take hospital outputs as exogenous and inputs as endogenous causing the analysis to fall on the "cost side of the equation" and perform a cost minimization analysis:

$$C(Y,W) = min_x\{W^T X :\ F(Y,X) = 0\} \quad (3)$$

More specifically, in a multi input-output production typical of hospitals nature, and following the studies of Carreira (1999) and Gonçalves (2008) the use of a *Translog Cost Function*, with the former defined requisites, assuming hypothesis of scale and scope economies, as well as homogeneity and substitution elasticity is preferred.[7]

However, a problem arises in the function, for it does not admit value zero (0) for missing outputs, since the natural logarithm of 0 is undefined. Caves et al. (1980) overcome this problem using the *Box and Cox (1964) Metrics.* Other authors, Cowing

[5] Needs to present several characteristics: *i)* non-negative *ii)* linearly homogeneous in input prices *iii)* non-decreasing in input prices *iv)* concave in *w v)* non-decreasing in output levels and *vi)* differentiable in *w* (Diewert; 1982)
[6] Barros; (1999)
[7] Almeida; (1994) , Azevedo; (2011)

and Holtman (1983), Akridge and Hertel (1986), Rebelo (1992) and Given (1996), choose to use a constant proxy value close to zero (approach used in this study)[8].

Other formulas such as generalized quadratic formulas, allow the use of output with 0 value, although they are not as coherent as *Translog* functions since its formulae does not represent accurately technology in hospital production The sample is sometimes not large enough, which will lead to the use of this type of model reducing efficiency of the estimators.

Methodology

The methods chosen to approach the cost minimization problem are based in optimization. *SFA* and *DEA* have the same objective despite technical differences: An efficient scale where the efforts of cost minimization (in this case) are comparable among firms. Using both models will allow us to correct technical deficiencies.

4.1 Stochastic Frontier Analysis[9]

Stochastic Frontier Analysis or *SFA* is a technique that is based in the stochastic error term in the econometric model. Basically, it divides the error term into two components: inefficiency and residuals. *SFA* computes the inefficiency component in the model's error term. The *Stochastic Frontier Cost Function* is written as:[10]

$$logTOC_{i,j} = \alpha_0 + \sum_{i,j} \alpha_1 Y_{i,j} + \sum_{j,k} \alpha_2 W_{j,k} + \left(V_{i,j,k} + U_{i,j,k}\right) \tag{7}$$

$TOC_{i,j}$ is the Total Operating Cost of the i-th firm for the j-th area

$Y_{i,j}$ is a vector of outputs ; $W_{j,k}$ vector input prices for the k-th input

[8] The value of 0.1 is used by a wide range of authors, among them, Cowing and Holtman (1983), Rebelo (1992) and Carreira (1999). Given (1996) used 0.00001

[9] *Theoretical computation of Translog Function in Complementary Appendix 1.*

[10] Coelli et al; (1979, 1996)

$V_{i,j,k}$ random variables assumed to be iid $N(0,\sigma^2_V)$, independent of

$U_{i,j,k}$ random variables assumed to account for inefficiency and are $|N(0,\sigma^2_U)|$

Thinking about the cost function, U_i is the distance from the firm to the efficiency frontier. U_i gives us both allocative and technical inefficiencies (if allocative efficiency is assumed from the beginning it gives us only technical inefficiency).

Individual efficiencies estimated from the *Stochastic Cost Frontier*, represent the so called distance from the inefficiency point to the perfect efficiency.[11]

$$EEF_i = \frac{E(TOC_i|U_i,X_i)}{E(TOC_i|U_i=0,X_i)} \qquad (8)$$

EEF ranges between 0 and 1, being 1 total efficiency and 0 none efficiency.

Due to data nature, the cost function takes hospital outputs and input prices as exogenous but input quantities as endogenous, since the population attending hospitals is not controlled by the hospital itself, but the resources used are.

The same rationale follows with the use of the *Translog* function.[12] However, due to data restrictions a simpler version of the *Translog* function had to be used. This version forfeits the cross variables in the *Translog* function, leading to the fall of these variables from the original model.

$$\ln TOC = \alpha_0 + \sum_{i=1}^{5} \alpha_1 \ln y_i + \sum_{k=1}^{6} \beta_k \ln w_k + \rho_1 \ln k + \sum_{i=1}^{5} \delta_i \, ln^2 y_i + \sum_{k=1}^{6} \varphi_1 ln^2 w_k + \rho_2 ln^2 k + \varepsilon \qquad (6)$$

[11] Coelli; (1996)

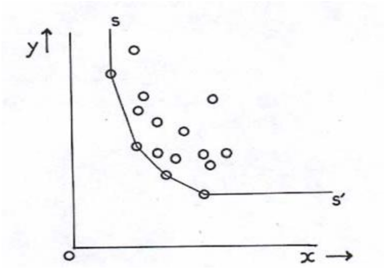
[12] see Complementary Appendix 10.1

4.2 Data Envelopment Analysis[13]

Data Envelopment Analysis or *DEA* was originally developed as a performance measurement technique, for use in public and non-profit sectors, where information about input prices is either missing or not accurate enough for a reliable analysis.

DEA methodology defines a *non-parametric* envelopment frontier, where firms lie on or below (above) the production (cost) frontier (Coelli, 1996), evaluating efficiency through output over input ratio (*Figure 2 and 3*).

Figure 2: Envelopment Frontier

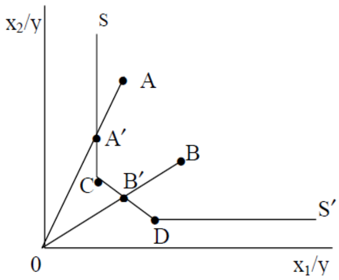


Source: Kittersen 2007

The maximum ratio possible represents complete efficiency since we maximize production with the pre-defined input weights, or vice-versa, the combination of inputs that will produce a determined output quantity.

4.2.1 CRS Assumption:

Figure 3: CRS Assumption



Source: Coelli 1996

The generalized model in Appendix, (Charnes et al. 1978) is only appropriate when all firms operate at an optimal scale. However, imperfect competition, financial constraints and market characteristics, can take the firm of this optimal scale (Coelli, 1996).

Follow-up studies led to the development of a *DEA* model that also regarded the influence of *Variable Returns to Scale* (*VRS*) in efficiency measures.

[13] *Theoretical Computation of DEA estimation in Complementary Appendix 2.*

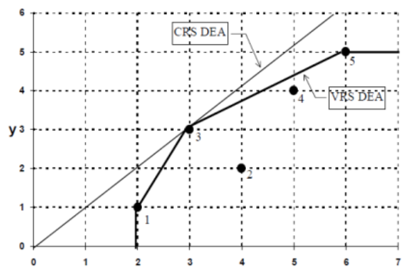
4.2.2 VRS Assumption: [14]

Charnes, Cooper and Banker developed in 1984, a model that allowed the phase-out of scale efficiencies from the efficiency scores. The *VRS* situation is easily achieved from adding one more constraint to the generalized model of 1978: N1'λ =1.[15]

The approach transforms the former conic hull into a set of convex intersecting planes, resulting in efficiency scores equal or higher to those in the *CRS* model.

The difference between the two assumptions indicates scale efficiency.

Figure 4: VRS Assumption



Source: Coelli 1996

With price information, we can consider cost minimization. This way, it is of the interest to measure allocative and technical efficiency.Used to compute Hospital Efficiency.

VRS assumption will emphasize the role of hospital size in the analysis. For so, it is necessary to run first a *VRS* input oriented *DEA* model to obtain technical efficiencies, and run the *Cost Minimization* simulation (13).[16]

Total Cost efficiency (*CE*) is computed by the statistical program as the ratio of the minimum cost to observed cost: $CE = \frac{w'_i x*_i}{w'_i x_i}$. (Coelli, 1996)

From here one can compute the *Allocative Efficiency*: $AE = \frac{CE}{TE}$. (Coelli 1996)

The values obtained for the efficiencies range from 0 to 1. Being 1, total efficiency, with the firm at the frontier, and a value bellow 1inefficiency in the firm.

[14] *Complementary Appendix 2*
[15] Coelli, (1996) ; Banker et al, (1984)
[16] *Complementary Appendix 2*

4.3 SFA vs DEA

Differences arise from the theoretical outlook of both estimations. Jacobs et al (2006) and Gonçalves (2008) summarize these in their empirical project.[17]

Gonçalves (2008), Jacobs (2001) and Jacobs et al. (2006) present two main reasons for the different results from the methods. *i)* differences in building the efficient frontier *ii)* differences in computing the distances to the efficient frontier.[18]

For all these reasons, it is no surprise that efficiency scores obtained from each estimation methods are different.

4. Data

The data for the present work project was retrieved from the NHS Accounting Authority; *Administração Central dos Sistemas de Saúde* (ACSS) database *Base Dados dos Elementos Analíticos* (BDEA), from the National Health Reports from *Direcção Geral da Saúde* and a compiled *Deloitte* database.

The number of observations should be in accordance with the general rule: #Hospitals = (#input variables + #output variables) x 3. Unfortunately, the unavailability of data, requested to the proper authorities at the beginning of the project, led to a restriction in timeframe and sample size, limiting the project to a cross-section analysis (2008) and 46 hospitals;[19] a sample smaller than the optimal, according to the rule of thumb, which will caused biased results in the estimations (specially in the Stochastic Frontier Analysis).

[17] *Table 1 in Appendix*
[18] *See Complementary Appendix 3*
[19] *Complete list in Complementary Appendix Table 1*

The independent variable used in the working project is *Total Operating Cost* (TOC), since output variables are taken as independent as hospitals do not control the demand for its products and considering that hospitals will treat every individual in need.

5.1 Output variables:

The complexity of hospital care results in a series of outputs. Inpatient Days, Clinical Surgery and Day Hospital, are weighted by the case mix index, retrieved from the different *"contratos-programa"* for each activity, to adjust for complexity.

Emergency episodes and External Medical Appointments are taken for its absolute value, since they are not weighted by case -mix.

5.2 Input variables:

Input variables in the estimation models reflect both quantities and prices of inputs used in hospital production. The wide range of hospital services leads to an input mix of far greater complexity.

Human Resources are many times one of the main assets of companies. Public hospitals are no different. Doctors and nurses supported by technicians and other staff, encompass the largest share in Hospital's Total Costs (40%)[20] and provide a delicate service with great responsibility and ethical issues. Due to the data available it was decided to de-aggregate as much as possible the numbers of different staff categories, and so, kept four different kinds of staff: doctors, nurses, technical staff and other staff.

Wages are interpreted as labor price. Unfortunately it is very difficult to know in reality the prices paid for each doctor, nurse or technician and so some assumptions

[20] Relatório e Contas Do Serviço Nacional de Saúde 2008; ACSS

needed to be made. An average wage was computed for each staff category in each hospital dividing the total cost with doctors, nurses, technicians and other staff by the corresponding numbers, following the approach of Carreira (1999).

Drugs and clinical materials are an important current expenditure for hospitals. However, knowing the exact quantities used is impossible. For so a compose good, gathering both clinical material and drugs, was computed.[21]

In order to further show disparity between hospitals, a size variable was included in the regression. The number of beds will be used as a proxy size.[22]

For data variables and variables description see Tables 2 and 3 in Appendix

5. Results

6.1 Stochastic Frontier Analysis

Considering that hospital costs are a function of outputs and input prices as stated before, the generalized function would be similar to equation (6).

However, estimation problems due to high correlation[23] between different variables led to model manipulation, where some first and second order variables were taken out.[24] Four hospitals were taken out of the sample for outlier reasons[25].

Using *STATA 11* as a support tool, the estimated coefficients are the following: (*for full estimation, see Table 7 in Appendix*)

[21] Variable was not used due to heavy correlation between variables and to avoid multicollinearity. If used, a relative price based on average price where every good had a certain weight, needed to be computed
[22] Schuffman et al. (1996), Vita (1990) and Carreira (1999)
[23] Correlation of (0,75) was used as criteria to define high correlation
[24] *Table 5 and 6 in Complementary Appendix for Correlation Matrix and SFA Model Possibilities*
[25] Outlier criteria was based on the ratio TOC/K, representing the rational that larger the hospital (k) , larger the cost TOC; I.O Dr. Gama Pinto, Hosp. Magalhães Lemos, CH Psiquiátrico Lisboa and CH Psiquiátrico Coimbra were taken out of the sample.

Table 8: SFA Estimation

SFA Estimation			Translog Estimation					
lnTOC	Coefficient	SE	lnTOC	Coefficient	SE	lnTOC	Coefficient	SE
lnWT *	1684	2.05	lninpD	0.075	3605.47	lnER^2	0.006	1.6e^4
lnInpD^2 *	0.005	4.03	lnClS	-	-	lnEA^2	0.043	-1.5e^4
lnClS^2	-0.002	-0.87	lnER	-0.53	-1.8e^4	lnDH^2	0.08	3.3e^4
lnER^2	0.0003	0.55	lnEA	-0.366	-5966.24	lnWD^2	0.061	1.5e^4
lnEA^2 *	0.029	11.49	lnDH	-0.033	-2e^4	lnWN^2	-0.29	-4.3e^4
lnDH^2 *	0.007	3.82	lnWD	-1.2	-1.3e^4	lnWT^2	0.276	3.2e^4
lnWD^2	0.005	-1.79	lnWN	6.268	4.6e^4	lnWCMDT ^{2}	-	-
lnWN^2	0.011	4.83	lnWT	-5.672	-3.2e^4	lnK^2	0.031	4.2e^4
lnWT^2 *	-0.083	-2.03	lnWCMDT	-	-	Constant	15.192	2.7e^4
lnWCMDT ^{2}	-0.003	-0.19	lnK	-	-			
lnK^2 *	0.0163	2	lnInpD^2	0.002	1465.07			
Constant	2022	0.51	lnClS^2	-0.006	-5.1e^4			

*significant at 5%

Source: Own Source

An Analysis of the estimators shows that some variables have unexpected signal. Looking at the influence of Clinical Surgery and the price of Complementary Means of Diagnose and Treatment it is expected that with an increase in activity, TOC should increase. However by looking at the estimator values we can see that in fact it is not that linear. In fact, the signal may result from high correlation between variables and when taking the model as a whole, some variables might present a negative influence in TOC. One example is shown when one largely discussed input variable, Doctor Wage, increases by 1 percentage point will cause an increase in TOC of 0.005 percentage points. Observing now one output variable, External Appointments, we can see how it influences Total Operational Cost, causing an increase of 0.029 percentage points when *EA* varies only 1 percentage point, showing the weight of this area in the final cost.

STATA allows the user to obtain inefficiency scores[26] for the different hospitals by computing the residuals in the error term. Scores were mathematically manipulated inverting the results obtained from *STATA*, normalizing the scores to represent an

[26] Total or Cost Efficiency, since full allocative efficiency was not considered the scores do not represent Technical Efficiency

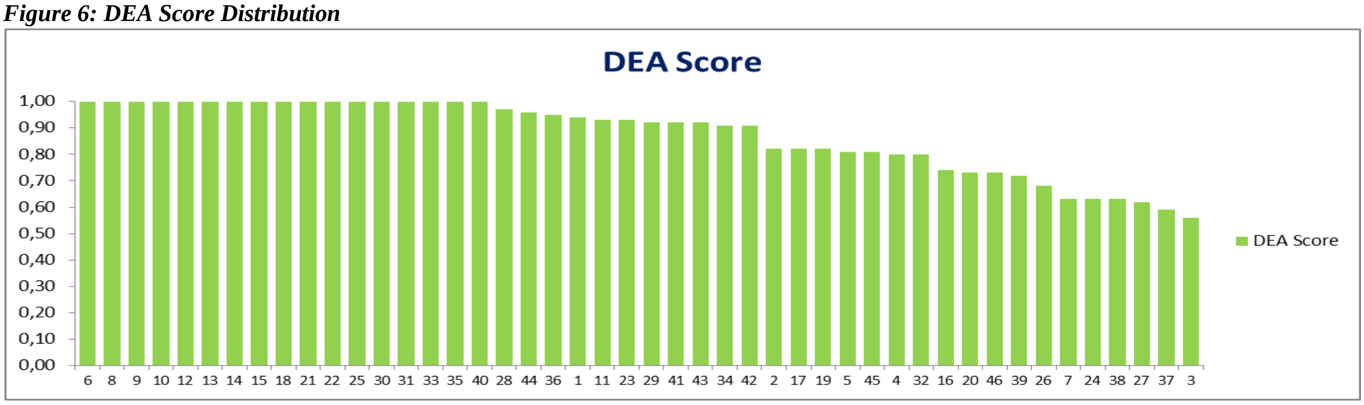
efficiency measure from 0 to 1, with 1 being perfect efficiency. Results can now be compared to the *DEA* scores. *(Table 9 in the Appendix shows full rank).*

6.2 Data Envelopment Analysis

The non-parametric *DEA* model takes in account the same variables used in *SFA* in order to maintain coherence and achieve the cost minimizing input quantities.

The statistical tool used is the software developed by T. Coelli, *DEAP*. In first phase input oriented *VRS* analysis, we obtain optimal input quantities. Secondly, we take these optimal quantities and input prices to perform the cost option and obtain the economic efficiency scores. The program's output presents efficiency scores from 0 to 1, with 1 being total efficiency. It os easy to see that there are a lot of hospitals considered to reach total efficiency (*Table 9 in Appendix for full rank*).

Figure 6: DEA Score Distribution



Source: Own Source

6.3 Comparing results

Table 10: SFA vs DEA

Comparing Results		
SFA		DEA
46	Observations	46
0.86	Mean	0.87
0.11	Standard Deviation	0.14

Source: Own Source

Results show similar statistics despite ranks being different.[27] Gonçalves (2008) performs two tests to check result similarity and statistical significance.[28]

[27] *Figure 7 in Complementary Appendix* , and figure 8 in page 16

A *Correlation test* suggests that the estimations are not that similar (0.24), in fact, by looking at the values we can see that the ranks are really different, contrary to the results of Gonçalves (2008). This happens not only because of the different models but also of the variables chosen.[29]

Clinical Surgery was used not only to differentiate the working paper but also since it has been the area to which specialists have been turning to (around forty percent of hospital programmed surgery was Clinic Surgery in 2008[30]). Recent developments have led former procedures to become simpler, allowing hospitals to transit patients to clinical surgery. Small and fast surgical procedures which do not require inpatient treatment and decrease costs massively, are preferred to extensive and more expensive procedures if allowed by complexity and patients pathology.

Difference in ranks is justified by the differences in models' specification. Analyzing the peer effects from DEA, it is curious that the peers for both the top and bottom 25% efficient hospitals are themselves, which suggest perfect technical efficiency. However, combinations of input prices and quantities stated highly inefficient resource allocation decreasing the total score. Hospitals may have an over or under-usage of doctors, nurses, and other inputs, which is one way to say that the costs with several inputs are too high (low) given what is produced, affecting scores and considering efficient hospitals in *SFA* inefficient in *DEA*.[31]

The *Wilcoxon Test* for statistical significance analyses consistency between scores of both estimations.[32] Following the results from *Table 11,* there is consistency between scores of both estimations, however there are still some differences in ranks for

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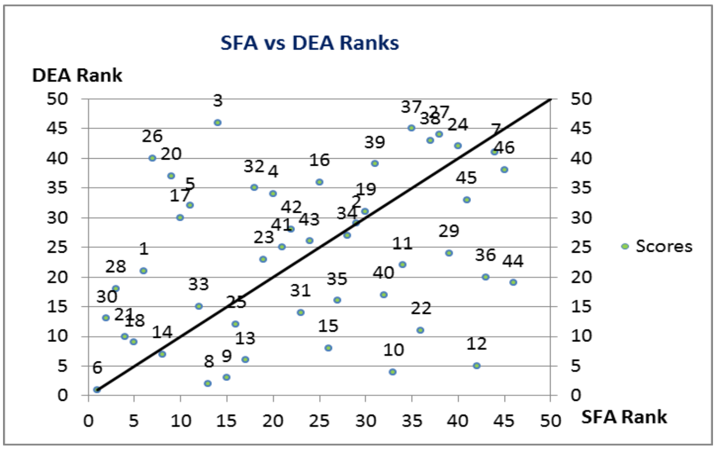
[28] *Correlation test* and a *Wilcoxon Rank test*
[29] Gonçalves considered a maximization problems and different input variables
[30] Relatório e Contas Do Serviço Nacional de Saúde 2008; ACSS
[31] *Difference in ranks of CH Trás-dos-Montes e Alto Douro, Table 9*
[32] *Table 11 in Complementary Appendix*

several hospitals, as seen in the graph below. This happens due to model differences. *DEA* considers firms that do not have a valid peer (similar carachteristics but different outputs) to be tottaly efficient, and suffers from outlier influence[33], which explains so many highly efficient hospitals and differences between ranks.

Figure 8: Rank Disparity



Source: Own Source

6. *Discussion*

The analysis of efficiency scores for both estimations led to several efficiency-oriented conclusions. Increasing Returns to scale give possibility to increase hospital sizing; DEAP output shows the transfer of hospital services between units could increase efficiency; Managerial skills were also draw to analysis.

Following the results obtained in the Stochastic Frontier Analysis, and to show the importance of hospital efficiency in a frugal economic context, savings estimations were made to see how much *Serviço Nacional de Saúde* was to save if every hospital performed at a score of (at least) 90 in the SFA estimation. *Table 12 in Complementary Appendix* shows the results of such estimation. Despite hospitals below the score limit (90) being no more than a few (22), final results show savings of around 400 Million

[33] A follow up DEA estimation was done, dropping the 2 highest and lowest efficient hospitals. Correlation between scores of the new estimation vs original estimation is of about 0.64, clearly stating outlier influence in the DEA Estimation Model.

Euros each year (4.57% of SNS's Annual budget)[34] a reduction in Total Operation Costs of about ten percent! It is patent the emergency of a general efficiency increase.

The analysis of technical efficiency scores from the DEAP output led to the conclusion that hospitals are in *increasing returns to scale* (*IRS*) performance. Hospitals can take advantage of economies of scale, growing up to the optimal point reaching the *Minimum Efficient Scale* (Minimum Average Cost). This outcome is in line with several recent efforts, with mergers in several public hospitals (e.g. Merger of Hospital Santa Maria, EPE and Hospital Pulido Valente into CH Lisboa Norte, EPE).

One curious fact is however the reduction in the number of beds for some hospitals (*Table 13 in Complementary Appendix*). This does not mean that the hospitals should be smaller, it means that decision makers should emphasize policies towards patient treatment capacity instead of infrastructural increase.

One other efficiency-oriented measure would be the transfer of several services from some hospitals to other more efficient and with "room to grow" hospitals. A production area in *IRS* with growth potential in one hospital can be more efficient that the same production area in a nearby hospital. Results show that to achieve efficiency some of the hospitals should increase an output and decrease another (e.g. CH Coimbra should increase Clinical Surgery and decrease Day Hospital); of course, they cannot be replaced one by the other and also, depending also on the specificity of said hospital.[35]

However, we must bear in mind that aggregating two or more hospitals is not a linear process. Impact studies on the population served should be done to ensure that the social outcome of aggregating hospital is positive.

[34]Serviço Nacional de Saúde annual budget: 8.3075€ billion; source: Relatório e Contas SNS, 2008
[35] *Table 14 in Complementary Appendix*

Clinical surgery is a cost efficient activity. Health Care development has (as previously mentioned) created the conditions for complicated procedures to become simpler; the patient "becomes cheaper" and goes home earlier. A win-win situation.

Despite model limitations with regard to input usage, it is clear to observe that resource spending is, in some cases, excessive.[36]

A managerial approach to the results led to the realization of an informal enquiry in Delloite's Health Staff. The enquiry was asked to seven Deloitte Professionals and had for data the 25% best and worst performing hospitals, asking for an efficiency score based on the knowledge from working with a certain hospital. In order to disclose the reasons for such differences, causes for inefficiency (efficiency) were also asked. [37]

Different reasons were exposed as causes for inefficiency,[38] but great emphasis was brought upon Human Capital. Managerial skills of both decision makers and clinical staff were pointed out as the main cause for an inefficient score. Other reasons of insufficient supply diversification or poor effort in efficiency seeking led to lower scores than expected.[39] These reasons cannot be directly input in the models for obvious mathematical reasons (it is impossible to know exactly the skill of a doctor or health management), which can explain different results from the staff's expectations.

However, if a certain hospital is considered efficient (Hospital São João), the scores are high and reasons stated are satisfactory; Cost control, both in human resources and drug use, as well as a high specialization pattern help to achieve efficiency, according to Deloitte Professionals.

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[36] *Table 12 in Complementary Appendix*
[37] *Table 15 in Complementary Appendix*
[38] *Table 16 in Complementary Appendix*
[39] *Table 15 in Complementary Appendix*

7. Conclusions

In choosing one of the methods to perform the efficiency analysis, I would have to say that the choice should rest upon not only on data availability and quality but also on the final objectives of the analysis. *DEA* methods allow an easier estimation since it does not require a parametric background, with a multiple output analysis. *SFA* requires a parametric analysis with hypothesis testing, not perform well with small samples.

From the Discussion above, it is fair to conclude that recent policies have been in light with eficicency seeking goals. A more emphasys to clinical care and the understanding of taking advantage of economies of scale have been patent in the recent hospital mergers and service transfers between hospitals. Human resources are now carefully planned, hring new staff is now satrting to have in mind future health careneeds instead of onely present.[40]

Nevertheless, limitations have risen during the project sharing the same problems with others. Lack of Data availability led to some assumptions that may not be in line with reality (average prices and composite goods). Shortage in the number of observations influenced results in the SFA estimation, while outlier presence has had a similar effect in DEA. Other limitations intrinsic to the nature of the model do not take into account some crucial aspects that clearly influence public hospital reality; managerial capacity, the negotiation power of different players, and the supply of different services unajusted to reality, since it only takes in account global indicators.[41]

Given the present outlook of the Portuguese economy and *Serviço Nacional de Saúde's* budgetary perspectives, an efficiency ranking is more than welcome so that cost efficiency goals are achieved and full efficiency is reached at a national level.

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[40] ACSS (2011), "Actuais e Futuras Necessidades Previsionais de Médicos (SNS) "
[41] An analysis to each specialty would require more data

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9. Appendix

Table 1: Comparing Methodologies

Comparing Methods	SFA	DEA
Assumption about Functional Form	Strong*	None
Distinguish Random Error from Efficiency Variation	Yes	None
Test for Inclusion of variables	Imperfectly	No
Allow for Exogenous Factors	Yes	Yes
Allow for Multiple Outputs	Not readily	Yes
Provides Information on "Peer" Organizations	Not automatically	Yes
Vulnerable to Outliers	Moderately*	Yes
Problems of Multicollinearity	Yes*	No
Problems of Endogeneity	Yes*	Yes
Problems of Heteroscedasticity	Yes*	No
Vulnerable to Small Sample Size	Yes*	Moderately

Source: Jacob et al, 2006 ; Gonçalves 2008

*Assumption is testable

Table 3: Data Description

Variable	Name	Defintition
TOC	Total Operational Cost	Annual costs excluding capital expenses
InpD	Inpatient Days	Total inpatient days weighted by case mix
ClS	Clinical Surgery	Clinical Surgery episodes weighted by case mix
DH**	Day Hospital	Day Hospital Sessions weighted by case mix
ER	Emergency episodes	Number Emergency episodes
EA	External Appointments	Number of external appointments
DT	Doctors	Number of Doctors
NS	Nurses	Number of Nurses
T	Technicians	Number of Health technical staff
OS*	Other Staff	Number of other staff including supervisors
DClM	Drugs & Medical Material	Quantities of Drugs and Medical Materials used
CMDT	Complementary Means of Diagnose and Treatment	Number of weighted Means od Diagnose and Treatment
K	Beds	Number of Beds
WD	Doctor's Wage	Average Doctor's Anual salary
WN	Nurse's Wage	Average Nurses' Anual salary
WT	Tecnician's Wage	Average Technical Staff's Anual salary
WOS*	Other Sff's Wage	Average Other staff's Anual salary
WDCIM*	Drugs & Medical Material price	GDP Deflator
WCMDT	CMDT Price	Average CMDT Anual Price
WK	Price of Beds	Representative price of beds (=1)

Source: Own Source

*For purposes of correlation and collinearity these variables have been taken out for the SFA estimation, they were also taken out of the DEA estimation for both estimations so that they could be compared

** Correlated with other variables but significant for analysis, considered in the models

Table 4: Statistical Data

Variable	Mean	Standard Deviation	Minimum Value	Maximum Value
TOC	93078545,43	87647659,95	4518554,2	371560036,47
InpD	110352,66	109142,31	0	504351,55
ClS	2287,41	2594,65	0	11289,98
DH	3287,75	3701,92	0	16419,49
ER	135306,33	99267,65	0	235367
EA	174232,20	71424,41	96505	292260
DT	223,00	76,11	118	296
NS	371,67	104,22	255	508
TC	129,72	121,89	6	576
OS	640,54	543,22	64	2541
DCIM	26634,72	33258,75	291,97	145964,94
CMDT	2311,17	2522,45	41,09	10687,82
K	381,33	93,38	250	459
WD	35021,21	17035,90	5797,31	108549,21
WN	2390,26	12702,34	8981,2	66607,95
WT	19648,33	11152,46	7748,27	61174,42
WO	8798,64	3915,14	2314,50	28644,07
WDCIM	0,93	0	0,93	0,93
WCMDT	8,54	4,64	2,68	33,45
WK	1	0	1	1

Source: Own Source

Table 7: STATA Coefficients table*

```
Stoc. frontier normal/exponential model          Number of obs   =         46
                                                 Wald chi2(11)   =    4555.92
Log likelihood =  23.451291                      Prob > chi2     =     0.0000
```

lntoc	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lnwt	1.684462	.8204608	2.05	0.040	.0763881	3.292535
sqLNInpD	.0053542	.0013278	4.03	0.000	.0027517	.0079566
sqLNCls	-.0020596	.0023797	-0.87	0.387	-.0067237	.0026046
sqLNEA	.0288961	.002521	11.46	0.000	.023955	.0338372
sqLNER	.0003494	.0006328	0.55	0.581	-.0008909	.0015896
sqLDH	.0073535	.001926	3.82	0.000	.0035786	.0111283
sqLWD	.0054136	.0030215	1.79	0.073	-.0005084	.0113357
sqLWN	.0105936	.0021914	4.83	0.000	.0062984	.0148887
sqLWT	-.0829596	.0408565	-2.03	0.042	-.1630369	-.0028824
sqLWMCDT	-.0032452	.017136	-0.19	0.850	-.0368312	.0303408
sqLNWK	.0163808	.0082082	2.00	0.046	.0002931	.0324685
_cons	2.022454	3.934038	0.51	0.607	-5.688119	9.733028
/lnsig2v	-5.801039	.993428	-5.84	0.000	-7.748122	-3.853956
/lnsig2u	-3.580243	.4640659	-7.71	0.000	-4.489796	-2.670691
sigma_v	.0549946	.0273166			.0207738	.1455875
sigma_u	.1669399	.0387356			.1059384	.2630673
sigma2	.0308933	.0112744			.0087958	.0529908
lambda	3.035567	.0598813			2.918201	3.152932

Source: Own Source

*Exponential distribution of Hospital Cost (Complementary Appendix Figure 5), required the Exponential Distribution option on STATA estimation

Table 9: Efficiency Scores

SFA Efficiency Scores			RANK	DEA Efficiency Scores		
ID	Hospitals	SFA Score		ID	Hospitals	DEA Score
6	CH do Tâmega e Sousa, EPE	0.98	1	6	CH do Tâmega e Sousa, EPE	1
30	CH do Médio Tejo, EPE	0.97	2	8	H de S. João	1
28	H Santo André – Leiria	0.96	3	9	H Nossa Sra da Conceição - Valongo	1
21	H José Luciano de Castro - Anadia	0.96	4	10	IPO Francisco Gentil (Porto), EPE	1
18	H S. Teotónio – Viseu	0.96	5	12	CH de Vila Nova de Gaia/Espinho, EPE	1
1	H Sta Maria Maior, EPE - Barcelos	0.96	6	13	CH de Entre o Douro e Vouga, EPE	1
26	IPO Francisco Gentil (Coimbra), EPE	0.95	7	14	H Dr. Francisco Zagalo - Ovar	1
14	H Dr. Francisco Zagalo - Ovar	0.95	8	15	H Conde de Salreu - Estarreja	1
20	ULS Guarda, EPE	0.95	9	18	H S. Teotónio - Viseu	1
17	H de Águeda	0.95	10	21	H José Luciano de Castro - Anadia	1
5	CH do Alto Ave, EPE	0.95	11	22	H Arcebispo João Crisóstomo - Cantanhede	1
33	CH de Torres Vedras, EPE	0.95	12	25	Hospitais Universitários de Coimbra	1
8	H de S. João	0.94	13	30	CH do Médio Tejo, EPE	1
3	CH Trás-os-Montes Alto Douro, EPE	0.94	14	31	CH Oeste Norte, EPE	1
9	H Nossa Sra da Conceição - Valongo	0.94	15	33	CH de Torres Vedras, EPE	1
25	Hospitais Universitários de Coimbra	0.94	16	35	CH de Lisboa Norte, EPE	1
13	CH de Entre o Douro e Vouga, EPE	0.93	17	40	CH de Lisboa Central, EPE	1
32	H de Santarém, EPE	0.93	18	28	H Santo André - Leiria	0.97
23	CH da Cova da Beira, EPE	0.93	19	44	ULS do Baixo Alentejo, EPE	0.96
4	CH Nordeste, EPE	0.93	20	36	H Dr. José de Almeida, Cascais	0.95
41	CH Barreiro/Montijo, EPE	0.92	21	1	H Sta Maria Maior, EPE - Barcelos	0.94
42	CH de Setúbal, EPE	0.90	22	11	CH do Porto, EPE	0.93
31	CH Oeste Norte, EPE	0.90	23	23	CH da Cova da Beira, EPE	0.93
43	H Espírito Santo, EPE	0.90	24	29	H Amato Lusitano - castelo Branco	0.92
16	H Infante D. Pedro – Aveiro	0.90	25	41	CH Barreiro/Montijo, EPE	0.92
15	H Conde de Salreu – Estarreja	0.89	26	43	H Espírito Santo, EPE	0.92
35	CH de Lisboa Norte, EPE	0.65	27	34	H Reynaldo dos Santos	0.91
34	H Reynaldo dos Santos	0.88	28	42	CH de Setúbal, EPE	0.91
2	H S. Marcos – Braga	0.85	29	2	H S. Marcos - Braga	0.82
19	H Cândido Figueiredo - Tondela	0.83	30	17	H de Águeda	0.82
39	Maternidade Dr. Alfredo da Costa	0.83	31	19	H Cândido Figueiredo - Tondela	0.82
40	CH de Lisboa Central, EPE	0.83	32	5	CH do Alto Ave, EPE	0.81
10	IPO Francisco Gentil (Porto), EPE	0.83	33	45	H de Faro, EPE	0.81
11	CH do Porto, EPE	0.82	34	4	CH Nordeste, EPE	0.8
37	IPO Francisco Gentil (Lisboa), EPE	0.79	35	32	H de Santarém, EPE	0.8
22	H Arcebispo João Crisóstomo - Cantanhede	0.79	36	16	H Infante D. Pedro - Aveiro	0.74
38	H Curry Cabral, EPE	0.78	37	20	ULS Guarda, EPE	0.73
27	H Distrital de Pombal	0.76	38	46	CH do Barlavento Algarvio,EPE	0.73
29	H Amato Lusitano - castelo Branco	0.74	39	39	Maternidade Dr. Alfredo da Costa	0.72
24	CH de Coimbra, EPE	0.73	40	26	IPO Francisco Gentil (Coimbra), EPE	0.68
45	H de Faro, EPE	0.72	41	7	ULS de Matosinhos, EPE	0.63
12	CH de Vila Nova de Gaia/Espinho, EPE	0.68	42	24	CH de Coimbra, EPE	0.63
36	H Dr. José de Almeida, Cascais	0.68	43	38	H Curry Cabral, EPE	0.63
7	ULS de Matosinhos, EPE	0.67	44	27	H Distrital de Pombal	0.62
46	CH do Barlavento Algarvio,EPE	0.62	45	37	IPO Francisco Gentil (Lisboa), EPE	0.59
44	ULS do Baixo Alentejo, EPE	0.45	46	3	CH Trás-os-Montes Alto Douro, EPE	0.56

Source: Own Source

Hospital Efficiency

Directed Internship: Deloitte & NOVA SBE

Pedro Sucena e Castro,   #486

Complementary Appendix

1-Translog Function

The Translog function is given by an arithmetical computation applied to the short run cost function of the second order Taylor's Series,

$C^s = C(Y, W, K)$, being the short-run cost function (4)

Where k is the amount of the fixed input (In the specificity of the case, k is the dimension proxy)

$TOC = \alpha_0 + \sum_{i=1}^{3} \alpha_1 \ln y_i + \sum_{k=1}^{2} \beta_k \ln w_k + \rho_1 \ln k \; + + \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \alpha_{ij} \ln y_i \ln \; y_j +$

$12 \quad k=12l=12\beta kl \ln \quad yk \ln \quad wl+12 \quad \rho 11 \quad ln2$

$k+i=13l=12\delta il \ln yi \ln wk++i=13\varphi i1 \ln yi \ln k+k=12\mu k1 \ln wk \ln k+ \varepsilon$

(5)

Where *TOC* is the Total Operational Cost, y_i the outputs, w_k the input prices, k the fixed input and ε the error factor. It is easy to see the limit of the *Translog* functions when firms have output 0, $\ln y_i$, where $y_i = 0$, and so, $\ln y_i$ is unidentified.

Given the number of observations with null values it is easier to use, and does not influence negatively the coherence of the study, a proxy closer to zero (0.1)[42] to correct the limitation given by the *Translog* function.

As said before, to consider the *Translog* function as a valid cost function we need to impose the properties of a cost function: *i)* symmetry restrictions in the second order terms *ii)* linear homogeneity in input prices.

$\alpha_{ij} = \alpha_{ji}, \; \forall_{i \neq j}, \; i, j = 1, \dots, 4 \qquad \beta_{kl} = \beta_{lk}, \; \forall_{j \neq k}, \quad l, k = 1,2$

$$\sum_{l=1}^{2} \beta_k = 1 \qquad \sum_{l=1}^{2} \beta_{kl} = 0, \quad k = 1,2 \qquad \sum_{k=1}^{2} \delta_{ik}, \quad i = 1, \dots, 4 \qquad \sum_{k=1}^{2} \mu_{k1}$$

[42] The value of 0.1 is also used by a wide range of authors, among them, Cowing and Holtman (1983), Rebelo (1992) and Carreira (1999). Given (1996) used 0.00001

The Translog function computes a large series of estimates for a small number of inputs and outputs. For that fact it is normal to present strong correlation between variables originating problems of multicollinearity. To minimize this possibility, a set of *share equations* for a multivariate regression system is adjusted to the model, increasing the efficiency of the estimators. Using the *Shephard's Lemma* and differentiating equation (5) to each input price we have the required *Share Equations.*

2- DEA Method

Choice of optimal input weights uses Linear Programming: (Coelli, 1996)

$$\boldsymbol{max_{u,v}}\ (\boldsymbol{u'y_i}/\boldsymbol{v'x_i}),$$

$$\boldsymbol{st}:\ \ u'y_j/v'x_j \ \leq 1,\ \ j = 1,2,3,\dots,N \qquad ; \qquad u,v \ \ \geq 0 \qquad (9)$$

Statistical programs allows us to compute the values for u and v that maximize efficiency measures for the different firms, subject to the constraint that this measures need to be less or equal to one.

The duality property allows us to achieve the desired DEA model:

$$\boldsymbol{min_{\theta,\lambda}\ \theta},$$

$$\boldsymbol{st}:\ -y_i + Y\lambda \geq 0, \qquad ; \qquad \theta x_i - X\lambda\ \geq\ 0, \qquad ; \qquad \lambda\ \ \geq 0 \qquad (11)$$

Where θ is a scalar and λ is a vector of Nx1 constants. θ is the efficiency score for the firm. It is smaller than one, with one being total efficiency (on the frontier), and zero, total inefficiency. (Farrell, 1957), (Coelli,1996).

<u>The VRS Constraint</u>

$$\boldsymbol{min_{\theta,\lambda}\ \theta},$$

$$\boldsymbol{st}:\ -y_i + Y\lambda \geq 0, \quad ; \quad \theta x_i - X\lambda \geq 0, \quad ; \quad N1'\lambda = 1 \quad ; \quad \lambda \geq 0 \qquad (12)$$

Cost minimization simulation: (Coelli 1996)

$$\boldsymbol{min_{\lambda,xi*}\ w_i{}'x_i\ *},$$

$$\boldsymbol{st}:\ -y_i + Y\lambda \geq 0, \quad ; \quad x_i * - X\lambda \geq 0, \quad ; \quad N1'\lambda = 1 \quad ; \quad \lambda \geq 0 \qquad (13)$$

Given that w_i is a vector input prices for the i^{th} firm and x_i* (computed in the first regression) is the vector of the optimal input quantities, given input prices, and output quantities y_i

3- SFA vs DEA

Following Gonçalves (2008),

When building the efficiency frontier, *DEA* assumes a correct specification by drawing the frontier in sections with real information (peers) and data is observed without errors (Gonçalves, 2008). *SFA* estimation considers errors and so, even if the frontiers are the same, the scores will be different from *DEA*. In situations where the error component is important, the best method to use is *SFA* since it inputs the error in the analysis whilst *DEA* may consider units to be wrongly efficient.

In computing distances and defining efficiency scores, *DEA* builds efficiency scores for each firm by comparison with other firms (peers) of comparable outputs (Gonçalves 2008). This brings along two major problems, one is that if there is no comparable firm within the sample, the analyzed firm is considered to be technically efficient even if it is not. Other problem is that when an inefficiency score is attained, it is so because it is being compared with the firm's peer and not any other inefficient firms. *SFA* considers all the information in the sample. (Gonçalves, 2008)

The *SFA* estimation requires, opposite to *DEA*, a functional form of a production function. Basically, it requires the analyst to theoretically define the problem and not just compute the solution through the statistical software. This allows the user to test the validity of the data in hand. (Banker, 1996; Grosskopf, 1996; Gonçalves, 2008)

However, *DEA* presents one important advantage: it allows the analysis of a multi-output problem, whereas *SFA* requires a two-step estimation. (Jacobs et al, 2006)

Regarding outlier influence, *DEA* estimations are easily influenced, taking outliers as super-efficient firms. *SFA* scores are highly dependent of information present in the sample; the problem is simply overcome by sample manipulation.

Sample dimension also has its influence in the different methods. *SFA* requires a large sample, with size highly dependent of the number of parameters. The non-parametric nature of the *DEA* estimation, allows the use of small samples.

Table 2: Sample list

Sample	
Hospital	**ID**
H Sta Maria Maior, EPE - Barcelos	1
H S. Marcos - Braga	2
CH Trás-os-Montes Alto Douro, EPE	3
CH Nordeste, EPE	4
CH do Alto Ave, EPE	5
CH do Tâmega e Sousa, EPE	6
ULS de Matosinhos, EPE	7
H de S. João	8
H Nossa Sra da Conceição - Valongo	9
IPO Francisco Gentil (Porto), EPE	10
CH do Porto, EPE	11
CH de Vila Nova de Gaia/Espinho, EPE	12
CH de Entre o Douro e Vouga, EPE	13
H Dr. Francisco Zagalo - Ovar	14
H Conde de Salreu - Estarreja	15
H Infante D. Pedro - Aveiro	16
H de Águeda	17
H S. Teotónio - Viseu	18
H Cândido Figueiredo - Tondela	19
ULS Guarda, EPE	20
H José Luciano de Castro - Anadia	21
H Arcebispo João Crisóstomo - Cantanhede	22
CH da Cova da Beira, EPE	23
CH de Coimbra, EPE	24
Hospitais Universitários de Coimbra	25
IPO Francisco Gentil (Coimbra), EPE	26
H Distrital de Pombal	27
H Santo André - Leiria	28
H Amato Lusitano - castelo Branco	29
CH do Médio Tejo, EPE	30
CH Oeste Norte, EPE	31
H de Santarém, EPE	32
CH de Torres Vedras, EPE	33
H Reynaldo dos Santos	34
CH de Lisboa Norte, EPE	35
H Dr. José de Almeida, Cascais	36
IPO Francisco Gentil (Lisboa), EPE	37
H Curry Cabral, EPE	38
Maternidade Dr. Alfredo da Costa	39
CH de Lisboa Central, EPE	40
CH Barreiro/Montijo, EPE	41
CH de Setúbal, EPE	42
H Espírito Santo, EPE	43
ULS do Baixo Alentejo, EPE	44
H de Faro, EPE	45
CH do Barlavento Algarvio,EPE	46

Source: Own Source

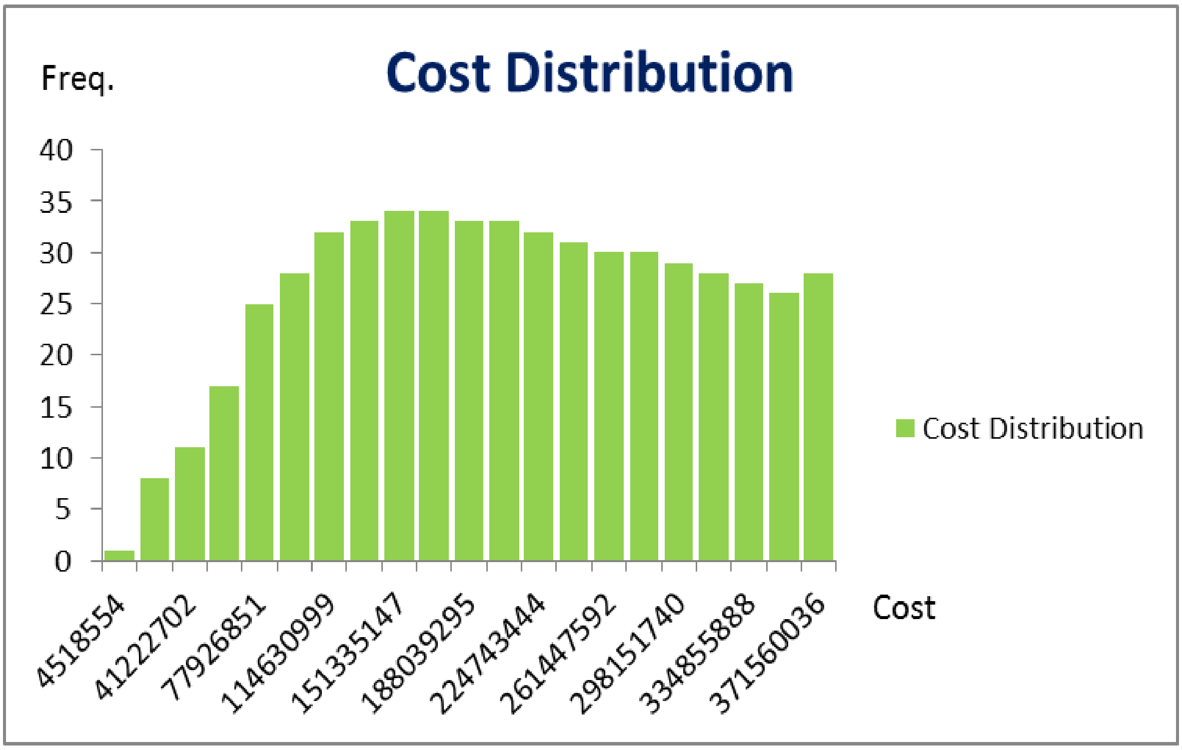
Table 5: Correlation Matrix

	lncust~l	lndias~n	lnciru~o	lncons~a	lnurge~a	lnhosp~a	ln~dicos	lnpr~ros	ln~nicos	lnpreo~l	lnpreo~t	lnk
lncustototal	1.0000											
lndiasintern	0.6524	1.0000										
lncirurgam~o	0.5268	0.3189	1.0000									
lnconsulta~a	0.9807	0.5864	0.5015	1.0000								
lnurgencia	0.4327	0.3572	0.6433	0.3904	1.0000							
lnhospdia	0.8163	0.5345	0.5180	0.7945	0.4190	1.0000						
lnpreomdicos	0.2022	-0.1417	-0.0090	0.1892	0.0247	0.1359	1.0000					
lnpreoenfe~s	0.0816	-0.1473	0.0647	0.0713	0.1032	0.0721	0.6045	1.0000				
lnpreotcni~s	-0.1014	-0.3514	-0.0543	-0.0921	0.0304	-0.0487	0.6480	0.6534	1.0000			
lnpreooutr~l	-0.0499	-0.4065	-0.0232	-0.0281	0.0042	-0.0224	0.7958	0.6233	0.7515	1.0000		
lnpreomcdt	-0.1136	-0.3031	-0.2804	-0.0885	-0.2541	-0.3869	0.1877	0.2293	0.2445	0.2553	1.0000	
lnk	0.9692	0.6663	0.5173	0.9474	0.5062	0.7796	0.1675	0.0428	-0.1047	-0.0796	-0.1133	1.0000
sqlndiasin~n	0.7176	0.9925	0.3681	0.6563	0.3978	0.5844	-0.1049	-0.1214	-0.3268	-0.3689	-0.2777	0.7348
sqlncirurg~o	0.7797	0.4847	0.8780	0.7760	0.5389	0.6262	0.0359	0.0255	-0.0957	-0.0461	-0.1377	0.7636
sqlnconsul~a	0.9780	0.5768	0.5125	0.9983	0.3868	0.7764	0.1759	0.0625	-0.0956	-0.0290	-0.0787	0.9456
sqlnurgencia	0.5494	0.4127	0.6693	0.5134	0.9713	0.4891	0.0909	0.0979	0.0290	0.0149	-0.2149	0.6183
sqlnhospdia	0.8787	0.5785	0.5129	0.8518	0.3788	0.9233	0.1260	-0.0123	-0.1101	-0.0613	-0.2839	0.8476
sqlnpreomd~s	0.2031	-0.1599	-0.0044	0.1925	0.0293	0.1383	0.9986	0.6075	0.6566	0.8011	0.2042	0.1685
sqlnpreoen~s	0.0856	-0.1482	0.0682	0.0751	0.1102	0.0771	0.5958	0.9989	0.6470	0.6062	0.2276	0.0492
sqlnpreotc~s	-0.0989	-0.3560	-0.0498	-0.0892	0.0364	-0.0428	0.6347	0.6431	0.9988	0.7356	0.2420	-0.1000
sqlnpreoou~l	-0.0435	-0.4244	-0.0157	-0.0198	0.0134	-0.0147	0.7916	0.6176	0.7524	0.9981	0.2682	-0.0720
sqlnpreomcdt	-0.1285	-0.3128	-0.3456	-0.0994	-0.2929	-0.4111	0.1899	0.2087	0.2442	0.2565	0.9883	-0.1196
sqlnk	0.9625	0.6548	0.5390	0.9446	0.4935	0.7505	0.1456	0.0257	-0.1148	-0.0850	-0.0916	0.9947

	sqlndi~n	sqlnci~o	sqlnco~a	sqlnur~a	sqlnho~a	sq~dicos	sqln~ros	sq~nicos	sqlnpr~l	sqlnpr~t	sqlnk
sqlndiasin~n	1.0000										
sqlncirurg~o	0.5521	1.0000									
sqlnconsul~a	0.6478	0.7901	1.0000								
sqlnurgencia	0.4634	0.6314	0.5099	1.0000							
sqlnhospdia	0.6371	0.6747	0.8421	0.4749	1.0000						
sqlnpreomd~s	-0.1207	0.0414	0.1792	0.0962	0.1263	1.0000					
sqlnpreoen~s	-0.1215	0.0275	0.0657	0.1052	-0.0102	0.5999	1.0000				
sqlnpreotc~s	-0.3304	-0.0929	-0.0930	0.0346	-0.1068	0.6448	0.6385	1.0000			
sqlnpreoou~l	-0.3835	-0.0375	-0.0208	0.0265	-0.0548	0.7998	0.6024	0.7388	1.0000		
sqlnpreomcdt	-0.2883	-0.1801	-0.0911	-0.2500	-0.3032	0.2063	0.2074	0.2426	0.2700	1.0000	
sqlnk	0.7254	0.7888	0.9477	0.6075	0.8263	0.1463	0.0312	-0.1108	-0.0781	-0.1018	1.0000

Source: Own Source

Figure 5: Hospital Cost Distribution



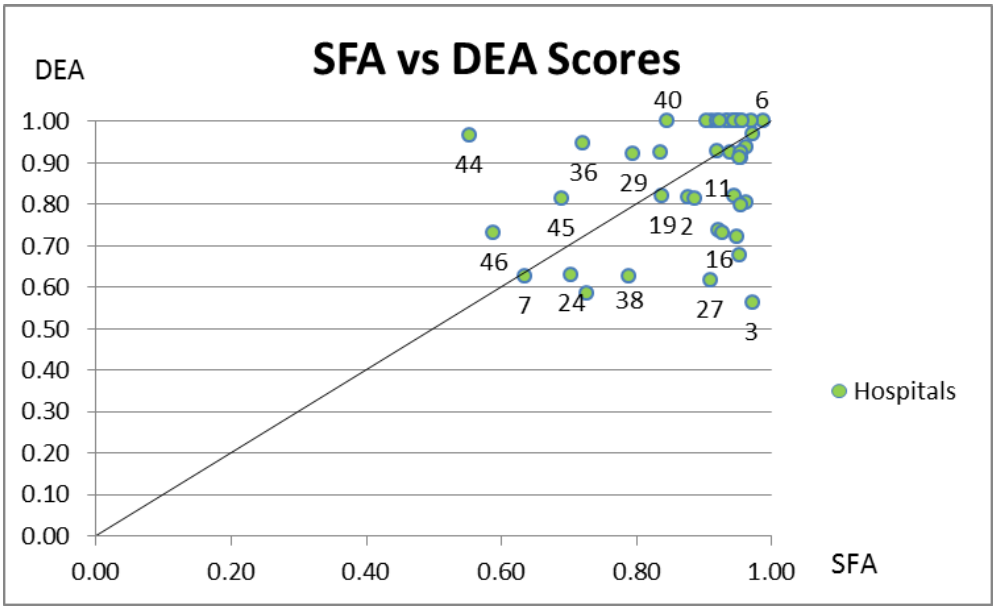
Source: Own Source

Table 6: SFA model comparison

SFA Models' specification				
lnTOC	Coef. (z) Total Model	Cobb-Douglas	Chosen Model	Chosen Model (significant vars)
lnInpDays	0.075 (3605.47)	0.124 (3.93)	-	-
lnClinC	-	0.684 (7.80)	-	-
lnER	-0.053 (-1.8e+04)	-0.0001 (-0.01)	-	-
lnExtAp	-0.366 (-5966.24)	0.684 (7.80)	-	-
lnDH	-0.033 (-2.0e+04)	0.023 (1.37)	-	-
lnWD	-1.200828 (-1.3e+04)	0.154 (1.70)	-	-
lnWN	6.268 (4.6e+04)	0.120 (1.40)	-	-
lnWT	-5.672 (-3.2e+04)	-0.010 (-0.10)	1.684 (2.05)	1.628 (1.76)
lnWCMDT	-	0.140 (1.41)	-	-
lnk	-	0.227 (1.96)	-	-
sqlnInpDays	0.002 (1465.07)	-	0.005 (4.03)	0.006 (4.55)
sqlnClinC	-0.006 (-5.1e+04)	-	-0.0021 (-0.87)	-
sqlnER	0.006 (1.6e+04)	-	0.003 (0.55)	-
sqlnExtAp	0.043 -1.50E+04	-	0.029 (11.49)	0.028 (10.77)
sqlnDH	0.008 (3.3e+04)	-	0.007 (3.82)	0.008 (5.22)
sqlnWD	0.061 (1.5e+04)	-	0.0054 -1.79	0.005 (1.65)
sqlnWN	-0.290 (-4.3e+04)	-	0.011 (4.83)	0.011 (4.69)
sqlnWT	0.276 (3.2e+04)	-	-0.083 (-2.03)	-0.081 (-1.76)
sqlnWCMDT	-	-	-0.003 (-0.19)	-
sqlnk	0.031 (4.2e+04)	-	0.016 (2)	0.014 (2.22)
_cons	15.192 (2.7e+04)	4.015 (3.85)	2.022 (0.51)	2.400 (0.54)
/lnsig2v	-38.220 (-0.13)	-3.577 (-6.69)	-5.801 (-9.31)	-5.286 (-9.79)
/lnsig2u	-3.675 (-12.46)	-5.655 (-1.39)	-3.580 (-9.99)	-3.775 (-8.83)
sigma_v	0.000	0.167	0.055	0.071
sigma_u	0.159	0.059	0.167	0.151
sigma2	0.025	0.031	0.031	0.028
lambda	31700000.000	0.354	3.036	2.128

Source: Own Source

Figure 7 : SFA vs DEA Scores



Source: Own Source

Table 11: Wilcoxon Test

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	19	468	540.5
negative	27	613	540.5
zero	0	0	0
all	46	1081	1081

Ho: te = deascore
z = -0.683
Prob > |z| = 0.4947

Source: Own Source

Results show that in 19 observation the values of the SFA estimation are superior to the scores of the *DEA*, and vice-versa for other 27 observations. A joint significance test (null hypothesis: Ho: te = DEA, meaning the median of scores is equally centered for both models), being *te* the *SFA* efficiency scores presented with the *Wilcoxon Test*, shows results that the null hipothesis is not rejected for the pvalues, evidencing consistency between scores of both estimations

Table 12: Economic result of achieving a (at least) 90/100 efficiency (drawn from SFA)

Economic Turnout for a (at least) 90/100 efficiency					
Hospital	**ID**	**Actual Costs**	**Costs (min 90/100 efficiency)**	**Savings**	**Savings (%)**
H Sta Maria Maior, EPE - Barcelos	1	23,653,351 €	23,653,351 €	0 €	-
H S. Marcos - Braga	2	117,095,915 €	108,835,704 €	8,260,211 €	7.05%
CH Trás-os-Montes Alto Douro, EPE	3	111,907,176 €	111,907,176 €	0 €	-
CH Nordeste, EPE	4	60,800,143 €	60,800,143 €	0 €	-
CH do Alto Ave, EPE	5	81,632,621 €	81,632,621 €	0 €	-
CH do Tâmega e Sousa, EPE	6	73,900,775 €	73,900,775 €	0 €	-
ULS de Matosinhos, EPE	7	127,019,837 €	92,606,510 €	34,413,327 €	27.09%
H de S. João	8	307,051,505 €	307,051,505 €	0 €	-
H Nossa Sra da Conceição - Valongo	9	8,713,649 €	8,713,649 €	0 €	-
IPO Francisco Gentil (Porto), EPE	10	110,926,370 €	99,867,014 €	11,059,355 €	9.97%
CH do Porto, EPE	11	247,055,914 €	220,451,880 €	26,604,033 €	10.77%
CH de Vila Nova de Gaia/Espinho, EPE	12	145,690,458 €	107,750,616 €	37,939,842 €	26.04%
CH de Entre o Douro e Vouga, EPE	13	91,918,861 €	91,918,861 €	0 €	-
H Dr. Francisco Zagalo - Ovar	14	8,355,438 €	8,355,438 €	0 €	-
H Conde de Salreu - Estarreja	15	5,173,331 €	5,043,193 €	130,138 €	2.52%
H Infante D. Pedro - Aveiro	16	64,725,272 €	63,373,731 €	1,351,541 €	2.09%
H de Águeda	17	13,187,779 €	13,187,779 €	0 €	-
H S. Teotónio - Viseu	18	99,492,820 €	99,492,820 €	0 €	-
H Cândido Figueiredo - Tondela	19	7,273,869 €	6,572,928 €	700,941 €	9.64%
ULS Guarda, EPE	20	24,462,238 €	24,462,238 €	0 €	-
H José Luciano de Castro - Anadia	21	4,518,554 €	4,518,554 €	0 €	-
H Arcebispo João Crisóstomo - Cantanhede	22	5,364,171 €	4,608,763 €	755,408 €	14.08%
CH da Cova da Beira, EPE	23	60,041,493 €	60,041,493 €	0 €	-
CH de Coimbra, EPE	24	155,189,852 €	122,423,879 €	32,765,973 €	21.11%
Hospitais Universitários de Coimbra	25	277,134,909 €	277,134,909 €	0 €	-
IPO Francisco Gentil (Coimbra), EPE	26	45,878,421 €	45,878,421 €	0 €	-
H Distrital de Pombal	27	7,983,455 €	6,562,289 €	1,421,166 €	17.80%
H Santo André - Leiria	28	64,382,098 €	64,382,098 €	0 €	-
H Amato Lusitano - castelo Branco	29	47,657,354 €	38,366,683 €	9,290,671 €	19.49%
CH do Médio Tejo, EPE	30	93,822,808 €	93,822,808 €	0 €	-
CH Oeste Norte, EPE	31	56,188,614 €	56,188,614 €	0 €	-
H de Santarém, EPE	32	74,022,312 €	74,022,312 €	0 €	-
CH de Torres Vedras, EPE	33	43,888,923 €	43,888,923 €	0 €	-
H Reynaldo dos Santos	34	36,580,825 €	34,838,373 €	1,742,453 €	4.76%
CH de Lisboa Norte, EPE	35	371,560,036 €	332,546,414 €	39,013,622 €	10.50%
H Dr. José de Almeida, Cascais	36	54,920,366 €	40,311,607 €	14,608,759 €	26.60%
IPO Francisco Gentil (Lisboa), EPE	37	113,563,765 €	97,704,678 €	15,859,087 €	13.96%
H Curry Cabral, EPE	38	109,152,362 €	92,508,399 €	16,643,963 €	15.25%
Maternidade Dr. Alfredo da Costa	39	32,539,349 €	29,401,022 €	3,138,327 €	9.64%
CH de Lisboa Central, EPE	40	360,850,739 €	325,482,743 €	35,367,996 €	9.80%
CH Barreiro/Montijo, EPE	41	85,748,325 €	85,748,325 €	0 €	-
CH de Setúbal, EPE	42	106,602,565 €	106,602,565 €	0 €	-
H Espírito Santo, EPE	43	73,683,172 €	73,683,172 €	0 €	-
ULS do Baixo Alentejo, EPE	44	75,483,737 €	36,918,008 €	38,565,729 €	51.09%
H de Faro, EPE	45	119,358,841 €	93,601,784 €	25,757,057 €	21.58%
CH do Barlavento Algarvio, EPE	46	75,458,746 €	50,813,875 €	24,644,871 €	32.66%
TOTAL		4,281,613,113€	390,157,864€	380,034,471€	**8.88%**

Table 13: Variation in actual to optimal input

Variation from actual to optimal intputs					
Hospital	DT	NS	T	CMDT	K
1	31%	5%	24%	-41%	0%
2	-31%	-12%	12%	-17%	-3%
3	11%	-27%	-23%	-63%	-29%
4	45%	-38%	-17%	-33%	-30%
5	-10%	41%	14%	-51%	42%
6	0%	0%	0%	0%	0%
7	-26%	-14%	-12%	-59%	5%
8	0%	0%	0%	0%	0%
9	0%	0%	0%	0%	0%
10	0%	0%	0%	0%	0%
11	24%	19%	28%	-24%	31%
12	0%	0%	0%	0%	0%
13	0%	0%	0%	0%	0%
14	0%	0%	0%	0%	0%
15	0%	0%	0%	0%	0%
16	2%	-3%	5%	-55%	4%
17	3%	-3%	304%	-51%	-7%
18	0%	0%	0%	0%	0%
19	15%	-31%	-3%	-26%	-6%
20	44%	-39%	-13%	-44%	-19%
21	0%	0%	0%	0%	0%
22	0%	0%	0%	0%	0%
23	5%	-23%	-25%	-63%	-10%
24	-36%	-28%	-21%	-58%	3%
25	0%	0%	0%	0%	0%
26	-6%	25%	-45%	-58%	29%
27	-13%	8%	12%	-65%	0%
28	43%	10%	24%	-37%	-5%
29	13%	-22%	-22%	-24%	-25%
30	0%	0%	0%	0%	0%
31	0%	0%	0%	0%	0%
32	-28%	-19%	31%	-23%	12%
33	0%	0%	0%	0%	0%
34	-41%	-28%	44%	19%	23%
35	0%	0%	0%	0%	0%
36	-24%	-16%	10%	7%	16%
37	-19%	69%	-51%	-63%	9%
38	-41%	20%	-59%	-59%	-4%
39	-38%	-15%	4%	-33%	23%
40	0%	0%	0%	0%	0%
41	-29%	-25%	-5%	-9%	10%
42	-19%	65%	-38%	-50%	2%
43	-10%	-18%	-32%	-42%	-10%
44	-4%	-36%	-47%	-56%	-15%
45	-10%	-22%	-32%	-55%	1%
46	28%	-21%	-36%	-53%	12%
MEAN	-3%	-4%	1%	-26%	1%

Source: Own Source

Table 14: Variation in actual to optimal output

Variation from actual to optimal Outputs					
Hospital	InpD	ClS	EA	ER	DH
1	0%	0%	0%	0%	-85%
2	0%	0%	0%	0%	-83%
3	0%	0%	0%	0%	-83%
4	0%	0%	0%	0%	-72%
5	0%	0%	0%	0%	-84%
6	0%	0%	0%	0%	-84%
7	0%	0%	0%	0%	-85%
8	0%	0%	0%	0%	-83%
9	0%	0%	0%	0%	0%
10	0%	0%	0%	0%	-87%
11	0%	0%	0%	0%	-84%
12	0%	0%	0%	0%	-80%
13	0%	0%	0%	0%	-85%
14	0%	0%	0%	0%	-75%
15	0%	0%	0%	0%	0%
16	0%	0%	0%	0%	-84%
17	0%	0%	0%	0%	-53%
18	0%	0%	0%	0%	-84%
19	0%	0%	0%	0%	-100%
20	0%	0%	0%	0%	-83%
21	0%	0%	0%	0%	-83%
22	0%	0%	0%	0%	0%
23	0%	304%	0%	51%	-85%
24	0%	28%	1%	11%	-82%
25	0%	0%	0%	0%	-88%
26	0%	0%	0%	0%	-88%
27	0%	0%	0%	0%	-84%
28	0%	0%	0%	0%	-85%
29	0%	202%	34%	24%	-83%
30	0%	0%	0%	0%	-85%
31	0%	0%	0%	0%	-85%
32	0%	0%	0%	0%	-85%
33	0%	0%	0%	0%	-85%
34	0%	0%	0%	0%	-68%
35	0%	0%	0%	0%	-79%
36	0%	0%	0%	0%	-76%
37	0%	0%	0%	0%	-88%
38	0%	0%	0%	0%	-77%
39	0%	0%	0%	0%	-72%
40	0%	0%	0%	0%	-81%
41	0%	62%	0%	246%	-36%
42	0%	0%	0%	0%	-81%
43	0%	388%	0%	58%	-83%
44	3%	0%	0%	20%	-76%
45	0%	64%	2%	26%	-86%
46	0%	0%	0%	0%	-84%
MEAN	0%	23%	1%	9%	-76%

Source: Own Source

Table 15: Comparing results with Market Knowledge

Estimated Results vs Market Knowledge			
Hospitals	SFA Rank	DEA Rank	Enquiry Rank
CH de Setúbal, EPE	3	7	7
CH do Barlavento Algarvio,EPE	8	6	9
H Curry Cabral, EPE	6	8	3
H de Faro, EPE	7	4	6
H de S. João	2	1	1
H Infante D. Pedro - Aveiro	4	5	4
H Santo André - Leiria	1	2	2
IPO Francisco Gentil (Lisboa), EPE	5	9	5
ULS do Baixo Alentejo, EPE	8	3	7
Correlation to Expected Results	72%	38%	-

Source: Own Source

Table 16: Efficiency Inefficiency Reasons

Enquiry	
Efficiency Drivers	**Inefficiency Causes**
- Cost control in human resources and drug use	- Unadjusted supply to the diversity of medical needs
- Focus on financial results and cost control	- Unskilled decision makers and clinical leaders with weak management capacity
- Specialization in certain service areas may contribute to efficiency seeking	- Disconnected goals between administration and clinical decision makers
- Potential for service differentiation	- Policy followed at a national level may cause a comfortable stagnation in the services provided
- Cases of high case mix index, benefits Hospital funding	- Unadjusted number of medical staff (more than needed)
- Power of Negotiation	- Lack of Physical Resources

Source: Own Source