ESSAYS ON DISCRIMINATORY PRICING IN OLIGOPOLISTIC DIFFERENTIATED MARKETS

by

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Contents

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Preface

This dissertation is composed of four essays which address discriminatory pricing in oligopolistic differentiated markets.

Under the most varied setups, a vast literature has explored the impact of different pricing policies upon various economic issues such as welfare, entry deterrence, locational choice and long-run market structure. The first two essays extend the discriminatory versus non-discriminatory pricing's literature to a setup where *equilibrium* prices are nonlinear. Both essays focus on a spatial competitive model where consumer's preferences differ over two dimensions: brand preferences or horizontal preferences (which are typically related with consumer's location in the market) and quality preferences or vertical preferences (which are related to consumer's marginal valuation of product quality). Discrete *vertical uncertainty* is modeled by assuming that at each location there are two types of consumers with different preferences for quality, either low or high valuation, and that these consumer's vertical preferences are not observable by the competing firms whereas consumer's brand preferences are complete information. In these circumstances nonlinear prices arise in *equilibrium*. Within this framework two different nonlinear pricing policies are compared: delivered nonlinear pricing, where firms offer different price-quality nonlinear contracts at each location and mill nonlinear pricing, where firms are not allowed to discriminate among consumers at different locations and they must simultaneously offer a unique price-quality nonlinear contract. All degrees of vertical differentiation (given by the ratio of the types' quality preference parameters) and degrees of horizontal differentiation (*i.e.*, the typical spatial models' differentiation parameter) where competition is effective are analyzed.

The first essay presents a short-run analysis under a linear city model. Considering two identical firms, full characterization of *equilibrium* contracts is studied for each nonlinear pricing policy. The comparison of the *equilibrium* nonlinear contracts under the two pricing policies shows that the quality offered to high valuation consumers under both pricing policies is the socially efficient one. The quality offered to low valuation consumers is the socially optimal one under mill nonlinear pricing but it may be suboptimal under delivered nonlinear pricing. The two policies may also differ in terms of market coverage of low valuation consumers. There is only a small set of degrees of vertical and horizontal differentiation where there are more low valuation consumers being served under mill nonlinear pricing.

Previous nonlinear pricing studies showed that when firms use delivered nonlinear pricing the type of contracts offered depends on the location and there can exist different market regions: monopoly, intermediate and competitive regions. The complete analysis of delivered nonlinear pricing shows that these three market regions may not always exist. In fact, we show that in order to have the three market regions we must have a relatively low vertical differentiation degree and a relatively high horizontal differentiation degree. For high degrees of vertical differentiation, the monopoly region does not exist even for high degrees of horizontal differentiation.

Apart from a complete characterization of *equilibrium* nonlinear contracts, the first essay also presents a short-run welfare comparison of the two nonlinear pricing policies. Taking into account contracts' efficiency as well as market coverage, the short-run welfare under delivered nonlinear pricing may be smaller, equal or larger than under mill nonlinear pricing.

The welfare comparison performed in the first essay considers a given market structure. However, complete welfare comparison should also include the effect of pricing policies on market structure since the entry incentives differ in the two entry policies. The second essay discusses the impact of nonlinear pricing policies on the free entry long-run *equilibrium.* Using a circular city model, the free entry long-run *equilibrium* number of firms is computed for each pricing policy when firms' relocation is costless, *i.e.*, when there is spatial contestability.

It is shown that the free entry long-run *equilibrium* number of firms under delivered nonlinear pricing may be smaller, equal or higher than under mill nonlinear pricing, depending on the degree of vertical and horizontal differentiation. The main contribution of this essay is to show that delivered nonlinear pricing may increase the free entry long-run *equilibrium* number of firms. This result only occurs when vertical differentiation degree is low and horizontal differentiation degree is close to the limit where local monopolies for both types of consumer start to appear. We also prove that under spatial contestability there is excess of entry under both mill nonlinear pricing and delivered nonlinear pricing.

This framework applies to the study of firms' pricing policy regulation since it revisits the issue of whether regulatory authorities should prohibit firms from practicing price discrimination among consumers who differ according to some observable characteristic (consumer's location). We extend the analysis of the economic justification of firms' pricing policies regulation to a nonlinear pricing setup. The analytical results show that delivered nonlinear pricing yields higher long-run welfare than mill nonlinear pricing when *(i)* fixed costs are low and when *(ii)* fixed costs are intermediate and consumer types are sufficiently different. Numerical results suggest that this also holds for other parameters' values, supporting the view that discriminatory nonlinear prices should not be prohibited.

A paper based on the first two essays (with more emphasis on the second essay) was submitted to the International Journal of Industrial Organization who required a revision of the paper. This revision has already been sent and a decision over the publication of this paper is forthcoming.

The third essay focuses in the impact of firms' pricing policies upon entry decision. We extend the analysis of the impact of pricing policies upon entry to a framework where price competition and differentiated products are present. We consider a model where an incumbent firm serves two distinct and independent geographical markets and an entrant firm may enter in one of the markets offering a differentiated product. Entry under discriminatory pricing is *more likely* than under uniform pricing when we have entry costs levels where entry is profitable under discriminatory pricing but unprofitable under uniform pricing.

We compare the impact of firms' pricing policies upon entry decision. We analyze how this comparison is affected by the difference in the competitive threat across markets, the difference in market demands and the degree of product substitutability. This latter effect was not present in previous studies.

It is shown that the impact of pricing policies upon entry is ambiguous since entry under discriminatory pricing may be *more*, *less* or *equally likely* than under uniform pricing. For all degrees of product substitutability, there is always some level of degree of difference in market demands where under discriminatory pricing entry is *more likely* than under uniform pricing. This happens for a larger interval of degree of difference in market demands when there are lower degrees of product substitutability.

We reinforce the idea that any regulation decision should be based on a complete evaluation of the market conditions in terms of markets' demand differences and degree of product substitutability.

The fourth essay is on discriminatory signaling prices to deter entry. In a two-period framework, a multimarket incumbent firm faces a single potential entrant firm in one of the two markets where he operates. We develop a model of cost-based informational asymmetry where the incumbent has private information about his production costs, either low or high. The entrant observes the incumbent's first period prices in both markets and decides whether to enter or not. If entry occurs in the second period firms are price-setting duopolists in a differentiated product market. The incumbent may use both pre-entry prices as *predatory signals* for his production costs. We thus have *discriminatory limit pricing*.

To date, the use of differentiated products has not been widely explored in cost-based informational asymmetry setups. We show that the degree of product substitutability affects the low cost incumbent firm's multiple signaling strategy.

Due to the lack of restrictions upon *off-the-equilibrium* path posterior beliefs, we find multiple pure strategy perfect bayesian *equilibria*. However, we show that there is always a unique reasonable perfect bayesian *equilibrium*. In some cases the only *equilibrium* that survives the domination criterion is the least cost separating *equilibrium*. In this separating *equilibrium*, the low cost incumbent's *complete information prices* may no longer hold. The low cost incumbent uses pre-entry prices in both markets to signal low production cost and there may be a downward distortion in both pre-entry prices. Moreover, we show that this distortion is identical in both markets for every relationship between markets' demand parameters. The price distortion is increasing with the discount factor, the degree of product substitutability and the efficiency of the entrant. For the remaining cases, a pooling *equilibrium* where both types of incumbents set the low cost incumbent's monopoly prices is the unique *equilibrium* that survives the Grossman and Perry (1986) criterion.

An implication of the analysis to predatory pricing detection is the fact that regulatory authorities should reach worldwide information since firms are becoming global and may price lower than the monopoly prices in more than one market in order to deter entry. Multinational firms may not be using purely discriminatory pricing but *discriminatory limit pricing*.

Chapter 1

Delivered versus Mill Nonlinear Pricing in a Duopoly

1.1 Introduction

Nonlinear pricing has been widely explored in the past two decades. Nonlinear pricing analysis within a competitive environment gave rise to a vast number of studies, starting with the notable examples of Katz (1984), Spulber (1989), Champsaur and Rochet (1989) and more recently Stole (1995), Hamilton and Thisse (1997), Armstrong and Vickers (2001) and Rochet and Stole (2002).

Focusing on a spatial competitive model, Stole (1995) considers a framework where consumer's preferences differ over two dimensions: brand preferences (horizontal preferences – which are typically related with each consumer's location in the market space) and quality preferences (vertical preferences – consumer's marginal valuation of product quality). Stole first presents *vertical uncertainty*: a setting where consumers' vertical preferences are unknown by the competing firms while consumers' brand preferences are common knowledge. In this setting nonlinear prices arise in *equilibrium*. Within this framework one can analyze two different nonlinear pricing policies: delivered nonlinear pricing – DNP – firms offer different nonlinear contracts at each location; or mill nonlinear pricing – MNP – firms are not allowed to discriminate among consumers at different locations and they must offer a unique nonlinear contract.

There are three relevant studies that discuss these pricing policies using a duopoly model considering two types of consumers at each location. Villas-Boas and Schmidt-Mohr (1999) study mill nonlinear contracts under discrete *vertical uncertainty*. They analyze a credit-market setting where banks offer a menu of credit contracts (which include a gross interest rate and a collateral requirement) and show that those contracts and their screening degree depend upon the degrees of horizontal differentiation (*i.e.*, the typical differentiation parameter in spatial models) and vertical differentiation (given by the ratio of the types' quality preference parameters).

Delivered nonlinear pricing was studied by Pires and Sarkar (2000) and Valletti (2002). Both analyze the locational *equilibrium*, the former considering price-quantity nonlinear contracts and the latter price-quality nonlinear contracts. The discrete case analysis showed that when firms use delivered nonlinear pricing the type of contracts offered depends on the location and there can be different market regions: monopoly, intermediate and competitive regions. In addition, Valletti studies what happens to these regions as the vertical differentiation degree increases but considering a fixed horizontal differentiation degree. However, it would be interesting to explore how the *equilibrium* contracts change with the degrees of horizontal and vertical differentiation since this would give a complete description of delivered nonlinear pricing.

Notice that these articles do not compare delivered nonlinear pricing with mill nonlinear pricing (since they study each pricing policy separately) and this is certainly an interesting comparison to do. In this chapter we use a discrete *vertical uncertainty* framework where at each location there are two types of consumers with different preferences for quality, either low or high valuation. We characterize the *equilibrium* solution contracts for delivered nonlinear pricing and mill nonlinear pricing for all degrees of vertical and horizontal differentiation where competition is effective. Our analysis has two objectives: firstly, to complete the analysis of previous studies on delivered nonlinear pricing; secondly, to compare the two pricing policies in terms of contracts' efficiency, market coverage and welfare.

Our analysis of delivered nonlinear pricing shows that the three market regions identified by Pires and Sarkar (2000) and by Valletti (2002) may not always exist. In fact, we show that in order to have all these regions we need to have a relatively low degree of vertical differentiation and a relatively high degree of horizontal differentiation. For a low degree of horizontal differentiation, only competitive contracts are offered. For a high degree of vertical differentiation, the monopoly region does not exist even for high degrees of horizontal differentiation.

The comparison of the nonlinear contracts under the two pricing policies shows that the quality offered to the high valuation consumers under both pricing policies is the socially efficient one. The quality offered to the low valuation consumers is the socially optimal one under mill nonlinear pricing but it may be suboptimal under delivered nonlinear pricing. The two pricing policies may also differ in terms of market coverage of the low valuation consumers. Taking into account the efficiency of the quality offered to the low valuation consumers as well as their market coverage, we compared welfare under the two pricing policies. The welfare results are ambiguous: welfare under delivered nonlinear pricing may be smaller, equal or larger than under mill nonlinear pricing.

The chapter will proceed as follows: Section 1.2 presents the linear city model with two identical firms and studies the *equilibrium* contracts for each pricing policy. Section 1.3 analyzes and compares both pricing policies in terms of contracts' efficiency, market coverage and welfare. Our main conclusions are presented in section 1.4.

1.2 Linear city duopoly model

1.2.1 The model

Consider two identical firms offering nonlinear contracts (price p_i, quality u_i) to consumers distributed on a unit length line (brand preferences space). At each location d, $d \in [0,1]$, there are two types of consumers characterized by their preference for quality, measured by θ: $\bar{\theta} > \underline{\theta} > 0$. The two types are uniformly distributed along the line, both having mass equal to 1. We assume that firms observe each consumer's location d whilst the quality preference parameter θ is consumer's private information (*i.e.*, there is *vertical uncertainty*). We assume that firms are located at $d_1 = 0$ and $d_2 = 1$ and have identical production cost for a good of quality u_i, given by $C(u_i) = \frac{u_i^2}{2}$. Each consumer type buys a single unit of a good and chooses the contract (p_i, u_i) that yields the maximum net surplus (purchasing at most from one firm). When a consumer is indifferent between buying from firm 1 or firm 2, we assume he buys from the nearest firm. If a type θ consumer located at d buys at price p_i a product of quality u_i produced by firm i, $i = 1, 2$, located at d_i, $d_i \in [0,1]$, his net surplus is given by:

$$V = U(\theta, d, u_i, d_i) - p_i = \theta u_i - t\,|d - d_i| - p_i, \tag{1.1}$$

where t is a differentiation parameter (horizontal differentiation degree where $t > 0$) and as in other spatial models $t\,|d - d_i|$ measures the consumer's loss of not consuming his ideal brand.[1] The horizontal differentiation degree, t, will be crucial for the competition analysis when firms use MNP and for the competition intensity when firms use DNP. Also consider k to be the asymmetric information degree or the vertical differentiation degree, *i.e.*, the dispersion of types' quality preference parameters: $k = \frac{\bar{\theta}}{\underline{\theta}}$.[2]

Notice that (1.1) implies that, independently of their brand preferences, consumers with higher θ will be willing to accept larger rises in price for a given increase in product quality.

1.2.2 Mill nonlinear pricing

Let us first consider the case where firms are not allowed to discriminate among consumers at different locations: they must simultaneously offer a unique nonlinear contract ($\bar{p}_i, \bar{u}_i$ and $\underline{p}_i, \underline{u}_i$) for all consumers distributed along the line. All prices and qualities must be non-negative.

In general, under *vertical uncertainty*, when firms set mill nonlinear contracts they face a multidimensional screening problem as they may use both θ and d as screening variables.[3] However, under our utility specification the sorting condition only holds with respect to θ since:

$$\frac{\partial\left[-\left(\partial V/\partial u\right)/\left(\partial V/\partial p\right)\right]}{\partial\theta} = 1 > 0 \quad \text{but} \quad \frac{\partial\left[-\left(\partial V/\partial u\right)/\left(\partial V/\partial p\right)\right]}{\partial d} = 0.$$

Therefore, in our model mill nonlinear contracts only screen consumers according to their quality preferences. Consumers differ in their relative preference for the two firms but these brand preferences are neutral with respect to quality preferences.[4]

Under monopoly mill nonlinear pricing (see Appendix 1.A.1), a monopolist located at $d_m = 0$ always stops selling to type $\bar{\theta}$ at a location further away than the one where he stops selling

[1] We consider a general specification given by $V = \theta u_i - \theta^r t\,|d - d_i| - p_i$ but restrict our analysis to $r = 0$ for technical simplicity since for $r \neq 0$ brand preferences and quality preferences interact. Thus, we may have changes in the *consumer ranking* since the type of consumer who is willing to pay more for a given u may be type $\underline{\theta}$ consumers for locations distant from the firm's location. These cases are interesting but require additional complex computations.

[2] Notice that θ can be reinterpreted as the inverse of the marginal utility of income in a setup where consumers have identical ordinal preferences but differ in their incomes. Wealthier consumers have lower marginal utility of income and consequently higher θ. In this case, a high k ($k \in [1; +\infty[$) implies a high asymmetry in income distribution.

[3] Rochet and Stole (2000) discuss in detail the multidimensional screening scenario.

[4] Valletti (2002) considers interaction between horizontal (brand) and vertical (quality) preferences, using the

to type $\underline{\theta}$ and there are no local monopolies *at least for one type* (type $\bar{\theta}$) as long as $t < \frac{\bar{\theta}^2}{2}$. Henceforth we call this the non-existence of local monopolies condition under duopoly – $NLMd$ condition.[5]

Assuming that t belongs to the previous interval, firms may cover the whole market for both types, cover the whole market for type $\bar{\theta}$ only or sell to type $\bar{\theta}$ only.[6]

Market fully covered

When firms cover the whole market for both types, firms' demand is determined by the indifferent consumer location for each type. From (1.1) if a type θ consumer located at d buys from firm 1 she gets a net surplus of $\theta u_1 - td - p_1$ while if she buys from firm 2 she gets $\theta u_2 - t(1-d) - p_2$. Hence, the location where a type θ consumer is indifferent between buying from firm 1 or from firm 2 is given by:

$$\tilde{d}^{\theta} = \frac{1}{2} + \frac{\theta(u_1 - u_2) + p_2 - p_1}{2t}.$$

Therefore, firm 1 solves the following problem:

$$\max_{(\bar{p}_1, \bar{u}_1)(\underline{p}_1, \underline{u}_1)} \left(\bar{p}_1 - \frac{\bar{u}_1^2}{2}\right) \tilde{d}^{\bar{\theta}} + \left(\underline{p}_1 - \frac{\underline{u}_1^2}{2}\right) \tilde{d}^{\underline{\theta}}$$

subject to:

$$\begin{aligned}
IRP^{\bar{\theta}} &: \bar{\theta}\bar{u}_1 - t\tilde{d}^{\bar{\theta}} - \bar{p}_1 \geq 0 \\
IRP^{\underline{\theta}} &: \underline{\theta}\ \underline{u}_1 - t\tilde{d}^{\underline{\theta}} - \underline{p}_1 \geq 0 \\
IC^{\bar{\theta}} &: \bar{\theta}\bar{u}_1 - \bar{p}_1 \geq \bar{\theta}\underline{u}_1 - \underline{p}_1 \\
IC^{\underline{\theta}} &: \underline{\theta}\ \underline{u}_1 - \underline{p}_1 \geq \underline{\theta}\bar{u}_1 - \bar{p}_1 \\
FE^{\theta} &: 0 \leq \tilde{d}^{\theta} \leq 1 \text{ for } \theta = \overline{\theta}, \underline{\theta}.
\end{aligned}$$

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following utility function:

$$V = U(\theta, d, u_i, d_i) - p_i = \theta\left(1 - |d - d_i|\right) u_i - p_i.$$

In this case screening is feasible both on d and θ. However, this would increase considerably the complexity of the mill nonlinear pricing problem and would require the use of recent techniques for solving multidimensional screening.

[5] Under monopoly delivered nonlinear pricing the condition for non-existence of local monopolies would be less restrictive (see Appendix 1.A.2). Therefore, we consider the limit given by the monopoly mill nonlinear pricing analysis.

[6] For all these cases we focus on symmetric Nash *equilibrium* solutions only.

The first two constraints are the individual rationality participation constraints (IRP) – the indifferent consumer, located at $\tilde{d}^{\theta}$, must have a non-negative net surplus (notice that this assures that any type θ consumer located to the left of $\tilde{d}^{\theta}$ gets a strictly positive net surplus). The third and fourth constraints are the incentive compatibility constraints (IC) – firm 1 will set $(\bar{p}_1, \bar{u}_1)$ and $(\underline{p}_1, \underline{u}_1)$ so that each type of consumer prefers the price-quality bundle designed for it. It is interesting to notice that the incentive compatibility constraints do not depend on d. Thus, if they are satisfied at one location, they are satisfied at every location. The last constraints are the feasibility constraints (FE) – $\tilde{d}^{\theta}$ must belong to the unit line for both types.

Obviously, firm 2 faces a similar problem with demands $(1 - \tilde{d}^{\bar{\theta}})$ and $(1 - \tilde{d}^{\underline{\theta}})$. Notice that if the incentive compatibility constraints of firm 2 are satisfied, then types $\overline{\theta}$ and $\underline{\theta}$ located to the left of the indifferent consumer do not want to buy from firm 2 at $(\underline{p}_2, \underline{u}_2)$ and $(\bar{p}_2, \bar{u}_2)$, respectively. Therefore, firm 1's demands are indeed $\tilde{d}^{\bar{\theta}}$ and $\tilde{d}^{\underline{\theta}}$.

There are two possible solutions to this problem. In one of them, indifferent consumers of both types get positive net surplus. In the other one, type $\underline{\theta}$ indifferent consumer gets zero net surplus (the Kuhn-Tucker conditions of firm 1's problem are described in Appendix 1.B.1):

$\mathbf{C}_1^{\bar{\theta}\underline{\theta}}$	$\underline{u} = \underline{\theta}$ and $\underline{p} = \frac{\underline{\theta}^2}{2} + t$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \frac{\bar{\theta}^2}{2} + t$
$\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$	$\underline{u} = \underline{\theta}$ and $\underline{p} = \underline{\theta}^2 - \frac{t}{2}$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \frac{\bar{\theta}^2}{2} + t$

The price contracts offered at these two solutions are very similar. In both cases the quality offered to each type is the socially efficient one.[7] The two contracts differ only in terms of the price charged to type $\underline{\theta}$ consumers, $\underline{p}$. At $\mathbf{C}_1^{\bar{\theta}\underline{\theta}}$ both prices ($\underline{p}$ and $\bar{p}$) are increasing with t thus, market power increases with the horizontal differentiation degree. However at $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$, $\underline{p}$ is decreasing with t. This last result is due to the fact that type $\underline{\theta}$ indifferent consumer has a zero net surplus. Thus when t increases, $\underline{p}$ has to decrease for the consumer to be willing to buy.[8]

As we will see afterwards in more detail, the market is fully covered for both types for low values of t. When t is close to zero, all consumers get positive net surplus ($\mathbf{C}_1^{\bar{\theta}\underline{\theta}}$). However, as t increases type $\underline{\theta}$ indifferent consumer starts getting zero net surplus ($\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$). When t increases

[7] Simple computations reveal that $u = \theta$ is the socially optimal quality.

[8] This pattern also holds for type $\overline{\theta}$ but for higher values of t.

even further the market is no longer completely covered for type $\underline{\theta}$. Thus, $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$ may be interpreted as an intermediate case between full competition for both types and local monopolies for type $\underline{\theta}$.

Market fully covered for type $\bar{\theta}$ only

When firms cover the whole market for type $\bar{\theta}$ only, demand from type $\underline{\theta}$ consumers is determined by the location where type $\underline{\theta}$ consumer is indifferent between buying and not buying at all. For firm 1, this consumer is located at $\hat{d}_1^{\underline{\theta}}$ where:

$$\hat{d}_i^{\underline{\theta}} = \frac{\underline{\theta}\ \underline{u}_i - \underline{p}_i}{t}. \tag{1.2}$$

In this case, firm 1 solves the following problem:

$$\max_{(\bar{p}_1,\bar{u}_1)(\underline{p}_1,\underline{u}_1)} \left(\bar{p}_1 - \frac{\bar{u}_1^2}{2}\right)\tilde{d}^{\bar{\theta}} + \left(\underline{p}_1 - \frac{\underline{u}_1^2}{2}\right)\hat{d}_1^{\underline{\theta}}$$

subject to:

$$\begin{aligned}
IRP^{\bar{\theta}} &: \ \bar{\theta}\bar{u}_1 - t\tilde{d}^{\bar{\theta}} - \bar{p}_1 \geq 0 \\
IC^{\bar{\theta}} &: \ \bar{\theta}\bar{u}_1 - \bar{p}_1 \geq \bar{\theta}\underline{u}_1 - \underline{p}_1 \\
IC^{\underline{\theta}} &: \ \underline{\theta}\ \underline{u}_1 - \underline{p}_1 \geq \underline{\theta}\bar{u}_1 - \bar{p}_1 \\
FE^{\bar{\theta}} &: \ 0 \leq \tilde{d}^{\bar{\theta}} \leq 1 \\
PC^{\underline{\theta}} &: \ 0 \leq \hat{d}_1^{\underline{\theta}} < \hat{d}_2^{\underline{\theta}}.
\end{aligned}$$

The last constraint is the Partially Covered constraint (PC). Again, firm 2 will solve an analogous problem.

Considering the demand for each type of consumer and maximizing profits subject to the usual asymmetric information constraints, we get the following solutions (the Kuhn-Tucker conditions of firm 1's problem are described in Appendix 1.B.2):

$\mathbf{S}_1^{\bar{\theta}}$	$u = \bar{\theta}$ and $p = \frac{\bar{\theta}^2}{2} + t$
$\mathbf{S}_2^{\bar{\theta}}$	$u = \bar{\theta}$ and $p = \bar{\theta}^2 - \frac{t}{2}$

$\mathbf{LM}_1^{\underline{\theta}}$	$\underline{u} = \underline{\theta}$ and $\underline{p} = \frac{3}{4}\underline{\theta}^2$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \frac{\bar{\theta}^2}{2} + t$
$\mathbf{LM}_2^{\underline{\theta}}$	$\underline{u} = \underline{\theta}$ and $\underline{p} = \frac{3}{4}\underline{\theta}^2$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \bar{\theta}^2 - \frac{t}{2}$

At $\mathbf{S}_1^{\bar{\theta}}$ and $\mathbf{S}_2^{\bar{\theta}}$ firms only sell to type $\overline{\theta}$ consumers. At $\mathbf{LM}_1^{\underline{\theta}}$ and $\mathbf{LM}_2^{\underline{\theta}}$ firms sell to both types of consumers but have local monopolies for type $\underline{\theta}$. Type $\overline{\theta}$ indifferent consumer gets a positive net surplus with solutions $\mathbf{S}_1^{\bar{\theta}}$ and $\mathbf{LM}_1^{\underline{\theta}}$ but no surplus at $\mathbf{S}_2^{\bar{\theta}}$ and $\mathbf{LM}_2^{\underline{\theta}}$.

When the two firms compete for type $\bar{\theta}$ consumers and have local monopolies for type $\underline{\theta}$, they offer the socially efficient qualities and charge $\underline{p} = \frac{3}{4}\underline{\theta}^2$, which is precisely the price that would be charged by a monopolist, and serve type $\underline{\theta}$ consumers located to the left of $\widehat{d^{\underline{\theta}}} = \frac{\underline{\theta}^2}{4t} < \frac{1}{2}$. The price contracts in these two solutions only differ for $\bar{p}$.

Notice that there is no distortion in the quality offered to both types of consumers in every solution under mill nonlinear pricing. Since we assume a perfectly separable utility function between θ and d (quality and brand preferences) we obtain precisely the same result as Armstrong and Vickers (2001) and Rochet and Stole (2002). However, under $\mathbf{LM}_1^{\underline{\theta}}$ and $\mathbf{LM}_2^{\underline{\theta}}$ type $\underline{\theta}$ consumers located further away from the firm are not served and under $\mathbf{S}_1^{\bar{\theta}}$ and $\mathbf{S}_2^{\bar{\theta}}$ type $\underline{\theta}$ consumers are not served. In these cases, as we will see later, there are too few type $\underline{\theta}$ consumers being served.

Until now we have described several possible *equilibrium* solution contracts under MNP but did not specify the set of parameters' values for which each of them holds. Lets analyze the symmetric Nash *equilibria* as a function of the parameters' values.

Nash Equilibrium solution

For some parameters' values, the Kuhn-Tucker conditions for the previous problems have multiple solutions.[9] In these cases, we consider the global maximum solution.[10] To achieve the

[9] There are also some undetermined solutions. But since their profits are equal to the profits of other determined solutions and their intervals' limits belong to the determined solutions intervals' limits, we can exclude them from analysis.

[10] Since we only analyze symmetric solutions of the Kuhn-Tucker conditions we cannot guarantee that the Nash *equilibrium* described is unique (there may exist Nash *equilibria* corresponding to local maxima). However, even if the Nash *equilibrium* described is not unique, it certainly is the most reasonable symmetric *equilibrium*.

maximum profits' *equilibrium* solution, we proceed by computing each solution's profits covering all t and k. We only have *equilibrium* solutions for all $t < \frac{\bar{\theta}^2}{2}$ if $k > \left(1 + \frac{\sqrt{5}}{5}\right)$.[11]

Figure 1.1 shows the set of parameters' values where each of the previously described solution contracts holds. Figure 1.1 illustrates several interesting features of the *equilibrium* solution contracts as a function of the horizontal and vertical differentiation degrees (t and k). First, for a given level of $k = \frac{\bar{\theta}}{\underline{\theta}}$, as t rises we move to less competitive solutions. Second, the solution contracts depend on k.

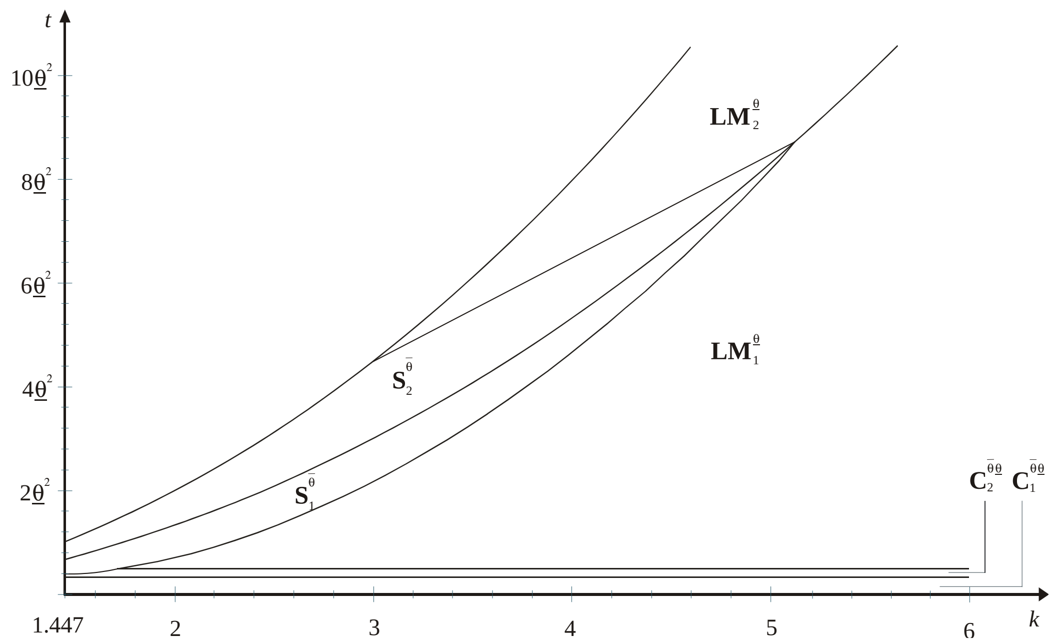

Figure 1.1: Duopoly MNP *equilibrium* contracts

Discussion of results under mill nonlinear pricing Under MNP firms set a unique nonlinear contract for the whole market. Curiously, some characteristics of the *equilibrium* nonlinear contracts are similar to the well known results of mill linear pricing: for low t, firms cover the whole market and the indifferent consumer gets positive net surplus; for intermediate t, firms cover the whole market but the indifferent consumer gets zero net surplus; and for high t, firms have local monopolies. When vertical differentiation is introduced, this phenomenon happens for both types but for different degrees of horizontal differentiation. As type $\underline{\theta}$ consumers

[11] Villas-Boas and Schmidt-Mohr (1999) also get similar non existence of (pure strategies) *equilibrium.*

are characterized by a lower preference for quality parameter, the previous phenomenon appear for low degrees of horizontal differentiation for this type of consumers.[12] As it is shown in Figure 1.1, for a given k and very low values of t, the market is fully covered for both types and as t rises, firms start having local monopolies for type $\underline{\theta}$. If t rises above the non-existence of local monopolies condition limit there is no competition between firms as they have local monopolies for both types.[13]

When t is very low, we get a separating *equilibrium* where firms screen consumers and cover the whole market for both types. For very low t, both types of indifferent consumers get positive net surplus but for slightly higher t, type $\underline{\theta}$ indifferent consumer gets zero net surplus.

When t is intermediate or high, the market is no longer completely covered for type $\underline{\theta}$ and the *equilibrium* contract depends on k. For low k, as t rises firms start offering only one price-quality bundle directed to type $\overline{\theta}$ consumers. This happens because when k is low, *i.e.*, the two types are very similar, it becomes difficult to set screening contracts (it becomes difficult to satisfy the incentive compatibility constraints). Therefore, for low k and high t, firms sell only to type $\overline{\theta}$ consumers (first leaving a positive net surplus and then capturing the entire net surplus from type $\overline{\theta}$ indifferent consumer). For high k, *i.e.*, when the two types are very different and separation of the two types is easier, as t rises firms continue to use screening and to sell to both types of consumers (going from full competition to local monopolies for type $\underline{\theta}$). Therefore, for high values of k, we get a separating *equilibrium* for all t. Finally, for intermediate values of k, firms offer two price-quality bundles for low and high values of t while for intermediate t firms sell only to type $\overline{\theta}$ consumers.[14]

[12] For instance, for some degree of horizontal differentiation, we may have type $\bar{\theta}$ indifferent consumer with positive net surplus but type $\underline{\theta}$ indifferent consumer with no net surplus or not buying at all.

[13] That level of differentiation depends on k. The higher k is, the higher is the level of horizontal differentiation needed for local monopolies to hold for both types. As mentioned earlier, we do not discuss the case where firms have local monopolies for both types.

[14] The previous results have similarities to Villas-Boas and Schmidt-Mohr (1999). They present an interesting exposition about the three effects that should be considered with mill nonlinear pricing: business stealing or demand effect, margin effect and screening effect.

1.2.3 Delivered nonlinear pricing

Suppose now that firms are able to offer different nonlinear contracts at each location in the line ($\bar{p}_i(d), \bar{u}_i(d)$ and $\underline{p}_i(d), \underline{u}_i(d)$). To simplify notation we will drop the argument d for the remaining of this chapter. However, one should keep in mind that a different nonlinear contract is set at each location. Since the two firms are identical, with DNP, the market will be split in two equal parts and each firm will have its own *market area*. Therefore, we will restrict the analysis to the optimization problem of firm 1 within half of the spatial market: $d \in \left[0, \frac{1}{2}\right]$ – firm 1 *market area*.[15] Also, for this pricing policy all prices and qualities must be non-negative.

When selling to both types of consumers firm 1 chooses, for each location d, the nonlinear contract that solves:

$$\max_{(\bar{p}_1, \bar{u}_1)(\underline{p}_1, \underline{u}_1)} \underline{p}_1 - \frac{\underline{u}_1^2}{2} + \bar{p}_1 - \frac{\bar{u}_1^2}{2}$$

subject to individual rationality and incentive compatibility constraints:

$$\begin{aligned}
IR^{\bar{\theta}} &: \quad \bar{\theta}\bar{u}_1 - td - \bar{p}_1 \geq \max\{0, \text{net surplus when buying from firm 2}\} \\
IR^{\underline{\theta}} &: \quad \underline{\theta}\ \underline{u}_1 - td - \underline{p}_1 \geq \max\{0, \text{net surplus when buying from firm 2}\} \\
IC^{\bar{\theta}} &: \quad \bar{\theta}\bar{u}_1 - \bar{p}_1 \geq \bar{\theta}\underline{u}_1 - \underline{p}_1 \\
IC^{\underline{\theta}} &: \quad \underline{\theta}\ \underline{u}_1 - \underline{p}_1 \geq \underline{\theta}\bar{u}_1 - \bar{p}_1.
\end{aligned}$$

The individual rationality constraints incorporate the fact the consumer has the outside options of buying from firm 2 or not buying. Thus, in order to induce the consumer to buy from firm 1, this firm has to offer a net surplus at least as high as the one the consumer can get with the best outside option.

In order to determine consumer's best outside option, we need to compute the net surplus when buying from firm 2 for each location d. Following an undercutting argument, firm 2 will offer a price equal to its marginal cost $p_2 = \frac{u^2}{2}$ (from the zero profit condition) to the consumers located at firm 1's *market area*. The maximum net surplus that a type θ consumer can get when buying from firm 2 is the solution to $\max_u \theta u - t(1-d) - \frac{u^2}{2}$. The optimal quality is $u = \theta$ and the maximum net surplus is then given by $\frac{\theta^2}{2} - t(1-d)$ which is only positive when $d_\theta > \frac{2t-\theta^2}{2t} = \check{d}_\theta$. Considering $k > \sqrt{2}$, type $\bar{\theta}$ consumers located at firm 1's *market area* always

[15] We would have a symmetric situation for the rest of the line but with firms' positions reversed.

get positive net surplus for $t < \frac{\bar{\theta}^2}{2}$ but that does not always occur for type $\underline{\theta}$ consumers.[16] More precisely, type $\underline{\theta}$ consumers net surplus when buying from firm 2 behaves as follows:

t	type $\underline{\theta}$ consumers located at d buying from firm 2
$0 < t < \frac{\underline{\theta}^2}{2}$	$d \in \left[0, \frac{1}{2}\right]$ have positive net surplus
$\frac{\underline{\theta}^2}{2} \leq t < \underline{\theta}^2$	$d \in \left[0, \breve{d}_{\underline{\theta}}\right]$ have negative net surplus $d \in \left[\breve{d}_{\underline{\theta}}, \frac{1}{2}\right]$ have positive net surplus
$\underline{\theta}^2 \leq t < \frac{\bar{\theta}^2}{2}$	$d \in \left[0, \frac{1}{2}\right]$ have negative net surplus

Therefore, when $0 < t < \frac{\underline{\theta}^2}{2}$ firm 1 must consider the following individual rationality constraints (*case 1*):

$$\begin{aligned} IR^{\bar{\theta}} \quad &: \quad \bar{\theta}\bar{u}_1 - td - \bar{p}_1 \geq \frac{\bar{\theta}^2}{2} - t(1-d) \\ IR^{\underline{\theta}} \quad &: \quad \underline{\theta}\, \underline{u}_1 - td - \underline{p}_1 \geq \frac{\underline{\theta}^2}{2} - t(1-d). \end{aligned}$$

When $\underline{\theta}^2 \leq t < \frac{\bar{\theta}^2}{2}$ (*case 2*) firm 1 must consider:

$$\begin{aligned} IR^{\bar{\theta}} \quad &: \quad \bar{\theta}\bar{u}_1 - td - \bar{p}_1 \geq \frac{\bar{\theta}^2}{2} - t(1-d) \\ IR^{\underline{\theta}} \quad &: \quad \underline{\theta}\, \underline{u}_1 - td - \underline{p}_1 \geq 0. \end{aligned}$$

Finally, when $\frac{\underline{\theta}^2}{2} \leq t < \underline{\theta}^2$, firm 1 considers *case 2* constraints for consumers located at $d \in \left[0, \breve{d}_{\underline{\theta}}\right]$ and *case 1* constraints for $d \in \left[\breve{d}_{\underline{\theta}}, \frac{1}{2}\right]$.

Notice that if individual rationality constraints hold then both types, $\bar{\theta}$ and $\underline{\theta}$, will not buy from firm 2 marginal cost pricing $(\underline{p}_2, \underline{u}_2)$ and $(\bar{p}_2, \bar{u}_2)$, respectively.

When firm 2 does not sell to type $\underline{\theta}$ at some location d (within firm 1's *market area*), firm 1 might consider to sell only to type $\bar{\theta}$ at that location. Firm 1 should maximize $\bar{p}_1 - \frac{\bar{u}_1^2}{2}$ subject to:

$$IR^{\bar{\theta}} : \bar{\theta}\bar{u}_1 - td - \bar{p}_1 \geq \frac{\bar{\theta}^2}{2} - t(1-d).$$

and $\underline{\theta}\bar{u}_1 - \bar{p}_1 < 0$.

[16] On account of the mentioned mill nonlinear pricing and local monopoly analysis, we focus on these t values.

Nash equilibrium solution

Since type $\underline{\theta}$'s individual rationality constraint change with t and d, we must solve two different maximization problems when firm 1 sells to both types of consumers[17] (the Kuhn-Tucker conditions optimization problems of firm 1 are described in Appendix 1.B.3 and 1.B.4).

***Case 1* − Low t** When $0 < t < \frac{\underline{\theta}^2}{2}$, consumer's best outside option is to buy from the competitor, for all locations in the firm 1's *market area*. In this case, the unique solution is characterized by the incentive compatibility constraints not binding and both types of consumers being indifferent between buying from firm 1 or firm 2. Firm 1 offers, at its *market area*, the following two bundles at location d:

$$\boxed{\begin{array}{c} \underline{u}_1 = \underline{\theta} \text{ and } \underline{p}_1 = \frac{\underline{\theta}^2}{2} - 2td + t \\ \bar{u}_1 = \bar{\theta} \text{ and } \bar{p}_1 = \frac{\bar{\theta}^2}{2} - 2td + t \end{array}}$$

Thus when the horizontal differentiation degree is low, firm 1 offers to both types of consumers the socially optimal qualities and matches the net surplus offered by firm 2. We call this solution contract the full competition (FC) since firms have to give consumers the positive net surplus they would attain when buying from the competitor. Since FC contracts are set at all locations of firm 1's *market area*, this scenario is called $1Reg$.

***Case 2* − High t** When $\underline{\theta}^2 \leq t < \frac{\bar{\theta}^2}{2}$, the characteristics of the *equilibrium* contracts may depend on the location, a result which is similar to Pires and Sarkar (2000) and Valletti (2002). Depending on t and k, there may be three different regions within firm 1's *market area*: 'monopolistic − M', 'intermediate − I' and 'competitive − C' regions (see Figure 1.2).

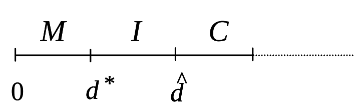

Figure 1.2: Three regions within firm 1's *market area*, where $d^* = \frac{6\bar{\theta}\underline{\theta} - 3\bar{\theta}^2 - 4\underline{\theta}^2 + 2t}{2t}$ and $\hat{d} = \frac{2\bar{\theta}\underline{\theta} - \bar{\theta}^2 - 2\underline{\theta}^2 + 2t}{2t}$.

[17] For more details on similar mathematical computations see Valletti (2002).

These regions are characterized by different *equilibrium* nonlinear contracts. In all of them firm 1 sells to both types of consumers and the contracts are given by:

Region	Contracts
M	$\underline{u}_1 = 2\underline{\theta} - \bar{\theta}$ and $\underline{p}_1 = \underline{\theta}(2\underline{\theta} - \bar{\theta}) - td$ $\bar{u}_1 = \bar{\theta}$ and $\bar{p}_1 = \bar{\theta}^2 + (\underline{\theta} - \bar{\theta})(2\underline{\theta} - \bar{\theta}) - td$
I	$\underline{u}_1 = \frac{\bar{\theta}^2 + 2td - 2t}{2(\bar{\theta} - \underline{\theta})}$ and $\underline{p}_1 = \frac{\underline{\theta}\bar{\theta}^2 + 2td(2\underline{\theta} - \bar{\theta}) - 2\underline{\theta}t}{2(\bar{\theta} - \underline{\theta})}$ $\bar{u}_1 = \bar{\theta}$ and $\bar{p}_1 = \frac{\bar{\theta}^2}{2} - 2td + t$
C	$\underline{u}_1 = \underline{\theta}$ and $\underline{p}_1 = \underline{\theta}^2 - td$ $\bar{u}_1 = \bar{\theta}$ and $\bar{p}_1 = \frac{\bar{\theta}^2}{2} - 2td + t$

Region M is the nearest to firm 1's location (until d^*). In this region, firm 1 offers typical monopolist asymmetric information contracts: only type $\underline{\theta}$'s individual rationality constraint and type $\bar{\theta}$'s incentive compatibility constraints are binding, type $\bar{\theta}$ consumes the socially efficient quality whilst type $\underline{\theta}$ consumes a suboptimal quality. Between d^* and $\hat{d}$, we have an intermediate region, region I, where type $\bar{\theta}$'s individual rationality constraint is also binding and firm 1 faces more competition (firm 2 offers a contract very attractive for type $\bar{\theta}$). In this region, firm 1 does not distort $\underline{u}$ as much as in region M, because its power to extract rent from type $\bar{\theta}$ consumers is limited. Finally, closer to the market center – region C, firm 1 faces vigorous competition. In region C, only the individual rationality constraints are binding and both consumer types get the socially optimal qualities.

When firm 1 sells to type $\bar{\theta}$ only, the contract offered to type $\bar{\theta}$ consumers is equal to the contract set in regions I and C for type $\bar{\theta}$ consumers Type $\underline{\theta}$ consumers do not want to buy the good from firm 1 under this contract thus we denote this region by $C^{\bar{\theta}}$. The profits achieved are never higher than profits obtained by selling to both types of consumers setting contract M. However, for high horizontal differentiation degrees (closer to $NLMd$ limit) the firm is better of selling only to type $\bar{\theta}$ consumers than selling to both types of consumers and setting contracts I and C.

Figure 1.3 shows the set of parameters' values where each different *equilibrium* solution contracts hold, considering the conditions for regions' existence and comparing profits of selling only to type $\bar{\theta}$ consumers or to both types of consumers.

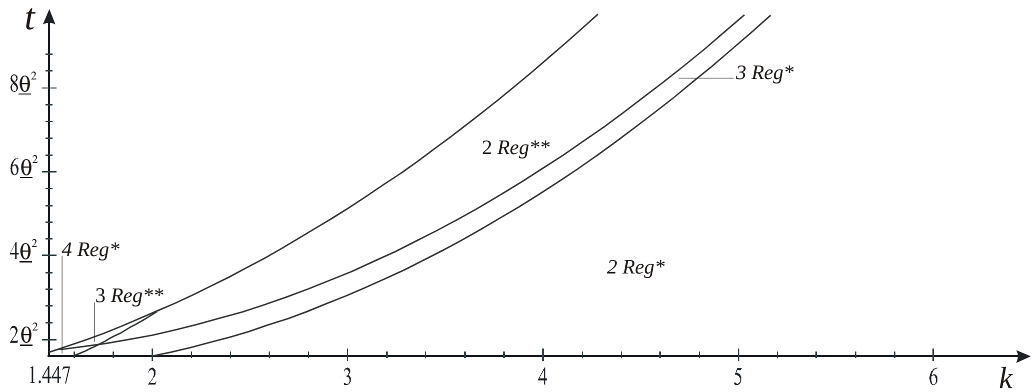

Figure 1.3: Duopoly DNP *equilibrium* solutions when t is High

The different scenarios in the parameter space are characterized as follows:

$2Reg^{*}$	C $([0, d^{C}[)$ and $C^{\bar{\theta}}$ $([d^{C}, \frac{1}{2}])$
$3Reg^{*}$	I $([0, \hat{d}[), C$ $([\hat{d}, d^{C}[)$ and $C^{\bar{\theta}}$ $([d^{C}, \frac{1}{2}])$
$4Reg^{*}$	M $([0, d^{*}[), I$ $[d^{*}, \hat{d}[, C$ $([\hat{d}, d^{C}[)$ and $C^{\bar{\theta}}$ $([d^{C}, \frac{1}{2}])$
$2Reg^{**}$	I $([0, d^{I}[)$ and $C^{\bar{\theta}}$ $([d^{I}, \frac{1}{2}])$
$3Reg^{**}$	M $([0, d^{*}[), I$ $([d^{*}, d^{I}[)$ and $C^{\bar{\theta}}$ $([d^{I}, \frac{1}{2}])$

where $d^{C} = \frac{\underline{\theta}^2}{2t}$ and $d^{I} = \frac{6\bar{\theta}\underline{\theta} - 3\bar{\theta}^2 - 4\underline{\theta}^2 + 2t + \sqrt{4\left(\bar{\theta}-\underline{\theta}\right)^2\left(2\bar{\theta}^2 + 4\underline{\theta}^2 - 4\bar{\theta}\underline{\theta} - 2t\right)}}{2t}$.

Figure 1.3 shows that when t is high, closer to the market center firms only sell to type $\overline{\theta}$ consumers, offering the efficient quality and matching the net surplus offered by the competitor. Moreover, it shows that the number of regions in a firm's *market area* depends on the parameters' values.

Notice that locations d^{*}, $\hat{d}$, d^{C} and d^{I} depend on t and k, which means that the way firm 1's *market area* is divided depends on the parameters' values. Figure 1.4 illustrates how firm 1's *market area* is divided for different sets of parameters' values. For example, when $k = 1.5$ and $t = \underline{\theta}^2$, firm 1 offers monopolist nonlinear contracts to consumers located between 0 and $d^{*} = 0.125$, intermediate contracts for consumers located between $d^{*} = 0.125$ and $\hat{d} = 0.375$ and competitive contracts for consumers located between $\hat{d} = 0.375$ and $d^{C} = 0.5$.

The first plot in Figure 1.4 shows that, for low k, there is a region M within firm 1's *market area* and as t rises this region becomes larger, *i.e.*, there are more locations where

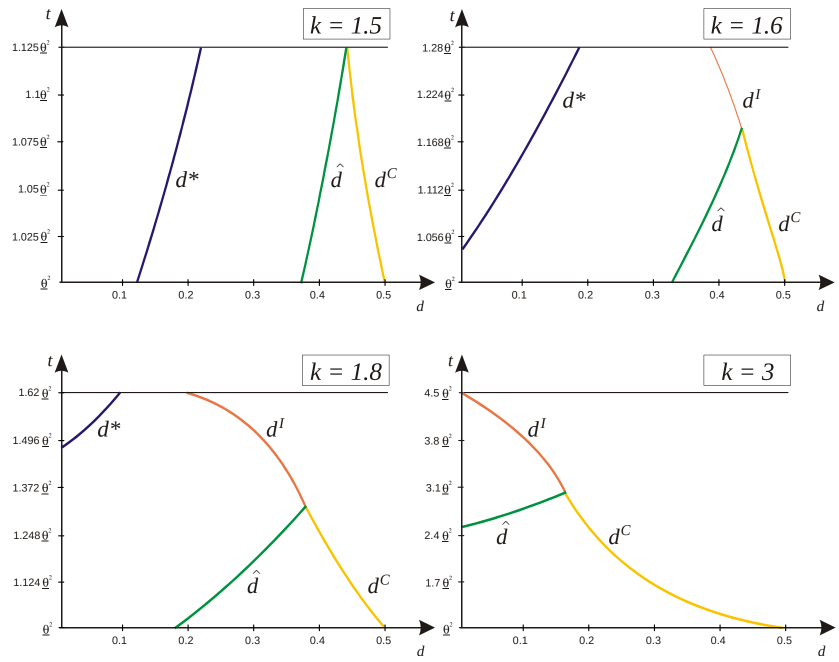

Figure 1.4: *Case 2* – Firm 1's *market area*

the monopolistic nonlinear contract is set. This result is quite intuitive, when the horizontal differentiation degree increases there are more locations where matching the net surplus offered by the competitor to type $\overline{\theta}$ consumers is not a binding constraint and thus competition is not effective.

However, when k increases, region M becomes gradually smaller until it disappears (see the various plots in Figure 1.4). This happens because when k is high, the two types are very different and the net surplus offered by the competitor to type $\overline{\theta}$ consumers is much higher than the one offered to type $\underline{\theta}$ consumers. This implies that type $\bar{\theta}$'s individual rationality constraint starts binding for locations closer to the firm. Notice that the set of locations closer to the market center, where firms sell only to type $\overline{\theta}$ consumers, increase with t but also with k (see curves d^C and d^I in Figure 1.4).

Case 3 **– Intermediate values of** t For $\frac{\underline{\theta}^2}{2} \leq t < \underline{\theta}^2$ the outside option of type $\underline{\theta}$ depends on consumer's location. For $d \in \left[0, \breve{d}\right]$ the best outside option is not to buy and we make the same reasoning as in the previous case. For $d \in \left[\breve{d}, \frac{1}{2}\right]$ the best outside option for $\underline{\theta}$ is to buy from the competitor and we get the same Nash *equilibrium* contract as in *case 1*. Figure 1.5

shows the Nash *equilibrium* for low and intermediate t values (*cases 1* and *3*).

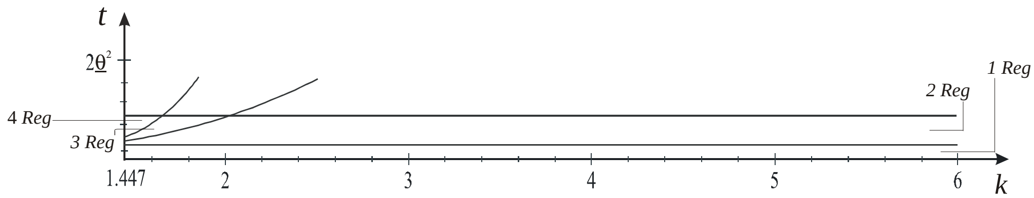

Figure 1.5: Duopoly DNP *equilibrium* solutions for low and intermediate t

The different scenarios in the parameter space are characterized as follows:

$1Reg$	FC $([0, \frac{1}{2}])$
$2Reg$	C $([0, \breve{d}_{\underline{\theta}}[)$ and FC $([\breve{d}_{\underline{\theta}}, \frac{1}{2}])$
$3Reg$	I $([0, \hat{d}[), C$ $([\hat{d}, \breve{d}_{\underline{\theta}}[)$ and FC $([\breve{d}_{\underline{\theta}}, \frac{1}{2}])$
$4Reg$	M $([0, d^*[), I$ $[d^*, \hat{d}[, C$ $([\hat{d}, \breve{d}_{\underline{\theta}}[)$ and FC $([\breve{d}_{\underline{\theta}}, \frac{1}{2}])$

As expected, for low and intermediate t values, firms offer FC contracts closer to the market center.

It is interesting to analyze how firm 1's *market area* is divided depending on the parameters' values (see Figure 1.6), for intermediate t values.

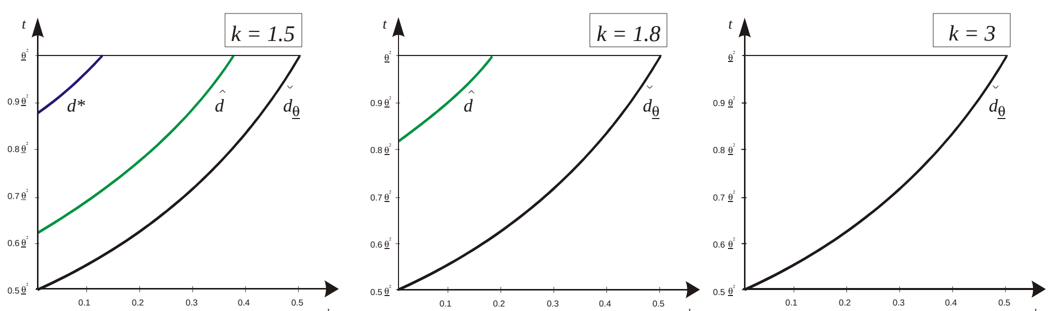

Figure 1.6: *Case 3* – Firm 1's *market area*

For low k, there may be up to four submarket regions (M, I, C and FC). However, the monopolistic region only appears for high t. As k increases, the less competitive contracts (regions M and I) are offered at a smaller set of locations and for relatively high k, only C and FC contracts are offered (see Figure 1.6). Notice that the locations where FC contract is set do not change with k but decrease with t.

Discussion of results under delivered nonlinear pricing Under DNP the *equilibrium* contract solution may change with the location of the consumer. The market served by a firm may be subdivided in several regions according to the nonlinear contract offered. The nonlinear contracts may differ in terms of the degree of inefficiency in the quality offered to type $\underline{\theta}$ consumers and with respect to the net surplus offered to each type of consumer. The quality chosen is always the social optimal for type $\bar{\theta}$ consumers but may be optimal or suboptimal for type $\underline{\theta}$ consumers.

When t is very low, the outside option of buying from the competitor is very attractive at every location in a firm's *market area*. Consequently a large net surplus has to be offered to both consumer types. Moreover, the net surplus that has to be offered to type $\overline{\theta}$ consumers, in order for them not to buy from the competitor, is so high that this type of consumers do not want to buy the efficient quality bundle offered to type $\underline{\theta}$ consumers. Therefore, the quality offered to both types of consumers is the socially efficient one and the net surplus offered to each type is exactly the same they would receive if they bought from the competitor. In other words, for very low horizontal differentiation degrees we get full competition at all locations of the market.

When t is intermediate, $\frac{\underline{\theta}^2}{2} \leq t < \underline{\theta}^2$, closer to the market center, firms always offer FC contracts and consequently we have intense competition. This happens because, closer to the market center, the outside option of buying from the competitor is very attractive for both types of consumers. However, for locations further away from the competitor, the best outside option for type $\underline{\theta}$ consumers is not to buy the good. At these locations, the firm will be able to extract more consumer surplus from type $\underline{\theta}$ consumers (and also from type $\overline{\theta}$ consumers, whenever the incentive compatibility constraint is binding). The number of regions in each firm's *market area* depends on t and k. When k is very high, the two types are very different, implying that the net surplus offered by the competitor to type $\overline{\theta}$ consumers is much higher than the net surplus offered to type $\underline{\theta}$ consumers. In these circumstances, the relevant constraint with respect to type $\overline{\theta}$ consumers is to meet the competitor's net surplus offer (incentive compatibility is not an issue). Consequently when k is large, the efficient quality is offered to type $\underline{\theta}$ consumers at every location in a firm's *market area*. However, the contract set is not the same at all locations since the outside option for type $\underline{\theta}$ consumers depends on whether the consumer is close to the market center or not. In other words, for intermediate t and high k, contracts are either C (closer to the firm) or FC (closer to the market center). As k decreases, we may start having more submarket regions. The intuition is that for smaller k, the difference between the reservation net surplus of type $\overline{\theta}$ consumers and the reservation net surplus of type $\underline{\theta}$ consumers is relatively small. This implies that type $\overline{\theta}$'s incentive compatibility constraint may start to be binding, especially as we consider locations further away from the market center. As soon as this happens, one starts having contracts where the quality offered to type $\underline{\theta}$ consumers is suboptimal. For intermediate k, we may have a region where $\underline{u}$ is efficient and a region where $\underline{u}$ is suboptimal but with a smaller distortion than if there was a monopoly. Finally, for very low k we may have monopolistic, intermediate and competitive regions (but always with FC contracts closer to the market center).

Finally, when t is high, $\underline{\theta}^2 \leq t \leq \frac{\overline{\theta}^2}{2}$, type $\underline{\theta}$'s best outside option is not to buy (for every location in a firm's *market area*). In this case, closer to the market center firms only sell to type $\bar{\theta}$ consumers, offering the efficient quality and matching the net surplus offered by the competitor. Once again, the number of regions in a firm's *market area* depends on t and k. For low k, each firm's *market area* can be divided into four regions: monopolistic, intermediate, competitive and a region where only type $\overline{\theta}$ consumers are served while for larger k values, the monopolistic region does not exist. In addition, as t increases the region where firms only sell to type $\bar{\theta}$ consumers becomes larger and the competitive region becomes smaller. For high t values, there is no competitive region.

Notice that, similarly to MNP, for a given k, as t rises we move to less competitive contracts. On one hand, for low k values, as t rises region M becomes larger and region C smaller ($\frac{\partial d^*}{\partial t}$ and $\frac{\partial \hat{d}}{\partial t}$ are both positive). On the other hand, for high k values, as t rises competition is only relevant for type $\overline{\theta}$ consumers and firms do not sell to type $\underline{\theta}$ consumers located closer to the market center.

1.3 Welfare under MNP and DNP

Let us now compare welfare under the two pricing policies *for a duopoly.* Since the quality offered to type $\overline{\theta}$ consumers is always the efficient one and this type of consumer is always served, both pricing policies maximize social welfare for this type of consumer (they only differ in the price charged but this a transfer between consumers and firms that does not affect social welfare). Therefore, for a duopoly, welfare differences are explained by what happens with type $\underline{\theta}$ consumers.

There are two issues which are relevant in the welfare comparison for type $\underline{\theta}$ consumers: the first one is whether this type of consumer is offered the efficient quality or not; the second one is whether all consumers of this type are served or not. We know that under MNP, when a type $\underline{\theta}$ consumer is served he consumes the efficient quality. Conversely the quality offered to type $\underline{\theta}$ consumers under DNP may or may not be the efficient one (the scenarios with a region closer to the firms' location where the quality offered to type $\underline{\theta}$ consumers is suboptimal are $3Reg$, $4Reg$, $3Reg^{*}$, $4Reg^{*}$, $2Reg^{**}$ and $3Reg^{**}$). This seems to suggest that for a duopoly MNP is better in welfare terms. However, the previous analysis is incomplete since some type $\underline{\theta}$ consumers may not be served under MNP and/or under DNP. Figure 1.7 overlaps the DNP and MNP solutions for the various parameters' values[18] and shows the only subset of parameters' values where there are more type $\underline{\theta}$ consumers served under MNP (in grey).

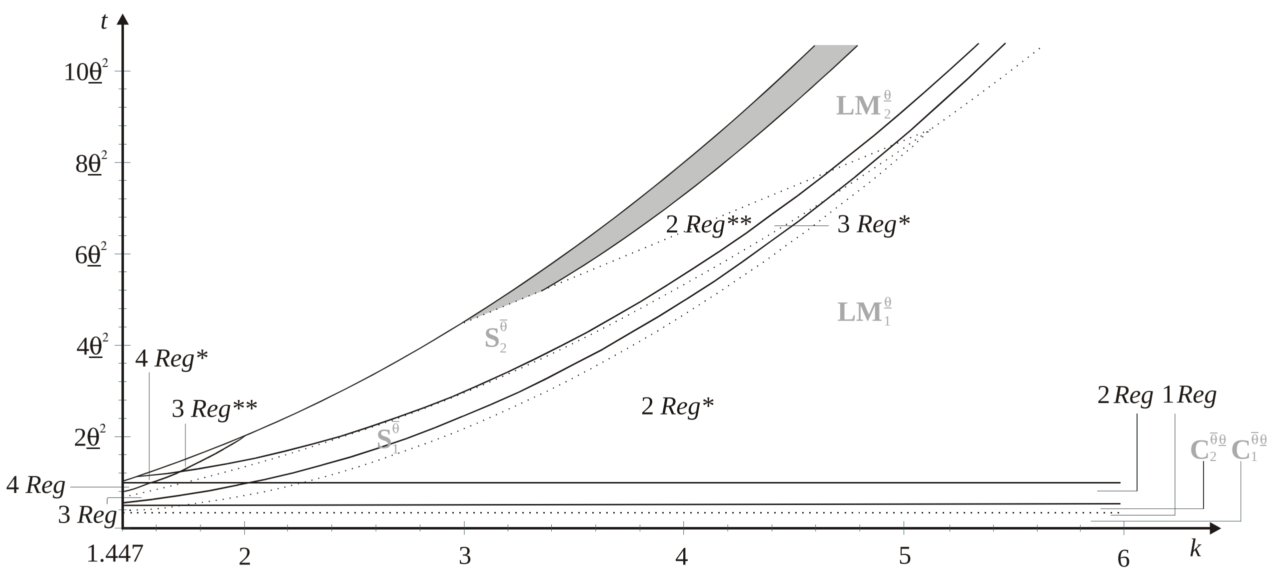

Figure 1.7: Duopoly DNP (solid and black) versus MNP (dots and grey) *equilibrium* solutions

One should notice that for high t, it may not be socially optimal to serve all type $\underline{\theta}$ consumers. The social net surplus for consumer θ at each location d is given by $\theta u - td - \frac{u^2}{2}$. Since $u = \theta$ is the socially optimal quality, the social net surplus from a consumer θ located at d is given

[18]For example, this figure shows that when t is low ($t < \frac{\theta^2}{2}$) we have fully competitive contracts ($1Reg$) under DNP and we may have either $\mathbf{C}_1^{\bar{\theta}\,\underline{\theta}}$, $\mathbf{C}_2^{\bar{\theta}\,\underline{\theta}}$ or $\mathbf{S}_1^{\bar{\theta}}$ contracts under MNP.

by $\frac{\theta^2}{2} - td$. Thus, type $\underline{\theta}$ consumers should be served only for locations to the left of $d^*_{\underline{\theta}} = \frac{\theta^2}{2t}$.

It turns out that there are many parameters' values for which we can easily compare welfare under MNP and DNP (see duopoly welfare functions in Appendix 1.C).

Proposition 1.1 *For a duopoly, welfare under DNP is:*

(i) *Equal to welfare under MNP when both types of consumers are completely covered under MNP (contracts $\boldsymbol{C}_1^{\bar{\theta}\underline{\theta}}$ and $\boldsymbol{C}_2^{\bar{\theta}\underline{\theta}}$).*

(ii) *Higher than welfare under MNP when only type $\overline{\theta}$ consumers are served under MNP (contracts $\boldsymbol{S}_1^{\bar{\theta}}$ and $\boldsymbol{S}_2^{\bar{\theta}}$); when only competitive and fully competitive contracts are offered under DNP ($2Reg$) and when under DNP firms serve only type $\overline{\theta}$ consumers closer to the market center but offer competitive contracts elsewhere ($2Reg^*$).*

(iii) *Lower than welfare under MNP when under DNP firms only serve type $\overline{\theta}$ closer to the market center and offer intermediate contracts elsewhere ($2Reg^{**}$) and under MNP there are local monopolies for type $\underline{\theta}$ and type $\overline{\theta}$ indifferent consumer gets no surplus (contracts $\boldsymbol{LM}_2^{\underline{\theta}}$) and there are less type $\underline{\theta}$ consumers served under DNP than under MNP.*

(iv) *When under MNP there are local monopolies for type $\underline{\theta}$ and type $\overline{\theta}$ indifferent consumer gets no surplus (contracts $\boldsymbol{LM}_2^{\underline{\theta}}$) and under DNP firms serve only type $\overline{\theta}$ closer to the market center and offer suboptimal quality to type $\underline{\theta}$ consumers at locations closer to the firm and there are more type $\underline{\theta}$ consumers served under DNP than under MNP, the welfare comparisons are ambiguous.*

Proof: **(i)** When both types of consumers are completely covered under MNP (contracts $\mathbf{C}_1^{\bar{\theta}\underline{\theta}}$ and $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$) we have full competition contracts under DNP. In this case, both pricing policies maximize social welfare since every consumer is served and in addition the efficient quality is offered to both types of consumers. Thus, they yield the same social welfare.

(ii) When under MNP only type $\overline{\theta}$ consumers are served, it is clear that DNP yields higher welfare. Type $\underline{\theta}$ consumers get no surplus under MNP whilst under DNP there are always some type $\underline{\theta}$ consumers who are served and for these consumers the social welfare is positive (even though quality may be suboptimal).

When we have competitive and fully competitive contracts under DNP ($2Reg$) we either have local monopolies for type $\underline{\theta}$ or type $\underline{\theta}$ is not served under MNP. In either case, DNP yields higher welfare since both types are completely covered and they both consume an efficient quality.

When under DNP firms only serve type $\overline{\theta}$ closer to the market center but offer competitive contracts elsewhere ($2Reg^*$), we either have local monopolies for type $\underline{\theta}$ or type $\underline{\theta}$ consumers are not served under MNP. In the latter case, DNP is clearly better in welfare terms. In the former case, since under DNP firms serve both types of consumers with the efficient qualities until $d^C = \frac{\underline{\theta}^2}{2t}$ whilst under MNP type $\underline{\theta}$ consumers are only served until $\widehat{d^{\underline{\theta}}} = \frac{\underline{\theta}^2}{4t} < d^C = d^*_{\underline{\theta}} = \frac{\underline{\theta}^2}{2t}$, one can conclude that DNP yields higher welfare.

(iii) When $\mathbf{LM}_2^{\underline{\theta}}$ and $2Reg^{**}$ simultaneously hold and $\widehat{d^{\underline{\theta}}} > d^I$ (notice that $d^*_{\underline{\theta}} > \widehat{d^{\underline{\theta}}}$), there are less type $\underline{\theta}$ consumers served under DNP than under MNP. Moreover, under DNP these consumers are offered a suboptimal quality. Consequently welfare is higher under MNP.

(iv) This case happens when $\mathbf{LM}_2^{\underline{\theta}}$ and $2Reg^{**}$ hold and $\widehat{d^{\underline{\theta}}} < d^I$, and when $\mathbf{LM}_2^{\underline{\theta}}$ and $3Reg^*$ hold. Under DNP, firms serve more type $\underline{\theta}$ consumers but on the other hand some of type $\underline{\theta}$ consumers are not offered the efficient quality. ■

Notice that welfare comparisons are unambiguous for most of the set of parameters' values that satisfy the $NLMd$ condition. We show that welfare under DNP is higher than welfare under MNP for a wider subset of parameters' values. In fact, welfare under MNP is higher than welfare under DNP only when the degree of horizontal differentiation is relatively high and the degree of vertical differentiation is not low. When the degree of horizontal differentiation is low, welfare is identical under both pricing policies. The only subset of parameters' values where the welfare comparisons are ambiguous is for high degree of horizontal differentiation and intermediate or high degree of vertical differentiation.

1.4 Conclusion

In this chapter we considered a model where two firms compete for two types of consumers with different preferences for quality, either low or high valuation, and where both types of consumers are distributed on a unit length line. We assumed that consumers' location is observable by the firms but preferences for quality are not. In this setup of discrete *vertical uncertainty*, we compared two pricing policies: delivered nonlinear pricing and mill nonlinear pricing. Our work provides a complete characterization of *equilibrium* nonlinear contracts for all sets of parameters' values where competition is effective. In other words, we analyzed the *equilibrium* contracts for all degrees of horizontal and vertical differentiation until the level of horizontal differentiation where firms stop competing for the high valuation consumer. Two important contributions stem from of our analysis: the first one is that it extends previous results on delivered nonlinear pricing, giving a complete description of how delivered nonlinear pricing changes with the horizontal and vertical differentiation degrees; the second one is that it provides a comparison between the two pricing policies with respect to the efficiency of the quality offered, market coverage and welfare.

As expected, the *equilibrium* nonlinear pricing contracts have relevant differences as we cover all degrees of horizontal and vertical differentiation. Our mill nonlinear pricing analysis has qualitative similarities to Villas-Boas and Schmidt-Mohr (1999) although our work focus on price-quality contracts. For very low degrees of horizontal differentiation, the market is fully covered for both types of consumers. For high degrees of horizontal differentiation the market is not completely covered for the low valuation consumers: when there is a low degree of vertical differentiation, firms sell only to the high valuation consumers whereas for high degrees of vertical differentiation, firms start having local monopolies for the low valuation consumer. Our delivered nonlinear pricing analysis identifies the sets of parameters' values where there are different market regions, including the ones described by Pires and Sarkar (2000) and by Valletti (2002): monopoly, intermediate and competitive regions. However, our work extends their analysis since Pires and Sarkar (2000) do not analyze what happens as the horizontal or vertical differentiation degrees change and Valletti (2002) does not consider the impact of changes in the horizontal differentiation degree. We show that the three market regions identified may not always exist. In fact, we show that in order to have all these regions we need to have a relatively low degree of vertical differentiation (as in Valletti (2002)) and a relatively high degree of horizontal differentiation. When the degree of vertical differentiation is low, the monopoly region does not exist for low degrees of horizontal differentiation. Moreover, for a high degree of vertical differentiation, this region does not exist even for high degrees of horizontal differentiation.

Using the same framework to analyze both pricing policies allowed us to compare the characteristics of each nonlinear pricing policy *equilibrium* contracts and their welfare. Our comparative analysis is based on the efficiency of the quality offered to the low valuation consumers and on whether all low valuation consumers who should be served are covered or not. We show that welfare under delivered nonlinear pricing may be either lower, equal or higher than welfare under mill nonlinear pricing. When the degree of horizontal differentiation is high and the degree of vertical differentiation is intermediate or high, welfare comparisons are ambiguous. For the remaining set of parameters' values, delivered nonlinear pricing yields higher welfare except when *(i)* the degree of horizontal differentiation is low, where welfare is identical under both pricing policies and when *(ii)* the degree of horizontal differentiation is relatively high and the degree of vertical differentiation is intermediate or high.

1.A Monopoly analysis

Consider a monopolist located at $d_m = 0$. In this section, we analyze the monopolist mill and delivered nonlinear contracts.

1.A.1 Mill nonlinear pricing

When the monopolist is not allowed to offer different nonlinear contracts at each location, his demand from type θ consumers is determined by the location, $\hat{d}_m^\theta$, where type θ is indifferent between buying or not buying at all:

$$\theta\ u_i - t\hat{d}_m^\theta - p_i = 0 \quad \Leftrightarrow \quad \hat{d}_m^\theta = \frac{\theta\ u_i - p_i}{t}.$$

If the monopolist sells to both types of consumers, he will offer $(\bar{p}_m, \bar{u}_m)$ and $(\underline{p}_m, \underline{u}_m)$ that solve the following problem:

$$\max_{(\bar{p}_m,\bar{u}_m),(\underline{p}_m,\underline{u}_m)} \left(\bar{p}_m - \frac{\bar{u}_m^2}{2}\right)\hat{d}_m^{\bar{\theta}} + \left(\underline{p}_m - \frac{\underline{u}_m^2}{2}\right)\hat{d}_m^{\underline{\theta}}$$

subject to incentive compatibility constraints and feasibility constraints:

$$\begin{aligned}
IC^{\bar{\theta}} &: \ \bar{\theta}\bar{u}_m - \bar{p}_m \geq \bar{\theta}\underline{u}_m - \underline{p}_m \\
IC^{\underline{\theta}} &: \ \underline{\theta}\ \underline{u}_m - \underline{p}_m \geq \underline{\theta}\bar{u}_m - \bar{p}_m \\
FE^{\theta} &: \ 0 \leq \hat{d}_m^\theta \leq 1 \text{ for } \theta = \bar{\theta}, \underline{\theta}.
\end{aligned}$$

The Kuhn-Tucker conditions have multiple solutions.[19] Obviously, the solution of the monopolist problem is the one which yields the highest profit. Therefore, the optimal nonlinear contracts which hold for different parameters sets are given by:

Sol. 1	$\underline{u}_m = 2\underline{\theta} - \bar{\theta}$ and $\underline{p}_m = \underline{\theta}(2\underline{\theta} - \bar{\theta}) - t$ $\bar{u}_m = \bar{\theta}$ and $\bar{p}_m = \bar{\theta}^2 + (\underline{\theta} - \bar{\theta})(2\underline{\theta} - \bar{\theta}) - t$

[19] We also achieve some undetermined solutions which could be represented by one of the stated solutions since the profits from these possible but undetermined solutions are equal to the profits of other determined solutions and their intervals' limits belong to the determined solutions intervals' limits.

Sol. 2	$u_m = \bar{\theta}$ and $p_m = \bar{\theta}^2 - t$
Sol. 3	$\underline{u}_m = \underline{\theta}$ and $\underline{p}_m = \frac{3}{4}\underline{\theta}^2$ $\bar{u}_m = \bar{\theta}$ and $\bar{p}_m = \bar{\theta}^2 - t$
Sol. 4	$u_m = \bar{\theta}$ and $p_m = \frac{3}{4}\bar{\theta}^2$
Sol. 5	$\underline{u}_m = \underline{\theta}$ and $\underline{p}_m = \frac{3}{4}\underline{\theta}^2$ $\bar{u}_m = \bar{\theta}$ and $\bar{p}_m = \frac{3}{4}\bar{\theta}^2$

Incentive compatibility constraints are not binding, except at **Sol. 1** (where type $\bar{\theta}$'s incentive compatibility constraint binds). At **Sol. 1**, **Sol. 2** and **Sol. 3**, the monopolist covers the whole market for type $\bar{\theta}$ (that is, $\hat{d}_m^{\bar{\theta}} = 1$). Notice that the quality offered by the monopolist to each type is socially optimal except for type $\underline{\theta}$ consumers at **Sol. 1**. However, for some parameters' values, the monopolist offers a product quality such that no type $\underline{\theta}$ consumer buys – **Sol. 2** and **Sol. 4**.

At **Sol. 4** and **Sol. 5**, the monopolist stops covering the whole market for type $\bar{\theta}$. Under these optimal nonlinear contracts $\hat{d}_m^{\bar{\theta}} = \frac{\bar{\theta}^2}{4t}$. Since $\hat{d}_m^{\bar{\theta}}$ is decreasing with t, for sufficiently high values of t we will have $\hat{d}_m^{\bar{\theta}} < \frac{1}{2}$:

$$\hat{d}_m^{\bar{\theta}} = \frac{\bar{\theta}^2}{4t} < \frac{1}{2} \;\; \Leftrightarrow \;\; t > \frac{\bar{\theta}^2}{2}.$$

For this horizontal differentiation degree interval, if we consider two identical firms located at the extremes of the unit length line, we get local monopolies (LM) for type $\bar{\theta}$ as each firm covers less than half of the market (**Sol. 4**^{LM} and **Sol. 5**^{LM}). Notice that, for a given t, $\hat{d}_m^{\bar{\theta}}$ is always higher than $\hat{d}_m^{\underline{\theta}}$. Thus, we may have local monopolies for type $\underline{\theta}$ for even lower t values. Local monopolies for type $\underline{\theta}$ arise when $t > \frac{\bar{\theta}^2}{4}$ (and $k > 3$).

Figure 1.8 shows the sets of parameters' values where each solution holds.[20]

Discussion of results under monopolist mill nonlinear pricing When k is low, the monopolist only sells to type $\bar{\theta}$ consumers except for very low t. When k is high, the monopolist still prefers to sell type $\bar{\theta}$ consumers only until an intermediate t level (since taking all the

[20] For k close to 1 and high t, there is a pooling *equilibrium* solution. This region is not represented in Figure 1.8.

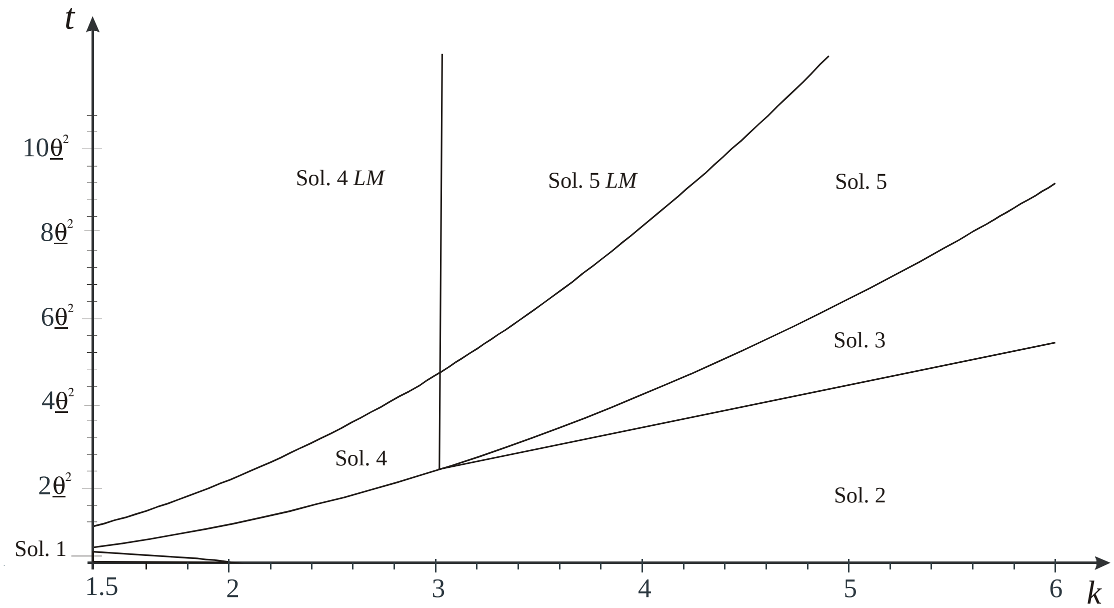

Figure 1.8: Monopolist *MNP equilibrium* contracts

possible type $\bar{\theta}$ consumers' net surplus maximizes its profits). However, when t increases, the monopolist starts selling to both types of consumers since screening becomes more profitable (first covering the whole market for type $\bar{\theta}$ and when t rises even more, not covering the whole market for both types).

1.A.2 Delivered nonlinear pricing

Let us now assume the monopolist is allowed to offer different nonlinear contracts at each location ($\bar{p}_m(d), \bar{u}_m(d)$ and $\underline{p}_m(d), \underline{u}_m(d)$). When he considers selling to both types, at each location d, he solves the following problem:

$$\max_{(\bar{p}_m,\bar{u}_m)(\underline{p}_m,\underline{u}_m)} \quad \underline{p}_m - \frac{\underline{u}_m^2}{2} + \bar{p}_m - \frac{\bar{u}_m^2}{2}$$

subject to:

$$\begin{aligned}
IRP^{\bar{\theta}} &: \bar{\theta}\bar{u}_m - td - \bar{p}_m \geq 0 \\
IRP^{\underline{\theta}} &: \underline{\theta}\ \underline{u}_m - td - \underline{p}_m \geq 0 \\
IC^{\bar{\theta}} &: \bar{\theta}\bar{u}_m - \bar{p}_m \geq \bar{\theta}\underline{u}_m - \underline{p}_m \\
IC^{\underline{\theta}} &: \underline{\theta}\ \underline{u}_m - \underline{p}_m \geq \underline{\theta}\bar{u}_m - \bar{p}_m.
\end{aligned}$$

Notice that type $\bar{\theta}$'s individual rationality constraint is redundant given type $\bar{\theta}$'s incentive compatibility constraint and type $\underline{\theta}$'s individual rationality constraint. When $k \leq 2$, we get the typical asymmetric information monopolist contract, with type $\bar{\theta}$'s incentive compatibility constraint and type $\underline{\theta}$'s individual rationality constraint binding, the quality offered to type $\bar{\theta}$ consumers is the socially efficient but the quality consumed by type $\underline{\theta}$ consumers is suboptimal. The optimal contract and profits are given by:

$\underline{u}_m = 2\underline{\theta} - \bar{\theta}$ and $\underline{p}_m = \underline{\theta}(2\underline{\theta} - \bar{\theta}) - td$ $\bar{u}_m = \bar{\theta}$ and $\bar{p}_m = \bar{\theta}^2 + (\underline{\theta} - \bar{\theta})(2\underline{\theta} - \bar{\theta}) - td$	$\pi_m = \bar{\theta}^2 - 2\bar{\theta}\underline{\theta} + 2\underline{\theta}^2 - 2td$

The monopolist will set this contract for a given t, until the location d where $\pi_m > 0$.

The monopolist can also choose to sell to type $\bar{\theta}$ consumers only and the maximization problem is:

$$\max_{(\bar{p}_m, \bar{u}_m)} \quad \bar{p}_m - \frac{\bar{u}_m^2}{2}$$

subject to:

$$IRP^{\bar{\theta}} : \bar{\theta}\bar{u}_m - td - \bar{p}_m \geq 0.$$

The unique solution has type $\bar{\theta}$'s individual rationality constraint binding with $\bar{u}_m = \bar{\theta}$ and $\bar{p}_m = \bar{\theta}^2 - td$. Type $\underline{\theta}$ consumers prefer not to buy under these conditions and profits at location d are equal to $\frac{\bar{\theta}^2}{2} - td$. Also, the monopolist will set this contract for a given t, until the location d where $\pi_m > 0$.

Comparing the profits in these two possible cases when $k \leq 2$, we conclude that for $t < \frac{(\bar{\theta} - 2\underline{\theta})^2}{2}$ the monopolist sells to both types at every location. If $t > \frac{(\bar{\theta} - 2\underline{\theta})^2}{2}$, the monopolist only serves both types until location $\bar{d} = \frac{(\bar{\theta} - 2\underline{\theta})^2}{2t}$.[21] Notice that for $t > (\bar{\theta} - 2\underline{\theta})^2$, the monopolist sells only to type $\underline{\theta}$ for locations to the left of the center of the market ($\bar{d} < \frac{1}{2}$). As a consequence, if we consider two identical firms operating at the extremes of the unit line, we may have local monopolies for type $\underline{\theta}$. Due to the non-negativity profits condition, as t rises the monopolist stops covering the whole market (even for type $\bar{\theta}$ consumers – the monopolist only set the contract described above until $\bar{d}^{\bar{\theta}} = \frac{\bar{\theta}^2}{2t}$). Therefore, for $t > \bar{\theta}^2$, we achieve local monopolies for both types.

[21] Afterwards, the monopolist sells only to type $\bar{\theta}$ consumers – by doing so the monopolist is able to extract a higher surplus from these consumers than if he also offered a price-quality contract to type $\underline{\theta}$ consumers.

For $k > 2$, it is optimal for the monopolist to sell to type $\bar{\theta}$ consumers only (for these k values that is the unique solution of the Kuhn-Tucker conditions). Also, due to the non-negativity profits condition, as t rises the monopolist stops covering the whole market (sells until $\bar{d}^{\bar{\theta}}$) and for $t > \bar{\theta}^2$ we achieve local monopolies for type $\bar{\theta}$.

Discussion of results under monopolist delivered nonlinear pricing When k is high, the monopolist only sells to type $\bar{\theta}$ consumers (first covering the whole market for type $\bar{\theta}$ and then, when t rises, will stopping to do so). Notice that the monopolist takes all the net surplus of type $\bar{\theta}$ consumers.

When k is low, the monopolist always sells to both types at locations closer to the monopolist's location (covering the whole market for low t). However, for locations near the market center, the monopolist only sells to type $\bar{\theta}$ consumers. As t rises, the locations where the monopolist sells to both types decrease since the monopolist prefers to sell only to type $\bar{\theta}$ consumers than selling to both types of consumers and thus leaving more net surplus to type $\bar{\theta}$ consumers.

1.B Kuhn-Tucker conditions of firm 1's problem

1.B.1 Mill nonlinear pricing – Market fully covered

Let σ_1, δ_1, λ_1, ρ_1 and μ_{1j} be the Lagrange multipliers ($j = 1, ..., 4$) associated with $IRP^{\bar{\theta}}$, $IRP^{\underline{\theta}}$, $IC^{\bar{\theta}}$, $IC^{\underline{\theta}}$ and FE, respectively. We can construct the Lagrangian function:

$$\begin{aligned}
\mathcal{L} \;=\;& \left(\bar{p}_1 - \frac{\bar{u}_1^2}{2}\right)\left(\frac{1}{2} + \frac{\bar{\theta}(\bar{u}_1 - \bar{u}_2) + \bar{p}_2 - \bar{p}_1}{2t}\right) + \left(\underline{p}_1 - \frac{\underline{u}_1^2}{2}\right)\left(\frac{1}{2} + \frac{\underline{\theta}(\underline{u}_1 - \underline{u}_2) + \underline{p}_2 - \underline{p}_1}{2t}\right) \\
&+\sigma_1\left[\bar{\theta}\bar{u}_1 - \frac{t}{2} - \frac{\bar{\theta}(\bar{u}_1 - \bar{u}_2) + \bar{p}_2 - \bar{p}_1}{2} - \bar{p}_1\right] + \delta_1\left[\underline{\theta}\ \underline{u}_1 - \frac{t}{2} - \frac{\underline{\theta}(\underline{u}_1 - \underline{u}_2) + \underline{p}_2 - \underline{p}_1}{2} - \underline{p}_1\right] \\
&+\lambda_1\left[\bar{\theta}\bar{u}_1 - \bar{p}_1 - \bar{\theta}\underline{u}_1 + \underline{p}_1\right] + \rho_1\left[\underline{\theta}\ \underline{u}_1 - \underline{p}_1 - \underline{\theta}\bar{u}_1 + \bar{p}_1\right] + \mu_{11}\left[\frac{1}{2} + \frac{\bar{\theta}(\bar{u}_1 - \bar{u}_2) + \bar{p}_2 - \bar{p}_1}{2t}\right] \\
&+\mu_{12}\left[\frac{1}{2} - \frac{\bar{\theta}(\bar{u}_1 - \bar{u}_2) + \bar{p}_2 - \bar{p}_1}{2t}\right] + \mu_{13}\left[\frac{1}{2} + \frac{\underline{\theta}(\underline{u}_1 - \underline{u}_2) + \underline{p}_2 - \underline{p}_1}{2t}\right] \\
&+\mu_{14}\left[\frac{1}{2} - \frac{\underline{\theta}(\underline{u}_1 - \underline{u}_2) + \underline{p}_2 - \underline{p}_1}{2t}\right].
\end{aligned}$$

The Kuhn-Tucker conditions, when all p and u must be non-negative, are given by:

$$\frac{\partial \mathcal{L}}{\partial \bar{p}_1} = t + \bar{\theta}(\bar{u}_1 - \bar{u}_2) + \bar{p}_2 - 2\bar{p}_1 + \frac{\bar{u}_1^2}{2} - t\sigma_1 - 2t\lambda_1 + 2t\rho_1 - \mu_{11} + \mu_{12} \leq 0;$$

$$\bar{p}_1 \geq 0; \quad \bar{p}_1 \frac{\partial \mathcal{L}}{\partial \bar{p}_1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \underline{p}_1} = t + \underline{\theta}(\underline{u}_1 - \underline{u}_2) + \underline{p}_2 - 2\underline{p}_1 + \frac{\underline{u}_1^2}{2} - t\delta_1 + 2t\lambda_1 - 2t\rho_1 - \mu_{13} + \mu_{14} \leq 0;$$

$$\underline{p}_1 \geq 0; \quad \underline{p}_1 \frac{\partial \mathcal{L}}{\partial \underline{p}_1} = 0$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \bar{u}_1} =& -\bar{u}_1\left(t + \bar{\theta}(\bar{u}_1 - \bar{u}_2) + \bar{p}_2 - \bar{p}_1\right) + \bar{\theta}\left(\bar{p}_1 - \frac{\bar{u}_1^2}{2}\right) + \bar{\theta}t\sigma_1 + 2t\bar{\theta}\lambda_1 - 2t\underline{\theta}\rho_1 \\
&+\bar{\theta}\mu_{11} - \bar{\theta}\mu_{12} \leq 0; \bar{u}_1 \geq 0; \quad \bar{u}_1 \frac{\partial \mathcal{L}}{\partial \bar{u}_1} = 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \underline{u}_1} =& -\underline{u}_1\left(t + \underline{\theta}(\underline{u}_1 - \underline{u}_2) + \underline{p}_2 - \underline{p}_1\right) + \underline{\theta}\left(\underline{p}_1 - \frac{\underline{u}_1^2}{2}\right) + t\underline{\theta}\delta_1 - 2t\bar{\theta}\lambda_1 + 2t\underline{\theta}\rho_1 \\
&+\underline{\theta}\mu_{13} - \underline{\theta}\mu_{14} \leq 0; \quad \underline{u}_1 \geq 0; \underline{u}_1 \frac{\partial \mathcal{L}}{\partial \underline{u}_1} = 0
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \sigma_1} &= \bar{\theta}\bar{u}_1 - \frac{t}{2} - \frac{\bar{\theta}(\bar{u}_1 - \bar{u}_2) + \bar{p}_2 - \bar{p}_1}{2} - \bar{p}_1 \geq 0; \quad \sigma_1 \geq 0; \quad \sigma_1 \frac{\partial \mathcal{L}}{\partial \sigma_1} = 0 \\
\frac{\partial \mathcal{L}}{\partial \delta_1} &= \underline{\theta}\ \underline{u}_1 - \frac{t}{2} - \frac{\underline{\theta}(\underline{u}_1 - \underline{u}_2) + \underline{p}_2 - \underline{p}_1}{2} - \underline{p}_1 \geq 0; \quad \delta_1 \geq 0; \quad \delta_1 \frac{\partial \mathcal{L}}{\partial \delta_1} = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda_1} &= \bar{\theta}\bar{u}_1 - \bar{p}_1 - \bar{\theta}\underline{u}_1 + \underline{p}_1 \geq 0; \quad \lambda_1 \geq 0; \quad \lambda_1 \frac{\partial \mathcal{L}}{\partial \lambda_1} = 0 \\
\frac{\partial \mathcal{L}}{\partial \rho_1} &= \underline{\theta}\ \underline{u}_1 - \underline{p}_1 - \underline{\theta}\bar{u}_1 + \bar{p}_1 \geq 0; \quad \rho_1 \geq 0; \quad \rho_1 \frac{\partial \mathcal{L}}{\partial \rho_1} = 0 \\
\frac{\partial \mathcal{L}}{\partial \mu_{11}} &= \frac{1}{2} + \frac{\bar{\theta}(\bar{u}_1 - \bar{u}_2) + \bar{p}_2 - \bar{p}_1}{2t} \geq 0; \quad \mu_{11} \geq 0; \quad \mu_{11} \frac{\partial \mathcal{L}}{\partial \mu_{11}} = 0 \\
\frac{\partial \mathcal{L}}{\partial \mu_{12}} &= \frac{1}{2} - \frac{\bar{\theta}(\bar{u}_1 - \bar{u}_2) + \bar{p}_2 - \bar{p}_1}{2t} \geq 0; \quad \mu_{12} \geq 0; \quad \mu_{12} \frac{\partial \mathcal{L}}{\partial \mu_{12}} = 0 \\
\frac{\partial \mathcal{L}}{\partial \mu_{13}} &= \frac{1}{2} + \frac{\underline{\theta}(\underline{u}_1 - \underline{u}_2) + \underline{p}_2 - \underline{p}_1}{2t} \geq 0; \quad \mu_{13} \geq 0; \quad \mu_{13} \frac{\partial \mathcal{L}}{\partial \mu_{13}} = 0 \\
\frac{\partial \mathcal{L}}{\partial \mu_{14}} &= \frac{1}{2} - \frac{\underline{\theta}(\underline{u}_1 - \underline{u}_2) + \underline{p}_2 - \underline{p}_1}{2t} \geq 0; \quad \mu_{14} \geq 0; \quad \mu_{14} \frac{\partial \mathcal{L}}{\partial \mu_{14}} = 0.
\end{aligned}
$$

The symmetric *equilibrium* solutions of the Kuhn-Tucker conditions are:

- when no constraint is binding:

s. I. $\mathbf{C}_1^{\bar{\theta}\underline{\theta}}$	$0 < t \leq \frac{\underline{\theta}^2}{3}$	$\underline{u} = \underline{\theta}$ and $\underline{p} = \frac{\underline{\theta}^2}{2} + t$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \frac{\bar{\theta}^2}{2} + t$

- when $IRP^{\underline{\theta}}$ is the only binding constraint:

s. II. $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$	$\frac{\underline{\theta}^2}{3} < t < \left(\frac{\underline{\theta}^2}{3} + \frac{(\bar{\theta}-\underline{\theta})^2}{3}\right)$	$\underline{u} = \underline{\theta}$ and $\underline{p} = \underline{\theta}^2 - \frac{t}{2}$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \frac{\bar{\theta}^2}{2} + t$

1.B.2 Mill nonlinear pricing – Market fully covered for type $\bar{\theta}$ only

Let σ_1, λ_1, ρ_1 and μ_{1j} be the Lagrange multipliers ($j = 1, ..., 4$) associated with $IRP^{\bar{\theta}}$, $IC^{\bar{\theta}}$, $IC^{\underline{\theta}}$, $FE^{\bar{\theta}}$ and $PC^{\underline{\theta}}$, respectively. We can construct the Lagrangian function:

$$\begin{aligned}
\mathcal{L} \;=\;& \left(\bar{p}_1 - \frac{\bar{u}_1^2}{2}\right)\left(\frac{1}{2} + \frac{\bar{\theta}(\bar{u}_1 - \bar{u}_2) + \bar{p}_2 - \bar{p}_1}{2t}\right) + \left(\underline{p}_1 - \frac{\underline{u}_1^2}{2}\right)\left(\frac{\underline{\theta}\ \underline{u}_i - \underline{p}_i}{t}\right) \\
& +\sigma_1\left[\bar{\theta}\bar{u}_1 - \frac{t}{2} - \frac{\bar{\theta}(\bar{u}_1 - \bar{u}_2) + \bar{p}_2 - \bar{p}_1}{2} - \bar{p}_1\right] + \lambda_1\left[\bar{\theta}\bar{u}_1 - \bar{p}_1 - \bar{\theta}\underline{u}_1 + \underline{p}_1\right] \\
& +\rho_1\left[\underline{\theta}\ \underline{u}_1 - \underline{p}_1 - \underline{\theta}\bar{u}_1 + \bar{p}_1\right] + \mu_{11}\left[\frac{1}{2} + \frac{\bar{\theta}(\bar{u}_1 - \bar{u}_2) + \bar{p}_2 - \bar{p}_1}{2t}\right] \\
& +\mu_{12}\left[\frac{1}{2} - \frac{\bar{\theta}(\bar{u}_1 - \bar{u}_2) + \bar{p}_2 - \bar{p}_1}{2t}\right] + \mu_{13}\left[\frac{\underline{\theta}\ \underline{u}_1 - \underline{p}_1}{t}\right] + \mu_{14}\left[1 - \frac{\underline{\theta}\ \underline{u}_2 - \underline{p}_2}{t} - \frac{\underline{\theta}\ \underline{u}_1 - \underline{p}_1}{t}\right].
\end{aligned}$$

We get the same Kuhn-Tucker conditions as in the previous case except for:

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial\underline{p}_1} &= \underline{\theta}\ \underline{u}_i - 2\underline{p}_i + \frac{\underline{u}_1^2}{2} - t\delta_1 + t\lambda_1 - t\rho_1 - \mu_{13} + \mu_{14} \le 0; \underline{p}_1 \ge 0;\quad \underline{p}_1\frac{\partial\mathcal{L}}{\partial\underline{p}_1} = 0 \\
\frac{\partial\mathcal{L}}{\partial\underline{u}_1} &= -\underline{u}_1\left(\underline{\theta}\ \underline{u}_i - \underline{p}_i\right) + \underline{\theta}\left(\underline{p}_1 - \frac{\underline{u}_1^2}{2}\right) - t\bar{\theta}\lambda_1 + t\underline{\theta}\rho_1 + \underline{\theta}\mu_{13} - \underline{\theta}\mu_{14} \le 0; \underline{u}_1 \ge 0; \underline{u}_1\frac{\partial\mathcal{L}}{\partial\underline{u}_1} = 0 \\
\frac{\partial\mathcal{L}}{\partial\mu_{13}} &= \frac{\underline{\theta}\ \underline{u}_1 - \underline{p}_1}{t} \ge 0;\quad \mu_{13} \ge 0;\quad \mu_{13}\frac{\partial\mathcal{L}}{\partial\mu_{13}} = 0 \\
\frac{\partial\mathcal{L}}{\partial\mu_{14}} &= 1 - \frac{\underline{\theta}\ \underline{u}_2 - \underline{p}_2}{t} - \frac{\underline{\theta}\ \underline{u}_1 - \underline{p}_1}{t} \ge 0;\quad \mu_{14} \ge 0;\quad \mu_{14}\frac{\partial\mathcal{L}}{\partial\mu_{14}} = 0.
\end{aligned}$$

The symmetric *equilibrium* solutions of the Kuhn-Tucker conditions are:

- when no constraint is binding:

s. III. $\mathbf{S}_1^{\bar{\theta}}$	$k > \frac{6}{5}$ and $\frac{\bar{\theta}(2\underline{\theta}-\bar{\theta})}{2} < t \le \frac{\bar{\theta}^2}{3}$	$\underline{u} = 0$ and $\underline{p} = 0$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \frac{\bar{\theta}^2}{2} + t$
s. VII.	$2 < k \le \left(6 + 2\sqrt{6}\right)$ and $0 < t < \frac{(\bar{\theta}-2\underline{\theta})^2}{2}$ or $k > \left(6 + 2\sqrt{6}\right)$ and $0 < t \le \frac{\bar{\theta}^2}{3}$	$\underline{u} = 2\underline{\theta}$ and $\underline{p} = 2\underline{\theta}^2$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \frac{\bar{\theta}^2}{2} + t$
s. V. $\mathbf{LM}_1^{\underline{\theta}}$	$\left(1 + \frac{\sqrt{2}}{2}\right) < k \le \left(3 + \frac{3\sqrt{2}}{2}\right)$ and $\frac{\underline{\theta}^2}{2} < t < \frac{(\bar{\theta}-2\underline{\theta})^2+\bar{\theta}^2-\underline{\theta}^2}{4}$ or $k > \left(3 + \frac{3\sqrt{2}}{2}\right)$ and $\frac{\underline{\theta}^2}{2} < t \le \frac{\bar{\theta}^2}{3}$	$\underline{u} = \underline{\theta}$ and $\underline{p} = \frac{3}{4}\underline{\theta}^2$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \frac{\bar{\theta}^2}{2} + t$

- when $IRP^{\bar{\theta}}$ is the only binding constraint:

s. IV. $\mathbf{S}_2^{\bar{\theta}}$	$k > \frac{6}{5}$ and $\frac{\bar{\theta}^2}{3} < t < 2\bar{\theta}(\bar{\theta} - \underline{\theta})$	$\underline{u} = 0$ and $\underline{p} = 0$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \bar{\theta}^2 - \frac{t}{2}$
s. VIII.	$2 < k \leq \left(6 + 2\sqrt{6}\right)$ and $4\underline{\theta}(\bar{\theta} - \underline{\theta}) < t < 2\bar{\theta}(\bar{\theta} - \underline{\theta})$ or $k > \left(6 + 2\sqrt{6}\right)$ and $\frac{\bar{\theta}^2}{3} < t < 2\bar{\theta}(\bar{\theta} - \underline{\theta})$	$\underline{u} = 2\underline{\theta}$ and $\underline{p} = 2\underline{\theta}^2$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \bar{\theta}^2 - \frac{t}{2}$
s. VI. $\mathbf{LM}_2^{\underline{\theta}}$	$2 < k \leq \left(3 + \frac{3\sqrt{2}}{2}\right)$ and $2\underline{\theta}\left(\bar{\theta} - \frac{3}{4}\underline{\theta}\right) < t < \frac{(\bar{\theta}-\underline{\theta})^2+\bar{\theta}^2}{2}$ or $k > \left(3 + \frac{3\sqrt{2}}{2}\right)$ and $\frac{\bar{\theta}^2}{3} < t < \frac{(\bar{\theta}-\underline{\theta})^2+\bar{\theta}^2}{2}$	$\underline{u} = \underline{\theta}$ and $\underline{p} = \frac{3}{4}\underline{\theta}^2$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \bar{\theta}^2 - \frac{t}{2}$

- when only $IC^{\bar{\theta}}$ and non-negativity $PC^{\underline{\theta}}$ constraints bind:

s. IX.	$2 < k \leq \left(6 + 2\sqrt{6}\right)$ and $0 < t < \frac{(\bar{\theta}-2\underline{\theta})^2}{2}$ or $k > \left(6 + 2\sqrt{6}\right)$ and $0 < t < \frac{\bar{\theta}^2}{3}$	$\underline{u} = \frac{2t-\bar{\theta}^2}{2(\underline{\theta}-\bar{\theta})}$ and $\underline{p} = \frac{2t-\bar{\theta}^2}{2(\underline{\theta}-\bar{\theta})}\underline{\theta}$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \frac{\bar{\theta}^2}{2} + t$

- when $IC^{\underline{\theta}}$ is the only non-binding constraint:

s. X.	$2 < k \leq \left(6 + 2\sqrt{6}\right)$ and $4\underline{\theta}(\bar{\theta} - \underline{\theta}) < t < 2\bar{\theta}(\bar{\theta} - \underline{\theta})$ or $k > \left(6 + 2\sqrt{6}\right)$ and $\frac{\bar{\theta}^2}{3} < t < 2\bar{\theta}(\bar{\theta} - \underline{\theta})$	$\underline{u} = \frac{t}{2(\bar{\theta}-\underline{\theta})}$ and $\underline{p} = \frac{t}{2(\bar{\theta}-\underline{\theta})}\underline{\theta}$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \bar{\theta}^2 - \frac{t}{2}$

1.B.3 Delivered nonlinear pricing $- t < \frac{\underline{\theta}^2}{2}$

Let σ_1, δ_1, $\lambda_1$and ρ_1 be the Lagrange multipliers associated with $IR^{\bar{\theta}}$, $IR^{\underline{\theta}}$, $IC^{\bar{\theta}}$ and $IC^{\underline{\theta}}$, respectively. We can construct the Lagrangian function:

$$\begin{aligned}
\mathcal{L} \;=\; & \left(\bar{p}_1 - \frac{\bar{u}_1^2}{2}\right) + \left(\underline{p}_1 - \frac{\underline{u}_1^2}{2}\right) + \sigma_1\left[\bar{\theta}\bar{u}_1 - td - \bar{p}_1 - \frac{\bar{\theta}^2}{2} + t(1-d)\right] \\
& + \delta_1\left[\underline{\theta}\;\underline{u}_1 - td - \underline{p}_1 - \frac{\underline{\theta}^2}{2} + t(1-d)\right] + \lambda_1\left[\bar{\theta}\bar{u}_1 - \bar{p}_1 - \bar{\theta}\underline{u}_1 + \underline{p}_1\right] \\
& + \rho_1\left[\underline{\theta}\;\underline{u}_1 - \underline{p}_1 - \underline{\theta}\bar{u}_1 + \bar{p}_1\right].
\end{aligned}$$

The Kuhn-Tucker conditions, when all p and u must be non-negative, are given by:

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial\bar{p}_1} &= 1-\sigma_1-\lambda_1+\rho_1\le 0;\;\; \bar{p}_1\ge 0;\;\; \bar{p}_1\frac{\partial\mathcal{L}}{\partial\bar{p}_1}=0\\
\frac{\partial\mathcal{L}}{\partial\underline{p}_1} &= 1-\delta_1+\lambda_1-\rho_1\le 0;\;\; \underline{p}_1\ge 0;\;\; \underline{p}_1\frac{\partial\mathcal{L}}{\partial\underline{p}_1}=0\\
\frac{\partial\mathcal{L}}{\partial\bar{u}_1} &= -\bar{u}_1+\bar{\theta}\sigma_1+\bar{\theta}\lambda_1-\underline{\theta}\rho_1\le 0;\;\; \bar{u}_1\ge 0;\;\; \bar{u}_1\frac{\partial\mathcal{L}}{\partial\bar{u}_1}=0\\
\frac{\partial\mathcal{L}}{\partial\underline{u}_1} &= -\underline{u}_1+\underline{\theta}\delta_1-\bar{\theta}\lambda_1+\underline{\theta}\rho_1\le 0;\;\; \underline{u}_1\ge 0; \underline{u}_1\frac{\partial\mathcal{L}}{\partial\underline{u}_1}=0\\
\frac{\partial\mathcal{L}}{\partial\sigma_1} &= \bar{\theta}\bar{u}_1-td-\bar{p}_1-\frac{\bar{\theta}^2}{2}+t(1-d)\ge 0;\;\; \sigma_1\ge 0;\;\; \sigma_1\frac{\partial\mathcal{L}}{\partial\sigma_1}=0\\
\frac{\partial\mathcal{L}}{\partial\delta_1} &= \underline{\theta}\;\underline{u}_1-td-\underline{p}_1-\frac{\underline{\theta}^2}{2}+t(1-d)\ge 0;\;\; \delta_1\ge 0;\;\; \delta_1\frac{\partial\mathcal{L}}{\partial\delta_1}=0\\
\frac{\partial\mathcal{L}}{\partial\lambda_1} &= \bar{\theta}\bar{u}_1-\bar{p}_1-\bar{\theta}\underline{u}_1+\underline{p}_1\ge 0;\;\; \lambda_1\ge 0;\;\; \lambda_1\frac{\partial\mathcal{L}}{\partial\lambda_1}=0\\
\frac{\partial\mathcal{L}}{\partial\rho_1} &= \underline{\theta}\;\underline{u}_1-\underline{p}_1-\underline{\theta}\bar{u}_1+\bar{p}_1\ge 0;\;\; \rho_1\ge 0;\;\; \rho_1\frac{\partial\mathcal{L}}{\partial\rho_1}=0.
\end{aligned}$$

1.B.4 Delivered nonlinear pricing – $\underline{\theta}^2\le t<\frac{\bar{\theta}^2}{2}$

Let σ_1, δ_1, $\lambda_1$and ρ_1 be the Lagrange multipliers associated with $IR^{\bar{\theta}}$, $IR^{\underline{\theta}}$, $IC^{\bar{\theta}}$ and $IC^{\underline{\theta}}$, respectively. We can construct the Lagrangian function:

$$\begin{aligned}
\mathcal{L} &= \left(\bar{p}_1-\frac{\bar{u}_1^2}{2}\right)+\left(\underline{p}_1-\frac{\underline{u}_1^2}{2}\right)+\sigma_1\left[\bar{\theta}\bar{u}_1-td-\bar{p}_1-\frac{\bar{\theta}^2}{2}+t(1-d)\right]\\
&\quad+\delta_1\left[\underline{\theta}\;\underline{u}_1-td-\underline{p}_1\right]+\lambda_1\left[\bar{\theta}\bar{u}_1-\bar{p}_1-\bar{\theta}\underline{u}_1+\underline{p}_1\right]+\rho_1\left[\underline{\theta}\;\underline{u}_1-\underline{p}_1-\underline{\theta}\bar{u}_1+\bar{p}_1\right].
\end{aligned}$$

We get the same Kuhn-Tucker conditions identical to the previous case except for:

$$\frac{\partial\mathcal{L}}{\partial\delta_1}=\underline{\theta}\;\underline{u}_1-td-\underline{p}_1\ge 0;\;\; \delta_1\ge 0;\;\; \delta_1\frac{\partial\mathcal{L}}{\partial\delta_1}=0.$$

1.C Welfare computations

In a duopoly, the aggregate total surplus under mill nonlinear pricing, W^{MNP}, is given by:

Contract	W^{MNP}
$\mathbf{C}_1^{\bar{\theta}\underline{\theta}} = \mathbf{C}_2^{\bar{\theta}\underline{\theta}}$	$\frac{\bar{\theta}^2}{2} + \frac{\underline{\theta}^2}{2} - \frac{t}{2}$
$\mathbf{S}_1^{\bar{\theta}} = \mathbf{S}_2^{\bar{\theta}}$	$\frac{\bar{\theta}^2}{2} - \frac{t}{4}$
$\mathbf{LM}_1^{\underline{\theta}} = \mathbf{LM}_2^{\underline{\theta}}$	$\frac{\bar{\theta}^2}{2} + \frac{3\underline{\theta}^4}{16t} - \frac{t}{4}$

On the other hand, welfare under delivered nonlinear pricing, W^{DNP}, is given by:

Scenario	W^{DNP}
$1Reg = 2Reg$	$\frac{\bar{\theta}^2}{2} + \frac{\underline{\theta}^2}{2} - \frac{t}{2}$
$3Reg$	$\frac{\bar{\theta}^2}{2} + \frac{\underline{\theta}^2}{2} - \frac{t}{2} - \frac{\left[2t+\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2\right)\right]^3}{24t(\bar{\theta}-\underline{\theta})^2}$
$4Reg$	$\frac{\bar{\theta}^2}{2} + \frac{\underline{\theta}^2}{2} - \frac{t}{2} - \frac{\left[2t+\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2\right)\right]^3}{24t(\bar{\theta}-\underline{\theta})^2} + \frac{\left[2t+\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right]^2\left[2t+\left(-6\bar{\theta}\underline{\theta}+3\bar{\theta}^2+2\underline{\theta}^2\right)\right]}{24t(\bar{\theta}-\underline{\theta})^2}$
$2Reg^*$	$\frac{\underline{\theta}^4}{4t} + \frac{\bar{\theta}^2}{2} - \frac{t}{4}$
$3Reg^*$	$\frac{\underline{\theta}^4}{4t} + \frac{\bar{\theta}^2}{2} - \frac{t}{4} - \frac{\left[2t+\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2\right)\right]^3}{24t(\bar{\theta}-\underline{\theta})^2}$
$4Reg^*$	$\frac{\underline{\theta}^4}{4t} + \frac{\bar{\theta}^2}{2} - \frac{t}{4} - \frac{\left[2t+\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2\right)\right]^3}{24t(\bar{\theta}-\underline{\theta})^2} + \frac{\left[2t+\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right]^2\left[2t+\left(-6\bar{\theta}\underline{\theta}+3\bar{\theta}^2+2\underline{\theta}^2\right)\right]}{24t(\bar{\theta}-\underline{\theta})^2}$
$2Reg^{**}$	$\frac{\bar{\theta}^2}{2} - \frac{t}{4} + \frac{1}{4(\bar{\theta}-\underline{\theta})^2}\left[\left(2t+(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2)\right)\frac{3*(\bar{\theta}^2-2t)(2t-(\bar{\theta}-2\underline{\theta})^2)+2*\left(2t+\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right)^2}{6t} + \frac{\left[-16\left(\bar{\theta}-\underline{\theta}\right)^2\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2+t\right)\right]\sqrt{4(\bar{\theta}-\underline{\theta})^2((-4\bar{\theta}\underline{\theta}+2\bar{\theta}^2+4\underline{\theta}^2)-2t)}}{6t}\right]$
$3Reg^{**}$	$\frac{\bar{\theta}^2}{2} - \frac{t}{4} + \frac{\left[2t+\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right]^2\left[2t+\left(-6\bar{\theta}\underline{\theta}+3\bar{\theta}^2+2\underline{\theta}^2\right)\right]}{24t(\bar{\theta}-\underline{\theta})^2} + \frac{1}{4(\bar{\theta}-\underline{\theta})^2}\left[\left(2t+(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2)\right)\frac{3*(\bar{\theta}^2-2t)(2t-(\bar{\theta}-2\underline{\theta})^2)+2*\left(2t+\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right)^2}{6t} + \frac{\left[-16\left(\bar{\theta}-\underline{\theta}\right)^2\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2+t\right)\right]\sqrt{4(\bar{\theta}-\underline{\theta})^2((-4\bar{\theta}\underline{\theta}+2\bar{\theta}^2+4\underline{\theta}^2)-2t)}}{6t}\right]$

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Chapter 2

Delivered versus Mill Nonlinear Pricing with Endogenous Market Structure

2.1 Introduction

Regulation theory has been one of the most active areas of research in the last decade. However, regulation of firms' pricing policies has been almost neglected within this wave of research. This chapter revisits the issue of whether regulatory authorities should prohibit firms from practicing price discrimination among consumers who differ according to some observable characteristic.

The currently accepted view that unregulated markets are more competitive has been justified by economic theory using spatial pricing models. Most existing studies compare spatial discriminatory pricing with non-discriminatory mill pricing[1] for a given market structure (see, for example, Norman (1983) and Thisse and Vives (1988)). Under spatial discriminatory pricing, firms can price discriminate across locations. Hence, firms compete in each of them. On the other hand, if firms have to set the same price at every location, competition only occurs at the boundary of each firm's market. As a consequence, for a given market structure, discrimi-

[1] Implicit in this comparison is the idea that without pricing regulation firms will practice discriminatory prices. Thisse and Vives (1988) have shown that if firms are free to choose their pricing policy, in *equilibrium*, they will in fact price discriminate even though this implies lower profits for all firms.

natory pricing is more competitive than mill pricing. However, as pointed out by Norman and Thisse (1996), the previous analysis is incomplete if one does not consider the effect of pricing policies on market structure since entry incentives are not the same under both pricing policies.

Norman and Thisse (1996) analyze the economic justification of firms' pricing policies regulation taking into account the effect of pricing policies on welfare for a given market structure and the effect of pricing policies on market structure. They consider a circular model of horizontal product differentiation where the location of each consumer is observable and firms may price discriminate (delivered pricing) or set a mill price (mill pricing). The free entry long-run *equilibrium* and its number of firms (number of product varieties[2]) is computed for each pricing policy and degree of spatial contestability (when firms' relocation is costless we have spatial contestability; when location is once-for-all we have spatial non-contestability). Their work illustrates an important trade-off: as delivered pricing implies fiercer competition and lower profits, in the long-run it acts as an entry deterrent and reduces the free entry number of firms. Therefore, mill pricing always leads to more variety than delivered pricing. However, this higher variety is not necessarily welfare improving. When relocation is costless, both mill and delivered pricing have too much product variety. Thus, the *equilibrium* number of firms under discriminatory pricing is closer to the socially optimal one and discriminatory pricing leads to higher social surplus. Under spatial non-contestability the welfare comparisons are less clear. In this case, there is too much product variety under mill pricing but too little product variety with delivered pricing.

The previously mentioned works consider linear prices: mill linear price is compared to discriminatory linear price, in a setup where consumers differ according to one observable characteristic (location). However, if consumers differ simultaneously according to one unobservable (e.g. quality preference) and one observable characteristic (e.g. location – brand preference), we would expect nonlinear prices to arise in *equilibrium*. Therefore, the issue of whether firms should be allowed to practice different nonlinear prices at each location or should be forced to set the same nonlinear contract in every location is relevant.

In order to study this previously unexplored issue, our work adds vertical differentiation to

[2] The free entry long-run number of firms has been interpreted as the number of product varieties (MacLeod *et al.* (1988) and Anderson and Palma (1988)). When firms use delivered pricing, they customize the basic products in order to offer consumer's optimal product specification. Since firms' location can be seen as the basic product characteristic, the free entry long-run number of firms measures the degree of product variety.

the Norman and Thisse (1996) setup. This is an important extension since horizontally differentiated firms competing in product lines are notorious in various markets (e.g. car, telecommunications, airline, etc.). Within this more realistic setup, we assume that firms face uncertainty about consumers' quality preferences but observe their location (or brand preferences' parameter). Under these circumstances, firms set price-quality nonlinear contracts that screen consumers according to their preference for quality. The regulatory issue[3] is then whether firms should be allowed to practice different nonlinear price contracts at each location (*delivered nonlinear pricing*) or, on the contrary, should be forced to set a unique nonlinear price contract (*mill nonlinear pricing*).

One interesting example where our discussion applies and which has been the subject of sizable debate within the European Union is the car market. The recent policy stance strongly acts upon its distribution sector to stop the practices upon which car manufacturers rely to prevent arbitrage (therefore acting upon a crucial feature for price discrimination). Although not acting upon price regulation, the economic justification of the European Commission is the need to create more competition to the advantage of European consumers. The European Commission claims that "the consumer should be in the driver's seat" and thus building a single market may put pressure on price differentials.[4] By comparing delivered nonlinear pricing and mill nonlinear pricing, our analysis uncovers some features that may be felt in the car market after its distribution sector liberalization.

Our work builds on previous results on nonlinear pricing under oligopoly. The framework we use is similar to the one first presented by Stole (1995), who named it *vertical uncertainty*. Stole studies delivered nonlinear price contracts considering a continuum of consumer types. Delivered nonlinear pricing with a discrete number of consumer types was studied by Pires and Sarkar (2000) and Valletti (2002). On the other hand, Villas-Boas and Schmidt-Mohr (1999) studied mill nonlinear contracts under discrete *vertical uncertainty*.

Focusing on the discrete case, with two types of consumers at each location, the economic justification for pricing policies' regulation is discussed in a model of horizontal and vertical dif-

[3]Regulation always acts upon discrimination based on some observable characteristic. In our model this characteristic is consumer's location.

[4]Brenkers and Verboven (2002) discuss the impact on competition from liberalizing the European car market distribution system and show empirically that there may be a reduction in international price discrimination and price differentials.

ferentiation where firms set price-quality nonlinear contracts. Considering spatial contestability, we study the impact on welfare and market structure of two pricing policies: *delivered nonlinear pricing* – DNP – and *mill nonlinear pricing* – MNP.

Our results show that delivered nonlinear pricing may bring smaller, equal or larger product variety (free entry long-run *equilibrium* number of firms) than mill nonlinear pricing. Moreover, our analytical results show that long-run welfare is higher under delivered nonlinear pricing for low fixed costs and for intermediate fixed costs and sufficiently different consumer types. Our numerical results suggest that this result also holds for other parameters' values.

This chapter will proceed as follows: Section 2.2 presents the circular city model with n identical firms, studies the *equilibrium* contracts and compares profits and welfare for both pricing policies for a given market structure. Section 2.3 analyzes the free entry long-run *equilibrium* under spatial contestability and compares both pricing policies in terms of long-run product variety and welfare. Our main conclusions are presented in Section 2.4.

2.2 Equilibrium price contracts for a given market structure

Our main objective is to study the impact of pricing policies on market structure. We address this issue in two steps. First, for a given market structure (fixed number of firms, n), we study the *equilibrium* under each pricing policy. This analysis is similar to the one presented in Chapter 1. Second, we determine the free entry long-run *equilibrium* under each pricing policy. We use the circular city oligopoly model to analyze our problem.

Consider n identical firms simultaneously offering nonlinear contracts (price p_i, quality u_i) to consumers distributed on a unit length circle. There are two types of consumers at each location d, $d \in [0, 1]$ and both types are uniformly distributed along the circle, both having mass equal to 1. These consumers are characterized by their preference for quality, measured by θ: $\bar{\theta} > \underline{\theta} > 0$. We assume *vertical uncertainty*, *i.e.*, firms observe consumer's location d while the quality preference parameter θ is consumer's private information. All firms have a fixed cost F and firms' relocation is costless (*i.e.*, we focus on spatial contestability). In addition, all firms have identical production costs for a good of quality u_i, given by $C(u_i) = \frac{u_i^2}{2}$. We assume a location pattern where firms are *equidistantly* located around the circle:[5] an arc distance of $\frac{1}{n}$

[5] Notice that this assumption is appealing since this location pattern is one of the possible locational *equilibrium*

separates every two *neighbor* firms in the market.

Each firm i will always compete directly with two *neighboring* firms (the first firm located at its left and the first at its right) implying that every firm deals with two independent symmetric problems. In this section, we will look for the *equilibrium* contracts *between two of the n firms* since the reasoning would be similar for all other pairs of firms. Notice that each firm's profit function is achieved by duplicating the profits attained in the market region under analysis.

Let us consider firm 1 and firm 2, located at $d_1 = 0$ and $d_2 = \frac{1}{n}$, respectively. We consider that each consumer type buys a single unit of a good and chooses the contract (p_i, u_i) that yields the maximum net surplus (purchasing at most from one firm). When a consumer is indifferent between buying from firm 1 or firm 2, we assume that he buys from the nearest firm. If a type θ consumer located at $d \in \left[0, \frac{1}{n}\right]$ buys at price p_i a product of quality u_i produced by firm i, $i = 1, 2$, located at d_i, his net surplus is given by:

$$V = U(\theta, d, u_i, d_i) - p_i = \theta u_i - t\,|d - d_i| - p_i, \tag{2.1}$$

where t is a differentiation parameter (horizontal differentiation degree – $t > 0$) and $t\,|d - d_i|$ measures the consumer's loss of not consuming his ideal brand. Also consider k to be the asymmetric information degree or the vertical differentiation degree, *i.e.*, the dispersion of types' quality preference parameters: $k = \frac{\bar{\theta}}{\underline{\theta}}$.

In this section we describe the nonlinear contracts for a given market structure. Since mill and delivered nonlinear pricing have been previously analyzed in Chapter 1 considering a

configurations. When firms practice delivered nonlinear pricing, firms choose locations at the median of their sales distribution (Pires and Sarkar (2000), Pires (2002) and Hamilton and Thisse (1992)). If firms are equally spaced in the circle, each firm's *equilibrium* sales distribution will be symmetric around its location. But then the firm's *equilibrium* location is at the median of its sales distribution. Thus the equally spaced configuration is an *equilibrium* of the two stage location-price game. Kats (1995) shows that equally spaced location configuration is one of the possible *equilibria* when firms practice mill linear pricing. When firms practice mill nonlinear pricing and are equidistantly located around the circle, if a firm i gets closer to firm j (located at the right of firm i) it will rise the distance between firm i and firm l (located at the left of firm i). As firms have competing firms at both sides of their location, the business stealing effect from getting closer to one of the *neighboring* firms will cancel out by getting further away from the other firm. When equally spaced in the circle, firms will minimize transportation costs and steal the biggest possible share of the consumers' surplus. For more discussion on symmetric locational *equilibrium* see Eaton and Wooders (1985), Lederer and Hurter (1986), MacLeod *et al.* (1988) and Novshek (1980).

duopoly, we do not present a detailed deduction of the *equilibrium* contracts.

2.2.1 Mill nonlinear pricing

Lets us look at the case between two of the n firms when they simultaneously offer a unique nonlinear contract – similar to Subsection 1.2.2. From the local monopoly analysis (similar to Appendix 1.A.1 but considering a monopolist selling in a market whose length is $\frac{1}{n}$) there are no local monopolies *at least for one type* (type $\bar{\theta}$) as long as $0 < t < \frac{n\bar{\theta}^2}{2}$ – NLM condition.[6] Following the same procedure as in Chapter 1 and assuming that t belongs to the previous interval, firms may cover the whole market for both types, cover the whole market for type $\bar{\theta}$ only or sell to type $\bar{\theta}$ only.

When firms cover the whole market for both types, firms' demand is determined by the indifferent consumer location for each type. Following an optimization procedure as in Chapter 1, the *equilibrium* solutions are similar to the linear city duopoly model. We achieve the following *equilibrium* solutions:

$\mathbf{C}_1^{\bar{\theta}\underline{\theta}}$	$\underline{u} = \underline{\theta}$ and $\underline{p} = \frac{\underline{\theta}^2}{2} + \frac{t}{n}$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \frac{\bar{\theta}^2}{2} + \frac{t}{n}$
$\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$	$\underline{u} = \underline{\theta}$ and $\underline{p} = \underline{\theta}^2 - \frac{t}{2n}$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \frac{\bar{\theta}^2}{2} + \frac{t}{n}$

When firms cover the whole market for type $\bar{\theta}$ only, demand from type $\underline{\theta}$ consumers is determined by the location where type $\underline{\theta}$ is indifferent between buying and not buying at all. Following an optimization procedure as in Chapter 1, we obtain the following *equilibrium* solutions:

$\mathbf{S}_1^{\bar{\theta}}$	$u = \bar{\theta}$ and $p = \frac{\bar{\theta}^2}{2} + \frac{t}{n}$
$\mathbf{S}_2^{\bar{\theta}}$	$u = \bar{\theta}$ and $p = \bar{\theta}^2 - \frac{t}{2n}$
$\mathbf{LM}_1^{\underline{\theta}}$	$\underline{u} = \underline{\theta}$ and $\underline{p} = \frac{3}{4}\underline{\theta}^2$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \frac{\bar{\theta}^2}{2} + \frac{t}{n}$
$\mathbf{LM}_2^{\underline{\theta}}$	$\underline{u} = \underline{\theta}$ and $\underline{p} = \frac{3}{4}\underline{\theta}^2$ $\bar{u} = \bar{\theta}$ and $\bar{p} = \bar{\theta}^2 - \frac{t}{2n}$

[6] Under monopoly delivered nonlinear pricing in a market whose length is $\frac{1}{n}$, the condition for non-existence of local monopolies would be less restrictive. Therefore, we consider the limit given by the monopoly mill nonlinear pricing analysis.

We only have *equilibrium* solutions for all $t < \frac{n\bar{\theta}^2}{2}$ if $k > \left(1 + \frac{\sqrt{5}}{5}\right)$. Figure 2.1 illustrates the set of parameters' values where each of the previously described solution contracts holds. Figure 2.1 is similar to Figure 1.1 but here the number of firms affects the scale of the vertical axis. Thus, the values of t such that a given solution holds change with n. For a given t and k, when n rises we move towards more competitive solutions.

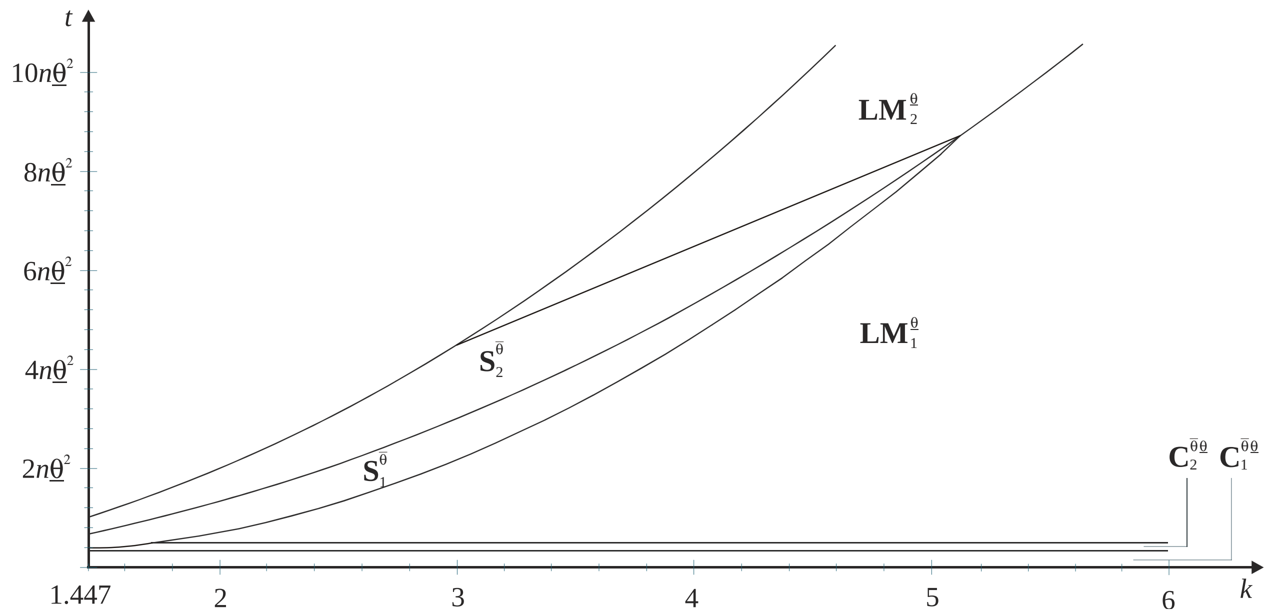

Figure 2.1: *Equilibrium* contracts under MNP

2.2.2 Delivered nonlinear pricing

Suppose now that firms are able to offer *different* nonlinear contracts *at each location* in the circle ($\bar{p}_i(d), \bar{u}_i(d)$ and $\underline{p}_i(d), \underline{u}_i(d)$) – similar to Subsection 1.2.3. To simplify notation, we will drop the argument d for the remaining of this subsection. Since the two firms are identical, with DNP the arc between firm 1 and firm 2 will be split into two equal parts and each firm has its own *market area*. We will restrict the analysis to firm 1's optimization problem within $d \in \left[0, \frac{1}{2n}\right]$, firm 1's *market area*, since the analysis would be symmetric for $d \in \left[\frac{1}{2n}, \frac{1}{n}\right]$ but with firms' positions reversed. The reasoning is similar to the linear city duopoly case but here the maximum net surplus attained when buying from the most direct competitors is equal to $\frac{\theta^2}{2} - t(\frac{1}{n} - d)$, which is positive when $d_\theta > \frac{2t - n\theta^2}{2tn} = \breve{d}_\theta$. Considering $k > \sqrt{2}$, type $\bar{\theta}$ consumers located at firm 1's *market area* always get positive net surplus for $t < \frac{n\bar{\theta}^2}{2}$. On the other hand, type $\underline{\theta}$ consumers net surplus when buying from firm 2 may be positive or negative. The change in the net surplus of the consumers' best outside option lead to the following types of contracts:

Region	Contracts
M	$\underline{u}_1 = 2\underline{\theta} - \bar{\theta}$ and $\underline{p}_1 = \underline{\theta}(2\underline{\theta} - \bar{\theta}) - td$ $\bar{u}_1 = \bar{\theta}$ and $\bar{p}_1 = \bar{\theta}^2 + (\underline{\theta} - \bar{\theta})(2\underline{\theta} - \bar{\theta}) - td$
I	$\underline{u}_1 = \frac{n\bar{\theta}^2+2tdn-2t}{2(\bar{\theta}-\underline{\theta})n}$ and $\underline{p}_1 = \frac{n\underline{\theta}\bar{\theta}^2+2tdn(2\underline{\theta}-\bar{\theta})-2\underline{\theta}t}{2(\bar{\theta}-\underline{\theta})n}$ $\bar{u}_1 = \bar{\theta}$ and $\bar{p}_1 = \frac{\bar{\theta}^2}{2} - 2td + \frac{t}{n}$
C	$\underline{u}_1 = \underline{\theta}$ and $\underline{p}_1 = \underline{\theta}^2 - td$ $\bar{u}_1 = \bar{\theta}$ and $\bar{p}_1 = \frac{\bar{\theta}^2}{2} - 2td + \frac{t}{n}$

Figure 2.2 shows these three possible regions at firm 1's *market area* where $d^* = \frac{6n\bar{\theta}\underline{\theta}-3n\bar{\theta}^2-4n\underline{\theta}^2+2t}{2tn}$ and $\hat{d} = \frac{2n\bar{\theta}\underline{\theta}-n\bar{\theta}^2-2n\underline{\theta}^2+2t}{2tn}$.

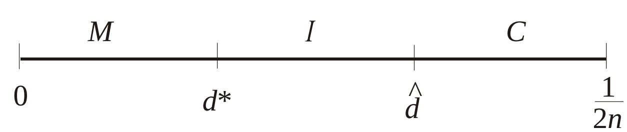

Figure 2.2: Three regions at firm 1's *market area*

The conclusions and contract analysis are analogous to the linear city duopoly case. Figure 2.3 shows the set of parameters where different *equilibrium* solution contracts hold. The bottom part (in grey) corresponds to low and intermediate values of t. The different scenarios in the parameters' space are characterized as follows:

$1Reg$	$FC\ ([0, \frac{1}{2n}])$
$2Reg$	$C([0, \breve{d}_{\underline{\theta}}[)$ and $FC([\breve{d}_{\underline{\theta}}, \frac{1}{2n}])$
$3Reg$	$I([0, \hat{d}[), C([\hat{d}, \breve{d}_{\underline{\theta}}[)$ and $FC([\breve{d}_{\underline{\theta}}, \frac{1}{2n}])$
$4Reg$	$M([0, d^*[), I[d^*, \hat{d}[, C([\hat{d}, \breve{d}_{\underline{\theta}}[)$and $FC([\breve{d}_{\underline{\theta}}, \frac{1}{2n}])$

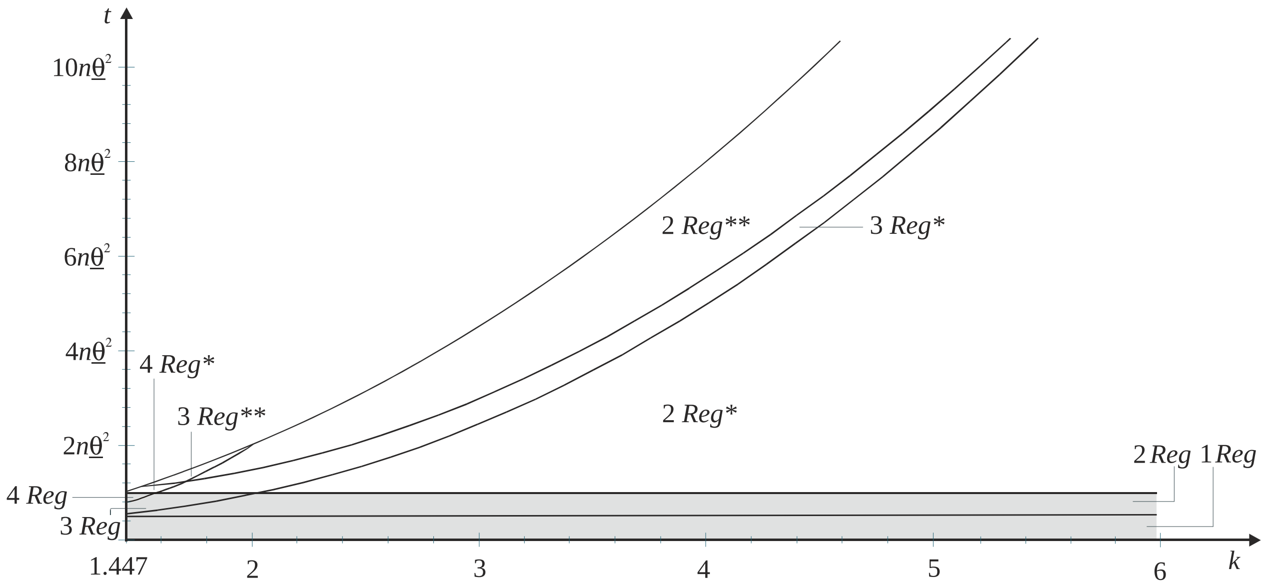

Figure 2.3: *Equilibrium* solutions under DNP

The top part (in white) in Figure 2.3 corresponds to high values of t. The different scenarios in the parameters' space are characterized as follows:

$2Reg^{*}$	$C\ ([0,d^{C}[)$ and $C^{\bar{\theta}}\ ([d^{C},\frac{1}{2n}])$
$3Reg^{*}$	$I\ ([0,\hat{d}[),C\ ([\hat{d},d^{C}[)$ and $C^{\bar{\theta}}\ ([d^{C},\frac{1}{2n}])$
$4Reg^{*}$	$M\ ([0,d^{*}[),I\ [d^{*},\hat{d}[,C\ ([\hat{d},d^{C}[)$ and $C^{\bar{\theta}}\ ([d^{C},\frac{1}{2n}])$
$2Reg^{**}$	$I\ ([0,d^{I}[)$ and $C^{\bar{\theta}}\ ([d^{I},\frac{1}{2n}])$
$3Reg^{**}$	$M\ ([0,d^{*}[),I\ ([d^{*},d^{I}[)$ and $C^{\bar{\theta}}\ ([d^{I},\frac{1}{2n}])$

where $d^{C}=\frac{\theta^{2}}{2t}$ and $d^{I}=\frac{6n\bar{\theta}\underline{\theta}-3n\bar{\theta}^{2}-4n\underline{\theta}^{2}+2t+\sqrt{4n\left(\bar{\theta}-\underline{\theta}\right)^{2}\left(2n\bar{\theta}^{2}+4n\underline{\theta}^{2}-4n\bar{\theta}\underline{\theta}-2t\right)}}{2nt}$.

In Figure 2.3 the number of firms affects the scale of the vertical axis. As in MNP, the values of t such that a given solution holds change with n and, for a given k and t, when n rises we move towards more competitive solutions.

2.2.3 MNP versus DNP

In this subsection we compare the two pricing policies for a given market structure, both in terms of profits and welfare. Figure 2.4 shows DNP and MNP solutions for the various parameters' values and illustrates that when n rises the NLM limit also rises. For a given k, when more firms enter the market we must have a higher horizontal differentiation degree (higher t values) in order to get local monopolies for both types of consumers.

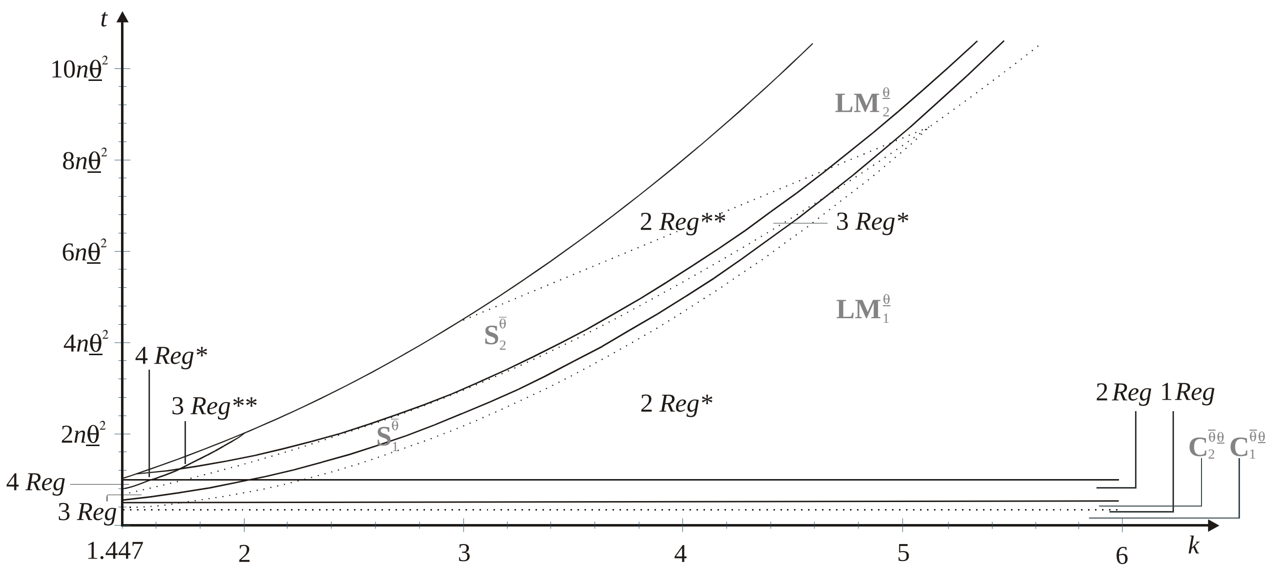

Figure 2.4: DNP (solid and black) versus MNP (dots and grey) *equilibrium* solutions

Profits comparison

Using the *equilibrium* contracts, for a given market structure, one can compute the *equilibrium* profit functions (see Appendices 2.A.1 and 2.A.2 for MNP and DNP, respectively). As expected, the *equilibrium* profit functions are all decreasing with n in the relevant range (i.e., for the parameters' values where they are the *equilibrium* solution).

Under mill nonlinear pricing, for a given n, the profit functions of $\mathbf{C}_1^{\bar{\theta}\underline{\theta}}$, $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$, $\mathbf{S}_1^{\bar{\theta}}$ and $\mathbf{LM}_1^{\underline{\theta}}$ are increasing with t whereas $\mathbf{S}_2^{\bar{\theta}}$ and $\mathbf{LM}_2^{\underline{\theta}}$ profits are decreasing with t. There are discontinuities in the profit function between $\mathbf{LM}_1^{\underline{\theta}}$ and $\mathbf{S}_1^{\bar{\theta}}$, $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$ and $\mathbf{S}_1^{\bar{\theta}}$ and between $\mathbf{S}_2^{\bar{\theta}}$ and $\mathbf{LM}_2^{\underline{\theta}}$.

Under delivered nonlinear pricing, for a given n, the profit functions of $1Reg$, $2Reg$ and

$2Reg^*$ are always increasing with t whereas in all the other profit functions there are some parameters' values where profits are decreasing with t and other parameters' values where they are increasing with t. However, under DNP there is no profits discontinuity**.**

If we compare profits for a given set of exogenous parameters, $t, \underline{\theta}, \overline{\theta}$ and F, and for a given market structure (*i.e.*, given n), we get ambiguous results:

Proposition 2.1 *For a given market structure, the profit under DNP may be either smaller, equal or larger than under MNP. That is, for certain sets of parameters' values MNP yields higher profits. For other sets of parameters' values the two pricing policies yield the same profits. Finally, there are some sets of parameters' values for which DNP has higher profits.*

Proof: Our proof does not compare DNP and MNP for every set of parameters' values. We only prove the existence of sets of parameters' values such that the result holds.

Let us denote by Π_i^d and Π_i^m, the profit of firm i under DNP and MNP, respectively. For $t < \frac{n\underline{\theta}^2}{2}$ and under DNP, full competition contracts are offered in every location and each firm gets a profit equal to $\Pi_i^d = \frac{t}{n^2} - F$. On the other hand, under MNP we may have $\mathbf{C}_1^{\bar{\theta}\underline{\theta}}$, $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$ or $\mathbf{S}_1^{\bar{\theta}}$ contracts. For the set of parameters' values where contracts $\mathbf{C}_1^{\bar{\theta}\underline{\theta}}$ and $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$ hold each firm's profit is either $\Pi_i^m = \frac{2t}{n^2} - F$ or $\Pi_i^m = \frac{\underline{\theta}^2}{2n} + \frac{t}{2n^2} - F$. In this case, it is trivial to show that $\Pi_i^m > \Pi_i^d$. On the other hand, when $\mathbf{S}_1^{\bar{\theta}}$ holds, each firm's profit is $\Pi_i^m = \frac{t}{n^2} - F$, hence $\Pi_i^m = \Pi_i^d$.

For low k and high t (close the limit where MNP yields local monopolies), DNP yields higher profits than MNP. Here we just show a single case where this happens. However, there are other sets of parameters' values where $\Pi_i^d > \Pi_i^m$. Let us consider the set of parameters' values where $\mathbf{S}_2^{\bar{\theta}}$ and $4Reg$ both hold and let $\underline{t}$ and $\bar{t}$ denote the lower and upper boundaries of this set (in terms of t), respectively. For $k < 1.507$, it is simple to show that $\Pi_i^d(\underline{t}) > \Pi_i^m(\underline{t})$ and $\Pi_i^d(\bar{t}) > \Pi_i^m(\bar{t})$, *i.e.*, at the lower and upper boundaries profits under DNP are higher than profits under MNP. Furthermore, in this set of parameters' values both profit functions are continuous and Π_i^m is always decreasing with t whilst Π_i^d initially increases and then decreases with t. Thus, we can conclude that $\Pi_i^d > \Pi_i^m$, for all $t \in \left[\underline{t}, \bar{t}\right]$. ■

Using the type of arguments applied in the previous proof we were able to compare profits in every subset of the parameters' space. Profits under DNP are higher only when $\mathbf{S}_2^{\bar{\theta}}$ holds and t is close to the NLM condition.

The fact that DNP may yield higher profits than MNP seems to contradict Norman and Thisse's results under linear pricing. However, Norman and Thisse (1996) limited their analysis to the case where no firm has monopoly power in any segment of its *market area*. If they had compared delivered and mill linear pricing for parameter values such that there are local monopolies under mill pricing, they would have concluded that profits under delivered pricing are higher than under mill pricing. This conclusion is obvious when we have local monopolies under both pricing policies, but it also holds when we have local monopolies under mill pricing but firms still compete, closer to market center, under delivered pricing. This happens because under delivered pricing, at locations closer to the firm, the firm is able to charge delivered monopoly prices (which, with inelastic demands means charging a price equal to the consumer reservation price). Consequently for a given market structure we may have higher profits under delivered linear pricing than under mill linear pricing.

In a footnote, Norman and Thisse state, without proof, that their analysis is not significantly affected by their assumption. Since they were interested in the free entry long-run *equilibrium*, we agree with this statement. In fact, if we ignore integer constraints (as Norman and Thisse did) in a free entry *equilibrium* we can not have local monopolies. Under local monopolies each firm's variable profit does not depend on the number of firms. But then, either variable profits are smaller than fixed costs or variable profits are higher than fixed costs. In the first case, every firm will exit the market. In the second case, each firm gets positive profit and thus more firms will enter the market. This is true as long as local monopolies hold (since profit does not depend on n). Thus, we will continue to have entry until the local monopoly situation disappears.[7] However, we should notice that since monopoly delivered profits are higher than monopoly mill profits, we may have values of F such that no entry occurs under linear mill pricing while entry occurs under delivered linear pricing. But for values of F such that entry occurs under both pricing policies, Norman and Thisse's result is valid.

The previous argument justifies why we did not study the case where there are local monopolies for both types of consumers. However, with two types of consumers and under MNP, we may have firms competing for type $\bar{\theta}$ consumer and having local monopolies, or even not serving

[7] If F is large, the argument that one cannot have local monopolies under free entry is not valid. In this scenario, ignoring integer constraints is not an adequate assumption and we may have local monopolies under free entry. Thus, we may have higher profits under delivered linear pricing.

at all, for type $\underline{\theta}$. Simultaneously, under DNP firms may be able to charge monopoly nonlinear prices at locations closer to the firm. Moreover, since each firm's profit depends on n under both pricing policies (firms are competing for type $\overline{\theta}$ under MNP and competition is effective under DNP for locations closer to the market center) there seems to be no reason for this situation not to happen in a free entry *equilibrium*. Whilst in the Norman and Thisse's framework the local monopolies case was not interesting in the free entry *equilibrium*, with two types of consumers, we may have free entry *equilibria* where firms are competing only for type $\bar{\theta}$ consumer.

Since, for a given market structure, profits under MNP may be higher, equal or smaller than profits under DNP, we also expect ambiguous results when comparing the free entry long-run *equilibrium* number of firms under the two pricing policies. This result will be shown in Section 2.3.

Welfare comparison

Let us now compare welfare under the two pricing policies, for a given market structure (*i.e.*, given n). Since the quality offered to type $\overline{\theta}$ consumers is always the efficient one and this type of consumer is always served, both pricing policies maximize social welfare for this type of consumer. Therefore, for a given market structure, welfare differences are explained by what happens to type $\underline{\theta}$ consumers. There are two issues which are relevant in the welfare comparison for type $\underline{\theta}$ consumers: the first one is whether this type of consumer is offered the efficient quality or not; the second one is whether consumers of this type are served or not.

Using an analysis similar to that in Section 1.3 but considering n firms' *equilibrium* solution contracts, we conclude that there are many parameters' values for which we can easily compare welfare under MNP and DNP.

Proposition 2.2 *For a given market structure, i.e., n fixed, welfare under DNP is:*

(i) *Equal to welfare under MNP when both types of consumers are completely covered under MNP (contracts $\boldsymbol{C}_1^{\bar{\theta}\underline{\theta}}$ and $\boldsymbol{C}_2^{\bar{\theta}\underline{\theta}}$).*

(ii) *Higher than welfare under MNP when only type $\overline{\theta}$ consumers are served under MNP (contracts $\boldsymbol{S}_1^{\bar{\theta}}$ and $\boldsymbol{S}_2^{\bar{\theta}}$); when only competitive and fully competitive contracts are offered under DNP ($2Reg$) and when under DNP firms serve only type $\overline{\theta}$ consumers closer to the market center but offer competitive contracts elsewhere ($2Reg^*$).*

(iii) *Lower than welfare under MNP when under DNP firms only serve type $\overline{\theta}$ closer to the market center and offer intermediate contracts elsewhere ($2Reg^{**}$) and under MNP there are local monopolies for type $\underline{\theta}$ and type $\overline{\theta}$ indifferent consumer gets no surplus (contracts $\boldsymbol{LM}_2^{\underline{\theta}}$) and there are less type $\underline{\theta}$ consumers served under DNP than under MNP.*

(iv) *When under MNP there are local monopolies for type $\underline{\theta}$ and type $\overline{\theta}$ indifferent consumer gets no surplus (contracts $\boldsymbol{LM}_2^{\underline{\theta}}$) and under DNP firms serve only type $\overline{\theta}$ closer to the market center and offer suboptimal quality to type $\underline{\theta}$ consumers at locations closer to the firm and there are more type $\underline{\theta}$ consumers served under DNP than under MNP, the welfare comparisons are ambiguous.*

Proof: The reasoning is similar to the one presented in the proof of Proposition 1.1 in Chapter 1. ■

In other words, for a given market structure, welfare under DNP may be either lower, equal or higher than under MNP. When the degree of horizontal differentiation is high and the degree of vertical differentiation is intermediate or high, welfare comparisons are ambiguous. For the remaining set of parameters' values, welfare under DNP is higher than welfare under MNP except when the degree of horizontal differentiation is low (where welfare is identical under both pricing policies) and when the degree of horizontal differentiation is relatively high and the degree of vertical differentiation is not low.

2.3 Pricing policies and market structure

In this section, we want to address several questions. First, for given values of the exogenous parameters, t, $\underline{\theta}$, $\overline{\theta}$ and F, which is the free entry long-run *equilibrium* number of firms for MNP (n^m) and DNP (n^d)? Second, for the same parameters' values, what is the socially optimal number of firms and how does it compare with the free entry *equilibrium* under the two pricing policies? Finally, which of the two pricing policies brings higher long-run welfare?

In the first part of this section we present some analytical results concerning the free entry *equilibrium*. We are able to show that for low and intermediate fixed costs the free entry long-run *equilibrium* number of firms is larger or equal under MNP but long-run welfare is higher under DNP. In the second part of this section we present some numerical results which confirm that the free entry number of firms under DNP may be smaller, equal or higher than under MNP and suggest that long-run welfare is always higher under DNP.

2.3.1 Some analytical results

Socially optimal market structure

The social planner's problem is easily described by:

$$\max_{n^s, d^*_{\bar{\theta}}, d^*_{\underline{\theta}}} \quad 2n^s \left[\int_0^{d^*_{\bar{\theta}}} (\frac{\bar{\theta}^2}{2} - tx)dx + \int_0^{d^*_{\underline{\theta}}} (\frac{\underline{\theta}^2}{2} - tx)dx \right] - n^s F$$

subject to $d^*_{\bar{\theta}} \leq \frac{1}{2n}$, $d^*_{\underline{\theta}} \leq \frac{1}{2n}$ and $n^s, d^*_{\bar{\theta}}, d^*_{\underline{\theta}}$ non-negative.

Notice that if $d^*_{\theta} < \frac{1}{2n}$ the market is not fully covered for type θ. Thus, the previous formulation allows for the possibility of not covering the whole market for one or for both types.

The socially optimal number of firms depends on the parameters' values as described in the following proposition.

Proposition 2.3 *When $F \leq \frac{\underline{\theta}^4}{2t}$ it is socially optimal to cover the whole market for both types and the optimal number of firms is given by:*

$$n^s = \sqrt{\frac{t}{2F}}.$$

When $\frac{\underline{\theta}^4}{2t} \leq F \leq \frac{\bar{\theta}^4 + \underline{\theta}^4}{4t}$ the market should not be fully covered for type $\underline{\theta}$ and the optimal number of firms is:

$$n^s = \frac{t}{\sqrt{4Ft - \underline{\theta}^4}}.$$

Finally, for $F > \frac{\bar{\theta}^4 + \underline{\theta}^4}{4t}$ it is socially optimal not to serve the market ($n^s = 0$).

Proof: These results are obtained from the solution of the social planner's problem. ■

Notice that it can never be socially optimal to have both types of consumers only partially covered. Why? In this case, the variable welfare (*i.e.*, excluding F) generated by each firm does not depend on n. Thus, either the variable welfare generated by each firm is higher than F or smaller than F. In the former case, it is socially optimal to keep increasing n until the market is covered for at least one type. In the latter case, it is socially optimal not to have entry. However, for intermediate values of F, it may be socially optimal not to serve all type $\underline{\theta}$ consumers. This provides an additional argument why analyzing the case where not all type $\underline{\theta}$ consumers are served is important (this case is relevant from a social welfare point of view).

In the next subsection we compare the socially optimal number of firms, n^s, with the free entry long-run *equilibrium* number of firms under the two pricing policies, n^m and n^d.

Equilibrium market structure

As it was mentioned in Section 2.2.3 the profit function is continuous with n under DNP but not under MNP. Consequently there may exist some parameters' values where there is no n^m such that $\Pi_i^m(n^m) = 0$. However, these discontinuities only happen when one «jumps» between $\mathbf{S}_1^{\bar{\theta}}$ and $\mathbf{LM}_1^{\underline{\theta}}$, $\mathbf{S}_1^{\bar{\theta}}$ and $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$ and between $\mathbf{LM}_2^{\underline{\theta}}$ and $\mathbf{S}_2^{\bar{\theta}}$. Everywhere else the MNP profit function is continuous. Moreover, for the purpose of *equilibrium* existence, discontinuity with n is only a problem when the profit function «jumps» down[8] when n increases (which only happens between $\mathbf{LM}_2^{\underline{\theta}}$ and $\mathbf{S}_2^{\bar{\theta}}$).

In the next proposition we compare the free entry number of firms and social welfare[9], for low F and for intermediate F and intermediate or high k (for these parameters' values *equilibrium* existence is guaranteed).

Proposition 2.4 *If* ***(i)*** $F \leq \frac{\underline{\theta}^4}{4t}$ *or* ***(ii)*** $k \geq 2$ *and* $\frac{\underline{\theta}^4}{4t} \leq F \leq \frac{3\underline{\theta}^4}{4t}$*, the free entry number of firms under* DNP *is smaller or equal than under* MNP*. In addition, the long-run social welfare under* DNP *is larger than under* MNP*.*

Proof: **(i)** If $F \leq \frac{\underline{\theta}^4}{4t}$, under DNP full competition contracts hold in every location of a firm's *market* ($1Reg$) and the free entry *equilibrium* number of firms under DNP is $n^d = \sqrt{\frac{t}{F}} = \sqrt{2}n^s$. Thus, there exists too much entry. Proposition 2.1 states that, for a given n, profit under DNP with full competition contracts is either smaller or equal to profit under MNP (depending on whether $\mathbf{C}_1^{\bar{\theta}\underline{\theta}}$, $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$ or $\mathbf{S}_1^{\bar{\theta}}$ contracts hold). By definition, n^m is such that $\Pi_i^m(n^m) = 0$. Since $\Pi_i^d(n^m) \leq \Pi_i^m(n^m) = 0$ and Π_i^d is continuous and decreasing with n, this implies that $n^d \leq n^m$.

[8] When the profit function «jumps» up we may have a problem of multiplicity of *equilibria*. In one of these solutions n^m is equal to n^d.

[9] In a monopolistic competition model where there are informed and uninformed consumers, Katz (1984) shows that price discrimination impact on long-run social welfare is ambiguous and depends of the proportion of informed to uninformed consumers.

When $1Reg$ and $\mathbf{C}_1^{\bar{\theta}\underline{\theta}}$ or $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$ hold, every consumer is served and qualities are the socially efficient ones under both pricing policies. Thus, welfare differences are exclusively due to the number of firms. Since $n^m > n^d > n^s$ (MNP has more excess entry than DNP), one concludes that $W^d(n^d) > W^m(n^m)$. When $\mathbf{S}_1^{\bar{\theta}}$ contracts hold under MNP, the two pricing policies have the same long-run number of firms, but since type $\underline{\theta}$ consumers are not served under MNP, once again $W^d(n^d) > W^m(n^m)$.

(ii) If $k \geq 2$ and $\frac{\underline{\theta}^4}{4t} \leq F \leq \frac{3\underline{\theta}^4}{4t}$, under DNP we have competitive and fully competitive contracts ($2Reg$) and $n^d > n^s$. For a given n, profit under DNP is smaller than profit under MNP (whether $\mathbf{LM}_1^{\underline{\theta}}$ or $\mathbf{S}_1^{\bar{\theta}}$ hold). Since profits are decreasing with n, this implies that $n^d < n^m$. Notice that in the free entry *equilibrium* we may have $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$ contract under MNP (since $n^m > n^d$, $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$ holds for higher values of t than $1Reg$).

When $\mathbf{LM}_1^{\underline{\theta}}$ ($\mathbf{S}_1^{\bar{\theta}}$) and $2Reg$ hold the quality offered is optimal under both pricing policies. However, there is more excess entry under MNP since $n^m > n^d > n^s$ and under MNP there are too few (no) type $\underline{\theta}$ consumers being served. When $\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$ and $2Reg$ hold every consumer is served and qualities are the socially efficient ones under both pricing policies, thus welfare differences are exclusively due to the number of firms and consequently $W^d(n^d) > W^m(n^m)$. ■

For very low fixed costs our results are very similar to Norman and Thisse (1996). In fact, if we restrict the analysis to $F < \frac{2\underline{\theta}^4}{9t}$, solution $\mathbf{C}_1^{\bar{\theta}\underline{\theta}}$ holds under MNP and the free entry number of firms is $n^m = \sqrt{\frac{2t}{F}} = \sqrt{2}n^d$, which is precisely the result obtained by Norman and Thisse (1996). However, our previous proposition extends their results to higher values of F, including cases where the market is not completely covered for type $\underline{\theta}$ under MNP.

It is possible to extend the previous proposition even further to the set of parameters' values where $2Reg^*$ holds under free entry. However, in this case, there may exist parameters' values for which no free entry *equilibrium* exists under MNP. The next proposition is limited to the case where free entry *equilibrium* exists under both pricing policies.

Proposition 2.5 *If $k \geq 2$ and $\frac{3\underline{\theta}^4}{4t} \leq F \leq \frac{\left(\bar{\theta}^2+2\underline{\theta}^2-2\bar{\theta}\underline{\theta}\right)^2}{8t} + \frac{\underline{\theta}^4}{4t}$ and there exists a free entry equilibrium under MNP, then the free entry number of firms under DNP is smaller than under MNP. Moreover, long-run social welfare under DNP is larger than under MNP.*

Proof: If $k \geq 2$ and $\frac{3\underline{\theta}^4}{4t} \leq F \leq \frac{\left(\bar{\theta}^2+2\underline{\theta}^2-2\bar{\theta}\underline{\theta}\right)^2}{8t} + \frac{\underline{\theta}^4}{4t}$, under DNP we have competitive contracts until $d^C = d^*_{\underline{\theta}} = \frac{\underline{\theta}^2}{2t}$, closer to the market center only type $\overline{\theta}$ is served ($2Reg^*$) and $n^d > n^s$. Under MNP, we may have either $\mathbf{LM}_1^{\underline{\theta}}$, $\mathbf{S}_1^{\bar{\theta}}$, $\mathbf{LM}_2^{\underline{\theta}}$ or $\mathbf{S}_2^{\bar{\theta}}$ contracts. If there exists a free entry *equilibrium* under MNP, n^m is such that $\Pi_i^m(n^m) = 0$. Since $\Pi_i^d(n^m) < \Pi_i^m(n^m) = 0$ (no matter whether $\mathbf{LM}_1^{\underline{\theta}}$, $\mathbf{S}_1^{\bar{\theta}}$, $\mathbf{LM}_2^{\underline{\theta}}$ or $\mathbf{S}_2^{\bar{\theta}}$ contracts hold) and $\Pi_i^d(n)$ is continuous and decreasing with n, we know that $n^d < n^m$, thus there is more excess entry under MNP. Moreover, the quality offered is the socially optimal one under both pricing policies but under DNP all type $\underline{\theta}$ consumers who should be served are served ($d^C = d^*_{\underline{\theta}} = \frac{\underline{\theta}^2}{2t}$) whereas under MNP too few type $\underline{\theta}$ consumers are served. Consequently $W^d(n^d) > W^m(n^m)$. ■

Ideally, we would like to compare analytically the two pricing policies with endogenous market structure for every set of parameters' values. However, some of the profit functions are so complex that we can not find the *equilibrium* number of firms analytically. In the next section we present numerical results that allow us to make a more complete comparison of the two pricing policies.

2.3.2 Numerical Results

Due to the complexity of some profit functions and the need to guarantee that the free entry long-run *equilibrium* number of firms and the profit function used are internally consistent[10], we decided to compute numerically the free entry long-run *equilibrium* number of firms for each pricing policy (n^m and n^d). Using Gauss programs, we computed the free entry long-run number of firms for each of the possible *equilibrium* solution contracts for different sets of parameters t, $\underline{\theta}$, $\overline{\theta}$ and F (for MNP and DNP, separately).

In each of our simulations, we fixed $\underline{\theta}$ and F and computed the long-run *equilibrium* for a large grid of values for k and t. We ran several of these simulations, considering different values for $\underline{\theta}$ and F. Obviously, the free entry *equilibrium* number of firms and welfare levels under each pricing policy depend a lot on the parameters' values. However, the qualitative results on the comparison of the two pricing policies are the same in all the simulations we have performed.

[10] Notice that the profit function varies from solution contract to solution contract. The relevant solution contract for a given set of exogenous parameters also depends on n. This implies that, for each set of exogenous parameters' values, we need to identify *simultaneously* which is the relevant solution contract (and consequently the profit function to use) and the number of firms such that this profit is zero.

Free Entry number of firms

Figure 2.5 exemplifies the results of our simulations, when $\underline{\theta} = 2$ and $F = 0.2$. There are several interesting features which are common to all simulations we performed. The first one is that the socially optimal number of firms and the *equilibrium* number of firms under DNP are both continuous with t. On the other hand, the *equilibrium* number of firms under MNP may present one (for low k) or two (for intermediate k) discontinuities[11] and it is decreasing with t for values of t close to the NLM limit (where either $\mathbf{LM}_2^{\underline{\theta}}$ or $\mathbf{S}_2^{\bar{\theta}}$ contracts hold). Curiously, the fact that n^m is decreasing with t for high t values was already present in the linear price case (see Salop (1979)). Our results also emphasize the fact that there is excess of entry under both pricing policies once n^s is always lower than n^m and n^d.

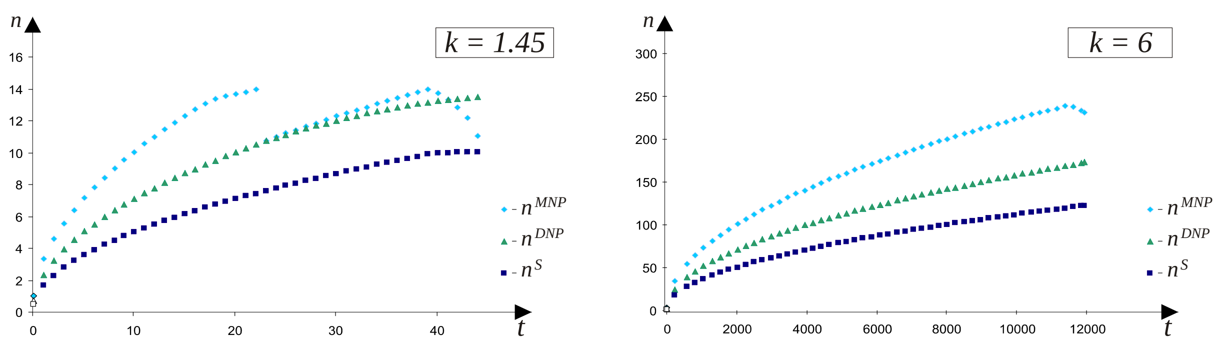

Figure 2.5: Free Entry Long-Run Number of Firms

Simulations for other values of F and $\underline{\theta}$ reveal the same type of pattern for n^s, n^d and n^m. In addition, these simulations show that the free entry number of firms is decreasing with F and non-decreasing with $\underline{\theta}$, under both pricing policies.

The first plot in Figure 2.5 shows that for low k (similar types) and high t (close to the NLM condition limit), n^d is higher than n^m, confirming that the free entry number of firms may be higher under DNP than under MNP.[12] For this set of parameters' values, DNP solution contracts have firms setting the typical asymmetric information monopolist contract at locations near the firms and this fact outweighs the competitive pressure of delivered pricing policies. Moreover, for this set of parameters' values, MNP solution contracts have firms selling

[11] The case of intermediate k is not shown in Figures 2.5 and 2.6.

[12] This plot also shows that we may have $n^d = n^m$, a result which we proved analytically.

only to type $\overline{\theta}$ consumers. As a consequence, for a given n, profits are higher under DNP, which leads to a larger number of firms in the long-run. Notice that, as F increases, the free entry *equilibrium* number of firms decreases, which implies that, for a given k, t does not have to be so high for this phenomenon to hold.

One should notice that $n^d > n^m$ holds only for a relatively small set of parameters' values. For higher k and/or smaller t, the long-run *equilibrium* number of firms is larger under MNP. If k is higher (or t is lower), the firm is not able to set monopoly contracts even for locations near the firm (there is no region M). Therefore, DNP has more competitive contracts and firms achieve lower profits, which leads to lower entry.

Welfare comparisons

Whether discriminatory nonlinear pricing should be prohibited depends on which of the two pricing policies is better in welfare terms. Therefore, comparing long-run welfare under the two pricing policies is extremely important. Propositions 2.4 and 2.5 tell us that long-run welfare under DNP is higher than under MNP for small F, and for intermediate F and k above a certain level. However, it would be interesting to learn something about the long-run welfare comparisons for other parameters' values.

Considering the results in the previous subsection and the results in Subsection 2.2.3 there are some sets of parameters' values for which the long-run welfare comparison is, *a priori*, ambiguous since welfare comparisons for a given market structure and the excess entry under the two pricing policies have welfare effects which run on opposite directions. For example, when k is low and t is high, n^m is closer to n^s but under MNP type $\underline{\theta}$ consumers are not served. Thus, it is not obvious whether welfare is higher under DNP or MNP. On the other hand, when both k and t are high, n^d is closer to n^s but under DNP type $\underline{\theta}$ consumers are offered a suboptimal quality and less consumers are served than under MNP. Once more one does not know whether long-run welfare is higher under DNP or MNP.

Unfortunately, for the set of parameters' values where there is uncertainty about long-run welfare comparisons, it is impossible to compute long-run welfare levels analytically, so we decided to do it numerically (even though we are aware of the limitations of this analysis). In order to compare long-run welfare, we first derived welfare function for each pricing policy (see Appendix 2.B). After computing n^m and n^d, for given values of t, $\underline{\theta}$, $\overline{\theta}$ and F, we then compared

$W^{MNP}(n^m)$ with $W^{DNP}(n^d)$.

Figure 2.6 illustrates the qualitative results when $\underline{\theta} = 2$ and $F = 0.2$. While the long-run welfare is always continuous with t under DNP, the long-run welfare presents one (for low k) or two (for intermediate k) discontinuities under MNP (for the other values of t, W^{MNP} is also continuous). Moreover, for values of t close to the NLM limit, long-run welfare is increasing with t under MNP, a feature which was already present in mill linear pricing.

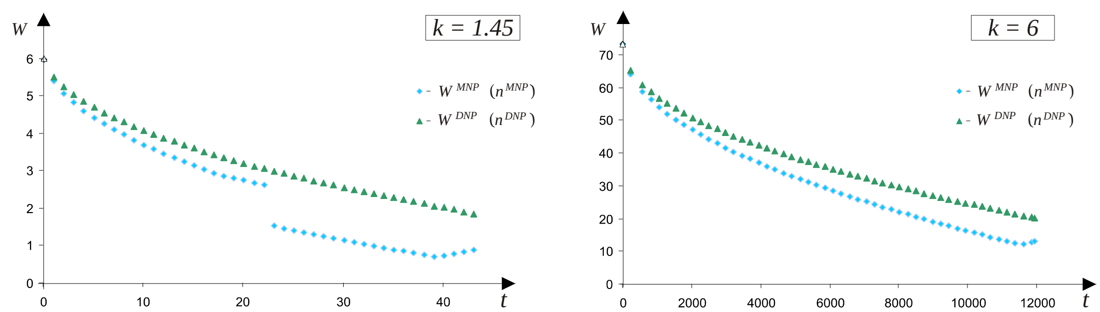

Figure 2.6: Free Entry Long-Run Welfare

In our simulations, we always reached higher free entry long-run social welfare with DNP than under MNP. Although this does not prove that long-run welfare is always higher under DNP, it seems to suggest that this is in fact the case.

2.4 Conclusion

The issue of whether price discrimination should be prohibited has been previously addressed in a setup where consumers differ according to one observable characteristic (location). Under this setup mill linear pricing and delivered linear pricing have been compared and it has been shown that when there is spatial contestability (firms can costlessly relocate), in the long-run delivered linear pricing yields higher welfare and consequently discriminatory prices should be allowed.

This chapter considers a model where consumers simultaneously differ according to one unobservable (quality preferences) and one observable characteristic (location – brand preferences). In these circumstances, nonlinear prices arise in *equilibrium*. The main question addressed in this work is whether firms should be allowed to discriminate according to the observable characteristic. In other words, should firms be allowed to practice different nonlinear contracts at each location or should they be forced to set a unique nonlinear contract?

We show that, when firms can costlessly relocate and set nonlinear contracts to screen consumers according to their quality preferences, delivered nonlinear pricing may bring smaller, equal or higher free entry *equilibrium* number of firms than mill nonlinear pricing, depending on the degree of horizontal and vertical differentiation. The fact that the number of product varieties (free entry number of firms) may be higher under delivered nonlinear pricing is not present in Norman and Thisse (1996) and occurs when consumer types are similar and horizontal differentiation degree is close to the level where local monopolies for both types start to appear.

Our results corroborate Norman and Thisse (1996), under spatial contestability, in terms of too much product variety (excess entry) with both mill nonlinear pricing and delivered nonlinear pricing, since the free entry socially optimal number of firms is always smaller than the free entry *equilibrium* number of firms. This also happens in terms of welfare, since delivered nonlinear pricing yields higher long-run welfare than mill nonlinear pricing when *(i)* fixed costs are low and when *(ii)* fixed costs are intermediate and consumer types are sufficiently different. In addition, our numerical results suggest that long-run welfare is higher under delivered nonlinear pricing than under mill nonlinear pricing for other parameters' values, supporting the view that discriminatory nonlinear prices should not be prohibited.

Two extensions seem worth considering. It would be interesting to study settings where there is a different proportion of types at each location or where brand preferences interact with quality preferences.

2.A Profit Functions

2.A.1 Mill nonlinear pricing – Firms' profit functions

$\mathbf{C}_1^{\bar{\theta}\underline{\theta}}$	$\frac{2t}{n^2} - F$
$\mathbf{C}_2^{\bar{\theta}\underline{\theta}}$	$\frac{\underline{\theta}^2}{2n} + \frac{t}{2n^2} - F$

$\mathbf{S}_1^{\bar{\theta}}$	$\frac{t}{n^2} - F$
$\mathbf{S}_2^{\bar{\theta}}$	$\frac{\bar{\theta}^2}{2n} - \frac{t}{2n^2} - F$
$\mathbf{LM}_1^{\underline{\theta}}$	$\frac{\underline{\theta}^4}{8t} + \frac{t}{n^2} - F$
$\mathbf{LM}_2^{\underline{\theta}}$	$\frac{\bar{\theta}^2}{2n} - \frac{t}{2n^2} + \frac{\underline{\theta}^4}{8t} - F$

2.A.2 Delivered nonlinear pricing – Firms' profit functions

$1Reg$	$\frac{t}{n^2} - F$
$2Reg$	$\frac{t}{n^2} - \frac{(2t-n\underline{\theta}^2)^2}{4tn^2} - F$
$3Reg$	$\frac{t}{n^2} - \frac{(2t-n\underline{\theta}^2)^2}{4tn^2} - \frac{\left[2t+n\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2\right)\right]^3}{24tn^3(\bar{\theta}-\underline{\theta})^2} - F$
$4Reg$	$\frac{t}{n^2} - \frac{(2t-n\underline{\theta}^2)^2}{4tn^2} + \frac{\left[2t+n\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right]^3-\left[2t+n\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2\right)\right]^3}{24tn^3(\bar{\theta}-\underline{\theta})^2} - F$
$2Reg^*$	$\frac{t}{2n^2} + \frac{\underline{\theta}^4}{4t} - F$
$3Reg^*$	$\frac{t}{2n^2} + \frac{\underline{\theta}^4}{4t} - \frac{\left[2t+n\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2\right)\right]^3}{24tn^3(\bar{\theta}-\underline{\theta})^2} - F$
$4Reg^*$	$\frac{t}{2n^2} + \frac{\underline{\theta}^4}{4t} + \frac{\left[2t+n\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right]^3-\left[2t+n\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2\right)\right]^3}{24tn^3(\bar{\theta}-\underline{\theta})^2} - F$
$2Reg^{**}$	$\frac{\left(2t+n\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right)\left[3*(n\bar{\theta}^2-2t)(2t-(\bar{\theta}-2\underline{\theta})^2n)+2*\left(2t+n\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right)^2\right]}{24t(\bar{\theta}-\underline{\theta})^2n^3} + {} + \frac{\left[-16n\left(\bar{\theta}-\underline{\theta}\right)^2\left(2n\bar{\theta}\underline{\theta}-n\bar{\theta}^2-2n\underline{\theta}^2+t\right)\right]\sqrt{4(\bar{\theta}-\underline{\theta})^2n(n(-4\bar{\theta}\underline{\theta}+2\bar{\theta}^2+4\underline{\theta}^2)-2t)}}{24t(\bar{\theta}-\underline{\theta})^2n^3} + \frac{12n(\bar{\theta}-\underline{\theta})^2t^2-24t(\bar{\theta}-\underline{\theta})^2n^3F}{24t(\bar{\theta}-\underline{\theta})^2n^3}$
$3Reg^{**}$	$\frac{\left(2t+n\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right)\left[3*(n\bar{\theta}^2-2t)(2t-(\bar{\theta}-2\underline{\theta})^2n)+3*\left(2t+n\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right)^2\right]}{24t(\bar{\theta}-\underline{\theta})^2n^3} + {} + \frac{\left[-16n\left(\bar{\theta}-\underline{\theta}\right)^2\left(2n\bar{\theta}\underline{\theta}-n\bar{\theta}^2-2n\underline{\theta}^2+t\right)\right]\sqrt{4(\bar{\theta}-\underline{\theta})^2n(n(-4\bar{\theta}\underline{\theta}+2\bar{\theta}^2+4\underline{\theta}^2)-2t)}}{24t(\bar{\theta}-\underline{\theta})^2n^3} + \frac{12n(\bar{\theta}-\underline{\theta})^2t^2-24t(\bar{\theta}-\underline{\theta})^2n^3F}{24t(\bar{\theta}-\underline{\theta})^2n^3}$

2.B Welfare computations

With n firms, aggregate total surplus under mill nonlinear pricing, $W^{MNP}(n)$, is given by:

Contract	$W^{MNP}(n)$
$\mathbf{C}_1^{\bar{\theta}\underline{\theta}} = \mathbf{C}_2^{\bar{\theta}\underline{\theta}}$	$\frac{\bar{\theta}^2}{2} + \frac{\underline{\theta}^2}{2} - \frac{t}{2n} - nF$
$\mathbf{S}_1^{\bar{\theta}} = \mathbf{S}_2^{\bar{\theta}}$	$\frac{\bar{\theta}^2}{2} - \frac{t}{4n} - nF$
$\mathbf{LM}_1^{\underline{\theta}} = \mathbf{LM}_2^{\underline{\theta}}$	$\frac{\bar{\theta}^2}{2} + \frac{3n\underline{\theta}^4}{16t} - \frac{t}{4n} - nF$

On the other hand, welfare under delivered nonlinear pricing, $W^{DNP}(n)$, is given by:

Scenario	$W^{DNP}(n)$
$1Reg = 2Reg$	$\frac{\bar{\theta}^2}{2} + \frac{\underline{\theta}^2}{2} - \frac{t}{2n} - nF$
$3Reg$	$\frac{\bar{\theta}^2}{2} + \frac{\underline{\theta}^2}{2} - \frac{t}{2n} - \frac{\left[2t+n\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2\right)\right]^3}{24tn^2(\bar{\theta}-\underline{\theta})^2} - nF$
$4Reg$	$\frac{\bar{\theta}^2}{2} + \frac{\underline{\theta}^2}{2} - \frac{t}{2n} - \frac{\left[2t+n\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2\right)\right]^3}{24tn^2(\bar{\theta}-\underline{\theta})^2} + \frac{\left[2t+n\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right]^2\left[2t+n\left(-6\bar{\theta}\underline{\theta}+3\bar{\theta}^2+2\underline{\theta}^2\right)\right]}{24tn^2(\bar{\theta}-\underline{\theta})^2} - nF$
$2Reg^*$	$\frac{n\underline{\theta}^4}{4t} + \frac{\bar{\theta}^2}{2} - \frac{t}{4n} - nF$
$3Reg^*$	$\frac{n\underline{\theta}^4}{4t} + \frac{\bar{\theta}^2}{2} - \frac{t}{4n} - \frac{\left[2t+n\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2\right)\right]^3}{24tn^2(\bar{\theta}-\underline{\theta})^2} - nF$
$4Reg^*$	$\frac{n\underline{\theta}^4}{4t} + \frac{\bar{\theta}^2}{2} - \frac{t}{4n} - \frac{\left[2t+n\left(2\bar{\theta}\underline{\theta}-\bar{\theta}^2-2\underline{\theta}^2\right)\right]^3}{24tn^2(\bar{\theta}-\underline{\theta})^2} + \frac{\left[2t+n\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right]^2\left[2t+n\left(-6\bar{\theta}\underline{\theta}+3\bar{\theta}^2+2\underline{\theta}^2\right)\right]}{24tn^2(\bar{\theta}-\underline{\theta})^2} - n\ F$
$2Reg^{**}$	$\frac{\bar{\theta}^2}{2} - \frac{t}{4n} - nF + \frac{1}{4(\bar{\theta}-\underline{\theta})^2n}\left[\left(2t+n(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2)\right)\frac{3*(n\bar{\theta}^2-2t)(2t-(\bar{\theta}-2\underline{\theta})^2n)+2*\left(2t+n\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right)^2}{6tn} + \frac{\left[-16n\left(\bar{\theta}-\underline{\theta}\right)^2\left(2n\bar{\theta}\underline{\theta}-n\bar{\theta}^2-2n\underline{\theta}^2+t\right)\right]\sqrt{4(\bar{\theta}-\underline{\theta})^2n(n(-4\bar{\theta}\underline{\theta}+2\bar{\theta}^2+4\underline{\theta}^2)-2t)}}{6tn}\right]$
$3Reg^{**}$	$\frac{\bar{\theta}^2}{2} - \frac{t}{4n} + \frac{\left[2t+n\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right]^2\left[2t+n\left(-6\bar{\theta}\underline{\theta}+3\bar{\theta}^2+2\underline{\theta}^2\right)\right]}{24tn^2(\bar{\theta}-\underline{\theta})^2} - nF + \frac{1}{4(\bar{\theta}-\underline{\theta})^2n}\left[\left(2t+n(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2)\right)\frac{3*(n\bar{\theta}^2-2t)(2t-(\bar{\theta}-2\underline{\theta})^2n)+2*\left(2t+n\left(6\bar{\theta}\underline{\theta}-3\bar{\theta}^2-4\underline{\theta}^2\right)\right)^2}{6tn} + \frac{\left[-16n\left(\bar{\theta}-\underline{\theta}\right)^2\left(2n\bar{\theta}\underline{\theta}-n\bar{\theta}^2-2n\underline{\theta}^2+t\right)\right]\sqrt{4(\bar{\theta}-\underline{\theta})^2n(n(-4\bar{\theta}\underline{\theta}+2\bar{\theta}^2+4\underline{\theta}^2)-2t)}}{6tn}\right]$

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Chapter 3

Entry Decision and Pricing Policies

3.1 Introduction

Entry incentives differ according to the pricing policy set in markets. Different pricing policies imply different market *equilibria* and thus different profit levels for the firms in the market (both entrants and incumbent firms). Therefore, the pricing policy is undoubtedly a determinant of competitors entrance in markets.

For very different setups, one can find various studies that illustrate the impact of firms' pricing policies upon the entry decision. Some of them show that discriminatory pricing tends to discourage entry. Armstrong and Vickers (1993) consider an incumbent firm that serves two identical independent geographical markets where entry can only occur in one of these markets. Both firms sell a homogeneous product and the entrant is price-taker.[1] They emphasize that the incumbent sets lower prices in the market where entry is possible when discriminatory pricing is effective. Thus, for intermediate levels of entry costs, there is less entry than under uniform pricing. In this analysis the impact on entry is due to the difference in the degree of competitive threat across markets since markets' demand is identical. Aguirre *et al.* (1998) explore the strategic choice of pricing policies under a spatial market model. When focusing on symmetric information, they show that discriminatory pricing is more aggressive and entry is more difficult than under uniform pricing. Motta (2004) also illustrates that discriminatory pricing always deters entry while uniform pricing may or may not deter entry using a very simple example with

[1] Armstrong and Vickers (1993) first study the case where the entrant firm's scale of entry is exogenous followed by the case where it is endogenous.

Bertrand competition and two identical independent geographical markets. In this example, the author explores the welfare impact of the two pricing policies.

On the other hand, allowing discriminatory pricing can increase profits and thus encourage more entry as shown in Katz (1984), which analyzes long-run monopolistic competition with a mixture of informed and uninformed consumers. In a research note, Cheung and Wang (1999) extend Armstrong and Vickers (1993) analysis considering non-identical demands. They showed that discriminatory pricing may have a positive or negative effect on entry since the impact on entry due to the difference in markets' demand may overwhelm the impact due to the difference in the degree of competitive threat across markets. Their analysis assumes that both markets are served by the monopolist under uniform pricing.

Our aim is to extend the analysis of the impact of pricing policies upon entry to a framework where product heterogeneity is present and thus add a third effect: the degree of product substitutability. This setup is more realistic since firms frequently use product differentiation as one of their strategies and consumers tend to be more diverse thus, *demanding* product differentiation in several markets. We show that the degree of product substitutability affects the impact of pricing policies upon entry decision.

Considering an incumbent firm that serves two distinct and independent geographical markets, an entrant firm may enter one of the markets with a differentiated product. If entry occurs the firms compete in prices. First we look for the price *equilibria* under monopoly and under entry for both discriminatory and uniform pricing. Then we compare the two pricing policies in terms of entry decision and analyze how this comparison is affected by the difference in the degree of competitive threat across markets, the degree of markets' demand difference and the degree of product substitutability.

Notice that our setup has two important differences with respect to Armstrong and Vickers (1993) and Cheung and Wang (1999). On one hand, they consider a dominant firm model with a price-taker entrant where firms sell homogeneous products whereas we use a duopoly price competition model with differentiated products. On the other hand, they just analyze the case where, under entry, the incumbent firm sells in both markets under uniform pricing whereas we also study the case where the incumbent's optimal decision when entry occurs is to abandon one of the markets.

We consider that entry under discriminatory pricing is *more likely* than under uniform pricing when we have levels of entry costs where entry is profitable under discriminatory pricing but unprofitable under uniform pricing, *i.e.*, when entrant's post-entry profit is higher under discriminatory pricing than under uniform pricing. Our results show that entry under discriminatory pricing may be *more, less* or *equally likely* than under uniform pricing. For all degrees of product substitutability, there is always some level of degree of markets' demand difference where under discriminatory pricing entry is *more likely* than under uniform pricing. This happens for a larger interval of degrees of markets' demand difference when there are lower degrees of product substitutability.

Previous studies do not take into consideration the effect of the degree of product substitutability. We show that this effect has an impact on determining which of the two pricing policy is more entry deterrent.

This chapter is organized as follows. In Section 3.2 we set up the model and compute the demand functions for each market and product. Section 3.3 studies the monopoly solution under discriminatory and uniform pricing whilst Section 3.4 analyzes these two pricing policies when a competitor enters one of the markets. Comparison of pricing policies, in terms of the impact upon entry decision, is presented in Section 3.5. Finally, Section 3.6 sets the conclusions.

3.2 The model and demand derivations

Consider an incumbent firm, I, operating in two distinct and independent geographical markets. In one of these markets entry is possible – the *competitive market*, henceforth we denote it by market A – and in the other market entrance is not possible – the *captive market*, henceforth we denote it by market B. The incumbent firm and the entrant firm, E, compete in prices: they choose their prices simultaneously and independently. The products sold by the two firms are differentiated, *i.e.*, products are imperfect substitutes. We assume linear demand in both markets. This type of demand function can be derived from the consumer's utility maximization problem with a quadratic utility function.

To derive demand in the *captive market*, where there is no product variety, we assume that the representative consumer's preferences are given by:

$$U(q_B^I) = q_B^o + a_B q_B^I - \frac{1}{2} b_B \left(q_B^I\right)^2, \tag{3.1}$$

where q_B^I represents the quantity sold in market B by the incumbent, q_B^o represents all other *captive market* products (with a price normalized to unity) and a_B, b_B are positive constants. The consumer's budget constraint is $M_B = p_B^I q_B^I + q_B^o$.

In the *competitive market*, the representative consumer's utility function takes the form of a standard quadratic utility function (similar to the one suggested by Dixit (1979)):

$$U(q_A^I, q^E) = q_A^o + a_A q_A^I + a_A q^E - \frac{1}{2}\left[b_A \left(q_A^I\right)^2 + 2d_A q_A^I q^E + b_A \left(q^E\right)^2\right], \tag{3.2}$$

where q_A^I and q^E represent the quantity sold in market A by the incumbent and by the entrant, respectively. All other *competitive market* products are represented by q_A^o (with a price normalized to unity) and a_A, b_A are positive constants. The degree of product substitutability can be measured by $d_A \in [0, b_A[$ where products become closer substitutes as $d_A \to b_A$ which implies intense price competition. When $d_A \to 0$ products are completely differentiated. We require that $d_A < b_A$ in order to assure that each product price is more sensitive to a change in its product quantity than to that of the other firm's product quantity. Let us define τ as the ratio between d_A and b_A, *i.e.*, $\tau = \frac{d_A}{b_A}$. Thus the degree of product substitutability can also be measured by τ with $\tau \in [0, 1[$. Values of τ close to 1 represent highly substitutable products and τ values close to 0 correspond to highly differentiated products. In the remaining analysis we will use τ to measure the degree of product substitutability. The utility function (3.2) assumes the two products have symmetric effects on consumer's utility. Notice that when entry does not occur, this market has no product variety since $q^E = 0$. In this case, the consumer's utility function in the *competitive market* will be similar to the *captive market*. The consumer's budget constraint is given by $M_A = p_A^I q_A^I + p^E q^E + q_A^o$.

From constrained optimization of the utility functions we obtain the inverse demand function in each market. In market B, we get:

$$p_B^I = a_B - b_B q_B^I.$$

In market A, when no entry occurs, we get:

$$p_A^I = a_A - b_A q_A^I.$$

Finally, in market A when entry occurs, we obtain:

$$\begin{aligned} p_A^I &= a_A - b_A q_A^I - d_A q^E \\ p^E &= a_A - b_A q^E - d_A q_A^I. \end{aligned}$$

In order to derive the Nash *equilibrium* of the price-game, we need to compute the demand functions. Solving the previous expressions with respect to the quantities demanded we get, respectively:

$$q_B^I = \frac{a_B}{b_B} - \frac{1}{b_B} p_B^I$$
$$q_A^I = \frac{a_A}{b_A} - \frac{1}{b_A} p_A^I$$

and finally:

$$\begin{aligned} q_A^I &= \frac{a_A (b_A - d_A)}{b_A^2 - d_A^2} - \frac{b_A}{b_A^2 - d_A^2} p_A^I + \frac{d_A}{b_A^2 - d_A^2} p^E \\ q^E &= \frac{a_A (b_A - d_A)}{b_A^2 - d_A^2} - \frac{b_A}{b_A^2 - d_A^2} p^E + \frac{d_A}{b_A^2 - d_A^2} p_A^I . \end{aligned}$$

To simplify computations, when there is a duopoly in market A we define:

$$\alpha = \frac{a_A (b_A - d_A)}{b_A^2 - d_A^2}, \beta = \frac{b_A}{b_A^2 - d_A^2} \text{ and } \gamma = \frac{d_A}{b_A^2 - d_A^2}.$$

Then the last two demand functions simplify to:

$$\begin{aligned} q_A^I &= \alpha - \beta p_A^I + \gamma p^E \\ q^E &= \alpha - \beta p^E + \gamma p_A^I . \end{aligned}$$

With this demand specification, the quantity demanded of a given product depends negatively on its own price and positively on the price of the other product. As $\tau = \frac{d_A}{b_A} \rightarrow 1$, the price of the other product affects more and more the quantity demanded of a firms' product.

For simplicity we assume that both firms' production costs are nil. The entrant's cost of entering market A is non-negative.[2]

Finally, we assume the following relationships between market B and A's parameters:

$$b_B = b_A \tag{3.3}$$

$$a_B = k a_A, k \in \left]0, +\infty\right[. \tag{3.4}$$

Parameter k measures the degree of markets' demand difference. Under no entry and assuming (3.3) and (3.4), when $k \rightarrow 1$ market A and B are identical. When $k > 1$ demand in

[2] We assume that the entrant's cost of entering in market B is so high that entry in this market would always be unprofitable.

market B is larger than demand in market A and for a given market price, demand in market B is less elastic than demand in market A. Conversely, when $k < 1$ demand in market A is larger than demand in market B and for a given market price, demand in market A is less elastic than demand in market B. Therefore, low k and high k stand for high degrees of markets' demand difference and k around 1 corresponds to low degrees of markets' demand difference.

3.3 Discriminatory versus uniform pricing under monopoly

In this section we compare discriminatory pricing with uniform pricing when the incumbent is a monopolist in both markets. Although we are mainly interested in studying the impact of pricing policies upon entry, the monopoly analysis is interesting for comparative purposes since it allows us to identify one important effect when we compare discriminatory and uniform pricing: differences in markets' demand of the various markets.

3.3.1 Discriminatory pricing

Under discriminatory pricing the incumbent may set different prices in each market. Since firms have null production costs, the incumbent solves two separate profit maximization problems.

In market A, the incumbent solves the following problem:

$$\max_{p^I_{Am}} \quad p^I_{Am}\left(\frac{a_A}{b_A} - \frac{1}{b_A}p^I_{Am}\right)$$

and similarly for market B:

$$\max_{p^I_{Bm}} \quad p^I_{Bm}\left(\frac{a_B}{b_B} - \frac{1}{b_B}p^I_{Bm}\right),$$

where p^I_{im} and q^I_{im} denote the incumbent's price and quantity in market i under discriminatory pricing and monopoly in both markets. Notice that all quantities must be non-negative.

The solutions to these problems are given by (first order conditions are described in Appendix 3.A.1 and 3.A.3):

$$p^I_{Am} = \frac{a_A}{2} \quad \text{and} \quad q^I_{Am} = \frac{a_A}{2b_A}$$

$$p^I_{Bm} = \frac{a_B}{2} = \frac{ka_A}{2} \quad \text{and} \quad q^I_{Bm} = \frac{a_B}{2b_B} = \frac{ka_A}{2b_A},$$

where the last equalities follow from the assumptions on a_B and b_B. Under our assumptions these solutions imply positive quantities. Thus, it is optimal for the incumbent to sell in both markets. Notice that if $k < 1$, the price is higher in market A while if $k > 1$, the price is higher in market B. This result is expected: a discriminatory monopolist charges a higher price in the less elastic market. When $k < 1$, market A is larger and less elastic thus, $p^I_{Am} > p^I_{Bm}$. Conversely, when $k > 1$, market B is larger and less elastic thus, $p^I_{Bm} > p^I_{Am}$.

We can compute the incumbent's profits under monopoly discriminatory pricing:

$$\pi^I_{Am} = \frac{a_A^2}{4b_A} \tag{3.5}$$

$$\pi^I_{Bm} = \frac{a_B^2}{4b_B} = \frac{k^2 a_A^2}{4b_A}. \tag{3.6}$$

Therefore the incumbent's total profit is given by:

$$\pi^I_{(Am+Bm)} = \frac{a_A^2(1+k^2)}{4b_A}.$$

3.3.2 Uniform pricing

In order to find the incumbent's *optimal* solution under uniform pricing we need to evaluate whether the incumbent is better of selling in both markets or selling only in one of the markets.

If the incumbent sells his product in both markets, he will choose a unique price so as to maximize his total profit. Therefore, the incumbent solves the following problem:

$$\max_{\overline{p}^I} \ \overline{p}^I \left(\frac{a_A}{b_A} - \frac{1}{b_A}\overline{p}^I\right) + \overline{p}^I \left(\frac{a_B}{b_B} - \frac{1}{b_B}\overline{p}^I\right)$$

subject to non-negativity constraints for quantities. We denote by $\overline{p}^I$, $\overline{q}^I_{Am}$ and $\overline{q}^I_{Bm}$ the incumbent's price and quantities produced in market A and B, respectively, under uniform pricing and monopoly in both markets.

The solution to this problem (first order conditions are described in Appendix 3.B.1) is:

$$\overline{p}^I = \frac{a_B b_A + a_A b_B}{2(b_A + b_B)} = \frac{a_A(k+1)}{4}$$

$$\overline{q}^I_{Am} = \frac{2a_A - a_B}{2(b_A + b_B)} + \frac{a_A b_B}{2b_A(b_A + b_B)} = \frac{a_A\,(3-k)}{4b_A}$$

$$\overline{q}^I_{Bm} = \frac{2a_B - a_A}{2(b_A + b_B)} + \frac{a_B b_A}{2b_B(b_A + b_B)} = \frac{a_A\,(3k-1)}{4b_A},$$

where the last equalities are obtained using the assumptions on a_B and b_B. Notice that $\overline{q}^I_{Am} > 0$ if $k < 3$ and $\overline{q}^I_{Bm} > 0$ if $k > \frac{1}{3}$.

The incumbent's profit under monopoly uniform pricing when selling in both markets is given by:

$$\bar{\pi}^I_{(Am+Bm)} = \frac{(a_B b_A + a_A b_B)^2}{4 b_A b_B (b_A + b_B)} = \frac{a_A^2 (k+1)^2}{8 b_A}.$$

If the incumbent abandons one of the markets, the *optimal* solution in that market is the one obtained under monopoly discriminatory pricing (Subsection 3.3.1). Therefore, the incumbent's profit function when selling in market A only is given by (3.5) whereas profit obtained when selling in market B only is given by (3.6).

Comparing the incumbent's profit in all three alternatives (selling in both markets, selling in market A only, selling in market B only) one concludes that:

- When $k < \sqrt{2} - 1$, the incumbent prefers to sell in market A only;
- When $k > \sqrt{2} + 1$, the incumbent prefers to sell in market B only;
- For $\sqrt{2} - 1 \leq k \leq \sqrt{2} + 1$, the incumbent prefers to sell in both markets.

Under uniform pricing selling in both markets brings lower profit than only selling in the largest market when there are large differences in markets' demand. For very low k values, demand in market A is considerably larger than demand in market B and the incumbent prefers to sell in market A only. On the other hand when demand in market B is considerably larger than demand in market A, for high levels of k, the incumbent abandons market A given its relatively smaller demand. Finally, the incumbent prefers to sell in both markets when markets are similar, *i.e.*, $\sqrt{2} - 1 \leq k \leq \sqrt{2} + 1$ and for these k values $\bar{q}^I_{Am}$ and $\bar{q}^I_{Bm}$ are non-negative.

3.3.3 Comparison of pricing policies

In this subsection we limit our analysis to the comparison of prices in market A under the two pricing policies since this is the market where entry is possible.

With discriminatory pricing the incumbent practices a higher price in the more inelastic market. Moreover, if the incumbent serves both markets the uniform price will be between the discriminatory prices in market A and B. When $\sqrt{2} - 1 < k < 1$, under uniform pricing the incumbent serves both markets and under discriminatory pricing the incumbent sets a higher price in market A, thus $p^I_{Am} > \bar{p}^I > p^I_{Bm}$. Under monopoly, if demand in market A is larger, discriminatory price in this market will be higher than uniform price as long as both markets are served under uniform pricing. This shows that one needs to take into account differences in markets' demand in order to know whether the incumbent's price in market A is higher under discriminatory or uniform pricing.

As we will see later this effect is also present when the entrant decides to enter market A. The intuition is that under duopoly the incumbent is a monopolist of the residual demand, thus he also needs to compare the elasticity of demand in market B with the elasticity of the residual demand in market A. If the residual demand in market A is less elastic than demand in market B, the incumbent will charge a higher price in this market than in market B under discriminatory pricing. This is precisely the message conveyed on Cheung and Wang (1999).

3.4 Discriminatory versus uniform pricing when E enters market A

Suppose now that the entrant decides to enter market A but the incumbent remains a monopolist in market B. In this section we analyze the Nash *equilibrium* under discriminatory and uniform pricing.

3.4.1 Discriminatory pricing

Since the incumbent may set a different price in each market and he continues to be a monopolist in market B, the optimal solution in this market is the same as in Subsection 3.3.1. However, market A is now a duopoly and we need to find the Nash *equilibrium* of the price-game.

Given the demands, the incumbent solves the following problem in market A:

$$\max_{p^I_{Ad}} \quad p^I_{Ad}\left(\alpha - \beta p^I_{Ad} + \gamma p^E\right)$$

and similarly the entrant solves:

$$\max_{p^E} \quad p^E\left(\alpha - \beta p^E + \gamma p^I_{Ad}\right),$$

where p^E and q^E denote the entrant's price and quantity when the incumbent uses a discriminatory pricing policy; p^I_{Ad} and q^I_{Ad} are the incumbent's price and quantity in market A under discriminatory pricing and duopoly in market A. Notice that all quantities must be non-negative.

The first order conditions (see Appendix 3.A.2) show that, under our assumptions, prices are strategic complements. Solving the system of first order conditions we obtain the following result:

Lemma 3.1 *Under discriminatory pricing, the unique Nash equilibrium in market A is symmetric and it is given by:*

$$p^I_{Ad} = p^E = \frac{\alpha}{2\beta - \gamma} = \frac{a_A (b_A - d_A)}{2b_A - d_A},$$

where the last expression is obtained from the definitions of α, β and γ.

The *equilibrium* quantities are given by:

$$q^I_{Ad} = q^E = \frac{\beta\alpha}{2\beta - \gamma} = \frac{a_A b_A}{(b_A + d_A)(2b_A - d_A)}.$$

These quantities are always positive under our assumptions.

Post-entry profits in market A under discriminatory pricing are given by:

$$\pi^I_{Ad} = \pi^E = \frac{\beta\alpha^2}{(2\beta - \gamma)^2} = \frac{b_A (b_A - d_A) a_A^2}{(b_A + d_A)(2b_A - d_A)^2}. \tag{3.7}$$

One can show that $p^I_{Ad} < p^I_{Am}$ and $\pi^I_{Ad} < \pi^I_{Am}$ as long as $\tau > 0$. Conversely $p^I_{Ad} = p^I_{Am}$ and $\pi^I_{Ad} = \pi^I_{Am}$ when $\tau = 0$. In other words, the incumbent's price and profit in market A is lower when the entrant decides to enter this market than when the incumbent is a monopolist unless the two products are completely differentiated. This result is quite obvious: as long as there exists some product substitutability, the incumbent looses with the competition of the entrant. Also notice that the difference between p^I_{Ad} and p^I_{Am} is bigger when products become highly substitutable, *i.e.*, when τ rises.

And the incumbent's total profit is given by:

$$\pi^I_{(Ad+Bm)} = \frac{b_A (b_A - d_A) a_A^2}{(b_A + d_A)(2b_A - d_A)^2} + \frac{k^2 a_A^2}{4b_A}.$$

3.4.2 Uniform pricing

In order to find the *equilibrium* solution under uniform pricing, we have to study the incumbent's alternatives of selling in both markets and of only selling in one of the markets when the entrant decides to enter market A.

If the incumbent sells in both markets, he will choose a unique price so as to maximize his profit function under uniform pricing considering that now there is a duopoly in market A. Given the demands, the incumbent solves the following problem:

$$\max_{\overline{p}_d^I} \ \overline{p}_d^I \left(\alpha - \beta \overline{p}_d^I + \gamma \overline{p}^E\right) + \overline{p}_d^I \left(\frac{a_B}{b_B} - \frac{1}{b_B} \overline{p}_d^I\right)$$

and similarly the entrant solves:

$$\max_{\overline{p}^E} \ \overline{p}^E \left(\alpha - \beta \overline{p}^E + \gamma \overline{p}_d^I\right)$$

where $\overline{p}^E$ and $\overline{q}^E$ denote the the entrant's price and quantity when the incumbent practices uniform pricing and sells in both markets and $\overline{p}_d^I$, $\overline{q}_{Ad}^I$ and $\overline{q}_{Bm}^{I'}$ are the incumbent's price and quantities in market A and B, respectively, under uniform pricing and duopoly in market A. Notice that all quantities must be non-negative (the first order conditions are described in Appendix 3.B.2).

If the incumbent abandons market B, he solves the following problem:

$$\max_{\overline{p}_d^I} \ \overline{p}_d^I \left(\alpha - \beta \overline{p}_d^I + \gamma \overline{p}^E\right)$$

while the entrant's problem remains the same. Notice that this problem is precisely the same as under discriminatory pricing (see Subsection 3.4.1). Finally, if the incumbent abandons market A, the entrant becomes a monopolist in this market whereas the incumbent remains a monopolist in market B.

The Nash *equilibrium* under uniform pricing depends on whether the incumbent is better of selling in both markets or just in one of the markets. Let k_A and k_B be defined as follows:

$$\begin{aligned} k_A &= \frac{\left(8 - 5\tau^2\right)\sqrt{(2-\tau^2)} - \left(2-\tau^2\right)\left(4-\tau^2\right)}{2\left(2-\tau^2\right)(1+\tau)(2-\tau)} \\ k_B &= \frac{(2+\tau)\left[8\left(1-\tau^2\right)\left(2-\tau^2\right) + 2\sqrt{(1-\tau^2)(2-\tau^2)}\left(8-5\tau^2\right)\right]}{(1+\tau)\left(9\tau^4 - 32\tau^2 + 32\right)}. \end{aligned}$$

The next lemma characterizes the unique Nash *equilibrium* under uniform pricing.

Lemma 3.2 *Under uniform pricing, the post-entry equilibrium is as follows:*

(i) *When $k < k_A$ the incumbent is better of selling in market A only and the Nash equilibrium prices are precisely the same as under discriminatory pricing:*

$$\overline{p}_d^I = \overline{p}^E = \frac{a_A (b_A - d_A)}{2b_A - d_A}.$$

(ii) *When* $k_A \leq k \leq k_B$ *the incumbent is better of selling in both markets and the Nash equilibrium prices are given by:*

$$\overline{p}_d^I = \frac{a_A (b_A - d_A) [2k (b_A + d_A) + (2b_A + d_A)]}{8b_A^2 - 5d_A^2}$$
$$\overline{p}^E = \frac{a_A (b_A - d_A) \left[b_A d_A (k+1) + 2 \left(2b_A^2 - d_A^2\right) + kd_A^2\right]}{b_A \left[8b_A^2 - 5d_A^2\right]}.$$

(iii) *When* $k > k_B$ *the incumbent is better of abandoning market A. Thus, the entrant becomes a monopolist and his optimal price is:*

$$\overline{p}^E = \frac{a_A}{2}.$$

Proof: The result is a direct consequence of solving the system of best response functions of the two firms, taking into account that the incumbent's best response function depends on k and τ. ■

If the incumbent abandons market B, the Nash *equilibrium* in market A will be the same as under discriminatory pricing and duopoly in market A. Therefore, the incumbent and entrant's profits are both given by (3.7). If the incumbent abandons market A, the entrant will be a monopolist in this market whereas the incumbent will be a monopolist in market B. Thus, the incumbent's total profit is given by (3.6) and the entrant's post-entry profit is given by:

$$\overline{\pi}^E = \frac{a_A^2}{4b_A}.$$

When the incumbent serves both markets the firms' *equilibrium* profits are:

$$\overline{\pi}^I_{(Ad+Bm)} = \frac{a_A^2 (b_A - d_A) \left(2b_A^2 - d_A^2\right) [2k (b_A + d_A) + (2b_A + d_A)]^2}{b_A (b_A + d_A) \left[8b_A^2 - 5d_A^2\right]^2}$$
$$\overline{\pi}^E = \frac{a_A^2 (b_A - d_A) \left[b_A d_A (k+1) + 2 \left(2b_A^2 - d_A^2\right) + kd_A^2\right]^2}{b_A (b_A + d_A) \left[8b_A^2 - 5d_A^2\right]^2}.$$

Figure 3.1 illustrates these results. For the set of parameters in dark grey, the incumbent's demand in market A is considerably larger than demand in market B. Thus, the incumbent prefers to sell in market A only even when there is a competitor in this market. On the other hand when demand in market B is considerably larger than the incumbent's demand in market A, the set of parameters in white, the incumbent prefers to sell in market B only. In this case, the entrant becomes a monopolist in market A.

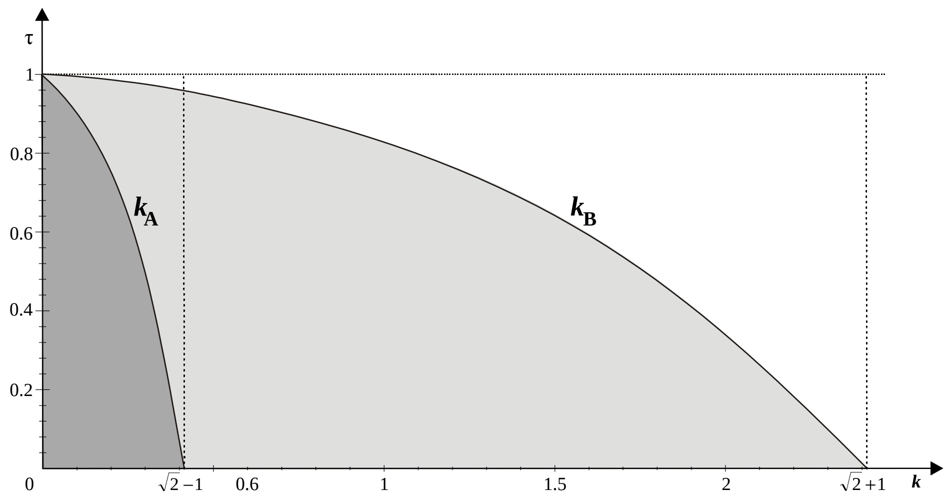

Figure 3.1: Uniform pricing *equilibrium* solutions under Duopoly at market A

The incumbent prefers to sell in both markets when there are smaller degrees of markets' demand difference, *i.e.*, $k_A \leq k \leq k_B$ (the set of parameters in light grey) and, for these values of k, the incumbent's quantities, $\bar{q}^{I}_{Ad}$ and $\bar{q}^{I'}_{Bm}$, are non-negative.

When the entrant decides to enter market A, the incumbent's decision of serving both or just one of the markets depends on the relative size of demand in market B and the incumbent's demand in market A under duopoly. This comparison depends on the degree of markets' demand difference but it also depends on the degree of product substitutability. The higher τ is, the larger is the reduction in the incumbent's residual demand in market A when entry occurs. Thus, as τ increases, demand in market A has to be much bigger in order for the incumbent to prefer to sell only in market A under duopoly, *i.e.*, the level of k below which the incumbent prefers to sell in market A only, k_A, is decreasing with τ. For the same reason, k_B is also decreasing with τ. As τ increases, market A becomes more competitive. Thus, the incumbent prefers to abandon this market for smaller values of k.

Notice that as products become virtually homogeneous, duopoly profits in market A are so low that selling in market A only or even selling in both markets is more unprofitable than selling in market B only. When products are highly substitutable ($\tau \to 1$), competition in market A is so fierce that even when market A is much larger (very low values of k) the incumbent prefers to sell in market B only. On the other hand when products are very differentiated ($\tau \to 0$), competition in market A is soft and consequently market B has to be much larger than market

A for the incumbent to start selling in market B only.

As expected, when the incumbent sells in market A under duopoly, $k < k_B$, the incumbent's price in market A, with competition, is lower than under monopoly except when there is complete differentiation, in which case the price is the same. The incumbent's profit under entry is also lower than under monopoly unless the two products are completely differentiated ($\tau = 0$).

3.4.3 Comparison of pricing policies

When the incumbent serves both markets under uniform pricing, one can show that the price decrease in market A due to competition is higher under discriminatory pricing than under uniform pricing. This happens because uniform pricing makes the incumbent softer. When the incumbent considers a marginal price decrease he has to take into account the reduction in profits in market A and market B. So he has less incentives to decrease the price. The competitive threat is higher under discriminatory pricing. This is the effect described by Armstrong and Vickers (1993).

However the previous effect does not mean that the incumbent's *equilibrium* price in market A is always lower under discriminatory than under uniform pricing. If demand in market A is much larger and less elastic than demand in market B, the discriminatory price may be higher than the uniform price even when there is a competitor in market A. This is the effect described by Cheung and Wang (1999) since the effect of the difference in markets' demand when k is low and lower than one (*i.e.*, when demand in market A is relatively larger than demand in market B) may overwhelm the effect of difference in competitive threat across markets. Thus, the price decrease in market A due to competition is higher under discriminatory pricing than under uniform pricing but the incumbent's *equilibrium* price in market A is higher under discriminatory than under uniform pricing.

As described in the next section, the degree of product substitutability affects the level of k such that the effect of the difference in markets' demand overwhelms the effect of the difference in competitive threat across markets. Thus, we add a third effect to Armstrong and Vickers (1993) and Cheung and Wang (1999) analysis.

3.5 Entry decision

The entrant's decision to enter market A depends on whether his post-entry profit is smaller or higher than entry costs. When the entrant's post-entry profit is higher under discriminatory pricing than under uniform pricing, there is an interval of entry costs values where entry is profitable under discriminatory pricing but unprofitable under uniform pricing. Therefore, for that interval we consider that entry under discriminatory pricing is *more likely* than under uniform pricing. From Subsection 3.4.1 we conclude that under discriminatory pricing the incumbent always sells in market A and the entrant's profit function is given by (3.7). However, under uniform pricing, the entrant's profit function depends on whether the incumbent sells in market A or not. For high k values, the incumbent prefers to abandon market A and consequently the entrant becomes a monopolist in market A. Therefore under uniform pricing, the entrant's post-entry profits are:

$$\bar{\pi}^E = \frac{a_A^2(b_A-d_A)\left[b_A d_A(k+1)+2\left(2b_A^2-d_A^2\right)+kd_A^2\right]^2}{b_A(b_A+d_A)\left[8b_A^2-5d_A^2\right]^2} \qquad \text{for } k \leq k_B$$

$$\bar{\pi}^E = \frac{a_A^2}{4b_A} \qquad \text{for } k > k_B.$$

The following proposition presents the main result of this chapter since it shows, for different degrees of markets' demand difference and product substitutability, which of the two pricing policies implies higher entrant's post-entry profit.

Proposition 3.1 *The entrant's profit under discriminatory pricing is:*

(i) *Equal to the entrant's profit under uniform pricing when $k \leq k_A$ and also when $k_A < k < k_B$ and there is complete differentiation.*

(ii) *Higher than the entrant's profit under uniform pricing when $k_A < k < k^E$ where $k^E = \frac{2(1-\tau)}{2-\tau}$.*

(iii) *Lower than the entrant's profit under uniform pricing when $k \geq k^E$ where for $k \geq k_B$ the entrant is the only firm in market A.*

Figure 3.2 illustrates this proposition where the area marked with circles stands for (i), the area marked with squares stands for (ii) and the area in white stands for (iii).

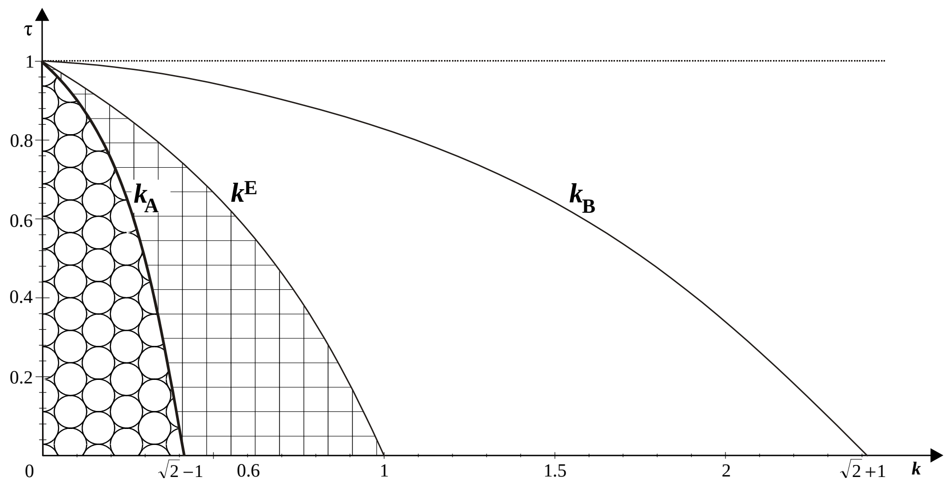

Figure 3.2: Entrant's profits comparison

This proposition shows that entry under discriminatory pricing may be *more, less* or *equally likely* than under uniform pricing. Entry under discriminatory pricing is *more likely* than under uniform pricing for $k_A < k < k^E$, *i.e.*, for the set of parameters corresponding to the area with squares. Thus our results corroborate Cheung and Wang (1999) since the impact on entry due to the difference in markets' demand overwhelms the impact due to the difference in the degree of competitive threat across markets. Notice that when products are highly substitutable there is only a small interval of k values where entry under discriminatory pricing is *more likely* than under uniform pricing. Entry under discriminatory pricing is *more likely* than under uniform pricing for large intervals of k when there are lower degrees of product substitutability. Thus, the degree of product substitutability affects the interval of the degrees of markets' demand difference where entry under discriminatory pricing is *more likely* than under uniform pricing. For a given k, if the degree of product substitutability rises, the entrant's profits under both pricing policies decrease except when the entrant is monopolist. For intermediate and high degrees of product substitutability, this decrease is bigger under discriminatory pricing and thus, entry under uniform pricing starts to be *more likely* than under discriminatory pricing.

Entry under uniform pricing is *more likely* than under discriminatory pricing for $k \geq k^E$, *i.e.*, for the set of parameters corresponding to the area in white. Notice that for $k \geq k_B$ the entrant has the highest profit possible since the entrant is a monopolist in market A.

Finally, when market A is much larger than market B, *i.e.*, $k \leq k_A$ the entrant has the same profit under both pricing policies since under uniform pricing the incumbent prefers to sell in market A only, which leads to the same Nash *equilibrium* in market A than under discriminatory pricing. Entry is also *equally likely* under both pricing policies when $k_A < k < k_B$ and there is complete differentiation.

3.6 Conclusion

Pricing policies imply different entry incentives. There are some studies which support the view that discriminatory pricing tends to discourage entry and conversely other studies show that discriminatory pricing can increase profits and thus encourage entry. We extended the analysis of the impact of pricing policies upon entry considering a framework where there exists price competition and differentiated products. In addition to the analysis of the impact of the difference in the degree of competitive threat across markets and of the degree of markets' demand difference upon entry decision, we analyze the effect of product differentiation measured by the degree of product substitutability.

We show that the impact of pricing policies upon entry is ambiguous. Entry under discriminatory pricing may be *more, less* or *equally likely* than under uniform pricing. For all degrees of product substitutability, there is always some level of degree of markets' demand difference where under discriminatory pricing entry is *more likely* than under uniform pricing. The degree of product substitutability has impact on the set of degrees of markets' demand difference where entry under discriminatory pricing is *more likely* than under uniform pricing.

Focusing solely on the effect of the degree of markets' demand difference as the determinant of when discriminatory pricing entry is *more likely* than under uniform pricing ignores an important effect – the effect of the degree of product substitutability. Therefore, we reinforce Cheung and Wang (1999) idea that there should always be a complete evaluation of the market conditions adding to their analysis, the impact of product differentiation. In addition, our analysis extends previous results since we study the cases where, under entry, the incumbent firm sells in both markets under uniform pricing as well as the case where the incumbent abandons one of the markets.

Interesting extensions could consider settings where there are complementary products and where firms have positive symmetric or asymmetric production costs.

3.A Discriminatory Pricing

We present the computations of the first order conditions for three different optimization problems under discriminatory pricing: Monopoly in market A, Duopoly in market A and Monopoly in market B.

3.A.1 Monopoly in market A

$$\frac{\partial \pi^I_{Am}}{\partial p^I_{Am}} = \frac{a_A}{b_A} - \frac{2}{b_A} p^I_{Am} = 0.$$

3.A.2 Duopoly in market A

$$\frac{\partial \pi^I_{Ad}}{\partial p^I_{Ad}} = \alpha - 2\beta p^I_{Ad} + \gamma p^E = 0$$

$$\frac{\partial \pi^E}{\partial p^E} = \alpha - 2\beta p^E + \gamma p^I_{Ad} = 0.$$

And thus the best response functions of the two firms are given by:

$$p^I_{Ad} = \frac{\alpha + \gamma p^E}{2\beta}$$

$$p^E = \frac{\alpha + \gamma p^I_{Ad}}{2\beta}.$$

3.A.3 Monopoly in market B

$$\frac{\partial \pi^I_{Bm}}{\partial p^I_{Bm}} = \frac{a_B}{b_B} - \frac{2}{b_B} p^I_{Bm} = 0.$$

3.B Uniform Pricing

We present the computations of the first order conditions for two different optimization problems under uniform pricing: sell both markets when there is a monopoly in market A and sell both markets when there is a duopoly in market A.

3.B.1 Sell both markets and monopoly in market A

$$\frac{\partial \overline{\pi}^I_{(Am+Bm)}}{\partial \overline{p}^I} = \left(\frac{a_A}{b_A} - \frac{2}{b_A}\overline{p}^I\right) + \left(\frac{a_B}{b_B} - \frac{2}{b_B}\overline{p}^I\right) = 0.$$

3.B.2 Sell both markets and duopoly in market A

$$\frac{\partial \overline{\pi}^I_{(Ad+Bm)}}{\partial \overline{p}^I_d} = \left(\alpha - 2\beta\overline{p}^I_d + \gamma\overline{p}^E\right) + \left(\frac{a_B}{b_B} - \frac{2}{b_B}\overline{p}^I_d\right) = 0$$

$$\frac{\partial \overline{\pi}^E}{\partial \overline{p}^E} = \alpha - 2\beta\overline{p}^E + \gamma\overline{p}^I_d = 0.$$

And thus the best response functions of the two firms are given by:

$$\overline{p}^I_d = \frac{\alpha b_B + a_B + \gamma b_B \overline{p}^E}{2\left(b_B\beta + 1\right)}$$

$$\overline{p}^E = \frac{\alpha + \gamma\overline{p}^I_d}{2\beta}.$$

The *equilibrium* prices are given by:

$$\overline{p}^I_d = \frac{2\beta a_B + \alpha b_B\left(2\beta + \gamma\right)}{4\beta + b_B\left(4\beta^2 - \gamma^2\right)} = \frac{a_A\left(b_A - d_A\right)\left[2k\left(b_A + d_A\right) + \left(2b_A + d_A\right)\right]}{8b_A^2 - 5d_A^2}$$

$$\overline{p}^E = \frac{\gamma a_B + 2\alpha + \alpha b_B\left(2\beta + \gamma\right)}{4\beta + b_B\left(4\beta^2 - \gamma^2\right)} = \frac{a_A\left(b_A - d_A\right)\left[b_A d_A\left(k+1\right) + 2\left(2b_A^2 - d_A^2\right) + k d_A^2\right]}{b_A\left[8b_A^2 - 5d_A^2\right]}$$

and *equilibrium* quantities are:

$$\overline{q}^I_{Ad} = \frac{\alpha\left(2\beta + \gamma\right)\left(2 + \beta b_B\right) + a_B\left(\gamma^2 - 2\beta^2\right)}{4\beta + b_B\left(4\beta^2 - \gamma^2\right)} =$$

$$= \frac{a_A\left[\left(2b_A + d_A\right)\left(3b_A^2 - 2\left(d_A\right)^2\right) + k\left(b_A + d_A\right)\left(d_A^2 - 2b_A^2\right)\right]}{b_A\left(b_A + d_A\right)\left[8b_A^2 - 5d_A^2\right]}$$

$$\overline{q}^{I'}_{Bm} = \frac{2\beta a_B + a_B b_B \left(4\beta^2 - \gamma^2\right) - \alpha b_B \left(2\beta + \gamma\right)}{b_B \left(4\beta + b_B \left(4\beta^2 - \gamma^2\right)\right)} =$$

$$= \frac{a_A \left[3k \left(2b_A^2 - d_A^2\right) - \left(b_A - d^A\right)\left(2b_A + d_A\right)\right]}{b_A \left[8b_A^2 - 5d_A^2\right]}$$

$$\overline{q}^E = \beta \frac{2\alpha \left(1 + \beta b_B\right) + \gamma a_B + \alpha\gamma b_B}{4\beta + b_B \left(4\beta^2 - \gamma^2\right)} =$$

$$= \frac{a_A \left[b_A d_A \left(k+1\right) + 2\left(2b_A^2 - d_A^2\right) + k d_A^2\right]}{\left(b_A + d_A\right)\left[8b_A^2 - 5d_A^2\right]}.$$

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Chapter 4

Discriminatory Limit Pricing

4.1 Introduction

Predation involves a dominant firm that sets low prices and thus sacrifices short-run profit in order to drive a rival out of the market or deter entry, thus achieving higher profit in the long-run – see Joskow and Klevoric (1979), Niels (1993), Tirole (1988) and Motta (2004). Thus one can either have models of predation where the incumbent firm forces exit or models where entry is discouraged.

Following McGee (1958) assessment of the rationality of predation, several models have been developed providing a convincing context where predation is a rational strategy. Most of these predation models are based on incomplete information.[1] This is the case of signaling models, reputation models and long purse predation models. Predation signaling models' type of reasoning was first developed by Milgrom and Roberts (1982). In this model, a potential entrant has imperfect knowledge about the incumbent's production cost and the incumbent exploits this uncertainty in order to make the entrant believe that entry is unprofitable (*limit pricing* model). Reputation models explore the incentive to prey if the incumbent sells in different markets (see Kreps and Wilson (1982)) while long purse predation models consider frameworks where there are differences between firms' financial constraints and thus the incumbent can survive longer using its *deep pockets* (see Benoit (1984)).

[1] Other contexts in which predation is feasible and rational are, for example, models with learning curves (Cabral and Riordan (1997)) and models with increasing returns (Ordover and Saloner (1989)).

This chapter focus on a monopolistic *limit pricing* model[2] but uses a multimarket setup in order to analyze *discriminatory limit pricing*. Considering different independent geographical markets, the use of third-degree price discrimination is optimal and studying how entry is affected when the incumbent sets discriminatory prices to deter entry is a relevant and interesting issue.[3]

We develop a two-period framework where a multimarket incumbent faces a single potential entrant in one of the two markets where he operates. The incumbent has private information about his production cost, which may either be low or high. In the first period, the incumbent sets his optimal third-degree discriminatory prices. The entrant observes the prices in both markets and decides whether to enter or not after updating his beliefs on the likelihood of the incumbent's production cost being low. If entry occurs, firms compete in prices. Naturally, the entrant will use the price information of every market where the incumbent operates in order to infer incumbent's production cost. Therefore, the incumbent may use pre-entry prices as *predatory signals* for his production cost. Our aim is to study the impact of an entry deterrence strategy in a third-degree price discrimination signaling model. We look for the determinants of these *discriminatory limit prices* when firms offer differentiated products. Notice that, the use of differentiated products has not been widely explored in these setups. Considering linear demands, our framework enables the analysis of different relationships between monopoly markets' demand parameters.

We analyze the pure strategy perfect bayesian *equilibria* and find multiple *equilibria*. This multiplicity is due to the lack of restrictions upon *off-the-equilibrium* path posterior beliefs. If, under the prior beliefs, the entrant's expected profit is higher than his entry costs then there are no pooling *equilibria* but there are multiple separating perfect bayesian *equilibria*. Using the *equilibrium* refinement that the entrant should put zero probability on any type of incumbent using a dominated strategy, we show that there is a unique perfect bayesian *equilibrium* which survives this criterion: the least cost separating *equilibrium*. When, under the prior beliefs, the entrant's expected profit is lower than his entry costs and the two types of incumbent's production cost are not too different, there are multiple pooling and separating *equilibria*. However, if

[2] Harrington (1987) and Bagwell and Ramey (1991) explore multiple incumbents' setups. For simultaneous olipolistic signaling see also Mailath (1989).

[3] Under a spatial market model, Aguirre *et al.* (1998) explore the strategic choice of pricing policies to deter entry when there is asymmetric information about incumbent firm's transportation costs. They show that the incumbent firm may use spatial price discrimination to deter entry.

the low cost incumbent's monopoly prices can not be supported as one of the pooling *equilibria*, the only perfect bayesian *equilibrium* that survives the domination criterion is also the least cost separating *equilibrium*. On the other hand, when the low cost incumbent's monopoly prices can be supported as one of the pooling *equilibria*, there are multiple perfect bayesian *equilibrium* that survive the *intuitive criterion*: the least cost separating *equilibrium* and a set of pooling *equilibria* in the line-segment from the low cost *equilibrium* prices in the least cost separating *equilibrium* to the low cost incumbent's monopoly prices. Using Grossman and Perry (1986) criterion, we show that only one perfect bayesian *equilibrium* survives: the pooling *equilibrium* where both types of incumbents set the low cost incumbent's monopoly prices. Therefore, there is always a unique perfect bayesian *equilibrium* which survives this *equilibrium* refinement upon *off-the-equilibrium* path posterior beliefs.

Our results show that, in the least cost separating *equilibrium*, the low cost incumbent's efforts to deter entry may or may not lead to a downward distortion in pre-entry discriminatory prices with respect to the incumbent's prices under a *complete information* setting. The low cost incumbent uses both pre-entry prices to signal low production cost. When there is a downward distortion in prices, the decrease from the low cost incumbent's monopoly prices is the same in both markets. We also explore the determinants of the downward distortion in pre-entry prices.

Other authors have explored the use of multiple signals to deter entry.[4] Bagwell and Ramey (1988) extend Milgrom and Roberts (1982) incomplete cost information model by allowing firms to use price and advertising as potential signals and show that, in a sequential *equilibrium*, low prices and high advertising are used. This paper is the closest to this chapter's analysis. Bagwell and Ramey (1990) explore a setup where the incumbent has private information concerning market's demand and study sequential *equilibria* for different alternatives of what the incumbent prefers that the market be believed to have (high or low demand). When the incumbent prefers to be believed to operate in a low demand market, low prices and low advertising are used to signal low demand whereas when the incumbent prefers to be believed to operate in a high demand market, both high prices and advertising are used. In both cases, dissipative advertising is never used. This chapter differs from this research as these authors

[4] Milgrom and Roberts (1986) present a detailed and noteworthy description of multiple potential signals used to signal product quality to consumers. An approach to refinements of the perfect bayesian *equilibrium* set with multiple potential signals is illustrated.

consider that firms operate in only one market and sell homogeneous products. We show that when the incumbent firm sells in more than one market, he uses every pre-entry price to signal his production cost and deter entry. We also show that the degree of product substitutability has impact on the *predatory* signals.

Our setup can be applied to anti-predation regulation theory since it reveals differences in discriminatory pricing behavior when there are entry deterrence objectives from when there is pure third-degree price discrimination (within a complete information setting). This is an important issue since mistaking pure third-degree price discrimination as predation may inhibit the use of price discrimination. Our results show that there is a set of parameters' values where there exists a downward distortion in pre-entry discriminatory prices and thus detection of predation would be possible. We show that the biggest distortion in prices occurs when products are highly substitutable.

Predatory dumping is also closely related to our analysis. If we consider the market where there is no entry as the domestic market and the other market as the external and potentially competitive market, the incumbent may set a lower price in the external market so as to deter entry (traditional definition of dumping). As we use a more general third-degree price discrimination model, this result is only valid for the set of parameters where demand in the domestic market is larger than demand in the external market . We extend the analysis to capture this scenario.

The chapter is organized in four sections. In Section 4.2 we set up the model and our main assumptions. Perfect bayesian *equilibria* are analyzed in Section 4.3 where the separating *equilibria* and the pooling *equilibria* are studied. Section 4.4 extends the initial model. Section 4.5 concludes and suggests other possible extensions.

4.2 The Model

Consider a two-period model where an incumbent firm, I, operates in two distinct and independent geographical markets, A and B. The incumbent can use discriminatory pricing in these markets. In the first period only this firm is operating. The incumbent faces a potential entrant firm, E, in market A only. The incumbent's constant marginal production cost can be high or low, denoted by $c \in \{\overline{c}, \underline{c}\}$, where $\overline{c} > \underline{c}$ and also $\underline{c} = 0$. To simplify notation we will drop the superscript I for the remaining of this chapter. However, one should keep in mind that when a price, quantity, profit function or marginal production cost has no superscript they represent incumbent's variables. The incumbent's production cost is private information. The entrant holds a prior belief, assumed to be common knowledge, concerning the probability, ρ_o, that the incumbent has low production cost, with $\rho_o \in]0,1[$. Knowing his production cost, the incumbent sets first period prices in both markets. After observing the incumbent's pre-entry price in each market, p_A and p_B, the entrant updates his beliefs concerning the probability that the incumbent has low production cost. We denote the posterior belief that the incumbent has low production cost by $\rho(p_A, p_B)$, with $\rho \in [0,1]$. Based upon this posterior belief, the entrant decides whether to enter market A. Since the entry decision is taken after the incumbent has set his first period price in each market, these pre-entry prices can be used to signal information about the type of incumbent's production cost. Second period profits are discounted by δ, with $\delta \in]0,1]$.

If entry occurs in the second period, firms are price-setting duopolists in a differentiated product market. The entrant also has constant marginal production cost, c^E, where $c^E \geq 0$. Let f^E denote the entrant's cost of entering market A, where $f^E \geq 0$.[5] Both c^E and f^E are known by the incumbent. Both firms have null fixed costs.

We assume linear demand in both markets. This type of demand function can be derived from the consumer's utility maximization problem using a quadratic utility function. To derive demand in market i when only the incumbent operates, we assume that the representative consumer's preferences are given by:

$$U(q_i) = q_i^o + a_i q_i - \frac{1}{2} b_i (q_i)^2, \tag{4.1}$$

[5] We assume that firm E's entry cost in market B is so high that entry in this market would always be unprofitable.

where q_i represents the quantity sold in market i by the incumbent, q_i^o represents all other market i products (with a price normalized to unity) and a_i, b_i are positive constants. The consumer's budget constraint is given by $M_i = p_i q_i + q_i^o$.

If there is entry in market A, in the second period firms choose their prices simultaneously and independently. These firms offer differentiated products, *i.e.*, firms' products are imperfect substitutes. In market A, the representative consumer's utility function takes the form of a standard quadratic utility function (similar to Dixit (1979)):

$$U(q_A, q^E) = q_A^o + a_A q_A + a_A q^E - \frac{1}{2}\left[b_A (q_A)^2 + 2d_A q_A q^E + b_A \left(q^E\right)^2\right], \tag{4.2}$$

where q_A and q^E represent the quantity sold in market A by the incumbent and the entrant, respectively. All other market A products are represented by q_A^o (with a price normalized to unity) and a_A, b_A are positive constants. The degree of product substitutability can be measured by $d_A \in [0, b_A[$ where products become completely differentiated as $d_A \to 0$. When $d_A \to b_A$, products become closer substitutes implying intense price competition. We require that $d_A < b_A$ in order to assure that each product price is more sensitive to a change in its product quantity than to that of the other firm's product quantity. Let us define τ as the ratio between d_A and b_A, *i.e.*, $\tau = \frac{d_A}{b_A}$. Therefore the degree of product substitutability can also be measured by τ with $\tau \in [0, 1[$. Values of τ close to 1 correspond to highly substitutable products whereas values of τ close to 0 correspond to highly differentiated products. In the remaining analysis we will use τ to measure the degree of product substitutability. Notice that the utility function (4.2) assumes that the two products have symmetric effects on the consumer's utility. The consumer's budget constraint is given by $M_A = p_A q_A + p^E q^E + q_A^o$.[6]

Since demand specifications are similar to those presented in Chapter 3, here we do not present their detailed deduction. From constrained optimization of the utility functions given by (4.1), we compute the demand function in each market. In market B:

$$q_B = \frac{a_B}{b_B} - \frac{1}{b_B} p_B$$

and in market A when there is a monopoly:

$$q_A = \frac{a_A}{b_A} - \frac{1}{b_A} p_A.$$

[6] Notice that when entry does not occur, this market has no product variety since $q^E = 0$. In this case, these consumer's specifications are identical to when there is a monopoly in market A.

If the incumbent with production cost c sets a price p_i, his monopoly profit in market i is given by:

$$\pi_i(p_i) = (p_i - c)\left(\frac{a_i}{b_i} - \frac{1}{b_i}p_i\right) = -\frac{1}{b_i}p_i^2 + p_i\left(\frac{a_i + c}{b_i}\right) - \frac{a_i}{b_i}c.$$

The unique maximizer of this profit function is the monopoly price in market i. Let $\overline{p}_{im}$ and $\underline{p}_{im}$ be the monopoly prices in market i when the incumbent has high and low production costs, respectively. Let $\bar{\pi}_{im}$ and $\underline{\pi}_{im}$ be the monopoly profits (see computation details in Appendix 4.A.1).

From constrained optimization of the utility function (4.2), demand functions under entry in market A are given by:

$$\begin{aligned} q_A &= \frac{a_A(b_A - d_A)}{b_A^2 - d_A^2} - \frac{b_A}{b_A^2 - d_A^2}p_A + \frac{d_A}{b_A^2 - d_A^2}p^E \\ q^E &= \frac{a_A(b_A - d_A)}{b_A^2 - d_A^2} - \frac{b_A}{b_A^2 - d_A^2}p^E + \frac{d_A}{b_A^2 - d_A^2}p_A. \end{aligned}$$

To simplify computations when there is a duopoly in market A we define:

$$\alpha = \frac{a_A(b_A - d_A)}{b_A^2 - d_A^2}, \beta = \frac{b_A}{b_A^2 - d_A^2} \text{ and } \gamma = \frac{d_A}{b_A^2 - d_A^2}.$$

and thus:

$$\begin{aligned} q_A &= \alpha - \beta p_A + \gamma p^E \\ q^E &= \alpha - \beta p^E + \gamma p_A. \end{aligned}$$

With this demand specification, the quantity demanded of a given product depends negatively on its own price and positively on the price of the other product. As $\tau = \frac{d_A}{b_A} \to 1$, the price of the other product affects more and more the quantity demanded of a firms' product. When we derive the Nash *equilibrium* of the price competition game where both firms have common knowledge of their costs, let $\bar{\pi}_{Ad}, \bar{\pi}^E$ and $\underline{\pi}_{Ad}, \underline{\pi}^E$ be the duopoly *equilibrium* profits of the incumbent and duopoly gross *equilibrium* profits of the entrant in market A when the incumbent has high and low production costs, respectively (see computation details in Appendix 4.A.2).

Finally, similarly to Chapter 3, we assume the following relationships between market B and market A's parameters:

$$a_B = ka_A \tag{4.3}$$

$$b_B = b_A. \tag{4.4}$$

Parameter k measures the degree of markets' demand difference. As described in Section 3.2, assuming (4.3) and (4.4) and under no entry, when $k \to 1$ market A and market B are identical. When $k < 1$, demand in market A is larger than demand in market B and demand in market A is less elastic than demand in market B for a given market price. Conversely, when $k > 1$, demand in market B is larger than demand in market A.

Our analysis focus on $k < 1$ only, thus demand in market A is less elastic than demand in market B for a given market price. For these k values, let $\mathbf{M}$ be the set of parameters' values where all monopoly quantities (market A and B) are non-negative and $\mathbf{D}$ be the set of parameters' values where all duopoly quantities are non-negative (see details in Appendix 4.B). Notice that, within the latter set of parameters' values, $\bar{\pi}_{Ad}, \bar{\pi}^E, \underline{\pi}_{Ad}$ and $\underline{\pi}^E$ are all non-negative. Therefore, we consider $k < 1$ and parameters' values $\left\{\bar{c}, c^E, k\right\} \in \mathbf{M} \cap \mathbf{D}$.

4.3 Perfect bayesian equilibria

In this section we analyze the pure strategy perfect bayesian *equilibria* of our signalling game. A perfect bayesian *equilibrium* (PBE) is fully characterized if we set strategies for each firm as a function of the information available at each decision point as well as beliefs for the entrant about the incumbent's production cost. The beliefs must be consistent with the information structure (using Bayes' rule) and with the hypothesis that the given strategies were being set. Given beliefs, all strategies must be best response strategies.

In the next subsection we analyze separating *equilibria* and subsection 4.3.2 studies the pooling *equilibria*. The *equilibria*' results are summarized in subsection 4.3.3 where, using *equilibrium* refinements, we are able to show that there always exists a unique reasonable PBE.

4.3.1 Separating equilibria

In a separating PBE, first period prices signal the type of incumbent's production cost and all information is revealed. Therefore, in the second period, when there is entry in market A, the entrant chooses his price under full information on the type of incumbent's production cost, *i.e.*, price competition in the second period occurs under complete information. The entrant achieves duopoly gross *equilibrium* profits given by, as mentioned, $\bar{\pi}^E$ and $\underline{\pi}^E$ when the incumbent has high and low production costs, respectively.

We assume that $\bar{\pi}^E > f^E > \underline{\pi}^E$, *i.e.*, if the entrant believes that the incumbent has low production cost, the optimal decision is not to enter. Conversely, the entrant optimally decides to enter if he believes that the incumbent has high production cost. Therefore, second period incumbent's profit are increasing in $\rho(p_A, p_B)$. If the posterior belief of the entrant is that the incumbent has low production cost, *i.e.*, $\rho(p_A, p_B)$ equals 1, then there is no entry in market A and the incumbent is a monopolist in the second period achieving π_{Am}. Since π_{Am} is never smaller than π_{Ad}, both types of incumbent would like to convince the entrant to be low cost. However, the entrant is aware of this and only believes the incumbent to be low cost if he sends a credible signal.

The main idea in a separating PBE is that the low production cost incumbent may use costly signals $(\underline{p}_A^*, \underline{p}_B^*)$ to separate from his high production cost counterpart and thus deter entry. That is, the low cost incumbent may use first period prices $(\underline{p}_A^*, \underline{p}_B^*)$ that would be unattractive were the incumbent to have high production cost. The second period provides the reward needed to justify the low cost incumbent's signaling cost. It follows that the low cost incumbent's prices $(\underline{p}_A^*, \underline{p}_B^*)$ typically differ from the prices chosen under complete information. If incumbent's production cost was known, in the first period the low cost incumbent would set the monopoly optimal price in each market, $(\underline{p}_{Am}, \underline{p}_{Bm})$, *i.e.*, the *complete information prices*. The question addressed is whether there is a distortion of these prices when they are used to signal incumbent's production cost.

In a separating PBE, first period prices charged by the high and low cost incumbents differ, $(\overline{p}_A^*, \overline{p}_B^*) \neq (\underline{p}_A^*, \underline{p}_B^*)$. Thus, observing first period prices allows the entrant to become fully informed. If $(\overline{p}_A^*, \overline{p}_B^*)$ is observed, the posterior beliefs must be $\rho(\overline{p}_A^*, \overline{p}_B^*) = 0$ whilst if $(\underline{p}_A^*, \underline{p}_B^*)$ is observed, $\rho(\underline{p}_A^*, \underline{p}_B^*) = 1$. *Off-the-equilibrium* path beliefs are not restricted as Bayes' rule is not applicable to events with zero probability.[7] We assume that for all other (p'_A, p'_B), the posterior beliefs are $\rho(p'_A, p'_B) = 0$ (so that neither type of incumbent wants to deviate to (p'_A, p'_B) since the entrant always decides to enter unless when he observes $(\underline{p}_A^*, \underline{p}_B^*)$).

In any separating PBE, the high cost incumbent charges the optimal monopoly price in each market, $(\overline{p}_A^*, \overline{p}_B^*) = (\overline{p}_{Am}, \overline{p}_{Bm})$. This can be shown by contradiction: if $(\overline{p}_A^*, \overline{p}_B^*) \neq (\overline{p}_{Am}, \overline{p}_{Bm})$, the high cost incumbent would have an incentive to deviate to $(\overline{p}_{Am}, \overline{p}_{Bm})$ as this would increase first period profit and it would not decrease second period profit since the entrant always decides to enter if he observes $(\overline{p}_A^*, \overline{p}_B^*)$.

On the other hand $(\underline{p}_A^*, \underline{p}_B^*)$ must be such that it is unprofitable for the high cost incumbent to mimic the low cost incumbent, *i.e.*:

$$\bar{\pi}_A\left(\underline{p}_A^*\right) + \bar{\pi}_B\left(\underline{p}_B^*\right) + \delta\left(\bar{\pi}_{Am} + \bar{\pi}_{Bm}\right) \leq \bar{\pi}_{Am} + \bar{\pi}_{Bm} + \delta\left(\bar{\pi}_{Ad} + \bar{\pi}_{Bm}\right).$$

Or equivalently:

$$\delta\left(\bar{\pi}_{Am} - \bar{\pi}_{Ad}\right) \leq \left(\bar{\pi}_{Am} + \bar{\pi}_{Bm}\right) - \left(\bar{\pi}_A\left(\underline{p}_A^*\right) + \bar{\pi}_B\left(\underline{p}_B^*\right)\right). \tag{4.5}$$

The left hand side of condition (4.5) is the second period gain if the high cost incumbent

[7] A perfect bayesian *equilibrium* requires Bayes' consistency of beliefs. These posterior beliefs were obtained from the prior beliefs using Bayes' rule and considering the incumbent's *equilibrium* strategies. Since any other vector of prices besides $(\overline{p}_A^*, \overline{p}_B^*)$ and $(\underline{p}_A^*, \underline{p}_B^*)$ is charged with probability zero, Bayes' rule cannot be applied for all other (p'_A, p'_B). Therefore, we may select any value for $\rho(p'_A, p'_B)$ for all other (p'_A, p'_B), *i.e.*, there is arbitrariness upon the *off-the-equilibrium* path posterior beliefs.

deviates from his *equilibrium* strategy and mimics the low cost incumbent. The right hand side of condition (4.5) is the first period loss if the high cost incumbent deviates from his *equilibrium* strategy. Let $\bar{\mathbf{C}}$ be the set of $(\underline{p}_A^*, \underline{p}_B^*)$ that satisfy the incentive compatibility condition (4.5). Figure 4.1 depicts the set $\bar{\mathbf{C}}$ in light grey.

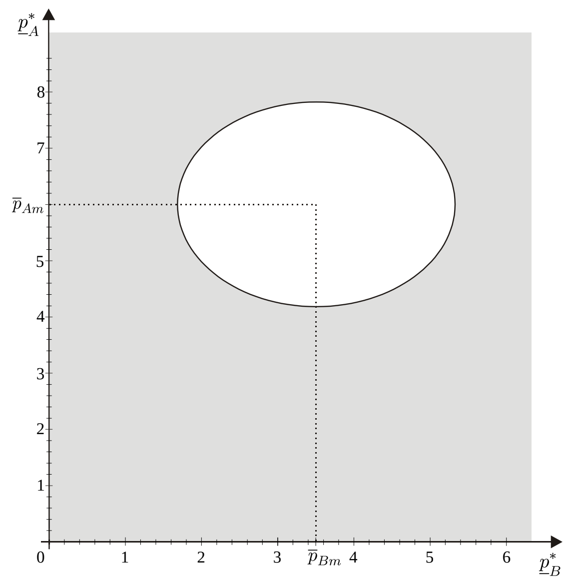

Figure 4.1: Set $\bar{\mathbf{C}}$ in light grey

Moreover, for a separating PBE to exist, it must be true that $(\underline{p}_A^*, \underline{p}_B^*)$ is optimal for the low cost incumbent, *i.e.*:

$$\underline{\pi}_A\left(\underline{p}_A^*\right) + \underline{\pi}_B\left(\underline{p}_B^*\right) + \delta\left(\underline{\pi}_{Am} + \underline{\pi}_{Bm}\right) \geq \underline{\pi}_{Am} + \underline{\pi}_{Bm} + \delta\left(\underline{\pi}_{Ad} + \underline{\pi}_{Bm}\right).$$

Or equivalently:

$$\delta\left(\underline{\pi}_{Am} - \underline{\pi}_{Ad}\right) \geq \left(\underline{\pi}_{Am} + \underline{\pi}_{Bm}\right) - \left(\underline{\pi}_A\left(\underline{p}_A^*\right) + \underline{\pi}_B\left(\underline{p}_B^*\right)\right). \tag{4.6}$$

Let $\underline{\mathbf{C}}$ be the set of $(\underline{p}_A^*, \underline{p}_B^*)$ that satisfy the incentive compatibility condition (4.6). Figure 4.2 depicts the set $\underline{\mathbf{C}}$ in grey.

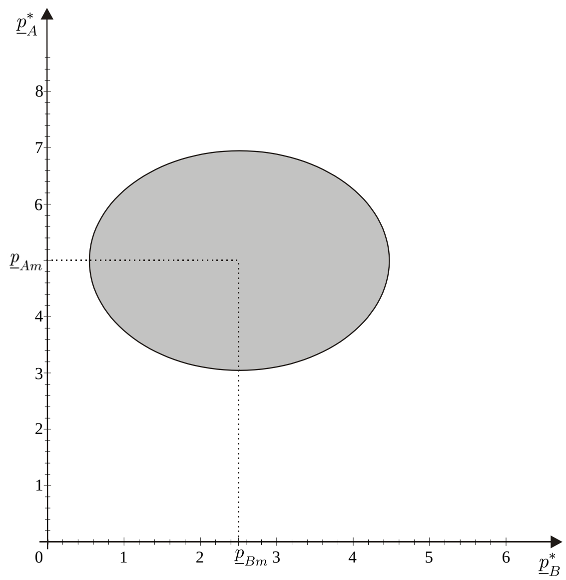

Figure 4.2: Set $\underline{\mathbf{C}}$ in grey

The following proposition sets a sufficient condition for the existence of a separating PBE.

Proposition 4.1 *A separating PBE exists for* $(\overline{p}_A^*, \overline{p}_B^*) = (\overline{p}_{Am}, \overline{p}_{Bm})$ *and for some non-negative prices* $(\underline{p}_A^*, \underline{p}_B^*)$ *set by the low cost incumbent if and only if the incentive compatibility conditions (4.5) and (4.6) both hold, i.e.,* $(\underline{p}_A^*, \underline{p}_B^*) \in \bar{\mathbf{C}} \cap \underline{\mathbf{C}}$. *Moreover if*

$$\underline{\pi}_{Am} - \underline{\pi}_{Ad} > \bar{\pi}_{Am} - \bar{\pi}_{Ad} \tag{4.7}$$

then the set $\bar{\mathbf{C}} \cap \underline{\mathbf{C}}$ *is non-empty and consequently there exists a separating PBE.*

Proof: To show necessity is enough to notice that if condition (4.5) or condition (4.6) fail, then either the high cost or the low cost incumbent is not behaving optimally in the proposed separating PBE for any specification of the *off-the-equilibrium* beliefs. Thus one can not have a separating PBE. To show that conditions (4.5) and (4.6) are sufficient for a separating PBE to exist it is important to notice that in a separating PBE, by Bayes' rule, $\rho(\underline{p}_A^*, \underline{p}_B^*) = 1$ and $\rho(\overline{p}_A^*, \overline{p}_B^*) = 0$. Assume that for all other (p'_A, p'_B), $\rho(p'_A, p'_B) = 0$. Given these beliefs and the fact that $\bar{\pi}^E > f^E > \underline{\pi}^E$, it is optimal for the entrant not to enter when he observes $(\underline{p}_A^*, \underline{p}_B^*)$ and to enter otherwise. But then, given the entrant's strategy, it is optimal for the high cost incumbent to set $(\overline{p}_{Am}, \overline{p}_{Bm})$ and the low cost incumbent to set $(\underline{p}_A^*, \underline{p}_B^*)$ if conditions (4.5) and

(4.6) both hold. Notice that beliefs given by $\rho(\underline{p}_A^*, \underline{p}_B^*) = 1$, $\rho(\overline{p}_A^*, \overline{p}_B^*) = 0$ and for all other (p_A', p_B'), $\rho(p_A', p_B') = 0$ are consistent with the described *equilibrium* strategies. Therefore, pricing strategies $(\overline{p}_A^*, \overline{p}_B^*) = (\overline{p}_{Am}, \overline{p}_{Bm})$ and $(\underline{p}_A^*, \underline{p}_B^*) \in \bar{\mathbf{C}} \cap \underline{\mathbf{C}}$ support a separating PBE.

The existence of a separating PBE depends on whether $\bar{\mathbf{C}} \cap \underline{\mathbf{C}}$ is non-empty or not. Condition (4.7) means that it is more profitable to generate favorable beliefs (which deter entry) for the low cost incumbent than to the high cost incumbent: the low cost incumbent has larger financial capacity to incur the costly signaling prices. Condition (4.7) is the typical single-crossing condition which guarantees existence of a separating PBE. The proof that condition (4.7) is a sufficient condition for the existence of a separating PBE is shown in Appendix 4.C. ■

Notice that if we assume that $\underline{\pi}_{Am} - \underline{\pi}_{Ad} > \bar{\pi}_{Am} - \bar{\pi}_{Ad}$, our set of parameters' values $\{\bar{c}, c^E, k\}$ becomes more restrict than $\mathbf{M} \cap \mathbf{D}$. For the remaining analysis we consider $k < 1$ and parameters' values $\{\bar{c}, c^E, k\} \in \mathbf{M} \cap \tilde{\mathbf{D}}$ (see details of set $\tilde{\mathbf{D}}$ in Appendix 4.D). For this set of parameters' values, there are multiple separating perfect bayesian *equilibria* since the subset $\bar{\mathbf{C}} \cap \underline{\mathbf{C}}$ is shown to be non-empty.

Figure 4.3 illustrates Proposition 4.1 where the subset $\bar{\mathbf{C}} \cap \underline{\mathbf{C}}$, in dark grey, is the set of all possible separating perfect bayesian *equilibria* $(\underline{p}_A^*, \underline{p}_B^*)$.

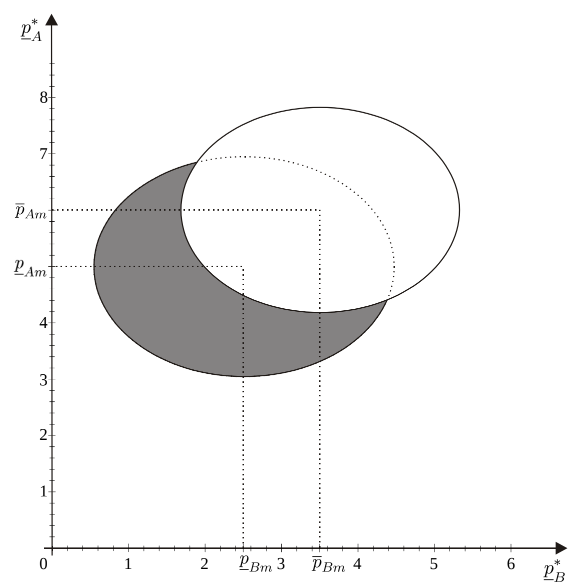

Figure 4.3: Subset $\bar{\mathbf{C}} \cap \underline{\mathbf{C}}$ in dark grey

The lack of restrictions upon *off-the-equilibrium* path posterior beliefs generates multiple separating perfect bayesian *equilibria*. The literature on refinements of perfect bayesian *equilibria* suggests plausibility criteria upon *off-the-equilibrium* path posterior beliefs. One of these criteria states that when the entrant observes an *off-the-equilibrium* path strategy, he should put probability zero on any type of incumbent for which the incumbent's strategy is dominated, *i.e.*, the entrant is not allowed to believe that the incumbent sets a dominated strategy. If we impose the domination criterion, there emerges a unique separating PBE for each possible set of parameters' values $\left\{\bar{c}, c^E, k, \delta\right\}$.

The pre-entry prices (p_A, p_B) are considered to be *dominated* for an incumbent whose production cost is i, i standing for high or low, if:

$$\pi_A^i\left(p_A\right) + \pi_B^i\left(p_B\right) + \delta(\pi_{Am}^i + \pi_{Bm}^i) < \pi_{Am}^i + \pi_{Bm}^i + \delta(\pi_{Ad}^i + \pi_{Bm}^i).$$

Thus, pre-entry prices (p_A, p_B) are *dominated* for type i if this type of incumbent has less profit with these prices under the most favorable entry conditions (*i.e.*, no entry) than with pre-entry monopoly prices in both markets under the least favorable entry conditions. If (p_A, p_B) is dominated for a high cost incumbent but not dominated for a low cost incumbent, the entrant should not believe that the incumbent has high production cost when he observes (p_A, p_B). Thus the only reasonable beliefs are $\rho(p_A, p_B) = 1$.

The set of separating perfect bayesian *equilibria*, $\bar{\mathbf{C}} \cap \underline{\mathbf{C}}$, contains pre-entry prices (p_A, p_B) which are *dominated* for a high cost incumbent (since they satisfy condition (4.5) in inequality) but are not *dominated* for a low cost incumbent (since they satisfy condition (4.6)). Therefore, the entrant's reasonable beliefs are given by $\rho(p_A, p_B) = 1$ for all $(p_A, p_B) \in \bar{\mathbf{C}} \cap \underline{\mathbf{C}}$ for which the incentive compatibility condition (4.5) is satisfied in inequality, *i.e.*, for all $(p_A, p_B) \in \bar{\mathbf{C}} \cap \underline{\mathbf{C}}$ which are not in the frontier of $\bar{\mathbf{C}}$ (we denote this set by $(int\bar{\mathbf{C}}) \cap \underline{\mathbf{C}}$) since the entrant excludes the possibility that the high cost incumbent sets any of those pre-entry prices. Given these beliefs, the entrant will not enter if he observes any (p_A, p_B) in $(int\bar{\mathbf{C}}) \cap \underline{\mathbf{C}}$ and he will also not enter if he observes the low cost incumbent's *equilibrium* prices $(\underline{p}_A^*, \underline{p}_B^*)$. The next lemma characterizes the low cost incumbent *equilibrium* strategy under these beliefs.

Lemma 4.1 *The low cost incumbent strategy in any separating PBE immune to the domination criterion is the solution* $(\underline{p}_A^*, \underline{p}_B^*)$ *of the following problem:*

$$\max_{(\underline{p}_A, \underline{p}_B)} \underline{\pi}_A\left(\underline{p}_A\right) + \underline{\pi}_B\left(\underline{p}_B\right) \tag{4.8}$$

subject to:

$$\overline{IC} : \bar{\pi}_A \left(\underline{p}_A \right) + \bar{\pi}_B \left(\underline{p}_B \right) \leq \bar{\pi}_{Am} + \bar{\pi}_{Bm} + \delta \left(\bar{\pi}_{Ad} - \bar{\pi}_{Am} \right)$$

$$\underline{IC} : \underline{\pi}_A \left(\underline{p}_A \right) + \underline{\pi}_B \left(\underline{p}_B \right) \geq \underline{\pi}_{Am} + \underline{\pi}_{Bm} + \delta \left(\underline{\pi}_{Ad} - \underline{\pi}_{Am} \right),$$

where the two constraints are the incentive compatibility conditions (4.5) and (4.6), respectively. For a given combination of parameters' values $\left\{ \bar{c}, c^E, k, \delta \right\}$ *there is a unique solution to this constrained optimization problem. Thus, there exists a unique separating PBE immune to the domination criterion.*

Proof: If the solution of problem (4.8) is in $(int\bar{\mathbf{C}}) \cap \underline{\mathbf{C}}$, it is immediate that only this solution can survive the domination criterion. Since the entrant does not enter for any (p_A, p_B) in $(int\bar{\mathbf{C}}) \cap \underline{\mathbf{C}}$, a low cost incumbent gets monopoly profit in the second period for any (p_A, p_B) in $(int\bar{\mathbf{C}}) \cap \underline{\mathbf{C}}$. But then for any $(p_A, p_B) \in \bar{\mathbf{C}} \cap \underline{\mathbf{C}}$ and $(p_A, p_B) \neq (\underline{p}_A^*, \underline{p}_B^*)$ the low cost incumbent is not behaving optimally.

If the solution of problem (4.8) is in the frontier of $\bar{\mathbf{C}}$ and we consider any (p_A, p_B) in $(int\bar{\mathbf{C}}) \cap \underline{\mathbf{C}}$, the low cost incumbent would always have an incentive to deviate to a pair of prices in $(int\bar{\mathbf{C}}) \cap \underline{\mathbf{C}}$ closer to $(\underline{p}_A^*, \underline{p}_B^*)$. Thus, none of the separating *equilibria* in $(int\bar{\mathbf{C}}) \cap \underline{\mathbf{C}}$ survives the domination criterion. Moreover, if we consider any $(p_A, p_B) \in \bar{\mathbf{C}} \cap \underline{\mathbf{C}}$ and (p_A, p_B) in the frontier of $\bar{\mathbf{C}}$ but different from $(\underline{p}_A^*, \underline{p}_B^*)$ the low cost incumbent would gain by deviating to a pair of prices in $(int\bar{\mathbf{C}}) \cap \underline{\mathbf{C}}$ arbitrarily close to $(\underline{p}_A^*, \underline{p}_B^*)$.

The uniqueness result is a direct consequence of solving the constrained optimization problem (4.8). ■

Henceforth, we will use the expression *«least cost separating equilibrium»* to denote the unique separating PBE immune to the domination criterion. Let us now examine this *equilibrium* for each combination of parameters' values $\left\{ \bar{c}, c^E, k, \delta \right\}$ where $k < 1, \left\{ \bar{c}, c^E, k \right\} \in \mathbf{M} \cap \tilde{\mathbf{D}}$ and $\delta \in \left]0, 1\right]$. The next proposition characterizes the three types of least cost separating *equilibria* where $\underline{IC}$ is never binding.

Proposition 4.2 *The three types of least cost separating equilibria are as follows:*

(i) *When no constraint is binding and both* $\underline{p}_A^*$ *and* $\underline{p}_B^*$ *are positive:*

Sol_1	$0 < \delta \leq \hat{\delta}$	$\underline{p}_A^* = \underline{p}_{Am}$ $\underline{p}_B^* = \underline{p}_{Bm}$

(ii) *When $\overline{IC}$ is the only binding constraint and both $\underline{p}_A^*$ and $\underline{p}_B^*$ are positive:*

Sol_2	$\hat{\delta} < \delta \leq \check{\delta}$	$\underline{p}_A^* = \overline{p}_{Am} - \sqrt{\frac{\delta(\bar{\pi}_{Am} - \bar{\pi}_{Ad})b_A}{2}}$ $\underline{p}_B^* = \overline{p}_{Bm} - \sqrt{\frac{\delta(\bar{\pi}_{Am} - \bar{\pi}_{Ad})b_A}{2}}$

(iii) *When $\overline{IC}$ is the only binding constraint and only $\underline{p}_A^*$ is positive:*

Sol_3	$\check{\delta} < \delta \leq 1$	$\underline{p}_A^* = \overline{p}_{Am} - \sqrt{\delta\left(\bar{\pi}_{Am} - \bar{\pi}_{Ad}\right)b_A - \frac{(a_B + \overline{c})^2}{4}}$

where:

$$\hat{\delta} = \frac{\overline{c}^2}{2b_A\left(\bar{\pi}_{Am} - \bar{\pi}_{Ad}\right)}$$
$$\check{\delta} = \frac{\left(a_B + \overline{c}\right)^2}{2b_A\left(\bar{\pi}_{Am} - \bar{\pi}_{Ad}\right)}.$$

Proof: The result is a direct consequence of solving the low cost incumbent's optimization problem. Kuhn-Tucker conditions are described in Appendix 4.E. ■

Notice that $\hat{\delta}$ and $\check{\delta}$ are always positive and may be larger than 1 for some subset of parameters' values $\left\{\overline{c}, c^E, k\right\} \in \mathbf{M} \cap \tilde{\mathbf{D}}$ and $k < 1$. For cases where $1 \leq \hat{\delta} < \check{\delta}$, which happen for subset $\mathbf{S_1}$ (see details in Appendix 4.F), the solution for all $k < 1$ and $\delta \in \left]0, 1\right]$ is given by Sol_1. At Sol_1, the least cost separating *equilibrium* equals the *complete information prices*, *i.e.*, the low cost incumbent pre-entry prices are the monopoly prices.

In addition, for cases where $\hat{\delta} < 1 < \check{\delta}$, which happen for subset $\mathbf{S_2}$ (see details in Appendix 4.F), then for $\delta \leq \hat{\delta}$, the solution is given by Sol_1 but for high δ, $\delta > \hat{\delta}$, the solution is given by Sol_2. In the latter solution, there is a downward distortion in prices at the least cost separating *equilibrium* when τ is positive since $\underline{p}_A^* < \underline{p}_{Am}$ and $\underline{p}_B^* < \underline{p}_{Bm}$.

Finally, if $\check{\delta} < 1$, which happens for subset $\mathbf{S_3}$ (see also details in Appendix 4.F), then for:

(i) $\delta \leq \hat{\delta}$, the solution is given by Sol_1;

(ii) $\hat{\delta} < \delta \leq \check{\delta}$, the solution is given by Sol_2;

(iii) $\delta > \hat{\delta}$, the solution is given by Sol_3.

At Sol_3, there is also a downward distortion in prices at the least cost separating *equilibrium* since $\underline{p}_A^* < \underline{p}_{Am}$ and $\underline{p}_B^* = 0 < \underline{p}_{Bm}$. Notice that both Sol_2 and Sol_3 present pre-entry prices lower than the *complete information prices*.

Therefore the least cost separating *equilibrium* may or may not exhibit a downward distortion in both pre-entry prices depending on parameters' values $\left\{\overline{c}, c^E, k, \delta\right\}$. This conclusion poses a relevant question: what determines the degree of the downward distortion in prices when it occurs? The next subsection presents a characterization of the factors which influence the downward distortion in prices.

Discussion of results

At Sol_1, the low cost incumbent sets the monopoly prices at the least cost separating *equilibrium.* In this case, the low cost incumbent does not use costly signals, *i.e.*, there is no distortion associated with *efficient* signaling. This solution holds for $\delta \leq \hat{\delta}$ where $\hat{\delta}$ is increasing with $\overline{c}$. Notice that the larger is $\overline{c}$, the larger is the difference between the two types of incumbent's production cost. But if the two types of incumbent are very different, which implies very different monopoly prices, the low cost incumbent does not need to distort prices in order to separate himself from the high cost incumbent. In other words, when the two types of incumbent's production cost are very different separation is easy.

For all other set of parameters' values $\left\{\overline{c}, c^E, k, \delta\right\}$, the least cost separating *equilibrium* exhibits a downward distortion in both pre-entry prices. Most curiously, at Sol_2 the downward distortion of the price in market A is equal to the downward distortion of the price in market B even under our assumption of demand in market A being less elastic than demand in market B for a given market price. Simple computations reveal that the relative price decrease is always higher in market B since $\overline{p}_{Am}$ is always higher than $\overline{p}_{Bm}$.

Notice that there are several factors which influence the degree of this identical downward distortion in prices given by:

$$\sqrt{\frac{\delta\left(\bar{\pi}_{Am} - \bar{\pi}_{Ad}\right) b_A}{2}}.$$

The downward distortion in prices is increasing with δ. As δ rises, the future becomes more important. Thus, if the high cost incumbent deviates and charges $(\underline{p}_A^*, \underline{p}_B^*)$, his gain increases, which means that he has more incentive to mimic the low cost incumbent. But this implies that the low cost incumbent has to increase the distortion in prices in order to separate from his high cost counterpart. On the other hand, when c^E rises, *i.e.*, when the entrant becomes less efficient, $\bar{\pi}_{Ad}$ increases and thus the downward distortion in prices is smaller.

Interestingly, the degree of product substitutability also affects the level of the downward distortion in prices. When $\tau \to 0$, *i.e.*, rival firms' products are completely differentiated, then

$\bar{\pi}_{Am} \rightarrow \bar{\pi}_{Ad}$ and there is no downward distortion in prices. Conversely when $\tau \rightarrow 1$, *i.e.*, rival firms' products are highly substitutable, then $\bar{\pi}_{Am}$ is much higher than $\bar{\pi}_{Ad}$ and thus the downward distortion in prices is the highest possible. Therefore, when the degree of product substitutability rises, the downward distortion in prices is bigger. All these factors similarly determine the downward distortion of the price in market A at Sol_3 but there is an additional factor. When k decreases, *i.e.*, demand in market A becomes much less elastic than demand in market B for a given market price, then the downward distortion of the price in market A is smaller.

4.3.2 Pooling Equilibria

We now analyze the pooling perfect bayesian *equilibria*. In a pooling PBE, first period prices of both types of incumbent are equal and thus do not signal the incumbent's type. Therefore, the entrant does not change his prior beliefs when he observes the *equilibrium* pair of prices. We assume that, if entry occurs in market A, the entrant learns the incumbent's cost before price decisions are taken in the second period. Thus, price competition in the second period occurs under complete information.

As in subsection 4.3.1, we assume that $\bar{\pi}^E > f^E > \underline{\pi}^E$. We further assume that $\rho_o \underline{\pi}^E + (1 - \rho_o) \bar{\pi}^E < f^E$, which implies that under the prior beliefs the entrant's optimal decision is not to enter. This condition is necessary for a pooling PBE to exist. In fact, if this condition failed and the two types charged the same prices, the entrant would enter when he observes the pooling pair of prices. Given this, each type of incumbent would have an incentive to charge his monopoly prices in the first period, thus the pooling strategy could not be optimal.

In a pooling PBE, first period prices charged by the high and low cost incumbents are the same, $(\overline{p}_A^*, \overline{p}_B^*) = (\underline{p}_A^*, \underline{p}_B^*) = (p_A^*, p_B^*)$. When (p_A^*, p_B^*) is observed, by Bayes' rule, the posterior beliefs are equal to the prior beliefs, *i.e.*, $\rho(p_A^*, p_B^*) = \rho_o$ and the entrant's optimal decision is not to enter. We assume that for all other (p'_A, p'_B) the posterior beliefs are $\rho(p'_A, p'_B) = 0$.

In any pooling PBE, (p_A^*, p_B^*) must be optimal for the high cost incumbent, *i.e.*:

$$\bar{\pi}_A (p_A^*) + \bar{\pi}_B (p_B^*) + \delta (\bar{\pi}_{Am} + \bar{\pi}_{Bm}) \geq \bar{\pi}_{Am} + \bar{\pi}_{Bm} + \delta (\bar{\pi}_{Ad} + \bar{\pi}_{Bm}) .$$

Or equivalently:

$$\delta (\bar{\pi}_{Am} - \bar{\pi}_{Ad}) \geq (\bar{\pi}_{Am} + \bar{\pi}_{Bm}) - (\bar{\pi}_A (p_A^*) + \bar{\pi}_B (p_B^*)) . \tag{4.9}$$

This condition is the reverse of condition (4.5) used in the analysis of separating PBE. Let $\bar{\mathbf{C}}'$ be the set of (p_A^*, p_B^*) that satisfy the incentive compatibility condition (4.9). Figure 4.1 depicts the set $\bar{\mathbf{C}}'$ in white since in light grey we have the set of (p_A^*, p_B^*) that satisfy condition (4.5).

Moreover, for a pooling PBE to exist, it must also be true that (p_A^*, p_B^*) is optimal for the low cost incumbent, *i.e.*:

$$\underline{\pi}_A(p_A^*) + \underline{\pi}_B(p_B^*) + \delta(\underline{\pi}_{Am} + \underline{\pi}_{Bm}) \geq \underline{\pi}_{Am} + \underline{\pi}_{Bm} + \delta(\underline{\pi}_{Ad} + \underline{\pi}_{Bm}).$$

Or equivalently:

$$\delta(\underline{\pi}_{Am} - \underline{\pi}_{Ad}) \geq (\underline{\pi}_{Am} + \underline{\pi}_{Bm}) - (\underline{\pi}_A(p_A^*) + \underline{\pi}_B(p_B^*)). \tag{4.10}$$

This is precisely the same condition as (4.6). Therefore, Figure 4.2 depicts in grey the set of (p_A^*, p_B^*) that satisfy the condition (4.10).

The following proposition summarizes the conditions for a pooling PBE.

Proposition 4.3 *A pooling PBE exists for some non-negative prices (p_A^*, p_B^*) set by the high and low cost incumbents if and only if the incentive compatibility conditions (4.9) and (4.10) both hold, i.e., $(p_A^*, p_B^*) \in \bar{\mathbf{C}}' \cap \underline{\mathbf{C}}$.*

Proof: To show necessity is enough to notice that if either condition (4.9) or (4.10) fail, then either the high cost or the low cost incumbent is not behaving optimally in the proposed pooling PBE for any specification of the *off-the-equilibrium* beliefs, thus one can not have a pooling PBE. To show that conditions (4.9) and (4.10) are sufficient for a pooling PBE to exist it is important to notice that in a pooling PBE, by Bayes' rule, $\rho(p_A^*, p_B^*) = \rho_o$. Assume that for all other (p'_A, p'_B), $\rho(p'_A, p'_B) = 0$. Given these beliefs and the assumptions over the entrant's profits, it is optimal for the entrant not to enter when he observes (p_A^*, p_B^*) and to enter otherwise. But then, given the entrant's strategy, it is optimal for the high and low cost incumbents to set (p_A^*, p_B^*) if conditions (4.9) and (4.10) both hold. Notice that beliefs given by $\rho(p_A^*, p_B^*) = \rho_o$ and for all other (p'_A, p'_B), $\rho(p'_A, p'_B) = 0$ are consistent with the described *equilibrium* strategies. Therefore, pricing strategies $(\overline{p}_A^*, \overline{p}_B^*) = (\underline{p}_A^*, \underline{p}_B^*) = (p_A^*, p_B^*)$ and $(p_A^*, p_B^*) \in \bar{\mathbf{C}}' \cap \underline{\mathbf{C}}$ support a pooling PBE. ■

Under the assumption that $\underline{\pi}_{Am} - \underline{\pi}_{Ad} > \bar{\pi}_{Am} - \bar{\pi}_{Ad}$, $\bar{\mathbf{C}}' \cap \underline{\mathbf{C}}$ is shown to be non-empty as long as the two types of incumbent are not too different ($\bar{c}$ is not too high). Thus, for this set of parameters' values, there are multiple pooling perfect bayesian *equilibria*. The lack of restrictions upon *off-the-equilibrium* path posterior beliefs generates multiple pooling perfect bayesian *equilibria*. The use of *equilibrium* refinements allows us to eliminate several perfect bayesian *equilibria*. The existence of pooling *equilibria* which survive the domination criterion depends on whether $(\underline{p}_{Am}, \underline{p}_{Bm})$ belongs or does not belong to $\bar{\mathbf{C}}' \cap \underline{\mathbf{C}}$. Next we analyze these two cases.

When $(\underline{p}_{Am}, \underline{p}_{Bm}) \notin \bar{\mathbf{C}}' \cap \underline{\mathbf{C}}$

Figure 4.4 illustrates an example where $(\underline{p}_{Am}, \underline{p}_{Bm}) \notin \bar{\mathbf{C}}' \cap \underline{\mathbf{C}}$. The set of $(p_A^*, p_B^*) \in \bar{\mathbf{C}}' \cap \underline{\mathbf{C}}$, in light grey, is the set of pooling perfect bayesian *equilibria*.

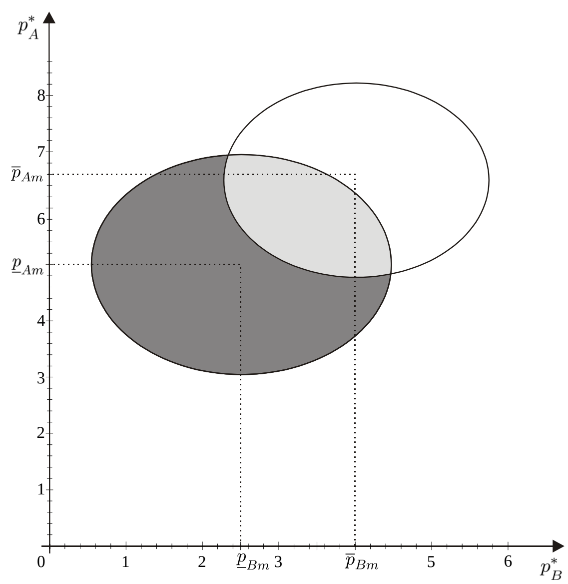

Figure 4.4: Subset $\bar{\mathbf{C}}' \cap \underline{\mathbf{C}}$ in light grey

In this case it is easy to show that no pooling PBE survives the domination criterion. Since any pre-entry pair of prices (p_A, p_B) in $(int\bar{\mathbf{C}}) \cap \underline{\mathbf{C}}$ is strictly *dominated* for a high cost incumbent but is not *dominated* for a low cost incumbent, then $\rho(p_A, p_B) = 1$ for such pair of prices. Given these beliefs, the entrant will not enter if he observes any (p_A, p_B) in $(int\bar{\mathbf{C}}) \cap \underline{\mathbf{C}}$. Therefore, in any $(p_A^*, p_B^*) \in \bar{\mathbf{C}}' \cap \underline{\mathbf{C}}$, the low cost incumbent would always have an incentive to deviate to $(\underline{p}_{Am}, \underline{p}_{Bm})$ since first period profit is higher and entry does not occur.

When $(\underline{p}_{Am}, \underline{p}_{Bm}) \in \bar{\mathbf{C}}'\cap\underline{\mathbf{C}}$

Figure 4.5 illustrates an example where $(\underline{p}_{Am}, \underline{p}_{Bm}) \in \bar{\mathbf{C}}'\cap\underline{\mathbf{C}}$. The set of $(p_A^*, p_B^*) \in \bar{\mathbf{C}}'\cap\underline{\mathbf{C}}$, in grey, is the set of pooling perfect bayesian *equilibria*.

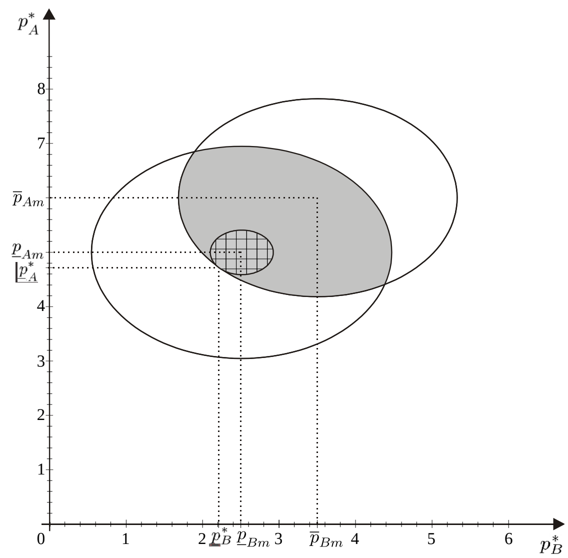

Figure 4.5: Subset $\bar{\mathbf{C}}'\cap\underline{\mathbf{C}}$ in grey

When $(\underline{p}_{Am}, \underline{p}_{Bm}) \in \bar{\mathbf{C}}'\cap\underline{\mathbf{C}}$, some pooling *equilibria* are immune to the domination criterion. Let $\underline{\pi}_{A,B}$ be the level of profit where the low cost incumbent's isoprofit is tangent to the frontier of the incentive compatibility condition (4.9). All the pooling perfect bayesian *equilibria* where pre-entry profits of the low cost incumbent are lower than $\underline{\pi}_{A,B}$ (outside the area in squares) do not survive the domination criterion. Consider any such pooling PBE. Since the entrant will not enter if he observes any (p_A, p_B) in $(int\bar{\mathbf{C}}) \cap \underline{\mathbf{C}}$ once $\rho(p_A, p_B) = 1$, the low cost incumbent would always have an incentive to deviate from the proposed pooling PBE to a pair of prices in $(int\bar{\mathbf{C}}) \cap \underline{\mathbf{C}}$ arbitrarily close to $(\underline{p}_A^*, \underline{p}_B^*)$. Therefore, the subset of pooling PBE in $\bar{\mathbf{C}}'\cap\underline{\mathbf{C}}$ that are immune to the domination criterion is given by all (p_A^*, p_B^*) where pre-entry profits of the low cost incumbent are higher than $\underline{\pi}_{A,B}$. We denote this subset by $\underline{\mathbf{I}}$, $\underline{\mathbf{I}} \subset \bar{\mathbf{C}}'\cap\underline{\mathbf{C}}$. Figure 4.5 depicts this subset in the area marked with squares.

If we further refine the *equilibrium* concept using Cho and Kreps (1987) *intuitive criterion* we can eliminate some more pooling perfect bayesian *equilibria* in subset $\underline{\mathbf{I}}$. Cho and Kreps criterion states that a type should not be expected to play an *equilibrium* dominated action.

An action a can be eliminated for type θ by *equilibrium* dominance if type θ does not gain by deviating from the *equilibrium* no matter what action the uninformed player chooses. Thus, if the uninformed player observes action a he should put zero probability on type θ. This reasonability criterion upon *off-the-equilibrium* path posterior beliefs may change the uninformed player's best response, which in turn may induce some other type to deviate from the proposed *equilibrium*. If this happens, the *equilibrium* does not survive the *intuitive criterion*. To understand better this criterion, let us consider some pooling PBE, $(p_A^*, p_B^*) \in \underline{\mathbf{I}}$. Suppose that there exists a deviant pair of prices $(\tilde{p}_A, \tilde{p}_B)$ such that a high cost incumbent would not want to deviate from (p_A^*, p_B^*) even if he was believed to be low cost by charging $(\tilde{p}_A, \tilde{p}_B)$ but, given these beliefs, the low cost incumbent's profit would be higher with $(\tilde{p}_A, \tilde{p}_B)$ than his *equilibrium* profit. That is, $(\tilde{p}_A, \tilde{p}_B)$ is an *equilibrium* dominated action for the high cost incumbent but not for the low cost incumbent, thus $\rho(\tilde{p}_A, \tilde{p}_B) = 1$ is the appropriate inference if $(\tilde{p}_A, \tilde{p}_B)$ is observed. Since the entrant will not enter if he observes $(\tilde{p}_A, \tilde{p}_B)$ the low cost incumbent gains by deviating from (p_A^*, p_B^*) and the *equilibrium* fails the *intuitive criterion*. In order for $(\tilde{p}_A, \tilde{p}_B)$ to exist the following two conditions have to hold:

$$\underline{\pi}_A(\tilde{p}_A) + \underline{\pi}_B(\tilde{p}_B) + \delta(\underline{\pi}_{Am} + \underline{\pi}_{Bm}) > \underline{\pi}_A(p_A^*) + \underline{\pi}_B(p_B^*) + \delta(\underline{\pi}_{Am} + \underline{\pi}_{Bm}) \Leftrightarrow$$

$$\Leftrightarrow \underline{\pi}_A(\tilde{p}_A) + \underline{\pi}_B(\tilde{p}_B) > \underline{\pi}_A(p_A^*) + \underline{\pi}_B(p_B^*) \tag{4.11}$$

and

$$\bar{\pi}_A(\tilde{p}_A) + \bar{\pi}_B(\tilde{p}_B) + \delta(\bar{\pi}_{Am} + \bar{\pi}_{Bm}) < \bar{\pi}_A(p_A^*) + \bar{\pi}_B(p_B^*) + \delta(\bar{\pi}_{Am} + \bar{\pi}_{Bm}) \Leftrightarrow$$

$$\Leftrightarrow \bar{\pi}_A(\tilde{p}_A) + \bar{\pi}_B(\tilde{p}_B) < \bar{\pi}_A(p_A^*) + \bar{\pi}_B(p_B^*). \tag{4.12}$$

Notice that these conditions represent the set of prices where pre-entry profits of the high cost incumbent are lower than his pooling PBE pre-entry profit and pre-entry profits of the low cost incumbent are higher than his pooling PBE pre-entry profit.

Using the *intuitive criterion* we eliminate most pooling perfect bayesian *equilibria*. Any pooling PBE above the isoprofit curve of the high cost incumbent with profit level $\bar{\pi}(\underline{p}_{Am}, \underline{p}_{Bm}) = \bar{\pi}_A\left(\underline{p}_{Am}\right) + \bar{\pi}_B\left(\underline{p}_{Bm}\right)$ fails the *intuitive criterion*, since $(\underline{p}_{Am}, \underline{p}_{Bm})$ is *equilibrium* dominated for the high cost incumbent and it is a profitable deviation for the low cost incumbent. Moreover, any pooling PBE with $\bar{\pi}(p_A^*, p_B^*) \leq \bar{\pi}(\underline{p}_{Am}, \underline{p}_{Bm})$ where the isoprofit curves of the two types intercept can not survive the *intuitive criterion* as one can find pairs of prices which are below the high cost isoprofit curve but above the low cost isoprofit curve. Figure 4.6 illustrates a pair of prices, $(\hat{p}^*_A, \hat{p}^*_B)$, where the isoprofit curves (in grey) intercept. The area marked with hexagons corresponds to the pairs of prices which are *equilibrium* dominated for the high cost incumbent and where the low cost incumbent gains by deviating from $(\hat{p}^*_A, \hat{p}^*_B)$. Thus $(\hat{p}^*_A, \hat{p}^*_B)$ fails the *intuitive criterion*.

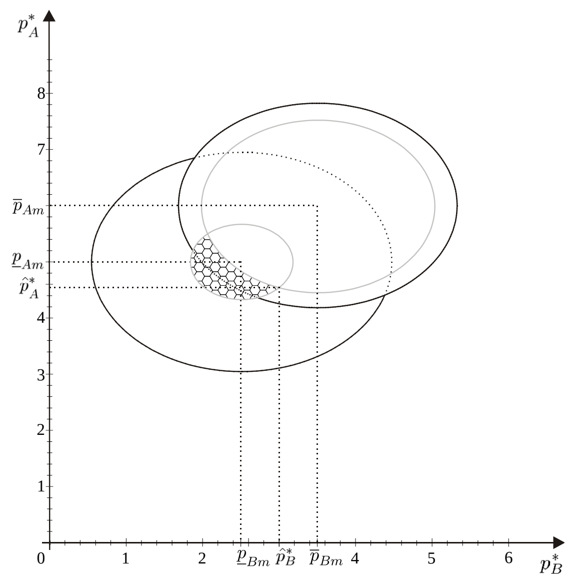

Figure 4.6: A pooling PBE that fails the *intuitive criterion*

However, the pooling *equilibria* where the two isoprofit curves are tangent survive the *intuitive criterion*. Since the low cost isoprofit curve is steeper than the high cost isoprofit curve, any $(\tilde{p}_A, \tilde{p}_B)$ below the high cost isoprofit curve is necessarily below the low cost isoprofit curve, hence one can not find a $(\tilde{p}_A, \tilde{p}_B)$ which is *equilibrium* dominated for the high cost incumbent but not for the low cost incumbent. The set of pooling *equilibria* where the isoprofits are tangent and thus survive the *intuitive criterion* is the line-segment from $(\underline{p}^*_A, \underline{p}^*_B)$ to $(\underline{p}_{Am}, \underline{p}_{Bm})$ (see Figure 4.7). It is interesting to notice that the pooling *equilibria* also entail a downward distortion with equal distortions in both markets.

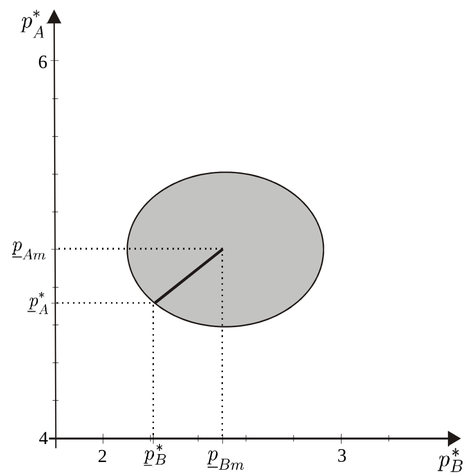

Figure 4.7: Subset $\underline{\mathbf{I}}$ in grey

4.3.3 Summary and further equilibrium refinements

At this point it is useful to summarize the *equilibria*' results. Assuming $\underline{\pi}_{Am} - \underline{\pi}_{Ad} > \bar{\pi}_{Am} - \bar{\pi}_{Ad}$ and $\bar{\pi}^E > f^E > \underline{\pi}^E$, there always exists a unique separating PBE which is immune to the domination criterion. If, in addition, $\rho_o \underline{\pi}^E + (1 - \rho_o)\,\bar{\pi}^E > f^E$, there are no pooling *equilibria*. Therefore, there exists a unique reasonable PBE which is the least cost separating *equilibrium* characterized in Proposition 4.2.

On the other hand, if $\rho_o \underline{\pi}^E + (1 - \rho_o)\,\bar{\pi}^E < f^E$ we also have a set of pooling *equilibria* as long as the two types of incumbent are not too different ($\bar{c}$ is not too high). In this case, if $(\underline{p}_{Am}, \underline{p}_{Bm})$ is a dominated strategy for the high cost incumbent, none of the pooling *equilibria* survives the domination criterion and the unique reasonable PBE is the least cost separating *equilibrium* where each type charges the monopoly prices in the first period.

Finally, when $\rho_o \underline{\pi}^E + (1 - \rho_o)\,\bar{\pi}^E < f^E$ and the two types of incumbent are so similar that $(\underline{p}_{Am}, \underline{p}_{Bm})$ can be supported as a pooling PBE, the set of pooling *equilibria* in the line-segment from $(\underline{p}_A^*, \underline{p}_B^*)$ to $(\underline{p}_{Am}, \underline{p}_{Bm})$ in Figure 4.7 survive the *intuitive criterion*. In this case, we have multiple perfect bayesian *equilibria* which survive the *intuitive criterion*: the least cost separating *equilibrium* and the set of pooling *equilibria* in the line-segment from $(\underline{p}_A^*, \underline{p}_B^*)$ to $(\underline{p}_{Am}, \underline{p}_{Bm})$. However, one can impose further refinements upon *off-the-equilibrium* path beliefs using Grossman and Perry (1986) criterion. According to this criterion if there exists a set K of types who are better of choosing action a than the *equilibrium* strategy while the remaining types are worse of, given that the uninformed player revises his beliefs to believe that $t \in K$ and according to Bayes' rule, then the proposed *equilibrium* is overturned. The next result shows that there exists a unique PBE which survives the Grossman and Perry (1986) criterion.

Proposition 4.4 *Suppose that* $\underline{\pi}_{Am} - \underline{\pi}_{Ad} > \bar{\pi}_{Am} - \bar{\pi}_{Ad}$ *and* $\bar{\pi}^E > f^E > \underline{\pi}^E$ *then there exists a unique PBE which survives the Grossman and Perry criterion which is equal to:*

(i) the least cost separating equilibrium characterized in Proposition 4.2, when $\rho_o \underline{\pi}^E + (1 - \rho_o)\, \bar{\pi}^E > f^E$.

(ii) the least cost separating equilibrium where $(\underline{p}_A^*, \underline{p}_B^*) = (\underline{p}_{Am}, \underline{p}_{Bm})$*, when* $(\underline{p}_{Am}, \underline{p}_{Bm})$ *is a dominated strategy for the high cost incumbent and* $\rho_o \underline{\pi}^E + (1 - \rho_o)\, \bar{\pi}^E < f^E$.

(iii) the pooling equilibrium with $(\bar{p}_A^*, \bar{p}_B^*) = (\underline{p}_A^*, \underline{p}_B^*) = (\underline{p}_{Am}, \underline{p}_{Bm})$*, when* $(\underline{p}_{Am}, \underline{p}_{Bm})$ *is not a dominated strategy for the high cost incumbent and* $\rho_o \underline{\pi}^E + (1 - \rho_o)\, \bar{\pi}^E < f^E$.

Proof: In case *(i)* and *(ii)* there exists a unique PBE which survives the domination criterion and consequently the *intuitive criterion*. Since Grossman and Perry criterion is a refinement of the *intuitive criterion*, if it exists a PBE which survives the Grossman and Perry criterion, it is necessarily the least cost separating *equilibrium*. In each case, it is also easy to prove that the least cost separating *equilibrium* is immune to Grossman and Perry criterion.

In case *(iii)*, the only PBE which is immune to Grossman and Perry criterion is the pooling PBE where both types choose first period prices $(\underline{p}_{Am}, \underline{p}_{Bm})$. Let us first eliminate the remaining pooling *equilibria*. Since Grossman and Perry criterion is a refinement of the *intuitive criterion*, we just need to show that no other vector of prices in the line-segment from $(\underline{p}_A^*, \underline{p}_B^*)$ to $(\underline{p}_{Am}, \underline{p}_{Bm})$ in Figure 4.7 survives the criterion. Consider any pooling PBE, (p_A^*, p_B^*), in the line segment between $(\underline{p}_A^*, \underline{p}_B^*)$ and $(\underline{p}_{Am}, \underline{p}_{Bm})$ such that $(p_A^*, p_B^*) < (\underline{p}_{Am}, \underline{p}_{Bm})$. Both types of incumbent would gain by deviating to $(\underline{p}_{Am}, \underline{p}_{Bm})$ if the entrant beliefs are such that he does not enter. Hence the entrant's posterior beliefs following $(\underline{p}_{Am}, \underline{p}_{Bm})$ should be equal to the prior beliefs, ρ_o, and his optimal decision is not to enter. Since there are consistent beliefs such that both types of incumbent gain by deviating, the proposed *equilibrium* fails the Grossman and Perry criterion. The least cost separating *equilibrium*, in which the low cost incumbent's prices are $(\underline{p}_A^*, \underline{p}_B^*)$, can also be eliminated using the same reasoning. Both types would prefer to deviate from their separating *equilibrium* strategies to $(\underline{p}_{Am}, \underline{p}_{Bm})$ if the entrant beliefs are such that he does not enter. Then beliefs following $(\underline{p}_{Am}, \underline{p}_{Bm})$ should be ρ_o, which implies that both types of incumbent gain by deviating from the least cost separating *equilibrium*. ■

It is interesting to notice that the unique PBE which survives the Grossman and Perry criterion is also the unique Pareto optimal *equilibrium* for the incumbent's types. Moreover, this unique PBE entails a downward distortion for at least one type, except in case *(i)* when $\delta \leq \hat{\delta}$ and in case *(ii)*.

4.4 Extension of results

In this section, we analyze the implications of changing two of the initial assumptions on results. Firstly, we explore what happens if $k \geq 1$, *i.e.*, demand in market B is equally elastic or less elastic than demand in market A for a given market price.[8] Secondly, we extend the initial assumptions on the relationships between market A and market B's parameters by considering that $b_B = \frac{b_A}{w}$. Our initial assumption is a particular case of this assumption since $b_B = \frac{b_A}{w}$ equals (4.4) when $w = 1$.

For $k \geq 1$, the set of parameters' values where monopoly and duopoly quantities are non-negative and where condition and where condition (4.7) holds is given by $\left\{\bar{c}, c^E, k\right\} \in \tilde{\mathbf{D}}$ once $\tilde{\mathbf{D}}$ is sufficient to guarantee that non-negative constraints are satisfied. For this set of parameters' values, there may occur two types of least cost separating *equilibrium*: Sol_1 and Sol_2. Sol_3 is impossible to occur since market B is now larger. Notice that Sol_1 and Sol_2 are both set, for different levels of δ, at the least cost separating *equilibrium* for a subset of $\tilde{\mathbf{D}}$ where $c^E < \hat{c}^E$. Conversely, when $c^E \geq \hat{c}^E$, only Sol_1 is set at the least cost separating *equilibrium* for all δ. For $k > 1$, the relative downward distortion in price is always higher in market A since $\overline{p}_{Am}$ is always lower than $\overline{p}_{Bm}$. When $k = 1$, the relative downward distortion in price is identical in both markets. The unique reasonable pooling PBE is $(\underline{p}_{Am}, \underline{p}_{Bm})$ where for $k \geq 1$, $\underline{p}_{Bm} \geq \underline{p}_{Am}$.

Lets explore the impact of the extended assumption over the relationships between market A and market B's parameters, $b_B = \frac{b_A}{w}$, but considering $k < 1$. Notice that this new assumption

[8] This is the set of parameters that may capture a predatory dumping behavior. If we consider market B as the domestic market and market A as the external market, the incumbent may set a lower pre-entry price in the external market so as to deter entry. But notice that the incumbent uses prices in both markets as *predatory signals*.

does not influence the relationship between markets' demand elasticity since given the quadratic utility function (4.2) and the resultant demand functions, demand in market A is less elastic than demand in market B for a given market price when $k < 1$. It certainly influences the relationship between consumer's marginal utility in market B and consumer's marginal utility in market A. Thus, we now face several different scenarios where the monopoly demand functions in market A and market B do not have identical slopes. Therefore, for the set of parameters' values $\left\{\bar{c}, c^E, k\right\} \in \mathbf{M} \cap \tilde{\mathbf{D}}$ and $k < 1$, there is impact upon two of the three types of the least cost separating *equilibrium* that may occur. More precisely upon the solutions where there is a downward distortion in prices. The downward distortion in price continues to be identical in both markets (since the Kuhn-Tucker conditions are similar) but this decrease in prices now depends on w. At Sol_2 the identical downward distortion in price is now given by $\sqrt{\frac{\delta(\bar{\pi}_{Am} - \bar{\pi}_{Ad}) b_A}{(1+w)}}$. If $w > 1$, there is a smaller downward distortion in price than when $w = 1$. Moreover as w rises, the downward distortion in prices becomes smaller. Also at Sol_3, the downward distortion in price is now given by $\sqrt{\delta\left(\bar{\pi}_{Am} - \bar{\pi}_{Ad}\right) b_A - \frac{w(a_B + \bar{c})^2}{4}}$. There is also an impact on these solutions' limits since $\hat{\delta}$ now equals $\frac{(1+w)\bar{c}^2}{4b_A(\bar{\pi}_{Am} - \bar{\pi}_{Ad})}$ and $\check{\delta}$ equals $\frac{(1+w)(a_B + \bar{c})^2}{4b_A(\bar{\pi}_{Am} - \bar{\pi}_{Ad})}$ and thus the subsets $\mathbf{S_1}$, $\mathbf{S_2}$ and $\mathbf{S_3}$ are different.

4.5 Conclusion

This chapter explores the incumbent's choice of a multiple signaling strategy when practicing *discriminatory limit pricing*. Considering potential entry in one of the two geographical independent markets where the incumbent operates, the incumbent uses third-degree price discrimination to deter entry when the entrant has incomplete knowledge about the incumbent's production cost. The entrant offers a differentiated product and decides whether to enter after observing incumbent's pre-entry prices in both markets. Thus, the incumbent may have an incentive to distort both pre-entry prices in order to convey information on production cost to the entrant so as to deter entry.

In the presence of this signaling effect we show that incumbent's *complete information prices* may no longer hold. We show that there is always a unique reasonable perfect bayesian *equilibrium*. There are some cases where the only *equilibrium* that survives the domination criterion is the least cost separating *equilibrium*. In this separating *equilibrium*, the low cost incumbent's pre-entry prices may have a downward distortion with respect to the *complete information* setting. The low cost incumbent uses pre-entry prices in both markets to signal low production cost. Moreover, we show that this distortion is identical in both markets for every relationship between markets' demand parameters. The price distortion is increasing with the discount factor, the degree of product substitutability and the efficiency of the entrant. For the remaining cases, we show that the unique *equilibrium* that survives the Grossman and Perry (1986) criterion is a pooling *equilibrium* where both types of incumbents set the low cost incumbent's monopoly prices.

We extend previous analysis of entry deterrence literature to a setup which considers third-degree price discrimination and product differentiation. In a multimarket setup, the incumbent uses multiple signaling. An implication of our analysis to predatory pricing detection is that policy regulation authorities should reach worldwide information since firms are becoming global and may price lower than the monopoly prices in more than one market in order to deter entry. This chapter shows that in some cases multinational firms may not be using purely discriminatory pricing but *discriminatory limit pricing*.

Further work should include the presence of an active regulator, for instance, in only one of the incumbent's operating markets. This may lead to the use of signaling exclusively in the unregulated market. Three other extensions seem worth considering. The analysis of other incumbent's strategies, such as accommodation strategies, which may lead to an upward distortion in prices. It would also be interesting to relax the assumption of constant marginal production cost. Finally, oligopolist incumbent firms would also be a relevant framework to consider since two-sided signaling arises.

4.A Firms' profit functions

For a detailed deduction of the *optimal* prices and profits see Subsections 3.3.1 and 3.4.1 but notice that these subsections consider that production cost of both firms are nil.

Due to the assumption of constant marginal production cost, the incumbent maximizes profits in each market independently when using third-degree price discrimination.

4.A.1 Monopoly in market A and B

The incumbent's monopoly profit in market i, $i = A, B$, when the incumbent has high production cost is given by:

$$\bar{\pi}_i = (p_i - \bar{c})\left(\frac{a_i}{b_i} - \frac{1}{b_i}p_i\right).$$

The unique maximizer of this profit function is the monopoly discriminatory price in market i:

$$\bar{p}_{im} = \frac{a_i + \bar{c}}{2}$$

and the respective monopoly discriminatory quantity and profit in market i are given by:

$$\bar{q}_{im} = \frac{a_i - \bar{c}}{2b_i} \text{ and } \bar{\pi}_{im} = \frac{(a_i - \bar{c})^2}{4b_i}.$$

Analogously, when the incumbent has low production cost and $\underline{c} = 0$, the monopoly discriminatory price, quantity and profit in market i simplify to:

$$\underline{p}_{im} = \frac{a_i}{2}, \underline{q}_{im} = \frac{a_i}{2b_i} \text{ and } \underline{\pi}_{im} = \frac{a_i^2}{4b_i}.$$

4.A.2 Duopoly in market A

When entry occurs in market A, firms' profit functions when the incumbent has high production cost are given by:

$$\bar{\pi}_{Ad} = (p_{Ad} - \bar{c})\left(\alpha - \beta p_{Ad} + \gamma p^E\right)$$

$$\bar{\pi}^E = \left(p^E - c^E\right)\left(\alpha - \beta p^E + \gamma p_{Ad}\right).$$

As firms choose their prices simultaneously and independently, firms maximize their profit functions. The second period Nash *equilibrium* prices and quantities are:

$$\bar{p}_{Ad} = \frac{\alpha(2\beta + \gamma) + \beta\gamma c^E + 2\beta^2\bar{c}}{4\beta^2 - \gamma^2}$$

$$\overline{p}^E = \frac{\alpha(2\beta+\gamma)+2\beta^2 c^E+\beta\gamma\overline{c}}{4\beta^2-\gamma^2}$$

$$\overline{q}_{Ad} = \frac{\beta\left(\alpha(2\beta+\gamma)+\beta\gamma c^E-(2\beta^2-\gamma^2)\overline{c}\right)}{4\beta^2-\gamma^2}$$

$$\overline{q}^E = \frac{\beta\left(\alpha(2\beta+\gamma)+\beta\gamma\overline{c}-(2\beta^2-\gamma^2)c^E\right)}{4\beta^2-\gamma^2}$$

and the firms' profits are given by:

$$\bar{\pi}_{Ad} = \frac{\beta\left[\alpha(2\beta+\gamma)+\beta\gamma c^E-\left(2\beta^2-\gamma^2\right)\overline{c}\right]^2}{\left(4\beta^2-\gamma^2\right)^2}$$

$$\bar{\pi}^E = \frac{\beta\left[\alpha(2\beta+\gamma)+\beta\gamma\overline{c}-\left(2\beta^2-\gamma^2\right)c^E\right]^2}{\left(4\beta^2-\gamma^2\right)^2}.$$

Similarly, when the incumbent has low production cost and as $\underline{c}=0$, the second period *equilibrium* prices, quantities and profits in market A simplify to:

$$\underline{p}_{Ad} = \frac{\alpha(2\beta+\gamma)+\beta\gamma c^E}{4\beta^2-\gamma^2}$$

$$\underline{p}^E = \frac{\alpha(2\beta+\gamma)+2\beta^2 c^E}{4\beta^2-\gamma^2}$$

$$\underline{q}_{Ad} = \frac{\beta\left(\alpha(2\beta+\gamma)+\beta\gamma c^E\right)}{4\beta^2-\gamma^2}$$

$$\underline{q}^E = \frac{\beta\left(\alpha(2\beta+\gamma)-(2\beta^2-\gamma^2)c^E\right)}{4\beta^2-\gamma^2}$$

and:

$$\underline{\pi}_{Ad} = \frac{\beta\left[\alpha(2\beta+\gamma)+\beta\gamma c^E\right]^2}{\left(4\beta^2-\gamma^2\right)^2}$$

$$\underline{\pi}^E = \frac{\beta\left[\alpha(2\beta+\gamma)-\left(2\beta^2-\gamma^2\right)c^E\right]^2}{\left(4\beta^2-\gamma^2\right)^2}.$$

4.B Non-negative constraints analysis

From the monopoly analysis in Appendix 4.A.1, if we assume (4.3) and (4.4) monopoly price, quantity and profit in market B for high and low production costs can be rewritten as:

$$\overline{p}_{Bm} = \frac{a_B + \overline{c}}{2} = \frac{ka_A + \overline{c}}{2}, \overline{q}_{Bm} = \frac{a_B - \overline{c}}{2b_B} = \frac{ka_A - \overline{c}}{2b_A}$$

$$\overline{\pi}_{Bm} = \frac{(a_B - \overline{c})^2}{4b_B} = \frac{(ka_A - \overline{c})^2}{4b_A}$$

$$\underline{p}_{Bm} = \frac{a_B}{2} = \frac{ka_A}{2}, \underline{q}_{Bm} = \frac{a_B}{2b_B} = \frac{ka_A}{2b_A} \text{ and } \underline{\pi}_{Bm} = \frac{a_B^2}{4b_B} = \frac{k^2 a_A^2}{4b_A}.$$

When $k < 1$, market A and market B monopoly quantities are non-negative if $\overline{c} \leq ka_A$. Let **M** be the set of parameters' values $\overline{c}$ and k such that $\overline{c} \leq ka_A$. Notice that if $k < 1$, monopoly discriminatory price is higher in market A. This result is expected: a discriminatory monopolist charges a higher price in the less elastic market. When $k < 1$, demand in market A is larger and less elastic than demand in market B. Thus $p^I_{Am} > p^I_{Bm}$ for both types of incumbent's production cost .

If we focus on the duopoly analysis in market A, presented in Appendix 4.A.2, from non-negativity constraints of duopoly *equilibrium* quantities it follows that, for different values of $\overline{c}$, the set of possible c^E changes. We use $\tau = \frac{d_A}{b_A}$, where τ is a measure of the degree of product substitutability, in the following conditions. Therefore, if:

$$\overline{c} < \tfrac{a_A(1-\tau)(2+\tau)}{(2-\tau^2)} \quad \text{then } c^E \in \left[0, \tfrac{a_A(1-\tau)(2+\tau)}{(2-\tau^2)}\right]$$

$$\tfrac{a_A(1-\tau)(2+\tau)}{(2-\tau^2)} \leq \overline{c} \leq \tfrac{a_A\left(1-\tau^2\right)\left(4-\tau^2\right)}{(2-\tau^2)^2} \quad \text{then } c^E \in \left[\tfrac{\left(2-\tau^2\right)\overline{c}-a_A(1-\tau)(2+\tau)}{\tau}, \tfrac{a_A(1-\tau)(2+\tau)}{(2-\tau^2)}\right]$$

We consider **D** to be this set of parameters' values $\overline{c}$ and c^E where duopoly *equilibrium* quantities are non-negative.

4.C Proof of Proposition 4.1

When $(\underline{p}_{Am}, \underline{p}_{Bm}) \in \bar{\mathbf{C}}$, the proof is immediate since $(\underline{p}_{Am}, \underline{p}_{Bm}) \in \underline{\mathbf{C}}$ and thus $\bar{\mathbf{C}} \cap \underline{\mathbf{C}}$ is non-empty. Suppose then that $(\underline{p}_{Am}, \underline{p}_{Bm}) \notin \bar{\mathbf{C}}$. Let (p'_A, p'_B) be a pair of prices in the frontier of the incentive compatibility condition of the high cost incumbent given by condition (4.5), *i.e.*:

$$\bar{\pi}_A\left(p'_A\right) + \bar{\pi}_B\left(p'_B\right) + \delta\bar{\pi}_{Am} = \bar{\pi}_{Am} + \bar{\pi}_{Bm} + \delta\bar{\pi}_{Ad} \Leftrightarrow$$

$$\Leftrightarrow \bar{\pi}_A\left(p'_A\right) + \bar{\pi}_B\left(p'_B\right) - \bar{\pi}_{Am} - \bar{\pi}_{Bm} + \delta\left(\bar{\pi}_{Am} - \bar{\pi}_{Ad}\right) = 0.$$

Let $p'_A < \underline{p}_{Am}$ and $p'_B < \underline{p}_{Bm}$. Notice that:

$$\underline{\pi}_A\left(p'_A\right) + \underline{\pi}_B\left(p'_B\right) - \underline{\pi}_{Am} - \underline{\pi}_{Bm} + \delta\left(\underline{\pi}_{Am} - \underline{\pi}_{Ad}\right)$$

$$= \underline{\pi}_A\left(p'_A\right) + \underline{\pi}_B\left(p'_B\right) - \underline{\pi}_{Am} - \underline{\pi}_{Bm} + \delta\left(\underline{\pi}_{Am} - \underline{\pi}_{Ad}\right) - \left[\bar{\pi}_A\left(p'_A\right) + \bar{\pi}_B\left(p'_B\right) - \bar{\pi}_{Am} - \bar{\pi}_{Bm} + \delta\left(\bar{\pi}_{Am} - \bar{\pi}_{Ad}\right)\right]$$

$$= \underline{\pi}_A\left(p'_A\right) - \bar{\pi}_A\left(p'_A\right) + \underline{\pi}_B\left(p'_B\right) - \bar{\pi}_B\left(p'_B\right) - \underline{\pi}_{Am} + \bar{\pi}_{Am} - \underline{\pi}_{Bm} + \bar{\pi}_{Bm} + \delta\left[\left(\underline{\pi}_{Am} - \underline{\pi}_{Ad}\right) - \left(\bar{\pi}_{Am} - \bar{\pi}_{Ad}\right)\right]$$

$$> \underline{\pi}_A\left(p'_A\right) - \bar{\pi}_A\left(p'_A\right) + \underline{\pi}_B\left(p'_B\right) - \bar{\pi}_B\left(p'_B\right) - \underline{\pi}_{Am} + \bar{\pi}_{Am} - \underline{\pi}_{Bm} + \bar{\pi}_{Bm}, \text{ as } \underline{\pi}_{Am} - \underline{\pi}_{Ad} > \bar{\pi}_{Am} - \bar{\pi}_{Ad}$$

$$> \underline{\pi}_A\left(p'_A\right) - \bar{\pi}_A\left(p'_A\right) + \underline{\pi}_B\left(p'_B\right) - \bar{\pi}_B\left(p'_B\right) - \underline{\pi}_{Am} + \bar{\pi}_A\left(p'_A\right) - \underline{\pi}_{Bm} + \bar{\pi}_B\left(p'_B\right), \text{ as } \bar{\pi}_i \text{ are the monopoly profits}$$

$$= \frac{\bar{c}}{b_A}\left(\underline{p}_{Am} - p'_A\right) + \frac{\bar{c}}{b_B}\left(\underline{p}_{Bm} - p'_B\right).$$

As $p'_A < \underline{p}_{Am}$ and $p'_B < \underline{p}_{Bm}$, then this last expression is positive. Therefore, if we assume condition (4.7), there is always some (p'_A, p'_B) where:

$$\underline{\pi}_A\left(p'_A\right) + \underline{\pi}_B\left(p'_B\right) - \underline{\pi}_{Am} - \underline{\pi}_{Bm} + \delta\left(\underline{\pi}_{Am} - \underline{\pi}_{Ad}\right) > 0.$$

This means that (p'_A, p'_B) is in the interior of $\underline{\mathbf{C}}$ and then $\bar{\mathbf{C}} \cap \underline{\mathbf{C}}$ is non-empty.

Therefore, condition (4.7) is a sufficient condition for the existence of a separating PBE.

4.D Single-crossing condition analysis

Assuming $\underline{\pi}_{Am} - \underline{\pi}_{Ad} > \bar{\pi}_{Am} - \bar{\pi}_{Ad}$ is equivalent to $c^E < \tilde{c}^E$ where:

$$\tilde{c}^E = \frac{\tau\left(\tau^4 - 5\tau^2 + 8\right)\bar{c} + 2a_A\left(1-\tau\right)\left(2+\tau\right)\left(\tau^3 - \tau^2 - 2\tau + 4\right)}{8\left(2-\tau^2\right)}, \text{ using } \tau = \frac{d_A}{b_A}.$$

This single-crossing condition has impact on possible parameter's values of c^E. Thus, let $\tilde{\mathbf{D}}$ be the following set of parameters' values $\bar{c}$ and c^E where if:

$$\begin{array}{rl}
\bar{c} < \frac{a_A(1-\tau)(2+\tau)}{(2-\tau^2)} & \text{then } c^E \in \left[0, \tilde{c}^E\right[\\
\frac{a_A(1-\tau)(2+\tau)}{(2-\tau^2)} \leq \bar{c} \leq \frac{2a_A\left(1-\tau^2\right)\left(4-\tau^2\right)}{(\tau^4-5\tau^2+8)} & \text{then } c^E \in \left[\frac{\left(2-\tau^2\right)\bar{c} - a_A(1-\tau)(2+\tau)}{\tau}, \tilde{c}^E\right[\\
\frac{2a_A\left(1-\tau^2\right)\left(4-\tau^2\right)}{(\tau^4-5\tau^2+8)} < \bar{c} \leq \frac{a_A\left(1-\tau^2\right)\left(4-\tau^2\right)}{\left(2-\tau^2\right)^2} & \text{then } c^E \in \left[\frac{\left(2-\tau^2\right)\bar{c} - a_A(1-\tau)(2+\tau)}{\tau}, \frac{a_A(1-\tau)(2+\tau)}{(2-\tau^2)}\right]
\end{array}$$

Therefore, adding non-negative constraints and single-crossing condition implies restricting the analysis to the parameters' values $\left\{\bar{c}, c^E, k\right\} \in \mathbf{M} \cap \tilde{\mathbf{D}}$.

4.E Low cost incumbent firm's Kuhn-Tucker conditions

Let λ and μ be the Lagrange multipliers associated with $\overline{IC}$ and $\underline{IC}$, respectively. We can construct the Lagrangian function:

$$\begin{aligned}\mathcal{L} &= \underline{\pi}_A\left(\underline{p}_A^*\right)+\underline{\pi}_B\left(\underline{p}_B^*\right)+\lambda\left[\bar{\pi}_{Am}+\bar{\pi}_{Bm}+\delta\left(\bar{\pi}_{Ad}-\bar{\pi}_{Am}\right)-\bar{\pi}_A\left(\underline{p}_A^*\right)-\bar{\pi}_B\left(\underline{p}_B^*\right)\right]\\ &\quad+\mu\left[\underline{\pi}_{Am}+\underline{\pi}_{Bm}+\delta\left(\underline{\pi}_{Ad}-\underline{\pi}_{Am}\right)-\underline{\pi}_A\left(\underline{p}_A^*\right)-\underline{\pi}_B\left(\underline{p}_B^*\right)\right].\end{aligned}$$

Or equivalently:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{b_A}\left(\underline{p}_A^*\right)^2+\left(\frac{a_A}{b_A}\right)\underline{p}_A^*-\frac{1}{b_B}\left(\underline{p}_B^*\right)^2+\left(\frac{a_B}{b_B}\right)\underline{p}_B^*\\ &\quad+\lambda\left[\begin{array}{c}\bar{\pi}_{Am}+\bar{\pi}_{Bm}+\delta\left(\bar{\pi}_{Ad}-\bar{\pi}_{Am}\right)+\\ +\frac{1}{b_A}\left(\underline{p}_A^*\right)^2-\left(\frac{a_A+\bar{c}}{b_A}\right)\underline{p}_A^*+\frac{a_A}{b_A}\bar{c}+\frac{1}{b_B}\left(\underline{p}_B^*\right)^2-\left(\frac{a_B+\bar{c}}{b_B}\right)\underline{p}_B^*+\frac{a_B}{b_B}\bar{c}\end{array}\right]\\ &\quad+\mu\left[\begin{array}{c}\underline{\pi}_{Am}+\underline{\pi}_{Bm}+\delta\left(\underline{\pi}_{Ad}-\underline{\pi}_{Am}\right)+\\ +\frac{1}{b_A}\left(\underline{p}_A^*\right)^2-\left(\frac{a_A}{b_A}\right)\underline{p}_A^*+\frac{1}{b_B}\left(\underline{p}_B^*\right)^2-\left(\frac{a_B}{b_B}\right)\underline{p}_B^*\end{array}\right].\end{aligned}$$

The Kuhn-Tucker conditions, when all $\underline{p}_A^*$ and $\underline{p}_B^*$ must be non-negative, are given by:

$$\frac{\partial\mathcal{L}}{\partial\underline{p}_A^*}=-\frac{2}{b_A}\underline{p}_A^*+\frac{a_A}{b_A}+\lambda\left(\frac{2}{b_A}\underline{p}_A^*-\frac{a_A+\bar{c}}{b_A}\right)+\mu\left(\frac{2}{b_A}\underline{p}_A^*-\frac{a_A}{b_A}\right)\leq 0;$$
$$\underline{p}_A^*\geq 0;\quad \underline{p}_A^*\frac{\partial\mathcal{L}}{\partial\underline{p}_A^*}=0$$

$$\frac{\partial\mathcal{L}}{\partial\underline{p}_B^*}=-\frac{2}{b_B}\underline{p}_B^*+\frac{a_B}{b_B}+\lambda\left(\frac{2}{b_B}\underline{p}_B^*-\frac{a_B+\bar{c}}{b_B}\right)+\mu\left(\frac{2}{b_B}\underline{p}_B^*-\frac{a_B}{b_B}\right)\leq 0;$$
$$\underline{p}_B^*\geq 0;\quad \underline{p}_B^*\frac{\partial\mathcal{L}}{\partial\underline{p}_B^*}=0$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\lambda} &= \bar{\pi}_{Am}+\bar{\pi}_{Bm}+\delta\left(\bar{\pi}_{Ad}-\bar{\pi}_{Am}\right)-\bar{\pi}_A\left(\underline{p}_A^*\right)-\bar{\pi}_B\left(\underline{p}_B^*\right)\geq 0;\quad \lambda\geq 0;\quad \lambda\frac{\partial\mathcal{L}}{\partial\lambda}=0\\ \frac{\partial\mathcal{L}}{\partial\mu} &= \underline{\pi}_{Am}+\underline{\pi}_{Bm}+\delta\left(\underline{\pi}_{Ad}-\underline{\pi}_{Am}\right)-\underline{\pi}_A\left(\underline{p}_A^*\right)-\underline{\pi}_B\left(\underline{p}_B^*\right)\leq 0;\quad \mu\leq 0;\quad \mu\frac{\partial\mathcal{L}}{\partial\mu}=0.\end{aligned}$$

4.F Characterization of $\mathbf{S_1}$, $\mathbf{S_2}$ and $\mathbf{S_3}$

Consider $\tau = \frac{d_A}{b_A}$ which measures the degree of product substitutability. For the following different subsets of parameters' values $\left\{\bar{c}, c^E, k\right\}$, the solution is:

- Sol_1 for all $\delta \in]0,1]$, when $\left\{\bar{c}, c^E, k\right\} \in \mathbf{S_1}$ and this subset is given by:

$\mathbf{S_1}$	$\left\{\bar{c}, c^E, k\right\} \in \mathbf{M} \cap \tilde{\mathbf{D}}$
	$c^E \geq \hat{c}^E$
	$k < 1$

- Sol_1 for $0 < \delta \leq \hat{\delta}$, Sol_2 for $\hat{\delta} < \delta \leq \check{\delta}$ and Sol_3 for $\check{\delta} < \delta \leq 1$, when $\left\{\bar{c}, c^E, k\right\} \in \mathbf{S_3}$ and this subset is given by:

$\mathbf{S_3}$	$\left\{\bar{c}, c^E, k\right\} \in \mathbf{M} \cap \tilde{\mathbf{D}}$
	$c^E < \check{c}^E$
	$k \leq \check{k}$

- Sol_1 for $0 < \delta \leq \hat{\delta}$ and Sol_2 for $\hat{\delta} < \delta \leq 1$, when $\left\{\bar{c}, c^E, k\right\} \in \mathbf{S_2}$ and this subset is given by:

$\mathbf{S_2}$	$\left\{\bar{c}, c^E, k\right\} \in \mathbf{M} \cap \tilde{\mathbf{D}}$
	$c^E < \hat{c}^E$
	$k < 1$
	$\left\{\bar{c}, c^E, k\right\} \notin \mathbf{S_3}$

where:

$$\hat{c}^E = \frac{\left(a_A(\tau-1)(2+\tau) - \left(2-\tau^2\right)\bar{c}\right)}{\tau} + \frac{\sqrt{(2+\tau)(1-\tau)\begin{pmatrix} -(1+\tau)(2+\tau)(2-\tau)^2\bar{c}^2 \\ +2a_A\left(-\tau^4+\tau^3-2\tau^2-4\tau+8\right)\bar{c} \\ +a_A^2(1+\tau)(2+\tau)(2-\tau)^2 \end{pmatrix}}}{2\tau},$$

$$
\begin{aligned}
\check{c}^E \quad = \quad & \frac{2\left(-a_A(1-\tau)(2+\tau)-\left(2-\tau^2\right)\bar{c}\right)}{\tau} \\
& + \frac{\sqrt{\begin{array}{c}\left(2a_A^2\left(1-\tau^2\right)\left(2-\tau^2\right)^2\right)k^2-4a_A\left(1-\tau^2\right)\left(2-\tau^2\right)^2\bar{c}k \\ +\left(\tau^2-1\right)\left(4-\tau^2\right)^2\bar{c}^2+a_A^2\left(1-\tau^2\right)\left(4-\tau^2\right)^2 \\ +2a_A(\tau-1)(2+\tau)\left(-8+4\tau+2\tau^2-\tau^3+\tau^4\right)\bar{c}\end{array}}}{2\tau}
\end{aligned}
$$

and

$$
\check{k} = \frac{\sqrt{\begin{array}{c}2(1+\tau)^2\left(4-\tau^2\right)^2\bar{c}^2+2a_A^2(1+\tau)^2\left(4-\tau^2\right)^2 \\ +4a_A(1+\tau)(2+\tau)\left(8-4\tau-2\tau^2+\tau^3-\tau^4\right)\bar{c}\end{array}}}{2a_A(1+\tau)\left(4-\tau^2\right)} - \frac{\bar{c}}{a_A}.
$$

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