A Work Project, presented as part of the requirements for the Award of a Master's degree in Finance from the Nova School of Business and Economics.

Banco Invest Consulting Project
«Delta-Gamma Value-at-Risk model for - »

Jannik Joel Wehrle (51221) - Portfolio of Altiplano options

Work project carried out under the supervision of:

Daniele D'arienzo

11/01/2022

Abstract (100 words maximum)

Banco Invest offers various over-the-counter (OTC) derivatives to institutional clients as part of its structured investment solutions. These derivatives are managed within the bank's Proprietary Trading Book. The focus of this consulting project is developing a Delta-Gamma Value-at-Risk (VaR) model that Banco Invest can implement to actively manage its equity derivative portfolio`s underlying risks. The first part contains the estimation of the portfolio delta and gamma. The second part consists of the quadratic approximation to calculate the portfolio standard deviation. In the last section, the authors calculate the Delta-Gamma Value-at-Risk and provide recommendations to Banco Invest.

Keywords: Value-at-Risk, Autocall option, Indicap option, Altiplano option, Call digital option, Capital protected option, Portfolio Delta, Portfolio Gamma, Delta-Gamma Value-at-Risk

This work used infrastructure and resources funded by Fundação para a Ciência e a Tecnologia (UID/ECO/00124/2013, UID/ECO/00124/2019 and Social Sciences DataLab, Project 22209), POR Lisboa (LISBOA-01-0145-FEDER-007722 and Social Sciences DataLab, Project 22209) and POR Norte (Social Sciences DataLab, Project 22209).

Contents

1 Value-at-Risk – Group part

Market risk describes the risk of a possible loss in a risk position due to collective adverse movements of market rates and prices. It is one of the most critical risks for institutions that actively trade in financial markets; quantifying and monitoring this risk is crucial for allocating capital and reserves needed to cover potential losses and assess their overall solvency. Market risks are determined by institutions using standard procedures or internal risk models; one of these procedures is the Value-at-Risk model. (Deutsche Bundesbank 2022)

1.1 Defining Value-at-Risk

The Value-at-Risk expresses the maximum potential loss, in absolute terms or as a percentage in the respective currency the asset is held, that results under normal market conditions from an adverse movement in the relevant market of an investment over a specified time horizon (H) at a given degree of confidence (α) during a fixed holding period of a risk position. The estimated maximum potential loss of the model, the VaR estimate, is only expected to be exceeded (1-α) % of the time. (Castellacci and Siclari 2003, pp. 531-532) (Fallon 1996, p. 2) The time horizon of interest for a VaR estimate can be one day or even months and is determined by the nature of the portfolio. The horizon should correspond to the most prolonged period needed for an orderly liquidation or the time to hedge an investment portfolio. (Bodie, Kane, and Marcus 2021, p. 138) The VaR estimate's horizon is determined by the liquidity profile of the assets in the underlying investment portfolio; the length relates to the time needed to sell these assets at average transaction volumes so that they have little impact on the market. Since the market impact of the liquidation scenario is not disregarded when choosing the horizon, the VaR estimate will be an estimate of a realizable loss and not only a loss on paper. (Wilmott 1998, p. 548) The confidence (α) level for a VaR estimate corresponds to the institution's risk profile, determined by its degree of risk aversion or regulatory requirements. (Fallon 1996, p. 2)

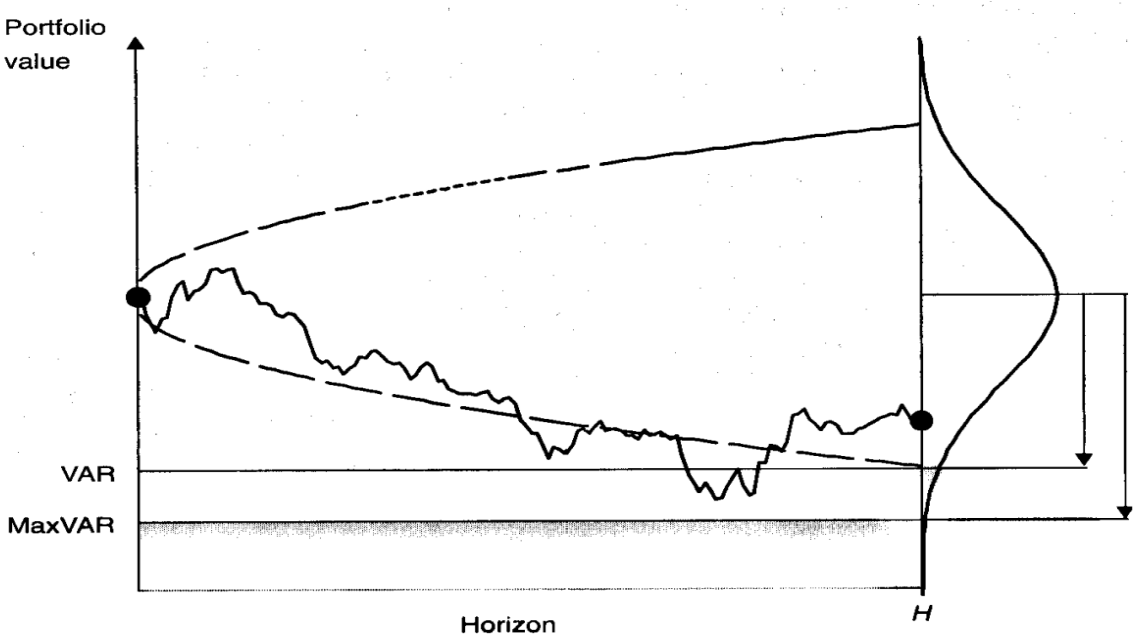


Figure 1: Development of VaR over time horizon H (Jorion 2007, p. 118)

A VaR calculation applies to all types of risky assets and can be applied to a single position and a whole portfolio of risky assets. Assessing VaR helps institutions evaluate the profitability of an investments in relation to the risk and identify investments with a higher-than-acceptable risk profile, allowing them to make changes or liquidate such investments. The VaR is used for active and passive risk measurement and defensive risk control. Ideally, it suits financial and non-financial institutions that engage in proprietary trading with significant exposure to market risks. (Jorion 2007, pp. 379-389) VaR estimates typically focus on 'tail events' where liquidity and large jumps are essential, as illustrated in ***Appendix 1*** below. (Wilmott 1998, p. 337) Therefore, confidence levels are typically set at 95%, 97.5%, and 99%. (Wilmott 1998, p. 547) An overview of which confidence levels translate into which z statics of the confidence interval can be found in ***Appendix 2***. The VAR statistic on portfolio losses is defined as a one-sided confidence interval:

$$Prob\left[\Delta\tilde{P}(\Delta t, \Delta\tilde{x}) > \ -VAR\right] = 1 - \alpha \tag{1}$$

In the above equation, $\Delta\tilde{P}(\Delta t, \Delta\tilde{x})$ stands for the change in the value of a portfolio that results from a function consisting of the forecasting period Δt and the vector $\Delta\tilde{x}$ of the random

variables, with α being the confidence level. The equation can be interpreted as the portfolio's value will not fall by more than VAR over Δt number of trading days with α % confidence. (Fallon 1996, p. 2) The degree of complexity and the computational requirements of the calculation of a VaR estimate depends in particular on how the price of the instrument changes in relation to the underlying. ***Appendix 3*** depicts the two different relationships. (Romano 2017) The calculation of a VaR estimate for non-linear (i.e., derivatives) assets is more complex than for a linear asset (i.e., a stock or bond). In the context of an option: nonlinearity implies that a price movement in the underlying asset causes a non-linear change in the option price. There are three major methodologies to calculate Value-at-Risk, the historical approach, the parametric or model-building approach, and performing a Monte Carlo simulation. ***Figure 2*** below provides an overview of the different methodologies and their advantages and disadvantages. (Hull 2021, pp. 293-297 & 317-340)

Type	Description	Advantages	Disadvantages
Historical	Estimates VaR using past distribution of returns to predict future returns	- Easy way to calculate VaR - Takes into account possible skeweness and fat tails - Accurate for non-linear products - No distributional assumptions necessary	- Assumes future returns dependend on the past (impractical) - Large amount of daily rate history required - Slow reaction to recent market events
Parametric	Estimates VaR using prespecified variables (volatility & correlation)	- Quick and easy to compute - Accurate for simple & linear products	- Assumption of normal distribution impractical - Less quick and accurate for non-linear derivatives
Monte Carlo	Estimates VaR by simulating random scenarios	- Accurate for linear & non-linear products - Flexibility to choose different distributions - Flexibilty on the choice of variables - Outputs full distribution of potential product values	- Massive computational power required to revalue the portfolio in each scenario - Accuracy dependend on number of simulation performed

Figure 2: Overview of different approaches for VaR calculation (Hull 2021, pp. 293-297 & 317-340)

1.2 Pitfalls and limitations of Value at Risk

Despite the widespread use of the Value-at-Risk model, it has several drawbacks that will be briefly discussed in the following. First and foremost, all methods require making assumptions and using them as inputs for the mode; this can result in different outcomes even if the same

modelling approach is used. Assumptions have to be made, e. g. about the applicable horizon and confidence level and the appropriate number of simulations. (Jorion 2007, pp. 542-557) Furthermore, all methods rely to some extent on historical data as a proxy to forecast future estimates. What has happened in the past does not necessarily imply that it will happen again in the future, so that estimation can be Inaccurate. (Jorion 2007, pp. 542-557) Second, there is yet to be an industry-wide standard to model VaR. The different approaches and models to calculate VaR can also lead to different estimates for the same portfolio. Hence, the correct interpretation is vital. (Jorion 2007, pp. 542-557) This brings us to the next limitation: a VaR estimate is calculated assuming normal market conditions, meaning extreme and rare events, such as so-called black swans, are not considered by the estimate. Because VaR only allows the risk manager to make statements about which value will not be exceeded with what degree of certainty, it does not tell anything about the worst outcome in case the VaR number is ex (Hull 2018, pp. 273-274) Additionally, the traditional VaR disregards intervening losses. These occur when the portfolio's value falls below VaR during the time horizon but eventually rises above it at the end of it. This can be an essential aspect for management if the portfolio is marked to market daily and faces potential margin calls that could result in liquidation in the worst-case scenario. (Jorion 2007, pp. 117-119) A VaR estimate provides the "big picture" of what is at risk regarding market risk effects. However, as it only accounts for this specific risk type, it has a narrow focus on what is really at risk. There are also risks which are not incorporated in the VaR framework, commonly referred to as "risks not in Value-at-risk" (RNIV): This can result in the actual Value at Risk of an investment being much higher than what the VaR model is predicting when capturing many of the other existing risk variables such as (geo-)political risks, liquidity risks, and regulatory risk. (Jorion 2007, pp. 542-557)

2 The "Greeks" - Group part

In option pricing, as well as for other derivatives, the "Greeks" are commonly used to measure

the sensitivity of a derivative's value to factors that might affect the price of an options contract. ***Appendix 4*** gives an overview of the existing Greeks and their definitions. (Leoni 2014) Within the frame of this work, the focus will be set on two risk metrics, delta (Chapter 3.1) and gamma (Chapter 3.2) risk, in relation to option pricing, as they are the most fundamental.

2.1 Delta Risk

The delta, designated with the symbol Δ, is the first-order partial derivative of the option pricing function c with respect to the underlying asset S. Therefore, it expresses the sensitivity of the option contract's price to changes in the price of the underlying asset while leaving all else constant (ceteris paribus). (Taleb 1997, p. 224) (Bouzoubaa and Osseiran 2010, p. 66)

$$\Delta = \frac{\partial c}{\partial S} \tag{2}$$

For vanilla options, the delta for long calls and short puts on standard options varies between 0 and 1. Vice versa, short calls and long puts have a delta ranging between 0 and -1. Graphically expressed is it the slope of the curve that links the option price to the underlying asset price. The higher the slope, the higher the delta and the more the derivative contract will change in response to price fluctuations of the underlying asset. ***Figure 3*** below depicts the change in delta with respect to the Strike price K and the time to maturity T for a European call option. With the option increasingly getting out of the money (OTM), a higher Strike K, and/or the option approaching its maturity date T, the delta tends to move towards 0. Conversely, with lower Strike K, the option being more in the money (ITM), and/or longer time until maturity T, delta approaches 1. (Hilpisch 2015, p. 78) The most significant change in delta can be observed with the option being at the money (ATM), S = K, close to its maturity date T. This is because theoretically, with the option being ATM a few seconds before it matures, one small move in either direction would result in the option being either in the money or out of the money, hence the considerable variation in delta. (Hilpisch 2015, p. 78)

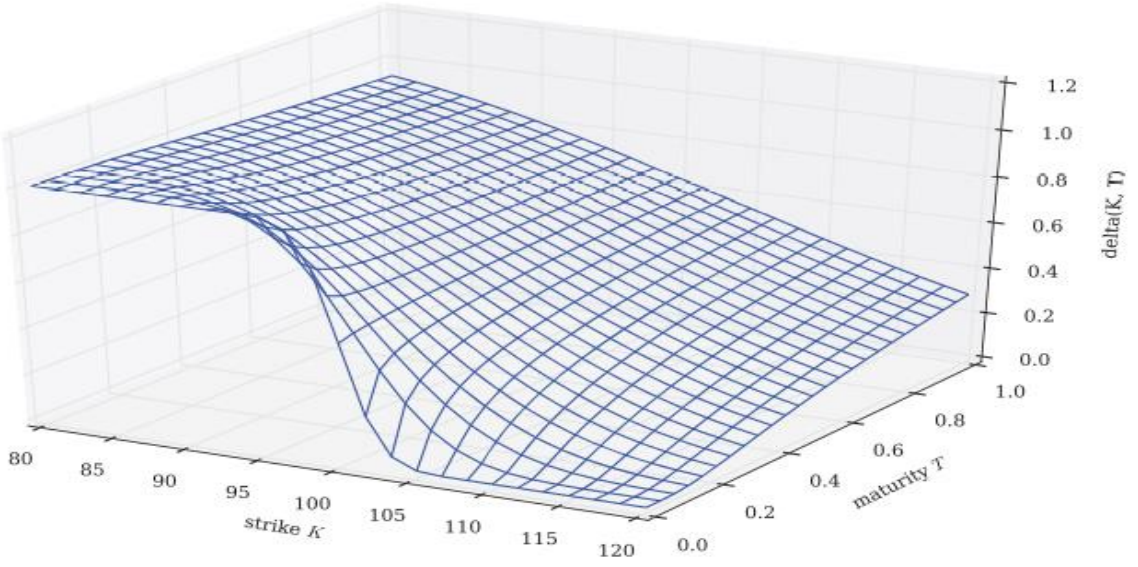


Figure 3: Delta of a European Call Option (Hilpisch 2015, p. 78)

Delta risk can be hedged to obtain a neutral position ($\Delta = 0$). How this can be achieved for a portfolio of derivatives will be explained in more detail in section 3.3, Hedging the Greeks.

2.2 Gamma Risk

For minor variations in the price of the underlying asset, delta proves to be good at estimating the change in the option's price. However, as soon as price changes become more severe, delta is extremely sensitive to changes in the underlying asset's price. This is because delta graphically represents a linear estimate for a non-linear option function. Hence, the actual option value might significantly differ from the proportion predicted by delta. (de Weert 2008, pp. 14-16) Gamma, Γ, measures by how much or how often a position or a portfolio of options needs to be re-hedged to maintain a delta-neutral position: it expresses by how much the Delta might change if the price of the underlying changes. It is the second-order derivative of the option pricing function c with respect to the underlying asset S.

$$\Gamma = \frac{\partial^2 c}{\partial^2 S} \tag{3}$$

The more curvature the option function entails, the higher the gamma and the more sensitive

the delta is towards changes in the underlying's price. An increase in the underlying's price could significantly increase the delta and vice versa for a low gamma. Considering plain vanilla options, the gamma is always positive for long positions, whereas for short positions, it is negative. (Bouzoubaa and Osseiran 2010, p. 72) ***Figure 4*** below shows that the gamma value is stable for most of the option's life as it hovers near zero. The most notable value changes in gamma happen around ATM options close to maturity. As previously stated in the preceding section, it is for at-the-money options close to maturity where one move in either direction has the most significant influence on delta as it determines whether the option is exercised. Hence, the high value in gamma. (Yen Jerome and Lai 2015, pp. 84-85).

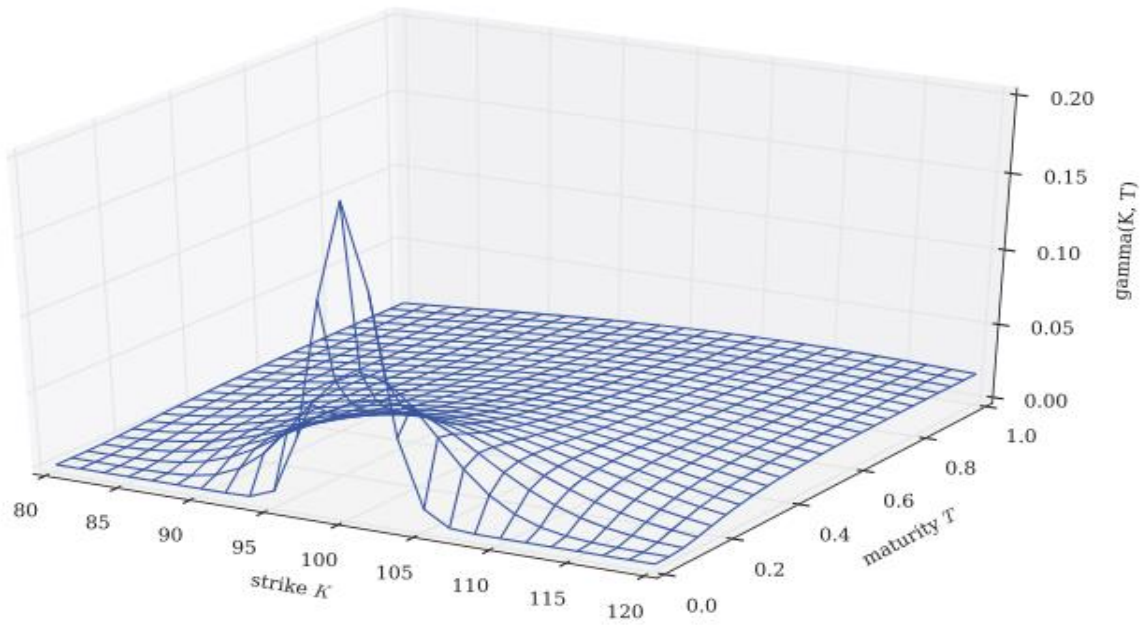


Figure 4: Gamma of a European Call option (Hilpisch 2015, p. 79)

How gamma is incorporated when hedging the respective portfolio's VaR will be explained in more detail in the next section.

2.3 Hedging the Greeks

As previously described, a portfolio's sensitivity to such is captured by the "Greek letters". The risk framework captures thresholds for each to ensure that these risks stay within the company's

tolerance. Exceeding the limits initializes a process known as hedging. This is where counter positions in the market are established to ensure that the exposure to a particular risk factor stays within its predefined limit. In the following, it will be presented how a portfolio is hedged against delta and gamma. (Hull 2018, p. 161) Hedging delta consists of establishing a counter position equal to Δ amount of the underlying. By combining the existing portfolio and the hedging trade, the new portfolio's exposure to delta is neutralized. (Hull 2018, pp. 161-162) For linear products, hedging delta turns out to be static as it protects against both small and large changes in the value of the underlying. Further, once a linear hedge is implemented, there is no need to adjust it over time. The delta for a linear portfolio stays constant. (Hull 2018, pp. 163-164) Neutralizing delta exposure for non-linear products such as options proves to be a more complex procedure due to the non-linear relationship between the price of the underlying and the options contract. As mentioned earlier in this work, eliminating a portfolio's delta only offers protection from small fluctuations in the price of the underlying. Additionally, once it is set up, the delta hedge has to be adjusted frequently, also known as dynamic hedging or "rebalancing". This is because Delta constantly evolves throughout a non-linear product's lifetime. (Hull 2018, pp. 165-168) In practice, rebalancing is costly as, e.g., hedging a long position on an option involves buying the underlying when its price increased and selling it when it dropped to consistently create a synthetical position opposite of that to neutralize the option's delta. This is usually reflected in the premiums that option buyers have to pay. (Hull 2018, p. 169) With more significant changes in the prices of the underlyings, a portfolio's gamma comes into play. There are two ways of adjusting for the additional gamma exposure of a non-linear portfolio that will be briefly described below. (Hull 2018, pp. 169-170) Firstly, the portfolio is made gamma neutral by trading options with opposite gammas on the same underlyings as the options in the existing portfolio. Non-linear products are needed as linear products do not have exposure to gamma. By doing this, the new and combined portfolio's delta also changes and would have to be re-adjusted by trading opposite positions in the underlyings

(Hull 2018, pp. 170-171) Implementing this in practice can be challenging as trading non-linear derivatives in the amounts needed often is impossible. Further, re-adjusting for the new delta of the combined portfolio is costly as it involves many transactions. (Hull 2018, p. 177) However, as described earlier, it makes economically more sense to see the gamma as a determinant of how often a portfolio needs to be re-hedged. In general, a portfolio with larger gamma would imply more frequent delta neutralization, whereas a smaller gamma results in less often adjustments to the portfolio, as changes in delta only tend to be small. (Hull 2018, pp. 169-170) Banco Invest hedges its equity derivatives portfolio with underlyings (delta neutralization) rather than options (gamma neutralization). The Bank does not take directional market risk, keeping the difference between the deltas (theoretical quantities) and the quantities held in the portfolio as close to zero as possible. These portfolio quantities are adjusted daily, at 30-minute intervals, based on market conditions, namely the evolution of the underlying shares.

3 Value-at-Risk for a Derivatives Portfolio - Group part

To begin with, calculating Value-at-Risk for a single asset is a straightforward process. Assuming linearity in the change of the portfolio's value to changes in the underlying and normally distributed returns, VaR is calculated as follows:

$$VaR = w_i S_i \left(\mu\, \delta t - \sigma_i \, (\delta t^{\frac{1}{2}}) \, \alpha(1-c) \right) \tag{4}$$

where w_i is the quantity of the asset i owned with price S_i. This is multiplied by the asset's drift over a predefined time horizon δt, with $\alpha(1-c)$ being the inverse cumulative distribution function of the standard normal distribution. This process is called delta approximation. (Wilmott 1998, pp. 548-550) Regarding a portfolio of assets, the calculation of VaR becomes more complex. First, the volatilities and covariances of all assets in the portfolio have to be computed. If this is done, the formula to calculate the VaR of a portfolio with M assets

consisting of w_i amount of asset i and w_j amount of asset j is:

$$VaR_{Portfolio} = -M\left(\alpha(1-c)(\delta t^{\frac{1}{2}})\sqrt{\sum_{i=1}^{M}\sum_{j=1}^{M} w_i w_j \sigma_i \sigma_j \rho_{ij}}\right) \quad (5)$$

with σ_i being the volatility of asset i and ρ_{ij} the correlation between asset i and j. (Wilmott 1998, pp. 551) Estimating VaR for a portfolio of derivatives, as mentioned earlier, the delta approximation would only be sufficient for portfolios where the underlyings show small movements in price. This is because the relationship between the portfolio's value and price changes in the underlyings can no longer be regarded as linear. For non-linear portfolios, the sensitivity to gamma additionally has to be considered. This is visually demonstrated in ***Figure 5*** below. It depicts the relationship between the price of an underlying asset to the corresponding value of a long call option on the same. While the underlying's price function is normally distributed, the option has a positively skewed probability distribution with a smaller tail on the left. (Hull 2018, pp. 333-334) This violates the initial premise that probabilities are normally distributed. If VaR were calculated based on this assumption, it would be excessively high. As a result, approximations for the portfolio's sensitivity to changes in the underlyings need to be reevaluated. (Wilmott 1998, pp. 550-551)

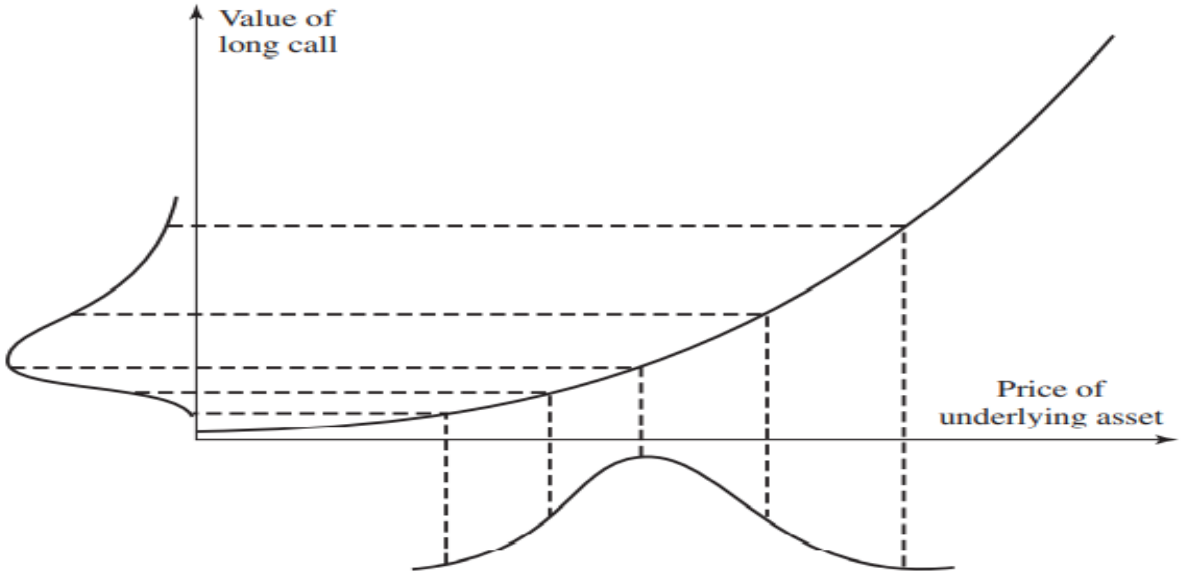


Figure 5: Translation of an Asset's normal probability distribution into that of a long call option (Hull 2018, p. 333)

To recapture, with larger swings in the prices of the underlyings of an options portfolio, the previous delta approximation to calculate VaR turns out to be inappropriate. A better estimation is achieved by incorporating the portfolio's sensitivity to gamma. Gamma exposure is particularly challenging as a second-order approximation is required. (Wilmott 1998, p. 551) This will be shown below. Assume a portfolio M consisting of a single option on an asset with price S. The change in the value of the portfolio δM compared to changes in the price of the underlying δS can be expressed as follows:

$$\delta M = \frac{\partial P}{\partial S}\delta S + \frac{1}{2}\frac{\partial^2 P}{\partial S^2}(\delta S)^2 + \frac{\partial P}{\partial \sigma}\delta t + \cdots \tag{6}$$

This can ultimately be reformulated into:

$$\delta M = \Delta\sigma S\, \delta t^{\frac{1}{2}}\, \phi + \delta t\left(\Delta\mu S + \frac{1}{2}\Gamma\sigma^2 S^2\phi^2 + \Theta\right) + \cdots \tag{7}$$

where Θ is the time drift of the option (Theta). (Wilmott 1998, p. 551) The quadratic term, the portfolio's exposure to gamma, is of specific interest above. ***Figure 6*** shows three different distribution functions. The distribution of the underlying with a standard deviation of $\sigma S\ \partial t^{\frac{1}{2}}$ is considered to be normal. The projected distribution for the change in the value of the options

portfolio according to the delta approximation. It is normally distributed with a standard deviation of $\Delta\sigma S\,\partial t^{\frac{1}{2}}$. Finally, the options portfolio's distribution using the delta-gamma approximation. (Wilmott 1998, pp. 551-552)

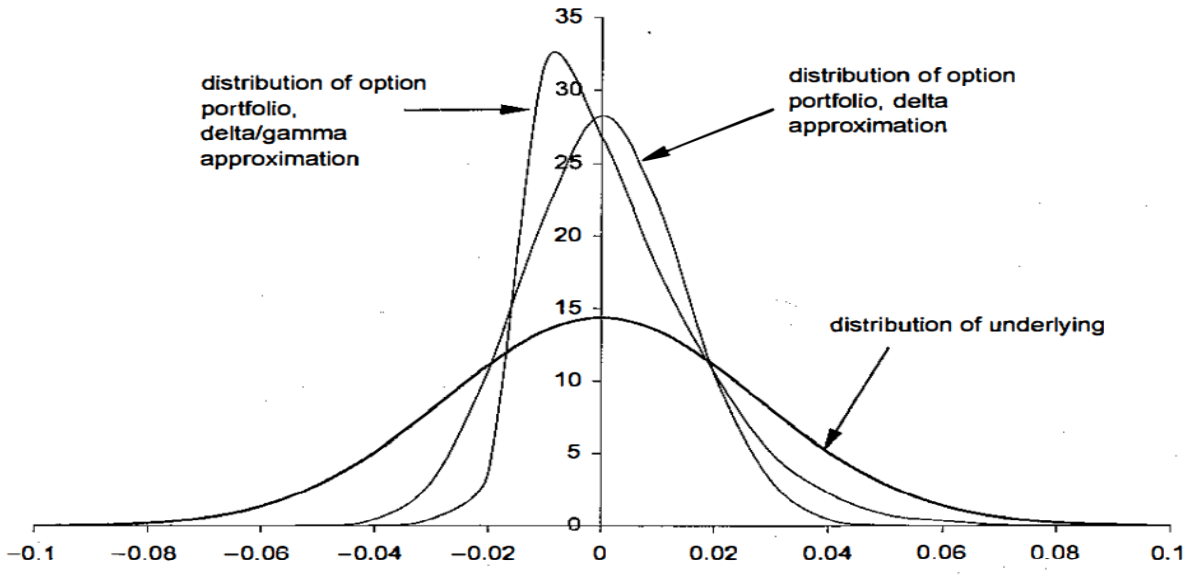


Figure 6: Relationship of an asset price's normal distribution to the distribution of an option portfolio according to the delta as well as the delta-gamma approximation (Wilmott 1998, p. 552)

By looking at the three different distributions, it is evident that the one for the delta-gamma approximation is not normally distributed compared to the other two. (Wilmott 1998, pp. 551-552)

4 Methodology used in Python - Group part

In the following, the Assumptions used to calculate the Delta-Gamma VaR in Python, as well as the fundamental parts of the code, are presented and explained. As the basis for all calculations of the various input statistics of the VaR model, the authors assume one year consisting of 252 trading days. Because of their ease of use for time series modelling, such as symmetry, time-additivity, and the log-normal distribution assumption, the various underlyings performances are transformed into logarithmic returns. Next, each option's volatility is calculated using equally weighted implied volatilities of the option's underlyings. In the absence

of implied volatility, the underlying's historical volatility on a 30-day basis is used. Furthermore, to determine the correlation, variance, and covariance of the different underlyings, a maximum lookback window of 2 years is assumed, the same as the option's time to maturity on the trade date. From there on, for each day that has progressed, the option's remaining time to maturity is used to calculate the above statistics until a predefined minimum of 30 days was reached. Below this, correlation, variance, and covariance are calculated on a 30-day basis until the option matures. At this point, it is referred to ***Appendix 5-6*** for the code example. The options in Banco Invest's portfolio are valued as of **30/06/2022** using Monte Carlo simulations. The first step of Monte Carlo involved calculating the geometric Brownian Motion. In finance, this is a stochastic process to model random behavior over a specific time frame (δt) that consists of two main components, drift, and a randomly generated variable. (Yan 2017, pp. 421-428) Drift indicates the direction of an asset's historical returns, allowing predictions on an asset's expected return. It is calculated as shown in ***equation (8)*** using the same receding time horizon as explained for the underlying's statistics, except for the time series' minimum requirement of 30 days.

$$Drift = \left(Mean\ (stock\ returns) - \frac{Variance\ (stock\ returns)}{2} \right) * \delta t \qquad (8)$$

Where underlyings are expected to pay dividends, the drift is adjusted further, as demonstrated in ***Appendix 7***. The next step is to obtain a random number by multiplying an asset's historical standard deviation with a random, standard normally distributed variable *(Z ([Rand (0;1)]))*.

$$Random\ variable = \left(Std.Dev. * Z([Rand(0;1)]) \right) * \sqrt{\delta t} \qquad (9)$$

As a result, the equation for predicting the future value of an asset *(S_{t+1})* sums up to the following:

$$S_{t+1} = S_t * e^{Drift\ +\ Random\ variable} \qquad (10)$$

However, when pricing options comprised of baskets of underlyings, Cholesky Decomposition is performed as an extension of the Monte Carlo simulation to account for the correlation

aspects between the various reference assets. A brief explanation of an example decomposition will be provided below. ***Appendix 8*** contains the code for the Cholesky decomposition performed for the different options. Assume a $2 * 2$ symmetric, positive definite correlation matrix Σ, where ρ is the correlation between $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$.

$$\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \tag{11}$$

The correlation matrix can then be decomposed into a $2 * 2$ lower triangular matrix L, where $LL^T = \Sigma$. (Wilmott 1998, pp. 682-683) This appears to be as follows:

$$L = \begin{pmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix} \tag{12}$$

Following the generation of L, the random variables with desired correlation can be expressed as LZ, where Z is a column vector of the independent standard normal random variables:

$$Z = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} \tag{13}$$

As a result, by setting $XL = Z$, we can sample from a bivariate normal distribution, indicating that: (Yen Jerome and Lai 2015, pp. 99-100)

$$X_1 = Z_1 \tag{14}$$

$$X_2 = \rho * Z_1 + \sqrt{1-\rho^2} * Z_2 \tag{15}$$

To generate a sufficient sample of possible future asset values for the different underlyings to calculate the option's payoffs appropriately, 200.000 simulations are run. Following this, the averaged payoffs are discounted using the respective's maturity Euribor 3-month forward. Where no forward for the maturity of the option's payoffs is readily available, linear interpolation is performed to compute the discount rate for the respective maturity's payoff, as shown in ***Appendix 9-10***. Further, each underlying's delta is estimated by changing its price by 1%, while leaving the other's prices constant, and calculating the new price of the option. The difference in both derivative prices is then divided by the relative changes in the prices of the

underlying. The option's delta is estimated as the weighted average of the underlying's deltas, assuming an equally weighted portfolio of underlyings. To calculate gamma, the above calculation is done a second time to get the change in delta. The difference in both deltas is then divided by the relative adjustment to obtain the gamma value. The equations used and the respective code for this can be found in ***Appendix 12-26***. In terms of VaR, the confidence level was set to 99,9 %. Calculations are performed initially for a one-day time horizon and then later multiplied by the square root of 252 to get the annualized VaR, as this is the requirement from the risk management department at Banco Invest. Detailed calculations performed for this in Python can be found in ***Appendix 26***.

5 Altiplano – Individual part (Jannik Wehrle)

An Altiplano option is an exotic multi-asset derivative (basket option) that belongs to the commonly referred "*Mountain Range*" options group. Other derivatives of this class are the so-called *Annapurna*, *Everest*, *Atlas* and *Himalaya* options. These derivatives were created and introduced in 1998 by the French bank Société Générale. The bank intended to create products which replicate specific portfolio strategies. The underlying assets in this basket option typically range from a low number of 4 to 20. (Gharavi 2010, p.674)

Usually, an Altiplano option also features characteristics of a Barrier Option. Therefore, this kind of derivative has two parameters that determine the remuneration of the options contract. First is the typical vanilla-type payoff when the barrier event occurs; second is a coupon payment *C* in case the barrier event does not occur. The barrier event usually means that at least one underlying asset must reach a certain price level. (Gharavi 2010, p.674)

To be more precisely, the expected payoffs for a basket of n underlying's can be expressed by the following function: *S(t)* represents the price of the underlying asset $i = 1 \ldots, n$ at time ***t***, with 0 being the effective date of the altiplano option and T its maturity date.

$$\max\left(\sum_{i=1}^{n}\frac{Si(T)}{Si(0)}-K,0\right)I_{\dot{B}}+(1-I_{\dot{B}})C \qquad (17)$$

$I_{\dot{B}}$ equals the value of one if the barrier event about barrier level B occurs. Otherwise, $I_{\dot{B}}$ takes the value of zero, in which case the altiplano option would pay the predetermined coupon payment. (Gharavi 2010, p.674)

Regarding the Altiplano Option type from Banco Invest, it can be highlighted that the structure of the derivative is slightly different. First, Banco Invest's Altiplano product is a structured deposit for investors who do not want to take any capital risks. The Altiplano-Type of Banco Invest is a derivatives product with guaranteed capital. For this option type, risks, as well as rewards, are predetermined. The option's remuneration depends on the performance of the underlying assets in the basket and the magnitude of the option's maximal possible payoff (Cap) and Coupon.

The underlying basket consists of five equity securities. In order to payout the Cap, the barrier event is a condition that all of the individual underlying stocks have to reach a certain price level equal or higher than the strike price. If only four of the five underlying assets are equal to or above the strike price, the investor will receive a coupon payment lower than the predetermined cap payment. If none of these conditions are met, the investor will receive his initial investment back. Regarding the last scenario, it is also possible for the investor to receive a floor payout in addition to his investment (Banco Invest 2022, pp. 4-5)

With being the closing price of the underlying Equity i on the start date of the product and P_i^n *the closing price of the underlying Equity i on the observation date of the product:*

1. If all underlying equity securities have a closing price at expiration greater than the strike price, the holder will receive the predetermined Cap as remuneration. In this case, the investor receives the maximum possible remuneration with this product.

$$P_i^n \geq P_0^n \; with \; n \in 1,2,3,4,5 \tag{17}$$

$$Remuneration = Deposit\ Amount\ x\ Cap \tag{18}$$

2. If four of the underlying equity securities have a closing price at maturity greater than the strike, the holder will receive the Coupon as remuneration (with $Coupon < Cap$)

$$P_i^n \geq P_0^n \; with \; n \in 1,2,3,4 \tag{19}$$

$$Remuneration = Deposit\ Amount\ x\ Coupon \tag{20}$$

3. If none of the previous two cases occurs: The initial deposited Amount will be remunerated. As the investor receives his deposited Amount back, his only loss, in this case, is the paid option premium.

$$Remuneration = Deposit\ Amount\ x\ Floor \tag{21}$$

The payoffs of the Altiplano option explained in this document have been translated into python according to the Bank's formula (17), (18), (19), (20), (21) and can be found in appendix 92-100.

As can be seen in ***Figure 16*** the derivative portfolio of Banco Invest contains six different Altiplano options: with the Floor constant at 0.1% for all options and the Cap in a range of [1.8% - 2.4%] and the Coupon in a range of [0.15% - 0.4%].

Portfolio - Payoff Type: Altiplano

Product ID	Name	Effective date	Maturity date	Cap	Floor	4 underlyings above
12/13/02	Invest Infraestruturas Nov-20	12/7/22	11/30/20	2.00%	0.10%	0.40%
2/12/03	Invest Clean Tech Mar-21	4/6/23	3/31/21	2.20%	0.10%	0.20%
3/13/03	Invest Nutrition Jun-21	7/7/23	6/30/21	2.00%	0.10%	0.20%
3/17/03	Invest Water Mai-21	6/7/23	6/1/21	2.40%	0.10%	0.20%
7/28/03	Invest Autonomous & EV Jan-22	2/7/24	1/31/22	1.85%	0.10%	0.20%
11/10/03	Invest Security Mai-22	6/7/24	5/31/22	1.80%	0.10%	0.15%

Figure 16: Basket Overview Altiplano option Banco Invest

5.1 Portfolio Delta

As Banco Invest offers and therefore also sells Altiplano options, the bank is short on all the underlying's. If the underlying's prices rise, the options' deltas will rise as well because it is now more likely that the first two payoff conditions are met. This would lead to a higher payoff and, therefore to a higher price of the derivative. To take a delta-neutral position, the bank must buy the underlying stocks.

Based on our calculations, which were explained in the previous chapters, we obtained the following delta values for the altiplano option ***Figure 17***. The delta of the aggregated Altiplano Portfolio is calculated by the sum of the weighted deltas and equals 0.0124. To take a delta-neutral position, the bank would therefore need to buy stocks worth 122,351.06€.

Delta (Δ) - Altiplano Products

Product ID	wi	Delta (Δ)	Numerical value (€)
1078	17.90%	0.0190	33,617.53 €
1139	11.27%	0.0229	25,449.41 €
1168	13.84%	0.0103	14,034.45 €
1172	11.74%	0.0076	8,783.81 €
1305	18.69%	0.0033	6,161.06 €
1410	26.56%	0.0131	34,304.80 €

Delta (Δ) - Aggregated Altiplano Portfolio

Product Type	Notional	Delta (Δ)	Numerical value (€)
Altiplano	9,867,209.32 €	0.0124	122,351.06 €

Figure 17: Altiplano - Overview of the deltas of the individual products and the portfolio

Figure 18 shows the Amount of the deltas graphically: Remarkably, products 1078 and 1139 take up almost 50% of the total capital needed for the hedge as their deltas (1078: 0.0190; 1139: 0.0229) and are the highest of the altiplano portfolio. Nevertheless, they only account for about 29% of the portfolio. It is also noticeable that the capital required for the hedge decreases with an increasing maturity date from product 1078 to 1305.

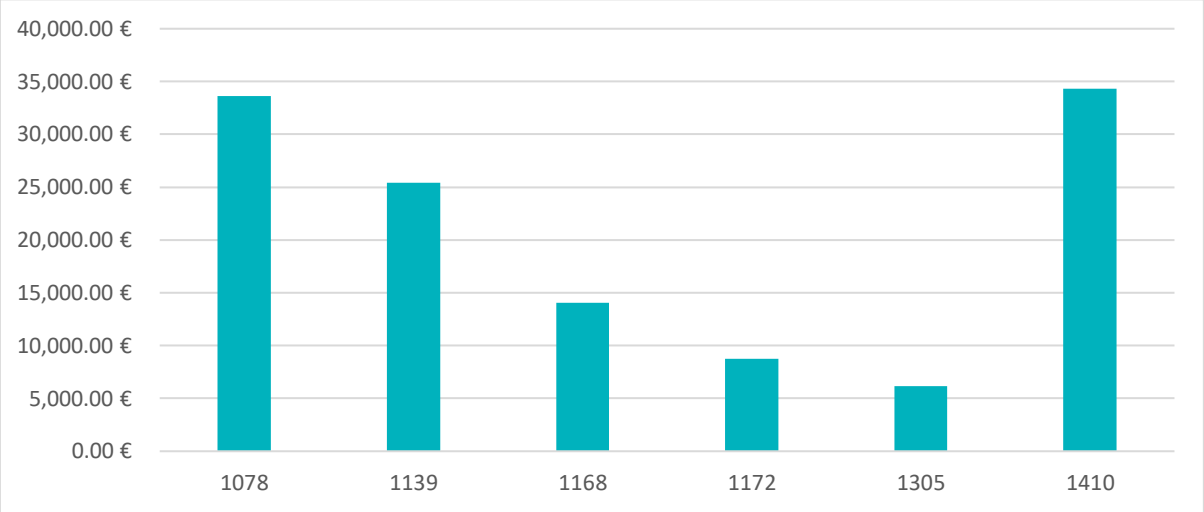


Figure 18: Altiplano - Amount of capital required for the delta hedge

5.2 Portfolio Gamma

Based on our calculations, which were explained in the previous chapters, we have obtained the following gamma values for the altiplano option (***Figure 19***). The Gamma of the aggregated Altiplano Portfolio is calculated by the sum of the weighted Gammas and equals 1.3464.

Gamma (Γ) - Altiplano Products

Product ID	wi	Gamma (Γ)
1078	17.90%	2.7362
1139	11.27%	1.1151
1168	13.84%	1.0084
1172	11.74%	3.0526
1305	18.69%	0.6203
1410	26.56%	0.4410

Gamma (Γ) - Aggregated Altiplano Portfolio

Product Type	Notional	Gamma (Γ)
Altiplano	9,867,209.32 €	1.3464

Figure 19: Altiplano - Overview of the gammas of the individual products and the portfolio

Except for the last two Altiplano Options in the portfolio, option Gamma is over one, implying that large prices of the underlyings do not strongly impact the option's deltas. As options 1305 and 1410 have the most extended maturity, this could be the reason for the low gammas: Delta may move suddenly close to the expiration date as the option can be in or out the money. A high Gamma expresses such significant moves. This explanatory approach would also be consistent with the decreasing Gammas from option 1078 to 1168. Nevertheless, option 1172 has the highest Gamma (3.0526), indicating that the option's delta is very sensitive to price changes of its underlyings. To protect its delta hedge against large price swings of the underlyings, the bank should perform a gamma hedge on the options with the highest Gammas, as the deltas of the respective options could change drastically, resulting in an ineffective delta hedge.

5.3 Non-linear Delta-Gamma-VaR

Based on the Value-at-Risk calculations we explained in the previous chapters, we calculated the Value-at-Risk for each Altiplano option and the value-at-risk for the aggregated portfolio of Altiplano options. The results are shown in the following (***Figure 20***):

Value-at-Risk (1d, 99,9%) - Altiplano Products

Product ID	VaR in %	Numerical value (€)	Numerical value (€)
1078	4.07%	1,162,084.00 €	71,925.32 €
1139	1.08%	1,297,321.00 €	12,061.62 €
1168	0.08%	1,364,224.00 €	1,098.22 €
1172	11.20%	1,373,584.00 €	129,796.78 €
1305	0.03%	1,703,025.00 €	634.75 €
1410	0.15%	1,988,100.00 €	3,889.04 €

Value-at-Risk (1d, 99,9%) - Aggregated Altiplano Portfolio

Product Type	Notional	VaR	Numerical value (€)
Indicap	9,867,209.32 €	0.0009	8,937.60 €

Figure 20: Altiplano - Overview of the results of the Vale-at-Risk calculation

The Value-at-Risk for the next trading day at a confidence level of 99,9% is 0.0009 in relative terms of the Altiplano Portfolio notional, which equals a numerical value of 8,937.6€ as an expected maximum loss: This Amount is only expected to be exceeded in 0,01% of all cases.

The Value-at-Risk calculated, refers to the undiversified Value-at-Risk estimate of the Portfolio. The sum of all individual Value-at-Risk values of the Altiplano options would result in the undiversified Value-at-Risk, which would higher than the undiversified. The undiversified Value-at-Risk for the next trading day at a confidence level of 99.9% is 219,405.73 €, it is expected that this value will only be exceeded 0.01% of all cases.

The diversified Value-at-Risk is used by banks that are active in trading to set limits for trading. Whereas the undiversified Value-at-Risk simulates a worst-case scenario loss in the event of a

market crash.

6 Recommendation - Group part

This chapter address how the bank's management should deal with the risk associated with the derivatives Portfolio. ***Figure 35*** below summarizes the delta, gamma, and Delta-Gamma Value-at-Risk for Banco Invest's overall options portfolio. The total derivatives portfolio of the bank has a notional of EUR 157.067.916, consisting of 53 different options. The 1-day Value-at-Risk at 99,9% confidence level for the bank's overall derivatives portfolio is EUR 372.773, implying a 99,9 % probability the portfolio will not lose more over the next trading day.

Banco Invest - Aggregated Derivatives Portfolio	
Notional	**157.067.916,00 €**
No. of option positions	**53**
Delta (Δ)	**0,0163**
Gamma (Γ)	**0,3166**
Volatility	**4,91%**
VaR (1d, @ 99,9%)	**0,24%**
VaR (1d, @ 99,9%)	**372.773,00 €**

Figure 35: Aggregated Portfolio Delta-Gamma VaR

As the bank does not take a directional risk on the market, the delta on combined option's portfolio must be neutralized with an appropriate hedging strategy. All five option types in the Banco Invest derivatives portfolio are basket options. The challenge of hedging, when facing options with a basket of underlying's, becomes evident in their correlated structure. This makes the evaluation of the contract's price but also the risks, e.g., delta, gamma, and their hedging a complex procedure. (Su 2006, pp. 3-5) This is because it is difficult to detangle the underlying basket's distribution. The correlation between the underlying tends to be volatile and can only be estimated. This further complicates the "perfect" hedging of basket options. As a result, in many cases, only a part of the underlying basket is used for hedging, or the payoffs of the basket are replicated "super-hedged". (Su 2008, pp. 19-23) Another difficulty arises from the number

of underlying assets: When following a standard dynamic hedging strategy, a hedging portfolio for the basket options should be related to the underlying assets in the basket. The larger the amount of underlying's the more difficult it is to implement such a dynamic strategy and the larger the transactions cost, caused by the continuous rebalancing, become. Since most of the options are "near-zero-gamma", which means that the directionality, the delta of the option is not greatly affected by changes in the underlying market prices, a dynamic hedging strategy can be implemented as major changes in the delta are not expected to be caused by changes in the underlying market prices. Transaction costs for rebalancing will occur but will be manageable as they do not occur very frequently. Lamberton and Lapeyre (1992) showed that a dynamic hedge on even a subset of the underlying's works well: they developed a method using multiple regression analysis to create a dynamic approximate hedging portfolio of plain-vanilla options on only a subset of the underlying's. For our "near-zero-gamma" options, such a dynamic hedge could further reduce the already low cost of rebalancing. A static hedging strategy has the advantage that transaction costs caused by continuous rebalancing can be avoided, and therefore this strategy could have a better hedging performance. (Su 2008, pp 2-4) Su (2006) used the Principal Components Analysis (PCA) to demonstrate that also a static hedge on a subset of the underlying's performs well: The PCA was used to determine a dominant subset of assets of the basket. Since a dynamic hedge of a basket option often only approximates the optimal hedge, the complete neutralization of the delta can only be achieved by a static hedge. Since Banco Invest instructs it takes no directional risk in the market, the only hedging strategy that fits this case is a static strategy as described above. Moreover, since the assets in the respective basket options are all in the same thematic investment universe, it is worthwhile to follow the approach of Su (2006) to determine whether it is sufficient to apply a static hedge only to a subset of the underlying assets, due to the high correlation between them.

Bibliography

Banco Invest. 2022. “Risk Management on the Equity Derivatives.”

Bodie, Zvi, Alex Kane, and Alan J. Marcus. 2021a. “Investments.” New York.

———. 2021b. *Investments*. New York.

Bouzoubaa, Mohamed, and Adel Osseiran. 2010a. “Exotic Options and Hybrids.” Chichester.

———. 2010b. *Exotic Options and Hybrids*. Chichester.

Castellacci, Giuseppe, and Michael J. Siclari. 2003. “The Practice of Delta–Gamma VaR: Implementing the Quadratic Portfolio Model.” *European Journal of Operational Research* 150 (3): 529–45. https://doi.org/10.1016/S0377-2217(02)00782-8.

Chuan, T. 2008. Capital Protected is not Capital Guaranteed. *Financial Planning Central*

Deng, Geng, Joshua Mallett, and Craig McCann. 2011. “Modeling Autocallable Structured Products.” *Journal of Derivatives and Hedge Funds* 17 (4): 326–40. https://doi.org/10.1057/jdhf.2011.25.

Deutsche Bundesbank. 2022. “Deutsche Bundesbank: Marktrisiko.” 2022. https://www.bundesbank.de/de/aufgaben/bankenaufsicht/einzelaspekte/eigenmittelanforderungen/marktrisiko/marktrisiko-598476.

Fallon, William. 1996. “Calculating Value-at-Risk.” Philadelphia. https://www.researchgate.net/publication/2428988_Calculating_Value-at-Risk.

Gharavi, Reza K. 2010. “Encyclopedia of Quantitative Finance” 4.

Guillaume, Tristan. 2015a. “Autocallable Structured Products.” *THE JOURNAL OF DERIVATIVES* 73. www.iijournals.com.

———. 2015b. “Analytical Valuation of Autocallable Notes.” *International Journal of Financial Engineering* 02 (02): 1550016. https://doi.org/10.1142/s2424786315500164.

Hilpisch, Yves. 2015. “Derivatives Analytics with Python.” Chichester.

Hull, John C. 2018. “Risk Management and Financial Institutions.” New York.

Hull, John C. 2021a. “OPTIONS, FUTURES, AND OTHER DERIVATIVES.” New York.

———. 2021b. *OPTIONS, FUTURES, AND OTHER DERIVATIVES*. New York.

Jorion, Philippe. 2007. *Philippe Jorion - Value at Risk, 3rd Ed._ The New Benchmark for Managing Financial Risk (2006, McGraw-Hill)*. Third. The McGraw-Hill Companies, Inc.

Leoni, Peter. 2014. *The Greeks and Hedging Explained*.

Romano. 2017. “Options Trading.” May 2017. https://romanornr.medium.com/options-trading-fd4d0bffb2c5.

Shefrin, Hersh. 2002. *Beyond Greed and Fear*. Oxford University PressNew York. https://doi.org/10.1093/0195161211.001.0001.

Su, Xia. 2006. “Hedging Basket Options by Using a Subset of Underlying Assets.” 14. Bonn. https://www.econstor.eu/bitstream/10419/22959/1/bgse14_2006.pdf.

———. 2008. “Essays on Basket Options Hedging and Irreversible Investment Valuation.” Rheinische Friedrich-Wilhelms-Universität Bonn. https://bonndoc.ulb.uni-bonn.de/xmlui/handle/20.500.11811/3322.

Taleb, Nassim Nicholas. 1997a. “Dynamic Hedging: Managing Vanilla and Exotic Options.” New York.

———. 1997b. *Dynamic Hedging: Managing Vanilla and Exotic Options*. New York.

Wealth Focus Pty Ltd. 2010. “Guide to Capital Protected Products.” Manly: Wealth Focus Pty Ltd. https://www.fundsfocus.com.au/managed-funds/pdfs/Capital-Protection.pdf.

Weert, Frans de. 2008. “Exotic Options Trading.” Chichester.
Wilmott, Paul. 1998. *Derivatives : The Theory and Practice of Financial Engineering*. John Wiley & Sons Ltd.
Yan, Yuxing. 2017a. *Python for Finance : Financial Modeling and Quantitative Analysis Explained*. Birmingham: Packt Publishing.
———. 2017b. *Python for Finance : Financial Modeling and Quantitative Analysis Explained*. Birmingham: Packt Publishing.
Yen Jerome, and Kin Keung Lai. 2015. “Emerging Financial Derivatives.” New York.
Z. Tong, Kevin. 2019. “A Recursive Pricing Method for Autocallables under Multivariate Subordination.” *Quantitative Finance and Economics* 3 (3): 440–55. https://doi.org/10.3934/QFE.2019.3.440.

Appendix

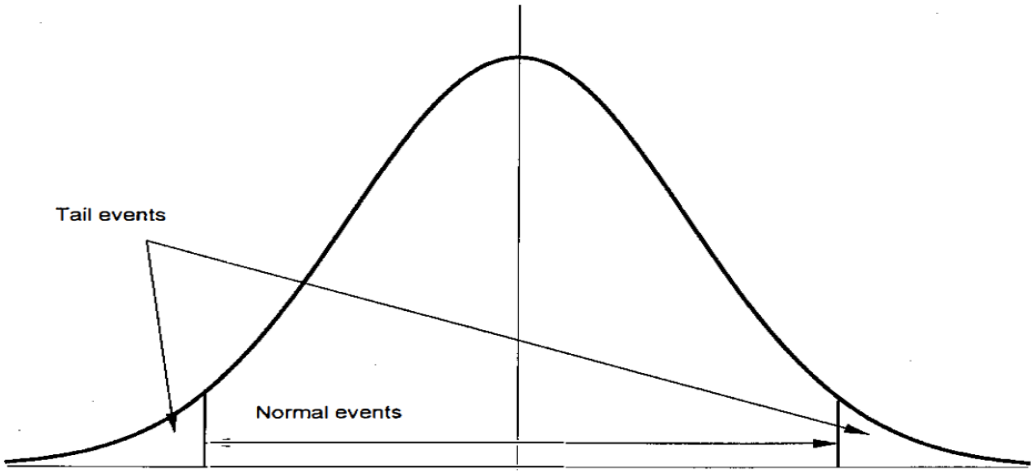
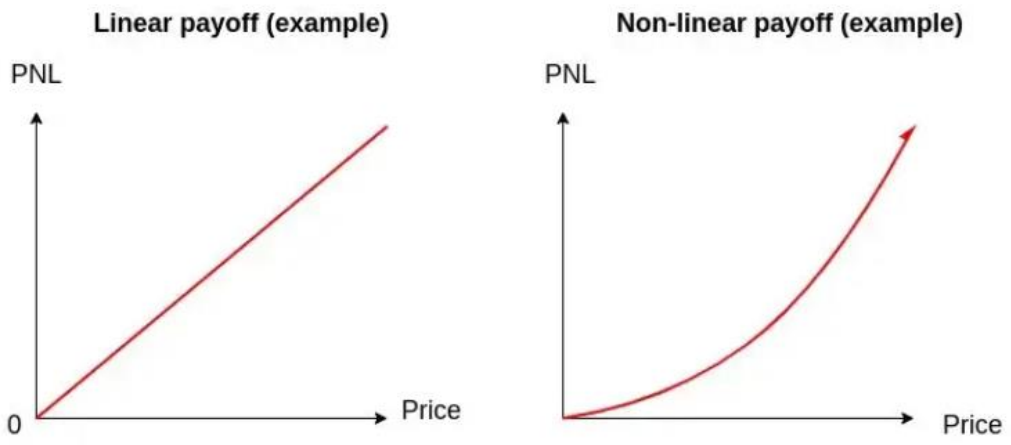


Figure 27.1 'Normal events' and 'tail events'.

Appendix 1: Value-at-Risk distribution showing possible tail events (Wilmott 1998, p. 338)

CI	Z
80%	1,282
85%	1,440
90%	1,645
95%	1,960
99%	2,576
99,5%	2,807
99,9%	3,291

Appendix 2: Overview of most common z-statistic for VaR calculation



Linear vs. Non-linear payoff

Appendix 3: Linear / Non-linear VaR (Romano 2017)

Name	Symbol	Derivative	Measures	Definition
Delta	Δ	$\frac{\partial c}{\partial s}$	Equity Exposure	Measures how much an option's price is estimated to shift in response to a change of a one unit in the underlying security
Gamma	Γ	$\frac{\partial^2 c}{\partial^2 s}$	Payout Convexity	Measures the amount of change in Delta if the price of the underlying security changes by one unit
Theta	Θ	$\frac{\partial c}{\partial T}$	Time Decay	Measures the change in the option price induced by the decrease of 1 day of the remaining time to maturity
Vega	V	$\frac{\partial c}{\partial \sigma}$	Volatility Exposure	Measures how much an option's price will change in response to a 1% change in the volatility of the underlying securities
Rho	P	$\frac{\partial c}{\partial r}$	Interest Rate Exposure	Measures how much the value of an option changes based on a 1% change in the interest rate

Appendix 4: Overview Greeks – In accordance with (Leoni 2014, pp. 85-97)

```python
def drift_calc(data, return_type='log'):
    if return_type == 'log':
        lr = log_returns(data)
    elif reutrn_type == 'simple':
        lr = simple_returns(data)
    u = lr.mean()
    var = lr.var()
    drift = u-(0.5*var)
    try:
        return drift.values
    except:
        return drift

drift = drift_calc(modified_data)

div = portfolio[self.id]['div']
# Drift adjusting if dividend paying (for Brownsche Motion)
if div > 0:
    drift = drift - div
```

Appendix 5: Drift calculation in Python

```python
covar = log_ret.cov() #covariance matrix.

chol = np.linalg.cholesky(covar) #create cholesky matrix from covariance matrix

uncorr_x = norm.ppf(np.random.rand(num_stocks, simulated_days)) #stocks, days

corr_x = np.dot(chol, uncorr_x)

corr_2 = np.zeros_like(corr_x) #Return an array of zeros with the same shape and type as a given array.
for i in range(num_stocks):
    corr_2[i] = np.exp(drift[i] + corr_x[i])
corr_2[0]

stock0 = pd.DataFrame()                                    #create new data frame
for s in range(len(ticks)):
    ret_reshape = corr_2[s]
    ret_reshape = ret_reshape.reshape(simulated_days) #Gives a new shape to an array without changing its data
    price_list = np.zeros_like(ret_reshape)
    price_list[0] = data.iloc[-1, s] #iloc = Purely integer-location based indexing for selection by position
    for t in range(1, simulated_days):
        price_list[t] = price_list[t-1]*ret_reshape[t]
```

Appendix 6: Cholesky decomposition in Python

```python
def get_vola(portfolio, volatility_file, stock_file):
    for a in range(2):
        for i in portfolio.keys():
            ticks = portfolio[i]['underlyings']
            today = "30-06-2022"
            today = pd.to_datetime(today)
            end = today
            #end = portfolio[i]['effective_date']
            vola = "VOLATILITY_30D"
            stock_vola = []

            if portfolio[i]["vol_type"] == "I": #calculation of implied volatility
                ids = portfolio[i]["underlyings"]
                for underlying in ids:
                    underlying_vola = volatility_file._get_value(vola, underlying)
                    stock_vola.append(underlying_vola)
                vol = (sum(stock_vola)/len(stock_vola))
                portfolio[i]["vol"] = vol
                #No Impl. vol.?, change vol_type to historical for second loop
                if  math.isnan(vol) == True:
                    portfolio[i]["vol_type"] = "H"
```

Appendix 7: Volatility calculation in Python (1/2)

```python
            else:
                portfolio[i]["vol_type"] = "H"
                def vola_data(tickers): #basically same funcion as used in cholesky
                    vol_data = pd.DataFrame() #create new data frame
                    for t in tickers: #loop through underlying tickers
                        vol_data[t] = stock_file[t].iloc[1:]
                    return(vol_data)

                data_ticks = vola_data(ticks)
                end_date = len(data_ticks.loc[:end]) #determine lenght of data frame for vola
                start_date = end_date - 30

                used_data = data_ticks.iloc[start_date:end_date]
                #print(used_data)

                def log_returns(data):
                    return (np.log(1+data.pct_change()))
                stdev = log_returns(used_data).std().values
                #monthly vola of the option = equal weighting vola of underlyings
                monthly_vol = sum(stdev)/len(stdev)
                vol = monthly_vol * sqrt(12) #annual vola
                portfolio[i]["vol"] = vol #append dictionary


get_vola(options_portfolio, file_vola, file_stocks)
```

Appendix 8: Volatility calculation in Python (2/2)

```python
def interpolation(rates, maturity_date):
    today = "30-06-2022"
    today = pd.to_datetime(today)
    today = today.to_pydatetime().date()

    maturity_day = maturity_date.day
    maturity_month = maturity_date.month
    maturity_year = maturity_date.year
    name = rates.columns[0]

    for a in range(len(rates)-1):
        rate_date = rates.index[a]
        prev_date = rates.index[a-1]
        next_date = rates.index[a+1]
        rate_day = rates.index[a].day
        rate_month = rates.index[a].month
        rate_year = rates.index[a].year

        if rate_date == maturity_date:
            r = rates._get_value(rate_date, name)
```

Appendix 9: Linear interpolation in Python to get discount rates for Option payoffs (1/2)

```python
        elif rate_month == maturity_month and rate_year == maturity_year:
            #structure: rate_day - maturity_date - next_rate
            if rate_day < maturity_day:
                #todays rate, shorter maturity
                r1 = rates._get_value(rate_date, name)
                #next rate, longer maturity
                r2 = rates._get_value(next_date, name)
                #day difference between rate_date and maturity_date
                t1 = abs(rate_date.to_pydatetime().date() - today)
                #day difference bewween next_date and maturity_date
                t2 = abs(next_date.to_pydatetime().date() - today)
                #day difference between rate_date and maturity_date
                tn = abs(today - maturity_date.to_pydatetime().date())



                r = r1 + (r2-r1)/((t2-t1).days)*((tn-t1).days)

            else:
                r1 = rates._get_value(prev_date, name)
                r2 = rates._get_value(rate_date, name)

                t1 = abs(prev_date.to_pydatetime().date() - today)
                t2 = abs(rate_date.to_pydatetime().date() - today)
                tn = abs(today - maturity_date.to_pydatetime().date())


                r = r1 + (r2-r1)/((t2-t1).days)*((tn-t1).days)
    return r
```

Appendix 10: Linear interpolation in Python to get discount rates for Option payoffs (2/2)

$$\Delta = \frac{S_t(\varepsilon) - S_t}{\varepsilon} \tag{18}$$

$$\Gamma = \frac{\Delta_t(\varepsilon) - \Delta_t}{\varepsilon} \tag{19}$$

Appendix 11: Equations used for Delta/Gamma calculation

```python
# Calculating per Option:
for i in options_portfolio.keys():
    print(i)
    # Determining the risk free rate by maturity matching the Swap Rate with the maturity of the option
    r = interpolation(swaps, options_portfolio[i]['maturity'])

    # Get infos
    ticks = options_portfolio[i]['underlyings']
    start = options_portfolio[i]['effective_date']
    S = options_portfolio[i]['spot']
    K = options_portfolio[i]['strike']

    today = "30-06-2022"
    today = datetime.strptime(today, '%d-%m-%Y').date()

    T = options_portfolio[i]['maturity'].to_pydatetime().date() - today
    T = T.days/365
    div = options_portfolio[i]['div']
    vol = options_portfolio[i]["vol"]

    price=0
    delta=0
    sym_delta = 0
    delta_2=0
    delta_3=0
    g=0
    var = 0
    z = 3.291
```

Appendix 12: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (1/15)

```
# Call different classes for payoffs and delta
    if options_portfolio[i]["payoff_id"] == 2:

        altiplano = Altiplano(options_portfolio,i)
        payoffs = altiplano.payoff()
        # Discounting the payoff with the maturity matched risk free rate
        price = payoffs[0] * math.exp(-r*T)

        #Deltas
        pos_price = payoffs[1] * math.exp(-r*T)
        neg_price = payoffs[2] * math.exp(-r*T)
        delta = (pos_price - price) / (percentage_change)
        sym_delta = (pos_price - neg_price)/(2*percentage_change)

        #Second & Third Deltas
        pos_price2 = payoffs[3] * math.exp(-r*T)
        neg_price2 = payoffs[4] * math.exp(-r*T)
        delta_2 = (pos_price2 - pos_price) / (percentage_change)
        delta_3 = (neg_price2 - neg_price) / (percentage_change)

        #Gamma
        delta_dif = delta_2 - delta
        g = abs(delta_dif)/percentage_change
        #sym_g = ()

        #print(pos_price)
        #print(neg_price)

        # 1d VaR and 99,9% interval (Z-score=3.291), calculated on 1y
        var = (delta*3.291*np.sqrt(1/252)*vol - g/2*(3.291*np.sqrt(1/252)*vol)**2)*np.sqrt(252)
```

Appendix 13: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (2/15)

```
    elif options_portfolio[i]["payoff_id"] == 4:
        auto = Autocall(options_portfolio, i)
        payoffs = auto.payoff()
        # Discounting the payoff with the maturity matched risk free rate
        price = payoffs[0] * math.exp(-r*T)

        #Deltas
        pos_price = payoffs[1] * math.exp(-r*T)
        neg_price = payoffs[2] * math.exp(-r*T)
        delta = (pos_price - price) / (percentage_change)
        sym_delta = (pos_price-neg_price)/(2*percentage_change)

        #Second & Third Deltas
        pos_price2 = payoffs[3] * math.exp(-r*T)
        neg_price2 = payoffs[4] * math.exp(-r*T)
        delta_2 = (pos_price2 - pos_price) / (percentage_change)
        delta_3 = (neg_price2 - neg_price2) / (percentage_change)

        #Gamma
        delta_dif = delta_2 - delta
        g = abs(delta_dif)/percentage_change
        #sym_g = ()

        #print(pos_price)
        #print(neg_price)

        # 1d VaR and 99,9% interval (Z-score=3.291), calculated on 1y
        var = (delta*3.291*np.sqrt(1/252)*vol - g/2*(3.291*np.sqrt(1/252)*vol)**2)*np.sqrt(252)
```

Appendix 14: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (3/15)

```
    elif options_portfolio[i]["payoff_id"] == 6:
        digi = Call_Digital(options_portfolio, i)
        payoffs = digi.payoff()
        # Discounting the payoff with the maturity matched risk free rate
        price = payoffs[0] * math.exp(-r*T)

        #Deltas
        pos_price = payoffs[1] * math.exp(-r*T)
        neg_price = payoffs[2] * math.exp(-r*T)
        delta = (pos_price - price) / (percentage_change)
        sym_delta = (pos_price-neg_price)/(2*percentage_change)

        #Second & Third Deltas
        pos_price2 = payoffs[3] * math.exp(-r*T)
        neg_price2 = payoffs[4] * math.exp(-r*T)
        delta_2 = (pos_price2 - pos_price) / (percentage_change)
        delta_3 = (neg_price2 - neg_price2) / (percentage_change)

        #Gamma
        delta_dif = delta_2 - delta
        g = abs(delta_dif)/percentage_change
        #sym_g = ()

        #print(pos_price)
        #print(neg_price)

        # 1d VaR and 99,9% interval (Z-score=3.291), calculated on 1y
        var = (delta*3.291*np.sqrt(1/252)*vol - g/2*(3.291*np.sqrt(1/252)*vol)**2)*np.sqrt(252)
```

Appendix 15: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (4/15)

```
    elif options_portfolio[i]["payoff_id"] == 11:
        indi = Indicap(options_portfolio,i)
        payoffs = indi.payoff()
        # Discounting the payoff with the maturity matched risk free rate
        price = payoffs[0] * math.exp(-r*T)

        #Deltas
        pos_price = payoffs[1] * math.exp(-r*T)
        neg_price = payoffs[2] * math.exp(-r*T)
        delta = (pos_price - price) / (percentage_change)
        sym_delta = (pos_price-neg_price)/(2*percentage_change)

        #Second & Third Deltas
        pos_price2 = payoffs[3] * math.exp(-r*T)
        neg_price2 = payoffs[4] * math.exp(-r*T)
        delta_2 = (pos_price2 - pos_price) / (percentage_change)
        delta_3 = (neg_price2 - neg_price2) / (percentage_change)

        #Gamma
        delta_dif = delta_2 - delta
        g = abs(delta_dif)/percentage_change
        #sym_g = ()

        #print(pos_price)
        #print(neg_price)

        # 1d VaR and 99,9% interval (Z-score=3.291), calculated on 1y
        var = (delta*3.291*np.sqrt(1/252)*vol - g/2*(3.291*np.sqrt(1/252)*vol)**2)*np.sqrt(252)
```

Appendix 16: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (5/15)

```
else:
    capprotect = Capital_Protected(options_portfolio,i)
    payoffs = capprotect.payoff()
    # Discounting the payoff with the maturity matched risk free rate
    price = payoffs[0] * math.exp(-r*T)

    #Deltas
    pos_price = payoffs[1] * math.exp(-r*T)
    neg_price = payoffs[2] * math.exp(-r*T)
    delta = (pos_price - price) / (percentage_change)
    sym_delta = (pos_price-neg_price)/(2*percentage_change)

    #Second & Third Deltas
    pos_price2 = payoffs[3] * math.exp(-r*T)
    neg_price2 = payoffs[4] * math.exp(-r*T)
    delta_2 = (pos_price2 - pos_price) / (percentage_change)
    delta_3 = (neg_price2 - neg_price2) / (percentage_change)

    #Gamma
    delta_dif = delta_2 - delta
    g = abs(delta_dif)/percentage_change
    #sym_g = ()

    #print(pos_price)
    #print(neg_price)

    # 1d VaR and 99,9% interval (Z-score=3.291), calculated on 1y
    var = (delta*3.291*np.sqrt(1/252)*vol - g/2*(3.291*np.sqrt(1/252)*vol)**2)*np.sqrt(252)
```

Appendix 17: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (6/15)

```
options_portfolio[i]['option_price'] = price
options_portfolio[i]['delta'] = delta
options_portfolio[i]["symmetric_delta"] = sym_delta
options_portfolio[i]['delta2'] = delta_2
options_portfolio[i]['delta3'] = delta_3
options_portfolio[i]['g'] = g
options_portfolio[i]["var"] = var
```

Appendix 18: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (7/15)

Per Option Type: Delta, Second Delta, Gamma and VaR

Weights

```
alti_notional = 0
auto_notional = 0
digi_notional = 0
indi_notional = 0
capprotect_notional = 0
sum_notional = 0
for key in options_portfolio.keys():

    if options_portfolio[key]["payoff_id"] == 4:
        notional = options_portfolio[key]["notional"]
        auto_notional += notional
        sum_notional += notional

    elif options_portfolio[key]["payoff_id"] == 4:
        notional = options_portfolio[key]["notional"]
        options_portfolio[key]["weight_type"] = notional/auto_notional
        options_portfolio[key]["weight_total"] = notional/sum_notional

        options_portfolio[key]["weighted_delta_type"] = options_portfolio[key]["weight_type"]*options_portfolio[key]["delta"]
        options_portfolio[key]["weighted_gamma_type"] = options_portfolio[key]["weight_type"]*options_portfolio[key]["g"]

        options_portfolio[key]["weighted_delta_total"] = options_portfolio[key]["weight_total"]*options_portfolio[key]["delta"]
        options_portfolio[key]["weighted_gamma_total"] = options_portfolio[key]["weight_total"]*options_portfolio[key]["g"]

#options_portfolio
options_type = {"Altiplano": {}, "Autocall": {}, "Call_Digital": {}, "Indicap": {}, "Capital_Protect": {}}
options_type["Altiplano"]["notional"] = alti_notional
options_type["Autocall"]["notional"] = auto_notional
options_type["Call_Digital"]["notional"] = digi_notional
options_type["Indicap"]["notional"] = indi_notional
options_type["Capital_Protect"]["notional"] = capprotect_notional
```

Appendix 19: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (8/15)

```
alti_delta = 0
auto_delta = 0
digi_delta = 0
indi_delta = 0
capprotect_delta = 0

alti_gamma = 0
auto_gamma = 0
digi_gamma = 0
indi_gamma = 0
capprotect_gamma = 0

for key in options_portfolio.keys():

    if options_portfolio[key]["payoff_id"] == 4:
        auto_delta += options_portfolio[key]["weighted_delta_type"]
        auto_gamma += options_portfolio[key]["weighted_gamma_type"]

options_type["Altiplano"]["delta"] = alti_delta
options_type["Autocall"]["delta"] = auto_delta
options_type["Call_Digital"]["delta"] = digi_delta
options_type["Indicap"]["delta"] = indi_delta
options_type["Capital_Protect"]["delta"] = capprotect_delta

options_type["Altiplano"]["gamma"] = alti_gamma
options_type["Autocall"]["gamma"] = auto_gamma
options_type["Call_Digital"]["gamma"] = digi_gamma
options_type["Indicap"]["gamma"] = indi_gamma
options_type["Capital_Protect"]["gamma"] = capprotect_gamma
```

Appendix 20: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (9/15)

```
alti_underlyings = []
auto_underlyings = []
digi_underlyings = []
indi_underlyings = []
capprotect_underlyings = []

alti_underlyings_weights = []
auto_underlyings_weights = []
digi_underlyings_weights = []
indi_underlyings_weights = []
capprotect_underlyings_weights = []

today = "30-06-2022"
for key in options_portfolio.keys():

    if options_portfolio[key]["payoff_id"] == 4:
        underlyings_1 = options_portfolio[key]["underlyings"]
        for underlyings in underlyings_1:
            single_weight = (1/5) * options_portfolio[key]["weight_type"] #stock weight in option * total option-type weight
            auto_underlyings_weights.append(single_weight)
            auto_underlyings.append(underlyings)

alti_underlyings_weights = np.array(alti_underlyings_weights)
auto_underlyings_weights = np.array(auto_underlyings_weights)
digi_underlyings_weights = np.array(digi_underlyings_weights)
indi_underlyings_weights = np.array(indi_underlyings_weights)
capprotect_underlyings_weights = np.array(capprotect_underlyings_weights)

options_type["Autocall"]["underlyings"] = auto_underlyings
options_type["Autocall"]["weights"] = auto_underlyings_weights

#options_type
```

Appendix 21: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (10/15)

For the whole Portfolio: Delta, Second Delta, Gamma and VaR

```
total_delta = 0
total_gamma = 0

for key in options_portfolio.keys():

    total_delta += options_portfolio[key]["weighted_delta_total"]
    total_gamma += options_portfolio[key]["weighted_gamma_total"]

total_portfolio = {}
total_portfolio["notional"] = sum_notional
total_portfolio["delta"] = total_delta
total_portfolio["gamma"] = total_gamma
```

Appendix 22: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (11/15)

Weights of the underlyings

```
list_underlyings = [] #== tickers
weights = []
today = "30-06-2022"
for key in options_portfolio.keys():
    underlyings_1 = options_portfolio[key]["underlyings"]
    for underlyings in underlyings_1:
        single_weight = (1/5) * options_portfolio[key]["weight_total"] #stock weight in option * total option weight
        weights.append(single_weight)
        list_underlyings.append(underlyings)

weights = np.array(weights)
total_portfolio["underlyings"] = list_underlyings
total_portfolio["weights"] = weights
```

Appendix 23: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (12/15)

```
weights = np.array(weights)

def vola_data(tickers): #basically same funcion as used in cholesky
    vol_data = pd.DataFrame(columns=tickers) #create new data frame
    for t in tickers: #loop through underlying tickers
        vol_data[t] = file_stocks[t].iloc[1:]
    return(vol_data)

tickerrs = list_underlyings


data_ticks = vola_data(tickerrs)

end_date = len(data_ticks.loc[:today]) #determine lenght of data frame for vola --> 30d volatility since effective date or today?
start_date = end_date - 30

used_data = data_ticks.iloc[start_date:end_date]


def log_returns(data):
    return (np.log(1+data.pct_change()))

returns = log_returns(used_data)
covar = returns.cov() * 12 #annualize monthly covariance

vol = np.sqrt(np.dot(weights.T, np.dot(covar, weights)))

total_portfolio["vol"] = vol
```

Appendix 24: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (13/15)

```
# 1d VaR and 99,9% interval (Z-score=3.291), calculated on 1y
var = (total_delta*3.291*np.sqrt(1/252)*vol - total_gamma/2*(3.291*np.sqrt(1/252)*vol)**2)*np.sqrt(252)
total_portfolio["var"] = var
```

Appendix 25: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (14/15)

```
elif key == "Autocall":
    notional = options_type[key]["notional"]
    tickerrs = options_type[key]["underlyings"]
    weights = options_type[key]["weights"]
    data_ticks = vola_data(tickerrs)
    end_date = len(data_ticks.loc[:today]) #determine lenght of data frame for vola --> 30d volatility since effective date or today?
    start_date = end_date - 30
    used_data = data_ticks.iloc[start_date:end_date]

    returns = log_returns(used_data)
    covar = returns.cov() * 12 #annualize monthly covariance

    vol = np.sqrt(np.dot(weights.T, np.dot(covar, weights)))
    options_type[key]["vol"] = vol

    delta = options_type[key]["delta"]
    g = options_type[key]["gamma"]
    # 1d VaR and 99,9% interval (Z-score=3.291), calculated on 1y
    var = (delta*3.291*np.sqrt(1/252)*vol - g/2*(3.291*np.sqrt(1/252)*vol)**2)*np.sqrt(252)
    options_type[key]["var"] = var
```

Appendix 26: Delta/ gamma & VaR Calculation for each option & overall portfolio in Python (15/15)

```
class Altiplano:

    def __init__(self,portfolio,id):
        self.id = id
        self.portfolio = portfolio


    def cholesky(self):

        def import_stock_data(tickers):
            data = pd.DataFrame()
            for t in tickers:
                data[t] = file_stocks[t].iloc[1:]
            return(data)

        def get_timeseries(data_frame): #comments are explanation from Flavio
            data = data_frame
            date = self.portfolio[self.id]["effective_date"]
            row_number = len(data.loc[:date])
            maturity_date = self.portfolio[self.id]["maturity"]

            date = date.strftime("%Y-%m-%d")
            maturity_date = maturity_date.strftime("%Y-%m-%d")

            delta = np.busday_count(date, maturity_date)

            #today = datetime.today()
            #today = today.strftime("%Y-%m-%d") --> activate for future use
            today = '2022-06-12'

            #print(delta)
            if delta >= 504: #2*252 --> 2 Years, Business Days, "take the life of the option at start (2 years)"
                calc = 504

            else:
                delta_today = np.busday_count(date, today) #"From there on you calculate it decreasing with every day"; calculation includes only businessdays
                #print(delta_today)
                days_used = 504 - delta_today
                minimum = 30    #"until you reach the pre-defined minimum of 30 days from which you keep calculating it on a 30-day basis until the option expires"
                calc = max(minimum, days_used)
```

Appendix 92: Altiplano Class in Python (1/10)

```
            dl = row_number - calc
            data = data.iloc[dl:row_number]
            return data


        ticks = self.portfolio[self.id]["underlyings"]
        num_stocks = len(ticks)
        data = import_stock_data(ticks)
        #print(f"full data: {data}")
        modified_data = get_timeseries(data)
        #print(f"modified data: {modified_data}")
        #print()

        def log_returns(data):
            return (np.log(1+data.pct_change()))

        log_return = log_returns(data)

        #Only for visualisation:


        #stock1_ret = log_return[ticks[0]][1:]
        #stock2_ret = log_return[ticks[1]][1:]
        #stock3_ret = log_return[ticks[2]][1:]
        #stock4_ret = log_return[ticks[3]][1:]
        #stock5_ret = log_return[ticks[4]][1:]

        #plt.figure(figsize=(12,7))
        #sns.distplot(stock1_ret, kde_kws={"color":"b","lw":1,"label":ticks[0]})
        #sns.distplot(stock2_ret, kde_kws={"color":"r","lw":1,"label":ticks[1]})
        #sns.distplot(stock3_ret, kde_kws={"color":"g","lw":1,"label":ticks[2]})
        #sns.distplot(stock4_ret, kde_kws={"color":"y","lw":1,"label":ticks[3]})
        #sns.distplot(stock5_ret, kde_kws={"color":"k","lw":1,"label":ticks[4]})


        #plt.xlabel("Daily Returns (Log)")
        #plt.show()
        #print(f"{ticks[0]} Average Daily Return: {round(stock1_ret.mean()*100,4):{10}}%\t {ticks[0]} Yearly Average Return: {round(stock1_ret.mean()*252*100,2)}%")
        #print(f"{ticks[1]} Average Daily Return: {round(stock2_ret.mean()*100,4):{10}}%\t {ticks[1]} Yearly Average Return: {round(stock2_ret.mean()*252*100,2)}%")
        #print(f"{ticks[2]} Average Daily Return: {round(stock3_ret.mean()*100,4):{10}}%\t {ticks[2]} Yearly Average Return: {round(stock3_ret.mean()*252*100,2)}%")
        #print(f"{ticks[3]} Average Daily Return: {round(stock4_ret.mean()*100,4):{10}}%\t {ticks[3]} Yearly Average Return: {round(stock4_ret.mean()*252*100,2)}%")
        #print(f"{ticks[4]} Average Daily Return: {round(stock5_ret.mean()*100,4):{10}}%\t {ticks[4]} Yearly Average Return: {round(stock5_ret.mean()*252*100,2)}%")
```

Appendix 93: Altiplano Class in Python (2/10)

```
def drift_calc(data, return_type='log'):
    if return_type == 'log':
        lr = log_returns(data)
    elif reutrn_type == 'simple':
        lr = simple_returns(data)
    u = lr.mean()
    var = lr.var()
    drift = u-(0.5*var)
    try:
        return drift.values
    except:
        return drift

drift = drift_calc(modified_data)

div = portfolio[self.id]['div']
# Drift adjusting if dividend paying (for Brownsche Motion)
if div > 0:
    drift = drift - div
#print(drift)
#print()

log_ret = log_returns(modified_data)

stdev = log_returns(modified_data).std().values
covar = log_ret.cov() #covariance matrix.
#print(covar)
#print()

chol = np.linalg.cholesky(covar) #create cholesky matrix from covariance matrix
#print(chol)
#print()

uncorr_x = norm.ppf(np.random.rand(num_stocks, simulated_days)) #stocks, days
#print(uncorr_x)
#print()

corr_x = np.dot(chol, uncorr_x)
#print(corr_x)
#print()
```

Appendix 94: Altiplano Class in Python (3/10)

```
corr_2 = np.zeros_like(corr_x) #Return an array of zeros with the same shape and type as a given array.
for i in range(num_stocks):
    corr_2[i] = np.exp(drift[i] + corr_x[i])
corr_2[0]

stock0 = pd.DataFrame()                                          #create new data frame
for s in range(len(ticks)):
    ret_reshape = corr_2[s]
    ret_reshape = ret_reshape.reshape(simulated_days) #Gives a new shape to an array without changing its data
    price_list = np.zeros_like(ret_reshape)
    price_list[0] = data.iloc[-1, s] #iloc = Purely integer-location based indexing for selection by position
    for t in range(1, simulated_days):
        price_list[t] = price_list[t-1]*ret_reshape[t]

    #print(f">------------------------------     {ticks[s]}     ------------------------------<")


    x = pd.DataFrame(price_list).iloc[-1]
    #fig, ax = plt.subplots(1,2, figsize=(14,4))
    #sns.distplot(x, ax=ax[0])
    #sns.distplot(x, hist_kws={'cumulative':True},kde_kws={'cumulative':True},ax=ax[1])
    #plt.xlabel("Stock Price")
    #plt.show()
    x = pd.DataFrame(price_list)
    stock0[ticks[s]]=x.loc[:,0]


last_date = data.last_valid_index() #get last date from historical data
last_date = pd.Timestamp(last_date) #transform to timestamp
dates = [last_date]                 #create new list with last date as starting point
a = 1                               #a = timedelta
while len(stock0) != len(dates):    #while loop for same length of simulated data and date-list
    next_date = last_date + timedelta(days = a)
    if next_date.weekday() < 5:     #if next_date is business day: append date list and add 1 to a for next day
        dates.append(next_date)
        a += 1
    else:
        a+=1                        #otherwiese (if day is on weekend), jump to next day/ a

stock0["Date"] = dates              #include dates list to the cholesky-output
stock0 = stock0.set_index("Date")   #set the dates as index

output_stocks_combined = data.append(stock0[1:]) #create one big data frame with complete data
```

Appendix 95: Altiplano Class in Python (4/10)

```
def drift_calc(data, return_type='log'):
    if return_type == 'log':
        lr = log_returns(data)
    elif reutrn_type == 'simple':
        lr = simple_returns(data)
    u = lr.mean()
    var = lr.var()
    drift = u-(0.5*var)
    try:
        return drift.values
    except:
        return drift

drift = drift_calc(modified_data)

div = portfolio[self.id]['div']
# Drift adjusting if dividend paying (for Brownsche Motion)
if div > 0:
    drift = drift - div
#print(drift)
#print()

log_ret = log_returns(modified_data)

stdev = log_returns(modified_data).std().values
covar = log_ret.cov() #covariance matrix.
#print(covar)
#print()

chol = np.linalg.cholesky(covar) #create cholesky matrix from covariance matrix
#print(chol)
#print()

uncorr_x = norm.ppf(np.random.rand(num_stocks, simulated_days)) #stocks, days
#print(uncorr_x)
#print()

corr_x = np.dot(chol, uncorr_x)
#print(corr_x)
#print()
```

Appendix 95: Altiplano Class in Python (5/10)

```
def drift_calc(data, return_type='log'):
    if return_type == 'log':
        lr = log_returns(data)
    elif reutrn_type == 'simple':
        lr = simple_returns(data)
    u = lr.mean()
    var = lr.var()
    drift = u-(0.5*var)
    try:
        return drift.values
    except:
        return drift

drift = drift_calc(modified_data)

div = portfolio[self.id]['div']
# Drift adjusting if dividend paying (for Brownsche Motion)
if div > 0:
    drift = drift - div
#print(drift)
#print()

log_ret = log_returns(modified_data)

stdev = log_returns(modified_data).std().values
covar = log_ret.cov() #covariance matrix.
#print(covar)
#print()

chol = np.linalg.cholesky(covar) #create cholesky matrix from covariance matrix
#print(chol)
#print()

uncorr_x = norm.ppf(np.random.rand(num_stocks, simulated_days)) #stocks, days
#print(uncorr_x)
#print()

corr_x = np.dot(chol, uncorr_x)
#print(corr_x)
#print()
```

Appendix 96: Altiplano Class in Python (6/10)

```
def drift_calc(data, return_type='log'):
    if return_type == 'log':
        lr = log_returns(data)
    elif reutrn_type == 'simple':
        lr = simple_returns(data)
    u = lr.mean()
    var = lr.var()
    drift = u-(0.5*var)
    try:
        return drift.values
    except:
        return drift

drift = drift_calc(modified_data)

div = portfolio[self.id]['div']
# Drift adjusting if dividend paying (for Brownsche Motion)
if div > 0:
    drift = drift - div
#print(drift)
#print()

log_ret = log_returns(modified_data)

stdev = log_returns(modified_data).std().values
covar = log_ret.cov() #covariance matrix.
#print(covar)
#print()

chol = np.linalg.cholesky(covar) #create cholesky matrix from covariance matrix
#print(chol)
#print()

uncorr_x = norm.ppf(np.random.rand(num_stocks, simulated_days)) #stocks, days
#print(uncorr_x)
#print()

corr_x = np.dot(chol, uncorr_x)
#print(corr_x)
#print()
```

Appendix 97: Altiplano Class in Python (7/10)

```
    return output_stocks_combined


def altiplano(self,stock_prices, strikes):

    stock_basket = stock_prices
    strike_prices = self.portfolio[self.id]["strikes"]
    #notional = self.portfolio[self.id]["notional"]
    notional = 1
    cap = self.portfolio[self.id]["cap_alti"]
    coupon = self.portfolio[self.id]["coupon_alti"]
    otherwise = self.portfolio[self.id]["floor_alti"]

    #print(id)
    #print(f"strikes:{strike_prices}")
    #print(f"prices:{stock_basket}")
    #print(portfolio[id]["maturity"])
    #print(notional)
    #print(cap)
    #print(coupon)
    #print(otherwise)

    j = 0
    for u in range(len(stock_prices)):
        if strikes[u] <= stock_prices[u]:
            j += 1

    if j == 5:
        payoff = notional * (1+cap)
        return payoff

    elif j == 4:
        payoff = notional * (1+coupon)
        return payoff

    else:
        payoff = notional * (1+otherwise)
        return payoff

def payoff(self):
    ticks = self.portfolio[self.id]["underlyings"]
    maturity_date = self.portfolio[self.id]['maturity']
    payoffs = [] #create empty lists
```

Appendix 98: Altiplano Class in Python (8/10)

```
payoffs_pos = [] #for Delta
payoffs_pos2 = [] #for second Delta

payoffs_neg = []
payoffs_neg2 = []


for i in range(number_simulations): #repeat simulations
    output = self.cholesky()        #every new loop, cholesky is called again --> new simulation



    stock_prices = []
    stocks_pos = []
    stocks_pos2 = []
    stocks_neg = []
    stocks_neg2 = []


    for k in ticks:
        maturity_date = self.portfolio[self.id]['maturity'] #filter for stock-prices on maturity date
        maturity_value = output._get_value(maturity_date, k)
        stock_prices.append(maturity_value)

    for k in range(len(stock_prices)): #for delta calculation: add 1 to every stock price & deduct 1 from every stock price
        pos_change = stock_prices[k] * (1 + percentage_change) #append to new price list
        pos_change2 = pos_change * ((1 + percentage_change)**2)
        stocks_pos.append(pos_change)
        stocks_pos2.append(pos_change2)

        neg_change = stock_prices[k] * (1 - percentage_change)
        neg_change2 = neg_change * (1 - percentage_change)
        stocks_neg.append(neg_change)
        stocks_neg2.append(neg_change2)


    #payoffs:
    normal_payoff = self.altiplano(stock_prices, self.portfolio[self.id]["strikes"])
    payoffs.append(normal_payoff)


    #Delta-Calculation-Payoffs
    pos_payoff = self.altiplano(stocks_pos, self.portfolio[self.id]["strikes"])
    payoffs_pos.append(pos_payoff)
```

Appendix 99: Altiplano Class in Python (9/10)

```
    neg_payoff = self.altiplano(stocks_neg, self.portfolio[self.id]["strikes"])
    payoffs_neg.append(neg_payoff)


    #Gamma-Calculation-Payoffs
    pos_payoff2 = self.altiplano(stocks_pos2, self.portfolio[self.id]["strikes"])
    payoffs_pos2.append(pos_payoff2)

    neg_payoff2 = self.altiplano(stocks_neg2, self.portfolio[self.id]["strikes"])
    payoffs_neg2.append(neg_payoff2)


average_payoff = sum(payoffs)/len(payoffs)
average_pos = sum(payoffs_pos)/len(payoffs_pos)
average_neg = sum(payoffs_neg)/len(payoffs_neg)
average_pos2 = sum(payoffs_pos2)/len(payoffs_pos2)
average_neg2 = sum(payoffs_neg2)/len(payoffs_neg2)

return [average_payoff, average_pos, average_neg, average_pos2, average_neg2]
```

Appendix 100: Altiplano Class in Python (10/10)

```
[ ] class Indicap():

        def __init__(self,portfolio,id):
            self.id = id
            self.portfolio = portfolio


        def cholesky(self):

            def import_stock_data(tickers):
                data = pd.DataFrame()
                for t in tickers:
                    data[t] = file_stocks[t].iloc[1:] #momentan mit iloc[1:]
                return(data)

            def get_timeseries(data_frame):
                data = data_frame
                date = self.portfolio[self.id]["effective_date"]
                row_number = len(data.loc[:date])
                maturity_date = self.portfolio[self.id]["maturity"]

                date = date.strftime("%Y-%m-%d")
                maturity_date = maturity_date.strftime("%Y-%m-%d")

                delta = np.busday_count(date, maturity_date)

                #today = datetime.today()
                #today = today.strftime("%Y-%m-%d")
                today = '2022-06-12'

                #print(delta)
                if delta >= 504: #2*252 --> 2 Years, Business Days, "take is the life of the option at start (2 years)"
                    calc = 504

                else:
                    delta_today = np.busday_count(date, today) #"From there on you calculate it decreasing with every day"
                    #print(delta_today)
                    days_used = 504 - delta_today
                    minimum = 30    #"until you reach the pre-defined minimum of 30 days from which you keep calculating it on a 30-day basis until the option expires"
                    calc = max(minimum, days_used)
```

Appendix 101: Altiplano Class in Python (1/7)

```
[ ]
                d1 = row_number - calc
                data = data.iloc[d1:row_number]
                return data


            ticks = self.portfolio[self.id]["underlyings"]
            num_stocks = len(ticks)
            data = import_stock_data(ticks)
            #print(f"full data: {data}")
            modified_data = get_timeseries(data)
            #print(f"modified data: {modified_data}")
            #print()

            def log_returns(data):
                return (np.log(1+data.pct_change()))

            log_return = log_returns(data)


            #stock1_ret = log_return[ticks[0]][1:]
            #stock2_ret = log_return[ticks[1]][1:]
            #stock3_ret = log_return[ticks[2]][1:]
            #stock4_ret = log_return[ticks[3]][1:]
            #stock5_ret = log_return[ticks[4]][1:]

            #plt.figure(figsize=(12,7))
            #sns.distplot(stock1_ret, kde_kws={"color":"b","lw":1,"label":ticks[0]})
            #sns.distplot(stock2_ret, kde_kws={"color":"r","lw":1,"label":ticks[1]})
            #sns.distplot(stock3_ret, kde_kws={"color":"g","lw":1,"label":ticks[2]})
            #sns.distplot(stock4_ret, kde_kws={"color":"y","lw":1,"label":ticks[3]})
            #sns.distplot(stock5_ret, kde_kws={"color":"k","lw":1,"label":ticks[4]})


            #plt.xlabel("Daily Returns (Log)")
            #plt.show()
            #print(f"{ticks[0]} Average Daily Return: {round(stock1_ret.mean()*100,4):{10}}%\t {ticks[0]} Yearly Average Return: {round(stock1_ret.mean()*252*100,2)}%")
            #print(f"{ticks[1]} Average Daily Return: {round(stock2_ret.mean()*100,4):{10}}%\t {ticks[1]} Yearly Average Return: {round(stock2_ret.mean()*252*100,2)}%")
            #print(f"{ticks[2]} Average Daily Return: {round(stock3_ret.mean()*100,4):{10}}%\t {ticks[2]} Yearly Average Return: {round(stock3_ret.mean()*252*100,2)}%")
            #print(f"{ticks[3]} Average Daily Return: {round(stock4_ret.mean()*100,4):{10}}%\t {ticks[3]} Yearly Average Return: {round(stock4_ret.mean()*252*100,2)}%")
            #print(f"{ticks[4]} Average Daily Return: {round(stock5_ret.mean()*100,4):{10}}%\t {ticks[4]} Yearly Average Return: {round(stock5_ret.mean()*252*100,2)}%")
```

Appendix 102: Altiplano Class in Python (2/7)

```
        def drift_calc(data, return_type='log'): #adjust for dividends
            if return_type == 'log':
                lr = log_returns(data)
            elif reutrn_type == 'simple':
                lr = simple_returns(data)
            u = lr.mean()
            var = lr.var()
            drift = u-(0.5*var)
            try:
                return drift.values
            except:
                return drift

        drift = drift_calc(modified_data) #QUESTION: use data or modified data for drift

        div = portfolio[self.id]['div']
        # Drift adjusting if dividend paying (for Brownsche Motion)
        if div > 0:
            drift = drift - div
        #print(drift)
        #print()

        log_ret = log_returns(modified_data)

        stdev = log_returns(modified_data).std().values
        covar = log_ret.cov() #covariance matrix.
        #print(covar)
        #print()

        chol = np.linalg.cholesky(covar) #create cholesky matrix from covariance matrix
        #print(chol)
        #print()

        uncorr_x = norm.ppf(np.random.rand(num_stocks, simulated_days)) #stocks, days
        #print(uncorr_x)
        #print()

        corr_x = np.dot(chol, uncorr_x)
        #print(corr_x)
        #print()
```

Appendix 103: Altiplano Class in Python (3/7)

```
        corr_2 = np.zeros_like(corr_x) #Return an array of zeros with the same shape and type as a given array.
        for i in range(num_stocks):
            corr_2[i] = np.exp(drift[i] + corr_x[i])
        corr_2[0]

        stock0 = pd.DataFrame()                                          #create new data frame
        for s in range(len(ticks)):
            ret_reshape = corr_2[s]
            ret_reshape = ret_reshape.reshape(simulated_days) #Gives a new shape to an array without changing its data
            price_list = np.zeros_like(ret_reshape)
            price_list[0] = data.iloc[-1, s] #iloc = Purely integer-location based indexing for selection by position
            for t in range(1, simulated_days):
                price_list[t] = price_list[t-1]*ret_reshape[t]

            #print(f">------------------------------     {ticks[s]}     ------------------------------<")


            x = pd.DataFrame(price_list).iloc[-1]
            #fig, ax = plt.subplots(1,2, figsize=(14,4))
            #sns.distplot(x, ax=ax[0])
            #sns.distplot(x, hist_kws={'cumulative':True},kde_kws={'cumulative':True},ax=ax[1])
            #plt.xlabel("Stock Price")
            #plt.show()
            x = pd.DataFrame(price_list)
            stock0[ticks[s]]=x.loc[:,0]


        last_date = data.last_valid_index()
        last_date = pd.Timestamp(last_date)
        dates = [last_date]
        a = 1
        while len(stock0) != len(dates):
            next_date = last_date + timedelta(days = a)
            if next_date.weekday() < 5:
                dates.append(next_date)
                a += 1
            else:
                a+=1

        stock0["Date"] = dates
        stock0 = stock0.set_index("Date")
```

Appendix 104: Altiplano Class in Python (4/7)

```
            output_stocks_combined = data.append(stock0[1:]) #create one big data frame with complete data

            return output_stocks_combined


        def indicap(self,stock_prices,strikes):


           # notional = self.portfolio[self.id]["notional"]
            notional = 1
            cap = self.portfolio[self.id]["cap_indi"]
            floor = self.portfolio[self.id]["floor_indi"]
            ticks = self.portfolio[self.id]["underlyings"]

            basket_return = 0
            cum_basket_return = 0
            for i in range(len(stock_prices)):
                return_on_equity = min(cap,max(0, (stock_prices[i]/strikes[i])-1))
                cum_basket_return += return_on_equity
                basket_return = (cum_basket_return/5)
            #print(f"basket_return:{basket_return}")

            remuneration = 0
            remuneration = notional*(1 + max(floor, min(cap, basket_return)))
            payoff = remuneration
            #print(f"payoff:{payoff}")
            return payoff

        def payoff(self):

            ticks = self.portfolio[self.id]["underlyings"]
            maturity_date = portfolio[self.id]['maturity']
            payoffs = []

            payoffs_pos = []
            payoffs_pos2 = []

            payoffs_neg = []
            payoffs_neg2 = []

            for i in range(number_simulations):
```

Appendix 105: Altiplano Class in Python (5/7)

```
                output = self.cholesky()

                stock_prices = []

                stocks_pos = []
                stocks_pos2 = []

                stocks_neg = []
                stocks_neg2 = []

                for k in ticks:
                    maturity_date = self.portfolio[self.id]['maturity']
                    maturity_value = output._get_value(maturity_date, k)
                    stock_prices.append(maturity_value)

                for k in range(len(stock_prices)):
                    pos_change = stock_prices[k] * (1 + percentage_change)
                    pos_change2 = pos_change * (1 + percentage_change)
                    stocks_pos.append(pos_change)
                    stocks_pos2.append(pos_change2)

                    neg_change = stock_prices[k] * (1 - percentage_change)
                    neg_change2 = neg_change * (1 - percentage_change)
                    stocks_neg.append(neg_change)
                    stocks_neg2.append(neg_change2)

                normal_payoff = self.indicap(stock_prices, self.portfolio[self.id]["strikes"])
                payoffs.append(normal_payoff)

                pos_payoff = self.indicap(stocks_pos, self.portfolio[self.id]["strikes"])
                payoffs_pos.append(pos_payoff)

                neg_payoff = self.indicap(stocks_neg, self.portfolio[self.id]["strikes"])
                payoffs_neg.append(neg_payoff)


                #Gamma-Calculation-Payoffs
                pos_payoff2 = self.indicap(stocks_pos2, self.portfolio[self.id]["strikes"])
                payoffs_pos2.append(pos_payoff2)
```

Appendix 106: Altiplano Class in Python (6/7)

```
                neg_payoff2 = self.indicap(stocks_neg2, self.portfolio[self.id]["strikes"])
                payoffs_neg2.append(neg_payoff2)


            average_payoff = sum(payoffs)/len(payoffs)
            average_pos = sum(payoffs_pos)/len(payoffs_pos)
            average_neg = sum(payoffs_neg)/len(payoffs_neg)
            average_pos2 = sum(payoffs_pos2)/len(payoffs_pos2)
            average_neg2 = sum(payoffs_neg2)/len(payoffs_neg2)

            return [average_payoff, average_pos, average_neg, average_pos2, average_neg2]
```

Appendix 107: Altiplano Class in Python (7/7)