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Pedro Pita Barros
Universidade Nova de Lisboa and CEPR
Xavier Martinez-Giralt
Universitat Autònoma de Barcelona

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Pedro Pita Barros
Universidade Nova de Lisboa and *CEPR*

Xavier Martinez-Giralt
Universitat Autònoma de Barcelona

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Abstract

We present a simple model of spatial competition to analyze the impact of a structural change in transaction (transportation) costs on the location of firms.

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MAILING ADDRESSES:

CODE
Departament d'Economia
Universitat Autònoma de Barcelona
Edifici B
08193 Bellaterra
Spain.
Fax: 34-3-581 24 61
e-mail:xmg@cc.uab.es

Faculdade de Economia
Universidade Nova de Lisboa
Travessa Estêvão Pinto-Campolide
1070 Lisboa
Portugal.
351-1-388 60 73
ppbarros@fe.unl.pt

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1 Introduction

One of the events that will make the twentieth century be remembered is the building of the Channel Tunnel inaugurated on May 6th, 1994, after seven years of works.

In a more modest perspective, such a tunnel will have an impact on economic activity especially between the UK and France, but also on the rest of Europe. One particular aspect of this economic activity concerns the (re)location of production activities.

From the UK perspective, two arguments can be put forth. On the one hand, making the continental (big) market closer gives incentives to locate near the coast; on the other hand, the tunnel means (among other things) important savings, at least, in terms of time. Therefore, the need to locate close to the continental market is not as crucial as could have been before. The net effect may go either way.

The example of the Channel Tunnel ("Chunnel" for short) provides the intuition for a variety of problems. In particular, non-homogeneity of geographic space (mountains, valleys, rivers, etc ...) and construction of major highways would have a similar effect. Other examples besides the "Chunnel" are the high-speed train (TGV) from Lisbon to Madrid (and from there to the rest of Europe) or the projected connection through a system of mountain tunnels between Barcelona and Paris. Even within countries we can find examples, like the construction of highways connecting the Portuguese coast (more developed) to the inland (less developed).

An important interpretation of the model relates to barriers to economic integration among regions. Building of infra-structures in less developed regions often have the aim of inducing location of economic activities in these regions. Facilities that may become available in one region may give rise to positive externalities if they incentivate the location of new industries in the neighboring regions. They may also impose negative externalities if such facilities turn into a relocation of industries from the neighboring regions to the one providing the new facilities. Whether the latter is always a reasonable expected effect, or not, is the focus of the paper.

We propose a model where two firms are located at two distant points in a space (say Manchester and Paris[1]) described by a non-connected set, where one of the components of this space is larger in terms of potential demand than the other.

We look into the location choice of firms in a very stylized way. Nonetheless, we hope it will provide more economic ground to discuss some policies directed to change firms' location decisions (for example, the appearance of business parks close to the Tunnel were backed by local authorities).

We choose to cast our analysis in the simplest economic model of location choice – the location-price game introduced by Hotelling (1929) under quadratic transport costs (d'Aspremont et al., 1979). The distinctive features of our work are twofold. First, we introduce a set of possible locations that is non-connected. That is, there are some points in the space that cannot be used as locations for either consumers or firms. Second, consumers have to pay a (linear) transport cost to cross such space and quadratic transport costs in each market. Two different specifications of transport costs are investigated, yielding distinct implications to firms' location decisions.

More in general, this work addresses the question of the impact on firms' locations when the economy faces a structural change in transport costs. I this sense we can relate our paper to the literature on transaction costs. A particular instance of this more general body of literature is provided by Bouckaert and Degryse (1995) where conditions under which banks offer phonebanking are studied, where phonebanking constitutes a new technology for transport costs made available.

The structure of the paper is as follows. Next section presents the basic model and two alternative formulations of the transportation costs. Sections three and four analyze the equilibria in the price game and the tendencies for firm relocation under the alternative specifications of the transport costs. Section five contains a discussion of the features of the equilibria in the different model formulation and draws some conclusions.

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[1] We could perhaps refer to Bordeaux to avoid some chauvinism.

2 The model.

2.1 The basic structure.

The model we propose departs from a space composed of two separated markets and a "hole" between them. One of these markets is larger in terms of potential demand than the other. For the sake of clarity in the exposition, we will carry over the example of the Channel Tunnel. That is, we will refer to UK and the Continent as the markets and to the Channel as the space linking them. Construction of the Tunnel is our example of a structural change in transport costs.

We assume that there is a continuum of consumers uniformly distributed on each market. The density of consumers in each market is normalized to unity. All consumers are identical except for their location (best preferred variety). The (common) reservation price is high enough to allow all consumers to buy one unit of a (horizontally differentiated) commodity.

Transport costs within every market are modeled as a quadratic function of distance.[2] Exporting or importing the commodity implies to add to the transport cost a fixed amount associated with the crossing of the Channel.

We denote by 0 and L the extreme points of the small market; T denotes the length of the channel; $L + T$ and $kL + T$ denote the extremes of the big market (implying th[illegible] 2). All distances are measured from zero. The smaller country is termed country 1, the other is country 2. Finally, "firm d" is the firm located in country 1, or if both firms are located in the same country, it denotes the firm located closest to 0. The other firm is "firm f".[3]

The starting point will be the case of isolated markets (i.e. where the costs of crossing the Channel are too high), where the reservation price is high enough to sustain an equilibrium where each firm is located in the mid-point of its (domestic) market.[4]

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[2] We should [illegible] specify that the transport cost rate is defined per unit of commodity. Since all consumers buy [illegible]ctly one unit, such an assumption would be redundant.

[3] Symmetric [illegible]ses will be omitted, unless otherwise noticed.

[4] Although [illegible] an assumption is arbitrary, the location at the center of the market has the appealing feature of [illegible]mizing the aggregate transport costs.

We can approximate the opening of the "Chunnel" as a (partial) integration of the two markets, so that now the space becomes a connected set. The distribution of consumers on the space remains the same though (i.e. there are no consumers between Folkstone and Calais). Now firms in both markets (countries) have access to the neighboring market paying a fixed fee of £qT (i.e. a linear function of the distance). That is to say, the firm in country j willing to reach a consumer in market i has to pay a transport cost defined as the sum of a fixed fee and the corresponding costs associated to the distance between consumer i and firm j, net of the the Tunnel.

2.2 Transport costs

For the analysis of market integration and location of firms, specification of transport costs does matter. Previous literature has identified trade-offs, in location decisions, between market size, transport costs and wages.[5] Moreover, as it will be apparent below, different ways of computing transport costs gives substantially different equilibrium characterization.

(i) The first possible specification for transport costs is to have quadratic transport costs (cumulative) in all the path outside the Tunnel (and excluding the Tunnel distance). For example, if a firm is located at d in country 1 and serves a consumer located at i in country 2, the associated transport costs are

$$c(i - T - d)^2 + qT$$

This implies that the transport cost at the margin is the same at both ends of the Tunnel.

(ii) The alternative specification is to have independent quadratic transport costs in each market, besides the linear fee associated with the Tunnel. For the same firm and consumer of the previous example, we have

$$c(i - (L + T))^2 + qT + c(L + T - d)^2$$

[5] See Cordella and Grilo (1995), Horstmann and Markusen (1987, 1992), Krugman and Venables (1991), Motta (1992) and Rowthorn (1992), among other.

In this case, at the margin, the transport cost is lower at the other edge of the Tunnel (transport costs in the other country are computed with reference to the Tunnel mouth). Both cases are depicted in Figure 1.

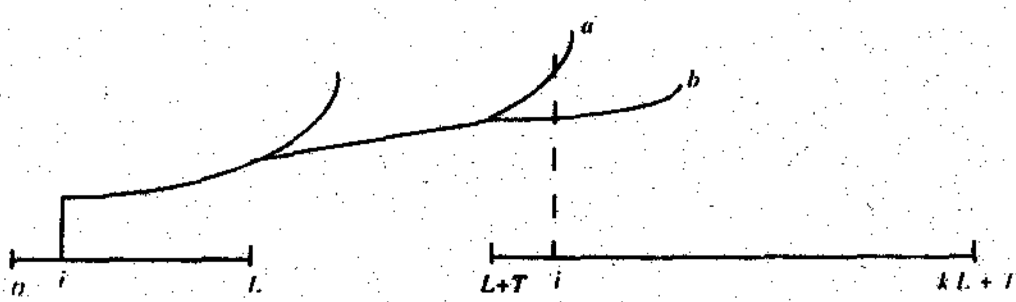

Figure 1: Transport costs.

To illustrate the first specification think of a consumer located at $i > L + T$ (Bordeaux) driving from his/her location to $L + T$ (Calais), taking the car into the train paying the corresponding (two-way) ticket at a price £qT, and driving again from L (Folkstone) to d (Manchester), and back. We will refer to this modeling as CAR transport costs (standing for "Continue After a Rest").

The second way of specifying the transport costs can be illustrated by our consumer living in i taking a taxi to Calais, getting in the train, and taking another taxi from Folkstone to Manchester. We will call this CAB transport cost (standing for "Change At the Border").

Formally, the total CAR transport cost function for firm d is

$$CAR(d,i) = \begin{cases} c(i - T - d)^2 + qT & \text{if } i \geq L + T. \\ c(i - d)^2 & \text{if } i \leq L \end{cases} \quad (1)$$

and

$$\left.\frac{\partial CAR(d,i)}{\partial(i-d)}\right|_{(L+T)^+} = \left.\frac{\partial CAR(d,i)}{\partial(i-d)}\right|_{L^-} = 2c(L-d) \tag{2}$$

where $(L+T)^+$ refers to the partial derivative at $L+T$ from the right, and L^- refers to the partial derivative at L from the left.

Total CAB transport-cost function for firm d is given by

$$CAB(d,i) = \begin{cases} c(i-(L+T))^2 + qT + c(L+T-d)^2 & \text{if } i \geq L+T, \\ c(i-d)^2 & \text{if } i \leq L \end{cases} \tag{3}$$

and, at the border, marginal transport costs differ:

$$\left.\frac{\partial CAB(d,i)}{\partial(i-d)}\right|_{(L+T)^+} \neq \left.\frac{\partial CAB(d,i)}{\partial(i-d)}\right|_{L^-} = 2c(L-d) \tag{4}$$

Note that the CAR transport costs constitute the more natural generalization of quadratic transport costs, in the sense that $T = 0$ recovers the traditional transport costs schedule.

3 Equilibrium under CAR transport costs.

In this section, the first formulation for transport costs is used. The alternative formulation receives attention in the next section. To simplify notation, we perform, in this section, two normalizations [illegible] $L = 1$. For expositional purposes the different cases will treated in separated subsections.

3.1 Firm f and firm d in their markets, z in the big market.

Suppose each firm is located in its domestic market and the indifferent consumer, z, is located in the big country. Then it is defined by the solution to

$$p_d + qT + [illegible] = p_f + c(f - z)^2$$

This gives

$$z = \frac{f^2 - d^2 \; [illegible] - qT - 2dT - T^2}{2c(f - T - d) \; [illegible]}$$

The profit function of each firm is, respectively,

$$\Pi_d = p_d \, [illegible] \qquad \Pi_f = p_f \, [illegible] + T - z)$$

The solution to the price game yields

$$p_d = \frac{(f-T-d)(f-T+d+2k)-qT}{3}$$
$$p_f = \frac{(f-T-d)(4k+T-d-f)+qT}{3}$$

It is easy to verify that p_d is always positive. Also, p_f is positive if

$$\frac{qT}{f-T-d} < f-T+d+2k \qquad (5)$$

Evaluating $z(p_d, p_f)$ at the candidate equilibrium price pair, we obtain,

$$z(p_d, p_f) = \frac{(f-d-T)(f+d+2k+5T)-qT}{6(f-d-T)}$$

and for the equilibrium to be well defined $z(p_d, p_f) \in [1+T, f]$, which holds if

$$2k+d+5T-5f \leq \frac{qT}{f-T-d} \leq f-T+d-6+2k \qquad (6)$$

It is trivial to check that conditions (6) imply condition (5). Also, $k > 2.5$ is sufficient to ensure that the upper bound in conditions (6) is positive. Thus, it will be assumed throughout.

Substituting the equilibrium price pair into profit functions defines the equilibrium profits accruing from the price subgame

$$\Pi_d(d,f) = \frac{p_d^2}{2(f-d-T)} \quad ; \quad \Pi_f(d,f) = \frac{p_f^2}{2(f-d-T)}$$

Next, we will examine tendencies in locations for firms d and f at those equilibrium prices.[6]

Consider the derivative of firm d's profits with respect to its location. It is given by,

$$\frac{\partial \Pi_d(d,f)}{\partial d} = \frac{p_d[(f-d-T)(f-3d-2k-T)-qT]}{9(f-d-T)^2} < 0.$$

[6]We cannot compute a subgame perfect equilibrium of a location price game, since demands are not quasi-concave. To see it, consider how demand for, say, firm d evolves given a price of firm f. It is easy to compute the minimum price that prevents firm d from capturing any demand; as it lowers the price its demand increases. At a certain price, firm d will capture all consumers in the national market. Further increases in its demand, requires to lower the price enough to compensate for the crossing of the channel. Therefore, there is a range of prices of firm d for which demand is completely inelastic. Such a feature provides a kink in the demand function that destroys its concavity.

where the negative sign follows from $k + T > f > 1 + T$ and $d < 1$. Thus, firm d towards zero, independently of the location of firm f in the larger market.

Take, in turn, firm d's location as given and consider the derivative of firm f's profits with respect to its location. It is given by,

$$\frac{\partial \Pi_f(d, f)}{\partial f} = \frac{p_f[(f - d - T)(d - 3f + 4k + 3T) - qT]}{6(f - d - T)^2}$$

Two cases may arise. Either this derivative is positive or negative.

Case I: $\dfrac{\partial \Pi_f(d, f)}{\partial f} < 0$. The derivative is negative if $(f - d - T)(d - 3f + 4k + 3T) - qT < 0$. This condition can be expressed as

$$\frac{qT}{f - T - d} > d - 3f + 4k + 3T. \tag{7}$$

Combining (7) and (6), we obtain

$$-3f + d + 4k + 3T < \frac{qT}{f - T - d} < f + d + 2k - T - 6.$$

This is a well-defined interval if $-2f + k + 2T + 3 < 0$.

Case II: $\dfrac{\partial \Pi_f(d, f)}{\partial f} > 0$. The derivative is positive if $(f - d - T)(d - 3f + 4k + 3T) - qT > 0$. That is to say, if

$$\frac{qT}{f - d - T} < d - 3f + 4k + 3T \tag{8}$$

The combination of (8) and (6) gives rise to the following condition

$$\frac{qT}{f - T - d} < \min\{-3f + d + 4k + 3T; f + d + 2k - T - 6\} \tag{9}$$

Putting together (9) and (6) we obtain.

$$\frac{qT}{f - T - d} < d - 5f + 2k + 5T$$
$$\frac{qT}{f - T - d} < f + d + 2k - T - 6 \text{ if } \Lambda(f) > 0$$
$$\frac{qT}{f - T - d} < d - 3f + 4k + 3T \text{ if } \Lambda(f) < 0$$

where $\Lambda(f) = 2(T - f) + k - 3$.

The above discussion can be summarized as.

Proposition 1. *Let each firm be located in its domestic market and the indifferent consumer, z, be located in the big country.*

(a) *Firm d tends to locate towards 0.*

(b) *Let $2(T-f)-k-3<0$. Then firm f tends to locate towards $k-T$.*

(c) *Let $2(T-f)-k-3>0$.*

- *firm f tends to locate towards $1+T$ if*

$$d-3f+4k+3T<\frac{qT}{f-T-d}<f+d+2k-T-6$$

- *firm f tends to locate towards $k+T$ if*

$$d-5f+2k+5T<\frac{qT}{f-T-d}<d-3f+4k+3T$$

As a reference point, note that for $T=0$, $\frac{\partial \Pi_f(d,f)}{\partial f}>0$, so that we recover the standard result of d'Aspremont et al. (1979).

3.2 Firm f and firm d in their markets, z in the small market.

Consider now the case of the indifferent consumer located in the small country. Suppose that firms are located in different countries. The position of the indifferent consumer is defined by

$$p_d+(z-d)^2=p_f+qT+(f-T-z)^2$$

or solving explicitly

$$z=\frac{f+d-T}{2}-\frac{p_f-p_d}{2(f-T-d)}+\frac{qT}{2(f-T-d)}$$

Maximization of profits, given locations, yields the following equilibrium prices:

$$p_d=\frac{(f-T)^2-d^2+2k(f-d-T)+qT}{3}$$

$$p_f=\frac{d^2-(f-T)^2+4k(f-d-T)-qT}{3}$$

Prices must be positive, which is guaranteed by the following condition:

$$\frac{qT}{f-T-d} < 4k - (f+d-T) \qquad (10)$$

The condition requires the Tunnel fee to be small or the size difference between countries to be large (k big).

We must now impose that, at equilibrium prices, the indifferent consumer position is consistent with initial assumptions: $1 \geq z \geq d$. This amounts to

$$6+T-f-d-2k \geq \frac{qT}{f-d-T} \geq 5d+T-f-2k \qquad (11)$$

provided country 2 is not too large (the exact condition being $2k < 6+T-f-d$). The interval is well-defined as $d \leq 1$.

So far, we have characterized the candidate for a price equilibrium given a pair of locations of firms (d, f) and the conditions under which this candidate is a *viable* equilibrium.

We now turn to the incentives of firms to change location. To this effect, we investigate the value of change in profits from a marginal change in the location of the firm. Equilibrium profits are

$$\Pi_d = \frac{p_d^2}{2(f-d-T)} \quad ; \quad \Pi_f = \frac{p_f^2}{2(f-d-T)}$$

A small change in the location of firm d implies

$$\frac{\partial \Pi_d}{\partial d} = \frac{p_d}{2(f-d-T)}\left[p_d - 4(f-d-T)\frac{d+k}{3}\right]$$

Then, it is straightforward to establish

$$\frac{\partial \Pi_d}{\partial d} < 0 \text{ if } \frac{qT}{f-d-T} < 2k+2d-(f-d-T) \qquad (12)$$

From the conditions for equilibrium to be well-defined (see expression (11)), it results that the above condition always holds. Thus, $\partial \Pi_d / \partial d < 0$, whatever the location of the other firm, relative size of countries size and Tunnel fee, provided equilibrium exists. Since this effect has a constant sign, firm d will locate away from the Tunnel (under the restriction that each firm locates in a different country and the indifferent consumer is in small country).

Consider a small change in the location of firm f. Computations reveal that

$$\frac{\partial \Pi_f}{\partial f} = \frac{p_f}{2(f-T-d)^2}\left[-p_f + 2(f-t-d)\frac{-2(f-T)+4k}{3}\right]$$

The sign of the expression hinges upon the sign of

$$(f+d-T)+4k+\frac{qT}{f-T-d}-4(f-T) \tag{13}$$

Combining this condition with the requirement of a well-defined equilibrium, the following basic characterization results:

$\frac{\partial \Pi_f}{\partial f} > 0$ if

(a) $2(f-T)-k-3d < 0$;

(b)

$$3 > 2(f-T)-k > 3d \quad and$$
$$\frac{qT}{f-T-d} > 3(f-T)-4k-d$$

$\frac{\partial \Pi_f}{\partial f} < 0$ if

(c) $2(f-t)-k > 3$.

(d)

$$3 > 2(f-T)-k > 3d \quad and$$
$$\frac{qT}{f-T-d} < 3(f-T)-4k-d$$

We may now investigate in more detail the stated conditions. Take first $\partial\Pi_f/\partial f > 0$. It is straightforward to establish that

$$0 > 2(f-T)-k-3 > 3(f-T)-4k-d$$

Since $qT > 0$, the second part in condition (b) is redundant. Consider $\partial\Pi_f/\partial f < 0$. Then,

$$0 > 2(f-T)-k-3 > 3(f-T)-4-d$$

which means that condition (d) is never satisfied.

Proposition 2. *Let each firm be located in its domestic market and let the indifferent consumer, z, be located in the small country. Then*

(a) *firm d tends to locate towards 0;*

(b) *firm f locates towards*

1. $k + T$ *if* $3 > 2(f - T) - k$;
2. $1 + T$ *if* $3 < 2(f - T) - k$.

3.3 Both firms in the same market.

■ Firm f and firm d in the big country.

The indifferent consumer is defined implicitly by

$$p_d + (z - d)^2 = p_f + (f - z)^2$$

Solving explicitly

$$z = \frac{f + d}{2} + \frac{p_f - p_d}{2(f - d)}$$

Profits are defined by

$$\Pi_d = (z - T)p_d \quad ; \quad \Pi_f = (k + T - z)p_f$$

for firm d and firm f, respectively. Maximizing and solving for the equilibrium prices, we get

$$\begin{aligned} p_d &= \frac{(f - d)(d + f + 2k - 2T)}{3} \\ p_f &= \frac{(f - d)(4k - f - d + 2T)}{3} \end{aligned}$$

Prices are always positive. It is, nevertheless, necessary that $z \in [d, f]$, which amounts to

$$5f - d - 4T > 2k > 5d - f - 4T. \tag{14}$$

It implies a very narrow range for k. Equilibrium profits are

$$\Pi_d = \frac{(f - d)(d + f + 2k - 2T)^2}{18} \quad ; \quad \Pi_f = \frac{(f - d)(4k - f - d + 2T)^2}{18}$$

The tendencies for changes of location, at the margin, can be obtained after straightforward derivations:

$$\begin{aligned}\text{sign}\ \frac{\partial\Pi_d}{\partial d} &= \text{sign}\ (f - 3d - 2k + 2T) < 0\\ \text{sign}\ \frac{\partial\Pi_f}{\partial f} &= \text{sign}\ (4k - 3f + d + 2T) > 0\end{aligned}$$

Therefore, also in this case firms tend towards the endpoints of the market.

■ **Firm f and firm d in the small market.**

The indifferent consumer is defined implicitly by

$$p_d + (z - d)^2 = p_f + (f - z)^2$$

Solving explicitly

$$z = \frac{f + d}{2} + \frac{p_f - p_d}{2(f - d)}$$

Profits are defined by

$$\Pi_d = zp_d \quad ; \quad \Pi_f = (k - z)p_f$$

for firm d and firm f, respectively. Maximizing each firm's profit with respect to own price and solving for the equilibrium prices, we get

$$p_d = \frac{(f - d)(d + f + 2k)}{3} \quad ; \quad p_f = \frac{(f - d)(4k - f - d)}{3}$$

Prices are always positive. It is, nevertheless, necessary that $z \in [d, f]$ for the equilibrium to be well defined, which amounts to $k \leq (5f - d)/2$. It implies a very narrow range for k, as at the most k can be 2.5 (by definition, k has a lower bound at 2). Equilibrium profits are

$$\Pi_d = \frac{(f - d)(d + f + 2k)^2}{18} \quad ; \quad \Pi_f = \frac{(f - d)(4k - f - d)^2}{18}$$

The tendencies for changes of location, at the margin, can be obtained after straightforward derivations:

$$\text{sign}\ \frac{\partial\Pi_d}{\partial d} = \text{sign}\ (f - 3d - 2k) < 0 \qquad (15)$$

$$\text{sign}\ \frac{\partial\Pi_f}{\partial f} = \text{sign}\ (4k - 3f + d) > 0 \qquad (16)$$

The signs of expressions result from $d, f \leq 1$ and $k \geq 2$. Thus both firms tend towards the endpoints of the market.

Summarizing,

Proposition 3. *Let both firm locate in the same country. Then firms tend towards different endpoints of the market.*

4 Equilibrium under CAB transport costs.

In this section we assume that transport costs are independent in each market, besides the linear fee associated with the Tunnel. This is illustrated by consumers changing taxis at the border. As in the previous section, we implement two normalizations: $c = 1, L = 1$.

4.1 Firm f and firm d in their markets, z in the big market.

Suppose each firm is located in its domestic market and the indifferent consumer, z, is located in the big country. Then, the indifferent consumer position is defined by the solution to

$$p_d + (1-d)^2 + qT + (z-1-T)^2 = p_f + (f-z)^2$$

This gives

$$z = \frac{f^2 - (1+T)^2 - (1-d)^2 - qT + p_f - p_d}{2(f-T-1)}$$

The profit function of each firm is, respectively,

$$\Pi_d = p_d(z-T) \quad ; \quad \Pi_f = p_f(k+T-z)$$

The solution to the price game yields

$$\begin{aligned} p_d &= \frac{(f-1-T)(f+1-T+2k) - (1-d)^2 - qT}{3} \\ p_f &= \frac{(f-1-T)(T+4k-1-f) + (1-d)^2 + qT}{3} \end{aligned}$$

Define $\Delta \equiv \dfrac{qT + (1-d)^2}{f-1-T}$. Prices are positive if

$$1 - T + f - 4k < \Delta < f + 1 - T + 2k \tag{17}$$

Evaluating $z(p_d, p_f)$ at the candidate equilibrium price pair, we obtain,

$$z(p_d, p_f) = \frac{(f - 1 - t)(2k + 1 + f + 5t) - (1 - d)^2 - qt}{6(f - 1 - t)}$$

and $z(p_d, p_f) \in [1 + T, f]$ if

$$2k - 5f + 5T + 1 < \Delta < 2k + f - T - 5 \tag{18}$$

It is trivial to check that conditions (18) imply condition (17). Also, we will assume that $k > 2.5$ to ensure that the upper bound in (18) is positive.

Substituting the equilibrium price pair into profit functions defines the equilibrium profits accruing from the price subgame

$$\Pi_d(d, f) = \frac{p_d^2}{2(f - 1 - T)} \quad : \quad \Pi_f(d, f) = \frac{p_f^2}{2(f - 1 - T)}$$

Next, we will examine tendencies in locations for firms d and f at those equilibrium prices.

Consider the derivative of firm d's profits with respect to its location. It is given by,

$$\frac{\partial \Pi_d(d, f)}{\partial d} = \frac{2p_d(1 - d)}{3(f - 1 - T)} > 0.$$

Thus, whatever the location of firm f, firm d always tends to move towards the mouth of the Tunnel.

Take firm d's location as given and consider the derivative of firm f's profits with respect to its location. It is given by,

$$\frac{\partial \Pi_f(d, f)}{\partial f} = \frac{p_f[(f - 1 - T)(1 + 3T + 4k - 3f) - (1 - d)^2 - qT]}{6(f - 1 - T)^2}$$

Two cases may arise. Either this derivative is positive or negative.

Case I: $\frac{\partial \Pi_f(d, f)}{\partial f} > 0$. The derivative is positive if $(f - 1 - T)(1 + 3T + 4k - 3f) - (1 - d)^2 - qT > 0$. This condition can be expressed as

$$\Delta < 1 + 3T + 4k - 3f. \tag{19}$$

Combining conditions (19) and (18), we obtain that if $2f - k - 2T - 3 < 0$, then condition (18) implies that the above derivative is positive. If on the contrary $2f - k - 2T - 3 > 0$ then the derivative is positive if

$$2k - 5f + 5T + 1 < \Delta < 1 + 3T + 4k - 3f \tag{20}$$

Case II: $\frac{\partial \Pi_f(d,f)}{\partial f} < 0$. The derivative is negative if $(f-1-T)(1+3T+4k-3f)-(1-d)^2-qT<0$. That is to say, if

$$\Delta > 1 + 3T + 4k - 3f \tag{21}$$

The combination of (21) and (18) gives rise to the following condition

$$1 + 3T + 4k - 3f < \Delta < 2k + f - T - 5 \tag{22}$$

The above discussion can be summarized in the following figure.

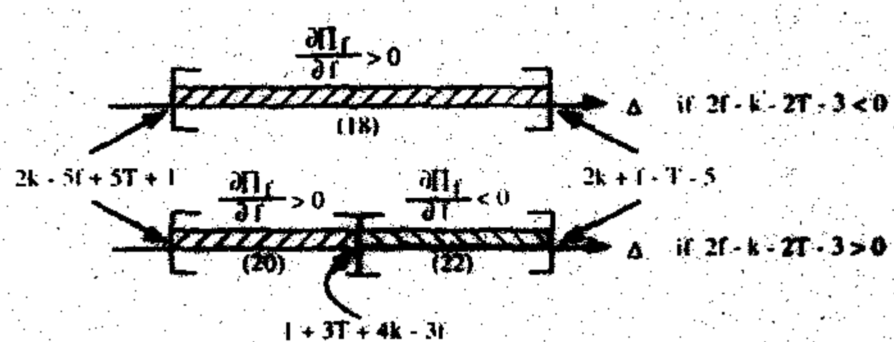

Figure 2:

Formally,

Proposition 4. *Let each firm be located in its domestic market and the indifferent consumer, z, be located in the big country.*

$$\frac{\partial \Pi_d(d,f)}{\partial d} > 0 \tag{23}$$

$$\frac{\partial \Pi_f(d,f)}{\partial f} < 0 \quad \text{if (22) holds,} \tag{24}$$

$$\frac{\partial \Pi_f(d,f)}{\partial f} > 0 \quad \text{otherwise.} \tag{25}$$

4.2 Firm f and firm d in their markets, z in the small market.

We now proceed to the case of firms are located in different countries and the indifferent consumer located in the small country. The position of the indifferent consumer is defined by

$$p_d + (z-d)^2 = p_f + qT + (f-(T+1))^2 + (1-z)^2$$

Solving yields

$$z = \frac{p_f - p_d + (f - 1 - T)^2 - d^2 + qT}{2(1 - d)}$$

Maximization of profits, given the fixed locations, yields the following equilibrium prices:

$$p_d = \frac{(2k + 1 + d)(1 - d) + (f - T - 1)^2 + qT}{3}$$

$$p_f = \frac{(1 - d)(4k - 1 - d) - (f - T - 1)^2 - qT}{3}$$

Define $\Phi \equiv \frac{qT + (f - 1 - T)^2}{1 - d}$. Prices must be positive, which is guaranteed by the following condition:

$$\Phi < 4k - 1 - d \qquad (26)$$

We must now impose the following conditions so that, at equilibrium prices, the indifferent consumer position is consistent with initial assumptions ($1 \geq z \geq d$):

$$5 - d - 2k > \Phi > 5d - 1 - 2k \qquad (27)$$

It is easy to check the interval is well defined. In addition, it is necessary that $k < (5 - d)/2$, as the Tunnel fee is positive.[7] It is straightforward to establish that the left-hand side condition on Φ also implies positive prices.

We now look at the incentives of firms to change location. Equilibrium profits are

$$\Pi_d = \frac{p_d^2}{2(1 - d)} \quad ; \quad \Pi_f = \frac{p_f^2}{2(1 - d)}$$

A small change in the location of firm d implies

$$\text{sign}\,\frac{\partial \Pi_d}{\partial d} = \text{sign}[(1 - d)(1 - 3d + 2k) + (f - 1 - T)^2 + qT] > 0 \qquad (28)$$

Consider now a small change in the location of firm f. Computations reveal

$$\frac{\partial \Pi_f}{\partial f} = \frac{p_f}{3(1 - d)}\frac{\partial p_f}{\partial f} = \frac{2p_f}{3(1 - d)}(f - 1 - T) < 0$$

Thus, we have

[7] Although it is not sufficient for existence of equilibrium.