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Teachers' knowledge and technology in the teaching of functions: A view from Portugal and Brazil

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ABSTRACT

Technology integration although highly recommended has proved to be complex. This study aims to characterize the perceptions of secondary mathematics teachers (grades 7th to 12th), from Portugal (Braga region) and Brazil (Campinas region), regarding the teaching of Functions and the use of technology. We also intend to consider the context of the two countries, reflecting on justifications for the differences identified (if any). The study adopts a mixed methodology based on a questionnaire applied to 129 Portuguese teachers in Braga region and 136 Brazilian teachers in Campinas region (quantitative dimension) and on an interview to six Portuguese teachers and six Brazilian teachers (qualitative dimension). The results point to similarities between teachers from Portugal and Brazil, but also to some differences. Technology potential is recognized by teacher from both countries, however teachers from Brazil tend to assume a more traditional use of technology, based on showing and more teacher-centered. This difference can be related to the Portuguese curriculum, valuing a relevant use of technology for more than three decades.

1. Introduction

The concept of function plays a central role in secondary mathematics education (Modabbernia et al., 2023), serving as a unifying idea across multiple domains such as algebra, calculus, and statistics (Uscanga et al., 2024). Understanding functions is not only fundamental for progressing in mathematics study but also essential for developing broader analytical and problem-solving skills that are increasingly valued in a range of academic and professional fields (Falcade et al., 2007; Trujillo et al., 2023).

Given its foundational importance, the teaching and learning of functions should be approached with methodologies that promote conceptual understanding, flexibility in representation, and connections between mathematical ideas. In this context, the use of digital technologies can offer significant pedagogical advantages. These tools allow students to explore functions dynamically, visualize relationships between variables, and engage in mathematical experimentation that would be difficult to achieve using traditional, paper-and-pencil methods alone (Rocha, 2020).

However, despite the recognized potential of technology to enrich the teaching of functions, its integration into classroom practice remains irregular and inconsistent. One of the factors influencing this variability are the political decisions, namely the official curricular documents, such as national programs and assessment frameworks. These documents often impact teachers' instructional priorities and decisions, either by explicitly encouraging the use of technological tools or, on the contrary, by limiting their use through

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high-stakes exam formats or rigid content guidelines (Sacristán et al., 2024).

Portugal and Brazil share a deep historical, linguistic, and cultural connection, rooted in centuries of colonial and post-colonial interaction. However, despite these common foundations, the two countries have followed significantly different trajectories in terms of current curricular policies, especially concerning the integration of technology in the teaching and learning of mathematics (MEC, 2018; Brasil, 2018). Nevertheless, the linguistic and cultural proximity between the two countries offers a valuable opportunity for comparative educational research (Jablonka et al., 2018). The combination of shared foundations and divergent present-day realities makes the study of teachers' professional knowledge particularly relevant. Understanding how teachers from Portugal and Brazil develop and enact their TPACK – Technological Pedagogical and Content Knowledge (Mishra & Kohler, 2006) within these differing curricular contexts can offer important insights into the role of policy environments in shaping instructional practices.

In this light, comparing the professional knowledge of mathematics teachers from Portugal and Brazil, especially regarding the integration of technology, provides a fertile ground for research. The contrast in curricular frameworks, assessment policies, and institutional expectations allows us to explore how context mediates teachers' decisions, while the cultural and linguistic affinities between the countries make such a comparison both feasible and meaningful. These factors reinforce the pertinence of investigating how curriculum, policy, and professional knowledge intersect in shaping technology use in mathematics education across these two Lusophone nations.

In this study we intend to characterize the perceptions of secondary mathematics teachers (grades 7th to 12th), from Portugal (Braga region) and Brazil (Campinas region), regarding the teaching of Functions and the use of technology. We also intend to consider the context of the two countries, reflecting on justifications for the differences identified (if any). Specifically, we intend to answer the following research questions:

(1) What are the perceptions of mathematics teachers from Portugal and Brazil regarding the use of technology to teach Functions?
(2) What are the differences (if any) between the perceptions of mathematics teachers from Portugal and Brazil regarding the teaching of Functions and the use of technology?

This is a study on the teachers' professional knowledge. However, being mainly a quantitative study, data collection was based on a questionnaire. As so, the information collected consists of what the teachers shared about their practice. As there is no direct observation to infer the teachers' knowledge, we chose to refer to perceptions of the teachers' knowledge and not simply to teachers' knowledge (although, for simplicity of writing, in some parts we simply refer to knowledge).

Besides this, when we speak about Portuguese/Brazilian teachers, we are referring to teachers from that country participating in the study and we do not intend to refer to all the teachers in the considered country or to suggest generalization of our results (as detailed in the methodology section).

2. Teachers' professional knowledge

Research in teachers' knowledge gained relevance with the work of Shulman (1987), who organized it into several dimensions,

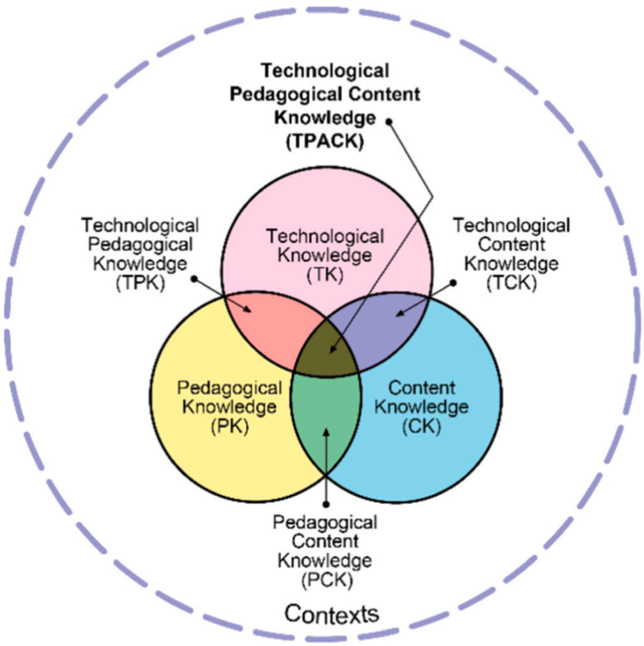

Fig. 1. TPACK model by Mishra and Koehler (reproduced by permission of the publisher, © 2012 by tpack.org).

highlighting PCK – *Pedagogical Content Knowledge* as the teachers' specific professional knowledge for teaching. PCK results from a mix between *Content Knowledge* and *Pedagogical Knowledge*, whose value is placed on the teachers' own ability to transform "the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (Shulman, 1987, p. 15).

Although PCK has become almost immediately a reference in the field, several authors came forward with several critics, such as the difficulties in distinguishing between the domains of knowledge, the blurred nature of the knowledge, and its statics characteristic with no attention to the context. One of the authors who criticized the construct was Deborah Ball. Together with her research group she worked on the development of MKT – *Mathematical Knowledge for Teaching* (Ball et al., 2008) clarifying and subdividing the domains in subdomains, developing at the same time an instrument to measure the teachers' knowledge.

Besides the critics received and the many different interpretations of the construct, it continued to be used (and still is) in many studies. Very clear evidence of that is present on the systematic literature review by Grigaliūnierė et al. (2025), identifying 237 studies in Mathematics using PCK, published between 2013 and 2022.

The use of technology in the mathematics classrooms and the recognition of the need for a specific type of knowledge to integrate it in the teachers practice led to new developments. Inspired by PCK, Mishra and Koehler (2006) proposed the model TPACK – *Technological Pedagogical Content Knowledge* (Fig. 1).

For Mishra and Koehler (2006), CK – *Content Knowledge* encompasses the structures of the content to be taught or learned; PK – *Pedagogical Knowledge* integrates the processes and practices or methods of teaching and learning; and PCK concerns the knowledge to transform CK into appropriate forms that are better understood by the students. TK – *Technology Knowledge* consists of knowledge and proficiency on the use of technology. TCK – *Technological Content Knowledge* is directly linked to the way technology and content influence each other. This knowledge allows the teacher to select which technology is most appropriate to teach a certain content and which contents are suitable for using or not using the technology (Koehler & Mishra, 2008). TPK – *Technological Pedagogical Knowledge* is knowledge relating to the potential of technology and the way in which teaching can be changed depending on a given technology. TPACK knowledge emerges from the three knowledge domains (CK, PK and TK) and from the knowledge domains resulting from these (TCK and TPK).

Due to its origin, TPACK was the target of the same criticisms previously directed at PCK. Intending to overcome them, some authors proposed new models. According to a systematic review conducted by Rocha (2023), TPACK, KTMT – *Knowledge for Teaching Mathematics with Technology* (Rocha, 2020), and PTK – *Pedagogical Technology Knowledge* (Thomas & Hong, 2013) are the knowledge models identified in the literature. Both, KTMT and PTK, value elements from the teachers practice, what makes it difficult to use them in this study. PTK integrates elements from the teachers' instrumental genesis, and KTMT integrates two sets of interdomains knowledge related to the teachers' options in their classroom practice. On the contrary, TPACK is a model often used in studies based on questionnaires (Rocha, 2024). However, this study, although not including observation of teachers' practice, intends to get information about it. As so, we adopt the TPACK model, combining it with elements inspired by the KTMT model.

The two sets of interdomains in KTMT are MTK – *Mathematics and Technology Knowledge*, and TLTK – *Teaching and Learning and Technology Knowledge*. TLTK includes a strong focus on the characteristics of the tasks proposed by the teacher, the role assumed by the teacher/students, and the role of the technology. MTK includes a strong focus on how technology impacts the students' interactions with mathematical concepts (e.g., use of different representations, impact over the concepts as experienced by the students). As KTMT values the interdomains knowledge, the *TPACK-inspired by KTMT* values in PCK the *strategies to teach functions* and the *options adopted to involve the students*; in TPK the *pedagogical use of the technology*, the *consequences of technology use*, and *the use of technology versus the use of an approach based on paper and pencil*; in TCK the *mathematical use of technology*, and the *development of mathematical skills*.

2.1. Methodology

This study aims to characterize the perceptions of Portuguese and Brazilian mathematics teachers in Braga region and Campinas region, respectively, regarding the teaching of Functions and the use of technology. It also intends to consider the context of the two countries, reflecting on justifications for the differences identified (if any). To achieve this goal, data were collected through questionnaires (quantitative approach), and interviews (qualitative approach).

2.2. Participants

In the quantitative part of the study, the participants were Portuguese and Brazilian teachers who taught Mathematics in public schools in the region of Braga (Portugal), and in the region of Campinas (Brazil). In Portugal, all the public schools in the region of Braga were contacted. This was done using school teachers collaborating with the University of Minho, as well as school teachers that are former students from this university. Through these teachers, the questionnaire was disseminated in each school. Based on this process, we obtained 129 responses, corresponding to approximately 80 % of the mathematics teachers in the region of Braga.

Given that Portugal and Brazil are considerably different in terms of area and population, and consequently in the number of teachers, we decided to adopt procedures in Brazil that would allow us to obtain a sample of similar size to that of Portugal. To this end, we selected a group of schools in the Campinas region, where we adopted a similar procedure to that used in Portugal (through school teachers with ties to the University of Campinas, we distribute the questionnaires in the schools of the region of Campinas). In this way, we received 136 questionnaire responses.

The choice of the region to study in each country was guided by two factors. First, both regions are among the economically wealthiest ones in each country, and also both host a technological hub, innovative research centers in science and technology, and a well-known university offering initial training for teachers; and, in the case of Braga, hosting the only university in Portugal that offers teacher education for all school levels. Secondly, the Campinas region has a significant history regarding in-service teachers' education. In Brazil, the *Lei de Diretrizes e Bases* (Law of Guidelines and Bases), approved in 1996, establishes in-service teacher education as a right. Although this formalization occurred in 1996, Campinas has had a teacher training center since 1980, i.e., since the very beginning of such centers in the country (Alvez & Dri, 2013). Like Campinas region, the Braga region as a significative history in in-service teacher education. This led to the establishment of the *Scientific-Pedagogical Council for Teachers' In-service Education* in this region – the national authority responsible for accreditation of teachers' in-service education in Portugal. Thus, these regions might not be representative of their respective countries, but they share similar socioeconomic contexts (in the sense of being above the national average), place a strong emphasis on both initial and in-service teacher education, and were pioneers in teacher training. These circumstances not only offer a similar context, but also make them particularly relevant for this study. The strong value given to teacher training in the regions considered in the study, together with the economical situation of the regions, gives support to the idea that the participants in the study can be assumed as representative of the best teachers in Portugal and Brazil.

In the teachers from Portugal, with an average age of 47 years old (ranging from 34 to 63), the female gender predominates (79.8 %). Most of these teachers have a degree (72.1 %), a permanent position (76 %), and have been teaching from 16 to 25 years (61 %). Some teach at the lower secondary – 7th to 9th grade (49.6 %) – and some at the upper secondary – 10th to 12th grade (50.4 %).

As for the teachers from Brazil, with an average age of 46 years old (ranging from 22 to 73), the female gender predominates (50.7 %). Most of these teachers also have a degree (73.5 %), – with some having a Master's (17.6 %), a Bachelor's (3.7 %) and a Doctorate (5.1 %) –, a permanent position (66.2 %) and has been teaching for over 16 years (51.5 %). Some teach at the lower secondary (39.7 %) and some at the upper secondary (60.3 %).

The qualitative part of the study involved 12 teachers. Six teachers were from Portugal, being three from lower secondary and three from upper secondary (Cristina, 18 years of teaching experience; Ana, 17 years of teaching experience; Celeste, 36 years of teaching experience; Isabel, 23 years of teaching experience; Clara, 30 years of teaching experience; Alice, 20 years of teaching experience). The other six teachers were from Brazil, being three from the lower secondary and three from the upper secondary (Maria, 11 years of teaching experience; Carlos, 12 years of teaching experience; Rute, 26 years of teaching experience; Roberto, 23 years of teaching experience; Izabel, 25 years of teaching experience; Lisa, 24 years of teaching experience). The choice of these teachers was based on geographic criteria (Portugal and Brazil) and the willingness to participate in the study. Besides this, due to the focus of the study, we valued some teachers' characteristics. As so, we selected teachers with different teaching experiences in relation to the levels they usually teach and to the time in the profession.

2.3. Data collection method

We adopted a quantitative approach, based on a questionnaire, which was validated by three experts in the field of Mathematics Education regarding its structure and clarity of content. For its application to Brazilian teachers, we took into account linguistic differences (e.g., the term used for technological resources), differences in school organization (e.g., the name given to basic and secondary school), as well as cultural aspects (e.g., the role ascribed to the textbook).

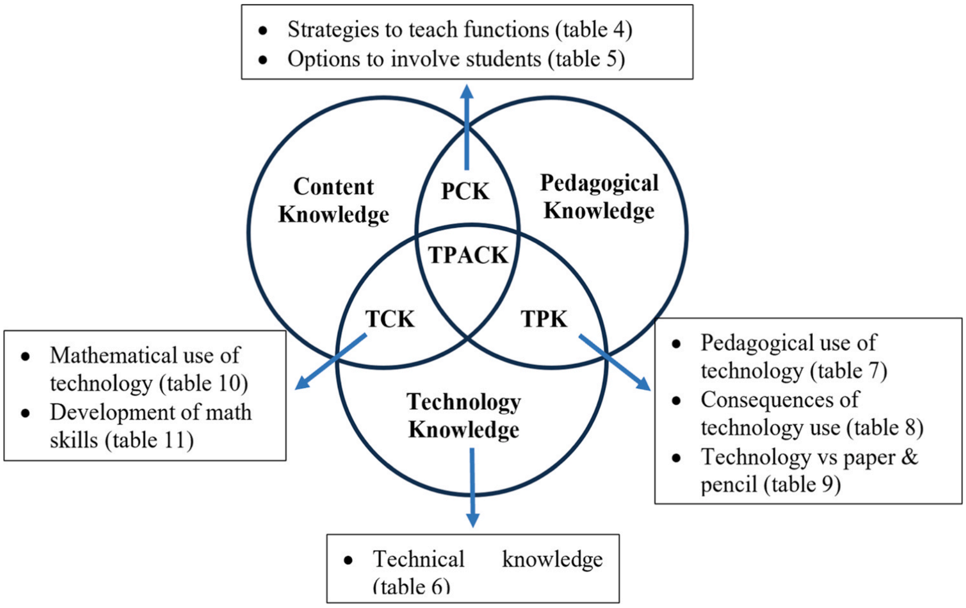

Fig. 2. The questionnaire items and the *TPACK-inspired by KTMT* model.

Besides a short characterization of the participants, the questionnaire includes closed items organized around two dimensions: (i) Pedagogical Content Knowledge (11 items); and (ii) Knowledge related to technology (16 items). These items are related to the *TPACK-inspired by KTMT* model as illustrated in Fig. 2.

To answer the questions in dimension (i), teachers should select one of the options *Never or Rarely* (NR), *Sometimes* (ST), *Often* (O), *Almost Always or Always* (AAA); and in dimension (ii), *Totally Disagree* (TD), *Partially Disagree* (PD), *Neither agree nor disagree* (NAD), *Partially Agree* (PA), *Totally Agree* (TA). These answers were then respectively coded as 1, 2, 3, 4 (dimension (i)), and 1, 2, 3, 4, 5 (dimension (ii)) and the average and standard deviation of these values was determined for each item. The option for different scales in the two dimensions is related to the content of each one. In dimension (i), the intention was to make the teacher take a position, not offering a neutral option. In the other dimension, aware that the use of technology is socially valued and intending to prevent the occurrence of false positive responses, a neutral answer was included.

To clarify and deepen some aspects, we cross-referenced the results of the questionnaire with data from semi-structured interviews. These interviews were carried out face to face for a period of 45 min and included 5 questions about teacher education, 7 questions about their professional activities, and 9 questions about the teaching of functions with technology. The interview script was validated by two experts in the field of Mathematics Education regarding the clarity of its content. All the participants in the study were informed about the goals and the prospective use of data, giving their informed consent.

Besides this, we conducted a documental analysis of Portuguese and Brazilian official documents related to the curriculum in each of the countries and some political decisions.

2.4. Quantitative data analysis

The reliability of the teachers' perceptions about the defined dimensions was analyzed using Cronbach's alpha coefficient. The internal consistency of the dimensions was classified according to the following criteria: very good ($\alpha \geq 0.9$), good ($0.8 \leq \alpha < 0.9$), reasonable ($0.7 \leq \alpha < 0.8$), weak ($0.6 \leq \alpha < 0.7$), and unacceptable ($\alpha < 0.6$) (Pallant, 2020; Pestana and Gageiro, 2014). The calculus of Cronbach's alpha coefficient showed the questionnaire's good level of internal consistency (Alpha=0.833), with dimension (i) and (ii) having respectively a reasonable level of consistency (Alpha=0.766) and a good level of consistency (Alpha=0.829).

The means (M) and standard deviations (SD) were calculated to describe the participants' perceptions about each item. To compare the perception of "dependent variable" between Portuguese and Brazilian participants across the entire scale and each dimension, the Student's *t*-test for independent samples was utilized. Prior to conducting the *t*-test, the normality assumption was validated (Marôco, 2021; Pallant, 2020). The normality assumption for each dependent variable was assessed using the Kolmogorov-Smirnov test. In cases where the assumption of normality was not met, the central limit theorem was applied (Field, 2018; Marôco, 2021). This theorem ensures that with sufficiently large samples ($n \geq 30$), the sampling distribution of the mean tends to be normal, allowing the use of the *t*-test even in the absence of normality in the data. Therefore, the normality assumption was assumed to hold (Marôco, 2021; Pallant, 2020; Pestana & Gageiro, 2014). The effect size for the Student's *t*-test for independent samples was calculated using Cohen's d, and the effect size was classified as follows (Marôco, 2021; Pallant, 2020): small ($d \leq 0.2$), medium ($0.2 < d \leq 0.5$), large ($0.5 < d \leq 0.8$), and very large ($d > 0.8$).

All statistical analysis for a 5 % significance level ($p < 0.05$) was performed using the IBM Statistical Package for the Social Sciences (SPSS, version 28) software.

2.5. Qualitative data analysis

Regarding the qualitative data, a content analysis was carried out on the teachers' interviews. The information from this analysis was used to deepen the interpretation of the data obtained from the quantitative part of the study, to identify underlying reasons for the patterns observed.

The Portuguese and Brazilian official documents were analyzed to identify, in both countries, the characteristics of the scholar system, the school level and the Functions' content studied, the recommended use of technology, and the assessment regulations, namely the ones related to external assessment. Based on this analysis, similarities and differences between the two countries were identified. A reflection on these suggested two areas to consider as potential justification for the reality of each country: curricular emphasis and teacher training. The first area stems in a very evident way from the differences of the official documents. The second one is the result of the different conceptions of the teachers' training programs as well as the different opportunities for in-service teacher training. These two areas were than used to interpret the teachers' ideas expressed in the interviews looking for possible reasons to justify the results from the analysis of the questionnaires.

As in the quantitative approach, ethical issues were considered, ensuring anonymity and informed consent of the participants.

3. Results

The first part of the results focuses on the curricular decisions and the policies implemented in each country. The analysis of the of the data from the questionnaires and interviews is then presented considering the dimensions of the study: Pedagogical Content Knowledge; Knowledge related to Technology.

3.1. Portuguese and Brazilian curriculum and policies

The school in Portugal and Brazil includes 12 grades, being the first nine the basic school and the last three the high school or upper secondary, as detailed in Table 1.

Teachers' training (for both, pre-service and in-service teachers) is assumed as central. In Portugal, teachers' training for future secondary mathematics teachers (7th to 12th grade) comprises a three-year degree in mathematics and a two-year master's degree in teaching, including a pedagogical internship. In Brazil, teachers' training for all school levels comprises a four-year degree in mathematics, which includes a pedagogical internship.

In both countries, in-service teachers periodically attend training programs to update their knowledge and as a requirement for a career promotion. In Portugal, it has been common to offer teachers training when a new syllabus starts to be implemented.

Regarding Functions, in Portugal its study starts at 7th grade by the notion of function (MEC, 2018). Supported by previous learning, the study of Functions develops students' algebraic language and thinking, broadening and deepening the study of mathematical relationships. Starting by recognizing a function in different representations, interpreting it as a relationship between variables and as a univocal correspondence between two sets, the study of the topic continues with the graphic representation of linear and quadratic functions, relating the graphical representation to the algebraic one and vice versa. The importance of representing and identifying these types of functions using graphic, algebraic expressions and tables is highlighted, as well as the relation among these representations. In Brazil, the same topics are addressed, however its study starts only at the 9th grade and continues at the beginning of the lower secondary (Table 2).

For the learning of Functions, the Portuguese secondary syllabus recommends the adoption of problem solving, namely in a modeling context, and the development of projects, in mathematical contexts or in interdisciplinary contexts. Technology is assumed as a powerful resource, being clearly recommended, namely the use of graphing technology. In Brazil, there is no obligation to use technology in the classroom, being its use merely suggested (Brasil, 2018) (Table 3). Most schools do not have adequate technological infrastructures, making it difficult for teachers to integrate it. Besides this, most initial training courses do not address the use of technology to teach mathematics.

The Portuguese school system includes exams at certain moments, although they have different goals. The exams at the end of 2nd, 5th and 8th grades intend to provide information about the students learning and have no impact on students' grades. The exam at the end of 9th grade certifies the completion of this level of education. The final exam at the end of 12th grade certifies the conclusion of high school and ranks the students intending to continue their studies.

In Brazil, every two years there is a national exam with a sample of students from the 5th, 9th and 12th grades, intending to offer information to political decisions. The grades achieved in this exam can, in some cases, be used to access universities in both countries. The state's own assessment systems implement tests at the 2nd, 5th, 9th and 12th grades, intending to ensure the quality of education.

3.2. Pedagogical content knowledge

3.2.1. Strategies to teach Functions and options to involve students

The teachers' PCK guides their choices about the strategies to teach functions. The use of examples to establish definitions or properties seems to be one of the most used strategies for teachers from both countries. Nevertheless, the introduction of concepts is mainly done based on tabular or algebraic representations, in the case of teachers from Brazil, and based in more than one representation in the case of the teachers from Portugal (Table 4).

In the classroom, teachers from both countries often allow students to solve the proposed tasks. Brazilian teachers tend to value the resolution by the teacher on the board a little more than Portuguese teachers. This relates to an appreciation of the students' coping what the teacher does on the board, aligned with a concern about fulfilling curricular programs. However, this does not prevent the teachers from encouraging students to present their own resolutions. When different approaches and results occur, the teachers tend to

Table 1
Organization of the school levels in Portugal and Brazil.

Portugal					Brazil				
	Grade	Age	Level	Math	Grade	Age	Level	Math	
Compulsory education	1–4	6–10	Primary school (single teacher)	National curriculum Same Math for all	1–5	6–11	Primary school (single teacher)	National curriculum Same Math for all	Compulsory education
	5–6	10–12	Middle school (several teachers)						
	7–9	12–15	Lower secondary (several teachers)		6–9	11–15	Lower secondary (several teachers)		
	10–12	15–18	Upper secondary (several teachers)	National curriculum Different options for Sciences, Arts, etc.	10–12	15–18	Upper secondary (several teachers)		Non compulsory education

Table 2
The study of Functions according to the school level in Portugal and Brazil.

Grade	Mathematical content	
	Portugal	Brazil
7	Definition of function Different representations of functions Functions to represent and analyze situations	
8	Linear function	
9	Inverse proportionality function Quadratic function $y = ax^2$, $a \neq 0$	Definition of function Different representations of functions Functions to represent and analyze situations
10	Basic elements of real functions of a real variable Quadratic function, absolute value function, piecewise function Polynomials	Linear function Inverse proportionality function Quadratic function $y = ax^2$, $a \neq 0$
11	Sequences Rational functions Limits and derivatives of polynomial and rational functions	Quadratic function, absolute value function, piecewise function Polynomials
12	Continuity and asymptotes Derivatives, monotony and concavities Exponential and logarithm functions Trigonometric functions	Exponential and logarithmic functions Trigonometric functions

Table 3
The technologies officially recommended in Portugal and Brazil.

	Portugal	Brazil
Lower secondary	Calculators Applets GeoGebra Scratch Statistics data bases Spreadsheets Internet	GeoGebra** Internet**
Upper secondary	Graphing calculators* Graphing software Applets GeoGebra Python Sensors Spreadsheets Internet	

**The use is not compulsive
* Also used in the final exam

Table 4
Strategies to teach Functions.

Items	Portugal		Brazil		Cohen's d	Effect Size
	$\bar{x}$	s	$\bar{x}$	s		
▪ I use examples to establish definitions/properties	3.05	0.629	3.24	0.845	0.748	Large
▪ I explain the theoretical contents and then the students apply them to solve exercises/problems	2.95	0.856	2.89	0.866	0.861	Very large
▪ I involve students in developing definitions/properties of the concepts studied	3.04	0.700	3.09	0.839	0.774	Large
▪ I introduce the concepts of Functions using tables	2.36	0.837	2.91	0.873	0.856	Very large
▪ I introduce the concepts of Functions using graphs	2.74	0.721	2.70	0.905	0.821	Very large
▪ I introduce the concepts of Functions connecting more than one representation (algebraic, tabular, graphical)	3.41	0.633	3.34	0.702	0.669	Large
▪ I introduce the concepts of Functions using algebraic expressions	2.43	0.818	2.91	0.865	0.842	Very large

Table 5
Options for involving students when teaching functions.

Items	Portugal		Brazil		Cohen's d	Effect Size
	$\bar{x}$	s	$\bar{x}$	s		
▪ In the classroom I solve exercises/problems on the board to fulfil the program	3.40	0.734	3.63	0.594	0.666	Large
▪ I solve the exercises/problems on the board and the students copy to their notebooks	2.53	0.893	2.69	0.931	0.913	Very large
▪ I encourage students to present their resolutions and results	3.37	0.613	3.46	0.666	0.641	Large
▪ When students obtain different results, I involve the class in validating the answers	2.83	0.867	3.13	0.901	0.885	Very large

involve students in their validation (Table 5).

3.3. *Curricular emphasis*

The use of examples is valued by teachers in both countries based in the same reason: to facilitate the understanding of concepts. Functions is not considered an easy topic by the teachers, and examples are seen as a good option to help students, as mentioned by one of the teachers: “allowing students to see an example allows a better understanding of the concepts involved” (Izabel, B). Actually, the use of examples is recognized in the literature and explicitly considered by Shulman (1986) in is characterization of PCK, and assumed as essential to promote students understanding (Tirosh et al., 2019).

Despite this similarity, there are differences regarding the characteristics of the examples (Tirosh et al., 2019). Teachers from Brazil tend to refer to numerical examples, while Portuguese teachers explicitly consider graphical examples as well. This choice is certainly influenced by the characteristics of the Portuguese curriculum, which significantly emphasizes the graphical component of functions. In the words of the teachers themselves:

“Through numerical examples, students are able to better understand the mathematical definition/property we are teaching” (Lisa, B).

“The use of examples is fundamental to clarify definitions/properties and facilitate learning. The topic of Functions is one of the least liked by students and where they show the most difficulties. Through examples, whether numerical or graphical, it is possible to concretize what seems abstract to them” (Alice, P).

The value that teachers from both countries place on student activity aligns with current international guidelines for Mathematics curricula, which encourage “students to explain what they did and to discuss ideas” (Clara, P). This reflects a teaching approach grounded in constructivist principles, which emphasizes what the student says and does, valuing mathematical thinking (Muir, 2025) and considering “error as a learning resource, and the validation of responses as a means to develop mathematical thinking by reflecting on and reasoning about both mistakes and correct answers” (Izabel, B).

Teachers from Brazil value the introduction of concepts through tabular and algebraic representations. In such cases, the use of tables can facilitate understanding: “The use of tables to represent functions helps visually” (Rute, B). While the simplicity of this approach may be one reason for its use, it is a particularly relevant choice that can also be justified by somewhat limited access to technology (Rocha, 2016). The use of tables is clearly referenced as an intermediate step in constructing a graph, which is a common process when technology is not available: “a process for students to understand the concept of function; (…) we try to transcribe the information into a table, and then build the graph” (Maria, B). This is a step that tends to disappear from the teachers practices when the technology becomes available, as identified by Rocha (2016).

In contrast, teachers from Portugal (where not only the access to technology is defined as mandatory by the official documents, but it is generally readily available) the use of multiple representations is clearly embraced: “Introducing concepts through multiple representations allows for a more comprehensive understanding” (Alice, P).

During the classroom activities, teachers from Brazil place greater emphasis on the central role of the teacher in task resolution, with the aim of “addressing all the topics in the curriculum. Depending on the school we work at, this is something that is heavily demanded (…) so that the school administration does not question our teaching methods” (Izabel, B).

The difference between teachers from both countries regarding the teacher's role in the classroom, may be related to the greater presence of technology in the Portuguese curriculum. Once technology enables the use of “multiple representations, with a more conceptual focus and the development of mathematical thinking” (Ana, P), it is easier to give students a more exploratory role, reducing the central role of the teacher.

3.4. *Teacher training*

Some teachers express a preference for practices that allow them to guide, constrain, or control students' activity: “I noticed that the students work well when I give them some guidance. Doing open-ended work out of nowhere is difficult, as students get lost” (Carlos, B). The contrast between a teaching perspective that values student activity and another that emphasizes the teacher's central role can be linked to the training and professional experiences that teachers have developed throughout their careers, but also to their beliefs. However, as noticed by Muir (2025), practices characterized by memorization and a repetition of mechanical procedures are still common in the mathematics classrooms.

The Portuguese curriculum (in terms of methodological guidelines and recommended resources) has an impact on classroom practices in Portugal. Besides this, for the past 30 years, most of the Portuguese curricular reforms have been accompanied by teacher training programs, which is a highly relevant factor (see Faggiano et al., 2021 for some details). This approach is likely responsible for the recognition of the contribution that different representations can make to learning: "(…) some [representations] have more advantages than others. If students can jump from one representation to another, they have a deeper understanding of what they learn" (Cristina, P). Teachers' training programs also influence the recognition for certain classroom approaches, as illustrated by one teacher: "they [technologies] promote more dynamic lessons, with a stronger conceptual focus and the development of mathematical thinking" (Alice, P).

The central role of the teacher's tends to result in lessons where "the teacher defines a sequence of instructions and the students use them as a basis for solving tasks, observing the procedures and strategies used by the teacher" (Rute, B). This is a teaching perspective deeply rooted in the practice of many teachers (Muir, 2025) and largely shaped by the socialization during the time the teachers were students, as Clara explains:

"In the universities most of the courses follow this method: explain the theory and then solve application exercises/problems. Teachers who learned this way, tend to replicate the method. However, with the changes implemented in education over the years, new teachers tend not to use this traditional methodology so much" (Clara, P).

4. Knowledge related to technology

This section presents the results of dimension (ii) of the study, corresponding to the TPACK knowledge domains with a focus on technology.

4.1. Knowledge of technology

4.1.1. Technical knowledge

Nowadays, mathematics teachers have a range of technologies that they can integrate into their practice, such as graphing calculators and specific software. For this to happen, the teachers need to know how to operate the technology. At this level, the teachers from Portugal express a deeper level of knowledge (Table 6).

4.1.2. Curricular emphasis

The mandatory use of graphing calculators at upper secondary in Portugal and the strong recommendation for the use of several technologies at all levels might justify the confidence expressed by these teachers in comparation to the Brazilian ones:

"Nowadays, teachers are very used to using technology in the classroom. The graphing calculator and GeoGebra are very common. I would say that all the teachers know how to use them. It maybe to a greater or lesser extent, but they all know. Schools also now offer better conditions". (Clara, P)

The impact of the Portuguese political decisions, namely the use of the graphing calculator in the final exam, is recognized by the teachers: "The national exam always includes an exercise or problem that requires its use [of graphing calculators]. Therefore, teachers are used to working with this tool" (Celeste, P).

4.1.3. Teacher training

The diversity of technological resources available to teachers is one of the reasons why, in Portugal, teachers seek professional training, for example in tools such as "Excel, Moodle, Sketchpad and GeoGebra" (Ana, P). The encouragement provided by the national curriculum "is really something that leads teachers to use technology, what is possible due to the opportunities for in-service training. The result is that more and more teachers are using and know how to use technology." (Clara, P)

In contrast, the willingness of Brazilian teachers to incorporate technology into their practice is not the same, mainly due to a lack of training and inadequate school conditions:

"Teachers do not frequently use technological resources to teach functions. This may be related to the lack of infrastructure and student access, lack of knowledge about the tools, and lack of time due to the need to cover the curriculum skills, among other reasons." (Lisa, B)

4.2. Technological pedagogical knowledge

4.2.1. Characteristics of technology use in the classroom

The use of technology to teach Functions is referred by teachers in both countries, being its use more common for the teachers from Portugal. However, approaches with paper and pencil are still usual (Table 7).

Table 6
Knowledge of technology.

Items	Portugal		Brazil		Cohen's d	Effect Size
	$\bar{x}$	s	$\bar{x}$	s		
▪ I feel comfortable using a graphing calculator/GeoGebra to teach functions	4.06	1.164	3.35	1.331	1.252	Very large
▪ I feel comfortable using specific software to teach functions	3.60	1.034	3.52	1.328	1.194	Very large

Table 7
Use of resources to teach Functions.

Items	Portugal		Brazil		Cohen's d	Effect Size
	$\overline{x}$	s	$\overline{x}$	s		
▪ I use the graphing calculator/GeoGebra to explore Functions	3.49	1.073	2.53	1.014	1.043	Very large
▪ I favor paper and pencil and blackboard and chalk activities	3.71	0.717	3.71	0.769	0.744	Large

Table 8
Consequences of technology use.

Items	Portugal		Brazil		Cohen's d	Effect Size
	$\overline{x}$	s	$\overline{x}$	s		
▪ The use of technology favors a teaching of Functions less expositive and more participative	4.22	0.739	3.93	0.971	0.866	Very large
▪ The use of technology makes the teacher look for tasks about Functions in other sources besides the textbook	3.57	1.088	4.00	0.935	1.012	Very large

The use of technology impacts the teachers' practice. Teachers from both countries agree that its use favors the teaching of Functions, reducing the lecture time and making learning more student centered (Table 8). The use of technology also impacts the source for tasks used by the teachers, with the teachers from Brazil assuming a stronger need to look for tasks beyond the textbook.

Technology can be used in very different ways, depending on the characteristics of the task and the strategy adopted. Portuguese teachers tend to agree with the use of technology when an analytical approach is not possible, while Brazilian teachers tend to assume a more neutral position. The same trend applies to the possibility of confirming with technology a paper and pencil approach, as well as the reverse process (Table 9).

4.2.2. Curricular emphasis

The teachers from Portugal tend to take a deeper advantage of the technology than the teachers from Brazil. Besides this, the use of the technology and the potential of that use seems to be understood in different ways by the teachers from the two countries. The teachers from Portugal refer to the characteristics of the work in the classroom that the use of technology can promote. They highlight the development of practices that encourage student participation in classroom activities, using "technology to promote discussion and exploration" (Clara, P). The shift in the student's role enabled by technology seems to be a point of consensus:

"Graphing calculators or similar software allow students, for example, to explore the properties of functions. With well-designed tasks, students can investigate the behavior of functions more quickly, less monotonously, and in a much more engaging and, above all, productive way. Technology enables a level of interactivity that cannot be achieved with just paper and pencil." (Isabel, P)

The teachers from Brazil recognize the potential of technology but, unlike teachers from Portugal, feel the need to explicitly emphasize its articulation with traditional approaches:

"To complement lessons with technological methods that, together with traditional methods, support students' learning of Functions. Just as it is important to construct a graph using paper and pencil, it is also important to construct it using technology, as both approaches contribute to the observation of the characteristics and variations of functions". (Roberto, B)

This difference can be justified by the different syllabus. In Portugal, the use of technology is mandatory for more than 30 years, as so, the teachers are already used to integrate it in their practices, and more knowledgeable about how to integrate the approaches with and without technology. The literature (Faggiano et al., 2018; Komatsu & Jones, 2020) recognizes the potential of this integration although, as pointed by Komatsu and Jones (2020), more research is needed to better understand the best way to do it. Still, it is recognized the potential that the different environments can bring to the mathematical learning.

Despite the several studies pointing to the potential of technology integration, the articulation between an approach with technology and with paper and pencil is often associated with the idea of a reduction in the development of procedural skills and a concern

Table 9
Technology versus paper and pencil.

Items	Portugal		Brazil		Cohen's d	Effect Size
	$\overline{x}$	s	$\overline{x}$	s		
▪ The resolution of tasks on Functions must be done with paper and pencil and checked with technology	3.69	0.942	3.33	0.997	0.970	Very large
▪ The resolution of tasks on Functions must be done with technology and checked with paper and pencil	3.46	1.031	3.27	1.145	1.091	Very large
▪ When solving tasks on Functions one must resort to technology when it is impossible to solve using analytical processes	4.14	0.958	3.29	1.161	1.067	Very large

for students' dependence (Sevimli, 2016). This is one of the points addressed by one of the teachers from Brazil: "We have a certain concern about students' technological dependence when solving problems related to functions. However, this does not mean the exclusion of technology, as we do not want to reduce lessons to mechanized problem-solving processes" (Lisa, B).

Teachers from Portugal refer to how technology can contribute to work with different students and different learning rhythms: "Depending on the type of students, different approaches have their advantages and disadvantages. For students with learning difficulties or a slower learning pace, starting with paper and pencil can be more frustrating and demotivating"(Alice, P).

They also refer the use of technology when an analytical approach is not possible: "There is a general consensus on using technology when analytical methods are not available. Finding the zeros of functions or identifying maximum points through graphs displayed on a graphing calculator has already become commonplace" (Clara, P).

Once again, the comments from the teachers reflect the political decisions and the use of technology as one of the resources to promote differentiation in the classroom, but also the direction that has been taken for quite some time and the impact it already has in making some options common for the teachers. Although the need to articulate the technological and the paper and pencil approaches is always present, there is an awareness that, in some cases, there is something else more important:

"Using technology, students can intuitively grasp the problem and understand how to solve it. However, verification or validation should always be done with paper and pencil, as students need to be aware of the limitations of technology, which can sometimes suggest results that are not accurate. Still, in many problems, what matters is not the analytical solution itself, but the interpretation of the behavior of the function". (Alice, P)

This is the recognition of the relevance of the conceptual understanding over the procedural approach, a point of view clearly aligned with the potential of the technology identified in the literature (Rocha, 2020; Roschelle et al., 2017).

However, the difference between the teachers from the two countries is deeper. In fact, when the technology is used, there is a difference related to the type of technology used. While in Portugal graphing calculators and GeoGebra are the technologies that teachers tend to speak about, in Brazil the teachers speak about GeoGebra but also about "platforms like Quizizz and Kahoot used to help the interaction with the students" (Maria, B). This means Portuguese teachers are considering technologies somehow connected to the mathematical topic and Brazilian teachers are using general technologies that have no relation to Mathematics and can be used in all the scholar disciplines. This is a substantial difference in the role of the technology: technology to do and learn Mathematics versus technology to show and motivate. As so, the teachers from Portugal seem to be aligned with the didactical use of technology as characterized by Roschelle et al. (2017), what is not necessarily the case of the teachers from Brazil. It is important to notice that, as documented by Sacristán el al. (2024), the main use of technology in the mathematics classroom is still for presentation and communication.

Textbooks are, of course, influenced by the characteristics of the curriculum. Besides this they have the important role of offering the teachers an interpretation of the new curricula (Fan et al., 2025). Therefore, it is not surprising that, since technology is one of the resources designated for use in Portuguese classrooms, textbooks include suitable tasks incorporating it. In contrast, teachers in Brazil tend to feel the need to seek tasks beyond what is provided in the textbooks.

4.2.3. Teacher training

In Portugal, the use of technology in teaching Functions is a consequence of the curricula recommendations, but it is also influenced by the professional development opportunities available to teachers (Faggiano et al., 2021), and to the permission of technology use in exams (Kastberg & Leatham, 2005), as highlighted by Clara:

"It is the continuous training that leads to learning how to use it, followed by programs that encourage its use. It is also connected to the exams, we need to work with our students so that they have enough practice to perform well". (Clara, P).

In the Brazilian context, however, there is a need to promote training initiatives on technology with teachers. This is recognized by one of the teachers: "It represents a rather trivial use of technology, without much reflection. It is used, but not critically or thoughtfully incorporated into teaching" (Izabel, B). And the teacher continues, expressing she believes she should be doing more: "Because I do show the tool. I build two or three functions for the students to see… but it's not in every lesson, and the students don't really explore or engage deeply with these materials" (Izabel, B). Here it is possible to identify the impact of the access to technology, related to the conditions in the schools. However, it is evident a difference in the use of technology, with a use from this Brazilian teacher centered in herself and mainly to show to the students, and the use of the Portuguese teacher willing to teach students how to use the technology by

Table 10
Technology use.

Items	Portugal		Brazil		Cohen's d	Effect Size
	$\bar{x}$	s	$\bar{x}$	s		
▪ Technology should be used mainly to introduce concepts of functions	3.20	0.971	3.48	0.981	0.976	Very large
▪ Technology helps to do transformations on graphs of functions	4.37	0.791	4.22	0.987	0.897	Very large
▪ Technology is mainly useful for drawing graphs of functions	3.18	1.093	3.55	1.179	1.138	Very large
▪ Technology allows establishing relations between the different representations of functions (algebraic/graphical/tabular)	4.32	0.696	4.26	0.888	0.800	Large

themselves. One important difference can be the fact the students will have to use it in the final exam, but there is also a need for training, as recognized by the teacher, to be able to explore the technology in a deeper way. The teachers need opportunities to understand the didactical use of technology (Roschelle et al., 2017), namely getting familiar with the different types of tasks allowed by the technology and the related roles (from teachers and students) (Rocha, 2020).

4.3. Technological content knowledge

4.3.1. Characteristics of technology use to teach mathematics

Regarding the use of technology for teaching Functions, teachers from both countries express agreement on its use to study the transformations of functions' graphs and to establish relationships between the different representations (Table 10).

The same level of agreement is not observed when it comes to using technology to introduce function concepts and to graphically represent a function. The neutral stance of the teachers, slightly more evident among the teachers from Portugal, may be due to the usefulness of technology for introducing concepts from other mathematical topics and for exploring representations beyond the graphical one.

From the teachers' perspective, the use of technology by students promotes the development of visualization skills and challenges students to think (Table 11).

Less importance is given to the use of technology in the development of symbolic manipulation skills, specially by the teachers from Portugal.

4.3.2. Curricular emphasis

In the use of technology, there are statistically significant differences between the positions of teachers from both countries, with teachers from Brazil standing out in their view of technology as a tool to introduce concepts and to draw the graph of a function. These results suggest that, for teachers from Brazil, technology is more often perceived as an introductory and motivational tool (Menezes, 2022). In contrast, for teachers from Portugal, technology is seen as a very powerful tool. Based on the guidelines of the national curriculum, teachers from Portugal make use of technology in several different ways, involving students actively in their learning and enhancing their understanding of the content, as illustrated by Clara:

"The use of technology to study families of functions has become a common option. It helps connecting the function's expression and its graph in a way it was not possible before. The curriculum also reinforces this. But it does much more than that. It allows us to introduce content in a way that captures students' attention, stimulates their engagement, and fosters their understanding". (Clara, P)

The Portuguese curricula ascribe a strong relevance to the graphical approach to the study of Functions. This fact, together with the experience of several years in the use of technology, makes the teachers from Portugal recognize the potential of technology to introduce the topic and to draw graphics, but also makes them attribute a higher relevance to other points, such as the ability to work with different representations and develop relations among them. The teachers from Brazil have a different curriculum, as so, they see technology more as a resource useful to introduce the topic and to draw graphs.

Globally, the teachers recognize advantages on the use of technology. Among these advantages, teachers from Portugal stand out slightly in their use of technology for studying transformations of function's graphs and the relationships between different representations (Zentgraf & Prediger, 2024). In this context, Alice highlights the dynamic nature that technology brings to visualizing the effect of parameter changes on graphs and the connections among different representations:

"Technology is a great ally in studying function transformations. By using the dynamic features of a calculator (for example), it is possible to see in real time what happens to the graph when various parameters are changed. Besides, with technology, it becomes possible to simultaneously observe the different representations of functions". (Alice, P)

The recognition by teachers from both countries that technology enables connections between different representations aligns with studies that emphasize the importance of technology in promoting representational fluency, which is essential for the development of functional thinking (Günster & Weigand, 2020; Lichti & Roth, 2018; Lichti & Roth, 2019).

Regarding perceptions of the impact of technology use on the development of competencies aligned with international recommendations (e.g., visualization, critical thinking, and symbolic manipulation), the results recognize technology as an effective tool for promoting the visualization, assuming it "develops students' ability to visualize and interpret the behavior of functions" (Celeste, P). This result is consistent with studies that highlight the role of software's such as GeoGebra in building mental images and supporting the understanding of abstract concepts through dynamic representations (Ainsworth, 2006; Chechan et al., 2023; Hohenwarter & Jones, 2007; Mushipe & Ogbonnaya, 2019).

As for the idea that technology challenges students to think, both groups show a high level of agreement. This finding reinforces current recommendations for mathematics teaching, which suggest that well-structured digital environments can foster problem-

Table 11
Development of mathematical skills.

Items	Portugal		Brazil		Cohen's d	Effect Size
	$\bar{x}$	s	$\bar{x}$	s		
▪ The use of technology develops the visualization skill.	4.15	0.751	4.34	0.880	0.820	Very large
▪ The use of technology challenges the student to think.	3.93	0.831	4.08	0.935	0.886	Very large
▪ The use of technology develops skills relevant for symbolic manipulation.	3.36	0.967	3.94	0.925	0.946	Very large

solving abilities, experimentation, and critical reflection (Azra, 2025; Drijvers et al., 2010; Koyunkaya & Dede, 2024).

4.3.3. Teacher training

Teachers from both countries recognize the potential of technology to develop essential mathematical competencies, although with different emphases. While teachers from Brazil appear to place greater value on the role of technology in symbolic manipulation and visualization, teachers from Portugal maintain a more balanced view across different types of competencies. The results reinforce the importance of teacher training policies that promote the use of digital technologies in mathematics education (Sacristán el al., 2024), as advocated by Clara from the Portuguese context and Izabel from the Brazilian context:

"In-service training leads to learning how to use the technology encouraged by the curriculum. The train is very important, so that the teachers can learn how to balance everything and properly prepare their students." (Clara, P)

"Even though I recognize that using technology is important and necessary, I don't always use it due to a lack of mastery of the tools and students' difficulties in accessing software and equipment. Therefore, in-service professional development is needed to enable the use of technology". (Izabel, B)

The connection between content knowledge and technology knowledge, developed through teachers' professional experience and the dynamics in which they engage, enables them to perceive the relevance of technology in studying the characteristics of functions through the connection of different representations (Lindenbauer et al., 2024; Thurm & Barzel, 2020).

4.4. Differences between portuguese and brazilian teachers

A more in-depth analysis of the average frequency of responses from teachers in both countries to the items that structure the first dimension, allowed us to determine statistically significant differences in five items (Table 12).

Between the two groups, it is the Brazilian teachers who have a higher average frequency in the items related to solving exercises/problems on the board with the purpose of fulfilling the program, in introducing the concepts of Functions based on tables and algebraic expressions, in starting from solving examples to establish definitions/properties, and in involving students in validating the class's answers.

Regarding the analysis of the teachers' average agreement with the items in the dimensions about knowledge involving technology, statistically significant differences were determined in nine items (Table 13).

Regarding the items on the use of technology to teach Functions, the comparison of the means of the two defined groups determined that Portuguese teachers had a higher average in terms of willingness to work with graphing calculators, use of technology to confirm the results of tasks solved with paper and pencil, use of technology to solve tasks when an analytical approach is not possible. Besides this, the use of technology is assumed as favoring a less expositive and more participative approach to teaching. In the remaining items, it is Brazilian teachers who most value them. This is the case of using technology to draw graphs, and to introduce concepts. It is also the case of a need to search for tasks in other places beyond the textbook, when the technology is used. Besides this, the use of technology is assumed in a more significative way as having the potential to develop skills relevant for symbolic manipulation.

Fig. 3 presents the items with differences statically significative, using colors to highlight the results according to the country.

The analysis of the items related to the teaching of Functions and the use of technology in which significative differences were found, highlight different perspectives on how to teach. Teachers from Brazil stand out in items related to content, in a way that reflects a teaching conception emphasizing the central role of the teacher in the classroom and valuing traditional practices focused on mechanization and suggesting procedural approaches. For these teachers, the use of technology seems to be mainly associated to routine tasks involving symbolic manipulation.

Teachers from Portugal, on the other hand, stand out in technological knowledge, as they feel confident in using different technological tools. They also stand out in TPK by valuing the use of technology to engage students in classroom activities (Biber et al., 2022; Pepin et al., 2024), as well as in exploring different ways of integrating it with other artifacts (such as paper and pencil) in order to help students make meaning of what they learn.

5. Conclusion

In this study, we considered a group of teachers from Portugal (Braga region) and another from Brazil (Campinas region), and we analyzed their knowledge for teaching Functions with technology, drawing on a version of the TPACK model inspired by KTMT.

Table 12
Items with statistically significant differences regarding the 1st dimension.

Dimension		p-value	Cohen's d	Effect Size
1. Pedagogical Content Knowledge	▪ I use examples to establish definitions/properties	0.048*	0.748	Large
	▪ I introduce the concepts of Functions using tables	0.000*	0.856	Very large
	▪ I introduce the concepts of Functions using algebraic expressions	0.000*	0.842	Very large
	▪ In the classroom I solve exercises/problems on the board to fulfil the program	0.006*	0.666	Large
	▪ When students obtain different results, I involve the class in validating the answers	0.006*	0.913	Very large

Note: *statistically significant differences to $p < 0.05$.

Table 13
Items with statistically significant differences regarding the 2nd dimension.

Dimension		p-value	Cohen's d	Effect Size
2. Knowledge related to technology	▪ I feel comfortable using a graphing calculator/GeoGebra to teach functions	0.000*	1.252	Very large
	▪ I use the graphing calculator/GeoGebra to explore Functions	0.000*	1.043	Very large
	▪ The use of technology favors a teaching of Functions less expositive and more participative	0.006*	0.866	Very large
	▪ The use of technology makes the teacher look for tasks about Functions in other sources besides the textbook	0.000*	1.012	Very large
	▪ The resolution of tasks on Functions must be done with paper and pencil and checked with technology	0.003*	0.970	Very large
	▪ When solving tasks on Functions one must resort to technology when it is impossible to solve using analytical processes	0.000*	1.067	Very large
	▪ Technology is mainly useful for drawing graphs of functions	0.008*	1.138	Very large
	▪ Technology should be used mainly to introduce concepts of functions	0.022*	0.976	Very large
	▪ The use of technology develops skills relevant for symbolic manipulation	0.000*	0.946	Very large

Note: *statistically significant differences to $p < 0.05$.

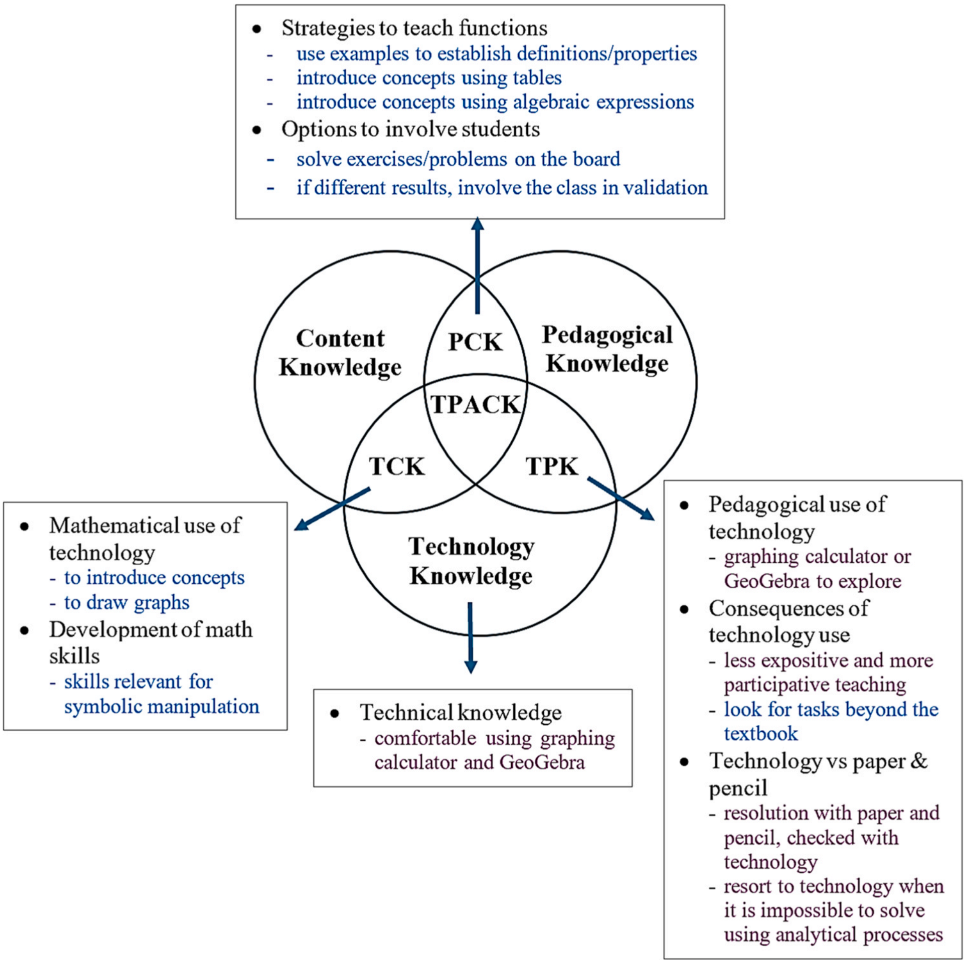

Fig. 3. Statistical significative differences according to the knowledge domains (legend: higher $\overline{x}$ for Portugal; higher $\overline{x}$ for Brazil).

(1) What are the perceptions of mathematics teachers from Portugal and Brazil regarding the use of technology to teach Functions?

The conclusions reached regarding PCK suggest that Brazilian teachers adopt a more teacher-centered approach, favoring more traditional methods (i.e., less common in contexts where technology is available, such as the use of tables or the blackboard). Nevertheless, they still value opportunities to interact with students and promote discussion or engagement in the formulation of concepts.

As for TK, Portuguese teachers report having more knowledge regarding the use of technology than Brazilian teachers.

Regarding TPK, Portuguese teachers are characterized by a less teacher centered approach and by the use of technology for exploration. These dimensions are not so present among teachers from Brazil, who express a need to look for tasks that go beyond the textbook in order to integrate technology. Notably, teachers from both countries equally value working with paper and pencil.

Concerning TCK, the potential of technology to explore and work with different representations is clearly highlighted by Portuguese teachers. In contrast, Brazilian teachers emphasize the potential of technology to introduce concepts, draw graphs, and even to improve skills relevant to symbolic manipulation.

(2) What are the differences (if any) between the perceptions of mathematics teachers from Portugal and Brazil regarding the teaching of Functions and the use of technology?

The main differences between teachers from the two countries relate to their knowledge of technology and pedagogical issues concerning its use. Portuguese teachers tend to adopt a more student-centered and less lecture-based approach, and they give greater importance to the integration of technology with traditional tools such as paper and pencil.

In contrast, Brazilian teachers place more emphasis on the interaction between mathematics and technology, such as using technology to draw graphs, and on strategies that are largely independent of technology, including the value attributed to the use of tables, a strong focus on algebraic expressions and analytical approaches, and the use of examples.

Overall, aspects related to the Portuguese curriculum and the availability of teacher training opportunities in the country, appear to be plausible explanations for the identified differences. However, it is worth questioning whether these factors alone are sufficient to account for the observed divergence (Yang et al., 2025), raising broader questions about the impact of curriculum on changing teaching practices.

In Portugal, the issue goes beyond the present curriculum itself, it reflects decades of a trajectory that, with few exceptions, has consistently valued the integration of technology in mathematics education and has implemented curriculum reforms as part of a broader process of teacher professional development.

Naturally, it would be necessary to expand the present study to achieve greater representativeness of each country's educational reality and to better understand the underlying causes of the results obtained.

CRediT authorship contribution statement

Floriano Viseu: Conceptualization, Methodology, Formal analysis, Writing – original draft. **Helena Rocha:** Conceptualization, Methodology, Formal analysis, Writing – original draft, Translation to English, Writing – review & editing. **Sandra Menezes:** Methodology, Formal analysis, Writing – original draft.

Ethical approval

The authors declared that the study adhered strictly to standard ethical principles, ensuring the anonymity of those involved. Participants were informed about the study and how the data collected would be used for academic purposes, and voluntarily agreed to participate.

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Declaration of Competing Interest

None.

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Data availability

Data will be made available on request.

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