Fiscal Federalism in Continuous Time Stochastic Economies

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Abstract

This paper examines the different types of incentives that countries face when deciding to take part in a federal fiscal system. The optimal degree of participation depends on the structural parameters of the economy and on the properties of the federal fiscal system. Firstly, the paper examines the case of federal fiscal systems that work as simple insurance arrangements. Secondly, a mixed insurance and redistribution system is considered. Thirdly, fiscal system participation is added as a parameter of the utility function. Finally, the paper examines the case of a fiscal federation that responds to shocks on the terms of trade. The models are developed in a continuous time stochastic framework and simple closed form solutions are obtained. Simulation methods are used to illustrate the stationary equilibrium path of variables. The paper concludes that risk sharing is a strong motive for taking part in a fiscal federation. Fiscal system participation increases when the country benefits from redistribution or when participation is a parameter of the utility function. Finally, it is shown that there are incentives to participate in fiscal federations that respond to shocks on the terms of trade. In this case, the higher the volatility of the shocks on the terms of trade, the lower the variance of the capital stock growth rate in the optimum.

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Contents

1 Introduction

Fiscal federations stand as an advanced stage in a process of economic integration. This type of integrated fiscal system exists within each country, enabling the transfer of resources among regions and playing a vital role in macroeconomic stabilisation policy. The economic integration process that is underway in the European Union has revived the interest in the role and functioning of fiscal federations. This literature is closely linked to the theory of optimal currency areas, which dates back to seminal contributions by Mundell (1961), Mckinnon (1963) and Kenen (1969). The main point is that countries which take part in a monetary union lack the instruments to offset the effects of negative asymmetric shocks that might affect them. In fact, when the shocks are symmetric, the common central bank can run the monetary policy in a way that benefits all countries. The problem arises when the shocks are asymmetric. In this case the adequate monetary policy is different for each country. In fact, central banks in monetary unions do not normally intervene to offset the effects of country specific shocks. Instead, they tend to run a pre-defined monetary policy, the aim of which is to assure price stability. In such a context, fiscal policy becomes an important instrument for offsetting the effects of negative asymmetric shocks. Nevertheless, fiscal policy in monetary unions is not free of constraints. In fact, excessive budget deficits can generate problems in terms of the sustainability of public finances. Such problems can jeopardise the monetary union and are avoided by imposing limits on the budget deficits that countries can run.

At this point two questions can be raised. The first is to know how frequent and how serious asymmetric shocks are in monetary unions. The second question is to know what the proper mechanisms are to fight the consequences of negative asymmetric shocks. The answer to the first question has been addressed by empirical studies focused on Europe and the United States, such as Emerson et al. (1990), Bayoumi and Eichengreen (1993) and Krugman (1993). The results obtained are not conclusive, but other studies which have focused on European regions (such as De Grauwe and Vanhaverbeke (1991) and De Nardis, Goglio and Malgarini (1996)) conclude that there are asymmetric shocks at the regional level. In terms of the mechanisms that enable the adjustment to the effects of asymmetric shocks, the answer is twofold. Firstly, if the labour market is sufficiently flexible, then a negative shock that affects a particular country can be adjusted through a decrease in real wages in that same country. Secondly, the effects of asymmetric shocks can be totally or partially offset by a federal fiscal authority by transfering funds to the country affected. We will focus on this last type of solution, which can be seen as a risk sharing arrangement. There is an extensive literature on the desirability and effectiveness of fiscal federations in Europe and the Unites States. A short and incomplete list of such studies includes Bayoumi and Masson (1995), von Hagen and Hammond (1995), Persson and Tabellini (1996), Fatás (1997), Sϕrensen and Yosha (1998), von Hagen (1998) and Forni and Reichlin (1999). Most of these studies are empirical and conclude that there is significant intranational risk sharing in the United States, but it is of much smaller magnitude than the redistributive effect of the federal fiscal system. These studies also indicate that intranational risk sharing is variable across European countries and that the national fiscal and tax systems provide substantial protection against asymmetric shocks. However, the restrictions imposed on domestic fiscal policy will change this reality and the need for a fiscal federation will increase.

In this paper we take a theoretical approach and examine the different types of incentives that countries face when deciding to take part in a federal fiscal system. The models are developed in a continuous time stochastic framework, where the control variables are constant proportions of the capital stock in the stationary equilibrium. This approach generates simple closed form solutions that depend on the structural parameters of the economy, such as technology, preferences and uncertainty. This also allows a good analysis of the different effects at work and provides explanations for some of the situations observed in reality. The paper concludes that risk sharing is a strong motive for taking part in a fiscal federation. Therefore, the degree of relative risk aversion and the covariance between domestic and aggregate federal shocks play a vital role in the fiscal system participation decision. In addition, fiscal system participation increases when the country benefits from redistribution or when participation is a parameter of the utility function. Finally, it is shown that there are incentives to participate in fiscal federations that allow a reduction in the effect of shocks on the terms of trade. In this case, the higher the volatility of these shocks, the lower the variance of the capital stock growth rate in the optimum.

The paper is organised as follows. In the next section, we introduce the continuous time stochastic framework and the structural parameters of the economy. Then, in the third section, we model a fiscal federation that operates as a simple insurance system. In such a system, the single country optimally decides the level of participation, receiving transfers when it faces an output shock that is more damaging than the average output shock in the rest of the federation. On the contrary, the country transfers funds if the shock is less damaging than the average shock in the system. In the fourth section we model an alternative fiscal system where there are both insurance and redistribution components. In this section it is assumed that transfers depend on the output growth rate of the fiscal federation, which contains deterministic and stochastic components. Next, in the fifth section, it is assumed that countries take utility from participating in the fiscal system. In particular, the degree of participation in the system will be included as a parameter of the utility function. Then, Section Six presents a model where there are domestic and foreign consumption goods, as well as uncertainty on the terms of trade. In this case the fiscal federation is assumed to respond to shocks on the terms of trade. Finally, Section Seven presents some concluding remarks.

2 The Continuous Time Stochastic Economy

In this section we introduce the general framework and describe the main features of the model. Firstly, we assume that in this continuous time stochastic economy there are two sources of uncertainty. The first source of uncertainty lies on the domestic production function. Thus, it is assumed that there are technological shocks, which are described by a Brownian motion. The second source of uncertainty comes from the output shocks that affect the fiscal federation. The covariance between these two sources of uncertainty is generically defined, allowing us to examine the effects of asymmetric shocks. Secondly, it is also assumed that this is a non-monetary economy. This assumption is not limitative if we centre on the case of a monetary union with a centrally defined and low-inflation-oriented monetary policy. In this case money plays no important role in macroeconomic performance. The production function of the economy depends on a single input, which is the capital stock available in the country. We assume a technology with constant returns to scale, to which we add a technology shock, proportional to the deterministic output. Thus:

$$dY = \alpha K dt + \alpha K dy \tag{1}$$

where K is the domestic capital stock available to production in each period, α is a technological parameter and dy is a stochastic process, defined as a Brownian motion with zero mean and variance σ_y^2.

Next we introduce the problem of the representative consumer, which maximises the expected discounted value of a constant relative risk aversion utility function[1]. This utility function depends on consumption in each period. Therefore, the consumer problem is:

$$Max\ E_o \int_0^\infty \frac{1}{\gamma} C(t)^\gamma e^{-\beta t} dt \tag{2}$$

where $\gamma \leq 1$ and $\gamma \neq 0$. This parameter reflects the behaviour towards risk, while β is the intertemporal discount rate. The control variables of the model are consumption and the degree of participation in the federal fiscal system. The degree of participation in the federal fiscal system is measured as the amount of funds transferred to the central authority in each period. These funds have a given rate of return, which depends on the characteristics of the fiscal federation. This type of continuous time stochastic model, which was developed by Turnovsky (1995), sets the control variables as proportions of the state variable. Therefore, our model will consider the control variables to be the consumption-capital ratio and the fiscal system participation-capital ratio.

[1] The elasticity of substitution between consumption at any two points in time is constant and equal to $\frac{1}{1-\gamma}$. The empirical evidence suggests that this elasticity lies around or below unity. In fact, the bulk of the empirical evidence suggests a relatively low value for the elasticity of substitution. When attitudes towards risk are described, $(1-\gamma)$ has an alternative interpretation. It is then the coefficient of relative risk aversion, defined as $-\frac{u''(C)C}{u'(C)}$. Therefore, this function is also called the constant relative risk aversion utility function (CRRA). Note also that when $\gamma = 0$, the logarithmic utility function is obtained. This property will be used at the end of the paper to simplify some results.

3 A Fiscal Federation as a Simple Insurance System

3.1 The Insurance System

This section presents the intertemporal resource constraint for the case of a federal fiscal system that works as a simple insurance system. As described above, in such a system, the single country receives funds when it faces an output shock that is more damaging than the average output shock affecting the rest of the fiscal federation. When the reverse happens, the country transfers funds to the federation. This can be easily interpreted as an insurance system where the degree of participation is equivalent to the quantity of insurance that is bought. Therefore, the fiscal system is described by the following equation:

$$dR = -T\left(dy - dr\right) \tag{3}$$

where dR represents the amount of transfers made with the central fiscal authority in each period, T is the quantity of insurance that is bought in each period, dy is the domestic technology shock and dr is the average shock affecting the fiscal federation. Finally, dr is defined as a Brownian motion stochastic process, with zero mean and variance σ_r^2.

Therefore, the intertemporal resource constraint is written as:

$$dK = \alpha K dt + \alpha K dy - sT dt - T\left(dy - dr\right) - C dt \tag{4}$$

where s is the price paid for each unit of insurance. As noted above, the control variables are set as proportions of the state variable. Therefore, equation 4 is divided by the capital stock K and the intertemporal resource constraint is rewritten as:

$$\frac{dK}{K} = \alpha - \frac{C}{K} - sn_T + dk \tag{5}$$

where the stochastic components have been put together and presented simply as:

$$dk = \alpha dy - n_T\left(dy - dr\right) \tag{6}$$

with n_T defined as T/K. Note that, given the form of the production function, equation 5 also defines the growth rate of the economy.

3.2 Optimum Conditions

In this section we proceed with the technical details of the solution of the model. Not all steps will be mentioned because detailed explanations have already been presented elsewhere[2].

[2] For more details on the technical solution of the model see, for instance, Turnovsky (1995), Malliaris and Brock (1982) or Chow (1979).

First, it is necessary to write the stochastic Lagrangian function, which is given by:

$$£ = \frac{1}{\gamma} C(t)^{\gamma} e^{-\beta t} + L_K \left[e^{-\beta t} X(K) \right] \tag{7}$$

where $L_K \left[e^{-\beta t} X(K) \right]$ is the differential generator of the value function. Furthermore, the differential generator of the value function can be written as:

$$L_K \left[e^{-\beta t} X(K,t) \right] = \frac{\partial V(K,t)}{\partial t} + \left(\alpha - \frac{C}{K} - s n_T \right) K \frac{\partial V(K,t)}{\partial K} + \frac{1}{2} \sigma_k^2 K^2 \frac{\partial^2 V(K,t)}{\partial K^2} \tag{8}$$

where $V(K,t)$ is the value function of the problem and σ_k^2 is the variance of capital stock. Note that the utility function is time separable, so the value function depends only t for the effect of time discounting. Thus, it is assumed that $V(K,t) = e^{-\beta t} X(K)$, which is the way it is presented in the stochastic Lagrangian function.

At this point we compute the variance of total capital stock as:

$$\sigma_k^2 = (\alpha - n_T)^2 \sigma_y^2 + n_T^2 \sigma_r^2 + 2(\alpha - n_T) n_T \sigma_{yr} \tag{9}$$

where σ_{yr} is the covariance between the domestic output shocks and the fiscal federation output shocks. From here on it is necessary to assume a specification for the value function. This is a necessary step in order to use equation 7 as an optimum condition. Given the structure of our problem the value function is assumed to be:

$$X(K) = \delta K^{\gamma}$$

where δ is an unknown parameter. This *a priori* formulation for the value function is later confirmed to be correct.

Taking all hypotheses together, and deriving equation 7 in order to C/K and n_T we obtain respectively:

$$\left(\frac{C}{K} \right)^{\gamma - 1} - \gamma \delta = 0 \tag{10}$$

$$-s + \left[(-\alpha + n_T) \sigma_y^2 + n_T \sigma_r^2 + (\alpha - 2n_T) \sigma_{yr} \right] (\gamma - 1) = 0 \tag{11}$$

which are the first order conditions of the problem. Additionally, the Bellman Equation of the problem, evaluated at the optimum, can be written as:

$$\frac{1}{\gamma} K^{\gamma} \left(\frac{C}{K} \right)^{\gamma} - \beta \delta K^{\gamma} + \left(\alpha - \frac{C}{K} - s n_T \right) \gamma \delta K^{\gamma} + \frac{1}{2} \sigma_k^2 \gamma (\gamma - 1) \delta K^{\gamma} = 0 \tag{12}$$

The previous three conditions 10, 11 and 12, completely define the stationary equilibrium. In fact, the optimal values for C/K and n_T are constant and depend only on the uncertainty, technology and preferences parameters. Next, we proceed with the determination of the optimum values of C/K and n_T. Firstly, writing equation 11 in order to n_T gives the optimal quantity of insurance as a proportion of total capital stock, that is:

$$n_T = \frac{-s + \alpha\left(\sigma_y^2 - \sigma_{yr}\right)(1-\gamma)}{\left(\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}\right)(1-\gamma)} \tag{13}$$

Secondly, using equations 10 and 12, it is possible to determine the optimal consumption-capital stock ratio. The expression is:

$$\begin{aligned}\frac{C}{K} = \frac{\beta}{1-\gamma} - \frac{\gamma}{1-\gamma}\left(\alpha - s n_T\right) + \\ \frac{\gamma}{2}\left[\left(\alpha - n_T\right)^2 \sigma_y^2 + n_T^2 \sigma_r^2 + 2\left(\alpha - n_T\right) n_T \sigma_{yr}\right]\end{aligned} \tag{14}$$

Finally, there is the no-ponzy game condition. It states that when $t \to \infty$, the expected discounted value of the capital stock is zero.

$$\lim_{t\to\infty} E\left[K e^{-\beta t}\right] = 0 \tag{15}$$

Note that, given the structure of the model, none of these equations depend on the value function parameter δ. Therefore, its determination is not necessary and the assumed value function proves to be adequate. Furthermore, it is necessary to guarantee that C/K and n_T are non-negative values. In fact, consumption must always be positive and we rule out the possibility of having a country selling insurance, which would be equivalent to $n_T < 0$. Thus, a necessary condition for fiscal system participation is that:

$$\alpha\left(\sigma_y^2 - \sigma_{yr}\right)(1-\gamma) > s \tag{16}$$

At this point we briefly analyse the intuition behind equation 13. A country is willing to participate in the federal fiscal system when the expected costs that occur in the event the domestic shock is less damaging than the federal shock $(dy > dr)$, are smaller than the expected benefits that occur when the reverse happens $(dy < dr)$. When the covariance between these shocks is negative, the country knows that negative domestic shocks tend to be accompanied by positive shocks at the federal level. The transfers received in this situation clearly reduce the negative effect of the domestic output shock. On the contrary, positive domestic shocks tend to be accompanied by negative federal shocks. In this case the country transfers funds to the federation. Nevertheless, for a risk averse country, the net effect is positive. This point is illustrated in Figure 3.1, which is the standard risk aversion diagram, where:

$$\mathrm{a} = \alpha K_t + dy \tag{17}$$

$$\mathrm{a}_1 = \alpha K_t + dy_1 \tag{18}$$

$$\mathrm{b} = \alpha K_t + dy - n_T\left(dy - dr\right) \tag{19}$$

$$\mathrm{b}_1 = \alpha K_t + dy_1 - n_T\left(dy_1 - dr\right) \tag{20}$$

Figure 1: Fiscal System and Risk Aversion

For simplicity, we assume a fixed regional shock dr and two possible domestic shocks dy and dy_1 with probability $1/2$ and $dy_1 > dr > dy$. In this case, federal fiscal system participation improves the utility of a risk averse country.

Nevertheless, if the covariance between shocks is positive, these benefits can be severely diminished. In this situation there is a higher probability of transferring funds to the fiscal federation even when there is a negative domestic output shock. Therefore, if the covariance between shocks is sufficiently high, the country may optimally decide not to participate in the federal fiscal system.

At this point it is also desirable to comment on the financial sustainability of the federal fiscal system. Taking the perspective of a single country, we can say that the federal fiscal system is intertemporally in surplus. In fact, the average value of shocks is zero, which means that the average value of the transfers made with the federation is also zero. However, the country is expected to pay for each unit of insurance that it buys. This means that unless $s = 0$, there are payments due to the federation in every period. Although the system runs an average surplus, there may be financing problems in the short run. In fact, in each period there are countries paying and receiving funds. However, their different dimensions, measured in terms of total capital stock, as well as their different shocks may cause some problems. Take the case of a big country which has a high degree of participation in the system and which faces a negative output shock that is more damaging than the average shock in the federation. In this case, the total amount of funds that it requires may be larger than what is received from smaller countries with low participation in the system and shocks above the average. These short-run imbalances may be adjusted by funds previously saved by the federal fiscal authority or provided by a central monetary authority.

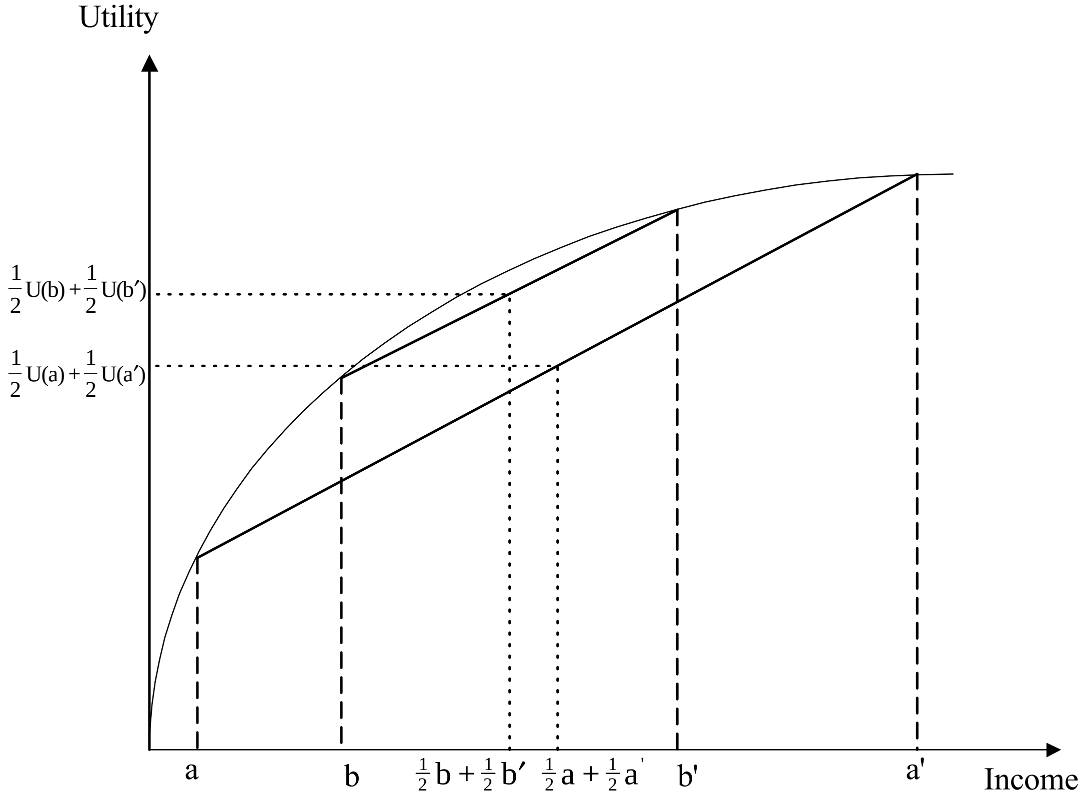
3.3 Equilibrium Analysis

At this point we proceed with the analysis of the equilibrium that has been obtained in the previous section. Our main interest is to examine what it is that determines the optimal quantity of insurance that is bought by a single country. Equation 13 sheds light on this. Firstly, we study the effect of technology on n_T. This effect is described by the derivative:

$$\frac{\partial n_T}{\partial \alpha} = \frac{\sigma_y^2 - \sigma_{yr}}{\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}} > 0 \tag{21}$$

which is always positive for the case of countries taking part in the fiscal federation. This result can be easily interpreted. In fact, the higher the parameter α, the stronger the effects of technology shocks on output. In addition, we

Figure 1



Utility
$\frac{1}{2}U(b)+\frac{1}{2}U(b')$
$\frac{1}{2}U(a)+\frac{1}{2}U(a')$
a
b
$\frac{1}{2}b+\frac{1}{2}b'$
$\frac{1}{2}a+\frac{1}{2}a'$
b'
a'
Income

know that when the domestic output shocks are highly volatile and negatively correlated with the shocks in the fiscal federation there are incentives to buy insurance. Therefore, in such situations, the higher the parameter α, the higher the optimal amount of insurance that is bought.

Secondly, the attitude towards risk, which is described by the value of parameter γ, also plays an important role. As mentioned above, $(1-\gamma)$ is the coefficient of relative risk aversion and the effect on the optimal amount of insurance is given by the derivative:

$$\frac{\partial n_T}{\partial (1-\gamma)} = \frac{s}{\left(\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}\right)(1-\gamma)^2} > 0 \tag{22}$$

As expected, the above expression is positive and states that the higher the relative risk aversion, the higher the optimal insurance-capital ratio. Next, we examine the effect of an increase in the price of insurance s on n_T. This is clearly a negative effect, which is described by the derivative:

$$\frac{\partial n_T}{\partial s} = -\frac{1}{\left(\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}\right)(1-\gamma)} < 0 \tag{23}$$

In this context, it is interesting to examine the effect of having a zero insurance price $s = 0$. Looking at equation 13 it is clear that the optimal amount of insurance would be:

$$n_T = \frac{\alpha\left(\sigma_y^2 - \sigma_{yr}\right)}{\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}} \tag{24}$$

It is important to note that this is not the quantity of insurance that provides full protection against domestic output shocks. This happens because the fiscal federation that we have been describing does not provide a pure type of insurance. Firstly, the uncertainty in the fiscal system affects the total uncertainty faced by the country. Secondly, when $\sigma_{yr} > 0$ there is a higher probability that, even in the case of a negative domestic output shock, the country may have to transfer funds to the federation. On the contrary, if we had assumed a pure insurance system, the full protection against domestic output shocks would arise. In this case we would have to consider $R = -Tdy$, which would mean:

$$n_T = \frac{-s}{\sigma_y^2(1-\gamma)} + \alpha \tag{25}$$

Thus, with $s = 0$, the country would optimally choose to have full protection against domestic output shocks, that is $n_T = \alpha$.

This section continues by examining the effects of uncertainty. We start with the effect of an increase in the variance of domestic output shocks on n_T. The derivative of equation 13 in order to σ_y^2 is:

$$\frac{\partial n_T}{\partial \sigma_y^2} = \frac{s + \alpha\left(\sigma_r^2 - \sigma_{yr}\right)(1-\gamma)}{\left(\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}\right)^2(1-\gamma)} \tag{26}$$

which is an expression without a definite sign. In fact, risk aversion assures that the denominator is positive but the sign of the numerator depends on the values of the parameters. There are several effects at work in this type of situation. Firstly, a higher variance of domestic technology shocks increases the total uncertainty that is associated with the federal fiscal system, which is given by $\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}$. For a risk averse country, this increased uncertainty decreases the optimal insurance-capital ratio. Secondly, the higher volatility of domestic shocks changes the expected benefits from participating in the fiscal system. As a matter of fact, when σ_{yr} is negative, the higher domestic output variance increases n_T. In this case, when there are stronger negative domestic shocks the country expects to receive larger transfers that reduce the negative effects on output. It is also true that when there are positive domestic shocks the country will tend to transfer more funds to the fiscal federation. However, for a risk averse country the net effect is positive. In other words, the benefits of receiving higher transfers when there are stronger negative domestic shocks are higher than the costs of making higher transfers when the reverse happens. This positive effect is stronger than the additional costs that come from facing a larger variance in the fiscal system. When σ_{yr} is positive a higher variance of domestic technology shocks may lead to a reduction in n_T. In such cases the domestic and federal shocks tend to have the same sign. Therefore, an increase in σ_y^2 makes the country receive less or even pay funds in the event of a positive shock, but also makes it pay less or even receive funds in the event of a negative shock. This has a net positive effect on n_T, which may not be strong enough to counter the negative effect that comes from a larger variance in the fiscal system.

Next, we analyse the effect of an increase in the variance of federal shocks σ_r^2 on the optimal insurance-capital ratio n_T. The corresponding derivative is:

$$\frac{\partial n_T}{\partial \sigma_r^2} = \frac{s + \alpha\left(\sigma_{yr} - \sigma_y^2\right)(1-\gamma)}{\left(\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}\right)^2 (1-\gamma)} < 0 \tag{27}$$

which, given the restriction that $n_T > 0$, has a negative sign. The first interesting point to note is that the expression above is similar to equation 26, except for the fact that it relates the covariance σ_{yr} with the variance of domestic shocks and not with the variance of the federal shocks. Therefore, the intuition of the result is close to what has been described above. In fact, an increase in the variance of the regional shocks decreases the optimal n_T because it increases the uncertainty in the fiscal system. This negative effect is stronger the higher is the initial quantity of insurance and the smaller is the uncertainty in the system. Such a result is clear when we rewrite 27 as:

$$\frac{\partial n_T}{\partial \sigma_r^2} = -\frac{n_T}{\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}} < 0 \tag{28}$$

Finally, we analyse the effect of an increase in covariance between domestic and federal shocks on the demand for insurance. The derivative of equation 13 in order to σ_{yr} is:

$$\frac{\partial n_T}{\partial \sigma_{yr}} = \frac{-2s + \alpha\left(\sigma_y^2 - \sigma_r^2\right)(1-\gamma)}{\left(\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}\right)^2 (1-\gamma)} \tag{29}$$

which is, again, an expression without a definite sign. It is clear that an increase in the covariance of shocks decreases the uncertainty in the fiscal system. This has a positive effect on the optimal insurance-capital ratio. However, an increase in this covariance also means that there are fewer opportunities for risk sharing, which has a negative effect on n_T. In the next section we develop a comparative dynamics analysis and plot the path of the variables in the stationary equilibrium.

3.4 Growth and Comparative Dynamics

At this point it is important to analyse how the federal fiscal system participation affects the mean and variance of the capital growth rate in the economy. Taking equation 5 we rewrite:

$$\frac{dK}{K} = \psi dt + dk \tag{30}$$

where ψ is defined as:

$$\psi = \alpha - \frac{C}{K} - s n_T \tag{31}$$

and dk is defined as in equation 6. Substituting the optimal values of the control variables n_T and C/K, the mean and variance of the capital growth rate are respectively:

$$\psi = \frac{\alpha - \beta}{1-\gamma} - \frac{1}{(1-\gamma)}\left(\frac{-s^2 + \alpha s\left(\sigma_y^2 - \sigma_{yr}\right)(1-\gamma)}{\left(\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}\right)(1-\gamma)}\right) - \frac{\gamma}{2}\sigma_k^2 \tag{32}$$

and:

$$\begin{aligned}\sigma_k^2 = \alpha^2\sigma_y^2 &+ \left(\frac{-s + \alpha\left(\sigma_y^2 - \sigma_{yr}\right)(1-\gamma)}{\left(\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}\right)(1-\gamma)}\right)^2 \left(\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}\right) \\ &- 2\alpha\left(\frac{-s + \alpha\left(\sigma_y^2 - \sigma_{yr}\right)(1-\gamma)}{\left(\sigma_y^2 + \sigma_r^2 - 2\sigma_{yr}\right)(1-\gamma)}\right)\left(\sigma_y^2 - \sigma_{yr}\right)\end{aligned} \tag{33}$$

One of the things that may look strange in these models is that, for $\gamma < 0$, the variance of the capital stock growth rate affects positively the average growth rate in the economy. This effect works through consumption. For example, an increase in σ_k^2 leads to a decrease in the consumption-capital ratio, which increases ψ. Therefore, uncertainty proves to have real effects on the economy. Next, we present three different scenarios and compare the simulated stationary equilibrium paths of variables for the cases of optimal system participation and no system participation. At this point it is necessary to stress that the purpose

Exogen.	α	β	γ	s	σ_y^2	σ_r^2	σ_{yr}
Values	0.3	0.2	-2	0.001	0.07	0.01	-0.009
Endogen.	n_T	C/K	ψ	σ_k^2	$C/K\ ns$	$\psi\ ns$	$\sigma_k^2\ ns$
Values	0.238	0.2659	0.0338	0.0006	0.2604	0.0396	0.0063

Table 1: Scenario 1

of these simulations is simply to illustrate some different situations that may occur within the framework of the federal fiscal system that we have examined.

The first scenario is presented in Table 3.1. Note that the second and fourth rows of the table contain respectively the values assumed by the exogenous and endogenous parameters in the model. Finally, the letters ns designate the value assumed by the endogenous variable in the case of no participation in the federal fiscal system. The first scenario describes a situation where domestic shocks are more volatile than the regional shocks and there is a negative covariance between both. The model was calibrated in order to generate useful illustrations and does not intend to describe any particular economy. Therefore, the values assumed by some parameters are intentionally exaggerated so as to produce clear illustrations. Note that the value of parameter s was chosen to be very small. In fact, large values for s strongly diminish the benefits of the federal fiscal system, so countries would choose not to participate. In addition, the value of γ is chosen to be within the interval suggested by Hall (1988).

The simulation is made for 150 periods, following a monthly basis. The domestic and regional shocks were randomly generated and the Cholesky factorisation was used to assure that they exhibit the desired covariance. Figure 3.2 plots the results of the simulation. As expected, Scenario 1 produces a positive optimal degree of participation in the federal fiscal system. It can be observed that the country sacrifices average growth in exchange for a substantial reduction in uncertainty. Therefore, there is less volatility in the total amount of resources that is available in each period, which translates into more-stable paths for capital stock and consumption. In addition, it is clear that the path for consumption is identical to the path for capital. As demonstrated above, the consumption-capital ratio is constant in the stationary equilibrium. Finally, we examine the reason why average growth has decreased. Firstly, fiscal system participation had a net positive impact on consumption, which was mainly due to the reduction in σ_k^2. Secondly, fiscal system participation also means that the country has to spend resources to buy insurance, which also has a negative impact on average growth.

The second scenario is illustrated in Table 3.2. Comparing with the first scenario there are changes on the exogenous parameters that describe the uncertainty environment. In fact, the situation has been reversed. This time, the domestic shocks are less volatile than the regional shocks and there is a positive covariance between them. The results of the second simulation are plotted in Figure 3.3.

The analysis of the results obtained provides some interesting insights. The

Exogen.	α	β	γ	s	σ_y^2	σ_r^2	σ_{yr}
Values	0.3	0.2	-2	0.001	0.03	0.05	0.009
Endogen.	n_T	C/K	ψ	σ_k^2	$C/K\ ns$	$\psi\ ns$	$\sigma_k^2\ ns$
Values	0.096	0.2645	0.0354	0.0020	0.2640	0.0360	0.0027

Table 2: Scenario 2

Exogen.	α	β	γ	s	σ_y^2	σ_r^2	σ_{yr}
Values	0.3	0.2	-2	0.001	0.6	0.01	0
Endogen.	n_T	C/K	ψ	σ_k^2	$C/K\ ns$	$\psi\ ns$	$\sigma_k^2\ ns$
Values	0.2945	0.2656	0.0341	0.0009	0.2127	0.0873	0.054

Table 3: Scenario 3

optimal degree of participation in the federal fiscal system is much less than in the previous situation. Consequently, the reduction in the volatility of the amount of resources that is available in each period is also much less. As can be observed in Figure 3.3, the paths of the capital stock and consumption are close in the situations of system participation and no system participation. The effects on average growth and on the consumption-capital ratio are similar to those of scenario 1, but of smaller magnitude.

Finally, the third scenario is described in Table 3.3. It presents a situation where the volatility of domestic technology shocks is extremely high when compared with the volatility of regional shocks. In fact, the value assumed by the variance of domestic shocks is clearly exaggerated and it serves only for illustrative purposes. Furthermore, it is considered that the domestic and federal shocks are independent.

In this last scenario the optimal degree of participation in the fiscal system is relatively high. As a consequence, the country manages to switch from a very high volatility situation to a very low volatility situation. Therefore, the stationary equilibrium paths of capital and consumption are very stable. Figure 3.4 illustrates these results. The effects on average growth and consumption-capital ratio are, again, similar to those obtained in the previous scenarios but of greater magnitude. This is due to the larger values for n_T. We conclude this section with a remark. It is obvious that changes in technology, preferences and insurance cost parameters lead to changes in the optimal insurance-capital ratio. We have chosen not to present scenarios involving changes in these parameters because we are mainly interested in the effects of different uncertainty environments on optimal system participation.

Figure 3.2: (-) Operating Fiscal System (- -) No Fiscal System

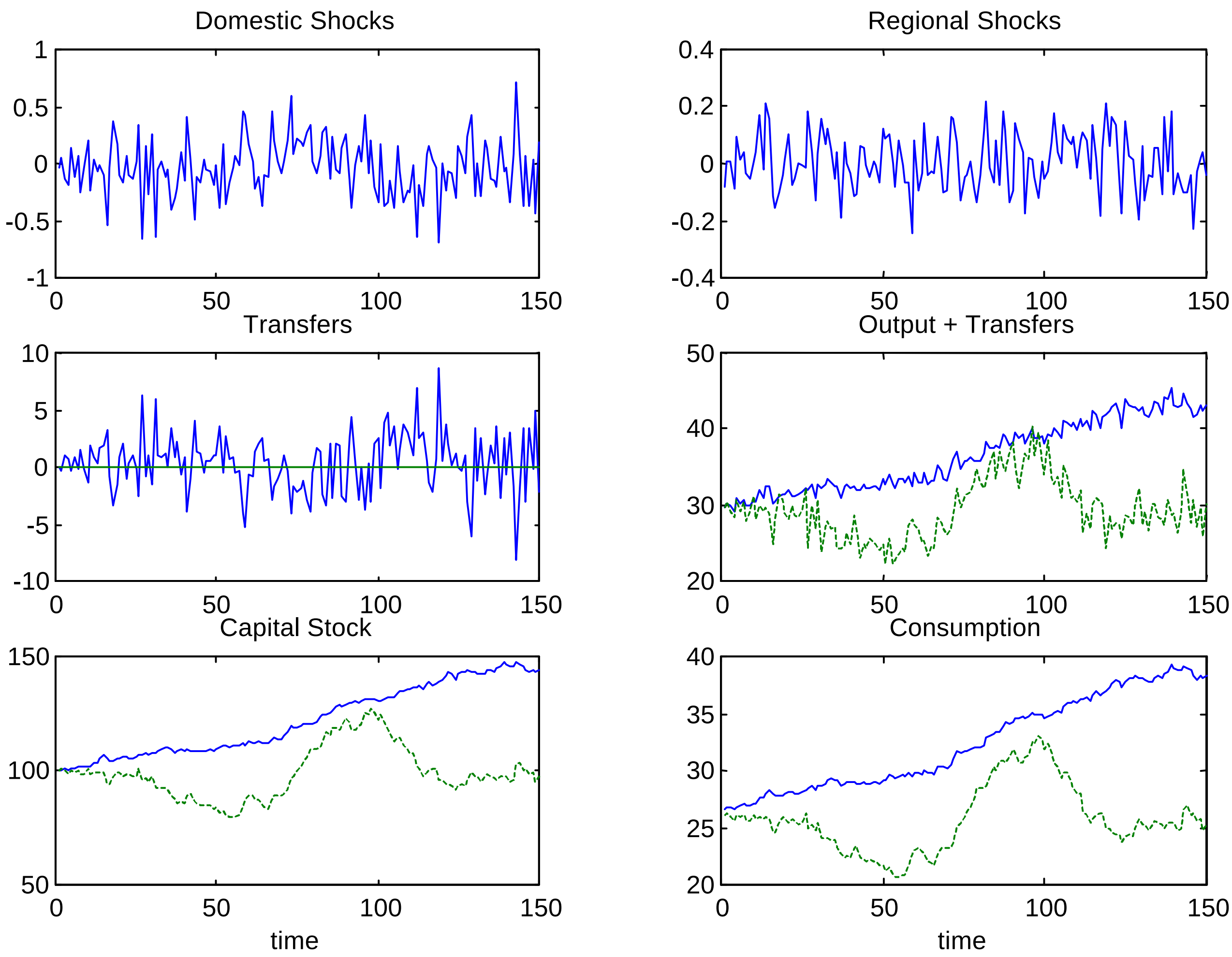


Figure 3.3: (-) Operating Fiscal System   (- -) No Fiscal System

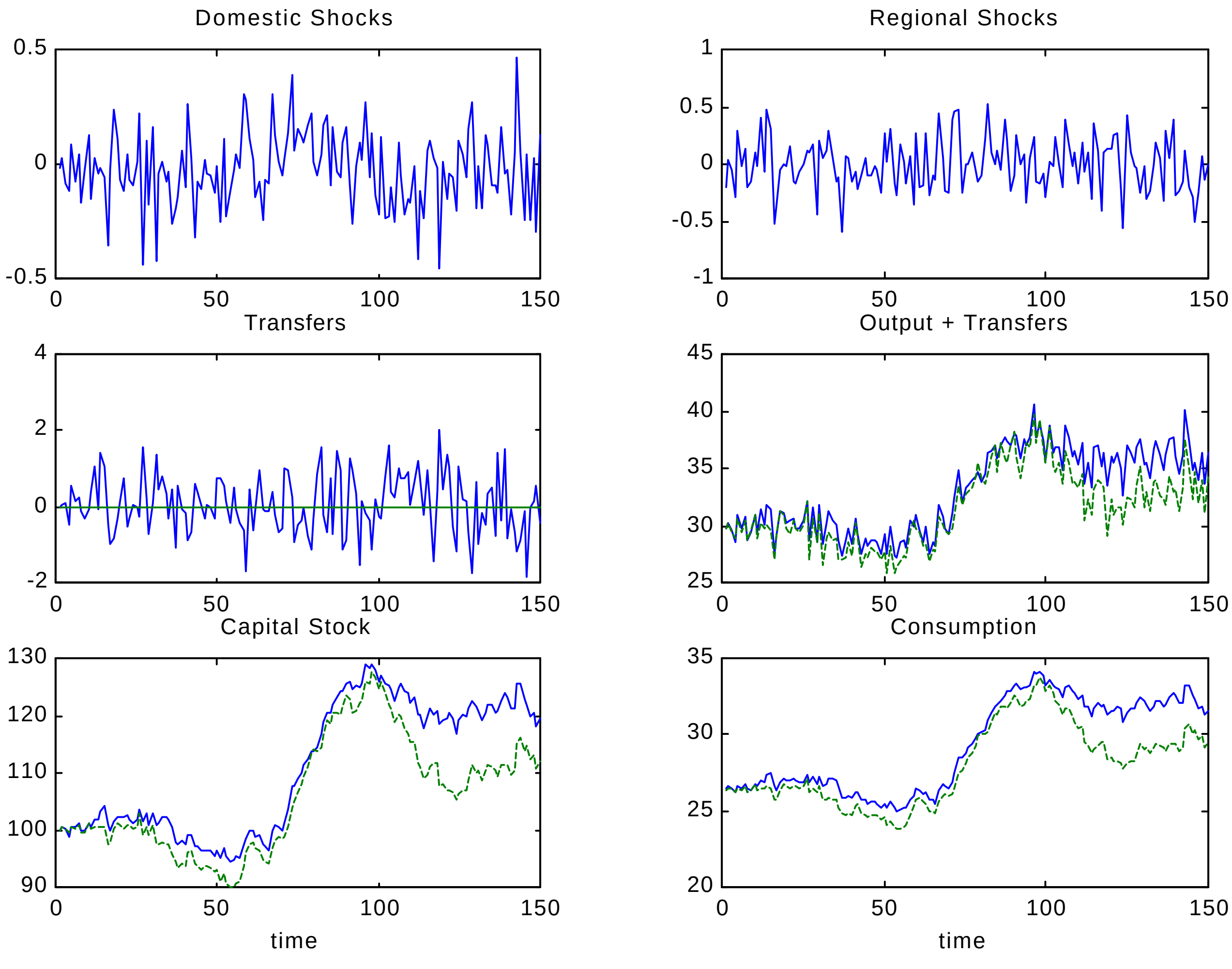
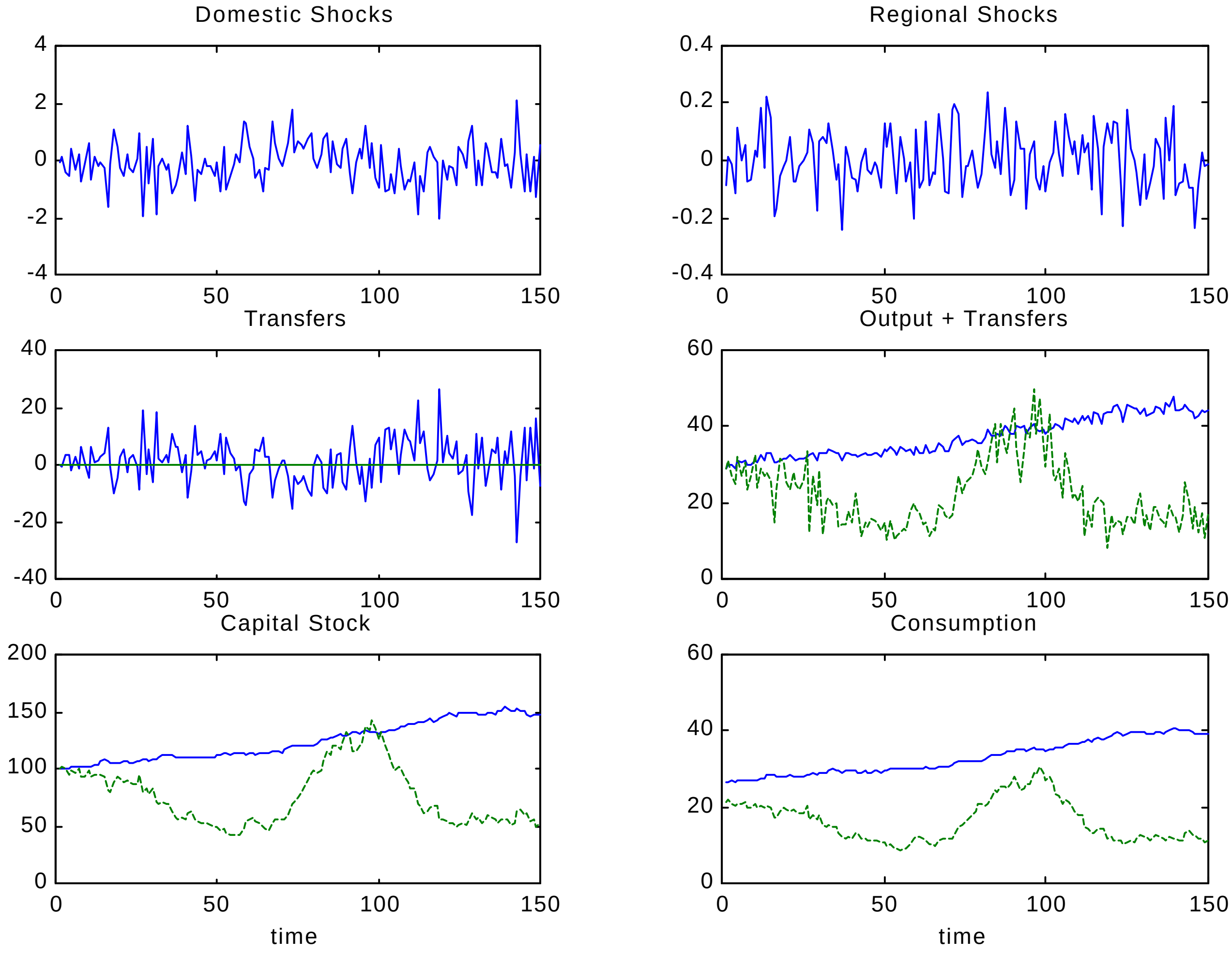


Figure 3.4: (-) Operating Fiscal System   (- -) No Fiscal System



4 A Fiscal Federation with a Redistribution Component

4.1 The Fiscal System with Redistribution

In this section a different type of federal fiscal system is examined. This time we assume that the country faces a federal fiscal system whose returns depend on the capital stock growth rate in the rest of the federation. In other words, the decision to take part in the federal fiscal system links the path of domestic capital stock to the general economic performance in the rest of the fiscal federation. The amount of transfers received from the federation in each period is defined as:

$$dR = \theta T \left(\frac{dK^*}{K^*} \right) \tag{34}$$

where, T stands for the degree of participation in the fiscal system and dK^*/K^* is the capital stock growth rate in the fiscal federation. This growth rate is described by a Brownian motion defined as:

$$\frac{dK^*}{K^*} = rdt + dr \tag{35}$$

where r is the deterministic trend and dr is the stochastic component with variance σ_r^2. Finally, θ is a scale parameter which is positive but inferior to one. This parameter has no qualitative effect on the results. It is included to scale down the benefits that the country may take from participating in the fiscal federation. These benefits should not be too high in order to assure that the federation runs balanced budgets. Next, we analyse equation 34 and discuss the essence of the fiscal system that is being proposed. The assumption that the transfers depend on the path of the capital stock of the federation means that the system includes both risk sharing and redistribution components. On the one hand, the system allows a reduction in uncertainty because it provides the possibility of risk diversification. In fact, when the country takes part in the system, the total amount of resources that is available in each period depends not only on the domestic shock but also on the federal shock. Despite being different from the previous model, this feature makes a risk averse country willing to participate in the federal fiscal system.

On the other hand, the system allows for redistribution between the country and the federation. In fact, the difference between what is being paid for each unit of fiscal system participation and the deterministic rate of return θr, determines the size and direction of the redistribution. If the price of each unit of participation is lower than the deterministic rate of return, it means that, on average, the country is a net receiver of funds. If the reverse is true, then the country will be, on average, a net contributor to the fiscal system. A crucial point is to know what it is that may lead a net contributor to take part in the fiscal federation. In fact, it is obvious that redistribution can take place only if some countries are net contributors. Although, these countries are being penalised in terms of redistribution, they still profit from the uncertainty reduction effect. Therefore, if the opportunities for risk sharing are strong enough, even net contributors may wish to participate in the federal fiscal system. This result has already appeared in the previous section. In fact, by saying that there is a payment for each unit of insurance, we actually considered countries as average net contributors. There is also an obvious analogy between this model and the optimal portfolio selection literature. In this context, one unit of federal fiscal system participation can be regarded as an asset whose rate of return contains deterministic and stochastic components.[3]

The intertemporal resource constraint in this model is then written as:

$$dK = \alpha K dt + \alpha K dy - sT dt + \theta T \left(\frac{dK^*}{K^*} \right) - C dt \tag{36}$$

where s is again the price paid for each unit of fiscal system participation. Next, grouping the stochastic components on the intertemporal resource constraint we define:

$$dk = \alpha dy + \theta n_T dr \tag{37}$$

As mentioned in Section Three, in this type of model the optimal values for the control variables are fixed proportions of the state variable. Therefore, the intertemporal resource constraint must be rewritten as:

$$\frac{dK}{K} = \left(\alpha - s n_T - \frac{C}{K} + \theta n_T r \right) dt + dk \tag{38}$$

where $T/K = n_T$. In the next subsection we derive the optimum conditions and determine the stationary equilibrium. Closed form expressions for n_T and C/K are then obtained.

4.2 Optimum Conditions

Note that the model is similar to the one presented in Section Three. Thus, we take the same steps towards the solution. The stochastic Lagrangian function is given by:

$$\pounds = \frac{1}{\gamma} C(t)^{\gamma} e^{-\beta t} + L_K \left[e^{-\beta t} X(K) \right] \tag{39}$$

where $L_K \left[e^{-\beta t} X(K) \right]$ is the differential generator of the value function. Furthermore, this differential generator of the value function is given by:

$$L_K \left[e^{-\beta t} X(K) \right] = \frac{\partial V}{\partial t} + \left(\alpha - s n_T - \frac{C}{K} + \theta n_T r \right) K \frac{\partial V}{\partial K} + \frac{1}{2} \sigma_k^2 K^2 \frac{\partial^2 V}{\partial K^2} \tag{40}$$

[3]This discussion raises again the question of knowing if the risk sharing role of fiscal federations can be fulfilled by capital markets. As mentioned in the introduction, the evidence for the United States suggests that there is an important role for capital and credit markets in smoothing product shocks. In the European Union the situation varies across countries, but it seems that government smoothing is relatively more important. This may indicate the need for a fiscal federation.

where $V(K,t)$ stands for the value function of the problem and σ_k^2 is the variance of the capital stock. As before, the value function is assumed to depend on time only for the effect of discounting, that is, $V(K) = e^{-\beta t}X(K)$. At this point we compute the variance of the capital stock, which can be written as:

$$\sigma_k^2 = \alpha^2\sigma_y^2 + \theta^2 n_T^2 \sigma_r^2 + 2\alpha\theta n_T \sigma_{ry} \tag{41}$$

Additionally, the suggested value function will be again:

$$X(K) = \delta K^\gamma \tag{42}$$

where δ is a parameter to be determined. The first order conditions of the problem are obtained by deriving the stochastic Lagrangian function in order to C/K and n_T. That is:

$$K^\gamma \left(\frac{C}{K}\right)^{\gamma-1} - \gamma\delta K^\gamma = 0 \tag{43}$$

$$(-s+\theta r)\gamma\delta K^\gamma + \left(\theta^2 n_T \sigma_r^2 + \alpha\theta\sigma_{yr}\right)\delta\gamma(\gamma-1)K^\gamma = 0 \tag{44}$$

Additionally, we need the expression of the Bellman Equation evaluated at the optimum, which can be written as:

$$\frac{K^\gamma}{\gamma}\left(\frac{C}{K}\right)^\gamma - \beta\delta K^\gamma + \left(\alpha - sn_T - \frac{C}{K} + \theta n_T r\right)\gamma\delta K^\gamma + \\ + \frac{1}{2}\sigma_k^2\gamma(\gamma-1)\delta K^\gamma = 0 \tag{45}$$

Thus, rearranging equation 44 we obtain the expression for the optimal n_T as:

$$n_T = \frac{-s+\theta r - \alpha\theta\sigma_{yr}(1-\gamma)}{\theta^2\sigma_r^2(1-\gamma)} \tag{46}$$

Finally, combining equations 43 and 45, the optimum C/K is given as:

$$\frac{C}{K} = \frac{\beta}{1-\gamma} - \frac{\gamma}{1-\gamma}(\alpha - sn_T + \theta n_T r) + \\ \frac{\gamma}{2}\left[\alpha^2\sigma_y^2 + \theta^2 n_T^2\sigma_r^2 + 2\alpha\theta n_T \sigma_{ry}\right] \tag{47}$$

It is useful to make a few comments on equations 46 and 47. Firstly, it is necessary to impose restrictions on the values of the parameters in order to guarantee that n_T is non-negative. That condition is:

$$\sigma_{yr} < \frac{\theta r - s}{\alpha\theta(1-\gamma)} \tag{48}$$

which means that net contributors take part in the fiscal federation only if the covariance between domestic and federal shocks is negative. This is the situation that we have described above. An interesting point is that the variance of domestic technology shocks σ_y^2 does not affect the optimal degree of participation in the federal fiscal system. In fact, neither the costs nor the benefits of taking part in the system depend on the magnitude of the volatility that is faced domestically. Finally, if $\gamma < 0$, the consumption-capital ratio is negatively affected by increases in the variance of the capital stock growth rate and it is positively affected by increased redistribution in favour of the country. In the next subsection we turn to the study of the properties of the equilibrium.

4.3 Equilibrium Analysis

The analysis of the stationary equilibrium is divided into two parts. First, we analyse the effects of costs, technology and preferences. Then, we examine the parameters that describe uncertainty. We begin with changes in the price of system participation (s) and with changes in the deterministic rate of return (r). Taking the derivatives from equation 46 we obtain respectively:

$$\frac{\partial n_T}{\partial s} = -\frac{1}{\theta^2 \sigma_r^2 (1-\gamma)} < 0 \tag{49}$$

$$\frac{\partial n_T}{\partial r} = \frac{1}{\theta \sigma_r^2 (1-\gamma)} > 0 \tag{50}$$

These results are clear. When the price of each unit of participation increases, the optimum participation is lower. On the contrary, when the deterministic return of being in the system increases, the optimal participation is higher. The next derivative captures the effect of changes in the technology parameter α.

$$\frac{\partial n_T}{\partial \alpha} = -\frac{\sigma_{yr}}{\theta \sigma_r^2} \tag{51}$$

This effect depends on the covariance between domestic and federal shocks. Take for instance the case of a negative covariance. An increase in α means that there is a stronger effect of domestic shocks on output. Therefore, if the covariance between domestic and federal shocks is negative, the country would choose a higher participation in the system as a way of balancing the effects of domestic shocks. Next, the effect of an increase in the coefficient of relative risk aversion is given by:

$$\frac{\partial n_T}{\partial (1-\gamma)} = \frac{s - \theta r}{\theta^2 \sigma_r^2 (1-\gamma)^2} \tag{52}$$

$$= -\frac{n_T}{1-\gamma} - \frac{\alpha \theta \sigma_{yr} (1-\gamma)}{\theta^2 \sigma_r^2 (1-\gamma)^2} \tag{53}$$

which also has an indefinite sign. Looking at equation 53, it is clear that an increase in the coefficient of relative risk aversion decreases the optimal participation in the system, except for some situations where the covariance between shocks is negative. One the one hand, an increase in risk aversion raises the value of the additional uncertainty that country is facing by taking part in the fiscal federation. This effect leads to a lower n_T. On the other hand, an increase in risk aversion means that the country will want to take additional advantage of risk sharing opportunities. This second effect exists only if $\sigma_{yr} < 0$ and it leads to a higher n_T. Therefore, the net effect of a higher coefficient of relative risk aversion can only be positive if $\sigma_{yr} < 0$. Now we turn to the analysis of the parameters that describe uncertainty. The following derivative shows that the effect of an increase in the covariance between domestic and federal shocks is always negative.

$$\frac{\partial n_T}{\partial \sigma_{yr}} = -\frac{\alpha}{\theta \sigma_r^2} < 0 \tag{54}$$

In fact, the higher the covariance, the lower the opportunities for risk sharing, which means a lower n_T. Finally, the effect of an increase in the variance of federal shocks is given by:

$$\frac{\partial n_T}{\partial \sigma_r^2} = \frac{s - \theta r + \alpha \theta \sigma_{yr} (1-\gamma)}{\theta^2 \sigma_r^4 (1-\gamma)} < 0 \tag{55}$$

which is always negative, since n_T must be defined as a positive value. As expected, the higher variance of regional shocks turns the return of the fiscal system more uncertain, which reduces the optimal participation.

4.4 Growth and Comparative Dynamics

This section examines the growth rate of an economy that is taking part in the federal fiscal system and compares it to the situation where there is no participation. As before, we present some simulations to illustrate the different situations that may occur. The capital stock growth rate of the country is defined as:

$$\frac{dK}{K} = \psi dt + dk \tag{56}$$

where:

$$\psi = \alpha - s n_T - \frac{C}{K} + \theta n_T r \tag{57}$$

$$dk = \alpha dy + \theta n_T dr \tag{58}$$

Given the solution that was obtained above, we can rewrite ψ and σ_k^2 as:

$$\psi = \alpha + \left(\frac{\theta r - s}{1-\gamma}\right)\left(\frac{-s+\theta r}{\theta^2 \sigma_r^2 (1-\gamma)} - \frac{\alpha \sigma_{yr}}{\theta \sigma_r^2}\right) + \frac{\gamma \alpha - \beta}{1-\gamma} - \frac{\gamma}{2}\sigma_k^2 \tag{59}$$

and:

$$\sigma_k^2 = \alpha^2 \sigma_y^2 + \theta^2 \left(\frac{-s+\theta r}{\theta^2 \sigma_r^2 (1-\gamma)} - \frac{\alpha \sigma_{yr}}{\theta \sigma_r^2}\right)^2 \sigma_r^2 + 2\alpha \left(\frac{-s+\theta r}{\theta \sigma_r^2 (1-\gamma)} - \frac{\alpha \sigma_{yr}}{\sigma_r^2}\right)\sigma_{yr} \tag{60}$$

Parameters	α	β	γ	s	r	θ	σ_y^2	σ_r^2
Values	0.3	0.2	-2	0.07	0.09	0.75	0.01	0.01
Parameters	σ_{yr}	n_T	C/K	ψ	σ_k^2	$C/K\ ns$	$\psi\ ns$	$\sigma_k^2\ ns$
Values	-0.009	0.2119	0.2660	0.0335	0.0003	0.2658	0.0342	0.0009

Table 4: Scenario 4

Parameters	α	β	γ	s	r	θ	σ_y^2	σ_r^2
Values	0.3	0.2	-2	0.07	0.13	0.75	0.01	0.06
Parameters	σ_{yr}	n_T	C/K	ψ	σ_k^2	$C/K\ ns$	$\psi\ ns$	$\sigma_k^2\ ns$
Values	0.01	0.2049	0.2672	0.0385	0.0032	0.2658	0.0342	0.0009

Table 5: Scenario 5

Note again that the parameters that describe uncertainty affect the average growth rate of the capital stock.

Next, we present four numerical simulations that illustrate different types of stationary equilibria. Table 4.1 describes a situation that we designate as Scenario 4. In this scenario there are two opposing effects. On the one hand, there is a negative average rate of return from taking part in the fiscal system ($\theta r < s$), which does not provide incentives for participation. On the other hand, there is a negative covariance between domestic and federal shocks. Therefore, there are opportunities to diversify risk, which provide the incentives for system participation. In this case, the net effect is positive and the country decides to take part in the federal fiscal system. Figure 4.1 plots the results of the simulation. As expected, fiscal system participation leads to a smaller average capital growth rate but also to a smaller variance. Scenario 5 describes a very different situation. The values assumed by the endogenous and exogenous parameters are presented in Table 4.2. This time the fiscal system has a positive average rate of return, but shocks present a positive covariance. The results of the simulation are presented in Figure 4.2. In this case, the optimal participation in the fiscal system leads to an increase in the average capital growth rate that more than compensates the negative effects of the increased variance in capital, output and consumption.

Next, Table 4.3 presents Scenario 6. In this scenario the fiscal system average rate of return is positive, the variance of regional shocks is much smaller than the variance of domestic shocks and their covariance is zero. In this case, both the average and the variance of the capital stock growth rate slightly increase with system participation. Figure, 4.3 plots the stationary equilibrium paths of the variables. Note that despite the large optimal value for n_T, the participation and the no participation paths are not much different. In fact, a high degree of fiscal system participation does not necessarily mean large changes the path of the domestic variables. It means that the benefits of the federal fiscal system can only be enjoyed through a large participation, which is equivalent to saying that the benefit per unit of participation is small. In the present situation, there

Parameters	α	β	γ	s	r	θ	σ_y^2	σ_r^2
Values	0.3	0.2	-2	0.07	0.1	0.75	0.09	0.009
Parameters	σ_{yr}	n_T	C/K	ψ	σ_k^2	$C/K\ ns$	$\psi\ ns$	$\sigma_k^2\ ns$
Values	0.00	0.3292	0.2591	0.0425	0.0086	0.2586	0.0414	0.0081

Table 6: Scenario 6

Parameters	α	β	γ	s	r	θ	σ_y^2	σ_r^2
Values	0.3	0.2	-2	0.07	0.0933	0.75	0.05	0.0095
Parameters	σ_{yr}	n_T	C/K	ψ	σ_k^2	$C/K\ ns$	$\psi\ ns$	$\sigma_k^2\ ns$
Values	-0.02	0.8421	0.2660	0.0340	0.0007	0.2621	0.0378	0.0045

Table 7: Scenario 7

is no large redistribution effect and the additional uncertainty that comes from the federation is small when compared with the existing domestic uncertainty. Therefore, the large participation, which is necessary to take advantage of the small redistribution, does not affect the path of the domestic variables. Finally, Table 4.4 presents Scenario 7. In this case the average rate of return of the fiscal system is zero, the variance of domestic shocks is high, the variance of federal shocks is low and the covariance between both is clearly negative. As a result, the country chooses to have a high degree of participation in the federal fiscal system, reducing the variance and the average growth rate of the capital stock. This is a case where the fiscal system only offers risk diversifying opportunities. Figure 4.4 plots the results of this last simulation.

Figure 4.1: (-) Operating Fiscal System (- -) No Fiscal System

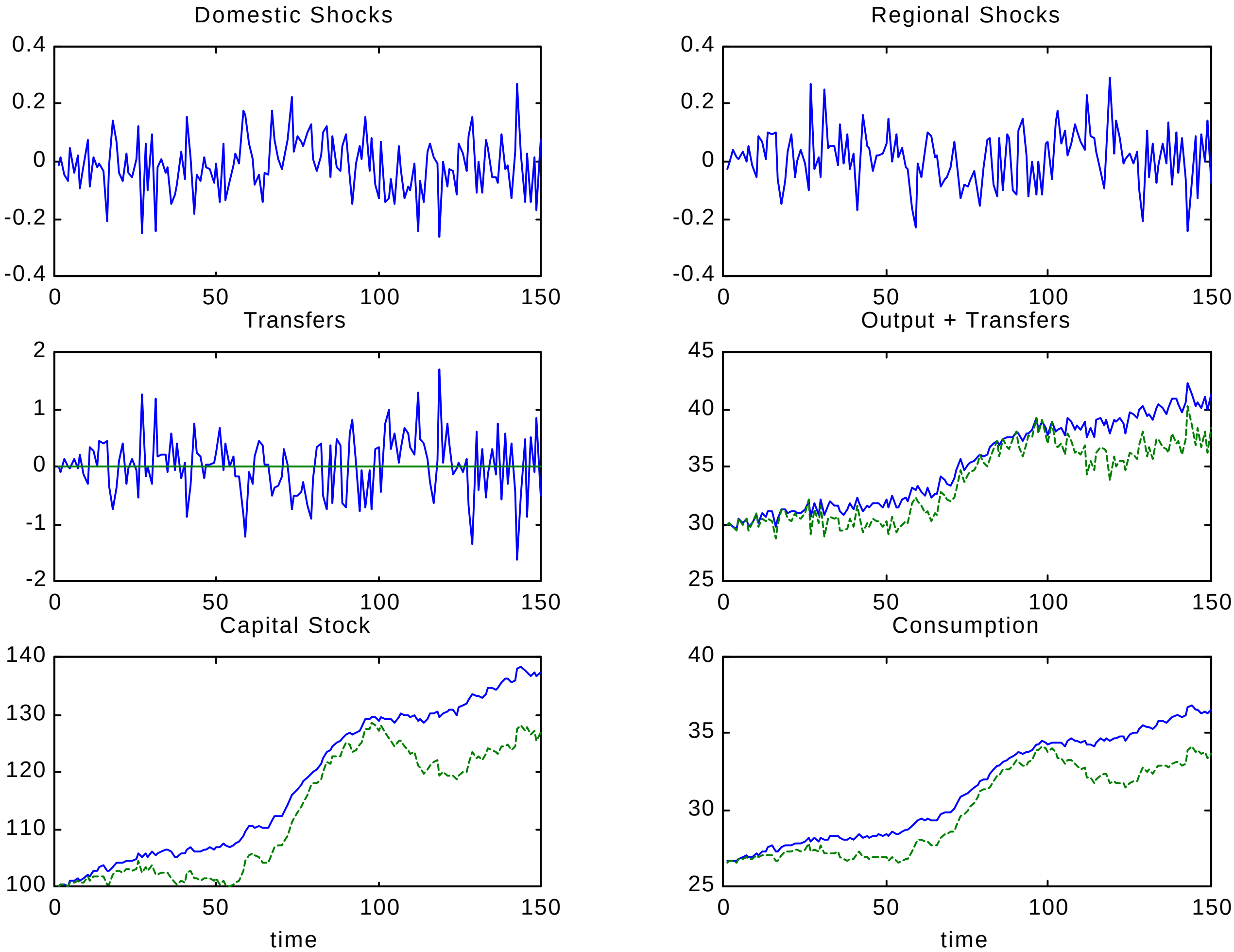


Figure 4.2: (-) Operating Fiscal System (- -) No Fiscal System

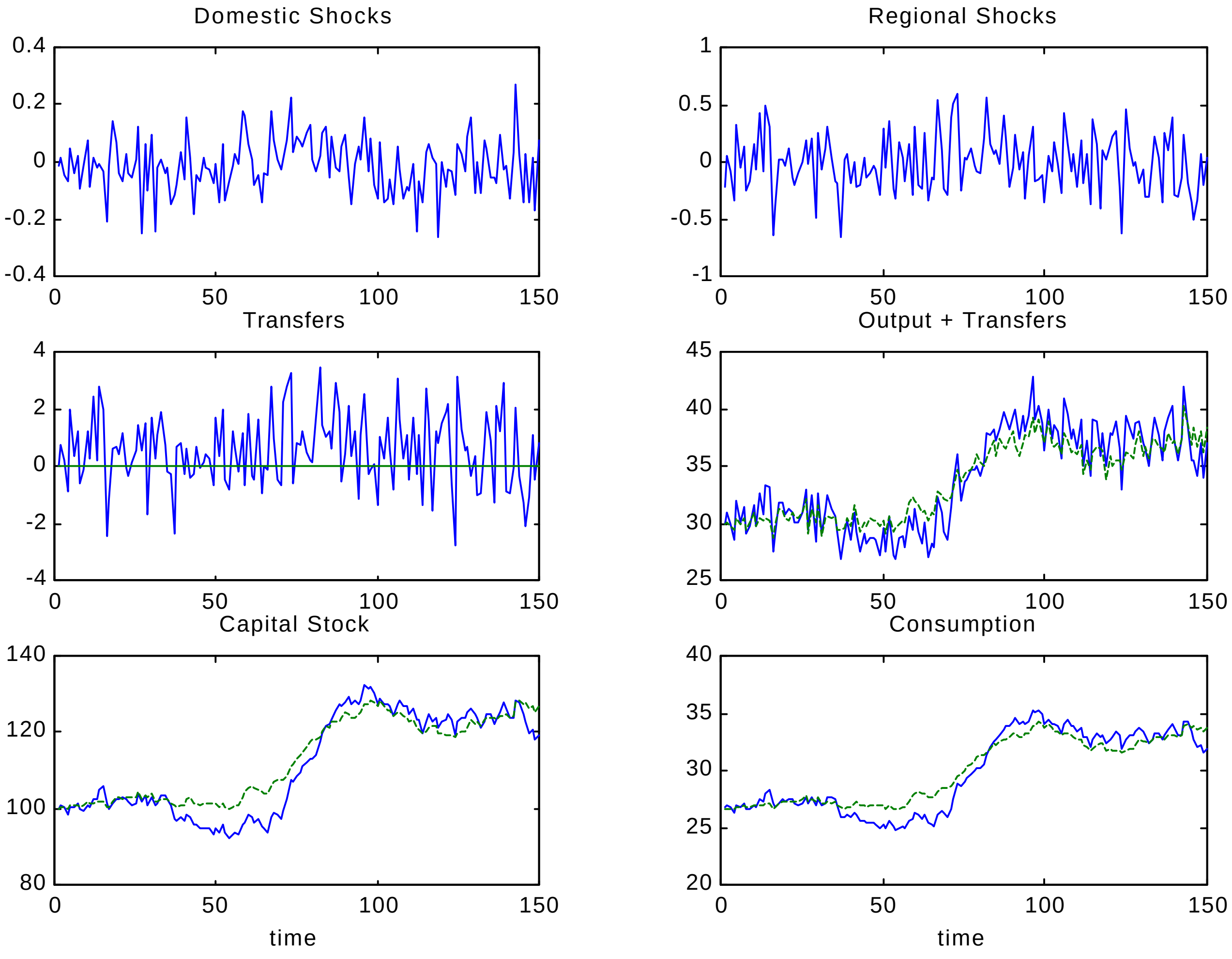


Figure 4.3: (-) Operating Fiscal System (--) No Fiscal System

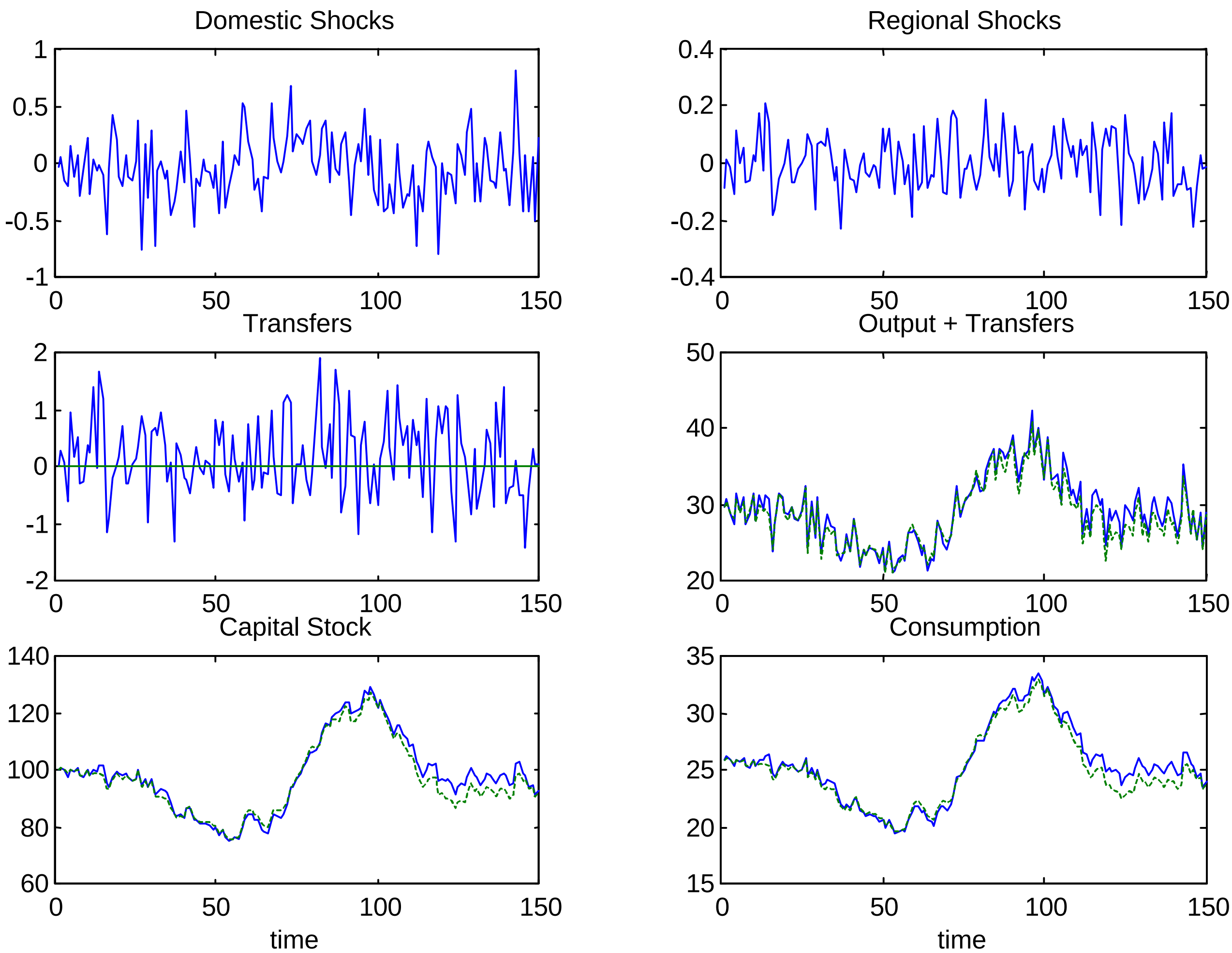
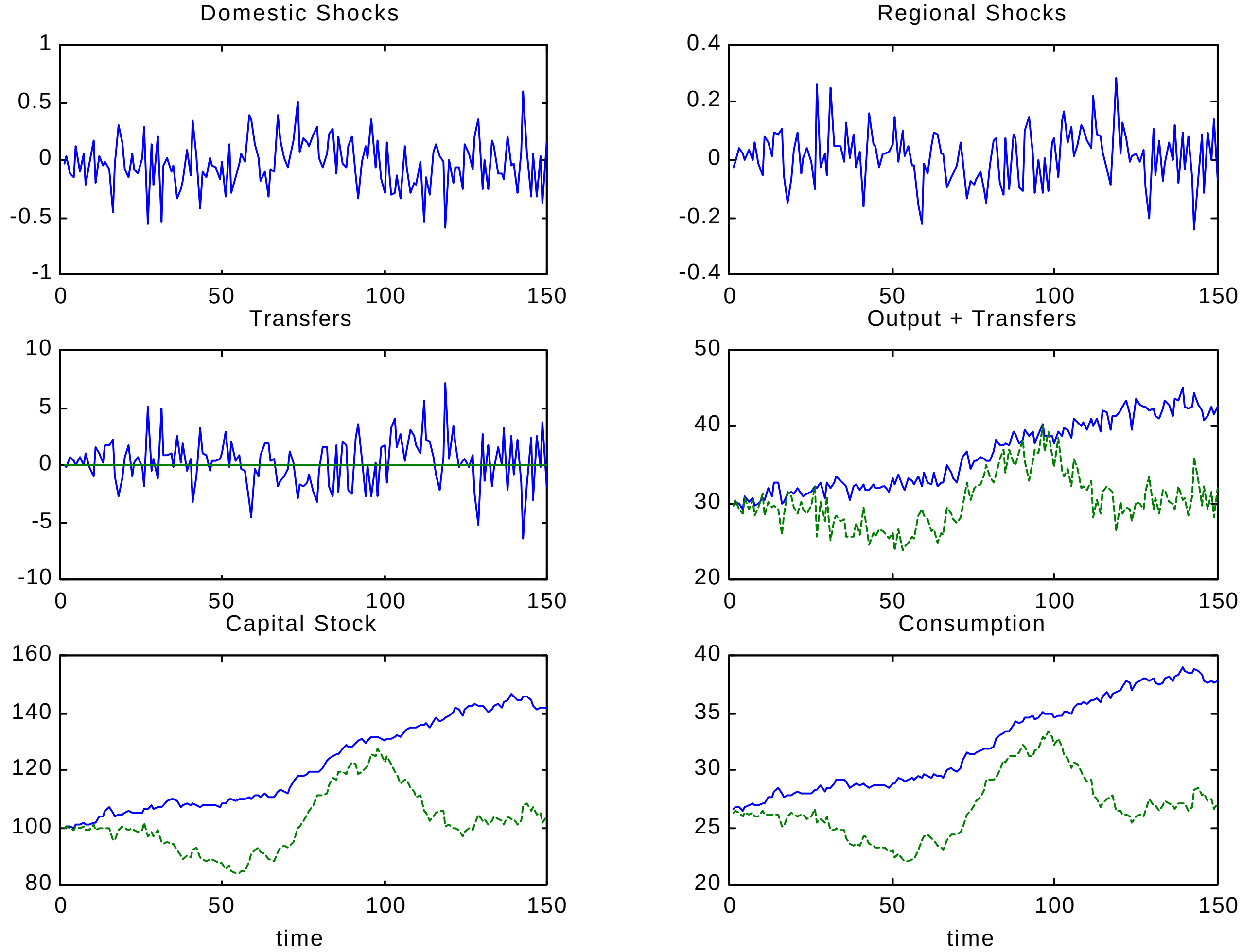


Figure 4.4: (-) Operating Fiscal System (--) No Fiscal System



In the next section we extend this model in order to include the parameter n_T as an argument of the utility function. Therefore, countries will have an additional motive for taking part in the federal fiscal system.

5 A Fiscal Federation Affecting the Utility Function

5.1 Fiscal System Participation as a Parameter of the Utility Function

This section considers that the degree of participation in the federal fiscal system is an additional argument in the utility function. In other words, countries place a positive value on fiscal system participation, which is independent of the benefits or losses that they may take from it. Consequently, countries that previously did not have enough incentives to take part in the system may now become participants. At this point it is important to discuss the motives why it may be adequate to place fiscal system participation in the utility function. Firstly, it can be argued that there are externalities or political motives, which are not included in the model, that can be captured by including participation in the utility function. Secondly, there may be some degree of altruism in the behaviour of countries. In some cases this may be a reason for taking part in the fiscal system. The problem of the single country will now be:

$$Max\ E_o \int_0^\infty \frac{1}{\gamma} \left[C(t)^\phi T(t)^{1-\phi} \right]^\gamma e^{-\beta t} dt \tag{61}$$

where $1 \geq \phi \geq 0$ is the parameter that measures the relative weight of consumption and fiscal system participation in the utility function. As before, $T(t)$ stands for the amount of participation in the federal fiscal system. The remaining variables of the model are similar to those of Section Four. Therefore, the intertemporal resource constraint is written again as:

$$dK = \alpha K dt + \alpha K dy - sTdt + \theta T \left(\frac{dK^*}{K^*} \right) - Cdt \tag{62}$$

In the next subsection we proceed with the solution of the model.

5.2 Optimum Conditions

In this case, the stochastic Lagrangian function is given by:

$$\pounds = \frac{1}{\gamma} \left[C(t)^\phi T(t)^{1-\phi} \right]^\gamma e^{-\beta t} + L_K \left[e^{-\beta t} X(K) \right] \tag{63}$$

where $L_K\left[e^{-\beta t}X(K)\right]$ is the differential generator of the value function. The differential generator of the value function is again:

$$L_K\left[e^{-\beta t}X(K)\right]=\frac{\partial V}{\partial t}+\left(\alpha-sn_T-\frac{C}{K}+\theta n_T r\right)K\frac{\partial V}{\partial K} \tag{64}$$
$$+\frac{1}{2}\sigma_k^2K^2\frac{\partial^2 V}{\partial K^2}$$

Following the same method that was used in the previous section, the first order conditions of the problem become:

$$\left[\left(\frac{C}{K}\right)^{\phi}n_T^{1-\phi}\right]^{\gamma-1}K^{\gamma}\phi\left(\frac{C}{K}\right)^{\phi-1}n_T^{1-\phi}-\gamma\delta K^{\gamma}=0 \tag{65}$$

$$\left[\left(\frac{C}{K}\right)^{\phi}n_T^{1-\phi}\right]^{\gamma-1}K^{\gamma}(1-\phi)\left(\frac{C}{K}\right)^{\phi}n_T^{-\phi}+ \tag{66}$$
$$+(-s+\theta r)\gamma\delta K^{\gamma}+\left(\theta^2n_T\sigma_r^2+\alpha\theta\sigma_{yr}\right)\delta\gamma(\gamma-1)K^{\gamma}=0$$

In addition, there is the expression of the Bellman equation evaluated at the optimum, which can be written as:

$$\frac{K^{\gamma}}{\gamma}\left[\left(\frac{C}{K}\right)^{\phi}n_T^{1-\phi}\right]^{\gamma}-\beta\delta K^{\gamma}+\left(\alpha-sn_T-\frac{C}{K}+\theta n_T r\right)\gamma\delta K^{\gamma}+ \tag{67}$$
$$\frac{1}{2}\sigma_k^2\gamma(\gamma-1)\delta K^{\gamma}=0$$

At this point, using equations 65, 66 and 67 we obtain:

$$n_T=\left(\frac{1-\phi}{\phi}\right)\left[\frac{1}{s-\theta r-\left(\theta^2n_T\sigma_r^2+\alpha\theta\sigma_{yr}\right)(\gamma-1)}\right]\left(\frac{C}{K}\right) \tag{68}$$

and also:

$$\frac{C}{K}=\left(\frac{\phi}{1-\gamma\phi}\right)\left[\beta-(\alpha-sn_T+\theta n_T r)\gamma-\frac{1}{2}\sigma_k^2\gamma(\gamma-1)\right] \tag{69}$$

These two equations define the optimal values for n_T and C/K. The solution of this two-equation non-linear system is computationally complex. Therefore, we revert to the case of a logarithmic utility function, which is the important benchmark that results from making $\gamma=0$. Recall that this situation corresponds to a coefficient of relative risk aversion equal to one. In this case the optimum conditions simplify to:

$$\frac{C}{K}=\phi\beta \tag{70}$$

$$n_T = (1-\phi)\left[\frac{\beta}{s - \theta r + \theta^2 n_T \sigma_r^2 + \alpha\theta\sigma_{yr}}\right] \tag{71}$$

Finally, solving the second order degree equation 71, two roots are obtained:

$$n_T = \frac{-\rho \pm \sqrt{\rho^2 + 4\left(1-\phi\right)\beta\theta^2\sigma_r^2}}{2\theta^2\sigma_r^2} \tag{72}$$

where $\rho = s - \theta r + \alpha\theta\sigma_{yr}$. At this point there are three important things to mention. Firstly, note that, $\rho^2 + 4\left(1-\phi\right)\beta\theta^2\sigma_r^2$ is always positive, which means that the solution is well defined. Secondly, the valid solution is the one generated by the positive root. In fact, the other solution presents negative values for n_T, which is something that we have ruled out. Finally, note that equations 72 and 46 generate the same solution for $\rho < 0$, with $\phi = 1$ and $\gamma = 0$. As expected, to put all the weight in consumption is equivalent to the case where $T(t)$ is not an argument of the utility function.

5.3 Equilibrium Analysis

This subsection examines the characteristics of the stationary equilibrium that was just obtained. First, we briefly comment on equation 70, which is the expression for the optimal consumption-capital ratio. In fact, making $\gamma = 0$ is equivalent to setting the elasticity of substitution between consumption at any two points in time equal to one. Therefore, it is natural to have the optimal consumption-capital ratio equal to the intertemporal discount rate β, adjusted by parameter ϕ.

Next, we examine how the optimal n_T responds to changes in the exogenous parameters. The effect of a marginal increase in the weight attributed to n_T in the utility function is given by the derivative:

$$\frac{\partial n_T}{\partial\left(1-\phi\right)} = \beta\left(\rho^2 + 4\left(1-\phi\right)\beta\theta^2\sigma_r^2\right)^{-\frac{1}{2}} > 0 \tag{73}$$

which is always positive. As expected, if the utility function puts greater importance on system participation the optimal n_T increases. The effect of a higher volatility in federal shocks σ_r^2 is more complex. The relevant derivative is:

$$\frac{\partial n_T}{\partial \sigma_r^2} = \frac{\left(\rho^2 + 4\left(1-\phi\right)\beta\theta^2\sigma_r^2\right)^{-\frac{1}{2}}\left(1-\phi\right)\beta}{\sigma_r^2} - \frac{\left(-\rho + \sqrt{\rho^2 + 4\left(1-\phi\right)\beta\theta^2\sigma_r^2}\right)}{2\theta^2\sigma_r^4} \tag{74}$$

It is not possible to completely define the sign of this expression, but for reasonable values of the parameters it tends to be negative. We complete this subsection with the effect of changes in ρ on the optimum n_T. The derivative is given by:

$$\frac{\partial n_T}{\partial \rho} = \frac{-1 + \rho\left(\rho^2 + 4\left(1-\phi\right)\beta\theta^2\sigma_r^2\right)^{-\frac{1}{2}}}{2\theta^2\sigma_r^2} < 0 \tag{75}$$

which is always negative. Recall that ρ is a function of other exogenous parameters. However, it is possible to obtain the effect of changes on those parameters. Take for instance the cost parameter s:

$$\frac{\partial n_T}{\partial s} = \frac{\partial n_T}{\partial \rho}\frac{\partial \rho}{\partial s} < 0 \tag{76}$$

The same thing can be done for the other parameters in ρ and the intuition of the results is similar to that of Section Four. In the next section we turn again to the analysis of the capital stock growth rate and simulate the stationary equilibrium path of variables.

5.4 Growth and Comparative Dynamics

As in the previous sections, the growth rate of the economy is defined as:

$$\frac{dK}{K} = \psi dt + dk \tag{77}$$

where:

$$\psi = \alpha - sn_T - \frac{C}{K} + \theta n_T r \tag{78}$$

$$dk = \alpha dy + \theta n_T dr \tag{79}$$

Substituting equations 70 and 72, we rewrite:

$$\psi = \alpha + (\theta r - s)\left(\frac{-\rho + \sqrt{\rho^2 + 4(1-\phi)\beta\theta^2\sigma_r^2}}{2\theta^2\sigma_r^2}\right) - \phi\beta \tag{80}$$

and:

$$\sigma_k^2 = \alpha^2\sigma_y^2 + \left(\frac{-\rho + \sqrt{\rho^2 + 4(1-\phi)\beta\theta^2\sigma_r^2}}{2\sigma_r^2}\right)^2 \sigma_r^2 + \left(\frac{-\rho + \sqrt{\rho^2 + 4(1-\phi)\beta\theta^2\sigma_r^2}}{\theta\sigma_r^2}\right)\alpha\sigma_{yr} \tag{81}$$

Note that in this case the average capital stock growth rate is independent of the variance of domestic shocks. This results directly from the assumption that $\gamma = 0$ and it means that total domestic uncertainty does not affect the long run path of the economy. Next, we present a set of simulations for this model. First we take Scenario 8, which is described in Table 5.1. In this situation there is a negative deterministic rate of return in the federal fiscal system. This is compensated by the existence opportunities to diversify risk ($\sigma_{yr} < 0$) and by the existence of altruism ($1 - \phi = 0.7$). Figure 5.1 plots the stationary equilibrium path of variables. The first thing to observe is that the transfers are negative and become more negative as time goes on. This happens because there is both a strong negative deterministic rate of return in the system and a strong

Parameters	α	β	ϕ	s	r	θ	σ_y^2	σ_r^2
Values	0.3	0.2	0.7	0.07	0.005	0.75	0.01	0.01
Parameters	σ_{yr}	n_T	C/K	ψ	σ_k^2	$C/K\ ns$	$\psi\ ns$	$\sigma_k^2\ ns$
Values	-0.005	0.8578	0.14	0.1032	0.0031	0.2	0.1	0.0009

Table 8: Scenario 8

Parameters	α	β	ϕ	s	r	θ	σ_y^2	σ_r^2
Values	0.3	0.2	0.7	0.1	0.001	0.75	0.01	0.04
Parameters	σ_{yr}	n_T	C/K	ψ	σ_k^2	$C/K\ ns$	$\psi\ ns$	$\sigma_k^2\ ns$
Values	0.008	0.5310	0.14	0.1073	0.0092	0.2	0.1	0.0009

Table 9: Scenario 9

average capital growth rate. Therefore, as time goes on, the higher values for the capital stock increase the negative absolute level of transfers. In addition, note that despite the reduction in volatility, the fiscal system participation strongly reduces consumption. This is obviously compensated by the gains in utility that come from system participation. Finally, note that the average capital growth rate ψ slightly increases with the fiscal federation. In this case the negative effect, which is caused by the negative deterministic rate of return in the system, is more than compensated by the positive effect due to the decrease in the consumption-capital ratio. The economy sacrifices consumption but assures long-run growth.

Table 5.2 describes a different situation, named Scenario 9. In this situation there is not only a negative deterministic rate of return in the system, but also a positive covariance between domestic and foreign shocks. Despite this negative situation, there is some participation in the federal fiscal system, which is entirely due to altruistic behaviour.

The results of the simulation are plotted in Figure 5.2. As expected, the higher volatility of federal shocks, together with the positive covariance between domestic and federal shocks, generates an increase in total uncertainty, which translates into the path of the variables. As in the previous scenario, there is a slight increase in the average capital growth rate and in the negative absolute value of transfers.

Finally, Table 5.3 presents Scenario 10. In this situation both the deterministic rate of return of the system and the covariance between domestic and federal shocks are zero. Furthermore, the variance of federal shocks is much higher than the variance of domestic shocks. Here again, there are no incentives for fiscal system participation other than the role played by n_T on the utility function. Figure 5.3 plots this last simulation. Both total uncertainty and growth rise sharply. Note that since there is no loss attached to the deterministic return of the system, the decrease in consumption spills entirely into additional growth.

Parameters	α	β	ϕ	s	r	θ	σ_y^2	σ_r^2
Values	0.3	0.2	0.7	0.07	0.0933	0.75	0.01	0.5
Parameters	σ_{yr}	n_T	C/K	ψ	σ_k^2	$C/K\ ns$	$\psi\ ns$	$\sigma_k^2\ ns$
Values	0.00	0.4619	0.14	0.16	0.0609	0.2	0.1	0.0009

Table 10: Scenario 10

[FIGURE 5.1]

Figure 5.1: (-) Operating Fiscal System (--) No Fiscal System

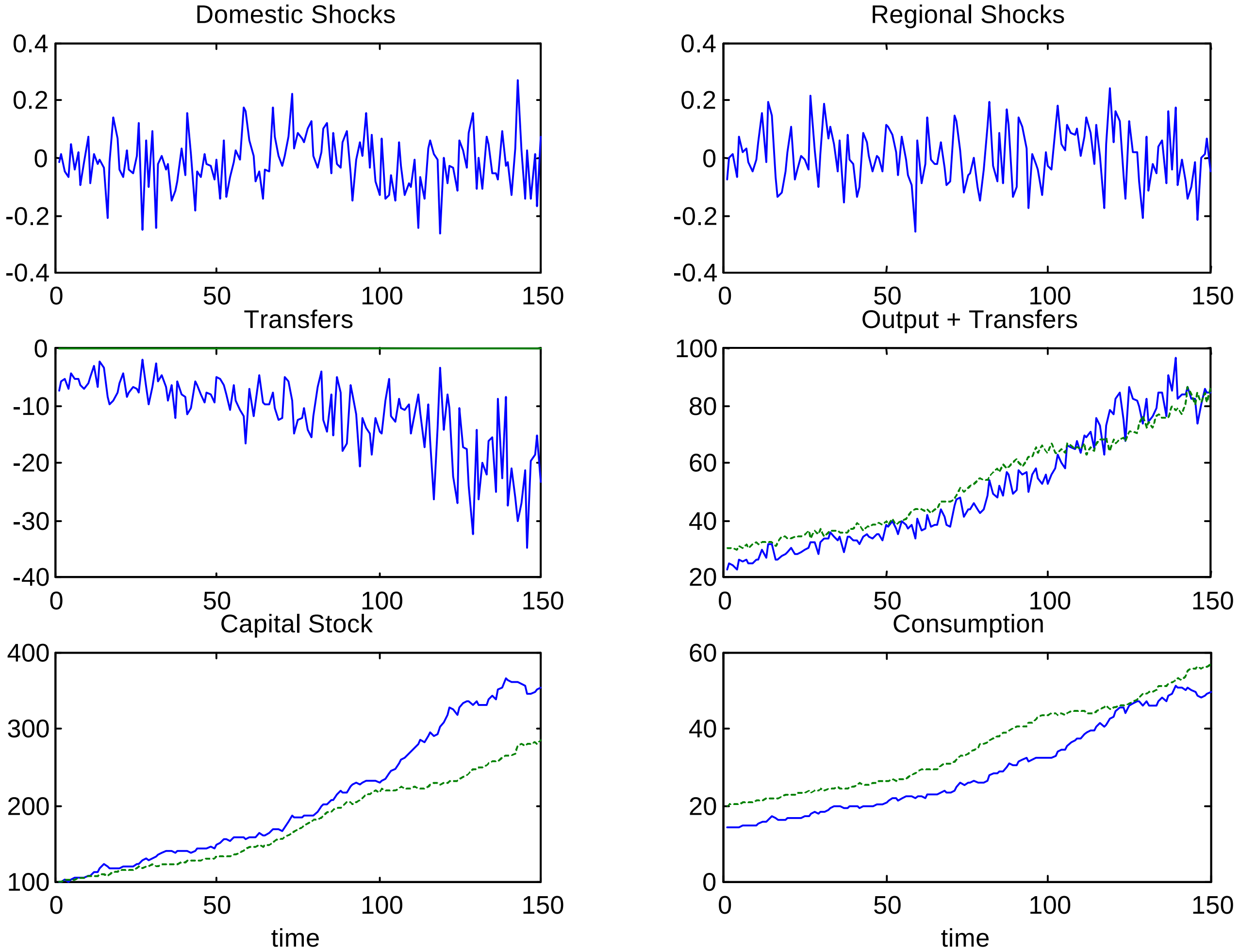


Figure 5.2: (-) Operating Fiscal System (--) No Fiscal System

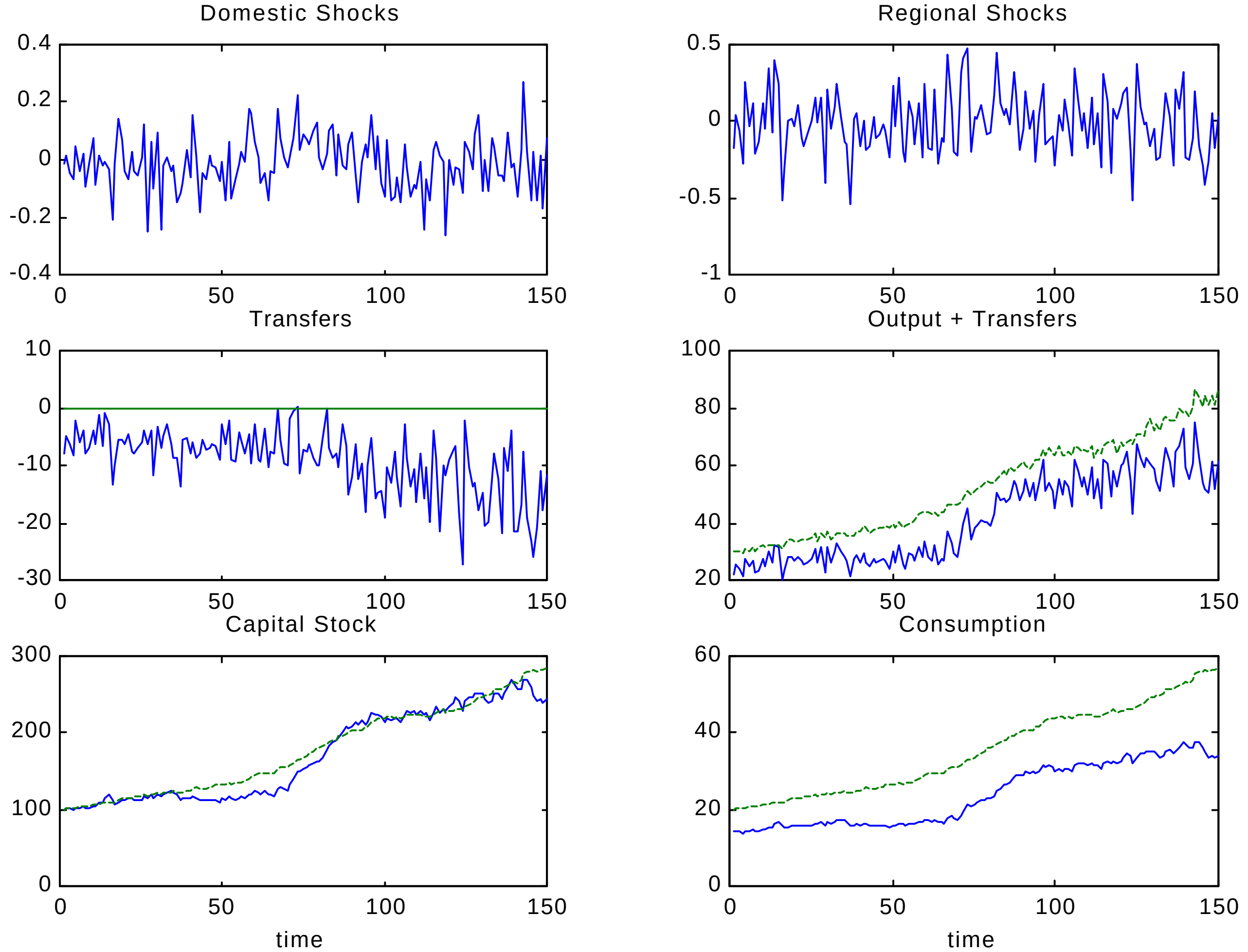
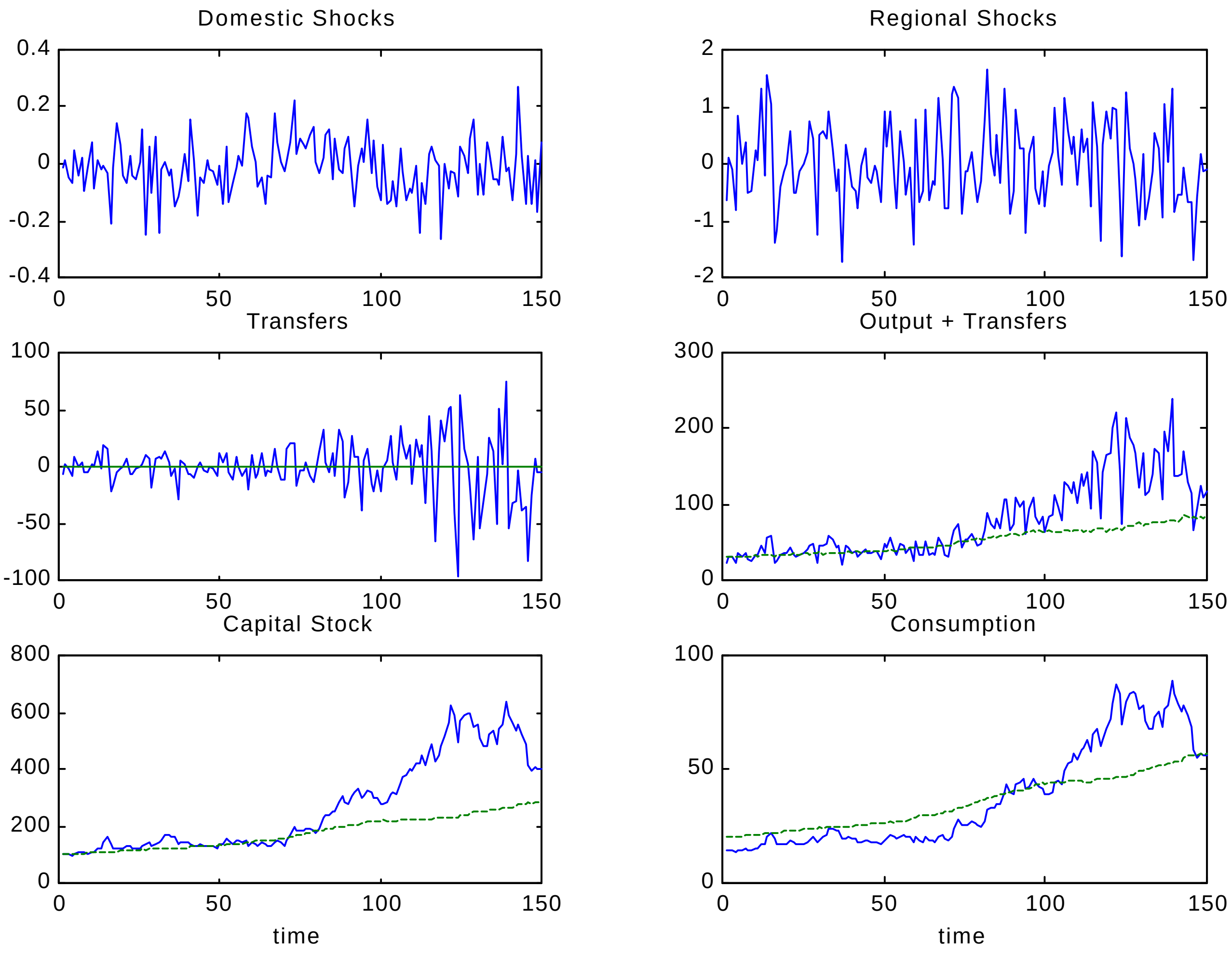


Figure 5.3: (-) Operating Fiscal System (--) No Fiscal System



6 A Fiscal Federation that Responds to Shocks on the Terms of Trade

6.1 The Fiscal System with Shocks on the Terms of Trade

This section presents an economy where the only source of uncertainty comes from shocks in the terms of trade. In addition, it is assumed that the federal fiscal system generates transfers, which partially offset the effects of these shocks on the terms of trade. This is substantially different from what is assumed in the previous sections, where domestic and federal uncertainty are attached to technology shocks. In fact, assuming that uncertainty is related with shocks on the terms of trade stresses the role of international linkages and allows different interpretations. In this sense, changes in trade policy or foreign consumer preferences can be discussed. Although, substantially different from those previously presented, the model in this section retains the same continuous time stochastic framework. First, it is assumed that there is a domestic and a foreign good, whose terms of trade follow a Brownian motion defined as $\varphi dt + \theta dz$. The variance of the shock dz is given by σ_z^2. Next, the problem of the representative consumer is assumed to be:

$$Max\ E_0 \int_0^\infty \frac{1}{\gamma}\left[C_1^\phi C_2^{1-\phi}\right]^\gamma e^{-\beta t} dt \tag{82}$$

where C_1 is the domestic good, C_2 is the foreign good and $1 \geq \phi \geq 0$ is the parameter that measures their relative weight in the utility function. Finally, the intertemporal resource constraint is given by:

$$\frac{dK}{K} = \alpha dt - \frac{C_1}{K}dt - \varphi\frac{C_2}{K}dt - sn_T dt + dk \tag{83}$$

where:

$$dk = -\theta\frac{C_2}{K}dz + \omega n_T dz \tag{84}$$

As in the previous sections, s stands for the price paid for each unit of federal fiscal system participation. The parameter n_T represents total participation and ω stands for the rate of return of the fiscal federation.

6.2 Optimum Conditions

The details of the optimisation parallel those of the previous sections. Consequently, the first order conditions of $\frac{C_1}{K}$, $\frac{C_2}{K}$ and n_T are:

$$\left[\left(\frac{C_1}{K}\right)^\phi \left(\frac{C_2}{K}\right)^{1-\phi}\right]^{\gamma-1} \phi \left(\frac{C_1}{K}\right)^{\phi-1} \left(\frac{C_2}{K}\right)^{1-\phi} - \gamma\delta e^{-\beta t} = 0 \tag{85}$$

$$\left[\left(\frac{C_1}{K}\right)^{\phi}\left(\frac{C_2}{K}\right)^{1-\phi}\right]^{\gamma-1}(1-\phi)\left(\frac{C_1}{K}\right)^{\phi}\left(\frac{C_2}{K}\right)^{-\phi}-\gamma\varphi\delta e^{-\beta t}- \tag{86}$$

$$\gamma(\gamma-1)\delta e^{-\beta t}\left(\theta\omega n_T-\theta^2\frac{C_2}{K}\right)\sigma_z^2=0$$

$$-s+(\gamma-1)\left(\omega^2 n_T-\theta\omega\frac{C_2}{K}\right)\sigma_z^2=0 \tag{87}$$

and the Bellman equation evaluated in the optimum is:

$$\frac{K^{\gamma}}{\gamma}\left[\left(\frac{C_1}{K}\right)^{\phi}\left(\frac{C_2}{K}\right)^{1-\phi}\right]^{\gamma}-\beta K^{\gamma}\delta e^{-\beta t}+K^{\gamma}\delta e^{-\beta t}\gamma\left(\alpha-\frac{C_1}{K}-\varphi\frac{C_2}{K}-sn_T\right)+$$

$$\frac{1}{2}\gamma(\gamma-1)K^{\gamma}\delta e^{-\beta t}\sigma_K^2=0 \tag{88}$$

The solution of the problem is then defined as:

$$n_T=\frac{-s}{(1-\gamma)\omega^2\sigma_z^2}+\frac{\theta}{\omega}\frac{C_2}{K} \tag{89}$$

$$\frac{C_2}{K}=\frac{\beta-\gamma s-\frac{1}{2}\frac{\gamma}{1-\gamma}\frac{s^2}{\omega^2\sigma_z^2}}{\frac{1-\gamma}{1-\phi}\left(\varphi+\frac{s\theta}{\omega}\right)} \tag{90}$$

$$\frac{C_1}{K}=\frac{\phi}{1-\phi}\frac{C_2}{K}\left[\varphi+(1-\gamma)\left(\theta^2\frac{C_2}{K}-\theta\omega n_T\right)\sigma_z^2\right] \tag{91}$$

In the next subsection we analyse the equilibrium that emerges from this set of equations and present the main results.

6.3 Equilibrium Analysis

The optimal degree of fiscal system participation is defined by equations 89 and 90. However, it is possible to obtain some results simply by looking at equation 89. In fact, if it is costless to take part in the federal fiscal system ($s=0$), it results that $n_T=(\theta C_2)/(\omega K)$. When we substitute this optimal n_T into equation 84 it is clear that the variance of the capital stock growth rate becomes zero. This means that the country makes use of the fiscal federation to become fully insured against shocks in the terms of trade. On the contrary, if $s>0$ the variance of the capital stock growth rate is positive. It is also clear that the higher the importance of trade (C_2/K), the higher the optimal participation in the fiscal federation. As a matter of fact, additional foreign good consumption means a larger exposure to the volatility in the terms of trade, which leads to a higher optimal n_T as a way of reducing uncertainty.

Next, we turn to the analysis of the equilibrium in the benchmark case of the logarithmic utility function, which is equivalent to setting $\gamma=0$. In this case the solution simplifies to:

$$n_T=\frac{-s}{\omega^2\sigma_z^2}+\frac{\theta\beta(1-\phi)}{\omega\varphi+s\theta} \tag{92}$$

$$\frac{C_2}{K} = \frac{\beta(1-\phi)}{\varphi + \frac{s\theta}{\omega}} \tag{93}$$

$$\frac{C_1}{K} = \beta\phi \tag{94}$$

To examine the effect on n_T of changes in the structural parameters of the model provides some additional interesting insights. We start with the effect of a marginal increase in the weight of C_2 in the utility function.

$$\frac{\partial n_T}{\partial(1-\phi)} = \frac{\beta\theta}{\omega\varphi + s\theta} > 0 \tag{95}$$

This derivative is always positive. In fact, a higher $(1-\phi)$ leads to an increase in the optimal consumption of the foreign good, which leads to an increase in the optimal n_T. The effect of an increase in parameter s is given by the following derivative:

$$\frac{\partial n_T}{\partial s} = -\frac{1}{\omega^2\sigma_z^2} - \frac{\theta^2\beta(1-\phi)}{(\omega\varphi + s\theta)^2} < 0 \tag{96}$$

which is always negative. This effect has already been discussed above. Increasing the volatility of the terms of trade is equivalent to adding uncertainty to the economy, which obviously increases the optimal participation in the fiscal federation. Therefore, the corresponding derivative is always positive:

$$\frac{\partial n_T}{\partial\sigma_z^2} = \frac{s}{\omega^2(\sigma_z^2)^2} > 0 \tag{97}$$

Recall that the shocks in the terms of trade are the only source of uncertainty in the model and taking part in the fiscal federation is the only way to reduce it. Next, we examine the effect of an increase in the average terms of trade.

$$\frac{\partial n_T}{\partial\varphi} = -\frac{\theta\beta\omega(1-\phi)}{(\omega\varphi + s\theta)^2} < 0 \tag{98}$$

It is clear that the higher the average terms of trade φ the lower the optimal n_T. First, an increase in the average terms of trade makes the foreign good more expensive, which leads to a reduction in imports. This situation reduces the exposure to the volatility in the terms of trade and consequently the need to participate in the fiscal federation. Finally, we look at the effect of changes in the parameters θ and φ. In both cases the sign of the derivative is uncertain.

$$\frac{\partial n_T}{\partial\theta} = \frac{\beta(1-\phi)}{\omega\varphi + s\theta} - \frac{\beta(1-\phi)\theta s}{(\omega\varphi + s\theta)^2} \tag{99}$$

$$\frac{\partial n_T}{\partial\omega} = \frac{2s}{\omega^3\sigma_z^2} - \frac{\beta(1-\phi)\theta\varphi}{(\omega\varphi + s\theta)^2} \tag{100}$$

An increase in parameter θ raises the effect of the shocks in the terms of trade. On the one hand, it is optimum to increase the participation in the fiscal federation as a way to reduce uncertainty. On the other hand, the higher θ leads to a reduction in the consumption of the foreign good, which has become less attractive due to the increased price volatility. This reduction in imports lowers total uncertainty and consequently the optimal n_T. Therefore, the net effect is uncertain. Finally, the net effect of an increase in parameter ω is also explained by different opposing forces. First, an increase in ω raises the attractiveness of the fiscal federation because it provides more stabilisation with the same n_T. However, this same effect can lead to a net reduction in total participation. Second, an increase in ω also increases the optimal C_2/K, which increases n_T.

6.4 Growth and Comparative Dynamics

This section continues by presenting the expressions for the variance and the average capital stock growth rate in an economy which is taking part in this type of fiscal federation. These expressions will allow us to make two final simulations and illustrate the stationary equilibrium path of the variables. Defining the capital stock growth rate as:

$$\frac{dK}{K} = \psi dt + dk$$

and substituting the solution generated by equations 92, 93 and 94 it results that:

$$\psi = \alpha - \beta + \sigma_k^2 \tag{101}$$

$$\sigma_k^2 = \frac{s^2}{\omega^2 \sigma_z^2} \tag{102}$$

Note that there is a trade-off between lower uncertainty and growth. In fact, lower uncertainty is achieved through fiscal system participation, which is costly and reduces growth. It is also interesting to note that the higher the volatility of the shocks on the terms of trade, the lower the variance of the capital stock growth rate in the optimum. In fact, a higher σ_z^2 leads to an increase in the optimal participation, which moves the country closer to the situation of full insurance. This insurance effect more than compensates the initial effect of the increase in σ_z^2. Next, we present scenarios 11 and 12, which are described in Tables 6.1 and 6.2. Scenario 11 illustrates a situation where the cost of participating in the system is zero. As a result, the optimal n_T is such that the effects of the shocks in the terms of trade are totally offset by the federal transfers. Therefore, the variance of the capital stock growth rate is zero. In addition, there is an increase in the consumption of the foreign good. This happens because the additional exposure to shocks on the terms of trade is now insured. Thus, the consumption of the foreign good does not have to be restrained as a way to reduce the effect of those shocks. Note that this result applies in any scenario. In fact, equation 102 does not depend on the consumption of the foreign good. Finally, note that the time series of output

Parameters	α	β	ϕ	s	φ	θ	ω	γ	σ_z^2
Values	0.3	0.2	0.5	0	0.2	3	8	0	0.001
Parameters	n_T	$\frac{C_1}{K}$	$\frac{C_2}{K}$	ψ	σ_k^2	$\frac{C_1}{K}$ ns	$\frac{C_2}{K}$ ns	ψ ns	σ_k^2 ns
Values	0.1875	0.1	0.5	0.1	0	0.1	0.489	0.1022	0.0022

Table 11: Scenario 11

Parameters	α	β	ϕ	s	φ	θ	ω	γ	σ_z^2
Values	0.3	0.2	0.5	0.02	1	10	8	0	0.005
Parameters	n_T	$\frac{C_1}{K}$	$\frac{C_2}{K}$	ψ	σ_k^2	$\frac{C_1}{K}$ ns	$\frac{C_2}{K}$ ns	ψ ns	σ_k^2 ns
Values	0.0595	0.1	0.098	0.1013	0.001	0.1	0.095	0.1046	0.005

Table 12: Scenario 12

plus transfers is more volatile when the country is taking part in the system than otherwise. In fact, there are no domestic technology shocks to offset, so federal transfers add only volatility to this series. Lastly, scenario 12 illustrates a situation where the country reduces only a part of the uncertainty. The variance of the shocks in the terms of trade is higher and their effect is stronger than in the previous scenario. However, the cost of each unit of fiscal system participation is also higher. In addition, the average terms of trade equals one, which means that the foreign good consumption-capital ratio is much smaller than in scenario 11. In the next section we conclude the paper with some final remarks.

Figure 6.1: (-) Operating Fiscal System (--) No Fiscal System

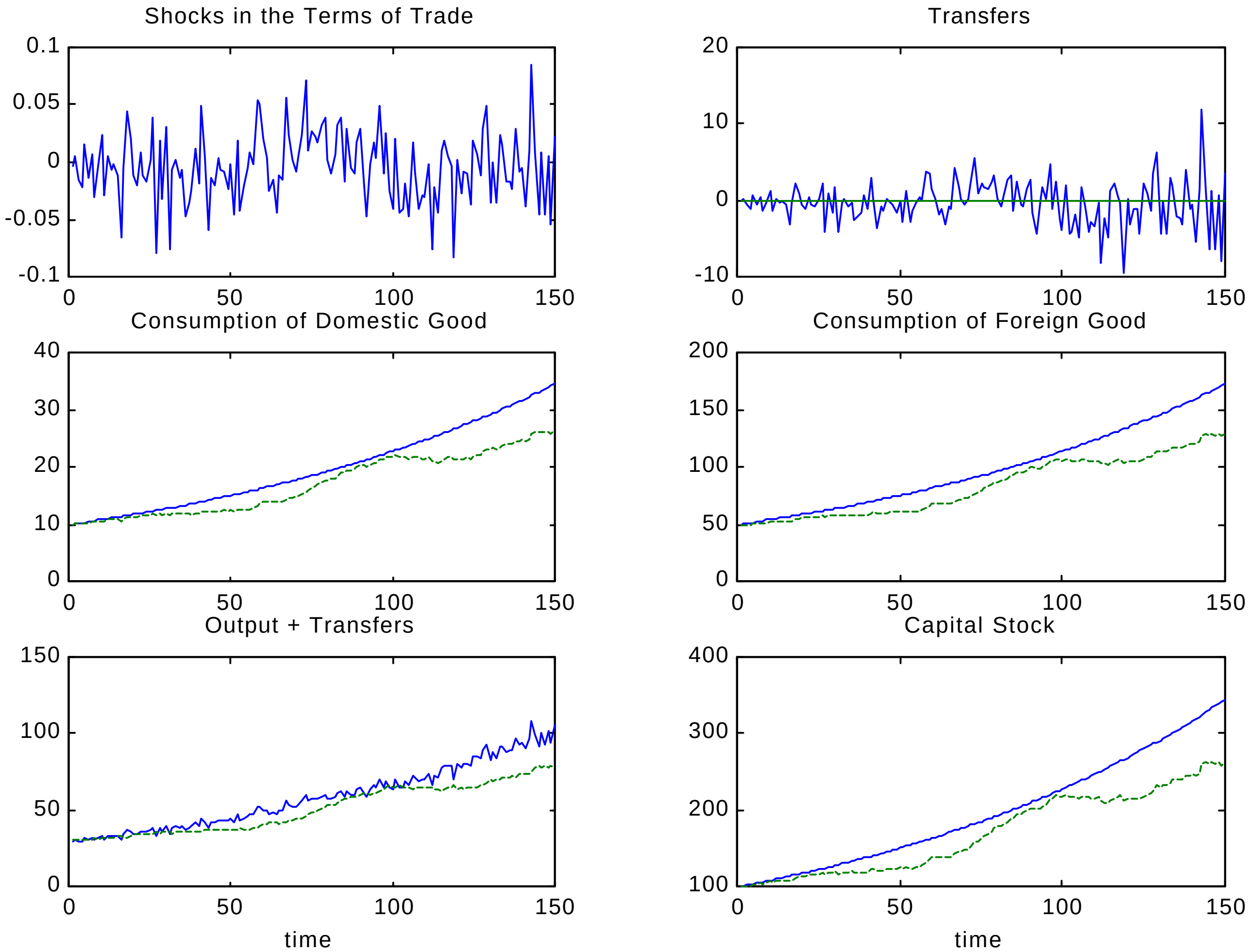
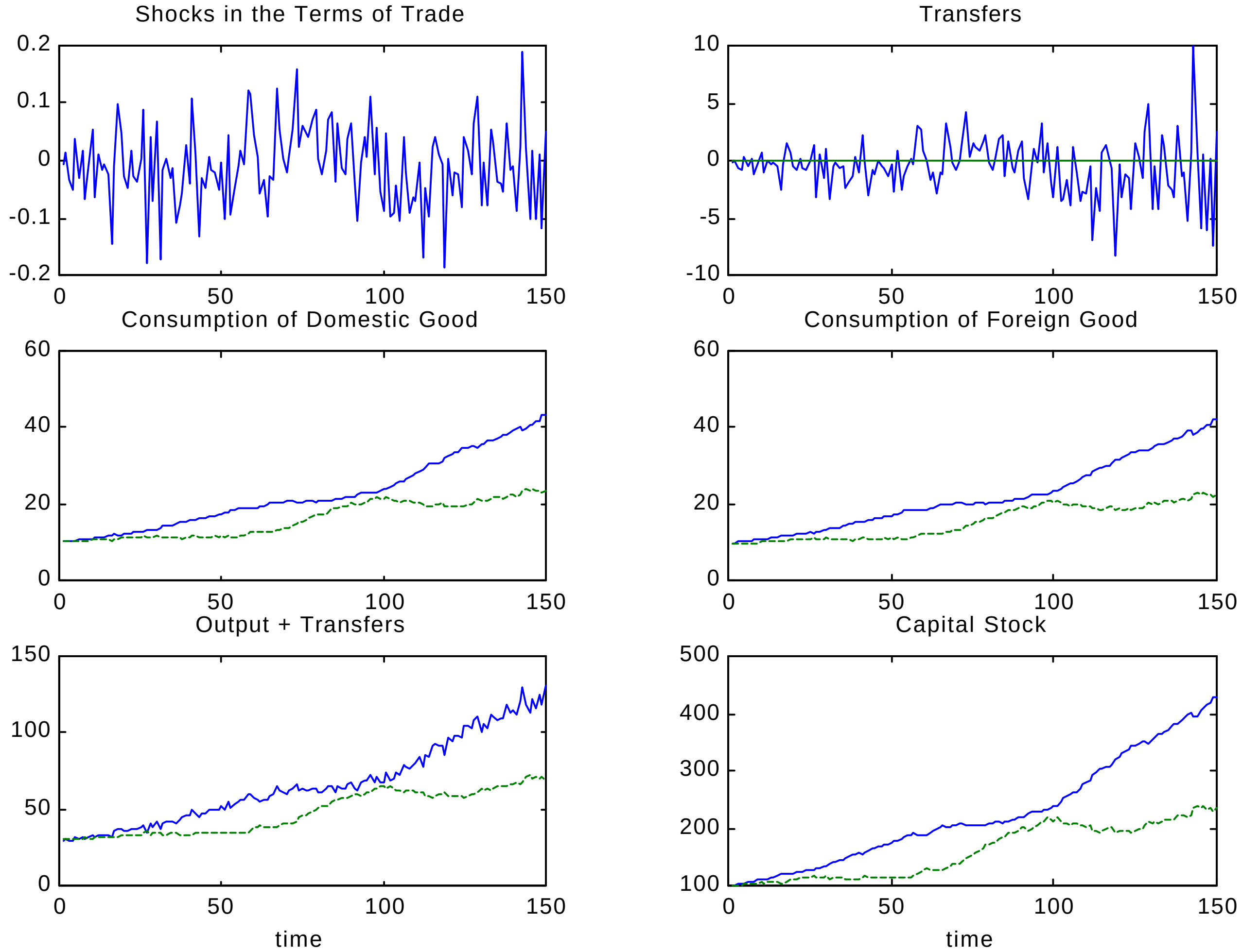


Figure 6.2: (-) Operating Fiscal System (--) No Fiscal System



7 Concluding Remarks

This paper has presented several types of federal fiscal systems and produced expressions for the optimal degree of participation that a country would choose to have. The paper presents the problem from the perspective of the individual country, which takes the characteristics of the federal fiscal system as given, and then takes the optimal participation decisions. These decisions obviously depend on the properties of the federal fiscal system, as well as on the characteristics of the country. It has been shown that there is a wide range of situations that may lead to federal fiscal system participation. For example, by optimally choosing to participate in the federation, some countries may be exploring risk sharing opportunities. In this cases countries may be willing to trade some output growth for a lower variance. In other cases, if the fiscal system contains redistribution mechanisms, countries may be willing to take a larger variance in exchange for a faster output growth. In addition, if fiscal system participation is included as a parameter of the utility function, it is possible to have situations where the participating country takes a higher variance and a lower growth simultaneously. In these case participation is due only to altruism. Finally, we have discussed the case of a fiscal federation which enables a partial offsetting of the shocks in the terms of trade. It was shown that there is again a trade-off between uncertainty and growth. The optimal fiscal system participation depends on the value of imports and typically increases it. This happens because the additional exposure to the shocks on the terms of trade is now insured. In addition, it was shown that the higher the volatility of the shocks on the terms of trade, the lower the variance of the capital stock growth rate in the optimum.

Further research could be done along the following lines. Firstly, other types of fiscal systems could be modelled. Secondly, it would be important to address the question of the financial sustainability of the federal fiscal system. Throughout the paper we have presented fiscal federations which have the potential to be intertemporally balanced or even in surplus. However, there is no guarantee that they run balanced budgets in each moment. It would be very important to model the budgeting process in the federation. Finally it would be interesting to estimate the stochastic processes that rule the path of shocks in European countries. This would allow a good calibration of the models and a clear assessment of the opportunities for the creation of a fiscal federation in the EU.

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