ISSN 0871-2573

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Working Paper nº 274
March, 1996

A FALSE START OF NEOCLASSICAL ECONOMICS IN PORTUGAL - THE WORK OF ANTÓNIO HORTA OSÓRIO *

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* paper presented at the Second European Conference on the History of Economic Thought, Lisboa, February 1996

I thank my colleague Joaquim Feio and Osório's family for information on the subject of this paper. I thank also professors António Manuel Pinto Barbosa, Carlos Bastien, Jorge Braga de Macedo, Luís Cunha, Manuel Farto, Manuel Jacinto Nunes and Nuno Valério for discussions about the same issue. Any errors and omissions are, of course, my own fault.

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Abstract:

This paper analyses the first attempt to introduce in Portugal economic studies based on a neoclassical approach. António Horta Osório wrote in 1910 a textbook, A Mathematica na economia pura [Mathematics in pure economics], in the context of an attempt to get a chair of Political Economy at the Lisbon Politechnical School. He failed to get the chair, and though the book was translated into French under the title Théorie mathématique de l'échange, and praised as a good elementary presentation of the basic framework of the general equilibrium theory of Léon Walras and Vilfredo Pareto, it had almost no effect on the Portuguese intellectual scene. Neoclassical economics would only become a standard paradigm in Portuguese universities in the 1940s.

Plan:

1. Introduction.

This paper analyses the first attempt to introduce in Portugal economic studies based on a neoclassical approach.

Such attempt was made by António Horta Osório, who wrote in 1910 a textbook, A Mathematica na economia pura [Mathematics in pure economics]. However, he failed to get the chair of Political Economy at the Lisbon Politechnical School he aimed at, and the book had almost no effect on the Portuguese intellectual scene. Thus, Osório's episode was a false start, and neoclassical economics would only become a standard paradigm in Portuguese universities in the 1940s. This happened first in the Technical University of Lisbon, thanks to the enthusiasm of a few young economists, namely António Manuel Pinto Barbosa, Francisco Pereira de Moura, Manuel Jacinto Nunes and Luís Teixeira Pinto. Even then there was some resistance to the new programmes from older professors. On this issue see NUNES, 1988 and BASTIEN, 1989.

Anyway, Osório's book was translated into French under the title Théorie mathématique de l'échange, and would be praised as a good elementary presentation of the basic framework of the general equilibrium theory of Léon Walras and Vilfredo Pareto. The most important reference is, of course, SCHUMPETER, 1954: 957, but other authors commended the book. (See section C of bibliography).

Two papers recently presented in Portuguese, NUNES, 1988 and FARTO, 1992, thoroughly explained Osório's contribution. This paper will try to sum up their contribution in English and to add a few remarks about Osório's position in the context of the debates going on among neoclassical economists in the early 20th century.

2. The man: António Horta Osório.

António de Sousa Horta Sarmento Osório was born in Lousã in a wealthy family in 1882. He was graduated in law at the University of Coimbra in 1903. In spite of good academic performance, he did not follow an

academic career at the University of Coimbra and came to Lisbon to start a career as a lawyer.

In 1910-1911, he tried to get a chair of political economy at the Lisbon Politechnical School, but failed. So, he went on as a lawyer until the 1930s. Then he retired, and dedicated his time to art studies. He died in Lisbon in 1960.

António Horta Osório published three books and a few brochures. The brochures are all about his cases as a lawyer. Of the three books, one is a textbook of pure economics, A Mathematica na economia pura [Mathematics in pure economics], which is analysed in section 3 below. Another is a report about his most famous case as a lawyer: he represented the Bank of Portugal in Portuguese courts against the men who obtained in 1925 duplicates of the notes of the Bank of Portugal from the house Waterlow & Sons, London, and prepared the case brought by the Bank of Portugal against Waterlow & Sons in English courts. The third one is a study about art written after he retired as a lawyer. (See section A of bibliography).

3. The book: A Mathematica na economia pura [Mathematics in pure economics].

- The origins of the book.

As already explained, António Horta Osório wrote his book A Mathematica na economia pura [Mathematics in pure economics] in the context of his attempt to get a chair of Political Economy in the Lisbon Politechnical School. As a matter of fact, he had become interested in economics and in mathematics during his studies at the University of Coimbra, where an eclectic perspective based on a diversified bibliography was used, particularly since 1900 with Professor Marnoco e Sousa(1) . Having red Pareto, he went once to Céligny to meet him, becoming an enthusiastic disciple of Walras and Pareto, whom he would consider his two masters in economic themes.

The announcement that applications for the chair of Political Economy in

(1) See Pedrosa, 1992 *in* Cardoso; Almodovar, 1992, p. 434.

the Lisbon Politechnical School were accepted, was made in 1910. According to the rules prevailing, each candidate to the chair should present a book related to the scientific field of the chair as the main element to be considered by the jury. Thus, Osório wrote A Mathematica na economia pura in 1910, what is confirmed by the fact that he refers to the Manuel d'économie politique by Vilfredo Pareto as having been published the previous year (1909).

Three candidates, all lawyers from the University of Coimbra, presented their works: Afonso Costa, a well known Republican politician, member of the first Republican government in 1910-1911; António Horta Osório, a monarchic; and António Lino Neto, a christian-democrat. Afonso Costa presented a study of applied political economy on emigration, in which he tried to examine Portuguese emigration in the context of European emigration. António Horta Osório presented A mathematica na economia pura. António Lino Neto presented a study about Portuguese municipalities, in which he supported the revival of mediaeval administrative principles adapted to modern times.

NUNES, 1988: 7-8 discusses the decision of the jury, formed by old monarchic conservative professors. They seem to have decided on a purely scientific basis, in spite of the fact that they had perhaps a strong political dislike for Costa and Osório, who were the most able candidates. The problem for Osório was that his approach to economic themes was rather odd for those old professors, and they chose a much less brilliant (from today's perspective), but much more familiar text, awarding the chair to Afonso Costa. Notice that the Lisbon Politechnical School was integrated in the University of Lisbon in 1911.

Curiously, António Lino Neto was the only of the three men to become a career professor, as he became professor of Political Economy at the Technical University of Lisboa. As a matter of fact, neither Afonso Costa, who went on with his career as a politician and became later several times minister and prime-minister, nor António Horta Osório, who went on with his career as a lawyer, would work as professors.

- The French translation and Pareto's introduction.

A Mathematica na economia pura was translated into French by José

d'Almada, who was a civil servant at the Portuguese Ministry of Foreign Affairs, under the title Théorie mathématique de l'échange, and was published in 1913 with an introduction by Vilfredo Pareto himself. (All the quotations in this paper are based in this edition).

This introduction starts by asking 'Quelle place occupe l'économie pure parmi les sciences sociales ? A quoi sert-elle ?" (p. V), and proceeds to a critique of the classical definitions (v. g. Smith), and of the classical illusions about the effects that the spread of the knowledge of political economy would have on practical affairs and on practical political decisions (v. g. Say), to conclude that pure economics has only a very indirect usefulness: "Toute personne qui veut tirer la solution d'un problème pratique exclusivement des théories de l'économie pure, ou même de celles de l'économie appliquée, est généralement dans le faux; il faut y ajouter des considérations d'un grand nombre d'autres sciences sociales" (p. XVII). Then, he makes some methodological remarks about the need to use abstraction and mathematics as instruments of research in pure economics and concludes: "Tout ce qu'on peut demander, c'est d'avoir des traités clairs, bien ordonnés, sachant allier la simplicité de la forme à la profondeur de la pensée. C'est de ce genre qu'est ce traité; et c'est pour cela que je crois qu'il pourra rendre de grands services aux personnes qui veulent étudier la science" (p. XVIII).

Thus, it is clear that, once the main purpose for which the book had been written failed, the translation and French edition had a new aim: to be an instrument of diffusion of the general equilibrium theory of Léon Walras and Vilfredo Pareto.

- The preface.

Osório's preface to his own book makes some warnings to the reader. Two sentences in p. 1 sum up these warnings: "Les quatre premiers chapitres sont à la porte de tout le monde. Les quatre derniers présupposent quelques connaissances d'algèbre, de géométrie analitique et de calcul infinitésimal".

Moreover, he says that economics did not made a large progress in the past and was still a young science at the moment. As a matter of fact, most of the available contributions at the time belonged, according to him, to one of

two categories:

- a set of practical discussions without any theoretical foundation;
- a set of doctrinal systems covered by scientific dresses, preaching political and ideological issues, as if scientists were religion priests aiming to influence public beliefs and to move social opinion. Osório included in this last category socialist writings, for example.

Of course, he wanted to make something very different. Economics should be a true science if it applied to empirical phenomena psychological and sociological knowledge in order to explain them and to formulate a generalised synthesis which could be expressed under a mathematical language. As he writes in p. 1, "Nous ne nous proposons ici qu'une étude de science pure et abstraite;".

- History of mathematical economics.

The first chapter (Histoire) is an attempt to sum up the evolution of mathematical economics. Osório mentions briefly the 18th century Italians Beccaria, Silio and Ortes and the works of the French Canard and of the English Whewell, and examines with some detail the works of Cournot, Dupuit, and Gossen. Then, he identifies Stanley Jevons and Léon Walras as the founders of modern mathematical economics. He acknowledges the role of the works written by Menger, Wieser and Böhm-Bawerk, to whom he adds Pantaleoni as forming the Austrian school (this may seem strange, but SCHUMPETER, 1954: 857 essentially agrees with Osório on the matter), though he deplores their insufficient use of mathematics. He lists Edgeworth, Marshall, Fisher, Lehr and Barone as the most important contemporary mathematical economists, praises the textbook by Auspitz and Lieben, and singles out Pareto as the leading figure of mathematical economics. (See section B of bibliography for Osório's references).

- Critique of Walras' definition of pure political economy.

Chapter II (L'économie pure selon Walras) is a critique of classical

definitions of political economy (v. g. Quesnay, Smith, Say) and of Walras' definition of pure political economy, as Osório does not follow the Walrasian thought uncritically. He clearly disagrees with Walras' distinctions between pure science, applied science and moral science, and between natural and human facts: " On est frappé, qu'un esprit aussi clair et aussi lucide que Walras ait pu concevoir le galimatias métaphysique, que nous venons d'exposer. (...) Walras a mal posé le problème et il l'a résolu encore plus mal" (p. 41). In the Elements d'economie politique Walras must have abandoned his definition and studied not only economic exchange, but also production, capitalisation and circulation. The discussion may be summed up by quoting a few concluding sentences (p. 53). He acknowledges that "Walras a su le premier poser les fondements de l'économie pure et en développer scientifiquement les divisions fondamentales", but says that "Il est surprenant qu'un penseur comme Walras ait pris pour point de départ une base si peu scientifique".

- Pareto's and Osório's definition of pure economics.

Chapter III (L'économie pure selon Pareto) is a presentation of Pareto's definition of pure economics, a definition that Osório accepts.

He starts by presenting the ideas of abstraction, generalisation, and successive approximations. Abstraction is the capacity to separate the arguments in a discussion in order to analyse each of them separately; generalisation is the human capacity to compare, to associate and to connect the arguments. Science uses both of these capacities, he remarks, and economics also, as it is no more an empirical subject or a practical art. Then , he states that pure economics gives a first approximation to economic phenomena by abstracting from real man the *homo œconomicus* . He defines the *homo œconomicus* using the distinction made by Pareto between logical and non-logical actions (of course, the *homo œconomicus* is concerned only with logical actions, although human actions are also religious actions, political actions, erotic actions, and so on). To develop the concept of logical action, the distinction between utility and ophelimity, and the Paretian analytical framework of preferences and obstacles is introduced.

In what concerns the problems studied by pure economics, Osório says that economics is an experimental science which catches an abstract theory for the human tastes from small empirical facts, a kind of rational mechanics for human desires, useful for practical applications, as well as rational mechanics is useful for engineering and for building. It should also be a "chapter on psychology" (p. 79), as all the human actions result from a personal estimation about desire and pleasure: "L'économie pure pourra devenir demain la science des actions humaines, en général, chapitre immense de la psychologie, duquel les sciences sociales ne constitueront qu'une subdivision" (p. 86), but acknowledges that, for the moment, it is only able to study the actions of exchange and production, in a large sense which includes production in a strict sense, transportation and storage.

He also accepts the division presented by Pareto - statics, comparative statics and dynamics - and acknowledges that only the first item was already thoroughly studied. To examine the concept of statics, he introduces the concept of equilibrium, and shows how a simple exchange problem with two persons and two commodities may be dealt with literary and mathematical tools.

- Methodology.

Chapter IV (De la nécessité de la méthode mathématique) is a pledge for the use of mathematics in the study of pure economics, as they were, in his opinion, not only a shorter and much more clear language, but also a much more comfortable way of reasoning.

On this methodological discussion Osório presents an eclectic conception for the economics methodology, remarking the need of learning from history, a wider field of human experiences which helps to keep the hypotheses according to reality. Although mathematical deductive power should be the key for scientific economics, inductive methods should be indispensable (See p. 118). Other methods - statistical, historical, etc. - are not wrong or useless. On the contrary, because the role of human feelings into the human action gives advantage to interdisciplinarity. However, deduction based on mathematics is also indispensable, as he proceeds to prove, though he notes that mathematics scare out many social scientists and even many

economists, as they lack the necessary knowledge to use it. According to him, this was, for example, the reason why Cournot's work had been so unsuccessful during his life.

Osório distinguishes causality relations and inter-dependence relations and says that ordinary logic is able to tackle with the first type of relations, but only mathematics is able to tackle with the second type of relations. On the way to prove this, he gets involved in somewhat complicated discussions about philosophy and psychology, which were examined in FARTO, 1989.

The proof also includes a rough literary presentation of the problem of general equilibrium, of its solution and of the 'tâtonnement' process. This presentation starts by the Walrasian analysis of social wealth as formed by three types of capitals (lands, personal abilities and capitals in a strict sense) and three types of revenues (consumption goods, raw materials, and services, including consumable services and productive services). Then, it proceeds to the distinction between consumers and producers, and the exam of the goals of their action, maximisation of ophelimity and maximisation of profit, respectively.

The next step is to identify the demand and supply functions of each good and service (depending on all prices), to introduce the roles of the accounting medium (numéraire) and circulating medium (monnaie) and to state the equilibrium condition of equality between demand and supply.

Finally, the 'tâtonnement' process is described and two warnings are made: First, that pure economics is able to determine what kind of influence different relevant variables have on one another, but not to compute the exact values of prices, quantities demanded and quantities supplied, being the formulation of the problem with equations the way to make it so precise as if it was a subject on astronomy or on mechanics; Second, that in the real world equilibrium is a limit state for which all markets tend, but not something that is permanently attained, because the relevant conditions are always changing.

That means that economic equilibrium is not a solution for practical purposes, says Osório. As it is a limit state, it is, however, the way to understand economic phenomena.

- The presentation of the theory of Walras.

Chapters V (Théorie de Walras) and VI (Théorie de Walras - continuation) present the theory of Léon Walras.

The first step is to present the utility curve (courbe d'utilité ou de besoin), what leads to the first Gossen law ("l'intensité du dernier besoin satisfait est une fonction décroissante de la quantité de marchandise consommée" - p. 166).

The second step is to present the maximum satisfaction theorem (théorème de la satisfaction maxima), or second Gossen law ("le maximum d'utilité effective a lieu [...] lorsque le rapport des intensités des derniers besoins satisfaits ou le rapport des raretés est égal au prix" - p. 177).

Chapter V concludes with a discussion of the concepts of rarity, price and value. In what concerns rarity, Osório clearly follows Walras: rarity, a property of scarce and useful goods, is defined as the intensity of the last satisfied need. Moreover, only rare goods have an exchange value in the market, only rare goods may be multiplied by using production methods, only rare goods may be appropriated.

Exchange value is dropped as an useless concept, because of the so large number of interpretations which had been given to this word in past, as Pareto had remarked. To discuss the prices of the goods and the exchange rate between them was much more useful than discussing the philosophical concept of value: "Walras emploie plusieurs fois les expressions *valeur*, et *valeur d'echange* , et d'autres fois le mot *prix* . On ne comprend pourtant pas l'utilité du premier de ces mots [...] le prix d'une marchandise est [...] égal au rapport direct des valeurs d'echangeet aussi comme il a été dit au rapport inverse des quantités échangées. L'entité *valeur* ou *valeur d'usage* , même ainsi définie ne presente pourtant aucun avantage. Elle est d'ailleurs tout à fait dispensable dans les théories de Walras, ce qui devrait suffire à la mettre de coté" (p.194-195). Thus, price stands out as the main concept of exchange theory, according to the standard neoclassical tradition.

The final equilibrium will result from successive adjustments occurring from the mutual interdependence of the economic phenomena.

Chapter VI opens with the distinction between partial (that is to say individual) and total (that is to say aggregate) demand curves (*courbes de*

demande partielle et totale), and with the presentation of the supply curve (courbe de l'offre). In the case of exchange, demand is a consequence of the attempt to attain the situation corresponding to the second Gossen law. Supply is a consequence of demand, because supply exists in the market because each person wants to demand, and demand exists in the market because there is supply.

It is now possible for Osório to present the solution to the exchange problem starting with the simple case of two persons and two commodities and ending with the general case of n individuals and m commodities. When prices of consumption goods are announced, goods are demanded and supplied, the equilibrium price is found and the exchange takes pace. The supplied quantity of a good multiplied by its price expressed on other good equals the demanded quantity of the other good. The same mechanism works in production for the markets of services and raw materials with landowners, workers and capitalists. In any exchange, if the price of one good changes, several adjustments will occur not only in the market of this good but also in the markets of all the other goods. It is like a stone falling into a lake's surface and producing consecutive waves on that surface, says Osório. This presentation repeats what was done in a rough way without mathematics in chapter IV using graphics and equations.

It includes the fundamental proposition of equality between demand and supply of each commodity, and the well known discussion of the number of equations and unknowns in a shorter way than Walras. For each good, demand must be equal to supply. Walras had preferred to say that excess demand curves must be equal to zero and spent much more time and pages to build his framework. Following Walras' system and his algebraic symbols (Compare OSÓRIO, 1913: p.197-253 and WALRAS, 1952: p.54 -163) Osório explains that there are so many variables as equations.

Arriving to this point Osório criticises Walras, as he never explained why does the economic system need the first mxn equations; will it be possible to assume that to attain the equilibrium it is enough that the demanded quantities and the supplied quantities of each good should be equal? Osório telis to his readers how the consumer makes his own accounting, considering his tastes and considering the prices, so that the m x n first equations can be respected, and leaving only to the market the role of solving the last m-1 equations: In the

market the price of a good will rise if its demand will exceed its supply and conversely. So, Osório believes that the solution for the equations equalising demand with supply supposes that the other equations were already respected.

In this way, the system of equations describes exactly the *tatônement* process, and the ups and downs of the market are, of course, the market's real attempts to find empirically the system' solution: "Toutefois il faut voir comment s'opèrent sur le marché les transformations successives des prix, qui peu à peu et par tâtonements s'approchent des prix d'équilibre, qui satisfont le système". (p. 243).

Osório omits the discussion of the economic equilibrium with non continuous demand functions, and forgets the Walrasian examples of goods consumable by units, like for example dressing and furniture. He always prefers to use continuous demand curves, what means that, although catching Walras ' contribution, he is selective to build the argument of his book.

It does not matter if the market is a monopolistic market, a competitive market or a mixed market. Producers only compare quantities of money. In a competitive market, producers' equilibrium is always in the limit, as new producers are attracted to successful sectors and higher demand of productive factors makes diminishing profits. If too much producers are attracted they will have losses in their business and some will leave the sector. The economic equilibrium discovered by Walras will prevail, says Osório, as a limit state.

Sometimes the adjustments resulting from one change in the market are occurring when a new change is produced in the market unchaining new reactions and new adjustments. On these issues Osório saves much time and paper, as he summarises much more briefly than Walras.

But he stressed how unfair it is to appreciate Walras' *Mathematical School* as a chimera. On the contrary, it is a realistic contribution describing reality through an experimental method.

Repeating Walras' system, and building an original discussion about it for the final equilibrium, Osório assumes that it is always obtained through the solution of a set of simultaneous equations, and the only way to solve them is using algebra.

Osório regrets that some economists don't have a clear idea about what really means the interdependence of economic phenomena and don't know

these new theories on Political Economy. I must stress that he is participating in the discussion of recent, fresh and polemic issues, advertising in Portugal a new conception for political economy and for economic phenomena.

- The presentation of the theory of Pareto.

Chapters VII (L'équilibre économique de l'échange selon Vilfredo Pareto - Cours d'Economie politique) and VIII (L'équilibre économique de l'échange selon Vilfredo Pareto - Manuel d'Economie politique) present the theory of Vilfredo Pareto.

The discussion begins by considering several critical assessments of Walras' theories. Osório rejects from the beginning as irrelevant any discussion based only on literary arguments, and underlines that even eminent mathematicians, like the French Bertrand, made surprising errors because they did not study carefully Walras' arguments.

Osório presents the Paretian concept of *ophelimities* , a property which does not necessarily mean utility but mainly does mean desirability and appetite. Pleasure and desire are related by utility, but he quotes good examples to oblige his readers to distinguish utility from desirability or *ophelimity* : The gun the suicide buys to kill himself is desired by him and is not useful; the morphine is of course *ophelime* for someone and is not also useful; the remedies to cure a child are useful for her, but she does not desire them.

Each consumer respects his *ophelimities* on assessing the agreeable and the disagreeable aspects for his decisions. (To decide to make a walk depends on the place where to go, on the weather, on the accompaniment, explains Osório). In the market man looks for the goods he most desires and exchanges one good for another whenever the *ophelimity* he receives is higher than that he looses on exchanging.

All goods are exchangeable, including capitals, productive services, raw materials, fertilisers, plough, seeds, domestic services, and so on. So, goods may be used to get direct *ophelimities* or to produce other goods , for example to produce wheat, to be exchanged for *ophelimities* like tobacco, or tea, or furniture, or anything else.

If someone refuses some *ophelimities* to produce wheat and looses more

ophelimities than those he gets on selling it against other goods, he will learn the lesson as consumer and as producer: In the next year he will produce less wheat. And his decisions will depend on the prices. In markets prices depend on the collective demand and supply resulting from individual decisions.

Osório was careful and did not forget to underline the limits for the exchange theory. Many *ophelimities* are not exchangeable as they can not be produced. Health, male strength, female glamour and beauty, love, personal character, or sexual virility, are very important factors for human happiness but can not be considered in the exchange theory, as they are gifts of Nature and non produced goods. On writing on these subjects Osório shows his humanity and his feelings, as well as an accurate capacity to understand life, joy and happiness.

On studying the economic equilibrium, economics is generalising the study on the individual *ophelimities* and on the obstacles to fulfil them. These obstacles were mainly the other men's tastes and *ophelimities* , as Pareto had explained.

While in Walras the intensity of the last satisfied necessity, or *rareté*, depended on the amount of the already consumed unities of that good, Pareto's *ophelimities* for a good depended not only on the amount of the already consumed unities of that good, but also on the amount of the already consumed unities of all other goods. The *ophelimity* was understood as a partial derivative of the utility function.

This important correction to Walras' theories was made, according to Osório, by Edgeworth: "Les *accroissements différentiels de l'utilité* ou les *accroissements du plaisir* , qui adviennent de la consommation d'une nouvelle petite quantité d'une marchandise quelconque, ne dépendent seulement de la plus ou moins grande quantité de cette même marchandise consommée antérieurement; ils dépendent aussi de plusieurs quantités plus ou moins grandes d'autres marchandises, consommées auparavant" (p. 259).

The empirical evidence on the Paretian concept is the following: the pleasure which someone gets from having a painting depends also from being well fed and comfortably dressed, says Osório. Pareto had shortly explained: "En resumé, l'ophélimité d'une consommation dépend de toutes les circonstances dans lesquelles se fait la consommation" (Manuel, p. 259).

This difference is quite significant: Summing up the *raretés* one could

get total utility, but one cannot get total utility integrating the elementary *ophelimities* . This is also empirically evident, Osório explains: Since it is important to consider the other consumption, even the order of consuming the goods is relevant.

If in the Walrasian system utility was maximised when the price was equal to the ratio between the rarities, in the Paretian equilibrium, utility is maximised when the price is equal to the ratio between the *ophelimities* (See Osório, 1913, p. 279).

These equations are similar, in Osório's opinion, to the Lagrangean equilibrium defined for mechanics. And he explains: There are also two forces (prices and quantities in this case) which are applied on two points with virtual velocities *dp* and *dq* ; one of them assures that man desires the goods which are *offelime* to him and which he does not have; the other assures that man desires at the same time to preserve those *ophelimities* he already has. The mechanical Lagrangean equilibrium is similar to the Paretian economic equilibrium.

Demand curves were obtained from maximising *ophelimity* to conclude they are also declining schedule functions, as Cournot and Walrasian demand curves. The main basic assumption is constant prices. Otherwise consumer's budget would be permanently changing. Even the order of consuming goods would make prices to change and would change the consumer's budget. The definition of the consumer's budget is the same as in Walras: Expenditures and receipts must be equal, because if a person wants to have more of one good he must give something in exchange to get it.

Paretian equilibrium has the same number of variables and the same number of equations as the Walrasian one.

Osório characterised perfectly the utility function as a growing function, (positive first order derivatives), because, even if the consumer is satiated with a good, if he has more unities of that good he may keep them for future consumption or exchange them against other goods; negative second derivatives, because of marginal diminishing utility.

Pareto had been very cautious on this methodological issue of marginal diminishing utility: "C'est un fait très générale que plus nous avons d'une chose, moins précieuse nous est chacune des unités de cette chose" (Manuel, page 266); "On a voulu faire dépendre cette propriété de la loi de Fechner

mais cela suppose nécéssairement la consommation et nous avons vu (§3) que cela entraînait beaucoup de difficultés" (Manuel, p. 265).

The reason for the problem was that: "Nous avons supposé qu'un homme peut comparer deux sensations; mais quand elles ne sont pas simultanées, et en verité il ne semble pas possible qu'elles le soient, il ne peut que comparer une sensation avec l'idé qu'il se fait d'une autre sensation" (Manuel, p. 262). "Il vaut mieux avoir directement recours à l'expérience " (Manuel, p. 265).

Osório had preferred to explain that this empirical law of diminishing utility results from human sensorial mechanics. Using the available knowledge on psychology he preferred to quote Spencer to confirm the biological character of pains and pleasure as well as their effects on the blood circulation and on sensibility. (p. 17)

Some details were considered by Osório as very relevant ones. Walras' theories were, according to Osório completed by Pareto. As a matter of fact, Walras studied only the case of perfect competition, in which consumers and producers do not aim at influencing the market equilibrium. However, in the real world, there is also a minority of consumers and producers that try to alter the equilibrium conditions or, at least, consider how much their actions may alter the equilibrium conditions. The study of this more complex situation, a blending of competition and monopoly was made by Pareto, who could show that either in a competitive market or in a monopolistic market the equilibrium is found in the same way, although the monopolist raises the price and consumers must consume less, catching less *ophelimities*, what means less wealth. Osório points that this complement to Walras' theories made by Pareto is a clear case of successive approximations.

Osório remarks how important it had been the evolution of Pareto's thought from the Cours d'economy politique (1896) to the Manuel d'économie politique (1909) because of the introduction of the ordinality hypothesis. On this subject the first important objection made to Walras' measurability hypothesis had been, according to Osório, the one made by Irving Fisher: "Fisher fit noter que la théorie de Walras repose entièrement sur l'*hypothèse* que le plaisir est une quantité *mésurable* et que cette hypothèse est indémontrable" (p. 256). The only definitive error in Walras, according to Osório, was in the following statement "Je suppose donc qu'il existe un étalon

de mesure pour l'intensité des besoins ou de l'utilité" (quoted by Osório, p. 392). Osório adds that "Le professeur Pareto a trouvé que l'argument était fondé et il a modifié en conformité la manière dont il a déduit la théorie de l'équilibre économique, qu'il a tâché de construire, sur des bases nouvelles" (p. 256), but says that "L'objection de Irving Fisher ne nous semble pas fondée. Mais le fait qu'elle a été admise par le savant Pareto nous porte à nous abstenir, pour le moment, du jugement que nous voudrions émettre à ce sujet. Et de plus nous n'avons pas pu obtenir, malgré tous les efforts, un exemplaire de l'œuvre de Fisher" (p. 256).

The same opinion about the less relevance of Fisher's criticism on Pareto may also be found in ZUMALACARREGUI, 1951 who preferred to stress the importance of the indifference curves, by Edgeworth, on Pareto's ordinality hypothesis. About the intellectual connections between Pareto and Edgeworth he even wrote: "And, however, I think that (...) Edgeworth is much more present spiritually in the Manuel... than what one may think" (p. 59). STIGLER, 1965: p.105 would remark, however, Fisher's contribution to the indifference curves.

Osório's conclusion is that Pareto's Manuel d'économie politique demonstrated that the possibility to measure utility (or, better, ophelimity) is irrelevant for the development of the theory of general equilibrium. STIGLER, 1965: p.126 would refer that two mathematicians consolidated this position that measurable utility could be eliminated from economics: W. E. Johnson in 1913 and E. E. Slutsky two years later. W. E. Johnson said in 1913: "This impossibility of measurement does not affect any economic problem" (quoted by STIGLER, 1965: p.126). Osório had sum up his ideas since his Portuguese edition (1911) by saying: "Nous ne tenons pas à faire de la littérature, par des critiques en l'air. Soulignons toutefois qu'il est parfaitement égal que l'argument de Fisher soit vrai ou faux, que Pareto ait raison ou tort de l'admettre; car ce qui importe d'être fixé, c'est que seule la méthode de Walras est visée. Le fond reste essentiellement le même, dans la dernière œuvre de Pareto, publiée après que l'auteur eût accepté l'objection de Fisher" (p. 257).

The rest of chapter VII summarises the theory of exchange as presented in Pareto's Cours... The main points to stress are:

a) The discussion of inferior goods.

b) The presentation of an analogy with mechanics.

c) The exam of monopoly situations.

In what concerns the discussion of inferior goods, Osório studies the Giffen effect, without adumbrating the deeper analysis that would be presented a few years later by Slutsky.

In what concerns the analogy with mechanics, it is interesting to note that Osório comes to agree with the opinion of Fisher without knowing directly his work. As a matter of fact, Fisher, a mathematician and an enthusiast of astronomy, wrote: "utility has an original 'common sense' meaning relating to feelings, when economics attempts to be a positive science, it must seek a definition which connects it with objective commodity" (p.17). (...) Scarcely a writer on economics omits to make some comparison between economics and mechanics. [...] In fact the economist borrows much of his vocabulary from mechanics. Instances are: equilibrium, stability, elasticity, expansion, inflation, contraction, flow, efflux, force, pressure, resistance, reaction, distribution, (price), levels, movement, friction" (FISHER, 1892: 24).

Osório underlined that "Il est intéressant de noter que les équations du maximum d'ophélimité ne sont qu'une application et une adaptation de l'équation de l'équilibre de Lagrange. [...] Nous passons de cette équation [de Lagrange] à l'antérieure [équation du maximum d'ophélimité] en remplaçant les *forces* par les *ophélimités élémentaires* et les *vitesses virtuelles* par les quantités *virtuellement échangeables* " and concluded that such a fact "vient confirmer l'exactitude de la concéption de Pareto qui, nous l'avons vu au chapitre III, considère l'économie pure comme la mécanique rationelle des désirs humains" (p. 282-283).

However, if Osório had red Fisher perhaps he would be less worried about the biological character of pains and pleasures as well as their effects on the blood circulation and on sensibility to explain marginal diminishing utility.

In what concerns the exam of monopoly situations, two points should be stressed. First, Osório underlines the disadvantages of monopolies: "Il va de soi que les avantages du monopoleur sont obtenus aux dépens des autres individus. [...] le consommateur jouira de moins d'ophélimités que dans un régime de libre concurrence" (p. 300); "Les monopoles de production, dont nous ne nous occupons pas [...] car leur étude concerne la *production* et non l'*échange* , impliquent une *destruction de richesse* comme on le démontre

mathématiquement" (p. 300-01). Second, he points out that when all persons involved act as monopolists the situation becomes indeterminate.

Chapter VIII explains how the results already presented in the previous chapters may be attained by means of the choice theory (théorie du choix) developed by Pareto in his Manuel...

To begin with, Osório presents the elegant methodology of the decreasing convex indifference curves, attributing them to Edgeworth , and abandoning the *ophelimities* because of the previous discussion on the irrelevance of measurability, and taking them as a fact, like Pareto, in his Manuel d'économie politique: "Cette expression est due au professeur F. Y. Edgeworth. Il supposait l'existence de l'utilité (ophélimité) et il en déduisait les courbes d'indifference; je considère au contraire comme une donné de fait les courbes d'indifference, et j'en déduis tout ce qui m'est nécéssaire pour la théorie de l'équilibre, sans avoir recours à l'ophélimité" (Manuel, footnote (1) p. 169).

As the preferred sets of goods represented by right hand indifference curves do correspond to higher *ophelimities*, they both prefer to attribute an ordinal index to the indifference curves to avoid any measure for the utility. Osório states three main properties ($dy/dx<0$, $d^2y/dx^2>0$, $d^3y/dx^3<0$), and considers the case of horizontal (or vertical) indifference curves, when one of the commodities is not ophelim. Then, ordinal ophelimity indexes are introduced to replace the hypothesis of cardinal utility. The basic conceptual framework is completed with the roads (*sentiers*) that are used by Pareto to describe the restrictions faced by the consumers and producers.

Osório could so conclude that Pareto's Manuel d'économie politique did not refuse the utility measure, but demonstrated that it is irrelevant for the economic equilibrium, as was said ahead.

Reading his book one must conclude he was participating in a thrilling polemical discussion, as the Manuel d'économie politique had been published in the last year, 1909. Cardinality versus ordinality was in the moment a scientific controversial issue and the Théorie mathématique de l'échange has chosen rightly.

Osório goes on to consider the question of discrete and continuous cases to justify the choice for a continuous approach as not too unrealistic, before solving the individual problem of maximising ophelimity. He stresses that this

usually leads to tangency solutions, but also considers the possibility of corner solutions. Then, he presents the graphical solution of the exchange of two commodities between two persons using the box-diagram, and proceeds to the discussion of increasingly complex cases using differential calculus.

Even so, Osório never refuses to Walras the scientific relevance of his contribution: "L'expression mathématique de l'utilit'e et la théory de l'échange ne relèvent pas de lui, mais c'est lui qui a su en tirer les déductions les plus importantes, c'est lui qui a su montrer, avec une clarté admirable, la multiplicité de ses conséquences; c'est lui qui a pour la première fois conçu l'équilibre économique (...); et c'est donc en toute justice que Walras est considéré come le fondateur de l'économie mathématique, connue aussi sous le nom d'*école de Lausanne*"(p. 21).

- General appraisal.

Four points should be stressed as a general appraisal of Osório's work.

First, it deserves to be praised because of its high scientific level. NUNES, 1988: 35 says that "Horta Osório's work (...) represented in the epoch it was written, the maximum updating in what concerns the theories of the new methodology represented by the mathematical economy". ZUMALACARREGUI, 1951: 81 suggests that Osório even might have known Pareto's Economie mathématique, published in 1911, "his last and definitive work", which elaborates his paper on the same subject published in the Giornale degli Economisti, November 1906.

Second, it deserves also to be praised because of its high teaching quality. Besides Pareto's comment in the introduction, several other references acknowledge that Osório's textbook was a good alternative to the study of elementary mathematical economics, as pointed out above. The book is organised from the easier to the most complex questions, and each subject is approached under three stages, or three languages, if we prefer: The current language, the graphical message and the mathematical formulae. Like Marshall, like Walras and like Pareto, Osório used all of them, although he preferred to mix much more the three styles in the text. (Pareto's Manuel separated mathematics to be used mainly in the appendices; Walras

separated the graphs to be included in the last pages, since the 1900 edition). First, Osório explains the matters talking about current life examples and uses a clear speech and an easy style to catch the main conclusions; sometimes he repeats the explanations using new examples to underline his message and he seems always worryring about convincing his readers. After these long explanations, Osório uses figures to put the questions under a geometric point of view and explains carefully each variable in the map or each theoretical concept, to reinforce his literary previous conclusion. At last, he repeats all the reasoning and all the results using mathematics, because in his opinion "la concision excessive de certaines déductions fondamentales est une des grandes difficultés que nous avons dû surmonter dans l'oeuvre des deux maîtres" (p. 7).

Third, the book keeps to the announced purpose of dealing only with pure economics and avoiding to mix up applied economics, or doctrinal systems. As a matter of fact, reading Osório one cannot conclude much neither about his political feelings nor about his social values and moral beliefs.

Although he appreciates the Pareto's conceptual framework of economics as a social and global science present in the Manuel d'économie politique, Osório did not adopt his sociological style and aimed strictly in his book to present the evolution of the available theories on economic equilibrium: "Ce livre n'est donc, par conséquent, qu'une étude de science pure" (Théorie mathématique de l'échange, page 7).

Fourth, Osório does not avoid a defence of the market mechanisms and their wonderful virtues. On this point he is following once more Walras' and Pareto's enthusiasm (Compare Osório,1913 p. 213, Walras, 1952 p. 65 and Pareto, 1927, p. 234). Pareto is the most detailed of them and stresses that for an human community composed of 100 people and 700 goods there will be 70 000 plus 6 999 equations to be solved; moreover for a community made of 40 million people! The interest of the theoretical analysis will not be the resolution of the general equilibrium equations, but the understanding of the mechanism underlying its existence. The resolution is made by the market itself, as "Le seul moyen accessible aux forces humaines pour les resoudre [tous ces équations] ce serait d'observer la solution pratique que donne le marché" and (...) "ce ne serait plus les mathématiques qui viendraient en aide à l'économie politique, mais l'économie politique qui viendrait en aide aux

mathématiques". Pareto, p. 234.

He stresses that the market mechanisms ensure the best satisfaction of human needs, under the condition of a single price for each good in the market, with the marvellous respect of the equilibrium between demand and supply. That is to say, the individual freedom is respected and the maximisation of utility is attained.

However, he does not repeat the Classical and Walrasian hope that the knowledge of economics would be a way to build an organised economic world in order to attain the universal economic equilibrium in a society full of happiness, harmony and peace for all the nations. This was, shortly, the highest goal in the Walrasian scientific works: the economic equilibrium would drive the nations to happiness and harmony. Walras had written: "When my teaching will be diffused and spread and when the political economy will substitute the actual doctrine of *laissez-faire* , one will see the chaos where agriculture and industry stay now, as they can not equilibrate their goods supply and demand at the selling prices. This disorder must give place to an economic world organised on the *Lagrangean equations* which already regulate the work of our machines and which could also take us, one day, to the peace era" - Quoted by Amzalak, 1934 (p. 45)

But Osório recognises, like Walras and Pareto that the traditional praise to the 'laisser-faire' principle was in the past nothing more than a doctrine. For him, from the general equilibrium theory on, it became a scientific principle and not a philosophical conclusion coming from metaphysical considering.

Of course, those who would not know what is a system of two equations on two variables would not understand how demonstrative is the new school, the *Mathematical school.* Osório underlines that it would be like to ask to someone blind-born to comment on the colours of the world.

Osório and Walras had similar difficulties to spread general equilibrium theories teaching. Walras could teach them in Lausanne, but never in France, as he would like to do, as he wrote in his memories: "(...) I never stopped to hanker after the knowledge and the discussion of my efforts in my country [France]. I tried without any success to send to the Academy of Moral and Political Sciences three papers on the exchange equations, production and

capitalisation. After then I made new attempts in order to make known my ideas in France, but I never could accomplish none of my hopes, as they always met the deaf and fierce hostility from certain hereditary mandarins, who have prevented the science progress" (quoted by Amzalak, 1934 on page 29).

At the same time Osório did not expect neither immediate personal utility nor social praise from publishing his book, as he states: "If one of the few readers we will have in Portugal, will think the issues are clearly exposed, we will feel compensated". (p. 10). Osório was writing for future.

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