

A Work Project, presented as part of the requirements for the Award of a Master's degree in Economics from the Nova School of Business and Economics.

SOURCES OF INEQUALITY IN PORTUGAL'S EDUCATIONAL ACHIEVEMENT

BEATRIZ CASACA CÉSAR

Work project carried out under the supervision of:

Ana Balcão Reis, Pedro Freitas and Ricardo Colaço

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## **Abstract**

Using PISA data, this work project examines the contribution of family characteristics and school inputs to the inequality in Portuguese students' academic achievement and assesses how this contribution has evolved in the last two decades. The results suggest that, consistently throughout the years, the family of the peers in the school is the main observable determinant of educational achievement. School inputs play a smaller role in explaining the variance in test scores. There is, however, a decrease in the part of the variance in test scores that is explained by these two factors. Some heterogeneity effects are observed between regions.

**Keywords:** Education Production Function, PISA, Educational Achievement, Sources of Inequality, Economics of Education

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## 1. Introduction

Inequality, a major concern in today's society, is often determined by the opportunities given at early stages in life. In this sense, education has a crucial role to play, as it influences people's labor market outcomes and socio-economic contexts at adult life. Hence, identifying the determinants of educational achievement is key to shape educational policy that, ultimately, will contribute to the improvement of individuals' socio-economic conditions and, consequently, enhance countries' economic growth (Reis, 2015). The discussion on the determinants of students' academic achievement dates back to the Coleman Report (1966). Looking at the performance of students in the United States, Coleman and his colleagues' results suggested that family background characteristics, especially those of one's peers, had a bigger impact on students' educational achievement than school resources. These results have been supported in the literature of the past 50 years, as research on the economics of education has focused towards finding the most accurate way to model the relationship of student achievement as a function of student-related and socio-economic factors — the Education Production Function (EPF). Innate ability, family background characteristics (socio-economic status, school level of the parents), school resources (class size, expenditure per student, teacher quality), and the institutions of the education system are portrayed in the literature as the main determinants of student performance (Hanushek and Woessmann, Ch. 2, 2010).

To explore the determinants of cognitive achievement, many studies have relied on International Tests of Educational Achievement. The International Association for Educational Achievement (IEA) started developing these tests in the late 1950s, including programs like FIMS<sup>1</sup>, TIMSS<sup>2</sup> and PIRLS<sup>3</sup>. In 2000, the Organization for Economic Co-operation and

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<sup>1</sup> First International Mathematics Study

<sup>2</sup> Third International Mathematics and Science Study

<sup>3</sup> Progress in International Reading Literacy Study

Development (OECD) started international testing by introducing the Programme for International School Assessment (PISA). On a three-year cycle, PISA assesses 15-year-old students' achievement on reading, mathematics, and science literacy.

Portugal started participating in PISA in 2000 and has participated in every wave since, presenting itself as a history of success. After ranking poorly compared to the other 28 OECD countries in the results reported in 2000 for reading, mathematics, and science literacy (OECD, 2001; Marôco, 2021), Portugal arises, in 2018, as one of the few countries presenting an improving result pattern throughout the years in all three domains of assessment, closing the gap in test scores to the OECD average. Despite this positive trend in results, there are still marked differences in test scores within the country, namely associated with socio-economic status: on average, students from an advantaged socio-economic background scored 95 points higher in the reading test than those of disadvantaged backgrounds in 2018. In 2009, this gap had been 6 points lower (OECD, 2019).

In light of these findings, this study aims to answer two questions: 1) How do school factors and family background characteristics contribute to the differences in the academic performance of Portuguese students, and, as a novelty to this literature, 2) how have these factors' contributions evolved in the last 20 years? The study extends the previous work by Carneiro (2008) for the 2000 wave of PISA through the waves of 2009 and 2018 and explores heterogeneity in different groups of students by type of institutions (public schools), educational track (academic track), and region (NUTS II<sup>4</sup>).

The remainder of the study is divided as follows: Section 2 reviews the underlying literature. Section 3 introduces the dataset. Section 4 explains the empirical strategy used. Section 5 presents and analyzes the results. Section 6 concludes.

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<sup>4</sup> Nomenclature of Territorial Units for Statistical purposes – level 2.

## **2. Literature Review**

### **2.1. The Education Production Function – Inputs and Estimation**

Todd and Wolpin (2003) propose a model of the EPF in which student achievement is an output of innate ability, family background, and school inputs. The authors argue that education is a cumulative process, implying that, ideally, we should have access to all past and present information on student, family background, and school inputs. Finding such rich datasets has posed a big challenge, as it is often the case that these lack longitudinal data on family background and historical school inputs. Missing data makes the estimation of the EPF susceptible to endogeneity problems such as omitted variable bias and measurement errors. Such problems usually come from the use of proxies to account for information on family and school inputs that are associated to unobserved family and school characteristics. To overcome these obstacles, many studies rely on value-added specifications (Woessmann, 2008), fixed-effects estimation (Woessmann, 2005) and, most recently, a wave of the literature takes advantage of quasi-experimental data (Jackson, 2020; Jackson et al., 2021).

### **2.2. Family Characteristics**

Among the literature relying on International Tests of Educational Achievement, Woessmann (2016) uses data from PISA 2012 and TIMSS 2011 to assess international differences in student performance by decomposing the variance of test scores into family background characteristics, school resources, and institutions. The results show that family background characteristics explain between 21% and 50% of the total cross-country variance in student achievement, with institutions playing a similar role, as this factor alone contributes to 26% to 53% of cross-country variation. School resources, in its turn, only account for between 4% and 18% of this variation.

In line with these results, the importance of family background has been consistently shown across the literature. One challenge regarding this input, nonetheless, has been to find the best way to measure it. Measures such as the number of books at home as a proxy for the educational and socio-economic background of the students' families, immigration status, parental education, and socio-economic status have been used to account for students' family characteristics. Woessmann (2003) explores TIMSS data from 1994/95 for more than 260,000 students in 39 countries, also concluding that student and family characteristics have substantial effects on educational performance, opposed to the lack of impact from school resources. The results show that native students, students living with both parents and students with at least one parent born in the country where they attend school obtain higher test scores.

### **2.3. School Inputs**

One simple measure of school inputs is the countries' levels of expenditure per student. Between 1970 and 1994, the increase in expenditure per student in OECD countries was not matched by a similar increase in student performance, which, in most cases, remained constant (Gundlach et al, 2001). The same analysis was performed for Asian countries, registering non-substantial improvements in students' results (Gundlach and Woessmann, 2001). Measures associated with school resource endowment, namely the quality of instructional material, teaching force (Hanushek and Luque, 2003), and class size (Lee and Barro, 2001) have also been used to account for the role of school inputs. Using measures of expenditure per student and student/teacher ratio, TIMSS (1995) results for 37 countries show a weak relationship between school resources and school achievement (Hanushek and Luque, 2003).

In contrast, Lee and Barro (2001) find a significant impact of both family background characteristics and school resources in 58 countries. In this case, school inputs, such as smaller class size, enhance educational outcomes. Similarly, Fuchs and Woessmann (2006) use PISA

2000 data from 32 countries, concluding on a positive relationship between test results and resource endowments and teacher characteristics, after controlling for family background and institutional effects.

This lack of consensus has been present in the literature for decades. In a review of the studies published prior to 1995 regarding the US, Hanushek (2003) shows that, across all types of school resources, the percentage of non-statistically significant estimations is 2 to 4 times bigger than the percentage of those with statistical significance.

The limitations associated with the estimation of the EPF have recently led to the development of a new strand of literature on school spending. This literature focuses on quasi-experimental methods that support a causal interpretation of results, and for which most evidence points towards a strong link between spending and students' outcomes (Jackson, 2020). One example relies on the Great Recession of 2008. During this crisis, public-school per-pupil spending in the US fell by 7%, coinciding with the end of the growth trend in test scores and college-going experienced until then. Jackson et al. (2021) use the interaction between the states' historical reliance on State taxes for public funding of school and the timing of the recession as instruments for the reductions in school spending to show that high spending cuts are associated with lower test scores and college-going rates.

#### **2.4. Results for Portugal**

Carneiro (2008) uses the 2000 wave of PISA to understand which factors explain Portuguese students' results. The author follows a two-step specification to estimate the variance decomposition of students' test scores. In a first step, the test scores are regressed on a set of family background variables and school dummies. In the second step, the school effects captured through the school dummies in the first regression are split into school quality effects and family background effects, the latter given by the average family characteristics in the

school, as to account for the family background of the peers. From this specification, the author first decomposes the variance of test scores, concluding that a large part of the inequality in test scores comes from differences between schools. Following this conclusion, the variance decomposition of school effects is performed, with results suggesting that inequality in educational achievement is mainly determined by the average socio-economic background of the students in a school — the family background of the peers explains 60% of the total variance in the school effects for the reading test scores (59% for mathematics and 65% for science) in contrast to the 3% of the total variance explained by school quality effects (2% and 3% for mathematics and science, respectively).

Besides the increasing trend in PISA results, in the last 20 years, Portugal has also seen improvements in schooling rates. Since 2000, the country has consolidated the gross enrollment rates at above 100% in grades 1 to 12. These numbers peaked in 2009, when mandatory education was extended to 12 years (or until age 18). Dropout rates have also seen a big improvement in this period. In 2000, this indicator registered a value of around 45% and, by 2018, it was close to 10% (Eurostat). Despite these positive figures, Portugal still faces a big challenge regarding completion rates for upper-secondary and higher education. In 2018, upper-secondary education (grades 10-12) registered a 79% schooling rate and only 24.8% of adults (age 25-64 years-old) had completed this level of education, 16.7 p.p. below the OECD average. Enrollment in higher education for 19 to 20-year-old individuals in this year was around 41% and its completion rate for adults aged 25 to 64 years-old was 25% (11.8 p.p. below the OECD average) (Marôco, 2021; OECD, 2021).

In light of the developments in the Portuguese educational framework in the last 20 years, this study picks up on the work by Carneiro (2008) and extends it to later waves of PISA. It adds to the literature on the determinants of student achievement in Portugal by bringing a more updated and complete picture of the country's sources of inequality in academic achievement.

### 3. Data

This analysis is conducted using the PISA dataset. Every three years, PISA assesses student achievement in reading, mathematics, and science literacy for 15-year-old students on both academic and vocational tracks. It includes very comprehensive questionnaires on student, family, and school background (answered by students, parents, and school principals, respectively) providing the information that allows the performance of this analysis.

To depict the evolution of the contribution of school and family factors on Portuguese students' performance, we use data from the 2000, 2009 and 2018 waves of PISA. The choice of the years relies upon 1) the interest to have an initial, intermediary, and final set of observations for comparison and 2) the fact that these three waves of PISA were all focused on reading literacy<sup>5</sup> and therefore guarantee consistency regarding the questionnaires and variables selected. Even though the focus was towards the aforementioned subject, our analysis is performed for mathematics and science results as well.

Given that not all students have the same probability of being selected to complete the test and because there may exist some difference in response rates between schools, a sample weight is associated to each school and student by the OECD to obtain a representative sample of the student population targeted in each year. These weights are used throughout the entirety of the analysis.

The chosen variables are divided into two groups: family and school variables. Family variables consist of indexes constructed by the OECD based on the answers given by the students in the student questionnaire that can be taken as proxies for the socio-economic

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<sup>5</sup> For each wave of PISA, one of the three subjects is the major domain of assessment, rotating and repeating accordingly. Reading literacy was the first major domain in 2000, followed by mathematics in 2003 and science in 2006.

background of the students' families. Table 1 below shows a list and description of the family variables chosen.

**Table 1.** Family background variables and description.

Variable	Description
HEDRES (Home Educational Resources)	Index based on the availability of the following items at home: dictionary, quiet place to study, a desk, textbooks, calculator, computer.
CULTPOSS (Cultural Possessions at Home)	Index based on the availability of the following items at home: classical literature, books of poetry, works of art, books on arts, music or design, musical instruments.
HISEI (Highest Parental Occupation Status)	Index of socio-economic status of the parents. Highest occupational status score of either parent.

Source: PISA's 2000, 2009 and 2018 Technical Reports.

School variables intend to measure the quality and availability of school resources. Such variables correspond to PISA indexes constructed by the OECD based on the answers given by school directors on the school questionnaire. The final list of school variables and their description can be found in table 2 below.

**Table 2.** School variables and description.

Variable	Description
SCHLSIZE (School Size)	Number of students in the school.
RTCOMP (Ratio of computers per student)	Ratio of computers available to 15-year-olds to the number of students in the modal grade for 15-year-olds.
STRATIO (Student-Teacher ratio)	Ratio of the number of students in the schools to the total number of teachers.
SCMATEDU (Quality of educational resources) <sup>i</sup>	Index of the school's capacity to provide instruction based on the availability and adequacy of school equipment and instructional materials. Positive scores indicate better quality of resources.
SCMATBUI (Quality of school infrastructure) <sup>ii</sup>	Index of the school's condition of infrastructures and instructional spaces. Positive scores indicate better quality of infrastructures.
EDUSHORT (Shortage of educational material) <sup>iii</sup>	Index of the school's availability and quality of educational material and availability and conditions of infrastructures. Positive scores correspond to worse school resources.
TCSHORT (Teacher Shortage) <sup>iv</sup>	Index of the school's availability and quality of teachers. <sup>v</sup>
STAFFSHORT (Staff shortage) <sup>vi</sup>	Index of the school's availability and quality of teachers and assisting staff. Positive scores mean less staff available.

TEACBEHA (Teacher Behaviour)	Index of teachers' expectations regarding students, absenteeism, strictness with students, relationship with the students and adaptability. Positive scores represent better teacher behaviour.
STUDBEHA (Student Behaviour)	Index of students' absenteeism, behaviour in classroom, respect for teachers, use of alcohol or drugs and bullying towards the peers. Positive scores represent better student behaviour.

(i) Index only available for the 2000 and 2009 waves of PISA

(ii) Index only available for the 2000 wave of PISA

(iii) Index only available for the 2018 wave of PISA

(iv) Index only available for the 2000 and 2009 waves of PISA

(v) Scaling for this item changed between 2000 and 2009. In 2000, positive scores indicate more teachers. In 2009, positive scores indicate fewer teachers.

(vi) Index only available for the 2018 wave of PISA

Source: PISA's 2000, 2009 and 2018 Technical Reports.

The choice of variables is based on the selection previously made by Carneiro (2008), which differs from our final set of variables because some of these were not constructed for later editions of PISA.<sup>6</sup> Also due to the availability of the variables in all waves of PISA, the set of school variables is not the same for the three years under analysis, as indicated in table 2. Nonetheless, this choice was carefully conducted as to assure we are measuring the same dimensions of school quality and resources throughout the years.

Finally, test scores in PISA are measured through plausible values.<sup>7</sup> In our analysis, we use all plausible values available for each year<sup>8</sup> to compute our final test scores.

As further explained in section 4, the nature of the estimation strategy requires an analysis of the data at both the student- and school-level.

The final dataset contains observations for 4,405 students for the reading domain, 2,431 students for the mathematics domain and 2,444 students for the science domain in 2000.<sup>9</sup> For

<sup>6</sup> We excluded the variables "Index of Social Communication of the Parents" and "Index of Cultural Communication of the Parents" from the original set of family variables due to the unavailability of these index in the 2009 and 2018 waves of PISA. The original set of school variables included "School size", "Ratio of computers per students", "Student-teacher ratio", "Proportion of teachers with degree in pedagogy" and "School hours per year". The latter two were excluded from our selection due to unavailability in the 2009 and 2018 waves of PISA. The remaining variables in table 2 were added.

<sup>7</sup> Plausible values are a representation of the range of abilities that a student might reasonably have. Instead of directly attributing a single test score to a student, a range of possible values with an associated probability for each of these values is estimated. Plausible values are random draws from this estimated distribution for a student's performance (for a detailed description of the methodology using plausible values, see appendix 2).

<sup>8</sup> The PISA dataset makes 5 plausible values available in 2000 and 2009; 10 plausible values in 2018.

<sup>9</sup> As this was PISA's first wave, there were more students taking the reading test, the domain of focus in 2000.

the remaining years, the number of students is the same for all domains of assessment. As such, the dataset includes observations for 6,128 students in 2009 and 5,443 students in 2018.<sup>10</sup> Appendix 1a displays detailed summary statistics of the variables at the student-level.

When it comes to data at the school-level, unlike what happens to the number of students, there was an increase in the number of schools throughout the three waves of PISA sampled. In 2000, our sample counts with 145 schools. This number increases to 199 schools in 2009 and 233 schools in 2018 (for detailed descriptive statistics at the school-level, see appendix 1b).

#### 4. Empirical Strategy

We use the same two-step estimation method as Carneiro (2008) to understand the sources of inequality in Portuguese students' test scores. For each subject in each year, we first estimate the following model of the EPF:

$$T_{ij} = \alpha + \beta * F_i + \theta * S_j + \varepsilon_{ij} \quad (1)$$

Where  $T_{ij}$  is the test score for student  $i$  in school  $j$ ,  $F_i$  is a vector of family characteristics, presented in table 1,  $S_j$  is a vector of school dummies and  $\varepsilon_{ij}$  is the error term which is orthogonal to  $F_i$  and  $S_j$ . Considering  $\Phi_i = \beta * F_i$  as family effects and  $\Psi_j = \theta * S_j$  as school effects, we can decompose test scores such that:

$$T_{ij} = \alpha + \Phi_i + \Psi_j + \varepsilon_{ij}$$

Using the variance as the measure of inequality, we perform the variance decomposition of students' test scores, given by the following equation:

$$\text{Var}(T_{ij}) = \text{Var}(\Phi_i) + \text{Var}(\Psi_j) + \text{Var}(\varepsilon_{ij}) + 2 * \text{Cov}(\Phi_i, \Psi_j) \quad (2a)$$

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<sup>10</sup> The original PISA databases contained observations for 4,585 students in 2000, 6,298 students in 2009 and 5,932 students in 2018. Some observations were dropped due to missing values in the variables chosen.

Which can be re-arranged into:

$$1 = \frac{\text{Var}(\Phi_i)}{\text{Var}(T_{ij})} + \frac{\text{Var}(\Psi_j)}{\text{Var}(T_{ij})} + \frac{\text{Var}(\varepsilon_{ij})}{\text{Var}(T_{ij})} + \frac{2*\text{Cov}(\Phi_i, \Psi_j)}{\text{Var}(T_{ij})} \quad (2b)$$

As to assess the relative contribution of each factor (family effects and school effects) to the total within-schools variance in test scores.

Inspired by the findings in the Coleman Report (1966), to understand the contribution of the student's peers' family background to their educational achievement, the school effects ( $\Psi_j$ ) captured through the school dummies in equation (1) are then separated into observed school characteristics ( $\Psi_j^S$ ) and family background characteristics ( $\Psi_j^F$ ). Using only one observation per school, we estimate the following model:

$$\Psi_j = \sigma + \varphi*S_j^S + \eta*S_j^F + \delta_j \quad (3)$$

Where  $\Psi_j$  is the vector of school effects obtained from equation (1),  $S_j^S$  is a vector of observed school characteristics composed of the variables presented in table 2 and  $S_j^F$  is a vector of the school-averaged family background characteristics in the school, given by  $F_i$  in equation (1), i.e., *Average Home Educational Resources (Average HEDRES)*, *Average Cultural Possessions at Home (Average CULTPOSS)*, and *Average Parental Socio-economic Background (Average HISEI)*. Similarly to what we did with equation (1), we can write equation (3) as:

$$\Psi_j = \sigma + \Psi_j^S + \Psi_j^F + \delta_j$$

With  $\Psi_j^S = \varphi*S_j^S$  and  $\Psi_j^F = \eta*S_j^F$ .

The variance decomposition of school effects between-schools is given by:

$$\text{Var}(\Psi_j) = \text{Var}(\Psi_j^S) + \text{Var}(\Psi_j^F) + \text{Var}(\delta_j) + 2*\text{Cov}(\Psi_j^S, \Psi_j^F) \quad (4a)$$

Alternatively,

$$1 = \frac{\text{Var}(\psi_j^S)}{\text{Var}(\Psi_j)} + \frac{\text{Var}(\psi_j^F)}{\text{Var}(\Psi_j)} + \frac{\text{Var}(\delta_j)}{\text{Var}(\Psi_j)} + \frac{2*\text{Cov}(\psi_j^S, \psi_j^F)}{\text{Var}(\Psi_j)} \quad (4b)$$

Once again, equation (4b) gives the relative contribution of each factor to the total variance of school effects between schools. In this sense, the term  $\frac{\text{Var}(\psi_j^S)}{\text{Var}(\Psi_j)}$  represents the share of this variance that is explained by the variance in school observable characteristics (inputs) between schools. In other words, it represents how much of the inequality in educational achievement is explained by differences in school quality and resources. The same reasoning is applied to terms  $\frac{\text{Var}(\psi_j^F)}{\text{Var}(\Psi_j)}$  and  $\frac{2*\text{Cov}(\psi_j^S, \psi_j^F)}{\text{Var}(\Psi_j)}$  for the family background of the peers in the schools and the covariance between the family background of the peers and school inputs, respectively. The term  $\frac{\text{Var}(\delta_j)}{\text{Var}(\Psi_j)}$  represents the share of the total variance in the school effects that comes from the residuals, and, therefore, is not explained by the model.

Although we use the same empirical strategy as Carneiro (2008), some adaptations are made to the author's methodology to ensure the robustness of results.<sup>11</sup> First of all, the test scores regressed in equation (1) are computed using all plausible values available in the PISA dataset for each year. Second, equation (1) is regressed using a weight for each student<sup>12</sup>, as recommended by the OECD (see appendix 2 for a detailed description of the methodology using plausible values). Lastly, to guarantee that each school is represented according to the response rates in PISA, a school weight<sup>13</sup> is attributed to each school in equation (3).

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<sup>11</sup> Carneiro (2008) uses only one plausible value to compute test scores. Neither equation (1) or (3) were regressed using weights in the authors original methodology.

<sup>12</sup> PISA's final student weight trimmed for non-response.

<sup>13</sup> PISA's final school weight trimmed for non-response.

## 5. Results

### 5.1. Main Results

Tables 3 and 4 report the coefficient estimates from equations (1) and (3), respectively. While a large part of the literature uses the number of books at home as a measure of the socio-economic status of the students' family (Woessmann, 2003; Schuetz et al, 2008), we use a direct measure of the socio-economic status of parents (HISEI) and the index of cultural possession at home (CULTPOSS) which, as stated in table 1, is based on the existence of some type of books at home. As seen in table 3, these two measures have a positive sign and are statistically significant at the 1% level for all years. A similar effect is seen for the index of home educational resources (HEDRES) except for the reading and science test scores in 2018, for which it is not significant. Overall, these results seem to be aligned with the previous literature for which family characteristics have a significant impact on student performance (Ammermueller et al., 2005; Shuetz et al., 2008; Woessmann, 2003 & 2005).

**Table 3.** Regression of test scores on family variables and school dummies

Variables	2000			2009			2018		
	Reading	Maths	Science	Reading	Maths	Science	Reading	Maths	Science
HEDRES	13.16*** (1.55)	14.20*** (2.250)	9.68*** (2.15)	4.76*** (1.30)	8.91*** (1.59)	5.95*** (1.20)	-0.43 (1.63)	4.08** (1.77)	1.51 (1.75)
CULTPOSS	9.83*** (1.40)	6.82*** (1.85)	5.23*** (1.89)	12.40*** (1.32)	7.23*** (1.39)	10.99*** (1.25)	11.77*** (1.85)	13.10*** (1.99)	11.56*** (1.68)
HISEI	0.87*** (0.09)	0.90*** (0.13)	0.80*** (0.12)	0.84*** (0.08)	1.22*** (0.09)	1.05*** (0.08)	0.83*** (0.07)	0.90*** (0.07)	0.87*** (0.07)
Constant	336.28*** (7.65)	337.13*** (17.10)	350.63*** (10.44)	409.92*** (7.56)	372.10*** (4.96)	413.72*** (6.67)	323.30*** (11.80)	328.99*** (22.437)	330.75*** (12.14)
N	4405	2431	2444	6128	6128	6128	5443	5443	5443
R <sup>2</sup>	0.444	0.396	0.397	0.387	0.397	0.369	0.352	0.369	0.360

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; Variables described in table 1; Weighted sample.

The coefficients for the regression of the school effects on the observed family and school characteristics in table 4 are also aligned with the results from previous literature. The average HEDRES, HISEI and CULTPOSS in the school — our measures of the socio-economic characteristics of the family of the peers — appear to be significant at the 5% and 1% levels

and positive for most cases. School inputs, in their turn, are not as consistent. Even though some of these variables are statistically significant for 2000, the same does not apply to 2009 and 2018. For the latter years, most school variables have no statistical significance.

**Table 4.** Regression of school effects on family observable variables and school observable variables.

Variables	2000			2009			2018		
	Reading	Maths	Science	Reading	Maths	Science	Reading	Maths	Science
SCHLSIZE	0.03*** (0.01)	0.02*** (0.01)	0.02*** (0.01)	0.01* (0.01)	0.02** (0.01)	0.02** (0.01)	0.01*** (0.00)	0.01** (0.00)	0.01*** (0.00)
RTCOMP	11.75 (8.97)	12.71 (7.75)	13.42 (8.584)	-23.31*** (7.96)	-18.35** (8.44)	-11.25* (6.78)	9.15 (7.65)	10.22 (6.85)	7.79 (6.69)
STRATIO	-4.42*** (0.980)	-3.11*** (0.85)	-3.02*** (0.94)	-1.59 (1.06)	-1.24 (1.12)	-1.34 (0.90)	-1.68** (0.81)	-0.46 (0.73)	-0.88 (0.71)
SCMATEDU	-9.79*** (3.36)	-8.66*** (2.90)	-6.56** (3.22)	6.95* (3.57)	3.76 (3.79)	5.07 (3.04)	-	-	-
SCMATBUI	16.06*** (3.66)	15.67*** (3.16)	12.71*** (3.50)	-	-	-	-	-	-
EDUSHORT	-	-	-	-	-	-	-15.34*** (3.13)	-12.95*** (2.80)	-12.69*** (2.74)
TCSHORT	0.64 (3.61)	3.35 (3.12)	1.68 (3.46)	-2.19 (5.21)	4.83 (5.52)	-3.65 (4.44)	-	-	-
STAFFSHORT	-	-	-	-	-	-	-2.90 (3.99)	-2.21 (3.58)	-3.83 (3.50)
TEACHBEHA	5.78 (4.20)	3.95 (3.62)	3.06 (4.02)	2.91 (3.08)	2.75 (3.26)	4.72* (2.62)	0.51 (3.67)	2.27 (3.28)	4.51 (3.21)
STUDBEHA	-14.46*** (3.60)	-16.45*** (3.11)	-12.62*** (3.45)	0.43 (3.22)	3.61 (3.41)	1.31 (2.74)	-2.68 (3.41)	-2.43 (3.05)	-0.20 (2.98)
Average HEDRES	45.62*** (13.98)	33.48*** (12.08)	43.79*** (13.38)	34.79*** (7.00)	40.45*** (7.42)	26.53*** (5.96)	31.19** (12.67)	29.69*** (11.35)	19.32* (11.09)
Average HISEI	0.88 (0.57)	0.84* (0.49)	0.20 (0.55)	2.33*** (0.43)	1.51*** (0.46)	1.59*** (0.37)	2.24*** (0.38)	1.92*** (0.34)	2.06*** (0.34)
Average CULTPOSS	37.06*** (11.67)	31.55*** (10.08)	45.02*** (11.17)	-3.51 (8.87)	2.40 (9.41)	5.13 (7.56)	-23.23* (12.91)	-4.43 (11.55)	-6.37 (11.30)
Constant	-32.12 (27.11)	-30.02 (23.42)	-2.49 (25.96)	-102.01*** (17.12)	-75.30*** (18.15)	-80.80*** (14.58)	-111.87*** (20.25)	-107.51*** (18.13)	-111.68*** (17.72)
N	145	145	145	199	199	199	233	233	233
R <sup>2</sup>	0.669	0.659	0.627	0.509	0.437	0.493	0.423	0.489	0.468

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; School variables described in table 2; Family variables are the per school average of the variables described in table 1; Weighted sample.

We now switch focus to table 5, which shows the results for the variance decomposition of test scores according to equations (2b) and (4b), in terms of relative contribution.

Looking at the variance decomposition of reading test scores on family effects and school effects (equation (2b)), we observe that, in 2000, the family characteristics of the students explain only 7.8% of the inequalities in test scores, 3 p.p. less than the covariance between the

family characteristics of the students and the school effects and much less than school effects. Results suggest that most of the variance in the reading test scores that is explained by the model in equation (1) comes from differences between the schools, as the variance in the school effects accounts for 28.3% of the total variance in test scores. We notice, nonetheless, that most of this variance cannot be explained by family or school observable characteristics, as 53.1% of the total variance comes from the residuals ( $\text{Var}(\varepsilon_{ij})$ ). Similar results can be observed for all subjects, in all years — for the part of the variance in test scores that is explained by our model, most inequality seems to be due to differences between the schools, given the larger contribution coming from the school effects.

**Table 5.** Variance decomposition of test scores (relative contribution).

	2000			2009			2018		
	Reading	Maths	Science	Reading	Maths	Science	Reading	Maths	Science
Regression of test scores on family variables and school dummies – Within-school (equation (2b))									
Total Var	8613.296	7244.231	6883.731	6800.819	7409.919	6110.454	8465.250	8185.111	7613.847
Var(F)	0.078	0.085	0.057	0.083	0.104	0.112	0.069	0.096	0.086
Var(S)	0.283	0.250	0.285	0.236	0.229	0.196	0.229	0.223	0.219
2*Cov(F, S)	0.108	0.096	0.093	0.095	0.104	0.101	0.075	0.089	0.086
Var( $\varepsilon_{ij}$ )	0.531	0.569	0.565	0.586	0.563	0.591	0.627	0.592	0.609
Regression of school effects on family observable variables and school observable variables – Between-school (equation (4b))									
Total Var	2704.875	1961.947	2201.496	2034.630	1993.340	1429.650	2985.214	2683.069	2480.310
Var( $\Psi_j^F$ )	0.338	0.321	0.378	0.370	0.291	0.320	0.314	0.362	0.355
Var( $\Psi_j^S$ )	0.197	0.215	0.140	0.057	0.061	0.068	0.118	0.094	0.092
2*Cov( $\Psi_j^F, \Psi_j^S$ )	0.134	0.123	0.109	0.082	0.085	0.105	-0.009	0.030	0.021
Var( $\delta_j$ )	0.331	0.341	0.373	0.491	0.563	0.507	0.577	0.514	0.532

Note: Var(F), Var(S), 2\*Cov(F, S) and Var( $\varepsilon_{ij}$ ) must sum to 1. The same applies to Var( $\Psi_j^F$ ), Var( $\Psi_j^S$ ), 2\*Cov( $\Psi_j^F, \Psi_j^S$ ) and Var( $\delta_j$ ).

To understand which school factors account for the variance in the school effects, we turn to the second part of table 5. In this part, we examine the between-school variance decomposition of school effects, according to equation (4b). The results are, once again, consistent with the literature: the family background of the peers in the school seems to be the main observable determinant of academic achievement.

Fuchs & Woessmann (2006) suggest that the reading domain is usually more affected by family characteristics. In our results, however, that is only verified in 2009. Overall, we do not observe any clear pattern between domains of assessment, as, for each year, there is a different subject for which the average family background in the school has a bigger contribution for the inequalities in educational achievement.

Moreover, the contribution coming from the peers' family background is quite constant throughout the years. The highest gap in this factor's contribution is registered for mathematics test scores, which increases by 7.1 p.p. between 2009 and 2018, after dropping by 3 p.p. from 2000 to 2009. For reading test scores, the family background of the peers in the school explains 33.8% of the total variance in the school effects in 2000, 37% in 2009 and 31.4% in 2018, a result that is similar for mathematics and science test scores.

As expected, school inputs have a smaller contribution to the variance of the school effects when compared to the family of the peers. In 2000, the year for which school inputs have the highest contribution, this factor explains 19.7% of the total variance between schools in the reading domain. This contribution is particularly small in 2009, when school inputs explain only 5.7% of the inequalities in student achievement between schools. Despite the increase by 6.1 p.p. from 2009 to 2018, this is not enough to close the gap between the contribution of school inputs and that coming from the family characteristics of the peers. By 2018, the contribution of the peers' family is 2.7 to 3.9 times larger than that of school inputs.

Another common aspect to all subjects throughout the years is the decline in the percentage of the total variance in the school effects that is explained by the covariance between the family characteristics of the peers and school observable characteristics, with a decrease that is particularly visible from 2009 to 2018. Note that this covariance indicates the relationship between the peers' family characteristics and school inputs. The results in table 5 suggest a

decrease in the relevance of this relationship to explain inequality between schools. By 2018, its contribution is practically null.

Lastly, the results show what we could already perceive from the  $R^2$ s in table 4. Throughout the years, we observe a decrease in the percentage of the variance of the school effects that is explained by family and school observable characteristics, as seen through the increase in the share of the total variance that comes from the variance in the residuals ( $\text{Var}(\delta_j)$ ). In 2000, school inputs and the average family background in the school explained close to 67% of the total variance in school effects for reading test scores, which decreased by 16 p.p. in 2009 and further decreased by 8.7 p.p. in 2018, explaining only 42.3% of the total variance in this year. Overall, the evolution of the total contribution of school inputs and the average family background in the school is similar for all subjects — in 2018, these factors explain less than 50% of the inequalities in student achievement between schools.

Although the main conclusions are similar, our results for 2000 differ in absolute terms from Carneiro (2008), as the author finds the contribution of the family of the peers to the variation in test scores to be between 59% and 65% in contrast to a very limited contribution of 2% to 3% coming from school resources. These differences reflect the changes in the set of variables chosen and adjustments in the methodology (mentioned in section 4) that we believe to be important for the analysis.

## **5.2. Heterogeneity Effects**

Given the heterogeneity that characterizes the Portuguese educational system, it is relevant to perform this analysis for some of its different dimensions. As such, the remainder of this study explores heterogeneity effects, splitting the sample by the type of institution (public schools), educational track (academic track) and region (NUTS II). In light of the findings of the Coleman Report (1966) and considering our previous findings in 5.1, subsections 5.2.1 to 5.2.3 will focus

on the inequality in educational achievement between schools, by performing the variance decomposition of school effects, according to equations (3) and (4).<sup>14</sup>

### **5.2.1. Decomposition of school effects for public schools**

The literature shows that institutions play a relevant role in explaining the variance of test scores between students (Fuchs et al., 2006; Woessmann, 2016) but how students' test scores vary within each type of institution is a question that also deserves to be addressed. Due to the low number of students in private schooling surveyed in PISA, we were only able to perform this analysis for public schools, which represent 88% of the schools in the whole sample (see appendices 3a and 3b for descriptive statistics). However, this analysis alone finds itself to be of great interest. According to Direção-Geral de Estatísticas da Educação e Ciência (2018), there are large inequalities in test scores and socio-economic contexts between Portuguese public schools.

Table 6 displays the variance decomposition of school effects for public schools.<sup>15</sup> We can observe some differences from the results for the whole sample (table 5), namely regarding the total variance between schools for each sample. While the variance between public schools was lower than that for the whole sample in 2000 and 2009, the contrary is verified for 2018. Despite the similarities verified in the increase of the unexplained part of the total variance of school effects that come from an increase in the residuals and the limited contribution of school factors, we see differences arising from the contribution of the family of the peers and the covariance between the latter and school inputs. In what concerns the contribution of the family of the peers, results in table 6 suggest that, for public schools in all domains of assessment, this

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<sup>14</sup> The total variance decomposition of test scores according to equations (1) and (2) can be found in appendices 3e, 4e and 5k. For all groups in analysis in sections 5.2.1. to 5.2.3. the results for the total variance decomposition were similar to those obtained in section 5.1.: most of the explained variance in test scores comes from the school effects. Regression coefficients according to equation (1) can be found in appendices 3c, 4c, 5c, 5e, 5g and 5i.

<sup>15</sup> Regression coefficients according to equation (3) for public schools can be found in appendix 3d.

contribution has decreased throughout the years. If in 2000 this contribution surpassed that verified for the whole sample, explaining close to 49% of the total variance in school effects for the reading test scores, in 2018, the family of the peers contributed to only 19.1% of the inequality between schools (and 12.3 p.p. lower than that registered for all the schools in the sample).

**Table 6.** Variance decomposition of school effects on family observable characteristics and school observable characteristics – Public Schools.

	2000			2009			2018		
	Reading	Maths	Science	Reading	Maths	Science	Reading	Maths	Science
Total Var	2533.798	1839.955	2027.978	1980.716	1948.777	1359.314	3280.520	2830.608	2754.017
$\text{Var}(\Psi_j^F)$	0.487	0.431	0.487	0.396	0.354	0.312	0.191	0.211	0.208
$\text{Var}(\Psi_j^S)$	0.105	0.162	0.093	0.070	0.058	0.085	0.155	0.117	0.120
$2*\text{Cov}(\Psi_j^F, \Psi_j^S)$	0.105	0.090	0.055	0.080	0.066	0.098	0.079	0.094	0.095
$\text{Var}(\delta_j)$	0.303	0.317	0.365	0.454	0.522	0.505	0.575	0.578	0.577

Note:  $\text{Var}(\Psi_j^F)$ ,  $\text{Var}(\Psi_j^S)$ ,  $2*\text{Cov}(\Psi_j^F, \Psi_j^S)$  and  $\text{Var}(\delta_j)$  must sum to 1.

As for the covariance between the family of the peers and school inputs, two conclusions can be taken. First, there seems to be a positive relationship between the two factors which increases the inequalities in student achievement between schools. Secondly, contrary to what happened for the whole sample, this relationship, and its contribution to the variance in school effects does not decrease throughout the years — it is, in fact, rather constant.

### 5.2.2. Decomposition of school effects for students in the Academic Track

Due to the reduced number of students in the vocational track surveyed in PISA, we decided to focus this analysis on the students enrolled in the academic track.<sup>16</sup> We can observe in appendices 4a and 4b that the test scores and the measures of family background for this sub-sample are higher than those observed for the entire sample (appendices 1a and 1b). This is in line with previous studies for Portugal that show that students enrolled in the vocational track tend to come from lower socio-economic backgrounds than those in the academic track (Duarte,

<sup>16</sup> Students in non-vocational-oriented tracks.

2012). Table 7 shows the variance decomposition of school effects for students enrolled in the academic track.<sup>17</sup>

If we compare these results to the ones in table 5, we do not observe a big contrast. In fact, the only noteworthy difference relies on the contribution coming from the family background of the peers. When compared to the whole sample (table 5), we can verify in table 7 that the family of the peers has an even bigger contribution to the inequality in test scores between the schools in 2009 and 2018. If we focus on reading test scores, this accounts for a difference 6.5 p.p. and 7 p.p. for each year, respectively. Once again, the results are very similar across the domains of assessment — the same pattern can be observed for the inequalities in mathematics and science test scores between schools.

**Table 7.** Variance decomposition of school effects on family observable characteristics and school observable characteristics – Academic Track.

	2000			2009			2018		
	Reading	Maths	Science	Reading	Maths	Science	Reading	Maths	Science
Total Var	2712.877	2005.616	2282.521	1808.848	1955.151	1409.335	3059.893	2928.842	2714.200
Var( $\Psi_j^F$ )	0.334	0.311	0.365	0.435	0.402	0.350	0.383	0.411	0.418
Var( $\Psi_j^S$ )	0.192	0.212	0.138	0.043	0.051	0.052	0.147	0.080	0.087
2*Cov( $\Psi_j^F, \Psi_j^S$ )	0.141	0.131	0.119	0.031	0.037	0.068	-0.001	0.035	0.019
Var( $\delta_j$ )	0.333	0.346	0.378	0.491	0.510	0.530	0.471	0.474	0.476

Note: Var( $\Psi_j^F$ ), Var( $\Psi_j^S$ ), 2\*Cov( $\Psi_j^F, \Psi_j^S$ ) and Var( $\delta_j$ ) must sum to 1.

### 5.2.3. Decomposition of school effects by Region (NUTS II)

Lastly, we performed the variance decomposition of test scores by region. Once again hindered by data limitations, we were only able to perform such analysis for 4 of the 7 regions delimited by NUTS II<sup>18</sup> and for the years of 2009 and 2018.<sup>19</sup> Having to exclude the Autonomous Regions of Azores and Madeira and the continental region of the Algarve, figures 1 and 2 show the

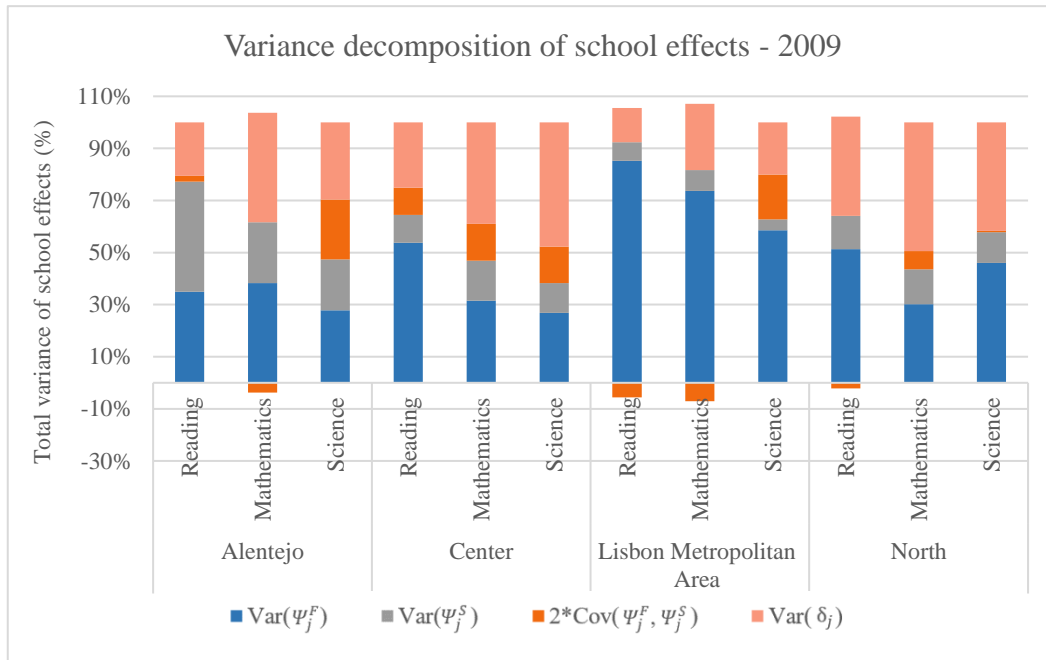
<sup>17</sup> Regression coefficients according to equation (3) for the academic track can be found in appendix 4d.

<sup>18</sup> NUTS II include 7 units: Algarve, Alentejo, Center, Lisbon Metropolitan Area, North, Autonomous Region of Azores and Autonomous Region of Madeira.

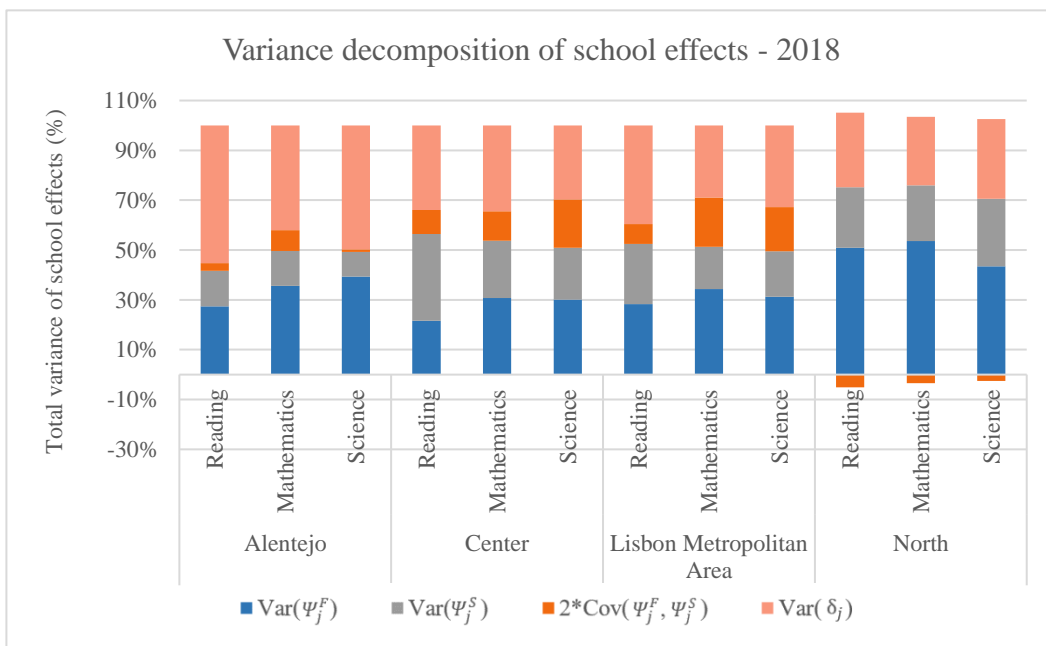
<sup>19</sup> The indicator of region was not available in the database for PISA 2000.

graphical representation of the variance decomposition of school effects for the remaining regions, for 2009 and 2018, respectively.<sup>20</sup>

**Figure 1.** Variance decomposition of school effects by region (NUTS II) – 2009.



**Figure 2.** Variance decomposition of school effects by region (NUTS II) – 2018.



<sup>20</sup> Regression coefficients according to equation (3) by region can be found in appendices 5d, 5f, 5h and 5j.

Despite being able to observe some heterogeneity between the 4 regions for the two years displayed in figures 1 and 2, the differences are particularly visible in 2009 (figure 1). Marked disparities are especially seen between Alentejo and the Lisbon Metropolitan Area. Alentejo, the region among the 4 with the lowest test scores and lowest measures of the socio-economic status of the family of peers in the school in both years (see appendices 5a and 5b) presents school inputs as the factor with the highest contribution for the variance in school effects. The opposite can be observed for the Lisbon Metropolitan area, where school inputs have the lowest contribution for the inequality in student achievement among the 4 regions. The family background of the peers in the school, in contrast, has a strong expression, especially in the reading domain, explaining 85.2% of the variance in school effects in this region.

In figure 2, similarities between regions are more evident. In 2018, the share of the inequality in test scores between schools that is explained by the family background of the peers seems to be more homogenous across regions. In the regions of Alentejo, Center, and Lisbon Metropolitan Area, this factor contributes to close to 30% of the variance between schools within each region (a similar result to that verified in table 5, for the whole sample). We notice, nonetheless, a big drop in the percentage of the variance in the school effects that is explained by the family background of the peers in the Lisbon Metropolitan Area between the two years. For the reading test scores, this consists in a 56.9 p.p. drop. Moreover, in this year, the family background of the peers seems to be particularly relevant in the North, where it explains close to 50% of the variance in the school effects.

The total share of the between-school variance that is explained by school inputs also seems to be more stable across regions in 2018. Furthermore, when compared to 2009, this factor's contribution suffers a slight increase, in all regions except Alentejo, where it decreases. School inputs appear to be particularly important in explaining the variance in reading test scores between schools in the Center region. In this region, the contribution of school inputs to

the variance in school effects for the reading domain is 13.1 p.p. bigger than the contribution of the family background of the peers.

Additionally, the results for 2018 suggest some relevance of the covariance between the two factors. A positive relationship between the family background of the peers and the school resources seems to be particularly visible in the Lisbon Metropolitan Area and the Center region, where it explains around 10% of the variance in school effects in all domains within each region.

## **6. Discussion and Concluding Remarks**

Identifying the determinants of educational achievement is a relevant subject in the context of education policy. This study picks up on the previous work by Carneiro (2008) and adapts its methodology to extend the analysis to more than one period and assess how family characteristics and school inputs contribute to the inequalities in 15-year-old Portuguese students' test scores. The analysis is further extended to study heterogeneity in different dimensions of the Portuguese educational system.

Our results suggest that the family of the peers in the school is the main observable determinant of students' educational achievement. School inputs, in contrast, have a smaller expression and their contribution to the inequalities in student performance suffers an overall decrease in the period under analysis. There is, nonetheless, a large part of the variance in students' test scores that is not explained by our model, which increases throughout the years. Different dimensions of the Portuguese educational system reveal some heterogeneity in the results. The contribution of the family characteristics of the peers to the differences in students test scores in public schools seems to decrease throughout the years. Stronger heterogeneity is, however, observed between regions. In 2009, school inputs are the main determinant of

students' academic achievement in the region of Alentejo. In 2018, results are more similar between regions and more aligned with the results found for the whole sample.

Due to the observational nature of the data from PISA, we should be careful when interpreting the results, particularly in what concerns inference of relationships of causality. Besides the possibility that we are in the presence of unobserved family and school characteristics that are correlated with the selected variables, we are also constrained by the reliability of the indexes constructed for PISA.

Understanding the reasons behind our results is a complex task, and even more so considering the limitations in the data. Further investigation may contribute to answer the challenges still faced in Portugal regarding enrollment and completion rates in upper-secondary and higher education. Also, it would be interesting to include institutions and the dimensions of school autonomy and operation in our estimation of the EPF. This could reduce the amount of the variation in students' academic achievement that is not explained by our model.

Despite the limitations addressed, our results provide an important understanding of the determinants of educational achievement. The relationships that were established deserve attention from an education policy perspective, as its goal should be to promote equal opportunities for individuals both at early ages and later in life, when entering the labor market. In the long-run, these policies should be translated into positive indicators of socio-economic development and growth.

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## Appendix<sup>21</sup>

**Appendix 1a.** Descriptive Statistics (Student-Level). Weighted Sample.

	2000				2009				2018			
	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max
<b>Test Scores</b>												
Reading	473.97	92.81	151.55	735.63	491.53	82.47	171.53	724.31	495.72	92.01	188.26	745.94
Mathematics	458.43	85.11	142.02	705.51	488.88	86.08	201.52	755.81	496.27	90.47	199.00	742.68
Science	462.52	82.97	192.31	701.01	494.88	78.17	195.67	729.24	494.74	87.26	146.34	739.59
<b>Family Variables</b>												
HEDRES	0.13	0.87	-4.34	0.76	0.20	1.04	-4.32	1.34	-0.15	0.87	-4.41	1.19
CULTPOSS	-0.09	0.99	-1.65	1.15	0.18	0.98	-1.29	1.35	0.01	0.84	-1.78	1.98
HISEI	43.88	15.91	16.00	90.00	44.19	16.06	16.00	90.00	49.52	23.07	11.01	88.96
Nr of Students	4405*				6128				5443			
Weight	95836				94255				90055			

Mathematics - 2431 students, weight: 52710; Science – 2444 students, weight: 53274

**Appendix 1b.** Descriptive Statistics (School-level). Weighted Sample.

	2000				2009				2018			
	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max
<b>Family Variables</b>												
Average HEDRES	0.02	0.26	-0.68	0.64	0.04	0.37	-0.98	0.87	-0.26	0.33	-1.20	0.47
Average CULTPOSS	-0.27	0.41	-1.05	0.88	0.08	0.37	-0.82	1.06	-0.11	0.36	-1.10	0.88
Average HISEI	40.81	7.43	26.10	67.51	40.67	9.05	23.67	71.59	44.27	12.97	22.10	77.84
<b>School Variables</b>												
SCHLSIZE	708.03	446.28	18	3724	643.47	402.29	73	2400	919.33	811.38	55	8150
RTCOMP	0.08	0.32	0.00	3.67	0.64	0.31	0.18	2.29	0.52	0.41	0.00	3.25
STRATIO	8.87	3.22	0.10	23.64	8.19	2.88	2.41	16.88	10.38	4.28	1.00	50.62
SCMATEDU	-0.11	0.96	-1.90	3.22	-0.13	0.85	-2.27	1.93	-	-	-	-
SCMATBUI	-0.35	0.86	-1.12	2.45	-	-	-	-	-	-	-	-
EDUSHORT	-	-	-	-	-	-	-	-	0.59	1.05	-1.42	2.96
TCSHORT	-0.13	0.87	-0.95	1.87	-0.82	0.46	-1.02	1.60	-	-	-	-
STAFFSHORT	-	-	-	-	-	-	-	-	0.58	1.06	-1.46	4.04
TEACHBEHA	0.24	0.87	-2.41	2.12	0.30	1.04	-2.25	2.12	0.22	1.07	-2.04	3.79
STUDBEHA	0.28	0.86	-2.61	2.22	0.18	1.06	-2.12	2.36	0.42	0.95	-4.09	3.31
Number of schools	145				199				233			
Weight of schools	1284				1496				1135			

<sup>21</sup> Some indexes in PISA are constructed using Item Response Theory scaling and are scaled to have an OECD average of 0 and standard deviation of 1. This applies to the following indexes: HEDRES, CULTPOSS, SCMATEDU, SCMATBUI, EDUSHORT, TCSHORT, STAFFSHORT, TEACHBEHA and STUDBEHA.

**Appendix 2.** Computation of regression coefficients and standard errors using plausible values.

In PISA, a set of plausible values is drawn for each student in each domain of assessment. In the PISA waves of 2000 and 2009, five plausible values were drawn. In 2018, this number was ten.

The estimation of regression coefficients and standard errors using plausible values was conducted following the procedure from the OECD's 2009 PISA data manual (OECD, 2009). First, regression coefficients are computed for *each plausible value and explanatory variable*, using a final student weight and the 80 replicate student weights provided in the PISA dataset following the Balanced Repeated Replication (BRR) method with Fay's modification of 0.5 to compute standard errors (detailed description of this procedure can be found in the 2009 PISA Data Manual).

Population statistics are estimated using each of the plausible values separately and, lastly, averaged to obtain one single population statistics. In a second step, the final regression coefficient is given by the average of the estimated coefficients per plausible value. Let  $\hat{\beta}$  be the final regression coefficient and M the number of plausible values.:

$$\hat{\beta} = \frac{1}{M} \sum_{i=1}^M \hat{\beta}_i$$

The final error variance for the regression coefficients using plausible values is given by the combination of the sampling variance and the imputation (measurement) variance.

The final sampling variance ( $\sigma_{\hat{\beta}}^2$ ) is the average sampling variance for all plausible values, such that:

$$\sigma_{\hat{\beta}}^2 = \frac{1}{M} \sum_{i=1}^M (\sigma_{\hat{\beta}_i}^2)$$

The imputation or measurement variance is the variance that arises from the uncertainty in the estimate of the regression coefficient due to the lack of precision in the measurement test. The final imputation variance ( $\sigma_{(test)}^2$ ) is given by:

$$\sigma_{(test)}^2 = \frac{1}{M-1} \sum_i^M (\hat{\beta}_i - \hat{\beta})^2$$

The final error variance ( $\sigma_{(error)}^2$ ) is given by the combination of the sampling variance and the imputation variance in the following way:

$$\sigma_{(error)}^2 = \sigma_{\hat{\beta}}^2 + \left( \left( 1 + \frac{1}{M} \right) \sigma_{(test)}^2 \right)$$

Lastly, the standard error for regression coefficients is the square root of the final error variance.

$$S.E = \sqrt{\sigma_{(error)}^2}$$

**Appendix 3a.** Descriptive Statistics (Student-Level) – Public Schools. Weighted Sample.

	2000				2009				2018			
	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max
<b>Test Scores</b>												
Reading	472.82	93.36	151.55	735.63	487.26	82.07	171.53	700.01	495.955	91.75	188.26	745.94
Mathematics	456.79	85.49	142.02	705.51	484.48	85.16	201.52	755.81	495.543	90.29	199.00	742.68
Science	461.44	83.04	192.31	701.01	491.23	77.80	195.67	729.24	494.402	86.60	146.34	739.59
<b>Family Variables</b>												
HEDRES	0.12	0.88	-4.34	0.76	0.18	1.04	-4.32	1.34	-0.15	0.87	-4.41	1.19
CULTPOSS	-0.10	0.99	-1.65	1.15	0.14	0.98	-1.29	1.35	-0.01	0.83	-1.78	1.98
HISEI	43.84	15.91	16.00	90.00	43.21	15.50	16.00	90.00	48.48	22.79	11.01	88.96
Nr of Students	4093*	2260	2273		5459				4712			
Weight	88828	48885	49420		80412				76777			

Mathematics – 2260 students, weight: 48885; Science – 2273 students, weight: 49420

**Appendix 3b.** Descriptive Statistics (School-level) - Public Schools. Weighted Sample.

	2000				2009				2018			
	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max
<b>Family Variables</b>												
Average HEDRES	0.01	0.26	-0.68	0.64	0.03	0.34	-0.98	0.87	-0.26	0.290	-1.20	0.46
Average CULTPOSS	-0.31	0.39	-1.05	0.88	0.024	0.33	-0.74	1.06	-0.16	0.343	-1.10	0.82
Average HISEI	40.59	7.32	26.10	67.51	39.28	7.14	23.67	67.94	42.78	10.606	24.04	72.79
<b>School Variables</b>												
SCHLSIZE	721.59	420.13	18	2700	677.83	353.41	138	2400	1163.52	845.48	91	8150
RTCOMP	0.08	0.34	0.00	3.67	0.61	0.33	0.18	2.29	0.41	0.41	0.00	3.25
STRATIO	8.21	2.51	0.10	23.64	7.20	1.79	2.41	11.62	9.22	3.90	1.00	50.62
SCMATEDU	-0.07	0.94	-1.90	3.22	-0.31	0.68	-2.27	1.93	-	-	-	-
SCMATBUI	-0.29	0.88	-1.12	2.45	-	-	-	-	-	-	-	-
EDUSHORT	-	-	-	-	-	-	-	-	0.84	0.93	-1.42	2.96
TCSHORT	-0.04	0.87	-0.95	1.87	-0.80	0.49	-1.02	1.60	-	-	-	-
STAFFSHORT	-	-	-	-	-	-	-	-	1.11	0.66	-1.46	4.04
TEACHBEHA	0.33	0.86	-2.41	2.12	0.08	0.95	-2.25	2.12	0.58	0.75	-1.29	3.79
STUDBEHA	0.41	0.78	-1.80	2.22	-0.09	0.94	-2.12	2.36	0.58	0.77	-1.73	2.65
Number of schools	134				174				193			
Weight of schools	1149				1165				752			

**Appendix 3c.** Regression of test scores on family variables and school dummies. Public Schools.

Variables	2000			2009			2018		
	Reading	Maths	Science	Reading	Maths	Science	Reading	Maths	Science
HEDRES	13.30*** (1.59)	14.95*** (2.326)	9.57*** (2.19)	4.68*** (1.32)	9.02*** (1.52)	6.21*** (1.24)	0.60 (1.84)	4.95** (1.96)	3.07* (1.86)
CULTPOSS	10.11*** (1.39)	6.90*** (1.857)	5.39*** (1.99)	11.89*** (1.41)	7.00*** (1.41)	10.52*** (1.33)	13.04*** (1.86)	13.68*** (2.15)	12.33*** (1.82)
HISEI	0.86*** (0.08)	0.91*** (0.137)	0.78*** (0.12)	0.86*** (0.08)	1.24*** (0.09)	1.06*** (0.08)	0.82*** (0.07)	0.93*** (0.07)	0.90*** (0.08)
Constant	336.86*** (7.70)	337.24*** (17.004)	351.44*** (10.10)	409.22*** (7.51)	371.07*** (5.10)	413.01*** (6.65)	324.87*** (11.83)	329.55*** (22.37)	332.20*** (12.22)
N	4093	2260	2273	5459	5459	5459	4712	4712	4712
R <sup>2</sup>	0.451	0.398	0.405	0.368	0.377	0.350	0.341	0.350	0.343

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; Variables described in table 1; Weighted sample.

**Appendix 3d.** Regression of school effects on family observable variables and school observable variables. Public Schools.

Variables	2000			2009			2018		
	Reading	Maths	Science	Reading	Maths	Science	Reading	Maths	Science
SCHLSIZE	0.01 (0.01)	0.01 (0.01)	0.00 (0.01)	0.02** (0.01)	0.02* (0.01)	0.02*** (0.01)	0.03*** (0.01)	0.02*** (0.01)	0.02*** (0.01)
RTCOMP	19.19*** (8.68)	16.51** (7.57)	22.63*** (8.51)	-21.52*** (7.94)	-14.85* (8.44)	-10.95 (6.92)	12.38 (8.46)	14.31* (7.89)	9.69 (7.77)
STRATIO	-0.78 (1.49)	-1.08 (1.30)	1.02 (1.47)	-2.96* (1.58)	-1.14 (1.68)	-1.45 (1.38)	-4.95*** (1.19)	-3.43*** (1.11)	-3.76*** (1.09)
SCMATEDU	-4.80 (3.41)	-3.46 (2.97)	-3.48 (3.35)	7.92** (3.67)	2.53 (3.90)	5.66* (3.20)	-	-	-
SCMATBUI	15.19*** (3.41)	14.78*** (2.98)	12.03*** (3.35)	-	-	-	-	-	-
EDUSHORT	-	-	-	-	-	-	-12.92*** (3.92)	-11.05*** (3.65)	-10.04*** (3.60)
TCSHORT	-1.10 (3.41)	2.33 (2.98)	0.49 (3.35)	-5.84 (5.12)	3.44 (5.45)	-6.50 (4.47)	-	-	-
STAFFSHORT	-	-	-	-	-	-	-5.28 (5.84)	-2.16 (5.45)	-4.01 (5.37)
TEACHBEHA	5.35 (3.90)	3.34 (3.40)	2.20 (3.82)	0.90 (3.07)	1.28 (3.26)	3.16 (2.68)	8.26* (4.57)	6.91 (4.26)	8.08* (4.19)
STUDBEHA	-13.08*** (3.58)	-15.25*** (3.12)	-11.74*** (3.51)	3.79 (3.17)	7.84** (3.37)	4.00 (2.77)	-2.20 (4.58)	-3.56 (4.27)	0.45 (4.20)
Average HEDRES	46.37*** (13.45)	29.07** (11.73)	42.41*** (13.19)	44.21*** (10.03)	50.73*** (10.67)	38.29*** (8.75)	37.63** (16.08)	20.54 (14.99)	24.15 (14.76)
Average HISEI	1.11* (0.66)	0.65 (0.58)	1.06 (0.65)	2.56*** (0.57)	1.79*** (0.60)	1.41*** (0.50)	2.56*** (0.46)	2.13*** (0.43)	2.18*** (0.42)
Average CULTPOSS	48.34 (14.34)	47.50*** (12.51)	41.68*** (14.06)	-13.40 (9.31)	-6.70 (9.90)	-2.76 (8.12)	-44.40 (15.19)	-8.23 (14.16)	-15.57 (13.95)
Constant	-55.64* (33.47)	-25.32 (29.20)	-59.96* (32.83)	-109.16*** (23.91)	-88.17*** (25.44)	-78.56*** (20.86)	-120.63*** (24.10)	-111.37*** (22.48)	-110.25*** (22.13)
N	134	134	134	174	174	174	193	193	193
R <sup>2</sup>	0.697	0.682	0.636	0.545	0.477	0.496	0.426	0.421	0.423

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; School variables described in table 2; Family variables are the per school average of the variables described in table 1; Weighted sample.

**Appendix 3e.** Variance decomposition of test scores. Public Schools.

	2000			2009			2018		
	Reading	Maths	Science	Reading	Maths	Science	Reading	Maths	Science
Regression of test scores on school family variables and school dummies									
Var Total	8715.662	7308.043	6895.256	6735.777	7252.342	6052.846	8418.007	8151.524	7499.346
Var(F)	0.079	0.088	0.056	0.078	0.104	0.108	0.072	0.102	0.095
Var(S)	0.287	0.247	0.292	0.233	0.224	0.193	0.222	0.211	0.205
2*Cov(F, S)	0.110	0.097	0.096	0.082	0.089	0.086	0.068	0.074	0.074
Var( $\varepsilon_{ij}$ )	0.524	0.567	0.556	0.606	0.583	0.612	0.639	0.614	0.627

Note: Var(F), Var(S), 2\*Cov(F, S) and Var( $\varepsilon_{ij}$ ) must sum to 1.

**Appendix 4a.** Descriptive Statistics (Student-Level) – Academic Track. Weighted Sample.

	2000				2009				2018			
	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max
<b>Test Scores</b>												
Reading	474.26	93.09	194.81	735.63	500.77	79.30	235.74	724.31	505.60	90.46	191.49	745.94
Mathematics	458.65	85.39	142.02	705.51	497.43	83.95	210.63	755.81	506.02	90.11	202.67	742.68
Science	462.52	83.24	198.58	701.01	504.13	75.50	243.05	729.24	504.81	85.88	223.28	739.59
<b>Family Variables</b>												
HEDRES	0.14	0.87	-4.34	0.76	0.27	1.00	-4.32	1.34	-0.12	0.87	-4.41	1.19
CULTPOSS	-0.08	0.99	-1.65	1.15	0.25	0.96	-1.29	1.35	0.04	0.83	-1.78	1.98
HISEI	43.96	16.00	16.00	90.00	45.45	16.40	16.00	90.00	51.19	22.97	11.01	88.96
Nr of Students	4139*				5237				4553			
Weight	90450				79813				74637			

\*Mathematics – 2291 students, weight: 49894; Science – 2292 students, weight: 50102

**Appendix 4b.** Descriptive Statistics (School-level) - Academic Track. Weighted Sample.

	2000				2009				2018			
	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max
<b>Family Variables</b>												
Average HEDRES	0.03	0.27	-0.68	0.66	0.11	0.37	-0.98	0.93	-0.23	0.311	-0.99	0.51
Average CULTPOSS	-0.271	0.42	-1.05	0.86	0.15	0.39	-0.74	1.06	-0.08	0.368	-1.10	0.89
Average HISEI	40.88	7.47	26.21	68.13	42.10	9.27	26.50	71.59	46.01	13.066	24.04	77.84
<b>School Variables</b>												
SCHLSIZE	708.03	446.28	18	3724	691.73	387.73	73	2400	1153	805.01	55	8150
RTCOMP	0.08	0.32	0.00	3.67	0.63	0.32	0.18	2.29	0.444	0.42	0	3
STRATIO	8.87	3.22	0.10	23.64	8.10	2.87	2.41	16.88	10.345	4.34	1.03	50.62
SCMATEDU	-0.11	0.96	-1.90	3.22	-0.10	0.87	-2.27	1.93	-	-	-	-
SCMATBUI	-0.35	0.86	-1.12	2.45	-	-	-	-	-	-	-	-
EDUSHORT	-	-	-	-	-	-	-	-	0.652	0.98	-1.42	2.96
TCSHORT	-0.13	0.87	-0.95	1.87	-0.85	0.41	-1.02	1.60	-	-	-	-
STAFFSHORT	-	-	-	-	-	-	-	-	0.874	0.89	-1.46	4.04
TEACHBEHA	0.24	0.87	-2.41	2.12	0.30	1.06	-2.25	2.12	0.445	0.86	-2.04	3.79
STUDBEHA	0.28	0.86	-2.61	2.22	0.16	1.10	-2.12	2.36	0.348	0.95	-4.09	2.65
Number of schools	145				192				209			
Weight of schools	1284				1366				855			

**Appendix 4c.** Regression of test scores on family variables and school dummies. Academic Track.

Variables	2000			2009			2018		
	Reading	Maths	Science	Reading	Maths	Science	Reading	Maths	Science
HEDRES	12.72*** (1.57)	13.70*** (2.49)	8.20*** (2.22)	2.40* (1.35)	6.75*** (1.61)	3.60*** (1.31)	0.28 (1.82)	4.60** (2.05)	2.44 (1.85)
CULTPOSS	10.28*** (1.34)	7.11*** (1.80)	6.08*** (2.05)	10.61*** (1.40)	5.91*** (1.40)	9.43*** (1.31)	12.16*** (1.97)	13.26*** (2.16)	11.85*** (1.81)
HISEI	0.87*** (0.09)	0.88*** (0.14)	0.80*** (0.13)	0.79*** (0.09)	1.21*** (0.10)	1.01*** (0.09)	0.83*** (0.07)	0.92*** (0.07)	0.88*** (0.07)
Constant	336.65*** (7.72)	338.21*** (17.17)	350.72*** (10.82)	425.39*** (13.77)	388.51*** (11.31)	435.99*** (11.32)	324.18*** (12.01)	329.16*** (22.59)	331.69*** (12.28)
N	4139	2291	2292	5237	5237	5237	4553	4553	4553
R <sup>2</sup>	0.452	0.402	0.410	0.377	0.390	0.362	0.340	0.365	0.352

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; Variables described in table 1; Weighted sample.

**Appendix 4d.** Regression of school effects on family observable variables and school observable variables. Academic Track.

Variables	2000			2009			2018		
	Reading	Maths	Science	Reading	Maths	Science	Reading	Maths	Science
SCHLSIZE	0.03*** (0.01)	0.02*** (0.01)	0.02*** (0.01)	0.014** (0.01)	0.02** (0.01)	0.02** (0.01)	0.02*** (0.00)	0.01** (0.00)	0.01*** (0.00)
RTCMP	9.85 (9.03)	12.44 (7.93)	16.53* (8.84)	-15.19** (7.41)	-12.39 (7.86)	-7.71 (6.80)	7.97 (7.01)	11.51* (6.89)	8.02 (6.64)
STRATIO	-4.27*** (0.98)	-2.99*** (0.86)	-2.79*** (0.96)	-2.55** (1.19)	-2.32* (1.26)	-2.03* (1.09)	-3.85*** (0.82)	-2.18*** (0.80)	-2.38*** (0.76)
SCMATEDU	-9.27*** (3.38)	-8.58*** (2.96)	-5.86* (3.30)	5.54 (3.38)	1.41 (3.59)	4.54 (3.10)	-	-	-
SCMATBUI	16.43*** (3.67)	16.74*** (3.22)	13.17*** (3.59)	-	-	-	-	-	-
EDUSHORT	-	-	-	-	-	-	-19.47*** (3.352)	-13.91*** (3.30)	-13.81*** (3.16)
TCSHORT	0.78 (3.62)	2.98 (3.17)	1.06 (3.54)	4.75 (5.61)	8.83 (5.95)	-0.35 (5.14)	-	-	-
STAFFSHORT	-	-	-	-	-	-	0.29 (4.22)	-0.94 (4.15)	-2.53 (3.99)
TEACHBEHA	4.37 (4.19)	2.47 (3.68)	1.76 (4.10)	-0.51 (2.99)	0.80 (3.17)	2.95 (2.74)	1.86 (3.78)	2.24 (3.72)	3.39 (3.58)
STUDBEHA	-13.45*** (3.62)	-15.32*** (3.18)	-12.17*** (3.54)	4.58 (3.02)	7.57** (3.20)	3.78 (2.77)	-1.06 (3.72)	-0.42 (3.66)	2.50 (3.52)
Average HEDRES	53.12*** (13.83)	42.35*** (12.13)	53.47*** (13.53)	42.34*** (6.72)	54.98*** (7.12)	34.80*** (6.16)	25.58** (12.56)	15.07 (12.34)	14.87 (11.89)
Average HISEI	0.78 (0.58)	0.72 (0.51)	0.18 (0.57)	1.64*** (0.45)	1.30*** (0.47)	1.262** (0.41)	2.78*** (0.33)	2.45*** (0.32)	2.50*** (0.31)
Average CULTPOSS	32.46*** (11.53)	26.74*** (10.12)	38.01*** (11.28)	10.24 (9.07)	4.75 (9.62)	7.80 (8.32)	-29.66*** (11.47)	-1.22 (11.28)	-7.28 (10.87)
Constant	-30.36 (27.56)	-27.22 (24.18)	-6.79 (26.97)	-73.35*** (17.17)	-62.66*** (18.21)	-67.72*** (15.75)	-124.99*** (17.23)	-124.68*** (16.93)	-124.39*** (16.32)
N	145	145	145	192	192	192	209	209	209
R <sup>2</sup>	0.667	0.654	0.622	0.509	0.489	0.470	0.529	0.525	0.524

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; School variables described in table 2; Family variables are the per school average of the variables described in table 1; Weighted sample.

**Appendix 4e.** Variance decomposition of test scores. Academic Track.

	2000			2009			2018		
	Reading	Maths	Science	Reading	Maths	Science	Reading	Maths	Science
Regression of test scores on school family variables and school dummies									
Var Total	8665.883	7291.047	6928.305	6288.964	7047.769	5700.690	8183.244	8119.377	7374.462
Var(F)	0.078	0.082	0.055	0.066	0.093	0.095	0.072	0.100	0.092
Var(S)	0.285	0.256	0.297	0.249	0.235	0.210	0.216	0.215	0.208
2*Cov(F, S)	0.113	0.099	0.096	0.090	0.102	0.097	0.074	0.087	0.084
Var( $\varepsilon_{ij}$ )	0.524	0.563	0.551	0.595	0.570	0.598	0.639	0.598	0.616

Note: Var(F), Var(S), 2\*Cov(F, S) and Var( $\varepsilon_{ij}$ ) must sum to 1.

**Appendix 5a.** Descriptive Statistics (Student-Level) – Regions (NUTS II). Weighted Sample.

	2009				2018			
	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max
<b>Test Scores</b>								
<b>Reading</b>								
Alentejo	480.44	79.33	207.60	664.97	473.37	96.08	188.26	705.69
Center	492.44	82.39	171.53	688.09	500.91	92.10	232.93	742.10
Lisbon Metrop. Area	501.09	82.10	231.58	724.31	499.62	92.91	191.49	745.94
North	490.55	82.86	215.14	708.89	496.47	89.41	201.40	724.84
<b>Mathematics</b>								
Alentejo	475.02	82.88	242.18	735.48	474.71	90.62	260.29	713.47
Center	495.19	86.17	235.87	715.54	505.78	91.63	216.27	739.79
Lisbon Metrop. Area	492.01	87.96	213.20	755.81	494.50	92.53	199.00	742.68
North	490.34	85.57	226.06	752.93	499.35	87.72	226.78	742.60
<b>Science</b>								
Alentejo	482.16	76.06	259.08	672.36	474.54	90.06	234.08	694.16
Center	493.09	78.21	195.67	721.04	500.90	87.41	242.82	729.40
Lisbon Metrop. Area	505.13	77.94	247.15	711.15	495.66	89.51	146.34	739.59
North	495.09	78.30	252.18	729.24	496.45	85.26	243.74	731.50
<b>Family Variables</b>								
<b>HEDRES</b>								
Alentejo	0.20	1.07	-4.32	1.34	-0.06	0.88	-2.83	1.19
Center	0.24	1.03	-3.38	1.34	-0.10	0.86	-4.08	1.18
Lisbon Metrop. Area	0.24	1.05	-4.32	1.34	-0.17	0.90	-4.41	1.18
North	0.17	1.03	-4.32	1.34	-0.17	0.84	-4.41	1.18
<b>CULTPOSS</b>								
Alentejo	0.18	0.97	-1.29	1.35	0.05	0.85	-1.78	1.97
Center	0.14	0.97	-1.29	1.35	0.09	0.89	-1.78	1.98
Lisbon Metrop. Area	0.36	0.97	-1.29	1.35	0.05	0.84	-1.78	1.87
North	0.13	0.97	-1.29	1.35	-0.07	0.82	-1.78	1.87
<b>HISEI</b>								
Alentejo	43.33	15.92	16	90	49.44	23.45	11.01	88.70
Center	42.74	15.29	16.00	90.00	49.37	22.92	11.01	88.96

Lisbon Metrop. Area	47.97	17.45	16.00	90.00	55.27	22.71	11.01	88.96
North	43.39	15.39	16.00	90.00	45.80	22.68	13.24	88.96
<hr/>								
Nr of Students Alentejo	798				720			
Weight	5767				5607			
Nr of Students Center	1362				1230			
Weight	21036				18521			
Nr of Students Lisbon M. A.	1208				1080			
Weight	24210				24990			
Nr Students North	1792				2028			
Weight	36459				33577			

**Appendix 5b.** Descriptive Statistics (School-level) - Regions (NUTS II). Weighted Sample.

	2009				2018			
	Mean	S. d.	Min	Max	Mean	S. d.	Min	Max
<b>Family Variables</b>								
Average HEDRES								
Alentejo	0.05	0.34	-0.56	0.84	-0.09	0.29	-0.85	0.46
Center	0.15	0.30	-0.26	0.87	-0.26	0.33	-0.99	0.45
Lisbon Metrop. Area	-0.08	0.50	-0.98	0.73	-0.20	0.27	-0.71	0.47
North	0.06	0.29	-0.58	0.70	-0.34	0.38	-1.20	0.41
Average CULTPOSS								
Alentejo	0.13	0.33	-0.29	0.79	-0.06	0.30	-1.02	0.49
Center	0.01	0.30	-0.74	1.06	-0.09	0.32	-0.66	0.82
Lisbon Metrop. Area	0.27	0.42	-0.58	1.01	-0.04	0.29	-0.55	0.88
North	0.02	0.34	-0.56	1.06	-0.19	0.44	-1.10	0.77
Average HISEI								
Alentejo	38.41	6.91	26.43	59.69	43.81	12.38	22.10	68.31
Center	39.52	5.72	33.27	67.94	41.39	10.37	25.36	68.90
Lisbon Metrop. Area	43.65	12.63	23.67	71.59	52.05	12.89	26.87	77.32
North	40.34	8.28	27.90	68.26	40.91	13.23	23.48	77.84
<b>School Variables</b>								
SCHLSIZE								
Alentejo	410.19	262.44	88.00	1067.00	624.45	605.25	107.00	2802.00
Center	498.67	317.93	73.00	1616.00	837.43	940.84	153.00	8150.00
Lisbon Metrop. Area	676.03	416.61	124.00	2036.00	1099.90	849.36	134.00	4354.00
North	773.51	408.88	218.00	2157.00	845.26	721.27	55.00	3201.00
RTCOMP								
Alentejo	0.75	0.29	0.26	1.39	0.82	0.69	0.04	3.25
Center	0.61	0.30	0.21	1.67	0.55	0.39	0.07	1.70
Lisbon Metrop. Area	0.62	0.16	0.29	0.95	0.45	0.29	0.00	1.00
North	0.61	0.33	0.18	1.31	0.53	0.39	0.07	2.35

STRATIO								
Alentejo	7.51	1.89	3.97	10.90	9.80	3.20	1.26	15.97
Center	7.39	2.05	2.78	12.50	10.00	5.05	1.27	50.62
Lisbon Metrop. Area	8.38	3.38	3.81	15.91	11.67	4.16	1.24	20.80
North	9.02	3.23	2.41	16.88	10.13	3.77	1.03	18.50
SCMATEDU								
Alentejo	-0.12	0.72	-1.35	1.93	-	-	-	-
Center	-0.35	0.77	-2.27	1.93	-	-	-	-
Lisbon Metrop. Area	0.07	0.87	-1.75	1.93	-	-	-	-
North	-0.07	0.92	-1.35	1.93	-	-	-	-
EDUSHORT								
Alentejo	-	-	-	-	0.57	0.64	-1.42	1.69
Center	-	-	-	-	0.48	1.13	-1.42	2.96
Lisbon Metrop. Area	-	-	-	-	0.81	1.16	-0.69	2.96
North	-	-	-	-	0.35	0.95	-1.42	2.96
TCSHORT								
Alentejo	-0.82	0.51	-1.02	1.60	-	-	-	-
Center	-0.85	0.37	-1.02	0.43	-	-	-	-
Lisbon Metrop. Area	-0.75	0.62	-1.02	1.60	-	-	-	-
North	-0.88	0.33	-1.02	0.66	-	-	-	-
STAFFSHORT								
Alentejo	-	-	-	-	0.81	0.90	-0.59	2.49
Center	-	-	-	-	0.53	1.20	-1.46	4.04
Lisbon Metrop. Area	-	-	-	-	0.70	0.93	-1.46	2.87
North	-	-	-	-	0.31	1.12	-1.46	2.49
TEACHBEHA								
Alentejo	0.19	1.13	-1.40	2.12	0.89	1.12	-0.91	3.79
Center	0.07	0.73	-1.18	2.12	0.28	1.18	-2.04	1.90
Lisbon Metrop. Area	0.06	0.97	-1.62	2.12	0.29	0.98	-2.04	2.20
North	0.79	1.11	-2.25	2.12	-0.11	1.00	-2.04	1.81
STUDBEHA								
Alentejo	0.11	0.98	-1.46	1.67	0.97	1.06	-0.54	3.31
Center	0.06	0.75	-1.67	1.67	0.59	0.97	-1.73	2.57
Lisbon Metrop. Area	0.06	1.18	-2.12	2.36	0.41	0.72	-1.36	1.87
North	0.51	1.17	-1.46	2.36	0.13	1.02	-4.09	2.32
<hr/>								
Nr of Students Alentejo	27				32			
Weight	144				109			
Nr of Students Center	45				54			
Weight	363				261			
Nr of Students.Lisbon M. A.	41				50			
Weight	404				293			
Nr Students North	56				76			
Weight	477				379			

**Appendix 5c.** Regression of test scores on family variables and school dummies. Alentejo.

Variables	2009			2018		
	Reading	Maths	Science	Reading	Maths	Science
HEDRES	9.51*** (1.99)	14.32*** (2.15)	8.87*** (2.60)	3.57 (3.92)	6.76* (4.01)	6.33 (4.04)
CULTPOSS	11.83*** (4.03)	5.91* (3.46)	8.38** (4.03)	5.18 (3.47)	7.24* (3.93)	5.73 (3.64)
HISEI	1.16*** (0.18)	1.41*** (0.18)	1.37*** (0.22)	1.23*** (0.19)	1.22*** (0.20)	1.24*** (0.17)
Constant	366.77*** (12.30)	413.41*** (21.15)	425.67*** (18.60)	346.93*** (8.95)	336.58*** (15.34)	357.33*** (12.19)
N	798	798	798	720	720	720
R <sup>2</sup>	0.391	0.401	0.372	0.395	0.384	0.380

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; Variables described in table 1; Weighted sample.

**Appendix 5d.** Regression of school effects on family observable variables and school observable variables. Alentejo.

Variables	2009			2018		
	Reading	Maths	Science	Reading	Maths	Science
SCHLSIZE	0.03 (0.04)	-0.01 (0.06)	0.03 (0.04)	0.03 (0.02)	0.013 (0.02)	0.01 (0.02)
RTCOMP	-27.92 (16.81)	-8.36 (25.25)	-0.74 (20.00)	-6.59 (23.18)	-12.01 (18.52)	-15.07 (19.71)
STRATIO	5.35 (3.79)	2.29 (5.69)	3.86 (4.51)	-0.10 (4.57)	2.05 (3.66)	0.46 (3.89)
SCMATEDU	-9.62 (7.48)	-23.31* (11.23)	-4.01 (8.90)	-	-	-
EDUSHORT	-	-	-	-0.63 (19.75)	0.08 (15.80)	3.37 (16.81)
TCSHORT	-3.53 (8.86)	1.19 (13.31)	-10.38 (10.54)	-	-	-
STAFFSHORT	-	-	-	-6.84 (15.93)	-2.47 (12.73)	-8.43 (13.55)
TEACHBEHA	14.36** (6.11)	5.69 (9.18)	9.52 (7.27)	2.35 (13.05)	6.20 (10.43)	4.28 (11.10)
STUDBEHA	-3.49 (7.40)	3.84 (11.11)	13.44 (8.80)	-12.83 (15.09)	-17.98 (12.06)	-7.12 (12.83)
Average HEDRES	30.69 (19.18)	26.57 (28.81)	13.77 (22.81)	5.99 (61.61)	16.48 (49.24)	6.33 (52.40)
Average HISEI	-0.30 (1.19)	-0.21 (1.79)	-0.10 (1.41)	0.88 (2.008)	1.69 (1.61)	1.74 (1.71)
Average CULTPOSS	40.94* (22.77)	50.33 (34.21)	45.91 (27.09)	66.06 (76.96)	30.17 (61.51)	37.96 (65.45)
Constant	-49.93 (39.74)	-24.27 (59.70)	-64.62 (47.27)	-61.39 (100.55)	-102.13 (80.36)	-93.40 (85.52)
N	27	27	27	32	32	32
R <sup>2</sup>	0.795	0.579	0.704	0.447	0.579	0.502

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; School variables described in table 2; Family variables are the per school average of the variables described in table 1; Weighted sample.

**Appendix 5e.** Regression of test scores on family variables and school dummies. Center.

Variables	2009			2018		
	Reading	Maths	Science	Reading	Maths	Science
HEDRES	4.69* (2.67)	10.12*** (3.37)	4.79* (2.80)	-1.05 (3.77)	1.71 (3.99)	-1.85 (3.70)
CULTPOSS	15.78*** (2.64)	7.71*** (2.64)	12.68*** (2.35)	9.30*** (3.44)	9.06** (3.69)	8.28*** (2.69)
HISEI	0.67*** (0.18)	1.10*** (0.18)	1.03*** (0.18)	0.95*** (0.12)	1.11*** (0.14)	0.99*** (0.14)
Constant	416.24*** (10.08)	376.05*** (6.53)	414.69*** (8.26)	430.61*** (5.86)	434.28*** (12.09)	431.43*** (9.27)
N	1362	1362	1362	1230	1230	1230
R <sup>2</sup>	0.359	0.385	0.376	0.343	0.374	0.365

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; Variables described in table 1; Weighted sample.

**Appendix 5f.** Regression of school effects on family observable variables and school observable variables. Center.

Variables	2009			2018		
	Reading	Maths	Science	Reading	Maths	Science
SCHLSIZE	0.02 (0.03)	0.05** (0.02)	0.02 (0.02)	0.01* (0.007)	0.00 (0.01)	0.00 (0.01)
RTCOMP	-17.77 (27.70)	13.94 (25.12)	-26.48 (22.51)	35.97*** (13.27)	37.42*** (12.91)	28.66** (11.87)
STRATIO	0.37 (3.26)	-0.28 (2.95)	0.57 (2.65)	-1.63 (1.34)	0.27 (1.30)	-0.29 (1.20)
SCMATEDU	3.26 (8.09)	-3.80 (7.33)	3.00 (6.57)	-	-	-
EDUSHORT	-	-	-	-17.04*** (6.07)	-14.30** (5.90)	-11.66** (5.43)
TCSHORT	29.73 (18.68)	29.71* (16.94)	14.62 (15.18)	-	-	-
STAFFSHORT	-	-	-	19.19*** (6.51)	15.95** (6.33)	16.68*** (5.82)
TEACHBEHA	14.64 (9.60)	10.52 (8.71)	12.62 (7.80)	-4.26 (5.73)	0.75 (5.58)	3.12 (5.13)
STUDBEHA	-0.49 (10.04)	-2.42 (9.10)	-0.21 (8.16)	-20.81*** (5.85)	-17.96*** (5.69)	-18.45*** (5.23)
Average HEDRES	-15.39 (26.35)	-37.80 (23.89)	-0.91 (21.41)	23.37 (18.11)	7.28 (17.63)	26.78 (16.21)
Average HISEI	4.65** (1.87)	4.96*** (1.69)	4.30*** (1.52)	2.29*** (0.67)	2.56*** (0.66)	2.25*** (0.60)
Average CULTPOSS	-8.05 (30.67)	13.18 (27.81)	-20.30 (24.92)	-45.80** (20.24)	-15.27 (19.69)	-25.72 (18.11)
Constant	-181.52** (81.56)	-228.78*** (73.94)	-175.43** (66.28)	-121.40*** (32.92)	-144.26*** (32.04)	-121.33*** (29.46)
N	45	45	45	54	54	54
R <sup>2</sup>	0.463	0.611	0.523	0.660	0.655	0.703

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; School variables described in table 2; Family variables are the per school average of the variables described in table 1; Weighted sample.

**Appendix 5g.** Regression of test scores on family variables and school dummies. Lisbon Metropolitan Area.

Variables	2009			2018		
	Reading	Maths	Science	Reading	Maths	Science
HEDRES	4.80* (2.81)	8.55*** (3.17)	6.37** (2.7)	2.58 (3.11)	9.18*** (2.63)	5.00 (3.07)
CULTPOSS	15.09*** (2.88)	8.02** (3.13)	11.86*** (2.66)	13.68*** (4.26)	15.70*** (4.08)	13.59*** (4.15)
HISEI	0.89*** (0.15)	1.37*** (0.18)	1.07*** (0.18)	0.71*** (0.15)	0.71*** (0.12)	0.72*** (0.12)
Constant	487.29*** (13.27)	470.99*** (11.35)	483.54*** (10.19)	329.48*** (13.47)	339.11*** (22.24)	338.20*** (12.76)
N	1208	1208	1208	1080	1080	1080
R <sup>2</sup>	0.409	0.423	0.387	0.331	0.373	0.342

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; Variables described in table 1; Weighted sample.

**Appendix 5h.** Regression of school effects on family observable variables and school observable variables. Lisbon Metropolitan Area.

Variables	2009			2018		
	Reading	Maths	Science	Reading	Maths	Science
SCHLSIZE	0.01 (0.01)	0.01 (0.02)	0.01 (0.01)	0.01 (0.01)	0.01 (0.01)	0.01 (0.01)
RTCOMP	4.12 (27.27)	-52.00 (36.97)	26.70 (28.49)	56.64* (28.05)	47.28** (22.56)	43.55* (23.32)
STRATIO	0.88 (2.42)	-0.72 (3.27)	1.28 (2.52)	-3.63** (1.67)	-1.47 (1.34)	-1.64 (1.39)
SCMATEDU	7.50 (8.09)	6.55 (10.96)	3.88 (8.45)	-	-	-
EDUSHORT	-	-	-	-4.34 (5.38)	-4.26 (4.33)	-4.77 (4.47)
TCSHORT	-6.86 (7.67)	18.05* (10.40)	-2.56 (8.02)	-	-	-
STAFFSHORT	-	-	-	2.06 (10.32)	-0.13 (8.29)	-3.41 (8.58)
TEACHBEHA	-15.63 (10.64)	-16.88 (14.42)	-1.01 (11.11)	12.08 (9.90)	15.55* (7.96)	16.18* (8.23)
STUDBEHA	-2.42 (8.90)	12.37 (12.07)	-5.52 (9.30)	-30.97** (14.68)	-21.30* (11.81)	-18.86 (12.21)
Average HEDRES	23.64** (8.82)	44.02*** (11.95)	18.71* (9.21)	-118.98*** (36.93)	-64.70** (29.70)	-75.66** (30.70)
Average HISEI	4.27*** (0.86)	3.58*** (1.17)	2.98*** (0.90)	2.20** (1.04)	1.93** (0.83)	1.84** (0.86)
Average CULTPOSS	-42.29** (17.49)	-56.27** (23.71)	-31.08* (18.27)	54.50 (38.94)	56.30* (31.32)	56.73* (32.38)
Constant	-219.04*** (32.00)	-122.51*** (43.37)	-180.72*** (33.43)	-138.03** (60.95)	-131.74** (49.01)	-125.50** (50.67)
N	41	41	41	50	50	50
R <sup>2</sup>	0.868	0.746	0.780	0.605	0.711	0.673

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; School variables described in table 2; Family variables are the per school average of the variables described in table 1; Weighted sample.

**Appendix 5i.** Regression of test scores on family variables and school dummies. North.

Variables	2009			2018		
	Reading	Maths	Science	Reading	Maths	Science
HEDRES	3.95* (2.13)	7.86*** (2.18)	5.61*** (2.10)	-3.25 (2.90)	0.87 (3.16)	-0.65 (2.67)
CULTPOSS	8.63*** (1.86)	6.31** (2.53)	9.25*** (2.19)	12.37*** (3.21)	15.17*** (3.67)	12.74*** (3.14)
HISEI	0.90*** (0.13)	1.22*** (0.15)	1.09*** (0.14)	0.78*** (0.10)	0.87*** (0.11)	0.90*** (0.12)
Constant	383.51*** (8.04)	310.28*** (11.09)	362.53*** (6.86)	471.00*** (6.18)	470.62*** (9.26)	451.89*** (9.55)
N	1792	1792	1792	2028	2028	2028
R <sup>2</sup>	0.383	0.386	0.351	0.339	0.358	0.359

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; Variables described in table 1; Weighted sample.

**Appendix 5j.** Regression of school effects on family observable variables and school observable variables. North.

Variables	2009			2018		
	Reading	Maths	Science	Reading	Maths	Science
SCHLSIZE	0.00 (0.01)	-0.00 (0.01)	-0.00 (0.01)	0.02** (0.01)	0.02*** (0.01)	0.03*** (0.01)
RTCOMP	-24.76 (15.75)	-29.71* (16.63)	-23.66* (12.92)	15.50 (11.94)	17.76 (11.43)	24.77** (11.43)
STRATIO	-0.86 (1.74)	-0.55 (1.84)	-1.40 (1.43)	-0.53 (1.34)	0.10 (1.28)	0.01 (1.28)
SCMATEDU	4.62 (8.13)	12.45 (8.58)	6.33 (6.66)	-	-	-
EDUSHORT	-	-	-	-33.64*** (5.69)	-29.73*** (5.45)	-28.52*** (5.45)
TCSHORT	-4.96 (15.49)	-6.72 (16.35)	-6.11 (12.70)	-	-	-
STAFFSHORT	-	-	-	0.08 (6.51)	-0.86 (6.24)	-1.82 (6.24)
TEACHBEHA	-4.02 (5.44)	-5.00 (5.74)	0.91 (4.46)	6.53 (5.73)	1.99 (5.49)	3.79 (5.49)
STUDBEHA	12.69** (5.72)	6.73 (6.04)	5.97 (4.69)	7.45* (4.34)	9.39*** (4.15)	10.61** (4.15)
Average HEDRES	110.16*** (25.70)	50.32* (27.13)	69.47*** (21.07)	107.68*** (18.84)	100.16*** (18.04)	66.96*** (18.05)
Average HISEI	0.92 (0.88)	0.64 (0.93)	1.04 (0.72)	1.40** (0.69)	1.34** (0.66)	1.67** (0.66)
Average CULTPOSS	-20.01 (24.51)	14.93 (25.88)	-9.87 (20.10)	-47.89*** (17.90)	-30.67* (17.14)	-26.23 (17.15)
Constant	-50.95 (41.49)	-30.75 (43.80)	-43.54 (34.02)	-77.93** (38.54)	-91.55** (36.90)	-116.66*** (36.91)
N	56	56	56	76	76	76
R <sup>2</sup>	0.619	0.506	0.583	0.700	0.725	0.680

Standard errors in parentheses; \* p<0.1; \*\* p<0.05; \*\*\* p<0.01; School variables described in table 2; Family variables are the per school average of the variables described in table 1; Weighted sample.

**Appendix 5k. Variance Decomposition of test scores by region (NUTS II)**

	2009			2018		
	Reading	Maths	Science	Reading	Maths	Science
Regression of test scores on school family variables and school dummies						
<b>Alentejo</b>						
Var Total	6293.501	6869.486	5784.761	9231.766	8212.768	8102.732
Var( $F$ )	0.142	0.158	0.157	0.110	0.139	0.137
Var( $S$ )	0.175	0.185	0.167	0.218	0.184	0.182
2*Cov( $F, S$ )	0.102	0.097	0.088	0.087	0.101	0.091
Var( $\varepsilon_{ij}$ )	0.580	0.560	0.588	0.585	0.577	0.589
<b>Center</b>						
Var Total	6787.758	7425.966	6116.743	8483.177	8395.158	7640.866
Var( $F$ )	0.084	0.096	0.109	0.076	0.105	0.086
Var( $S$ )	0.227	0.234	0.209	0.194	0.196	0.207
2*Cov( $F, S$ )	0.073	0.095	0.098	0.092	0.112	0.104
Var( $\varepsilon_{ij}$ )	0.616	0.575	0.584	0.638	0.587	0.603
<b>Lisbon Metropolitan Area</b>						
Var Total	6739.769	7737.025	6074.892	8632.808	8561.351	8012.715
Var( $F$ )	0.113	0.132	0.135	0.064	0.093	0.077
Var( $S$ )	0.203	0.196	0.176	0.205	0.208	0.201
2*Cov( $F, S$ )	0.122	0.136	0.117	0.081	0.110	0.094
Var( $\varepsilon_{ij}$ )	0.562	0.537	0.573	0.650	0.589	0.628
<b>North</b>						
Var Total	6865.779	7321.734	6131.390	7994.469	7693.924	7268.911
Var( $F$ )	0.061	0.091	0.097	0.062	0.093	0.090
Var( $S$ )	0.262	0.241	0.200	0.240	0.233	0.226
2*Cov( $F, S$ )	0.087	0.094	0.091	0.058	0.071	0.075
Var( $\varepsilon_{ij}$ )	0.590	0.575	0.611	0.640	0.603	0.609

Note: Var( $F$ ), Var( $S$ ), 2\*Cov( $F, S$ ) and Var( $\varepsilon_{ij}$ ) must sum to 1.