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Modelling the objective function of managers in the presence of overlapping shareholding☆

Duarte Brito[a], Einer Elhauge[b], Ricardo Ribeiro[c,*], Helder Vasconcelos[d]

[a] Universidade Nova de Lisboa, Faculdade de Ciências e Tecnologia and NOVASBE, Portugal
[b] Harvard Law School, United States
[c] Universidade Católica Portuguesa, Católica Porto Business School and CEGE, Portugal
[d] Universidade do Porto, Faculdade de Economia and CEF.UP, Portugal; Compass Lexecon, Spain

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ABSTRACT

The objective function of managers in the presence of overlapping shareholding may differ from the traditional own-firm profit maximization, as they may internalize the externalities their strategies impose on other firms. The dominant formulation of the objective function in such cases has, however, been criticised for yielding counter-intuitive profit weights when the ownership of non-overlapping shareholders is highly dispersed. In this paper, we examine this issue. First, we make use of a probabilistic voting model (in which shareholders vote to elect the manager) to microfound an alternative formulation of the objective function of managers, which solves the above-mentioned criticism. Second, we apply the two formulations to the set of S&P 500 firms. We show that ownership dispersion of non-overlapping shareholders is, in fact, a relevant empirical issue, which may induce an over-quantification of the profit weights computed from the dominant formulation, particularly under a proportional control assumption.

1. Introduction

The assumption of own-firm profit maximization is key, (at least) since Fisher (1930)'s separation theorem, to most literature in corporate finance and industrial organization. However, the validity of this assumption has been recently questioned due to the increase, documented for a multitude of industries and economies, particularly since 2000, of overlapping shareholding (Azar et al., 2018; Newham et al., 2019; Backus et al., 2021b; Azar et al., 2022). The reason being that if firms impose externalities on one another, overlapping shareholding may imply a failure of the competitiveness condition, established by Hart (1979) to be essential for shareholders, regardless of their preferences, to unanimously agree on own-firm profit maximization. In order to see why, note, for example, that if firm A imposes a negative externality on firm B, a shareholder of

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* Corresponding author.
E-mail addresses: dmb@fct.unl.pt (D. Brito), elhauge@law.harvard.edu (E. Elhauge), rribeiro@ucp.pt (R. Ribeiro), hvasconcelos@fep.up.pt (H. Vasconcelos).

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firm A who also holds shares in firm B typically wants the manager of firm A to pursue a less aggressive strategy than the strategy desired by a shareholder with no holdings in firm B.

The managers of firms with overlapping shareholders, rather than maximizing own profit, may therefore weigh the eventual conflicting objectives of their shareholders. This implies that they may internalize (to some degree) the externalities their strategies impose on other firms (Rotemberg, 1984; Hansen and Lott, 1996). This internalization can alter the incentives to compete and can naturally impact market competition.[1,2]

In order to empirically examine the impact of overlapping ownership on market outcomes, we must quantify the above-mentioned induced internalization. To do so, the formulation of the objective function of managers is key.[3] This formulation is, however, non-trivial. In order to see why, consider, for example, that firm A has four shareholders, each holding 25% of the firm, and that one of those shareholders also holds 20% of firm B. If firm A imposes an externality on firm B, what would the mathematical formulation of the objective function of the manager of firm A be? What weight would the manager of firm A assign to the profit of firm B?

The dominant formulation of the objective function of managers in the presence of overlapping shareholders is due to O'Brien and Salop (2000). Incorporating features from both Rotemberg (1984) and Bresnahan and Salop (1986), they assume that *the manager of a firm with overlapping shareholders would decide the strategy of the firm to maximize a control-weighted sum of the expected returns of the firm's shareholders*. Azar (2012, 2016, 2017), Brito et al. (2018a) and Moskalev (2019) show that this formulation can be microfounded through a voting model in which shareholders vote to elect the manager from two potential candidates, an incumbent and a challenger, with conceivably differing strategy proposals to the firm. This formulation, although heavily used in the literature, has also been criticised for yielding counter-intuitive profit weights when the ownership of non-overlapping shareholders is highly dispersed (see, for example, Gramlich and Grundl, 2017, pages 9–13; O'Brien and Waehrer, 2017, pages 760–761; Crawford et al., 2018, supplementary material, pages 12–13).[4] The argument can be generalized as follows. No matter how small the ownership of overlapping shareholders (and the corporate control induced from their voting rights) is in a firm, as the dispersion of the ownership of non-overlapping shareholders increases, the weight assigned by the manager to the profit of other firms (in which overlapping shareholders hold shares in) tends to reflect *solely* the interests of the (non-dispersed) overlapping shareholders. In other words, no matter how small the ownership of overlapping shareholders is, the dominant formulation yields that the manager would weigh solely the interests of the overlapping shareholders whenever the remaining ownership of the firm becomes diffuse, even if such dispersion does not yield overlapping shareholders the full control of the firm.

In this paper, we take the first step to examine this issue, from both a theoretical and an empirical perspective. From a theoretical perspective, we make use of a probabilistic voting model (in the lines of Azar, 2012; Azar, 2016; Azar, 2017; Brito et al., 2018a; and Moskalev, 2019) to microfound an alternative formulation of the objective function of managers in the presence of overlapping shareholders. According to this alternative formulation, the manager of a firm with overlapping shareholderswould decide the strategy of the firm to maximize a control-weighted sum of the *relative* expected returns of the firm' s shareholders. And the weight assigned by the manager to the profit of other firms (in which overlapping shareholders hold shares in) will never reflect solely the interests of the (the non-dispersed) overlapping shareholders, unless the dispersion yields overlapping shareholders the full control of the firm. As such, it solves the above-mentioned criticism regarding the dominant formulation. From an empirical perspective, we apply the two formulations to the set of S&P 500

[1] Bresnahan and Salop (1986), Dietzenbacher et al. (2000), Shelegia and Spiegel (2012), and Brito et al. (2019), among others, show that the internalization induced by intra-industry overlapping ownership (i.e., among firms with horizontal relationships which, in partial equilibrium, are likely to impose a negative externality on each other) can directly lead to higher product prices and lower output levels. Azar and Vives (2021a) show that the internalization induced by inter-industry overlapping ownership (i.e., among firms in different industries which, in general equilibrium, are likely to impose a positive externality on each other) increases the incentive for firms to expand production, reducing relative prices in their industry relative to other industries.

[2] Although non-overlapping shareholders may favor a different firm-specific strategy, that does not mean they are harmed by overlapping shareholding because overlapping shareholding may, for example, reduce the competitiveness of rival firms, and non-overlapping shareholders benefit from a reduction of competition between the firm and its rivals (please see Schmalz, 2018 for a formal model).

[3] The literature proposes three different approaches for this quantification (see Backus et al. 2020 for a review). The first approach measures (from different perspectives, but atheoretically) the extent to which shareholders hold shares in more than one firm. Consider two firms A and B. Examples of the atheoretical measures used in the literature to examine the impact of overlapping ownership in firms A and B on market outcomes include (a) the number of overlapping shareholders in firms A and B; (b) the sum (across overlapping shareholders) of the minimum holdings (in firms A and B) of each overlapping shareholder; (c) the sum (across overlapping shareholders) of the holdings of each overlapping shareholder in firms A and B, weighted by the market capitalization of each firm; and (d) the sum (across overlapping shareholders) of the product of the holdings of each overlapping shareholder in firms A and B. These measures are not, however, microfounded from any theoretical model. In that sense, they are atheoretical measures (see Appendix B in Gilje et al. 2020 for a review). The second approach places additional structure and maps shareholders overlapping ownership into the weight that managers would assign to the profit of other firms in which their firm' s shareholders also hold shares. In other words, it maps shareholders overlapping ownership into the objective function of managers (see, for example, O'Brien and Salop, 2000; Azar, 2012; Azar, 2016; Azar, 2017; Brito et al., 2018a; Crawford et al., 2018; Moskalev, 2019; and Gilje et al., 2020). Finally, the third approach maps the objective function of managers into equilibrium market outcomes, yielding generalizations of the two most traditional indicators used to screen unilateral anti-competitive effects: the Herfindahl-Hirschman Index and the Gross Upward Price Pressure Index (see, for example, O'Brien and Salop, 2000; Azar et al., 2018; Brito et al., 2018a; and Azar et al., 2022). The structure placed by the second and third approaches is instrumental in deriving economically meaningful claims. And under both those approaches, the formulation of the objective function of managers is key.

[4] These authors illustrate the counter-intuitive profit weights with several examples, labelling them as "pathological scenarios", "questionable implications" or "counter-intuitive implications".

firms (in the lines of Backus et al., 2021b; and Amel-Zadeh et al., 2022) from 2003 to 2019. We show that the theoretical issue noted above is indeed of empirical relevance for the set of S&P 500 firms.

The remainder of the paper is organized as follows. Section 2 reviews the related literature and clarifies the contribution of the paper. Section 3 introduces the generalized probabilistic voting model used to derive the objective function of managers and the implied profit weights. Section 4 applies the profit weights established in Section 3 to the S&P 500 index constituents. Section 5 presents extensions to the generalized probabilistic voting model to account for shareholders inattention and cross-ownership structures. Section 6 concludes and discusses policy implications of the results.

2. Related literature and contribution

Azar (2012, 2016, 2017), Brito et al. (2018a) and Moskalev (2019) microfound the dominant formulation of the objective function of managers in the presence of overlapping shareholders through a voting model in which shareholders vote to elect the manager from two potential candidates, an incumbent and a challenger, with conceivably differing strategy proposals to the firm. Candidates are assumed to care about holding office.[5] In turn, shareholders are assumed to care about the returns that result from the different strategy proposals and to have an additive *profit-irrelevant* bias for (or against) the challenger.[6] Voting is probabilistic in the sense that the bias, while known to voters, is unobserved by candidates, who treat it as random. This microfoundation (a) is consistent with empirical evidence establishing that shareholders voting impacts the objective function of managers (Aggarwal et al., 2019) and (b) provides an endogenous measure of shareholders corporate control within the firm.[7]

In this paper, we examine the role of the profit-irrelevance assumption of the bias of shareholders on the derivation of the objective function of managers in the presence of overlapping shareholders. This assumption is borrowed from the electoral competition literature (Lindbeck and Weibull, 1987; Persson and Tabellini, 2000) and while in a political electoral setting it is reasonable to assume that voters have an ideological bias towards a candidate, in a corporate finance setting, however, it may be less so. Although one can argue that the assumption can be rooted on the fact that the credibility (or lack of credibility) of the incumbent, being already in office, is known to shareholders, while that of the challenger is not, one can also argue that if shareholders indeed care about the returns that result from the different strategy proposals (and, thereby, ultimately care about profits), there is not an obvious reason why the bias should not be profit-relevant. To examine the role of this assumption, we generalize the probabilistic voting model in the literature to allow the bias of shareholders for (or against) the challenger to be *profit-irrelevant* or *profit-relevant*, while maintaining the remaining assumptions of the literature.[8] We show that the profit-irrelevance assumption plays a key role on how ownership dispersion (of both overlapping and non-overlapping shareholders) is mapped into profit weights.[9]

Backus et al. (2021b) apply the dominant formulation to the set of S&P 500 firms from 1980 to 2017. To do so, they parse the SEC 13-F form fillings by institutional shareholders (with over $100 million in assets under management) and show the average weight assigned by the managers of S&P 500 firms to the profit of the remaining S&P 500 firms has increased from 0.2 in 1980 to almost 0.7 in 2017. Amel-Zadeh et al. (2022) parse not only the SEC 13-F form fillings, but also five additional SEC form filings, accounting for holdings not only of institutional shareholders, but also of corporate insiders and blockholders. They apply the dominant formulation to the set of S&P 500 firms from 2003 to 2019 and show that once we account for the holdings of corporate insiders and blockholders, the average weight assigned by the managers of S&P 500 firms to the profit of the remaining S&P 500 firms is, in fact, lower.

In this paper, we examine the role of ownership dispersion (of both overlapping and non-overlapping shareholders) on profit weights. To do so, we make use of Amel-Zadeh et al. (2022)'s ownership data to apply the two formulations to the set of S&P 500 firms. We show that the dispersion of shareholders' ownership, particularly of non-overlapping shareholders, is a relevant empirical issue in the S&P 500 and that, in those cases, the dominant formulation may, in fact, over-quantify profit weights, particularly under a proportional control assumption. This issue is particularly relevant because if the dominant formulation is over-quantifying the magnitude of overlapping ownership when the ownership of non-overlapping shareholders is highly-dispersed, then empirical studies that use that dominant formulation will suffer from attenuation bias that will tend to create empirical results that understate the magnitude and statistical significance of the marginal effect of

[5] Azar (2012, 2017) considers the case in which candidates choose strategy proposals to maximize their *vote share* while Azar (2016), Brito et al. (2018a) and Moskalev (2019) consider the case in which candidates choose strategy proposals to maximize their *expected utility from corporate office*.

[6] Azar (2012, 2016, 2017) and Brito et al. (2018a) consider the case in which this bias is *independent* (and identically) distributed across shareholders while Moskalev (2019) considers the case in which the bias can be *correlated* across shareholders.

[7] Azar (2012, 2017) shows that the corporate control of shareholders can be microfounded to be endogenously measured by their voting rights (proportional control) while Azar (2016), Brito et al. (2018a) and Moskalev (2019) show it can be microfounded to be endogenously measured by the normalized Banzhaf power indices that result from their voting rights.

[8] We allow candidates to choose strategy proposals to maximize their *vote share* or their *expected utility from corporate office* and the bias of shareholders for (or against) the challenger to be *correlated* or *non-correlated* across shareholders.

[9] The assumptions regarding the objective function of candidates and the non-correlation or correlation of the bias of shareholders for (or against) the challenger impact solely the (endogenous) measure of the control rights of shareholders (computed from their voting rights). As such, their influence on how ownership dispersion (of both overlapping and non-overlapping shareholders) is mapped into profit weights, is indirect (via the influence of ownership dispersion on the (endogenous) measure of control rights).

overlapping ownership on market competition. This could cause analysts to incorrectly reject or underestimate an empirical connection between overlapping shareholding and anticompetitive effects (Elhauge, 2020).

3. Theoretical framework

This section introduces the generalized probabilistic voting model used to derive the objective function of managers. The general setting combines features from Azar (2012, 2016, 2017), Brito et al. (2014), Brito et al. (2018a) and Moskalev (2019).

3.1. Setup

There are K shareholders, indexed by $k \in \Theta \equiv \{1, \ldots, K\}$, and N firms, which impose externalities on one another, indexed by $j \in \Im \equiv \{1, \ldots, N\}$, whose total stock is composed of voting stock and non-voting (preferred) stock. Both stock give the holder the right to a share of the firm's profits, but only the former gives the holder the right to vote in the firm's shareholder assembly. The holdings $\phi_{kj} \in [0, 1]$ of total stock of shareholder k in firm j, regardless of whether it be voting or non-voting stock, capture her *financial rights* to the firm's profits. The holdings $\upsilon_{kj} \in [0, 1]$ of voting stock of shareholder k in firm j, capture her *voting rights* in the firm. These voting rights may not necessarily coincide with her *control rights* in the firm, $\gamma_{kj} \in [0, 1]$, which refer to her rights to influence the decisions of firm j, to be discussed below.[10]

The ownership structure of the different firms is such that a subset of shareholders can hold general financial and voting rights in multiple firms.[11] This overlapping shareholding can induce a conflict in the firm-specific interests of shareholders, which managers must weigh.

Finally, the profit of each of the different firms is assumed to be a function not only of the strategies of all the firms but also of a state of nature. This implies that firms' profits and, consequently, shareholders' returns - because they are a function of the profits of the firms in which they hold financial rights - are random.

3.2. Voting model

We follow Azar (2012, 2016, 2017), Brito et al. (2018a) and Moskalev (2019) in microfounding the objective function of managers through a voting model in which shareholders of each firm j vote at the firm's shareholder assembly to elect the manager from two potential candidates, an incumbent a_j and a challenger b_j, with the candidate receiving the majority of voting rights being elected manager of the firm.

Shareholders and candidates are assumed to play the following two-stage game. In the first stage, candidates to all firms, who are assumed to be opportunistic in the sense their only motivation is to hold office, compete for the voting rights of shareholders by - simultaneously and noncooperatively - proposing a strategy for their firm, which is assumed binding in line with the literature on electoral competition (Downs, 1957; Lindbeck and Weibull, 1987; Polo, 1998; Persson and Tabellini, 2000). Let $x_{a_j} \in \Omega_j$ and $x_{b_j} \in \Omega_j$ denote the strategy proposals of the incumbent and the challenger to firm j, respectively, where Ω_j denotes the strategy space available to the candidates, which can refer to any decision variable(s) - e.g., quantity, price, R&D investment, etc. - of firm j. In the second stage of the game, the shareholder assemblies of all firms are simultaneously held and shareholders vote to elect the manager of each firm. Let $m_j \equiv \left\{a_j, b_j\right\}$ denote the identity of the manager elected to firm j.

Naturally, this voting model constitutes a reduced form model of the decision making process and the knowledge structure within the firm. The manager may not be elected directly by shareholders and operational decision variable(s) may be often decided, not by top managers, but by middle managers, who may not know the extent of the holdings of the firm's shareholders in other firms. Antón et al. (2022) show that, even in those cases, managerial incentives can serve as a mechanism (which requires no communication or coordination between the different players) that links overlapping ownership with operational decision variable(s).

3.2.1. Shareholders voting

We begin by addressing the equilibrium regarding the voting behavior of shareholders. Shareholders are assumed to care about the utility derived from their expected returns and, as such, vote - simultaneously and noncooperatively - for the candidate whose strategy proposal maximizes their expected returns, randomizing between the two in case of indifference.

We consider that the utility $u_k(\mathbf{x}, \mathbf{m})$ of each shareholder k to be a function of her expectation regarding the return from her financial rights, which will be a function of the winning strategy proposals in all the firms $\mathbf{x} = \left(x_1, \ldots, x_j, \ldots, x_N\right)^\top$ and the identity of the corresponding elected managers $\mathbf{m} = \left(m_1, \ldots, m_j, \ldots, m_N\right)^\top$:

$$u_k(\mathbf{x}, \mathbf{m}) = \mathbb{E}_k(R_k(\mathbf{x}, \mathbf{m})),$$

[10] Short-sales are not allowed and so financial, voting and control rights are non-negative.

[11] We assume that shareholders are external in the sense that firms do not hold financial and voting rights in other firms. We extend the framework to internal shareholders in Section 5.

where $\mathbb{E}_k(R_k(\mathbf{x}, \mathbf{m}))$ denotes the expectation of shareholder k regarding her return $R_k(\mathbf{x}, \mathbf{m})$. We model this expectation to be the sum of two components: a common component and (b) an additive shareholder-specific expectation bias, as follows:

$$\mathbb{E}_k(R_k(\mathbf{x}, \mathbf{m})) = \mathbb{E}(R_k(\mathbf{x})) + B_k(\mathbf{m}),$$

where $\mathbb{E}(R_k(\mathbf{x})) = \sum_{j\in\Im} \left(\phi_{kj}\mathbb{E}\left(\Pi_j(\mathbf{x})\right)\right)$ denotes the common component, rooted on a common expectation $\mathbb{E}\left(\Pi_j(\mathbf{x})\right)$, across shareholders, regarding the profits $\Pi_j(\mathbf{x})$ of each firm j, assumed to be publicly generated by, for example, the documentation distributed and discussed in the shareholder assemblies of the different firms, and $B_k(\mathbf{m})$ denotes the expectation bias of shareholder k for (or against) the challengers of (potentially) all firms.

Azar (2012, 2016, 2017), Brito et al. (2018a) and Moskalev (2019) associate the expectation bias $B_k(\mathbf{m})$ to a profit-irrelevant differentiation of candidates. This assumption is borrowed from the electoral competition literature (Lindbeck and Weibull, 1987; Persson and Tabellini, 2000). However, while in a political electoral setting it is reasonable to assume that voters have an idealogical bias towards a candidate, in a corporate finance setting, it may be less so. Although one can argue that the assumption can be rooted on the fact that the credibility (or lack of credibility) of the incumbent, being already in office, is known to shareholders, while that of the challenger is not, one can also argue that if shareholders indeed care about the returns that result from the different strategy proposals (and, thereby, ultimately care about profits), there is not an obvious reason why the bias should not be profit-relevant.

We contribute to the literature by considering a more general formulation than the one in the extant literature which allows the expectation bias to be *profit-irrelevant* or *profit-relevant.* A profit-relevant bias can, for example, be rooted (a) on a difference of expectation of shareholders regarding the competence or the cost to exert effort of the two candidates, with an impact on the firm's profit (as in Gomes, 2000; and Goshen and Levit, 2020), or (b) on a difference of expectation of shareholders regarding the (dis)loyalty of the two candidates to shareholders, who may divert, once elected, resources from the firm for personal use, with an impact on the firm's profits (as in Bebchuk and Jolls, 1999; Pagano and Immordino, 2012; Amess et al., 2015; Noe et al., 2015; and Goshen and Levit, 2020), or (c) on a difference of expectation of shareholders regarding the effectiveness of the firm's governance mechanism in deterring illicit managerial diversion and enforce its reimbursement, with an impact on the firm's profit (as in Desai et al., 2007; Pagano and Immordino, 2012; Amess et al., 2015; Noe et al., 2015; and Li and Li, 2018). As such, the expectation bias of shareholder k for (or against) the challengers of (potentially) each firm is given by:

$$B_k(\mathbf{m}) = \sum_{j\in\Im} \left(\lambda\phi_{kj}1\left(m_j = b_j\right)\xi_{kg} + (1-\lambda)1\left(m_j = b_j\right)\xi_{kj}\right) = \sum_{j\in\Im} \left(1-\lambda+\lambda\phi_{kj}\right)1\left(m_j = b_j\right)\xi_{kj},$$

where $\xi_{kj} \lessgtr 0$ denotes the expectation bias of shareholder k for (or against) the challenger of firm j and $1(m_j = b_j)$ denotes a dummy variable that takes the value 1 if the challenger is elected manager of firm j. $\lambda \in \{0, 1\}$ controls the profit relevance of the bias. When $\lambda = 0$, we have that $B_k(\mathbf{m}) = \sum_{j\in\Im} 1\left(m_j = b_j\right)\xi_{kj}$, which implies that:

$$\mathbb{E}_k(R_k(\mathbf{x}, \mathbf{m})) = \sum\nolimits_{j\in\Im} \left(\phi_{kj}\mathbb{E}\left(\Pi_j(\mathbf{x})\right) + 1\left(m_j = b_j\right)\xi_{kj}\right),$$

and, as a consequence, that the bias is profit-irrelevant, as in Azar (2012, 2016, 2017), Brito et al. (2018a) and Moskalev (2019). When $\lambda = 1$, we have that $B_k(\mathbf{m}) = \sum_{j\in\Im} \phi_{kj}1\left(m_j = b_j\right)\xi_{kj}$, which implies that:

$$\mathbb{E}_k(R_k(\mathbf{x}, \mathbf{m})) = \sum\nolimits_{j\in\Im} \left(\phi_{kj}\left(\mathbb{E}\left(\Pi_j(\mathbf{x})\right) + 1\left(m_j = b_g\right)\xi_{kj}\right)\right),$$

and, as a consequence, that the bias is profit-relevant (as it impacts the shareholder's specific expectation regarding the firm's profit).[12]

We assume that shareholders vote, in each firm's shareholder assembly, for the candidate whose strategy proposal, given their bias, maximizes their utilities, randomizing between the two in case of indifference. Following Alesina and Rosenthal (1995), Azar (2012, 2016) and Brito et al. (2018a), we also assume the following regarding this voting behavior.

Assumption 1. Shareholders are conditionally sincere.

Assumption 1 implies that the vote of shareholders is, conditional on the equilibrium strategy proposals of the candidates to the remaining firms, deterministic, as follows: shareholder k will vote for firm j's incumbent with probability 1 if $u_k(\mathbf{x}_a, \mathbf{m}_a) > u_k(\mathbf{x}_b, \mathbf{m}_b)$, will vote for firm j's challenger with probability 1 if instead $u_k(\mathbf{x}_a, \mathbf{m}_a) < u_k(\mathbf{x}_b, \mathbf{m}_b)$, and will randomize between the two candidates with equal probability if $u_k(\mathbf{x}_a, \mathbf{m}_a) = u_k(\mathbf{x}_b, \mathbf{m}_b)$, where $\mathbf{x}_a = \left(x_1, \ldots, x_{aj}, \ldots, x_N\right)^\top$, $\mathbf{x}_b = \left(x_1, \ldots, x_{bj}, \ldots, x_N\right)^\top$, $\mathbf{m}_a = \left(m_1, \ldots, a_j, \ldots, m_N\right)^\top$ and $\mathbf{m}_b = \left(m_1, \ldots, b_j, \ldots, m_N\right)^\top$ condition on the equilibrium strategy proposals of the candidates to the remaining firms. Assumption 1 is presented for simplicity. It can be relaxed in line with Moskalev (2019) by considering an equilibrium refinement that excludes weakly dominated strategies of shareholders.

[12] The discreteness of λ does not impact the results. They remain virtually unchanged even if we allow $\lambda \in [0, 1]$ so that the bias is a weighted average of the two elements.

3.2.2. Candidates strategy proposals

Having described the second stage of the game, we now address the first stage, in which candidates simultaneously choose strategy proposals. To do so, we follow Lindbeck and Weibull (1987), Azar (2012, 2016, 2017), Brito et al. (2018a) and Moskalev (2019) in assuming that the bias of shareholders for (or against) the challenger, while known to shareholders, is unobserved by candidates, who treat it as a random utility shock. As a consequence, voting by shareholders is, from the perspective of candidates, probabilistic.

Azar (2012, 2016, 2017) and Brito et al. (2018a) assume that candidates consider the biases of shareholders to be independent (and identically) distributed across firms and shareholders according to a symmetric probability distribution with mean zero and cumulative distribution. However, there is not an obvious reason why they should be drawn separately for each shareholder of the same firm. If shareholders indeed care about the utility derived from their returns (and, thereby, ultimately about profits), and information about the profits of a firm is, to some extent, common across shareholders of the firm, then biases may be *correlated* across shareholders (Moskalev, 2019). To do so, we consider the bias ξ_{kj} of each shareholder k for (or against) the challenger of each firm j can be disaggregated, following the literature on electoral competition, into two independently drawn components (Lindbeck and Weibull, 1987; Persson and Tabellini, 2000; Ponzetto, 2011; Matějka and Tabellini, 2021):[13]

$$\xi_{kj} = \tilde{\xi}_j + \tilde{\xi}_{kj},$$

where $\tilde{\xi}_j$ denotes a component common to all shareholders of firm j, which induces a correlation among the biases of all shareholders of the firm, and $\tilde{\xi}_{kj}$ denotes a component specific to shareholder k and firm j. $\tilde{\xi}_j$ is independently drawn across firms from a distribution with density function $h_j(\cdot)$, cumulative distribution function $H_j(\cdot)$ and support in the interval $[-\psi_j/2, \psi_j/2]$, while $\tilde{\xi}_{kj}$ is independently drawn across shareholders and firms from a distribution with density function $g_j(\cdot)$, cumulative distribution function $G_j(\cdot)$ and support in the interval $[-\tau_j/2, \tau_j/2]$, where $\psi_j \geq 0$ and $\tau_j > 0$. We allow $\psi_j \geq 0$ to control the correlation of the biases of all shareholders in a firm. When $\psi_j = 0$, we have that $\xi_{kj} = \tilde{\xi}_{kj}$, which implies that there is no common component and the biases are independently distributed across firms and shareholders, as in Azar (2012, 2016, 2017) and Brito et al. (2018a). In turn, when $\psi_j > 0$, the biases combine the two components and, thereby, exhibit correlation among the shareholders of the firm.[14]

We consider that candidates choose their strategy proposals under two alternative assumptions (Azar, 2012; Azar, 2016; Azar, 2017; Brito et al., 2018a; Moskalev, 2019):[15]

Assumption 2. Candidates choose strategy proposals to maximize their vote share.

Assumption 3. Candidates choose strategy proposals to maximize their expected utility from corporate office.

We begin by addressing the choice of strategy proposals by candidates under Assumption 2. In this setting, the incumbent chooses x_{a_j} to solve:

$$\max_{x_{a_j}} \sum\nolimits_{k\in\Theta_j} Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)\upsilon_{kj},$$

while the challenger chooses x_{b_j} so to solve:

$$\max_{x_{b_j}} \sum\nolimits_{k\in\Theta_j} Pr_{kb_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)\upsilon_{kj},$$

where $Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)$ and $Pr_{kb_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)$ denote the probability that, in the candidates perspective, shareholder k votes for the incumbent and the challenger of firm j, respectively. Because $Pr_{kb_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b) = 1 - Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)$, it is straightforward to see that the solution to the maximization problem of the two candidates to firm j is symmetric. As such, we characterize - for simplicity of exposition - solely the incumbent's problem. To do so, we must derive $Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)$. Using the law of total probability, we can write $Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)$ as follows:

$$Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b) = \int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j} Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j\right)h\left(\tilde{\xi}_j\right)d\tilde{\xi}_j,$$

[13] This two-component structure is presented for simplicity. It can be relaxed in line with Moskalev (2019) by considering the bias of each shareholder of firm j to be a shareholder-specific weighted sum of M_j common biases.

[14] We do not allow $\tau_j = 0$, i.e, that $\xi_{kj} = \tilde{\xi}_j$, which would imply that the biases are independently distributed across firms, but perfectly correlated across all shareholders of each firm, so to rule out, as it will become apparent below, corner solutions for the voting (and election) probabilities.

[15] We are implicitly assuming that candidates do not derive any direct utility from the strategy proposal because, as established by Azar (2020), doing so breaks down the equivalence, when shareholders are fully diversified across firms, between the equilibrium in monopoly and oligopoly settings.

where $Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j\right)$ denotes the probability that, in the candidates perspective, shareholder k votes for the incumbent of firm j conditional on the common component of the bias $\tilde{\xi}_j$, which, in turn, is given by:

$$\begin{aligned} Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j\right) &= Pr(u_k(\mathbf{x}_a, \mathbf{m}_a) > u_k(\mathbf{x}_b, \mathbf{m}_b)) \\ &= Pr\Big(\mathbb{E}(R_k(\mathbf{x}_a)) > \mathbb{E}(R_k(\mathbf{x}_b)) + \big(1-\lambda+\lambda\phi_{kj}\big)\xi_{kj}\Big) \\ &= Pr\Big(\mathbb{E}(R_k(\mathbf{x}_a)) > \mathbb{E}(R_k(\mathbf{x}_b)) + \big(1-\lambda+\lambda\phi_{kj}\big)\big(\tilde{\xi}_j + \tilde{\xi}_{kj}\big)\Big) \\ &= Pr\left(\tilde{\xi}_{kj} < \frac{\mathbb{E}(R_k(\mathbf{x}_a)) - \mathbb{E}(R_k(\mathbf{x}_b))}{\left(1-\lambda+\lambda\phi_{kj}\right)} - \tilde{\xi}_j\right) \\ &= G_j\left(\frac{\mathbb{E}(R_k(\mathbf{x}_a)) - \mathbb{E}(R_k(\mathbf{x}_b))}{\left(1-\lambda+\lambda\phi_{kj}\right)} - \tilde{\xi}_j\right), \end{aligned}$$

where the second equality makes use of the fact that the biases for (or against) the challenger of other firms, $\sum_{g\in\Im, g\neq j}\left(\left(1-\lambda+\lambda\phi_{kg}\right)1\left(m_g = b_g\right)\xi_{kg}\right)$, enter the utility obtained from both candidates.

We now address the choice of strategy proposals by candidates under Assumption 3. In this setting, the incumbent chooses x_{a_j} so to solve:

$$\max_{x_{a_j}} Pr\Big(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\Big)\,\Xi_{a_j},$$

while the challenger chooses x_{b_j} so to solve:

$$\max_{x_{b_j}} Pr\Big(m_j = b_j | \mathbf{x}_a, \mathbf{x}_b\Big)\,\Xi_{b_j},$$

where $Pr(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b)$ and $Pr(m_j = b_j | \mathbf{x}_a, \mathbf{x}_b)$ denote the probability that the incumbent and the challenger, respectively, are elected while Ξ_{a_j} and Ξ_{b_j} denote the utility that the incumbent and the challenger, respectively, expect to accrue conditional upon being elected. Again, because $Pr(m_j = b_j | \mathbf{x}_a, \mathbf{x}_b) = 1 - Pr(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b)$, it is straightforward to see that the solution to the maximization problem of the two candidates to firm j is symmetric. As such, we characterize - for simplicity of exposition - solely the incumbent's problem. To do so, we must derive $Pr(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b)$. Let ℓ_j denote the number of shareholders with voting rights in firm j, $\wp_j$ denote all the $2^{\ell_j - 1}$ possible subsets of those shareholders that can award the majority of votes to a candidate and $\Theta^\iota_j \in \wp_j$ denote a particular subset of those shareholders. Given that the election of the incumbent is ensured with the votes of the shareholders in each subset in $\wp_j$, we have that $Pr(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b)$ just sums the probabilities with which she is elected by each subset Θ^ι_j, $Pr(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j)$, as follows:

$$Pr\Big(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\Big) = \sum_{\Theta^\iota_j \in \wp_j} Pr\Big(m_j = a_j \Big| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j\Big) = \sum_{\Theta^\iota_j \in \wp_j} \int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j} Pr\Big(m_j = a_j \Big| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\Big) h\Big(\tilde{\xi}_j\Big) d\tilde{\xi}_j,$$

where the last equality uses the law of total probability to write $Pr\Big(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j\Big)$ in terms of $Pr\Big(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\Big)$, which conditions on the common component of the bias $\tilde{\xi}_j$. Further, given that conditional on $\tilde{\xi}_j$, the shareholders-specific biases $\tilde{\xi}_{kj}$ for $k \in \Theta_j$ are independently distributed, we can write $Pr\Big(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\Big)$ as the product of the voting probabilities of the corresponding shareholders, as follows:

$$Pr\Big(m_j = a_j \Big| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\Big) = \prod_{k\in\Theta^\iota_j} Pr_{ka_j}\Big(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j\Big) \prod_{k\notin\Theta^\iota_j}\Big(1 - Pr_{ka_j}\Big(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j\Big)\Big).$$

3.2.3. Nash-equilibrium

Having described the maximization problem of the candidates, we now address the pure-strategy Nash equilibrium for the candidates strategy proposals' game. To do so, we follow Azar (2012, 2016, 2017) and Brito et al. (2018a) in making the following technical assumptions regarding the strategy space Ω_j available to the candidates of each firm j, the common expectation $\mathbb{E}(R_k(\mathbf{x}))$ of the return of each shareholder k, and the cumulative distribution functions $H_j(\cdot)$ and $G_j(\cdot)$.

Assumption 4. The strategy space Ω_j available to the candidates of each firm j is a nonempty compact subset of $\Re$.

Assumption 5. The common expectation $\mathbb{E}(R_k(\mathbf{x}))$ of the return of each shareholder k is (a) continuous and twice differentiable in $\mathbf{x}$, with continuous second derivatives; and (b) strictly concave in firm j's strategy $x_j \in \Big\{x_{a_j}, x_{b_j}\Big\}$, conditional on the strategies of the remaining firms.

Assumption 6. $H_j(\cdot)$ is the cumulative distribution function of an uniform distribution over the range $[-\psi_{j}/2, \psi_{j}/2]$ for each firm j.

Assumption 7. $G_j(\cdot)$ is the cumulative distribution function of an uniform distribution over the range $[-\tau_j/2, \tau_j/2]$, with τ_j sufficiently large for each firm j.

Assumptions 5, 6 and 7 can ensure that the objective function of candidates is strictly concave conditional on the strategy proposals of other candidates (to the firm and to other firms).[16] As strategy proposals are, under Assumption 4, defined in a convex set, this implies that the maximization problem of candidates has, conditional on the strategy proposals of other candidates, a unique maximum. Given the symmetry of the solution to the maximization problem of the two candidates to the firm, we have that they will choose best-response functions that are, conditional on the strategy proposals of the candidates to the remaining firms, symmetric with respect to the strategy proposal of the opponent candidate. This implies that the two candidates will choose the same strategy proposal for the firm, conditional on the strategies proposals of the candidates to the other firms, i.e., they will choose the same best-response function to the strategy proposals of the candidates to the other firms. Proposition 1 below characterizes the pure-strategy Nash equilibrium for the candidates strategy proposals' game $(x_{a_1}, x_{b_1}, \ldots x_{a_j}, x_{b_j}, \ldots, x_{a_N}, x_{b_N})$.

Proposition 1. *There exists a pure-strategy Nash equilibrium for the candidates strategy proposals' game that is entirely equivalent to the pure-strategy Nash-equilibrium in the case in which each candidate maximizes the following objective function:*

$$\max_{x_j} \sum\nolimits_{k\in\Theta_j} \gamma_{kj} \frac{\mathbb{E}(R_k(\mathbf{x}))}{1-\lambda+\lambda\phi_{kj}} \propto \max_{x_j} \sum\nolimits_{g\in\Im} w_{jg}\mathbb{E}(\Pi_g(\mathbf{x})),$$

where Θ_j denotes the subset of shareholders that hold financial rights in firm j and w_{jg} denotes the (normalized) weight that candidates assign to the expected profit of firm g for any $j, g \in \Im$:

$$w_{jg} = \frac{\sum_{k\in\Theta_j} \frac{\gamma_{kj}\phi_{kg}}{1-\lambda+\lambda\phi_{kj}}}{\sum_{k\in\Theta_j} \frac{\gamma_{kj}\phi_{kj}}{1-\lambda+\lambda\phi_{kj}}} \geq 0.$$

Under Assumptions1, 2, 4, 5 and 7, γ_{kj} is measured by the voting rights of shareholder k in firm j: $\gamma_{kj} = \upsilon_{kj}$. Under Assumptions 1,3, 4, 5, 6 and 7, γ_{kj} is measured by the following power index of shareholder kin firm j: $\gamma_{kj} = \beta_{kj}/\sum_{h\in\Theta_j}\beta_{hj}$, with β_{kj} given by:

$$\beta_{kj} = \sum\nolimits_{\Theta_j^\iota \in \wp_{kj}^p} \sum_{i=0}^{\#\Theta_j^\iota - 1} (-1)^i C_i^{\#\Theta_j^\iota - 1} m_{\ell_j - \#\Theta_j^\iota + i},$$

where $\wp_{kj}^p$ denotes the subsets of Θ_j that can award victory to a candidate of firm j in which shareholder k enters and is pivotal, $\#\Theta_j^\iota$ denotes the number of shareholders in Θ_j^ι, $C_i^{\#\Theta_j^\iota - 1}$ denotes the number of combinations of i elements taken from a set with $\#\Theta_j^\iota - 1$ elements, and $m_{\ell_j - \#\Theta_j^\iota + i}$ denotes the $\left(\ell_j - \#\Theta_j^\iota + i\right)$th raw moment of an uniformly distributed variable over the range $\left[1/2 - \psi_j/2\tau_j, 1/2 + \psi_j/2\tau_j\right]$, as follows:

$$m_{\ell_j - \#\Theta_j^\iota + i} = \frac{\sum_{n=0}^{\ell_j - \#\Theta_j^\iota + i} \left(\frac{1}{2} - \frac{\psi_j}{2\tau_j}\right)^n \left(\frac{1}{2} + \frac{\psi_j}{2\tau_j}\right)^{\ell_j - \#\Theta_j^\iota + i - n}}{\ell_j - \#\Theta_j^\iota + i + 1}.$$

Proof. See Appendix. □

Proposition 1 establishes that the two candidates would choose the same strategy proposal for each firm j, conditional on the strategies of the candidates to the remaining firms. In particular, they would choose the strategy proposal of each firm j to maximize a weighted sum of the expected profits of (potentially) all the firms. We now address the mathematical properties and the empirical applicability of this (unique) objective function of managers. Corollary 1 establishes the mathematical properties.

Corollary 1. *The objective function of managers established in Proposition 1 satisfies the following properties:*

(i) *Absent overlapping shareholding, managers do not weigh the expected profit of other firms.*
(ii) *In the presence of non-infinitesimal overlapping shareholding, managers weigh the interests of overlapping shareholders by assigning a positive weight to the expected profit of other firms (when those overlapping shareholders have control rights in the firm and financial rights in both firms).*

[16] Assumption 7 requires the support of $G_j(\cdot)$ to be sufficiently large (relative to their argument - not to profits) so to rule out corner solutions for probabilities $\Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j)$. This technical assumption is standard and explicit in the electoral competition literature (Persson and Tabellini, 2000; Ponzetto, 2011; Matějka and Tabellini, 2021) so that the behavior underlined by probabilities $\Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j)$ is not perfectly predictable on the basis of strategy proposals. It is also standard in the literature that microfounds the dominant formulation, implicitly in Azar (2012, 2016, 2017) and Brito et al. (2018a) while explicitly in Moskalev (2019).

(iii) *The weight that managers assign to the expected profit of other firms is continuous on the financial and control rights of the shareholders that have financial rights in the firm.*
(iv) *The weight that managers assign to the expected profit of other firms is one when all the shareholders that have financial rights in the firm are fully diversified across firms.*
(v) *If the bias of shareholders for (or against) the challenger is profit-relevant ($\lambda = 1$), managers will not weigh - solely - the interests of overlapping (non-overlapping) shareholders when those shareholders do not have full control, even if the ownership of each non-overlapping (overlapping) shareholder is dispersed among a collection of infinitesimal identical shareholders.*

Proof. See Appendix. □

The properties established in Corollary 1 constitute, in our view, attractive and intuitive properties for the objective function of managers. Property (*i*) implies that in the absence of overlapping shareholding, managers decide the strategy of their firm to maximize own expected profit. Property (*ii*) implies that in the presence of non-infinitesimal overlapping shareholding, managers internalize the impact of their firm's strategy on the expected profit of other firms. Property (*iii*) implies that the mathematical formulation of the weight that managers assign to the expected profit of other firms imposes no artificial ownership cut-offs. Property (*iv*) implies that when shareholders are fully diversified across firms, managers decide the strategy of their firm to maximize the sum of the expected profits. Finally, property (*v*) implies that the weight that managers assign to the expected profit of other firms will only reflect the interests of overlapping (non-overlapping) shareholders when those shareholders do have full control.

Properties (*i*) to (*iv*) are satisfied independently of the assumptions regarding the objective function of candidates and the bias of shareholders for (or against) the challenger. In contrast, property (*v*) is satisfied solely if the bias of shareholders for (or against) the challenger is profit-relevant ($\lambda = 1$).[17,18,19] In order to illustrate why, we now describe the objective function of the manager (and the implied profit weights) under the two assumptions. When the bias is profit-irrelevant ($\lambda = 0$), as in Azar (2012, 2016, 2017), Brito et al. (2018a) and Moskalev (2019), Proposition 1 establishes that managers would decide the strategy of the firm to maximize a *control-weighted sum of the expected returns of the firm's shareholders*, as follows:

$$\max_{x_j} \sum_{k\in\Theta_j} \gamma_{kj}\mathbb{E}(R_k(\mathbf{x})) \propto \max_{x_j} \sum_{g\in\Im} w_{jg}\mathbb{E}(\Pi_g(\mathbf{x})),$$

where the (normalized) weight that they assign to the expected profit of firm g for any $j, g \in \Im$ is given by:

$$w_{jg} = \frac{\sum_{k\in\Theta_j} \gamma_{kj}\phi_{kg}}{\sum_{k\in\Theta_j} \gamma_{kj}\phi_{kj}} \geq 0,$$

which then replicates the dominant formulation. Dividing the subset of shareholders Θ_j that hold financial rights in firm j in two smaller subsets: the subset of overlapping shareholders Θ_j^o and the subset of non-overlapping shareholders Θ_j^{no}, we can rewrite this profit weight as:

$$w_{jg} = \frac{\sum_{k\in\Theta_j^o} \gamma_{kj}\phi_{kg}}{\sum_{k\in\Theta_j^{no}} \gamma_{kj}\phi_{kj} + \sum_{k\in\Theta_j^o} \gamma_{kj}\phi_{kj}},$$

which makes clear, as discussed by Gramlich and Grundl (2017), O'Brien and Waehrer (2017) and Crawford et al. (2018), that it fails property (*v*). When the ownership of each non-overlapping shareholder becomes dispersed among a collection of infinitesimal identical shareholders that is equally large in aggregate, w_{jg} tends to reflect solely the interests of the (the non-dispersed) overlapping shareholders as $\sum_{k\in\Theta_j^{no}} \gamma_{kj}\phi_{kj} \to 0$. In this case, the candidates would weigh solely the interests of the overlapping shareholders: $w_{jg} \to \sum_{k\in\Theta_j^o} \gamma_{kj}\phi_{kg} / \sum_{k\in\Theta_j^o} \gamma_{kj}\phi_{kj}$, even when their voting rights do not induce full control of the firm. Similarly, when the ownership of each overlapping shareholder becomes dispersed among a collection of infinitesimal identical shareholders that is equally large in aggregate, w_{jg} tends to reflect solely the interests of the non-overlapping shareholders as $\sum_{k\in\Theta_j^o} \gamma_{kj}\phi_{kj} \to 0$ and $\sum_{k\in\Theta_j^o} \gamma_{kj}\phi_{kg} \to 0$. In this case, the candidates would weigh solely the interests of non-overlapping shareholders: $w_{jg} \to 0$, even when their voting rights do not induce full control of the firm.

[17] This implies that the assumption regarding the profit-irrelevance or profit-relevance of the bias of shareholders for (or against) the challenger is the only key assumption with a direct impact on how ownership dispersion (of both overlapping and non-overlapping shareholders) is mapped into profit weights. It does not imply, however, that the assumptions regarding the non-correlation or correlation of the bias of shareholders for (or against) the challenger does not have such an impact. They do, but it is indirect via the impact of ownership dispersion on the (endogenous) measure of control rights.

[18] If we allow $\lambda \in [0, 1]$ so that the bias is a weighted average of the two elements, this result remains valid, with property (*v*) failing for $\lambda < 1$.

[19] A profit-relevant bias is also instrumental in obtaining a formulation that is invariant to the distribution of ownership among non-overlapping shareholders, which may constitute an important empirical advantage.

In turn, when the bias is profit-relevant ($\lambda = 1$), Proposition 1 establishes that managers would decide the strategy of the firm to maximize a *control-weighted sum of the relative expected returns of the firm's shareholders*, as follows:[20]

$$\max_{x_j} \sum\nolimits_{k\in\Theta_j} \gamma_{kj} \frac{\mathbb{E}(R_k(\mathbf{x}))}{\phi_{kj}} = \max_{x_j} \sum\nolimits_{g\in\Im} w_{jg}\mathbb{E}(\Pi_g(\mathbf{x})),$$

where $\mathbb{E}(R_k(\mathbf{x}))/\phi_{kj}$ denotes the *relative* expected return of shareholder k and the (normalized) weight that they assign to the expected profit of firm g for any $j, g \in \Im$ is given by:

$$w_{jg} = \sum\nolimits_{k\in\Theta_j} \gamma_{kj} (\phi_{kg}/\phi_{kj}) \geq 0,$$

which, as ϕ_{kg}/ϕ_{kj} captures the *bliss point* of shareholder k regarding the expected profit of firm g (i.e., the weight that shareholder k would individually want candidates to assign to the expected profit of firm g), constitutes a *control-weighted sum of the bliss points of the firm's shareholders* regarding the expected profit of firm g. Because a dispersion of the ownership of each non-overlapping (overlapping) shareholder to a collection of infinitesimal identical shareholders that is equally large in aggregate does not impact shareholders bliss points, in order for candidates to weigh solely the interests of overlapping (non-overlapping) shareholders, their voting rights must induce full control of the firm. As a consequence, this alternative formulation copes better with ownership dispersion than the dominant formulation.

The two formulations of the objective function of managers established in Proposition 1 can be used in empirical quantifications of the impact of overlapping ownership on market outcomes, which, for example, (a) incorporate w_{jg} directly into a structural model as, for example, in Brito et al. (2014), Kennedy et al. (2017), Brito et al. (2018b), Park and Seo (2019), Backus et al. (2021a), and Azar and Ribeiro (2022); or (b) incorporate w_{jg} into equilibrium market outcomes as, for example, in Azar et al. (2018), Kennedy et al. (2017), and Azar et al. (2022). Corollary 2 examines the assumptions required to compute the elements of this objective function from observed data on the financial and voting rights of shareholders.

Corollary 2. *The elements of the objective function of managers established in Proposition 1 can be computed solely from the financial and voting rights of shareholders if:*

(i) *Candidates choose strategy proposals to maximize their vote share (Assumption 2).*
(ii) *Candidates choose strategy proposals to maximize their expected utility from corporate office (Assumption 3) and the bias of shareholders for (or against) the challenger does not exhibit correlation among the shareholders of the firm ($\psi_j = 0$).*

The computation of the elements of the objective function of managers established in Proposition 1 requires the financial rights and the control rights of shareholders to be measurable. The financial rights have clear empirical counterparts and can thus be measurable. However, the same is not necessarily true for the (endogenous) control rights of shareholders (which refer to their rights to influence the decisions of the firm). Proposition 1 establishes that the measure of control rights can (but not necessarily) depend on the assumptions regarding the objective function of candidates and the non-correlation or correlation of the bias of shareholders for (or against) the challenger.

When candidates choose strategy proposals to maximize their vote share, the control rights of shareholders are always endogenously measured by their voting rights, which corresponds to proportional control. As such, the elements of the objective function of managers established in Proposition 1 can be computed from observed data on the financial and voting rights of shareholders, as established in Corollary 2. Independently of whether there is no correlation (i.e., when there is no common component: $\psi_j = 0$) or there is correlation (i.e., when there is a common component: $\psi_j > 0$) in the bias.[21] The former, non-correlation case, replicates the results in Azar (2012, 2016, 2017). This measure of control rights has two unappealing properties: (a) it does not converge to 100% as the voting rights of a shareholder approach 50%; and (b) it does not depend on the voting rights of the firm's all other shareholders.[22] Nevertheless, it has been widely used in empirical applications (see, for example, Brito et al., 2014; Azar et al., 2018; Newham et al., 2019; Backus et al. (2021a,b); Azar et al., 2022; and Azar and Ribeiro, 2022).

When candidates choose strategy proposals to maximize their expected utility from corporate office, the endogenous measure of the control rights of shareholders depends on the assumption regarding the non-correlation or correlation of the bias of shareholders for (or against) the challenger. If there is no correlation in the bias (i.e., when there is no common component: $\psi_j = 0$), the control rights of shareholders are endogenously measured by their normalized Banzhaf (1965)'s power indices, which capture the fraction of coalitions for which a shareholder is pivotal. As such, the elements of the objective function of managers established in Proposition 1 can be computed from observed data on the financial and voting rights of shareholders, as established in Corollary 2. In order to see why, in this case, the control rights of shareholders are

[20] This microfounds (a version of) the objective function in Crawford et al. (2018), who - to deal with the counter-intuitive profit weights implied by the dominant formulation - suggest an ad-hoc formulation for the objective function of managers in which managers, as we derive here, maximize a control-weighted sum of the relative expected returns of the firm's shareholders, but where the *sum* of the weights that the manager of each firm j assigns to the expected profit of the remaining firms is normalized to one (and not, as we derive here, the *own profit weight*).

[21] Further, no assumption regarding the distribution and support of the common component of the bias is required to derive this result.

[22] As we may expect a shareholder who holds 10% of the voting rights in a firm to have effective control if each of the remaining shareholders hold a tiny amount of the firm's voting rights.

endogenously measured by their normalized Banzhaf (1965)'s power indices, note that we can rewrite the control rights of shareholder k in firm j established in Proposition 1 as follows:

$$
\begin{aligned}
\gamma_{kj} &= \frac{\sum_{\Theta^1_j \in \wp^p_{kj}} \sum_{i=0}^{\#\Theta^1_j-1} (-1)^i C_i^{\#\Theta^1_j-1} \left(\tfrac{1}{2}\right)^{\ell_j-\#\Theta^1_j+i}}{\sum_{h\in\Theta_j} \sum_{\Theta^1_j \in \wp^p_{hj}} \sum_{i=0}^{\#\Theta^1_j-1} (-1)^i C_i^{\#\Theta^1_j-1} \left(\tfrac{1}{2}\right)^{\ell_j-\#\Theta^1_j+i}} \\
&= \frac{\sum_{\Theta^1_j \in \wp^p_{kj}} \left(\tfrac{1}{2}\right)^{\ell_j-\#\Theta^1_j} \sum_{i=0}^{\#\Theta^1_j-1} C_i^{\#\Theta^1_j-1} \left(-\tfrac{1}{2}\right)^i}{\sum_{h\in\Theta_j} \sum_{\Theta^1_j \in \wp^p_{hj}} \left(\tfrac{1}{2}\right)^{\ell_j-\#\Theta^1_j} \sum_{i=0}^{\#\Theta^1_j-1} C_i^{\#\Theta^1_j-1} \left(-\tfrac{1}{2}\right)^i} \\
&= \frac{\sum_{\Theta^1_j \in \wp^p_{kj}} \left(\tfrac{1}{2}\right)^{\ell_j-\#\Theta^1_j} \left(1-\tfrac{1}{2}\right)^{\#\Theta^m_j-1}}{\sum_{h\in\Theta_j} \sum_{\Theta^1_j \in \wp^p_{hj}} \left(\tfrac{1}{2}\right)^{\ell_j-\#\Theta^1_j} \left(1-\tfrac{1}{2}\right)^{\#\Theta^m_j-1}} \\
&= \frac{\sum_{\Theta^1_j \in \wp^p_{kj}} \left(\tfrac{1}{2}\right)^{\ell_j-1}}{\sum_{h\in\Theta_j} \sum_{\Theta^1_j \in \wp^p_{hj}} \left(\tfrac{1}{2}\right)^{\ell_j-1}} \\
&= \frac{\lambda^p_{kj}}{\sum_{h\in\Theta_j} \lambda^p_{hj}},
\end{aligned}
$$

where the first equality makes use of the fact that $m_{\ell_j-\#\Theta^1_j+i} = \left(\tfrac{1}{2}\right)^{\ell_j-\#\Theta^1_j+i}$ under $\psi_j = 0$, the second equality makes use of the binomial theorem, which establishes that $(1-1/2)^{\#\Theta^m_j-1} = \sum_{i=0}^{\#\Theta^1_j-1} C_i^{\#\Theta^1_j-1} (-1/2)^i$, and λ^p_{kj} denotes the number of subsets of Θ_j that can award victory to a candidate of firm j in which shareholder k enters and is pivotal. This replicates the results in Azar (2016), Brito et al. (2018a) and Moskalev (2019). This measure of control rights addresses the two unappealing properties described above: (a) it does converge to 100% as the voting rights of a shareholder approach 50%; and (b) it does depend on the voting rights of all the firm's shareholders. Nevertheless, it has been relatively less used in empirical applications (Brito et al., 2018b; Brito et al., 2018a; Azar and Vives, 2021b). If, on the other hand, there is correlation in the bias (induced by a common component: i.e., $\psi_j > 0$), the endogenous measure of the control rights of shareholders requires a quantification of the correlated voting behavior of shareholders, captured by the ratio ψ_j/τ_j in $m_{\ell_j-\#\Theta^1_j+i}$. As such, the elements of the objective function of managers established in Proposition 1 can not be computed solely from observed data on the financial and voting rights of shareholders, as established in Corollary 2. The full characterization of the measure of the control rights of shareholders in this case and the definition of strategies to empirically estimate ψ_j/τ_j from shareholder observed *voting behavior* seem to be very interesting avenues for future research.

4. Overlapping ownership in the S&P 500

This section empirically applies the profit weights established in Proposition 1 to the S&P 500 index constituents, from 2003:Q3 to 2019:Q4, in the lines of Backus et al. (2021b) and Amel-Zadeh et al. (2022). We do so assuming there is no correlation in the bias (i.e., when there is no common component: $\psi_j = 0$) of shareholders for (or against) the challenger, so that the profit weights can be computed from observed data on the financial and voting rights of shareholders.

4.1. Data description

We combine data from multiple sources. First, we use historical quarterly ownership data, scraped (and made publicly available) by Amel-Zadeh et al. (2022) from all ownership reports required by the Securities Exchange Commission (SEC) for investors in US public corporations.[23] Specifically, Amel-Zadeh et al. (2022) parse not only the SEC 13-F form fillings (by institutional shareholders with over $100 million in assets under management), but also five additional SEC form filings: 3, 4 and 5 (by insiders and shareholders owning more than 10% of an equity share class), 13-D (by investors acquiring more than 5% of an equity share class) and 13-G (by investors acquiring more than 5% of an equity share class that are exempt, qualified institutional or passive). The data includes, for each quarter, the fraction of shares outstanding owned by institutional shareholders, corporate insiders as well as active and passive blockholders (identified by CIK) on S&P 500 firms (identified by PERMNO/CUSIP).[24,25] Second, we use historical annual horizontal relatedness scores from Hoberg and Phillips (2010, 2016). These scores are obtained by parsing the text in the business descriptions of the original SEC 10-K filings to form, for each year, vectors of the words associated to the product descriptions of each firm. These word vectors are then used to compute symmetric continuous measures of product similarity for each pair of firms in each year, capturing how similar

[23] The data is publicly available upon request from Amir Amel-Zadeh, Fiona Kasperk, and Martin Schmalz.

[24] We use this data instead of competing databases made available by Thomson Reuters or Backus et al. (2021b) because it includes ownership by institutional investors, corporate insiders as well as active and passive blockholders.

[25] We lightly cleaned the data by dropping, in each quarter, the firms in which investors, in aggregate, report holding more than 100% of shares outstanding.

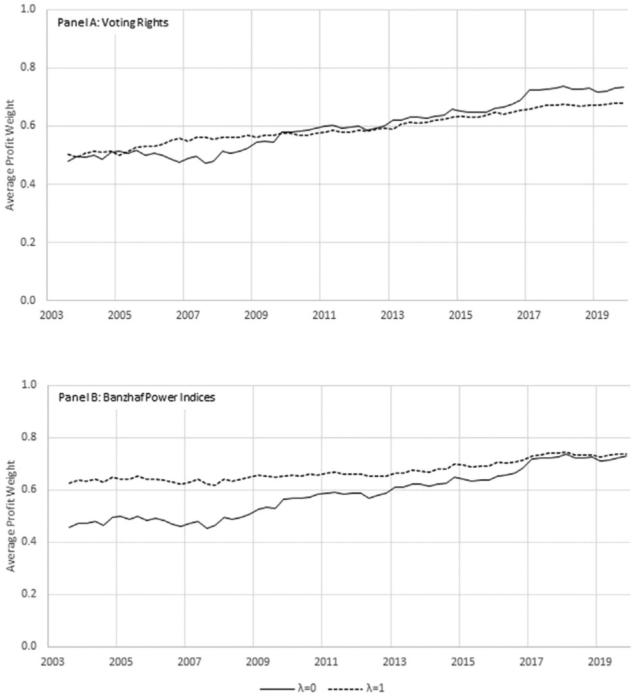

Fig. 1. S&P 500 intra-industry profit weights. Arithmetic average profit weight of all cross-pairs of S&P 500 firms with a strictly positive horizontal relatedness score (from Hoberg and Phillips, 2010; Hoberg and Phillips, 2016) in each quarter. The profit weights are computed according to the objective function of managers established in Proposition 1 for $\lambda = 0$ and $\lambda = 1$. Panel A considers the case in which the control rights of shareholders are measured by their voting rights (which are assumed coincident with their financial rights). Panel B considers the case in which the control rights of shareholders are measured by the Banzhaf power indices that result from their voting rights. In all cases, the retail share of each firm is assumed to be made up of an infinity of shareholders each holding infinitesimal financial and voting rights.

the product (descriptions) of two firms (identified by GVKEY) in a given year are. We use the scores from the TNIC-3 classification data which is constructed to ensure that the likelihood of two randomly drawn firms (from the CRSP/COMPUSTAT universe) being deemed related matches that of three digit SIC codes. Finally, using the Compustat Merged Database linking table, we link the PERMNO and GVKEY firm identifiers and merge Hoberg and Phillips (2010, 2016)'s relatedness score, for each year and firm, with Amel-Zadeh et al. (2022)'s ownership data.[26]

4.2. Profit weights

We use the data above to compute the weights that the managers of S&P 500 firms assign to the expected profit of each of the remaining S&P 500 firms, according to the objective function of managers established in Proposition 1 for both $\lambda = 0$ and $\lambda = 1$. The retail shareholders (not accounted for in SEC forms 3, 4, 5 13-D and 13-G) are not observed in the data. As such, in computing profit weights, we assume, following the practice in the literature that the (remaining) retail share of each firm is made up of an infinity of atomist shareholders.[27,28]

4.2.1. Intra-industry analysis

We begin by examining the pairs of firms in the same industry, which absent overlapping ownership are, in partial equilibrium, likely to impose a negative externality on one another. To do so, we focus on the pairs of firms that have a strictly positive horizontal relatedness score, as computed by Hoberg and Phillips (2010, 2016). Fig. 1 depicts the (arithmetic) average profit weight of all *intra-industry* cross-pairs of S&P 500 firms in each quarter according to the objective function of

[26] Firm-pairs with horizontal relatedness scores that are, in a given year, below the thresholds required to be included in the TNIC-3 classification data are assumed unrelated.

[27] The profit weight that managers assign to the expected profit of other firms according to the objective function of managers established in Proposition 1 when $\lambda = 1$ *and* the control rights of shareholders are measured by their voting rights, depends on the financial interest that overlapping shareholders as a group hold in other firms. As such, for this particular case, we assume also that retail shareholders are undiversified (and, as such, do not engage in overlapping ownership). For all the remaining cases, this assumption is innocuous.

[28] Instead of assuming that the (remaining) retail share of each firm is made up of an infinity of atomist shareholders (and that for the case when $\lambda = 1$ *and* the control rights of shareholders are measured by their voting rights, also that retail shareholders are undiversified and, as such, do not engage in overlapping ownership), we could have assumed that the (remaining) retail shareholders are inattentive and, as such, not accounted for by the manager.

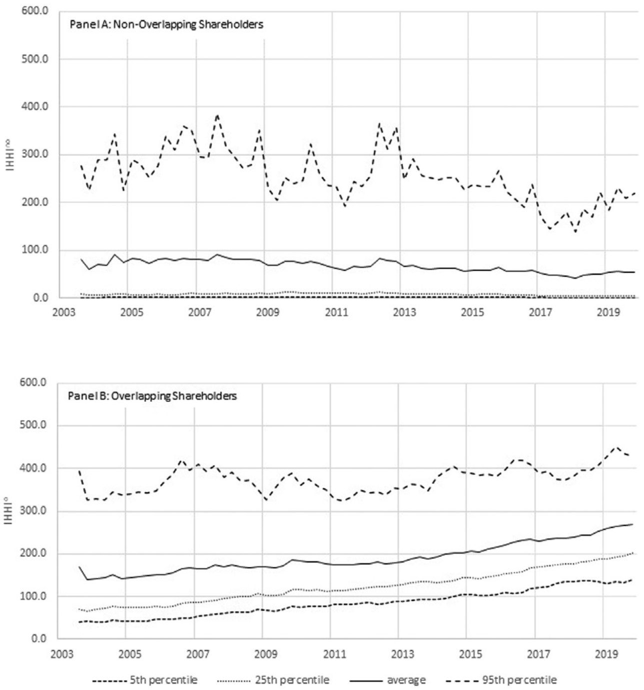

Fig. 2. Intra-industry shareholder concentration in the S&P 500. Arithmetic average and quantiles of the concentration of the financial rights, measured as 10,000 x component of the IHHI associated to non-overlapping (Panel A) and overlapping shareholders (Panel B) across the S&P 500 firms with a strictly positive horizontal relatedness score (from Hoberg and Phillips, 2010; Hoberg and Phillips, 2016) in each quarter.

managers established in Proposition 1 for both $\lambda = 0$ and $\lambda = 1$. Fig. 1, Panel A considers the case in which the control rights of shareholders are measured by their voting rights (which are assumed, also following the practice in the literature, to be coincident with their financial rights). The plot shows that the two formulations yield very similar average profit weights: the overall (across all quarters) average profit weight is 0.59 both under $\lambda = 0$ and $\lambda = 1$. The plot also shows that the two formulations yield very similar trends over time: the average profit weight has increased from 0.48 in 2003:Q3 to 0.73 in 2019:Q4 under $\lambda = 0$ and from 0.50 in 2003:Q3 to 0.68 in 2019:Q4 under $\lambda = 1$.

Fig. 1, Panel B considers the case in which the control rights of shareholders are measured by the normalized Banzhaf power indices that result from their voting rights.[29] The plot shows that the two formulations yield now slightly different average profit weights: the overall average profit weight is 0.58 under $\lambda = 0$, but 0.67 under $\lambda = 1$. The plot also shows that the two formulations yield slightly different trends over time: the average profit weight has increased from 0.46 in 2003:Q3 to 0.73 in 2019:Q4 under $\lambda = 0$, but from 0.63 in 2003:Q3 to 0.74 in 2019:Q4 under $\lambda = 1$. Finally, the plots also suggest that the difference between the two formulations has been decreasing over time: the average profit weight in 2019:Q4 is 0.73 under $\lambda = 0$ and 0.74 under $\lambda = 1$.

In order to examine whether shareholder ownership dispersion is an issue for the set of intra-industry S&P 500 firms, we apply a (common) measure of concentration, the Herfindahl-Hirschman index, to shareholder's financial rights, which following Backus et al. (2021b) and Amel-Zadeh et al. (2022) we label IHHI. We do so for each S&P 500 firm j under analysis and quarter, decomposing the IHHI on the components associated to the ownership of non-overlapping and overlapping shareholders, as follows:

$$IHHI_j = \sum_{k \in \Theta_j} \phi_{kj}^2 = \sum_{k \in \Theta_j^{no}} \phi_{kj}^2 + \sum_{k \in \Theta_j^{o}} \phi_{kj}^2 = IHHI_j^{no} + IHHI_j^{o}.$$

Fig. 2 depicts the (arithmetic) average and different quantiles of the components of the IHHI (multiplied by 10,000) over time associated to the ownership of non-overlapping shareholders (Panel A) and overlapping shareholders (Panel B). The plots show that shareholder ownership dispersion is a relevant empirical issue for the set of intra-industry S&P 500 firms, particularly for non-overlapping shareholders. The financial rights of non-overlapping shareholders are, in fact, heavily dispersed. The overall 5th and 25th percentiles are 1.22 and 7.39, respectively. The overall average is 67.96. Even the overall 95th percentile is solely 256.34. Further, this dispersion has increased slightly over time.

In order to examine the impact of the ownership dispersion of non-overlapping shareholders on the profit weight computed according to the objective function of managers established in Proposition 1 for $\lambda = 0$, which corresponds to the

[29] To do so, we compute, following Dubey and Shapley (1979), the normalized Banzhaf power indices using the set of observed shareholders.

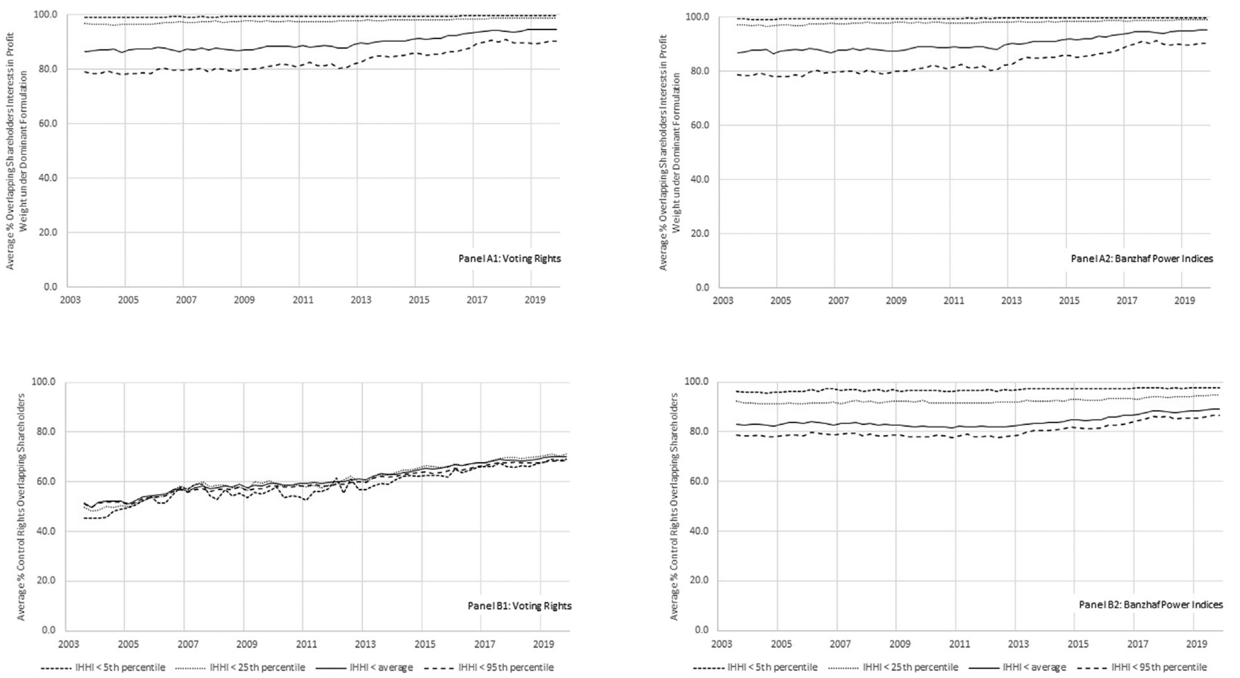

Fig. 3. Overlapping shareholders interests and control rights in S&P 500 inter-industry profit weights. Panels A1 and A2 depict the arithmetic average of $100(w_{jg}/w^o_{jg})$ across the subset of intra-industry S&P 500 firms with a strictly positive horizontal relatedness score (from Hoberg and Phillips, 2010; Hoberg and Phillips, 2016) for which the component of the IHHI associated to the ownership non-overlapping shareholders is below the average or different quantiles. Panels B1 and B2 depict the arithmetic average of $100\sum_{k\in\Theta^o_j}\gamma_{kj}$, the aggregate control rights of overlapping shareholders across the same subset of S&P 500 firms. Panels A1 and B1 consider the case in which the control rights of shareholders are measured by their voting rights (which are assumed coincident with their financial rights). Panels A2 and B2 consider the case in which the control rights of shareholders are measured by the Banzhaf power indices that result from their voting rights. In all cases, the retail share of each firm is assumed to be made up of an infinity of shareholders each holding infinitesimal financial rights.

dominant formulation, we quantify the fraction of the interests of overlapping shareholders that is reflected in the profit weight of each firm pair jg and quarter, as follows:

$$\frac{w_{jg}}{w^o_{jg}} = \frac{\frac{\sum_{k\in\Theta^o_j}\gamma_{kj}\phi_{kg}}{\sum_{k\in\Theta^{no}_j}\gamma_{kj}\phi_{kj}+\sum_{k\in\Theta^o_j}\gamma_{kj}\phi_{kj}}}{\frac{\sum_{k\in\Theta^o_j}\gamma_{kj}\phi_{kg}}{\sum_{k\in\Theta^o_j}\gamma_{kj}\phi_{kj}}},$$

where the numerator denotes the actual profit weight according to the dominant formulation while the denominator denotes the profit weight that would reflect solely the interests of overlapping shareholders. As w_{jg}/w^o_{jg} increases, the higher is the fraction of the interests of overlapping shareholders that is reflected in the actual profit weight.

Fig. 3, Panels A1 and A2 depict the (arithmetic) average of the above ratio (multiplied by 100) across the subset of intra-industry S&P 500 firms for which the component of the IHHI associated to the ownership of non-overlapping shareholders is below the average or different quantiles. Fig. 3, Panels B1 and B2 depict the (arithmetic) average aggregate corporate control of overlapping shareholders (multiplied by 100) across the same subset of S&P 500 firms. Fig. 3, Panels A1 and B1 consider the case in which the control rights of shareholders are measured by their voting rights. The plots show that the dominant formulation may reflect heavily the interests of overlapping shareholders even when those overlapping shareholders do not have full control. For the subset of firms for which the component of the IHHI associated to the ownership of non-overlapping shareholders is below the 5th and 25th percentiles, the overall average profit weight reflects 99.46% and 97.79%, respectively, of the interests of overlapping shareholders, which seems counter-intuitive as, on aggregate, those overlapping shareholders only hold, on average, 58.48% and 60.94%, respectively, of the control rights.[30] If we consider, instead, the subset of firms for which the component of the IHHI associated to the ownership of non-overlapping shareholders is below the average, the overall average profit weight reflects 89.57% of the interests of overlapping shareholders, although,

[30] Note that Fig. 3, Panels A1 and B1 consider the case in which the control rights of shareholders are measured by their voting rights, a measure of control rights that does not converge to 100% as the voting rights of a shareholder approach 50%. As such, when on aggregate, overlapping shareholders hold, on average, 58.48% or 60.94% (as described above) of the control rights, they do not have full control of the firm, even if pooled into a single shareholder.

on aggregate, those overlapping shareholders, again, only hold, on average, 60.66% of the control rights. And even if we consider, instead, the subset of firms for which the component of the IHHI associated to the ownership of non-overlapping shareholders is below the 95th percentile, the overall average profit weight reflects 82.91% of the interests of overlapping shareholders, although, on aggregate, those overlapping shareholders, again, only hold, on average, 59.44% of the control rights. This implies that, when control rights of shareholders are measured by their voting rights, the interests of overlapping shareholders reflected in the dominant formulation may be heavily dissociated from their aggregate corporate control. Moreover, this dissociation seems stronger when the ownership of non-overlapping shareholders is more dispersed.[31]

Fig. 3, Panels A2 and B2 consider the case in which the control rights of shareholders are measured by the normalized Banzhaf power indices that result from their voting rights. The plots show that the interests of overlapping shareholders reflected in the dominant formulation have, in this case, a stronger association to their aggregate corporate control. For the subset of firms for which the component of the IHHI associated to the ownership of non-overlapping shareholders is below the 5th and 25th percentiles, the overall average profit weight reflects 99.61% and 98.11%, respectively, of the interests of overlapping shareholders, which does not seem counter-intuitive as, on aggregate, those overlapping shareholders hold, on average, 96.92% and 92.50%, respectively, of the control rights. If we consider, instead, the subset of firms for which the component of the IHHI associated to the ownership of non-overlapping shareholders is below the average, the overall average profit weight reflects 90.05% of the interests of overlapping shareholders and, on aggregate, those overlapping shareholders hold, on average, 83.99% of the control rights. And even if we consider, instead, the subset of firms for which the component of the IHHI associated to the ownership of non-overlapping shareholders is below the 95th percentile, the overall average profit weight reflects 82.91% of the interests of overlapping shareholders and, on aggregate, those overlapping shareholders hold, on average, 80.16% of the control rights. This implies that using normalized Banzhaf power indices as a measure of corporate control seems to avoid, at least for set of intra-industry S&P 500 firms, the counter-intuitive profit weights that potentially the dominant formulation may yield in the presence of highly dispersed ownership by non-overlapping shareholders.

Fig. 3, as a whole, seems, therefore, to suggest that the dominant formulation may over-quantify profit weights for the set of intra-industry S&P 500 firms in the presence of dispersed ownership by non-overlapping shareholders, when control rights of shareholders are measured by their voting rights, but not when they are measured by the normalized Banzhaf power indices that result from their voting rights. In order to have an idea of the magnitude of the over-quantification of the dominant formulation under a proportional control assumption, we recompute the profit weights for the same subset of firms as in Fig. 3 using (a) voting rights as the measure of corporate control; and (b) the objective function of the managers established in Proposition 1 for $\lambda = 0$, which corresponds to the dominant formulation, and $\lambda = 1$, which corresponds to the alternative formulation and, as discussed above, will reflect solely the interests of overlapping shareholders only if they have full control. Fig. 4, Panels A to D depict the corresponding (arithmetic) average profit weight across the subset of S&P 500 firms for which the component of the IHHI associated to the ownership of non-overlapping shareholders is below the average or different quantiles. The plots show that the objective function of managers established in Proposition 1 for $\lambda = 0$ and $\lambda = 1$ respond to the ownership dispersion of non-overlapping shareholders differently. The plots also show that the difference between the two formulations decreases as the ownership dispersion of non-overlapping shareholders decreases. For the subset of firms for which the component of the IHHI associated to the ownership of non-overlapping shareholders is below the 5th (25th) percentile, the overall average profit weight is 0.82 (0.76) under $\lambda = 0$ and 0.60 (0.60) under $\lambda = 1$. This suggests that the dominant formulation may, in fact, over-quantify profit weights for the set of intra-industry S&P 500 firms in the presence of dispersed ownership by non-overlapping shareholders, particularly for firms for which the component of the IHHI associated to the ownership of non-overlapping shareholders is below 7.39 (the overall 25th percentile). For the subset of firms for which the component of the IHHI associated to the ownership of non-overlapping shareholders is below the average (95th percentile), the overall average profit weight is 0.66 (0.61) under $\lambda = 0$ and 0.59 (0.59) under $\lambda = 1$. This suggests that the difference between the two formulations disappears as the ownership dispersion of non-overlapping shareholders decreases.

4.2.2. Inter-Industry analysis

We now examine the pairs of firms in different industries, which absent overlapping ownership are, in general equilibrium, likely to impose a positive externality on one another (Azar and Vives, 2021a). To do so, we focus on the pairs of firms that have a null horizontal relatedness score, as computed by Hoberg and Phillips (2010, 2016). Fig. 5 replicates Fig. 1 above and depicts the (arithmetic) average profit weight of all *inter-industry* cross-pairs of S&P 500 firms in each quarter. The plots suggest that the qualitative and quantitative patterns are similar to those obtained for the set of intra-industry S&P 500 firms. This similarity is valid for all the remaining figures presented in the intra-industry analysis above, which are, for that reason, omitted here for brevity.

[31] The plots also show that the fraction of the interests of overlapping shareholders reflected in the dominant formulation has increased slightly over time, as both the dispersion of the ownership of non-overlapping shareholders and the aggregate corporate control of overlapping shareholders have also increased slightly over time.

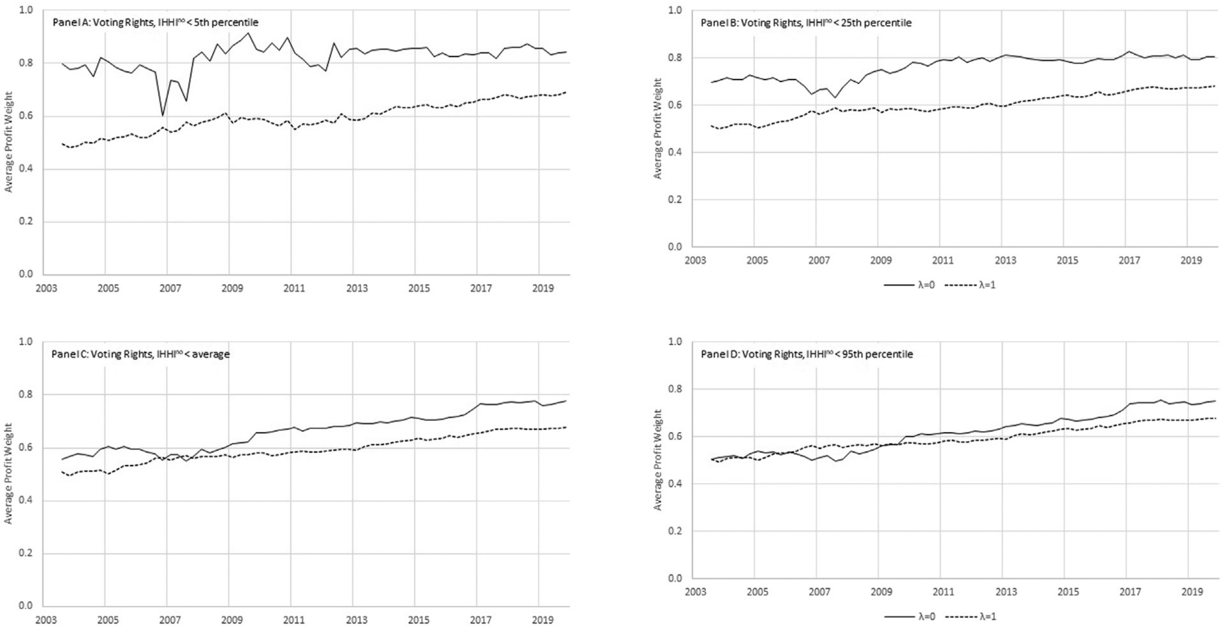

Fig. 4. S&P 500 intra-industry profit weights, voting rights and non-overlapping shareholders dispersion. Arithmetic average profit weight across the subset of S&P 500 firms with a strictly positive horizontal relatedness score (from Hoberg and Phillips, 2010; Hoberg and Phillips, 2016) for which the component of the IHHI associated to the ownership of non-overlapping shareholders is below the average or different quantiles. The profit weights are computed following the objective function of managers established in Proposition 1 for $\lambda = 0$ and $\lambda = 1$ considering that the control rights of shareholders are measured by their voting rights (which are assumed coincident with their financial rights) and that the retail share of each firm is made up of an infinity of shareholders each holding infinitesimal financial rights.

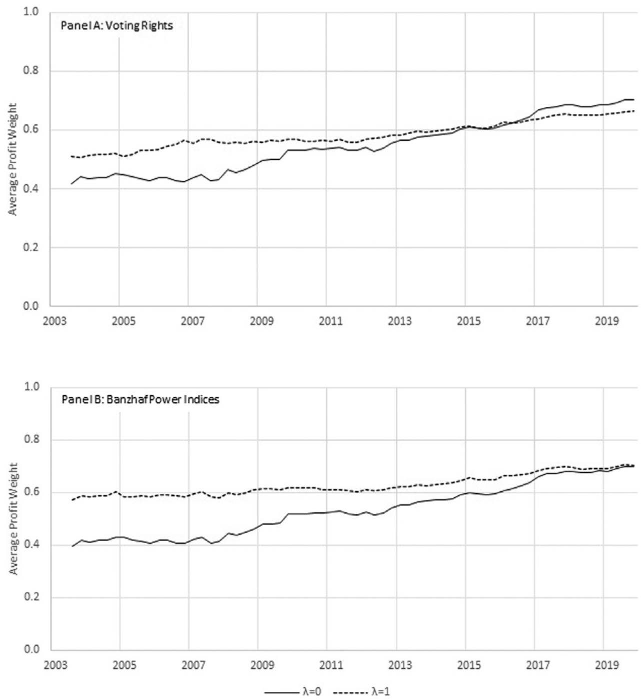

Fig. 5. S&P 500 inter-industry profit weights. Arithmetic average profit weight of all cross-pairs of S&P 500 firms with a null horizontal relatedness score (from Hoberg and Phillips, 2010; Hoberg and Phillips, 2016) in each quarter. The profit weights are computed according to the objective function of managers established in Proposition 1 for $\lambda = 0$ and $\lambda = 1$. Panel A considers the case in which the control rights of shareholders are measured by their voting rights (which are assumed coincident with their financial rights). Panel B considers the case in which the control rights of shareholders are measured by the Banzhaf power indices that result from their voting rights. In all cases, the retail share of each firm is assumed to be made up of an infinity of shareholders each holding infinitesimal financial and voting rights.

5. Extensions

In this section, we introduce and discuss extensions to the theoretical framework used to derive the objective function of managers established in Proposition 1.

5.1. Shareholders inattention

We have assumed a framework in which shareholders are fully attentive to the strategy proposals of candidates. In this section, we examine the robustness of the objective function of managers established in Proposition 1 by discussing an extension framework in which shareholders can either be attentive or inattentive to those proposals. In particular, we follow Gilje et al. (2020) in considering that each shareholder k is attentive to the strategy proposals of firm j's candidates with probability δ_{kj} and inattentive with probability $1-\delta_{kj}$. If attentive, as discussed above, shareholder k will vote for the incumbent with probability 1 if $u_k(\mathbf{x}_a, \mathbf{m}_a) > u_k(\mathbf{x}_b, \mathbf{m}_b)$, will vote for the challenger with probability 1 if instead $u_k(\mathbf{x}_a, \mathbf{m}_a) < u_k(\mathbf{x}_b, \mathbf{m}_b)$, and will randomize between the two candidates with equal probability if $u_k(\mathbf{x}_a, \mathbf{m}_a) = u_k(\mathbf{x}_b, \mathbf{m}_b)$. If inattentive, shareholder k will, irrespective of the strategy proposals of the candidates, vote for the incumbent with probability ρ_k and will vote for the challenger with probability $1-\rho_k$.

In this setting, it is relatively straightforward to show that the (normalized) weight that the manager of firm j would assign to the expected profit of firm g for any $j, g \in \Im$ would be given by:

$$w_{jg} = \frac{\sum_{k\in\Theta_j} \frac{\gamma^a_{kj}\phi_{kg}}{1-\lambda+\lambda\phi_{kj}}}{\sum_{k\in\Theta_j} \frac{\gamma^a_{kj}\phi_{kj}}{1-\lambda+\lambda\phi_{kj}}} \geq 0,$$

where γ^a_{kj} denotes the control rights of shareholder k in firm j, which now incorporate (additionally) the attention probabilities δ_{kj}. Under Assumption 2 and $\lambda = 0$, we obtain $\gamma^a_{kj} = \upsilon_{kj}\delta_{kj}$, yielding a weight that is qualitatively similar to the measure proposed by Gilje et al. (2020) to capture the impact of overlapping ownership on managerial incentives. Although the attention probabilities δ_{kj} do not have a direct empirical counterpart, they can be modeled to be a function of a multitude of observed firm and shareholder factors (for example, the importance of firm j in shareholder k's investment portfolio) and estimated using observed voting behavior (see Gilje et al., 2020 for an illustrative example and the references therein).

5.2. Cross-ownership structures

We have assumed a framework in which shareholders are external, in the sense that we do not allow firms to hold financial and voting rights in other firms of the same industry. In this section, we examine the robustness of the objective function of managers established in Proposition 1 by discussing an extension framework in which firms are also allowed to hold financial and voting rights in other firms within the industry. In particular, consider that there are K shareholders, indexed by $k \in \Theta \equiv \{1, \ldots, N, \ldots K\}$, who may include not just *external* shareholders (and can engage in common-ownership), but also *internal* shareholders from the set of firms (and can engage in cross-ownership), both of which can hold financial and voting rights in multiple firms.

In this setting, it is relatively straightforward to show that the (normalized) weight that the manager of firm j would assign to the expected profit of firm g for any $j, g \in \Im$ would be given by:

$$w_{jg} = \frac{\sum_{k\in\Theta_j} \frac{\gamma^u_{kj}\phi^u_{kg}}{1-\lambda+\lambda\phi^u_{kj}}}{\sum_{k\in\Theta_j} \frac{\gamma^u_{kj}\phi^u_{kj}}{1-\lambda+\lambda\phi^u_{kj}}} \geq 0,$$

where ϕ^u_{kj} and γ^u_{kj} denote the *ultimate* financial and control rights, respectively, of *external* shareholder k in firm j, which can be computed following the algorithm in Brito et al. (2018a).

6. Conclusions

We examine the objective function of managers in the presence of overlapping shareholding. We do so, from both a theoretical and an empirical perspective. From a theoretical perspective, we make use of a probabilistic voting model in which shareholders vote to elect the manager from two potential candidates (the incumbent and a challenger) with conceivably different strategy proposals to microfound a proposed alternative formulation of the objective function of managers in which (in contrast to the dominant formulation) the manager of a firm with overlapping shareholders would decide the strategy of the firm to maximize a control-weighted sum of the *relative* expected returns of the firm' s shareholders. In particular, to do so, we generalize the probabilistic voting model typically used in the literature to allow the bias of shareholders for (or against) the challenger to be both *profit-irrelevant* or *profit-relevant*. We show that a profit-relevant bias microfounds our proposed alternative formulation, which can cope better with ownership dispersion in the sense that (in contrast to the dominant formulation) it will never reflect solely the interests of a set of (non-dispersed) shareholders, unless the dispersion yields those shareholders the full control of the firm.

From an empirical perspective, we apply the two formulations to the set of S&P 500 firms. We show that (a) shareholder ownership dispersion, particularly of non-overlapping shareholders, is a relevant empirical issue for the set of S&P 500 firms; and (b) the dominant formulation, in such cases, may over-quantify profit weights, particularly when corporate control is measured by voting rights. This, in turn, suggests that empirical quantifications of the impact of overlapping ownership on market outcomes of S&P 500 firms using the dominant formulation when the ownership of shareholders is highly dispersed, should be avoided, particularly when corporate control is measured by voting rights, so to elude attenuation bias that causes them to underestimate the magnitude and statistical significance of the marginal effect of overlapping shareholding on market outcomes. If we consider counter-intuitive that the weight assigned by the manager to the profit of other firms (in which overlapping shareholders hold shares in) reflects solely the interests of the (non-dispersed) overlapping shareholders even if overlapping shareholders do not have the full control of the firm, then our proposed alternative formulation appears to be, in those cases, a preferable option that could result in more accurate estimates of the effects of overlapping shareholding.

Our findings are particularly relevant for competition agencies because profit weights can (and should) be incorporated into the traditional indicators used to help predict anti-competitive effects. Using our proposed alternative formulation could lead to more accurate conclusions in specific markets about whether overlapping shareholding is likely to produce anticompetitive effects. If so, our proposed alternative formulation will accurately conclude in more markets that the effects are not likely to be anticompetitive, thus more accurately distinguishing those markets from other markets where the effects are more likely to be anticompetitive. Our findings are also particularly relevant for antitrust policy because overlapping ownership "has stimulated a major rethinking of antitrust enforcement" (Elhauge, 2016; Scott Morton and Hovenkamp, 2018; Hemphill and Kahan, 2020), which must naturally be based on empirical evidence as accurate as possible.

We leave to future work the next steps: (a) which formulation more accurately predicts firm behavior? and (b) do the two formulations lead to different conclusions regarding the magnitude and statistical significance of the marginal effect of overlapping ownership on market competition?

CRediT authorship contribution statement

Duarte Brito: Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing. **Einer Elhauge:** Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing. **Ricardo Ribeiro:** Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing. **Helder Vasconcelos:** Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing.

Data Availability

Data will be made available on request.

Appendix A. Mathematical Appendix

In this mathematical appendix, we present the proofs of Proposition 1 and Corollary 1.

Proof of Proposition 1

The structure of this proof follows three steps.

First, we show that the objective function of the incumbent is strictly concave conditional on *the strategy proposal of the challenger to the firm* and on *the strategy proposals of the candidates to the other firms.* Given that strategy proposals are, under Assumption 4, defined in a convex set, this implies that the incumbent's maximization problem has a unique maximum conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the other firms. Given the symmetry of the solution to the maximization problem of the two candidates to the firm, we have that they will choose best-response functions that are, conditional on the strategy proposals of the candidates to the remaining firms, symmetric with respect to the strategy proposal of the opponent candidate. This implies that the two candidates will choose the same strategy proposal for the firm, conditional on the strategies proposals of the candidates to the other firms, i.e., they will choose the same best-response function to the strategy proposals of the candidates to the other firms. Because this common best-response function achieves, conditional on the strategies proposals of the candidates to the other firms, the unique maximum of the objective functions of the two candidates to the firm, there are no unilateral incentives to deviation.

Second, we show that this common best-response function is the same as the best-response function that would arise from maximizing, conditional on the strategy proposals of the candidates to the other firms, the objective function established in the proposition.

Finally, given that the strategy proposal of each candidate to the different firms is, under Assumption 4, defined in a convex set and the common expectation of each shareholder is, under Assumption 5, continuous, the best-response functions of the candidates to the different firms are guaranteed to be upper-hemicontinuous, which implies that we can apply Kakutani's fixed point theorem to ensure that the Nash equilibrium exists.

We now address the sub-proof of the remaining points: (a) that the objective function of the incumbent is strictly concave conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the other firms; and (b) that the common best-response function is the same as the best-response function that would arise from maximizing, conditional on the strategy proposals of the candidates to the other firms, the objective function established in the proposition. We do so considering, in turn, Assumptions 2 and 3.

Consider, first, Assumption 2. We begin by addressing (a) and show that the objective function of the incumbent is strictly concave conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the other firms. Let ϖ_{aj} denote the objective function of the incumbent, as follows:

$$\begin{aligned}\varpi_{aj} &= \sum_{k\in\Theta_j} Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)\upsilon_{kj}\\ &= \sum_{k\in\Theta_j}\int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j} Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j\right)h\left(\tilde{\xi}_j\right)d\tilde{\xi}_j\upsilon_{kj}\\ &= \sum_{k\in\Theta_j}\int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j} G_j\left(\frac{\mathbb{E}(R_k(\mathbf{x}_a)) - \mathbb{E}(R_k(\mathbf{x}_b))}{\left(1-\lambda+\lambda\phi_{kj}\right)} - \tilde{\xi}_j\right)h\left(\tilde{\xi}_j\right)d\tilde{\xi}_j\upsilon_{kj}\\ &= \sum_{k\in\Theta_j}\int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j}\left(\frac{1}{2} + \frac{\mathbb{E}(R_k(\mathbf{x}_a)) - \mathbb{E}(R_k(\mathbf{x}_b)) - \tilde{\xi}_j\left(1-\lambda+\lambda\phi_{kj}\right)}{\tau_j\left(1-\lambda+\lambda\phi_{kj}\right)}\right)h\left(\tilde{\xi}_j\right)d\tilde{\xi}_j\upsilon_{kj}\\ &= \sum_{k\in\Theta_j}\left(\frac{1}{2} + \frac{\mathbb{E}(R_k(\mathbf{x}_a)) - \mathbb{E}(R_k(\mathbf{x}_b))}{\tau_j\left(1-\lambda+\lambda\phi_{kj}\right)}\right)\upsilon_{kj} - \sum_{k\in\Theta_j}\int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j}\left(\frac{\tilde{\xi}_j}{\tau_j}\right)h\left(\tilde{\xi}_j\right)d\tilde{\xi}_j\upsilon_{kj},\end{aligned}$$

which makes use of the fact that, under Assumption 7, $G_j(\cdot)$ is the cumulative distribution function of an uniform distribution over the range $[-\tau_j/2, \tau_j/2]$ with τ_j sufficiently large such that:

$$0 < G_j\left(\frac{\mathbb{E}(R_k(\mathbf{x}_a)) - \mathbb{E}(R_k(\mathbf{x}_b))}{\left(1-\lambda+\lambda\phi_{kj}\right)} - \tilde{\xi}_j\right) < 1,$$

for all $\tilde{\xi}_j \in [-\psi_j/2, \psi_j/2]$, $x_{a_m} \in \Omega_m$ and $x_{b_m} \in \Omega_m$ $\forall j, m \in \Im$.

Under Assumption 1, shareholders are conditionally sincere, which implies that the incumbent of firm j can choose her strategy proposal taking the strategies of the candidates to the remaining firms as given. The first order condition of this problem is, thus, given by:

$$\frac{\partial\varpi_{aj}}{\partial x_{a_j}} = \sum\nolimits_{k\in\Theta_j}\frac{1}{\tau_j\left(1-\lambda+\lambda\phi_{kj}\right)}\frac{\partial\mathbb{E}(R_k(\mathbf{x}_a))}{\partial x_{a_j}}\upsilon_{kj},$$

while the second order condition is given by:

$$\frac{\partial^2\varpi_{aj}}{\partial x_{a_j}^2} = \sum\nolimits_{k\in\Theta_j}\frac{1}{\tau_j\left(1-\lambda+\lambda\phi_{kj}\right)}\frac{\partial^2\mathbb{E}(R_k(\mathbf{x}_a))}{\partial x_{a_j}^2}\upsilon_{kj},$$

which implies that, because under Assumption 5 we have that $\partial^2\mathbb{E}(R_k(\mathbf{x}_a))/\partial x_{a_j}^2 < 0$, the objective function of the manager is strictly concave in x_{a_j}, conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the other firms.

We now address (b) and show that the common best-response function is the same as the best-response function that would arise from maximizing, conditional on the strategy proposals of the candidates to the other firms, the objective function established in the proposition. To do so, note that because the two candidates will choose the same best-response function, in equilibrium, we have $\mathbb{E}(R_k(\mathbf{x}_a)) = \mathbb{E}(R_k(\mathbf{x}_b)) = \mathbb{E}(R_k(\mathbf{x}))$ for all $k \in \Theta_j$. As a consequence, we have from the first order condition above that:

$$\max_{x_{a_j}}\varpi_{aj} = \sum\nolimits_{k\in\Theta_j} Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)\upsilon_{kj} \propto \max_{x_j}\sum\nolimits_{k\in\Theta_j}\gamma_{kj}\frac{\mathbb{E}(R_k(\mathbf{x}))}{1-\lambda+\lambda\phi_{kj}},$$

where γ_{kj} is measured by the voting rights of shareholder k in firm j: υ_{kj}.

Consider, now, instead, Assumption 3. We begin by readdressing (a) and show that the objective function of the incumbent is strictly concave conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the other firms. Let ϖ_{aj} denote the objective function of the incumbent, as follows:

$$\varpi_{aj} = Pr\left(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b\right)\Xi_{a_j} = \sum_{\Theta'_j\in\wp_j} Pr\left(m_j = a_j\middle|\mathbf{x}_a, \mathbf{x}_b, \Theta'_j\right)\Xi_{a_j} = \int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j}\sum_{\Theta'_j\in\wp_j} Pr\left(m_j = a_j\middle|\mathbf{x}_a, \mathbf{x}_b, \Theta'_j, \tilde{\xi}_j\right)h\left(\tilde{\xi}_j\right)d\tilde{\xi}_j\Xi_{a_j}.$$

Under Assumption 1, shareholders are conditionally sincere, which implies that the incumbent of firm j can choose her strategy proposal taking the strategies of the candidates to the remaining firms as given. In order to compute the first order condition of this problem, it will help decompose, with reference to any given shareholder k, the set $\wp_j$ of all the 2^{ℓ_j-1} possible subsets of those shareholders that can award the majority of votes to a candidate into three subsets: (a) the subsets where shareholder k enters and is pivotal: $\wp^p_{kj}$, (b) the subsets where shareholder k enters and is not pivotal: $\wp^{np}_{kj}$, and (c) the subsets where shareholder k does not enter: $\wp^n_{kj}$. As such, we have, for any given shareholder k, that $\wp_j = \wp^p_{kj} \cup \wp^{np}_{kj} \cup \wp^n_{kj}$. This implies that we can write $\sum_{\Theta^\iota_j \in \wp_j} Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right)$ as follows:

$$\begin{aligned}\sum_{\Theta^\iota_j \in \wp_j} Pr\left(m_j = a_j \middle| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right) &= \sum_{\Theta^\iota_j \in \wp^p_{kj}} Pr\left(m_j = a_j \middle| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right)\\ &+ \sum_{\Theta^\iota_j \in \wp^{np}_{kj}} Pr\left(m_j = a_j \middle| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right)\\ &+ \sum_{\Theta^\iota_j \in \wp^n_{kj}} Pr\left(m_j = a_j \middle| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right).\end{aligned}$$

Because all the subsets in $\wp^n_{kj}$ can award the majority of votes to a candidate, if we add shareholder k to $\wp^n_{kj}$, the corresponding new subsets will be able to award as well the majority of votes to a candidate, with shareholder k not being pivotal. As such, we have that, for all $\Theta^\iota_j \in \wp^n_{kj}, \Theta^\iota_j \cup \{k\} \in \wp^{np}_{kj}$, with the number of subsets in $\wp^n_{kj}$ being equal to the number of subsets in $\wp^{np}_{kj}$. This implies - letting, for notation compactness, for the purposes of this proof, $Pr^c_{ra_j}$ denote $Pr_{ra_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b; \tilde{\xi}_j\right)$ - that:

$$\begin{aligned}\sum_{\Theta^\iota_j \in \wp^n_{kj}} Pr\left(m_j = a_j \middle| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right) &= \sum_{\Theta^\iota_j \in \wp^n_{kj}} \prod_{r\in\Theta^\iota_j} Pr^c_{ra_j} \prod_{r\notin\Theta^\iota_j} \left(1 - Pr^c_{ra_j}\right)\\ &= \sum_{\Theta^\iota_j \in \wp^n_{kj}} \left(\prod_{r\in\Theta^\iota_j} Pr^c_{ra_j} \prod_{r\notin\Theta^\iota_j, r\neq k} \left(1 - Pr^c_{ra_j}\right)\right)\left(1 - Pr^c_{ka_j}\right)\\ &= \sum_{\Theta^\iota_j \in \wp^n_{kj}} \prod_{r\in\Theta^\iota_j} Pr^c_{ra_j} \prod_{r\notin\Theta^\iota_j, r\neq k} \left(1 - Pr^c_{ra_j}\right)\\ &\quad - \sum_{\Theta^\iota_j \in \wp^{np}_{kj}} \prod_{r\in\Theta^\iota_j} Pr^c_{ra_j} \prod_{r\notin\Theta^\iota_j} \left(1 - Pr^c_{ra_j}\right)\\ &= \sum_{\Theta^\iota_j \in \wp^n_{kj}} \prod_{r\in\Theta^\iota_j} Pr^c_{ra_j} \prod_{r\notin\Theta^\iota_j, r\neq k} \left(1 - Pr^c_{ra_j}\right)\\ &\quad - \sum_{\Theta^\iota_j \in \wp^{np}_{kj}} Pr\left(m_j = a_j \middle| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right),\end{aligned}$$

where the second equality factors out the (conditional) probability associated to shareholder k. As a consequence, we can write $\sum_{\Theta^\iota_j \in \wp_j} Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right)$ as follows:

$$\sum_{\Theta^\iota_j \in \wp_j} Pr\left(m_j = a_j \middle| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right) = \sum_{\Theta^\iota_j \in \wp^p_{kj}} Pr\left(m_j = a_j \middle| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right) + \sum_{\Theta^\iota_j \in \wp^n_{kj}} \prod_{r\in\Theta^\iota_j} Pr^c_{ra_j} \prod_{r\notin\Theta^\iota_j, r\neq k} \left(1 - Pr^c_{ra_j}\right).$$

Having this result in mind, we can now address the first order condition of the problem. We can write this condition as follows:

$$\frac{\partial \varpi_{aj}}{\partial x_{a_j}} = \int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j} \sum_{\Theta^\iota_j \in \wp_j} \sum_{k\in\Theta_j} \frac{\partial Pr\left(m_j = a_j \middle| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right)}{\partial Pr^c_{ka_j}} \frac{\partial Pr^c_{ka_j}}{\partial x_{a_j}} h\left(\tilde{\xi}_j\right) d\tilde{\xi}_j \, \Xi_{a_j}.$$

Because there is always at least one shareholder that is pivotal, we can rewrite this condition, using the result above for $\sum_{\Theta^\iota_j \in \wp_j} Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right)$, as follows:

$$\begin{aligned}\frac{\partial \varpi_{aj}}{\partial x_{a_j}} &= \int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j} \sum_{\Theta^\iota_j \in \wp^p_{kj}} \sum_{k\in\Theta_j} \frac{\partial Pr\left(m_j = a_j \middle| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right)}{\partial Pr^c_{ka_j}} \frac{\partial Pr^c_{ka_j}}{\partial x_{a_j}} h\left(\tilde{\xi}_j\right) d\tilde{\xi}_j \, \Xi_{a_j}\\ &= \sum_{k\in\Theta_j} \sum_{\Theta^\iota_j \in \wp^p_{kj}} \int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j} \frac{\partial Pr\left(m_j = a_j \middle| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right)}{\partial Pr^c_{ka_j}} \frac{\partial Pr^c_{ka_j}}{\partial x_{a_j}} h\left(\tilde{\xi}_j\right) d\tilde{\xi}_j \, \Xi_{a_j},\end{aligned}$$

which makes use of the fact that the term $\sum_{\Theta^\iota_j \in \wp^n_{kj}} \prod_{r\in\Theta^\iota_j} Pr^c_{ra_j} \prod_{r\notin\Theta^\iota_j, r\neq k} \left(1 - Pr^c_{ra_j}\right)$ does not include the (conditional) probability associated to shareholder k. As a consequence, the second order condition is given by:

$$\frac{\partial^2 \varpi_{aj}}{\partial x^2_{a_j}} = \sum_{k\in\Theta_j} \sum_{\Theta^\iota_j \in \wp^p_{kj}} \int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j} \frac{\partial^2 Pr\left(m_j = a_j \middle| \mathbf{x}_a, \mathbf{x}_b, \Theta^\iota_j, \tilde{\xi}_j\right)}{\partial Pr^c_{ka_j} \partial x_{aj}} \frac{\partial Pr^c_{ka_j}}{\partial x_{a_j}} h\left(\tilde{\xi}_j\right) d\tilde{\xi}_j \, \Xi_{a_j}$$

$$+\sum_{k\in\Theta_j}\sum_{\Theta^{\text{!}}_j\in\wp^p_{kj}}\int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j}\frac{\partial Pr\left(m_j=a_j\middle|\mathbf{x}_a,\mathbf{x}_b,\Theta^{\text{!}}_j,\tilde{\xi}_j\right)}{\partial Pr^c_{ka_j}}\frac{\partial^2 Pr^c_{ka_j}}{\partial x^2_{a_j}}h\left(\tilde{\xi}_j\right)d\tilde{\xi}_j\,\Xi_{a_j}$$

$$=\sum_{k\in\Theta_j}\sum_{\Theta^{\text{!}}_j\in\wp^p_{kj}}\int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j}\frac{\partial^2 Pr\left(m_j=a_j\middle|\mathbf{x}_a,\mathbf{x}_b,\Theta^{\text{!}}_j,\tilde{\xi}_j\right)}{\partial\left(Pr^c_{ka_j}\right)^2}\left(\frac{\partial Pr^c_{ka_j}}{\partial x_{a_j}}\right)^2 h\left(\tilde{\xi}_j\right)d\tilde{\xi}_j\,\Xi_{a_j}$$

$$+\sum_{k\in\Theta_j}\sum_{\Theta^{\text{!}}_j\in\wp^p_{kj}}\int_{-\frac{1}{2}\varphi_j}^{\frac{1}{2}\varphi_j}\frac{\partial Pr\left(m_j=a_j\middle|\mathbf{x}_a,\mathbf{x}_b,\Theta^{\text{!}}_j,\tilde{\xi}_j\right)}{\partial Pr^c_{ka_j}}\frac{\partial^2 Pr^c_{ka_j}}{\partial x^2_{a_j}}h\left(\tilde{\xi}_j\right)d\tilde{\xi}_j\,\Xi_{a_j}.$$

In order to examine the sign of $\partial^2\varpi_{aj}/\partial x^2_{a_j}$, we begin by addressing $\partial^2 Pr^c_{ka_j}/\partial x^2_{a_j}$. Under Assumption 7, $G_j(\cdot)$ is the cumulative distribution function of an uniform distribution over the range $[-\tau_j/2,\tau_j/2]$ with τ_j sufficiently large such that:

$$0<Pr^c_{ra_j}=G_j\left(\frac{\mathbb{E}(R_k(\mathbf{x}_a))-\mathbb{E}(R_k(\mathbf{x}_b))}{\left(1-\lambda+\lambda\phi_{kj}\right)}-\tilde{\xi}_j\right)<1,$$

for all $\tilde{\xi}_j\in[-\psi_j/2,\psi_j/2]$, $x_{a_m}\in\Omega_m$ and $x_{b_m}\in\Omega_m$ $\forall j,m\in\Im$. This implies that:

$$Pr^c_{ka_j}=\frac{1}{2}+\frac{\mathbb{E}(R_k(\mathbf{x}_a))-\mathbb{E}(R_k(\mathbf{x}_b))-\tilde{\xi}_j\left(1-\lambda+\lambda\phi_{kj}\right)}{\tau_j\left(1-\lambda+\lambda\phi_{kj}\right)}$$

$$\frac{\partial Pr^c_{ka_j}}{\partial x_{a_j}}=\frac{1}{\tau_j\left(1-\lambda+\lambda\phi_{kj}\right)}\frac{\partial\mathbb{E}(R_k(\mathbf{x}_a))}{\partial x_{a_j}},$$

which in turn implies, because under Assumption 5 we have $\partial^2\mathbb{E}(R_k(\mathbf{x}_a))/\partial x^2_{a_j}<0$, that:

$$\frac{\partial^2 Pr^c_{ka_j}}{\partial x^2_{a_j}}=\frac{1}{\tau_j\left(1-\lambda+\lambda\phi_{kj}\right)}\frac{\partial^2\mathbb{E}(R_k(\mathbf{x}_a))}{\partial x^2_{a_j}}<0.$$

We now address $\partial Pr\left(m_j=a_j|\mathbf{x}_a,\mathbf{x}_b,\Theta^{\text{!}}_j,\tilde{\xi}_j\right)/\partial Pr^c_{ka_j}$. $Pr\left(m_j=a_j|\mathbf{x}_a,\mathbf{x}_b,\Theta^{\text{!}}_j,\tilde{\xi}_j\right)=\prod_{r\in\Theta^{\text{!}}_j}Pr^c_{ra_j}\prod_{r\notin\Theta^{\text{!}}_j}\left(1-Pr^c_{ra_j}\right)$ for $\Theta^{\text{!}}_j\in\wp^p_{kj}$. As such, because under Assumption 7, $0<Pr^c_{ra_j}<1$, for all $\tilde{\xi}_j\in[-\psi_j/2,\psi_j/2]$, $x_{a_m}\in\Omega_m$ and $x_{b_m}\in\Omega_m$ $\forall j,m\in\Im$, we have that:

$$\frac{\partial Pr\left(m_j=a_j|\mathbf{x}_a,\mathbf{x}_b,\Theta^{\text{!}}_j,\tilde{\xi}_j\right)}{\partial Pr^c_{ka_j}}=\prod\nolimits_{r\in\Theta^{\text{!}}_j,r\neq k}Pr^c_{ra_j}\prod\nolimits_{r\notin\Theta^{\text{!}}_j}\left(1-Pr^c_{ra_j}\right)>0.$$

Finally, we address $\partial^2 Pr\left(m_j=a_j|\mathbf{x}_a,\mathbf{x}_b,\Theta^{\text{!}}_j,\tilde{\xi}_j\right)/\partial\left(Pr^c_{ka_j}\right)^2$. Because $\partial Pr\left(m_j=a_j|\mathbf{x}_a,\mathbf{x}_b,\Theta^{\text{!}}_j,\tilde{\xi}_j\right)/\partial Pr^c_{ka_j}$ does not depend on $Pr^c_{ka_j}$, we have that $\partial^2 Pr\left(m_j=a_j|\mathbf{x}_a,\mathbf{x}_b,\Theta^{\text{!}}_j,\tilde{\xi}_j\right)/\partial\left(Pr^c_{ka_j}\right)^2=0$.

Combining the three results above, we have that the objective function of the manager is strictly concave in x_{a_j}, conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the other firms.

We now readdress (b) and show that the common best-response function is the same as the best-response function that would arise from maximizing, conditional on the strategy proposals of the candidates to the other firms, the objective function established in the proposition. To do so, note that because the two candidates will choose the same best-response function, in equilibrium, we have $\mathbb{E}(R_k(\mathbf{x}_a))=\mathbb{E}(R_k(\mathbf{x}_b))=\mathbb{E}(R_k(\mathbf{x}))$ for all $k\in\Theta_j$. This implies that $Pr^c_{ra_j}=1/2-\tilde{\xi}_j/\tau_j$ and:

$$\frac{\partial Pr\left(m_j=a_j\middle|\mathbf{x}_a,\mathbf{x}_b,\Theta^{\text{!}}_j,\tilde{\xi}_j\right)}{\partial Pr^c_{ka_j}}=\prod_{r\in\Theta^{\text{!}}_j,r\neq k}\left(\frac{1}{2}-\frac{\tilde{\xi}_j}{\tau_j}\right)\prod_{r\notin\Theta^{\text{!}}_j}\left(\frac{1}{2}+\frac{\tilde{\xi}_j}{\tau_j}\right)=\left(\frac{1}{2}-\frac{\tilde{\xi}_j}{\tau_j}\right)^{\#\Theta^{\text{!}}_j-1}\left(\frac{1}{2}+\frac{\tilde{\xi}_j}{\tau_j}\right)^{\ell_j-\#\Theta^{\text{!}}_j}.$$

As a consequence, the first order condition of this problem can be written as follows:

$$\frac{\partial\varpi_{aj}}{\partial x_{a_j}}=\sum\nolimits_{k\in\Theta_j}\sum\nolimits_{\Theta^{\text{!}}_j\in\wp^p_{kj}}\frac{1}{\psi_j}\int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j}\left(\frac{1}{2}-\frac{\tilde{\xi}_j}{\tau_j}\right)^{\#\Theta^{\text{!}}_j-1}\left(\frac{1}{2}+\frac{\tilde{\xi}_j}{\tau_j}\right)^{\ell_j-\#\Theta^{\text{!}}_j}d\tilde{\xi}_j\left(\frac{1}{\tau_j\left(1-\lambda+\lambda\phi_{kj}\right)}\frac{\partial\mathbb{E}(R_k(\mathbf{x}))}{\partial x_{a_j}}\right)\Xi_{a_j}$$

$$=\sum\nolimits_{k\in\Theta_j}\frac{\beta_{kj}}{\tau_j\left(1-\lambda+\lambda\phi_{kj}\right)}\frac{\partial\mathbb{E}(R_k(\mathbf{x}))}{\partial x_{a_j}}\Xi_{a_j},$$

where $\beta_{kj} = \sum_{\Theta^!_j \in \wp^p_{kj}} (1/\psi_j) \int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j} (1/2 - \tilde{\xi}_j/\tau_j)^{\#\Theta^!_j - 1} (1/2 + \tilde{\xi}_j/\tau_j)^{\ell_j - \#\Theta^!_j} d\tilde{\xi}_j$. This makes use of the fact that, under Assumption 6, $h(\tilde{\xi}_j) = 1/\psi_j$ and that $\partial Pr^c_{ka_j}/\partial x_{a_j}$ does not depend on $\tilde{\xi}_j$. In turn, this implies that:

$$\max_{x_{a_j}} \varpi_{aj} = Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right) \Xi_{a_j} \propto \max_{x_j} \sum\nolimits_{k \in \Theta_j} \beta_{kj} \frac{\mathbb{E}(R_k(\mathbf{x}))}{1 - \lambda + \lambda \phi_{kj}} \propto \max_{x_j} \sum\nolimits_{k \in \Theta_j} \gamma_{kj} \frac{\mathbb{E}(R_k(\mathbf{x}))}{1 - \lambda + \lambda \phi_{kj}},$$

where $\gamma_{kj} = \beta_{kj} / \sum_{h \in \Theta_j} \beta_{hj}$. This establishes the formulation of the objective function in the proposition. However, it remains to be shown that:

$$\beta_{kj} = \sum_{\Theta^!_j \in \wp^p_{kj}} \frac{1}{\psi_j} \int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j} \left(\frac{1}{2} - \frac{\tilde{\xi}_j}{\tau_j}\right)^{\#\Theta^!_j - 1} \left(\frac{1}{2} + \frac{\tilde{\xi}_j}{\tau_j}\right)^{\ell_j - \#\Theta^!_j} d\tilde{\xi}_j = \sum_{\Theta^!_j \in \wp^p_{kj}} \sum_{i=0}^{\#\Theta^!_j - 1} (-1)^i C_i^{\#\Theta^!_j - 1} m_{\ell_j - \#\Theta^!_j + i}.$$

To do so, we begin by rewriting β_{kj} as follows:

$$\begin{aligned}
\beta_{kj} &= \sum\nolimits_{\Theta^!_j \in \wp^p_{kj}} \frac{1}{\psi_j} \int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j} \left(\frac{1}{2} - \frac{\tilde{\xi}_j}{\tau_j}\right)^{\#\Theta^!_j - 1} \left(\frac{1}{2} + \frac{\tilde{\xi}_j}{\tau_j}\right)^{\ell_j - \#\Theta^!_j} d\tilde{\xi}_j \\
&= \sum\nolimits_{\Theta^!_j \in \wp^p_{kj}} \frac{1}{\psi_j} \int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j} \left(\frac{\tau_j - 2\tilde{\xi}_j}{2\tau_j}\right)^{\#\Theta^!_j - 1} \left(\frac{\tau_j + 2\tilde{\xi}_j}{2\tau_j}\right)^{\ell_j - \#\Theta^!_j} d\tilde{\xi}_j \\
&= \sum\nolimits_{\Theta^!_j \in \wp^p_{kj}} \frac{1}{\psi_j (2\tau_j)^{\ell_j - 1}} \int_{-\frac{1}{2}\psi_j}^{\frac{1}{2}\psi_j} \left(\tau_j - 2\tilde{\xi}_j\right)^{\#\Theta^!_j - 1} \left(\tau_j + 2\tilde{\xi}_j\right)^{\ell_j - \#\Theta^!_j} d\tilde{\xi}_j.
\end{aligned}$$

Now, let $\tau_j + 2\tilde{\xi}_j = y_j$, which implies that $\tau_j - 2\tilde{\xi}_j = 2\tau_j - y_i$ and $dy_j = 2d\tilde{\xi}_j$. If, under Assumption 6, $\tilde{\xi}_j$ follows an uniform distribution over the range $[-\psi_j/2, \psi_j/2]$ and density $1/\psi_j$, then y_j follows an uniform distribution over the range $\left[\tau_j - \psi_j, \tau_j + \psi_j\right]$ and density $1/2\psi_j$. As such, we can rewrite β_{kj} as follows:

$$\begin{aligned}
\beta_{kj} &= \sum_{\Theta^!_j \in \wp^p_{kj}} \frac{1}{2\psi_j (2\tau_j)^{\ell_j - 1}} \int_{\tau_j - \psi_j}^{\tau_j + \psi_j} \left(2\tau_j - y_j\right)^{\#\Theta^!_j - 1} y_j^{\ell_j - \#\Theta^!_j} dy_j \\
&= \sum_{\Theta^!_j \in \wp^p_{kj}} \frac{1}{2\psi_j (2\tau_j)^{\ell_j - 1}} \int_{\tau_j - \psi_j}^{\tau_j + \psi_j} \sum_{i=0}^{\#\Theta^!_j - 1} C_i^{\#\Theta^!_j - 1} \left(2\tau_j\right)^{\#\Theta^!_j - 1 - i} \left(-y_j\right)^i y_j^{\ell_j - \#\Theta^!_j} dy_j \\
&= \sum_{\Theta^!_j \in \wp^p_{kj}} \frac{1}{2\psi_j} \int_{\tau_j - \psi_j}^{\tau_j + \psi_j} \sum_{i=0}^{\#\Theta^!_j - 1} C_i^{\#\Theta^!_j - 1} \left(\frac{1}{2\tau_j}\right)^{\ell_j - \#\Theta^!_j + i} (-1)^i y_j^{\ell_j - \#\Theta^!_j + i} dy_j \\
&= \sum_{\Theta^!_j \in \wp^p_{kj}} \sum_{i=0}^{\#\Theta^!_j - 1} (-1)^i C_i^{\#\Theta^!_j - 1} \left(\frac{1}{2\tau_j}\right)^{\ell_j - \#\Theta^!_j + i} \int_{\tau_j - \psi_j}^{\tau_j + \psi_j} y_j^{\ell_j - \#\Theta^!_j + i} \frac{1}{2\psi_j} dy_j \\
&= \sum_{\Theta^!_j \in \wp^p_{kj}} \sum_{i=0}^{\#\Theta^!_j - 1} (-1)^i C_i^{\#\Theta^!_j - 1} \left(\frac{1}{2\tau_j}\right)^{\ell_j - \#\Theta^!_j + i} \mathbb{E}\left(y_j^{\ell_j - \#\Theta^!_j + i}\right) \\
&= \sum_{\Theta^!_j \in \wp^p_{kj}} \sum_{i=0}^{\#\Theta^!_j - 1} (-1)^i C_i^{\#\Theta^!_j - 1} \mathbb{E}\left(\left(\frac{y_j}{2\tau_j}\right)^{\ell_j - \#\Theta^!_j + i}\right) \\
&= \sum_{\Theta^!_j \in \wp^p_{kj}} \sum_{i=0}^{\#\Theta^!_j - 1} (-1)^i C_i^{\#\Theta^!_j - 1} m_{\ell_j - \#\Theta^!_j + i} \\
&= \sum_{\Theta^!_j \in \wp^p_{kj}} \sum_{i=0}^{\#\Theta^!_j - 1} (-1)^i C_i^{\#\Theta^!_j - 1} \frac{\sum_{n=0}^{\ell_j - \#\Theta^!_j + i} \left(\frac{1}{2} - \frac{\psi_j}{2\tau_j}\right)^n \left(\frac{1}{2} + \frac{\psi_j}{2\tau_j}\right)^{\ell_j - \#\Theta^!_j + i - n}}{\ell_j - \#\Theta^!_j + i + 1},
\end{aligned}$$

where the second equality makes use of the binomial theorem, which establishes that we can write $\left(2\tau_j - y_j\right)^{\#\Theta^{!}_j-1} = \sum_{i=0}^{\#\Theta^{!}_j-1} C_i^{\#\Theta^{!}_j-1}\left(2\tau_j\right)^{\#\Theta^{!}_j-1-i}\left(-y_j\right)^i$, and the last equality makes use of the fact that:

$$\mathbb{E}\left(\left(\frac{y_j}{2\tau_j}\right)^{\ell_j-\#\Theta^{!}_j+i}\right) = m_{\ell_j-\#\Theta^{!}_j+i} = \frac{\sum_{n=0}^{\ell_j-\#\Theta^{!}_j+i}\left(\frac{1}{2}-\frac{\psi_j}{2\tau_j}\right)^n\left(\frac{1}{2}+\frac{\psi_j}{2\tau_j}\right)^{\ell_j-\#\Theta^{!}_j+i-n}}{\ell_j-\#\Theta^{!}_j+i+1},$$

for an uniformly distributed variable over the range $\left[1/2-\psi_j/2\tau_j, 1/2+\psi_j/2\tau_j\right]$. Naturally, we could have computed $\mathbb{E}\left(\left(y_j/2\tau_j\right)^{\ell_j-\#\Theta^{!}_j+i}\right)$ using (at least, two simpler) alternative formulations. However, they would not nest the case in which $\psi_j = 0$.

Proof of Corollary 1

First, absent overlapping shareholding, the manager of each firm j would maximize the expected own-profit $\mathbb{E}\left(\Pi_j(\mathbf{x})\right)$, because $\phi_{kg} = 0$ for the subset of shareholders k who hold financial rights in firm j and all $j, g \neq j$. This implies $w_{jg} = 0$ for all $j, g \neq j$ and, thus, that property (*i*) holds.

Second, with non-infinitesimal overlapping shareholding, the manager of each firm j would internalize the impact of her firm's strategy on the expected profit of firm g when the shareholders that have financial rights in both firms have also control rights in the firm, because if $\gamma_{kj} \neq 0$ and $\phi_{kg} >> 0$ for at least one shareholder k, we have that $w_{jg} > 0$ for all $j, g \neq j$. This implies that the manager of each firm j would maximize $\mathbb{E}\left(\Pi_j(\mathbf{x})\right) + w_{jg}\mathbb{E}\left(\Pi_g(\mathbf{x})\right)$ and, thus, that property (*ii*) holds.

Third, the weight w_{jg} that the manager of each firm j assigns to the expected profit of firm g is continuous in ϕ_{kj}, γ_{kj} and ϕ_{kg} for the subset of shareholders k with financial rights in firm j, because the product, sum and quotient, respectively, of continuous functions is continuous. This implies that property (*iii*) holds.

Fourth, the manager of each firm j would maximize the sum of the expected profits when all shareholders that have financial rights in the firm are fully diversified across firms, because if those shareholders are fully diversified, for the subset of shareholders k who hold financial rights in firm j, we have $\phi_{kj} = \phi_{kg} = \phi_k$ and $\gamma_{kj} = \gamma_{kg} = \gamma_k$ for all $j, g \neq j$. This implies that:

$$w_{jg} = \frac{\sum_{k\in\Theta_j}\frac{\gamma_k\phi_k}{1-\lambda+\lambda\phi_k}}{\sum_{k\in\Theta_j}\frac{\gamma_k\phi_k}{1-\lambda+\lambda\phi_k}} = 1,$$

for all $j, g \neq j$ and that the manager of each firm j would maximize $\mathbb{E}\left(\Pi_j(\mathbf{x})\right) + \sum_{g\in\Im, g\neq j}\mathbb{E}\left(\Pi_g(\mathbf{x})\right)$. As such, property (*iv*) holds.

Finally, if $\lambda = 1$, the objective function of the manager of firm j will weigh solely the interests of the firm's overlapping (non-overlapping) shareholders as the ownership of each non-overlapping (overlapping) shareholder becomes dispersed (among a collection of infinitesimal identical shareholders) when the voting rights of the overlapping (non-overlapping) shareholders do induce full control. In order to see why, note that the objective function of the manager of firm j can be written as follows:

$$\max_{x_j} \sum_{k\in\Theta_j^o}\gamma_{kj}\mathbb{E}\left(\Pi_j(\mathbf{x})\right) + \sum_{k\in\Theta_j^{no}}\gamma_{kj}\mathbb{E}\left(\Pi_j(\mathbf{x})\right) + \sum_{g\in\Im, g\neq j}\sum_{k\in\Theta_j^o}\gamma_{kj}\frac{\phi_{kg}}{\phi_{kj}}\mathbb{E}(\Pi_g(\mathbf{x})).$$

As the ownership of each non-overlapping shareholder becomes dispersed among a collection of infinitesimal identical shareholders, the objective function of the manager will weigh solely the interests of overlapping shareholders when the voting rights of those horizontal shareholders do induce full control of the firm, i.e., when $\gamma_{kj} = 0$ for each $k \in \Theta_j^{no}$. Similarly, as the ownership of each overlapping shareholder becomes dispersed among a collection of infinitesimal identical shareholders, the number of shareholders in Θ_j^o increases, but the ratio ϕ_{kg}/ϕ_{kj} of each new infinitesimal identical shareholder will be identical to the corresponding ratio of each of the previous (non-dispersed) shareholders. As a consequence, the objective function of the manager will weigh solely the interests of non-overlapping shareholders when the voting rights of those non-overlapping shareholders do induce full control of the firm, i.e., when $\gamma_{kj} = 0$ for each $k \in \Theta_j^o$. As such, property (*vi*) is satisfied.

Note that the same would not be true if $\lambda = 0$. In this case, the objective function of the manager of firm j will approximate a weighted sum of (solely) the interests of the firm's overlapping (non-overlapping) shareholders when non-overlapping (overlapping) shareholders are highly dispersed, for any given value of the control rights of the overlapping (non-overlapping) shareholders. In order to see why, let the subset of shareholders that hold financial rights in firm j, Θ_j, be divided in two smaller subsets: the subset of overlapping shareholders, Θ_j^o, and the subset of non-overlapping shareholders, Θ_j^{no}. This implies that the objective function of the manager of firm j can be written as follows:

$$\max_{x_j} \sum_{k\in\Theta_j^o}\gamma_{kj}\phi_{kj}\mathbb{E}\left(\Pi_j(\mathbf{x})\right) + \sum_{k\in\Theta_j^{no}}\gamma_{kj}\phi_{kj}\mathbb{E}\left(\Pi_j(\mathbf{x})\right) + \sum_{g\in\Im, g\neq j}\sum_{k\in\Theta_j^o}\gamma_{kj}\phi_{kg}\mathbb{E}(\Pi_g(\mathbf{x})).$$

As the ownership of each non-overlapping shareholder becomes dispersed among a collection of infinitesimal identical shareholders, we have that $\sum_{k\in\Theta_j^{no}}\gamma_{kj}\phi_{kj}\to 0$. As such, the objective function of the manager will weigh solely the interests of overlapping shareholders, even when the voting rights of those overlapping shareholders do not induce full control of the firm:

$$\max_{x_j}\sum_{k\in\Theta_j^{o}}\gamma_{kj}\phi_{kj}\mathbb{E}\big(\Pi_j(\mathbf{x})\big)+\sum_{g\in\Im,g\neq j}\sum_{k\in\Theta_j^{o}}\gamma_{kj}\phi_{kg}\mathbb{E}(\Pi_g(\mathbf{x})).$$

Similarly, as the ownership of each overlapping shareholder becomes dispersed among a collection of infinitesimal identical shareholders, we have that $\sum_{k\in\Theta_j^{o}}\gamma_{kj}\phi_{kj}\to 0$ and $\sum_{k\in\Theta_j^{o}}\gamma_{kj}\phi_{kg}\to 0$. As such, the objective function of the manager will weigh solely the interests of non-overlapping shareholders (yielding an objective function proportional to the expected own-profit), even when the voting rights of those non-overlapping shareholders do not induce full control of the firm:

$$\max_{x_j}\sum_{k\in\Theta_j^{no}}\gamma_{kj}\phi_{kj}\mathbb{E}\big(\Pi_j(\mathbf{x})\big).$$

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