Received: 19 December 2022 | Revised: 9 March 2023 | Accepted: 3 April 2023

IET Communications

DOI: 10.1049/cmu2.12615

The Institution of Engineering and Technology WILEY

ORIGINAL RESEARCH

# Impact and compensation of carrier synchronization errors in OFDM signals with very large QAM constellations

Zahra Mokhtari[1] | Rui Dinis[1,2] | Luís Oliveira[2,3] | João Oliveira[2,3]

[1]Radio Systems, Institute de Telecomunicações, Lisbon, Portugal

[2]DEEC, FCT, Nova University of Lisbon, Caparica, Portugal

[3]UNINOVA-CTS and LASI, Caparica, Portugal

**Correspondence**
Zahra Mokhtari, Institute de Telecomunicações, Lisbon, Portugal.
Email: zahra.mokhtari@lx.it.pt

**Funding information**
Fundação para a Ciência e a Tecnologia, Grant/Award Number: foRESTER PCIF/SSI/0102/2017; Instituto de Desenvolvimento de Novas Tecnologias, Grant/Award Number: UIDB/00066/2020; Instituto de Telecomunicações, Grant/Award Number: UIDB/50008/2020; CTS - Centro de Tecnologia e Sistemas, Grant/Award Number: UIDB/00066/2020

**Abstract**

Low cost video sensors used for streaming video signals to help firefighters, require high bit rate due to uncompressed images. To increase spectral efficiency given a limited bandwidth, very high order constellations in high signal to noise ratio regimes can be used. However, noise is not the only factor effecting the high order constellations. These constellations are also sensitive to hardware impairments and system non-linearities. Therefore, in this paper, the effect of carrier frequency offset (CFO) on the performance of an orthogonal frequency division multiplexing (OFDM) system with high order quadrature amplitude modulation (QAM) is studied. A closed form expression is derived for the maximum normalized residual CFO that an OFDM system with $M$-QAM constellation can resist to have an error free symbol detection. Finally, the suitability of common previous CFO estimation techniques such as the cyclic prefix based technique and the Moose technique in these systems are investigate. The results show that the maximum residual CFO that an OFDM system with $M$-QAM constellation can resist is proportional to the inverse of $\sqrt{M-1}$. The results also show that very large order QAM constellations such as 4096-QAM are very sensitive to even small residual CFO values and their performance degrades, significantly. However, the bit error rate analysis indicate that the Moose CFO estimation technique can be used in these systems to compensate the CFO effect, accurately.

## 1 | INTRODUCTION

In a modern decision support system (DSS) low cost and reconfigurable IoT sensors are required. Wireless sensor networks (WSNs) are used in past years especially in the important area of detection of new wildfire ignitions and spots, as soon as possible [1]. The multisensor node includes infrared (IR) cameras to classify the fire front and spot fires. But, these sensors must be low cost since a huge number of sensors distributed over the terrain are required to cover the very large forest areas, which leads to many difficulties, since they are destroyed or need to be repaired due to the wildfires very high temperature in a very large area [2, 3].

The current generation of IR cameras achieves high-speed and high-resolution thermal imaging making them important tools in critical scenarios as in the case of firefighting emergencies or rescue missions. As shown in Figure 1, these IR cameras are added to the multisensory IoT nodes which can be installed in the vehicles involved in fire suppression, helping, and reducing the subjectivity of human-dependent analysis. The IR data imaging collected in real time provides important information that must be processed by high computing facilities to obtain advanced real-time fire classification that combined with other sensor data, can be used to predict fire evolution on the field. Due to the limited computing capabilities at the remote IoT node, all the IR raw imaging data must be sent wirelessly to the DSS cloud server.

Compared to other sensors included in the IoT sensing note, which only require connections with low transmission rates, the huge amount of data associated with the IR images requires the use of high-speed links. Since the typical bandwidth available in these types of applications is low, the development of new techniques using high QAM constellations is needed to support high video data rate and maintain the remaining characteristics

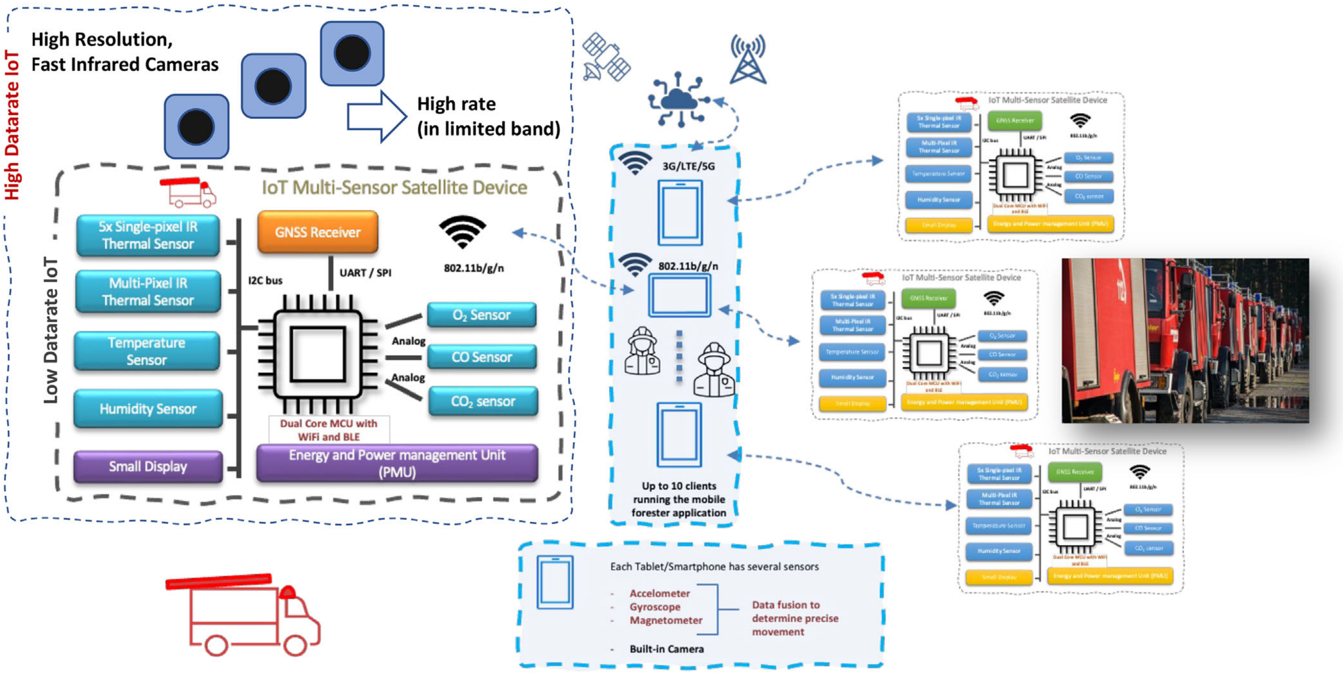


**FIGURE 1** The incorporation of high-resolution and high-speed infrared cameras demands high data rates in the wireless communication link.

of the remote IoT nodes, namely, flexibility. However, to have a cost effective network, low complexity sensors with no codec have to be used. Therefore, we have uncompressed images with large bit rate to be transmitted. One solution to increase spectral efficiency given a limited bandwidth is to use high order constellations in high signal to noise ratio (SNR) regimes [4, 5]. Thus, the investigation of new techniques high QAM constellations for low cost sensor and high video data rate is the main motivation for the investigation proposed in this paper.

High order constellations are very sensitive to system nonlinearity due to the closeness of the modulated symbols. Therefore, it is crucial to study the performance of these constellations in the presence of different hardware impairments. One of the common and inevitable hardware impairments in communication systems is the carrier frequency offset (CFO). Therefore, in this paper we will first study the performance of these systems in the presence of residual CFO and see how severe can this effect be. To have a good insight on the effect of CFO on different orders of $M$-QAM constellation, we derive a closed form expression for the maximum CFO that an $M$-QAM constellation in an OFDM system can have in order to have an error free detection. Later we investigate whether the previous CFO estimation techniques are sufficient for these systems or better estimation techniques have to be proposed. There are many estimation techniques proposed for CFO in the literature [6–16]. In this paper we will specifically investigate two common CFO estimation techniques named the Moose technique [6] and the cyclic prefix (CP) based technique [7]. Our results show that the performance of very high order constellations such as 4096-QAM is degraded significantly, even for small residual CFO values. The theoretical analysis show that the maximum normalized CFO that an $M$-QAM constellation in an OFDM system can have for an error free symbol detection is inversely proportional to $\sqrt{M-1}$. The results show the superiority of the Moose technique compared to the CP based technique, specially at large constellation orders. In fact, the results show that the Moose technique can estimate the CFO, accurately in these systems and there is no need for new complicated CFO estimation techniques to be proposed.

The paper is organized as follows. Section 2 reviews the high QAM constellation models and research, Section 3 describes the system model. In Section 4 analytical study on the effect of CFO on the system performance with large constellations is done. Also a closed form expression for the maximum CFO that an $M$-QAM constellation in an OFDM system can resist is derived. Section 5 introduces two common CFO estimation techniques. Section 6 evaluates the system performance through simulations and demonstrates how effective can the previous CFO estimation techniques be. Finally, in Section 7 the conclusions are drawn.

## 2 | HIGH ORDER QAM CONSTELLATIONS

Recently, there has been a great interest in high order constellations and some studies have been done on different forms of quadrature amplitude modulation (QAM) constellations [4, 17–21]. The different models that exist for QAM constellation are square QAM (SQAM), rectangular QAM (RQAM), star QAM, cross QAM (XQAM), and hexagonal QAM (HQAM). In [4] a comparative study on various QAM constellation models with different constellation orders has been done. The comparison is done in terms of average energy, peak to average power ratio, symbol error rate, bit mapping etc. In ref. [17] the authors have proposed a generalized semi definite programming relaxation based virtually antipodal detection approach for Gray-coded high order RQAM constellation in multiple input multiple output (MIMO) channels. In this paper, the structural regularity of Gray-coded high order RQAM is used to transform the symbol based MIMO detection model to a low complexity bit based detection model. To enhance the spectral efficiency of LTE system, 256-QAM was introduced in the 3GPP standard release 12.3. Therefore, ref. [18] studied the performance of LTE with 256-QAM constellation through MATLAB simulations. In ref. [19] the authors derived symbol error probability for orthogonal frequency division multiplexing (OFDM) systems with XQAM constellation with constellation order up to 512 over Gaussian and impulsive noise. In ref. [20], performance of HQAM, XQAM, and RQAM is studied for multi relay systems, by considering imperfect channel estimation and non-linear power amplifier at each relay. The application of QAM and adaptive modulation in free space optics (FSO) has been studied in ref. [21] and the results show the superiority of HQAM for a mixed radio frequency/FSO based relay network. In ref, [22] a MIMO detection algorithm for high-order QAM modulation single carrier transmissions in time dispersive channels has been proposed using a generalized form of the alternating direction method of multipliers (ADMM). The results indicate the suitability of the proposed technique for systems with high-order QAM modulation and large number of antennas. The authors in ref. [23] have also proposed two high order QAM detectors for uplink massive MIMO systems based on the penalty sharing ADMM. In ref. [24] a high-order QAM modulation and demodulation system, which controls the RF transceiver AD9361 based on FPGA has been proposed and the simulation experiments show the effectiveness of the system.

As it is clear, there is not much effort done on studying OFDM systems with very large order QAM constellations in the presence of hardware impairments. In wireless communication, to have a cost effective system, cheaper hardware are used. Using low cost hardware results in hardware impairments and system non-linearity which degrades the system performance. High order QAM constellations are sensitive to system non-linearity due to the closeness of the modulated symbols. CFO is one of the hardware impairments that is caused due to the mismatch between the local oscillators of the transmitter and the receiver. To the best of our knowledge there is no study on OFDM systems with very large order QAM constellations such as 4096-QAM in the presence of CFO, which is the main focus of this paper.

## 3 | SYSTEM MODEL

Figure 2 shows the block diagram of an OFDM transmitter and receiver. Assume $\boldsymbol{a} = [a_0, a_1, \dots, a_{P-1}]$ is the data bit vector to be transmitted, where $P$ is the number of bits. The bits are passed through an LDPC encoder and then interleaved. The resulting bits will be $\boldsymbol{b} = [b_0, b_1, \dots, b_{Q-1}]$, where $Q = P/r_c$ and $r_c$ is the LDPC code rate. Next, the coded bits are modulated with $M$-QAM modulation and the modulated symbols are $\boldsymbol{S} = [S_0, S_1, \dots, S_{N-1}]$, where $N = Q/\log_2(M)$ is the number of subcarriers in frequency domain that carry the data. After the data symbols are modulated in frequency domain they are passed through an inverse fast Fourier transform (IFFT) block as

$$s_n = \sum_{k=0}^{N-1} S_k e^{\frac{-j2\pi kn}{N}}, \quad n = 0, \dots, N-1 \tag{1}$$

and a cyclic prefix is added to the OFDM symbol in time domain. Therefore, the transmitted signal is

$$\begin{aligned} \boldsymbol{x} &= [x_{-N_{cp}}, \dots, x_{-1}, x_0, x_1, \dots, x_N] \\ &= [s_{N-N_{cp}}, \dots, s_{N-1}, s_0, s_1, \dots, s_{N-1}]; \end{aligned} \tag{2}$$

where $N_{cp}$ is the length of the CP. The signal is then passed through the channel and the $n$th time domain sample of the received signal, at the receiver side, is as

$$y_n = \sum_{l=0}^{L-1} h(l)x(n-l) + w_n, \quad n = -N_{cp}, \dots, N-1 \tag{3}$$

where $h(l)$ represents the $l$-path of the multipath Rayleigh fading channel and $w_n$ is the Gaussian noise for the $n$th time domain sample. The received signal is then passed through an $N$-point FFT block and the resulting signal in frequency domain will be

$$Y_k = \frac{1}{N} \sum_{n=0}^{N-1} y_n e^{\frac{j2\pi kn}{N}}, \quad k = 0, 1, \dots, N-1. \tag{4}$$

$Y_k$ can be rewritten as

$$Y_k = H_k S_k + W_k, \tag{5}$$

where, $H_k$ and $W_k$ are the channel coefficient and noise in frequency domain for the $k$th subcarrier. The resulting signal is then equalized in frequency domain with a single tap equalizer to compensate the effect of the channel as

$$\tilde{Y}_k = V_k Y_k = V_k H_k S_k + V_k W_k, \tag{6}$$

where, $V_k$ is the equalizer coefficient for subcarrier $k$. Next, the equalized symbols of each subcarrier are softly demodulated and their log-likelihood ratio (LLR) are computed. The LLRs are then given to the interleaver and LDPC decoder and the resulting data bits ($\hat{\boldsymbol{a}}$) are extracted.

## 4 | ANALYTICAL STUDY OF CFO EFFECT

In this section we explain the effect of CFO on OFDM systems to later see how severe this effect can be in very high order

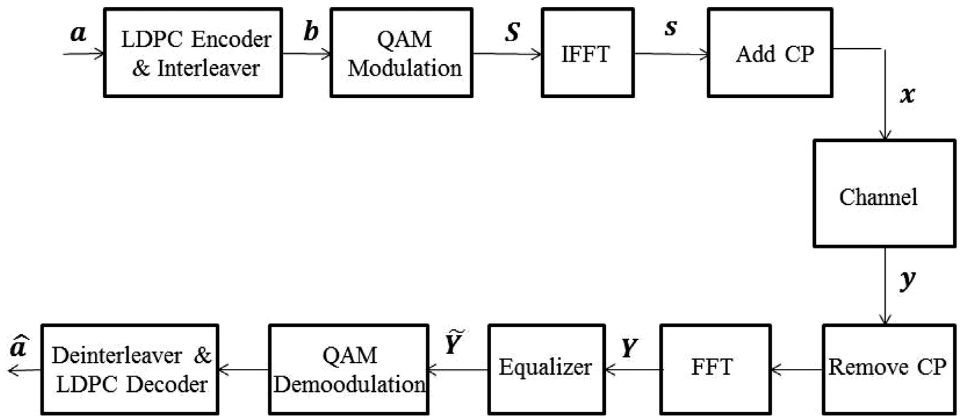


**FIGURE 2** Block diagram of an OFDM transceiver.

constellations. Also, to have a good insight, we derive a closed form expression for the maximum normalized residual CFO that an $M$-QAM constellation in an OFDM system can have to have an error free detection.

## 4.1 | CFO effect

In wireless communication systems, CFO occurs due to the mismatch between the transmitter and the receiver's local oscillators. OFDM systems are more sensitive to CFO than single carrier systems, since the orthogonality of the subcarriers is destroyed in the presence of CFO.

In the presence of CFO, Equation (3) will be as

$$y_n = \sum_{l=0}^{L-1} h(l)x(n-l)e^{\frac{j2\pi\epsilon n}{N}} + w_n e^{\frac{j2\pi\epsilon n}{N}}, \tag{7}$$

where $\epsilon$ is the normalized CFO with respect to subcarrier spacing. Therefore, in the frequency domain we will have

$$\begin{aligned} Y_k &= \sum_{m=0}^{N-1} C_{m-k}(H_m S_m + W_m) \\ &= C_0 H_k S_k + \sum_{m=0, m\neq k}^{N-1} C_{m-k} H_m S_m + \tilde{W}_k, \end{aligned} \tag{8}$$

where,

$$C_{m-k} = \frac{1}{N}\sum_{i=0}^{N-1} \exp\left(\frac{j2\pi i(m-k+\epsilon)}{N}\right) \tag{9}$$

and $\tilde{W}_k$ is the effective noise on subcarrier $k$ defined as

$$\tilde{W}_k = \sum_{m=0}^{N-1} C_{m-k} W_m.$$

In Equation (8), the first term is the desired symbol of subcarrier $k$ and the second term is the distortion caused by symbols of the other subcarriers on subcarrier $k$ due to the CFO. This term is called the inter carrier interference (ICI) term. Higher order constellations are more sensitive to system non-linearities and noise, since the symbols in the constellation are closer to each other. To see only the effect of CFO on the constellations and have an insight on how severe this effect can be, in the following we assume no channel and noise and consider only the CFO effect. Therefore, after the FFT block in the OFDM receiver we have

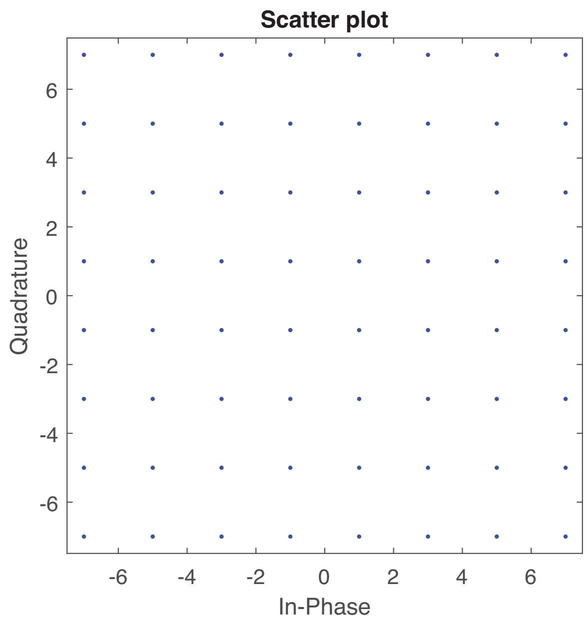


**FIGURE 3** symbols of a 64-QAM constellation with no CFO.

$$Y_k = \hat{S}_k = C_0 S_k + \sum_{m=0, m\neq k}^{N-1} C_{m-k} S_m, \tag{10}$$

Figures 3 and 4 show the modulated symbols of a 64-QAM constellation in the case of no CFO and the case of system hit by CFO equal to $\epsilon = 0.01$, respectively. Figures 5 and 6 show the modulated symbols of a 4096-QAM constellation in the case

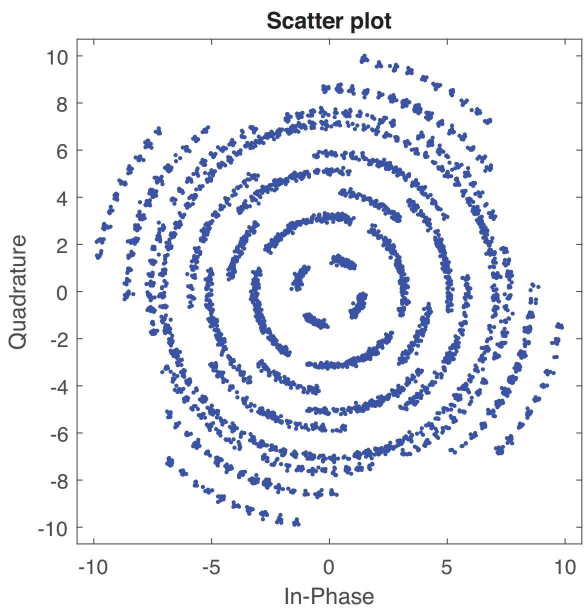


**FIGURE 4** symbols of a 64-QAM constellation hit by CFO $\epsilon = 0.01$.

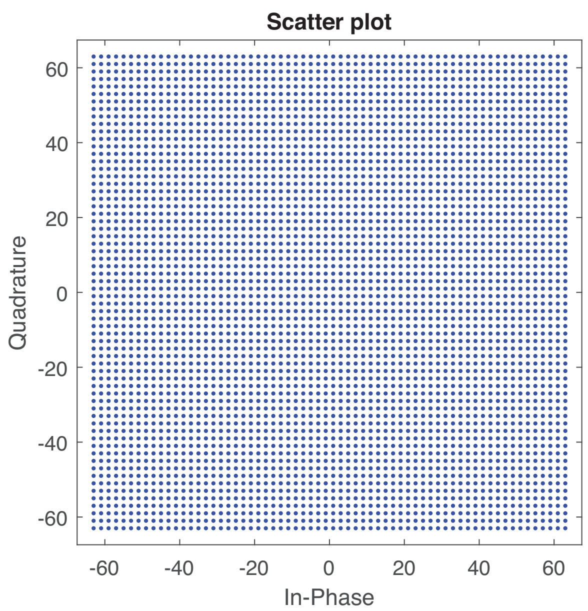


**FIGURE 5** Symbols of a 4096-QAM constellation with no CFO.

of no CFO and the case where we have normalized CFO of $\epsilon = 0.01$, respectively.

From Figures 3 and 4 it is clear that the CFO has caused a distortion due to phase rotation and ICI on the constellation. However, it seems we still have some information of the original data. By comparing Figures 4 and 6 we can see that the effect of CFO on 4096-QAM is much more severe than 64-QAM and the constellation looks like a noise and we have lost information of the original data even for a very small value of CFO. So, this shows the importance of the study of hardware impairments in very high order constellations.

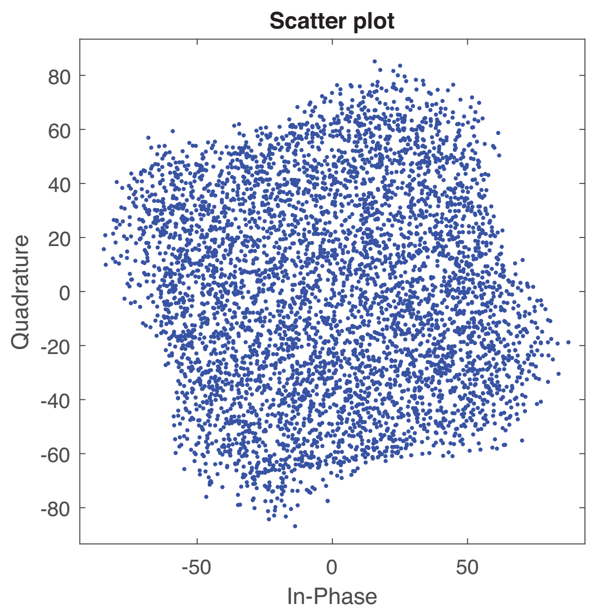


**FIGURE 6** Symbols of a 4096-QAM constellation hit by CFO $\epsilon = 0.01$.

## 4.2 | Maximum possible CFO for $M$-QAM error free detection

To have a better insight on the relation between the effect of CFO and the order of QAM constellation, in the following we will derive a closed form expression for the maximum normalized CFO that an OFDM system with $M$-QAM constellation can have in order to correctly detect $\hat{S}_k$ in Equation (10) as $S_k$. In an $M$-QAM modulation the real and imaginary parts of the symbol are selected from the set $\{\pm A, \pm 3A, \ldots, \pm(\sqrt{M}-1)\}$, where $A$ is a positive real number. Therefore, the minimum distance between two symbols will be $d_{\min} = 2A$ and in case that $M$ is even, the average symbol power of the constellation is computed as

$$\sigma^2 = \frac{4A^2}{\sqrt{M}} \sum_{m=1}^{\sqrt{M}/2} (2m-1)^2 = 2A^2 \frac{(\sqrt{M}+1)(\sqrt{M}-1)}{3}. \tag{11}$$

Thus for a fixed $\sigma^2$, the relation between the minimum distance and the order $M$ is as

$$d_{\min} = \sqrt{\frac{6\sigma^2}{M-1}}. \tag{12}$$

To detect $\hat{S}_k$ as $S_k$ with no errors, the difference between the real part of $\hat{S}_k$ and $S_k$ and the difference between imaginary part of $\hat{S}_k$ and $S_k$ should be smaller than half of the minimum distance, i.e. $\Delta R_k < d_{\min}/2$ and $\Delta I_k < d_{\min}/2$, where

$$\Delta R_k = \sum_{m=0}^{N-1} \left(\Re(C_{m-k})\Re(S_m) - \Im(C_{m-k})\Im(S_m)\right) - \Re(S_k) \tag{13}$$

and

$$\Delta I_k = \sum_{m=0}^{N-1} \left(\Re(C_{m-k})\Im(S_m) - \Im(C_{m-k})\Re(S_m)\right) - \Im(S_k). \tag{14}$$

In Equation (13) and (14), $\Re(Z)$ and $\Im(Z)$ indicate the real and imaginary part of $Z$, respectively. Considering the fact that the symbols on each subcarrier are independent with zero mean and variance $\sigma^2$, we have

$$E\left[|\Delta R_k|^2\right] = E\left[|\Delta I_k|^2\right] = \frac{\sigma^2}{2}\left(\sum_{m=0}^{N-1} |C_{m-k}|^2 + 1 - 2\Re(C_0)\right). \tag{15}$$

Therefore, to detect $\hat{S}_k$ correctly, the expression in Equation (15) has to be smaller than $d_{\min}^2/4$. For small CFOs we have

$$\begin{aligned}\sum_{m=0}^{N-1} |C_{m-k}|^2 &= \frac{1}{N}\sum_{m=0}^{M-1}\left|\sum_{i=0}^{N-1} \exp\left(\frac{j2\pi(m-k+\epsilon)}{N}\right)\right|^2 \\ &\simeq \frac{1}{N}\sum_{m=0}^{M-1}\left|\sum_{i=0}^{N-1} \exp\left(\frac{j2\pi(m-k)}{N}\right)\right|^2 = 1.\end{aligned} \tag{16}$$

Also, considering the Taylor series extension for cosine function we have

$$\begin{aligned}\frac{2}{N}\sum_{i=0}^{N-1} \cos\left(\frac{2\pi\epsilon i}{N}\right) &\simeq \frac{2}{N}\sum_{i=0}^{N-1}\left(1 - 2\left(\frac{\pi\epsilon i}{N}\right)^2\right) \\ &= 2 - \frac{2\pi^2}{3N^2}(N-1)(2N-1)\epsilon^2,\end{aligned} \tag{17}$$

for small CFOs. Thus, we can simplify Equation (15) for small CFOs as

$$E\left[|\Delta R_k|^2\right] = E\left[|\Delta I_k|^2\right] = \frac{\pi^2}{3N^2}\sigma^2(N-1)(2N-1)\epsilon^2 \tag{18}$$

Considering Equations (12) and (18), to detect $\hat{S}_k$ correctly the CFO has to satisfy the following inequality.

$$\epsilon \leq \frac{3N}{\pi}\sqrt{\frac{1}{2(N-1)(2N-1)(M-1)}} \overset{a}{\simeq} \frac{3}{2\pi}\sqrt{\frac{1}{M-1}}, \tag{19}$$

where $\overset{a}{\simeq}$ holds for large $N$. As it is seen, the maximum $\epsilon$ for an $M$-QAM constellation is proportional to the inverse of $\sqrt{M-1}$.

## 5 | CFO ESTIMATION TECHNIQUES

It is crucial to see whether previous CFO estimation techniques are sufficient to overcome CFO effects in very large order QAM modulations or better CFO estimation and compensation techniques need to be considered. In the following we will introduce two common used CFO estimation techniques to later investigate whether these techniques are efficient enough for high order constellation or not.

### 5.1 | CP based technique

The CP based technique is a blind technique. In this technique the correlation of the CP part of the received OFDM symbol and the end part of the received OFDM symbol which is associated to the CP is computed. Since these two parts are identical the CFO can be extracted from the correlation as

$$\hat{\epsilon} = \frac{1}{2\pi(N_{cp} - L + 1)}\sum_{i=-N_{cp}+L-1}^{-1} \angle y^*(n)y(N+n) \tag{20}$$

We should note that the first $L-1$ samples of the CP are not considered in the correlation, since the first $L-1$ samples are not equal to the first $L-1$ samples of the end $N_{cp}$ samples of the OFDM symbol due to the channel spread. The advantage of this technique is that it can be used any time during the transmission to follow and update the CFO estimation and synchronize the system without reducing the system spectral efficiency. The drawback of this technique is that it can only estimate the fractional normalized CFO. In other words it can only estimate $\epsilon$ for the range of $[-\frac{1}{2}, \frac{1}{2}]$. Another drawback of this technique is that its accuracy depends on the CP length and the channel spread. For larger CP lengths and smaller channel spread the estimation is better.

### 5.2 | Moose technique

In this technique two identical OFDM symbols are transmitted consecutively to estimate the CFO. Considering Equations (4) and (7) and keeping in mind the fact that the two transmitted OFDM symbols are identical and assuming the channel is slow fading, it can be shown that the two received signals in frequency domain in absence of noise are related as [6]

$$Y_k^{(1)} = Y_k^{(2)} e^{j2\pi\epsilon}, \tag{21}$$

where, $Y_k^{(1)}$ and $Y_k^{(2)}$ are the $k$th subcarrier data of first and second received OFDM symbol, respectively. Considering Equation (21), the CFO is estimated as

$$\hat{\epsilon} = \frac{1}{2\pi}\arctan\left(\frac{\Im\left(\sum_{k=0}^{N-1} Y_k^{(2)} Y_k^{*(1)}\right)}{\Re\left(\sum_{k=0}^{N-1} Y_k^{(2)} Y_k^{*(1)}\right)}\right). \tag{22}$$

**TABLE 1** Simulation parameters.

| Channel coding | LDPC |
|---|---|
| Code rate ($r_c$) | $\frac{1}{2}$ |
| FFT size ($N$) | 512 |
| CP size ($N_{cp}$) | 32 |
| Modulation type | QAM |
| Modulation order ($M$) | 64/ 256/4096 |
| Average symbol power ($\sigma^2$) | 1 |
| Channel type | Multipath Rayleigh fading with exponential power delay profile |
| Channel normalized delay spread | 3 |
| Number of multipath components ($L$) | 15 |
| Equalizer | Zero forcing |

The Moose technique is also able to estimate CFO in the range of $[-\frac{1}{2}, \frac{1}{2}]$. This technique has lower mean square error (MSE) and gives better CFO estimation compared to the CP based technique but at the cost of reducing spectral efficiency. This technique is usually used at the beginning of the transmission to synchronize the system. One of the drawbacks of this technique is that it cannot be used frequently to update the CFO estimation, since it reduces spectral efficiency.

## 6 | PERFORMANCE STUDY

In this section we will study the effect of carrier frequency offset on the performance of very high order QAM OFDM systems in terms of BER. We will also investigate whether the CP based and the Moose CFO estimation techniques are good enough for estimating and compensating the CFO in these systems or not.

Unless otherwise stated, for the simulations, we have assumed a $r_c = \frac{1}{2}$ LDCP code, an FFT size of 512 and a multipath Rayleigh fading channel with $L = 15$ paths and an exponential decaying power delay profile with normalized delay spread of 3. The considered equalizer is a single tap zero forcing (ZF) equalizer with coefficient $V_k = \frac{1}{H_k}$ at subcarrier $k$. We should note that the CFO estimation and compensation is done before the equalizer and we have considered perfect channel knowledge at the equalizer to solely study the effect of CFO on the BER. Table 1 summarizes the simulation parameters.

Figure 7 indicates the derived expressions for $E[|\Delta R_k|^2]$ (or $E[|\Delta I_k|^2]$) in Equations (15) and (18) versus $\epsilon$. As it is seen the approximation made in Equation (18) is very accurate. This figure also shows the maximum $\epsilon$ that each constellation order can have to have an error free detection of the symbols, according to Equation (19). According to this figure the maximum $\epsilon$ for 64-QAM, 256-QAM and 4096-QAM, is about 0.06, 0.03, and 0.007, respectively. To analyze the correctness of the theoretical studies done in Section 4.2, we have studied the BER performance of 64-QAM, 256-QAM, and 4096-QAM versus $\epsilon$ in Figure 8. In this figure we have assumed no channel and

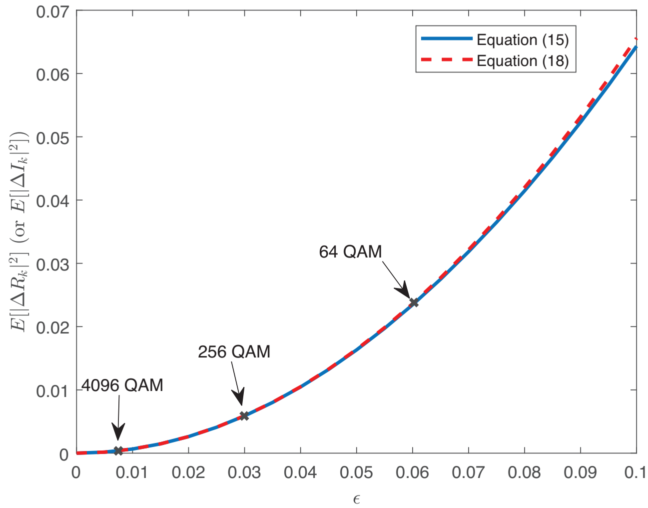


**FIGURE 7** $E[|\Delta R_k|^2]$ (or $E[|\Delta I_k|^2]$) derived in Equations (15) and (18) versus $\epsilon$.

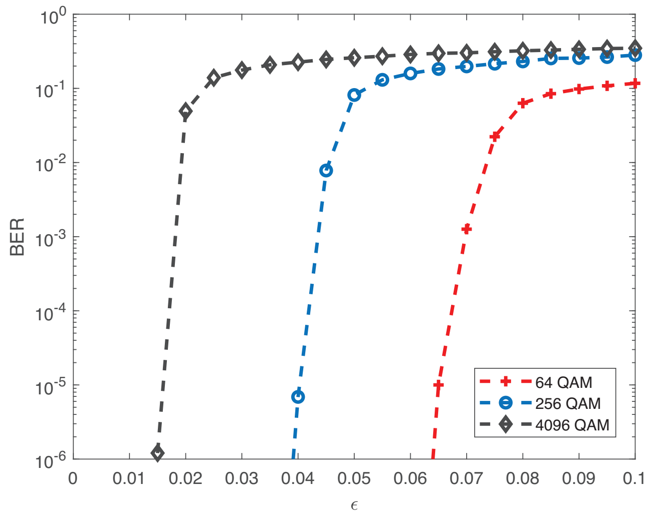


**FIGURE 8** BER versus $\epsilon$ for different $M$-QAM constellations.

noise to be consistent with the conditions considered in section 4. It can be seen that the dropping points of the BER curves are close to the maximum CFO values shown in Figure 7 for each constellation.

Figures 9–11 show the BER performance versus $E_b/N_0$ for 64-QAM, 256-QAM, and 4096-QAM, respectively. The red line in the figures shows the case where the system is completely synchronized and we have no CFO. This curve is the benchmark to see how good the residual CFO can be estimated and compensated with previous common CFO estimation techniques in the case of very large QAM constellations. The grey curve and the purple curve in the figures show the case where we have $\epsilon = 0.01$ and $\epsilon = 0.2$ that have not been estimated and compensated, respectively. The rest of the curves show the BER performance for different values of CFO that have been estimated and compensated either by the CP or the Moose technique. It can be concluded from the figures that the Moose

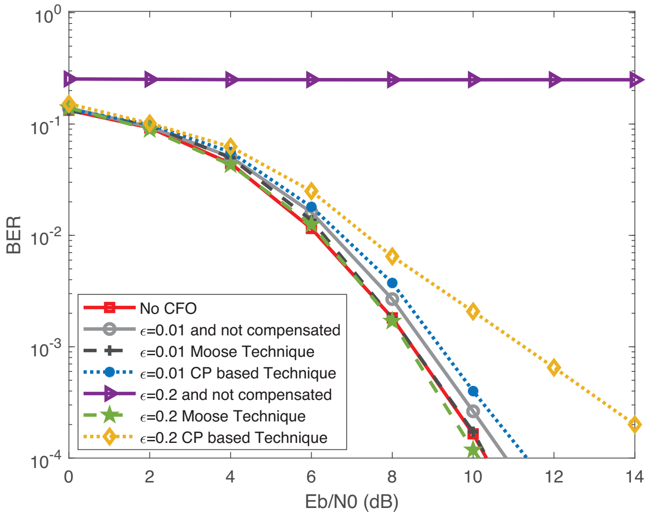


**FIGURE 9** BER versus $E_b/N_0$ for 64-QAM.

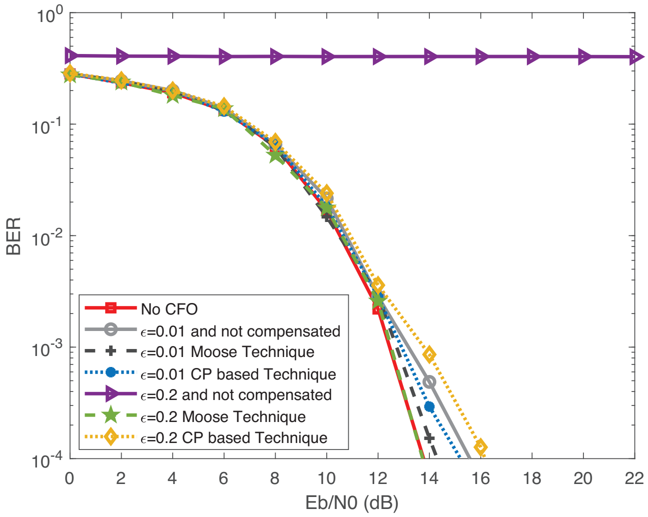


**FIGURE 10** BER versus $E_b/N_0$ for 256-QAM.

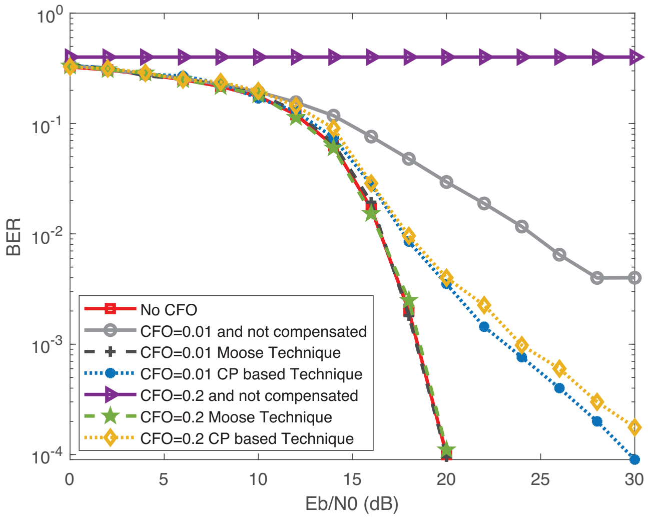


**FIGURE 11** BER versus $E_b/N_0$ for 4096-QAM.

technique outperforms the CP based technique, specially at higher order constellations. For example in 4096-QAM case, the moose technique can accurately estimate the CFO and compensate it for both cases, $\epsilon = 0.01$ and $\epsilon = 0.2$, and the BER performance is as the fully synchronized case. While for the CP based technique we need about 3 and 4 dB higher $E_b/N_0$, compared to the fully synchronized case, to have a BER of $10^{-3}$ in the case of $\epsilon = 0.01$ and $\epsilon = 0.2$, respectively.

## 7 | CONCLUSION

In this paper we studied the effect of CFO on the BER performance of an OFDM system with very high order QAM constellations. We derived a closed form expression for the maximum normalized CFO that an $M$-QAM constellation in an OFDM system can resist to have an error free detection. The results show that the performance of high order QAM constellations degrades greatly even for small residual CFO values such as $\epsilon = 0.01$. The results indicate the superiority of Moose technique over CP based technique in estimating and compensating the CFO, especially for higher order constellations. The Moose technique can estimate the CFO very well for these systems and there is no need for new complicated CFO estimation techniques to be proposed.

## AUTHOR CONTRIBUTIONS

Zahra Mokhtari: Conceptualization, formal analysis, investigation, software, writing - original draft, writing - review and editing. Rui Dinis: Conceptualization, funding acquisition, supervision, writing - review and editing. Luis Oliveira: Conceptualization, funding acquisition, supervision, writing - review and editing. Joao Oliveira: Conceptualization, funding acquisition, supervision, writing - review and editing.

## ACKNOWLEDGMENTS

This work was financially supported by FCT (Fundação para a Ciência e Tecnologia) under the scope of the projects CTS-Centro de Tecnologia e Sistemas UIDB/00066/2020, Instituto de Telecomunicações UIDB/50008/2020 and foRESTER PCIF/SSI/0102/201.

## CONFLICT OF INTEREST

The authors declared no conflict of interest.

## DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

## ORCID

*Zahra Mokhtari* https://orcid.org/0000-0002-3037-9555

## REFERENCES

1. Gheshlaghi, H.A., Feizizadeh, B., Blaschke, T., Lakes, T., Tajbar, S.: Forest fire susceptibility modeling using hybrid approaches. Trans. GIS 25(1), 311–333 (2020). doi: https://doi.org/10.1111/tgis.12688

2. Curva, J., Lourenço, M., Paulino, N., Oliveira, J.P., Oliveira, L., Oliveira, H.: Infrared fire alarm for vehicle protection. In: 2020 International Young Engineers Forum (YEF-ECE), pp. 19–24. IEEE, Piscataway, NJ (2020)
3. Abdullah, S., Bertalan, S., Masar, S., Coskun, A., Kale, I.: A wireless sensor network for early forest fire detection and monitoring as a decision factor in the context of a complex integrated emergency response system. In: 2017 IEEE Workshop on Environmental, Energy, and Structural Monitoring Systems (EESMS), pp. 1–5. IEEE, Piscataway, NJ (2017)
4. Singya, P.K., Shaik, P., Kumar, N., Bhatia, V., Alouini, M.S.: A survey on higher-order QAM constellations: technical challenges, recent advances, and future trends. IEEE Com. Soc. 2, 617–655 (2021)
5. Sharma, N., Anpalagan, A., Obaidat, M.S.: Evolutionary algorithms for wireless network resource allocation. In: Obaidat, M.S., Nicopolitidis, P., Zarai, F., (eds.) Modeling and Simulation of Computer Networks and Systems, pp. 629–652. Waltham: Morgan Kaufmann, Burlington, MA (2015)
6. Moose, P.H.: A technique for orthogonal frequency division multiplexing frequency offset correction. IEEE Trans. Commun. 42(10), 2908–2914 (1994)
7. van de Beek, J.J., Sandell, M., Borjesson, P.O.: ML Estimation of time and frequency offset in OFDM systems. IEEE Trans. Signal Process. 45(7), 1800–1805 (1997)
8. Schmidl, T.M., Cox, D.C.: Robust frequency and timing synchronization for OFDM. IEEE Trans. Commun. 45(12), 1613–1621 (1997)
9. Kay, S.: A fast and accurate single frequency estimator. IEEE Trans. Acoust, Speech, Signal Process. 37(12), 1987–1990 (1989)
10. Pedrosa, P., Dinis, R., Nunes, F.: Joint detection and CFO estimation for QAM constellations. In: 2011 IEEE 73rd Vehicular Technology Conference (VTC Spring), pp. 1–5. IEEE, Piscataway, NJ (2011)
11. Chang, K., Lee, S.: Robust OFDM-Based synchronization against very high fractional CFO and time-varying fading. IEEE Syst. J. 14(3), 4047–4058 (2020)
12. Meng, Y., Zhang, W., Stüber, G.L., Wang, W.: Blind fast CFO estimation and performance analysis for OFDM. IEEE Trans. Veh. Technol. 69(10), 11501–11514 (2020)
13. Mokhtari, Z., Sabbaghian, M., Eriksson, T.: Iterative channel and CFO estimation for SC-FDE and OFDM based massive MIMO systems. In: 2019 IEEE 89th Vehicular Technology Conference (VTC2019-Spring), pp. 1–5. IEEE, Piscataway, NJ (2019)
14. Zhu, J., Lee, W.: Carrier frequency offset estimation for OFDM systems with null subcarriers. IEEE Trans. Veh. Technol. 55(5), 1677–1690 (2006)
15. Zeng, X.N., Ghrayeb, A.: A blind carrier frequency offset estimation scheme for OFDM systems with constant modulus signaling. IEEE Trans. Commun. 56(7), 1032–1037 (2008)
16. Al-Dweik, A., Hazmi, A., Younis, S., Sharif, B., Tsimenidis, C.: Carrier frequency offset estimation for OFDM systems over mobile radio channels. IEEE Trans. Veh. Technol. 59(2), 974–979 (2009)
17. Yang, S., Lv, T., Hanzo, L.: Semidefinite programming relaxation based virtually antipodal detection for MIMO systems using gray-coded high-order QAM. IEEE Trans. Veh. Technol. 62(4), 1667–1677 (2013)
18. Halawa, T.N., Fathy, R.A., Zekry, A.: Performance analysis of LTE-A with 256-QAM. In: Sixth International Conference on Digital Information Processing and Communications (ICDIPC), pp. 141–145. IEEE, Piscataway, NJ (2016)
19. Carbonaro, K.B., Carrijo, G.A.: Analysis of cross M-QAM OFDM systems under Gaussian and impulsive noise. Int. J. Eng. Res. 4(7), 956–961 (2015)
20. Singya, P.K., Kumar, N., Bhatia, V., Alouini, M.S.: On performance of hexagonal, cross, and rectangular QAM for multi-relay systems. IEEE Access 7, 60602–60616 (2019)
21. Singya, P.K., Kumar, N., Bhatia, V., Alouini, M.S.: On the performance analysis of higher order QAM schemes over mixed RF/FSO systems. IEEE Trans. Veh. Technol. 69(7), 7366–7378 (2020)
22. Souto, N., Dinis, R.R.: Efficient MIMO detection for high-order QAM constellations in time dispersive channels. In: IEEE International Conference on Communications (ICC 2018), pp. 1–6. IEEE, Piscataway, NJ (2018)
23. Zhang, Q., Wang, J., Wang, Y.: Efficient QAM signal detector for massive MIMO systems via PS/DPS-ADMM approaches. IEEE Trans. Wireless Commun. 21(10), 8859–8871 (2022)
24. Changlong, L., Yinghui, C.: Design of high order QAM modulation and demodulation system. In: 2nd International Conference on Artificial Intelligence and Information Systems (ICAIIS), pp. 1–3. ACM, New York (2021)


**How to cite this article:** Mokhtari, Z., Dinis, R., Oliveira, L., Oliveira, J.: Impact and compensation of carrier synchronization errors in OFDM signals with very large QAM constellations. IET Commun. 17, 1118–1126 (2023). https://doi.org/10.1049/cmu2.12615