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ENVIRONMENT MONITORING AND ORGANIZATION STRUCTURE II

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1. Introduction

We have seen, in a companion paper (Lucena (1987,a), that the need to follow the dynamics of the environment will impose some restrictions on the set of the possible designs of organizations. There the analysis was restricted to hierarchies and polyarchies (and combinations of both) and the key concept was the idea of dynamically-independent partition. Such a partition represents a stable piece of information about the dynamics of the environment. It was shown that a department could operate independently of the others only if he was put in charge of keeping track of a piece of information represented by a dynamically--independent partition. A more general analysis is presented here, in which the class of designs allowed include the possibility of feed-back among departments. These designs can no longer be discussed on the basis of the concept of dynamic independence of partitions. The size of each department may be an important element in the cost of the organization. Take the limit case when this cost is overruling: i.e. it pays to have departments in charge of two-block partitions, rather than three or four block ones. It may be impossible to find a set of dynamically-independent two block partitions, whose product is "zero" partition. Then we must either accept more complex departments or accept the need of some communication between departments, and so work with partitions that are not dynamically-independent. In this last case the question arises of how to minimize the communication among departments.

2. Efficient Use of Information

The interest of dynamically-independent partitions results from the fact that they represent an amount of information that does not disappear in time. But that does not mean that we are using the information in the most efficient way. It may very well be that we are able to compute in which block of a partition, finer than the dynamically-independent one, we will be in, the next period. In a certain sense, the organizations discussed so far are informationally inefficient. A dual problem is whether, in order to compute the block of the dynamically-independent partition we will be in, next period, we need as much information as the information given by this partition. In general it is enough to have a smaller amount of information, represented by a coarser partition. Take an ordered pair, (e_1,e_2) of partitions. Call B_i the blocks of e_1 and B_j the blocks of e_2. We can generalize the idea of dynamic-independence to pair of partitions in the following way: each block of e_1 is mapped into a single block of e_2. Then, if we know e_1 we can compute the information e_2.

Definition: an ordered pair of partitions, (e_1,e_2) is dynamically-consistent if

$$\delta(z,s) \in B'_k \quad \text{when} \quad s \in B_j, \forall z \in Z$$

Notice that to say that $(e_1 . e_1)$ is a dynamically-consistent pair is equivalent to say that e_1 is a dynamically independent partition.

The definition refers to ordered pairs. The fact that (e_1,e_2) is a dynamically-consistent pair does not say anything about (e_2,e_1) also being such a pair.

If (e_1,e_2) is a dynamically-consistent pair, a department in charge of e_2 could operate if it received the information e_1. In general we cannot guarantee that $e_1=e_2$ (or better that $e_1 \leq e_2$) in which case it would need information from no other department. But e_1 may be too much information for that purpose. It is conceivable that (e_1',e_1), with $e_1' \geq e_1$, is also a dynamically-consistent pair.

We can introduce a partial ordering in the set of dynamically--consistent pairs. Take (e_1,e_2) and (e_1',e_2') as two such pairs. Then $(e_1,e_2) \geq (e_1,e_2')$ if $e_1 \geq e_1'$ and $e_2 \geq e_2'$. Also it can be easily proved that (e_1+e_1',e_2+e_2') and $(e_1 . e_1',e_2 . e_2')$ are also dynamically-consistent pairs. They define the greatest lower bound and least upper bound of the initial two pairs. The set of all dynamically-consistent pairs is a lattice under that partial ordering. (See Stearne and Hartmanis (1966)).

Now we can deal with the problem of the efficient use of information. Take a department in charge of the partition e_i. Then form all dynamically-consistent pairs (e_j,e_i), and compute $E(e_i)=\Sigma_j e_j$. This is the smallest amount of information the department in charge of e_i needs to operate. In partitions terms $E(e_i)$ is the largest partition with which e_i forms a pair.

Similarly take the partition E_i, form all dynamically-consistent pairs (E_i,E_j) and compute $e(E_i)=\pi_j E_j$. This is the largest amount of information about next state that can be extracted from E_i. A department receiving E_i as input can be in charge of any partition $e_i \geq e(E_i)$.

Definition 2: a pair of dynamically-consistent partitions, (E,e) is said to be informationally efficient if

i) e represents the maximum information that can be obtained about next period state, when E is the information about the current state.

ii) E is the minimum amount of information about the current state that will allow us to compute in which block of e we will be next period.

We assume that in these computations about next period state all the relevant information about the environment is available.

From any informationally efficient pair of partitions, (E,e) we can generate a lot of dynamically-efficient partitions, by refining E and/or coarsening e. In fact, by operating in this way in the set of all efficient pairs we can generate the set of all dynamically-consistent pairs. It is enough to see that any consistent pair has an informationally efficient pair associated.

3. Informational Efficiency and Structure

The concept of informational efficiency can be used to generate designs in which we do not need to impose the restriction of no feed-back among departments. To illustrate the uses of this concept take the case where we are interested in having only two-block partitions assigned to each department. Assume also that, by looking at the e partitions of the (E,e) set of informationally-efficient pairs we can find e_i, i=1,2,3, such that $\overset{3}{\underset{i=1}{\pi}} e_i=\pi_0$. Assign each one to a department. If, for example, e_1 is a dynamically-independent partition, then department 1 needs to receive no information from other departments, and $E_1 \leq e_1$. If $E_1\ e_1$, we must have some outside information fed into this department. Now, the department 1 knows always e_1. Then we must look if either $e_1e_2 \geq E_1$ or $e_1e_3 \geq E_1$. If it is the case that $e_1e_2 \geq E_1$ ($e_1e_3 \geq E_1$) then department one needs to receive information only from department 2(3). In general, then, given $\{e_i\}$, i=1...N as the two block-partitions used to define the departments, the minimization of the number of links obey the following systematic procedure:

i) check whether e_i is a dynamically-independent partition ($E_i \leq e_i$). If so it works independently.

ii) if not, try for $j \neq i$ if $e_i . e_j \geq E_i$. If this happens for any j, only department j needs to supply information to i.

iii) if this does not happen try $e_i e_j e_k \geq E_i$. If so only departments j and k need supply information to department i.

iv) ... an so on ...

In general the total information fed into department i does not coincide with E_i. There is still some waste. Call Ω_i to the partition representing this total information.

Notice that it may very well happen that department i feeds information to j and vice-versa. Notice also that the polyarchical and hierarchical decompositions studied in Lucena (1987,a) are particular cases of this procedure. In fact a pure polyarchy obtains if $E_i \leq e_i$:, $\forall i$. A pure hierarchy obtains if $E_1 \leq e_1; E_2 \leq e_1 e_2 \ldots E_N \leq e_1 \ldots e_N$.

It is not a matter of indifference how the partitions initially chosen are picked. The only condition they need obey, is that their product must be the zero partition. Otherwise we can chose any number of partitions with any number of blocks. Reducing the number of blocks in any partition - making departments "smaller" - must be traded off against reducing the number of partitions - reducing the number of departments. The reduction of the links among departments is obtained by picking with priority dynamically-independent partitions or, more generally, the second partitions of informationally efficient (E,e) pairs; and among these the ones for which the E element is as big as possible, leading to smaller needs in terms of information.

4. Reducing Costs of Observation

In all the discussion so far we assumed that each department observed the environment in full detail. It would be interesting if we could reduce that detail. That is indeed possible, and the procedure does not interfere with the reduction in dependencies between departments we just discussed. The idea is similar to the one present in Lucena (1987,a). Now we have a partition Ω_i, representing the information fed into department i. All states in same block of Ω_i are mapped into the same block of e_i, independently of the input. The image blocks may be different for different inputs. If two observations z_1 and z_2 map each block of Ω_i in the same block of e_i, it is not relevant for department i to distinguish between them.

Definition 3: two observations z_1 and z_2 are (Ω_i,e_i)-equivalent (or i-equivalent) if

$$\delta(z_1,B_j) \in B'_k \iff \delta(z_2,B_j) \in B'_k$$

when B_j is a block of Ω_i and B'_k a block of e_i.

We can now form a partition, λ_i, in the set of observations by having all the i-equivalent states in the same block of λ_i. This partition represents the amount of information about the environment that department i needs in order to keep track of the "dynamics of the system".

5. Decentralizing Decisions

The possibility of decentralizing decisions can be analysed much in the same way as was done for the simpler case discussed in

Lucena (1987,a). Notice that, by construction $\Omega_i \le e_i$, and we can say that Ω_i is the knowledge each department has about the state of nature. The concept of a partition sufficient for decision d_i can now be used by substituting the Ω_i for the π_i. For example, if $\Omega_i \le W_k$, the department i can decide alone about variable d_k. With this adaptation we can remake the analysis made before, changing the propositions in an obvious way.

6. Independence of Departments

In certain cases a big prize can be put in the independence of departments. However, if there are no dynamically-independent partitions, or not enough of them, this is impossible. But the use of partitions introduces a limitation in the description of information structures. Suppose that we have three states, and the information available allows to separate states one from two, but gives no information about state three. The way to represent this information is $\{\overline{13};\overline{23}\}$, which is a cover, but not a partition. A cover of S is a set of subsets (blocks) of S, $\{B_i\}$, such that (i) $\bigcup_i B_i = S$ and (ii) $B_i \subseteq B_j$ iff i=j.

The set of all covers of a set forms a lattice under the operations:

$$\gamma_1 + \gamma_2 = \text{el}\{B \mid B \in \delta_1 \ \text{ or } \ B \in \gamma_2\}$$

$$\gamma_1 \cdot \gamma_2 = \text{el}\{B \cap B' \mid B \in \gamma_1 \ \text{ and } \ B \in \gamma_2\}$$

where el{.} is the set of blocks obtained by eliminating the ones that are proper subsets of another, and B,B' are blocks of the covers. For more details about the algebra of covers see Stearne and Hartmanis (1966).

Just like a partition, a cover will be dynamically-independent if the states in the same block lead to states also in the same block, for any realization of current observation:

$$\delta(z,s) \subseteq B'_k \quad \text{if} \quad s \in B_j \ , \ \forall z$$

Again, notice that K may change with z.

The set of partitions being a subset of the set of covers, there may exist dynamically-independent covers, where no dynamically-independent partitions existed. It is immediate

PROPOSITION 1: a pure polyarchy with N departments is admissible if there are N dynamically-independent covers, γ_i, i 1...N, such that $\pi_i \gamma_i = \pi_0$.

PROPOSITION 2: a pure hierarchy with N departments is admissible if there are N dynamically-independent covers, γ_i, i=1...N, such that $\gamma_1 \geq \gamma_2 \geq \cdot \geq \gamma_N = \pi_0$.

In the first case department i is assigned the cover γ_i. In the second case define $\tau_1 = \gamma_1$, and $\gamma_{i-1}\tau_i = \gamma_i$, i=z,...,N; then assign τ_i to department i.

7. Efficient Use of Information with Covers

The set of dynamically-independent covers forms a sub-lattice of the lattice of all covers. Just like in the case of partitions a look to this (sub-)lattice will indicate the possible polyarchical--hierarchical designs. To avoid again the implicit restriction of no feed-back, we can generalize to the case of covers the idea of dynamically-consistent pair of partitions.

A pair of covers, (γ_1, γ_2) is dynamically-consistent if each block of γ_1 is mapped into a block of γ_2. Again, if (γ_1, γ_2) and (γ_1', γ_2') are dynamically-consistent pairs it can be seen that $(\gamma_1+\gamma_1', \gamma_2+\gamma_2')$ and $(\gamma_1 \cdot \gamma_1', \gamma_2 \cdot \gamma_2')$ are also dynamically-consistent pairs. Then if (γ_1, γ_j'), $j=1 \ldots N$ are pairs $(\gamma_1, \pi_j \gamma_j')$ is a pair; and if (γ_j, γ_1') are pairs $(\Sigma_j \gamma_j, \gamma_1)$ is a pair. This leads in a natural way to the notion of informationally-efficient pairs of covers, that can be used to generate additional designs. The technique is similar to the one developed in the first part of this paper, and it is not repeated here.

8. Decentralizing Decisions

The basic condition to decentralize decisions is that a department must have enough information about the states of nature to make that particular choice. If the optimal decision about d_i is the same for two states, then even someone who cannot distinguish among these two can pick the "right" value for d_i. This suggest very directly that the information structure (about the states) relevant for the analysis is a partition. In some circumstances, however, it may be useful to work with covers. The idea is the following. Suppose that we have three states of nature and that the best decisions about d_1 and d_2 (binary variables) are given by:

States	(d_1,d_2)
1	(1,0)
2	(0,0)
3	(–,1)

where (–,1) means that the payoff is independent of d_1 when state 3 materializes. In that case the decision about d_1 is irrelevant, and only d_2 counts.

The information needed to decide about d_2 is $\{\overline{12};\overline{3}\}$, a partition. But the information needed to decide about d_1 is given by $\{\overline{13};\overline{23}\}$, a cover. If in the first block decide $d_1=1$; if in the second decide $d_1=0$. In general the information structure induced by variable d_i into the state space is described by a cover (W_i). A department with information γ_i can then decide alone d_i iff $\gamma_i \leq W_i$. The analysis of decentralization of decisions is then similar to the one made with partitions (Lucena (1987,a)).

REFERENCES

HARTMANIS, J.; STEARNE, R. (1966) - Algebraic Structure Theory of Sequential Machines, Englewood-Cliffs: Prentice-Hall.

LUCEÑA, D. (1987) - Environment Monitoring and Organization Structure I, Working Paper nº 70, New University of Lisbon, 1987.

LUCENA, D. (1987) - A Note on the Representation of Information Structure, Working Paper nº 71, New University of Lisbon, 1987.

MARSCHAK, T. (1986) - Organization Design in Arrow, K. and Intriligator, M. (eds.), Handbook of Mathematical Economics, Vol. III, Chapter 27, North-Holland.

SAH, R.K. and STIGLITZ, J.E. (1985) - Human Fallibility and Economic Organizations, American Economic Review Proceedings, May 1985, 79, 292-297.