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ESSAY ON SECULAR STAGNATION

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Abstract

In this paper we show that a closed economy, with a balanced budget and unable to increase public spending, can avoid or leave a persistent slump through adequate and timely combination of monetary and fiscal policy based on distortionary taxation. We use a three generations OLG New Keynesian model in which a permanent slump is possible without any self-correcting force to full-employment. Complementing recent work on *Secular Stagnation* using lump-sum taxation and government spending as fiscal instruments, our contribution is to use distortionary taxes over labor, consumption and capital, in a balanced budget environment with constant (or decreasing) government spending.

Keywords: Secular stagnation, liquidity trap, distortionary taxes

1 Introduction and literature review

During IMF 14th annual research conference in November 2013 Larry Summers resurrected the expression “Secular Stagnation” firstly used in 1939 by Alvin Hansen, the President of the American Economic Association, where he stated that the Great Depression could be the start of an era of economic stagnation and of permanent and significant level of unemployment. An oversupply of savings and a low birth rate were then considered relevant causes of a restrained aggregate demand. Analogously, Summers suggests that similar conditions could be applied to the US after 2008, and could have been in place well before the crisis, although disguised by the recent housing bubble. The case of Japan during the last 20 years of economic stagflation may well also be regarded as part of the same phenomena, as referred by Krugman in the New York Times two months before Summers speech, where he mentioned that the “secular stagnation” hypotheses could be a natural epilogue of his research on the “liquidity trap”. Furthermore, in his recent book, “Le capital au XXIe siècle”, Piketty argues that the generous growth rates of output per capita during the period between 1970 and 2010 were one of a few exceptions in human history, and that we should expect growth rates not much greater than 1% during this century.

Although there is not a unanimous definition for “secular stagnation”, it is nevertheless also not consensual that it could be just a long period of slow growth, which by the way, may be a common situation from now on according to Piketty and other economists. Moreover there seems to be a fairly strong agreement among economists on some aspects of secular stagnation: When negative real interest rates are needed to balance savings and investment with full employment, which becomes difficult, and even impossible to achieve with low inflation due to a zero lower bound on nominal interest rates. This phenomenon may become persistent, and conventional monetary policy may be insufficient to achieve simultaneously full-employment, growth and financial stability, as economic bubbles, for example, become more likely in environments of low nominal interest rates, high propensity for savings, and low demand for loans.

Factors like higher borrowing restrictions from financial institutions due to financial crisis, or lower birth rates, contribute to lower demand for loans, creating a downward pressure on interest rates. In the loans market supply side, financial crisis increase the need and propensity for households, companies, and financial institutions to invest in safe assets rather than in riskier ones (see Caballero and Fahri 2014). Moreover, a decreasing trend of the relative price of durable goods, and thus of the cost of investment, leaves more funds left for savings, also creating a downward pressure on interest rates. Furthermore, when inflation is low, nominal wage rigidities may create an upward pressure on real wages further depressing output as well as the natural rate of interest consistent with full-employment. Nominal interest rate zero lower bound together with low inflation may prevent real interest rate to decrease to the level of the natural rate of interest, thus creating the conditions for a permanent slump. Explaining secular stagnation and its causes is the purpose of a recent paper by Eggertson and Mehrotra (2014) who propose a model where a slump is possible without any self-correcting force to full-employment, and where public spending together with lump-sum taxes are used as fiscal policy instruments.

The purpose of this paper is, instead, to show how to avoid a secular stagnation and to leave a liquidity trap once there, using a combination of monetary and fiscal policy. We use the model introduced by Eggertson and Mehrotra (2014) mentioned above, and our contribution is to introduce distortionary taxation on consumption, labor and capital, in a setting where increasing public spending may not be an option, and a period by period balanced budget an obligation ensured by a sort of *"Taylor rule"* for fiscal policy.

2 Description of the closed economy model

The simple OLG model we propose allows for steady state equilibriums with persistent negative real interest rates, when population growth is low enough or when credit constraints are present. We will present the model starting with (i) a simple endowment economy with population growth, (ii) adding credit constraints, (iii) endogenous output, (iv) fiscal policy, (v) monetary policy, and (vi) capital.

Endowment economy:

In the model, households go through three stages of life: young, middle aged and old. The young generation borrow from the middle aged; the middle aged save by lending to the young, and pay back their loans to the previous generation, then old. The old receive back with interest what they have lent to the young when middle aged. Lending is constrained by a binding debt limit faced by the young, exogenously determined and independent of any fiscal instrument. For simplification, income is only earned by the middle aged. For this simple endowment economy the household objective function and budget constraints are given below:

$$\max_{C_t^y, C_{t+1}^m, C_{t+2}^o} E_t\{\log(C_t^y) + \beta\log(C_{t+1}^m) + \beta^2\log(C_{t+2}^o)\} \tag{1}$$

$$\text{s.t.} \quad C_t^y = B_t^y \tag{2}$$

$$C_{t+1}^m = Y_{t+1} - (1 + r_t)B_t^y - B_{t+1}^m \tag{3}$$

$$C_{t+2}^o = (1 + r_{t+1})B_{t+1}^m \tag{4}$$

Furthermore, loan market equilibrium implies that total borrowing of young generation equals total savings of middle aged, or $N_t B_t^y = N_{t-1} B_t^m$, where N_t is the size of generation *t*. Then

$$(1 + g_t)B_t^y = B_t^m \tag{5}$$

where $g_t = N_t/N_{t-1}$ is population growth. For constant endowment the equilibrium level real interest rate is given by $1 + r_t = (1 + g_t)/\beta^2$, which can be negative if $1 + g_t < \beta^2$ without any further constraints to the model.

Adding Credit Constraints to the model:

By adding a lending constraint through an exogenously determined binding borrowing limit faced by the young, $$(1 + r_t)B_t^y \leq D_t \tag{6}$$

we get an expression for the borrowing of the young B_t^y, and savings of the middle aged B_t^m given by $$B_t^y = \frac{D_t}{1+r_t} \quad \text{and} \quad B_t^m = \frac{1+g_t}{1+r_t} D_t \tag{7}$$

and an equilibrium real interest rate, which can be negative, given by the expression:

$$1 + r_t = \frac{1+\beta}{\beta} \frac{(1+g_t)D_t}{Y_t - D_{t-1}} \tag{8}$$

(iii) Introducing Endogenous Output into the model

Expression (8) does not change when we add endogenous output to the model, by replacing constraint (3) by: $$C_{t+1}^m = \left[z_{t+1} + w_{t+1}L_{t+1}\right] - (1 + r_t)B_t \ - B_{t+1}^m \tag{9}$$

Where, for now, the firm problem has no capital and is given by:

$$Z_t = \max_{L_t} Y_t - w_t L_t \quad \text{s.t} \quad Y_t = a_t {L_t}^{\alpha}, \quad \text{then } w_t = \alpha \frac{Y_t}{L_t} \tag{10}$$

At full employment $L_t = \bar{L}$ and $Y_t^f = a_t \bar{L}^{\alpha}$. For simplicity we assume $a_t = a = 1$.Then Y_t^f is constant, which will not be the case when capital is introduced in the model. From (8), we can derive the expression for demand output:

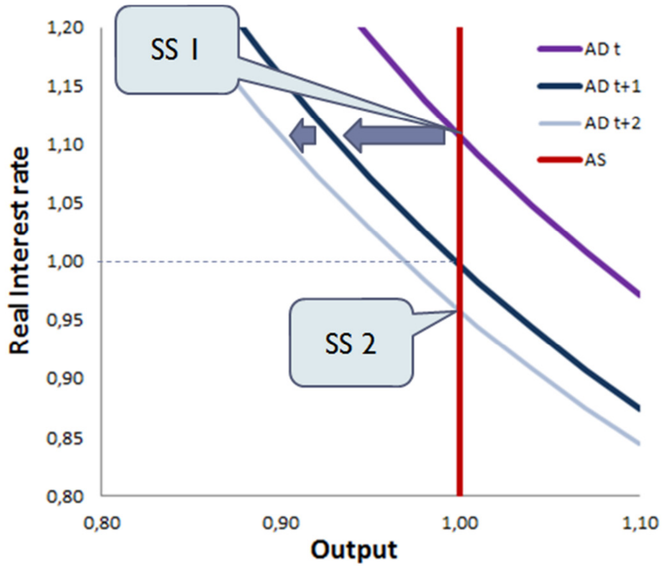
$$Y_t^d = \left(\frac{1+\beta}{\beta}\right) \frac{(1+g_t)D_t}{1+r_t} + D_{t-1} \tag{11}$$

And we can establish an expression for the real interest rate for a full employment equilibrium, which we will call the *natural rate of interest* r_t^f:

$$1 + r_t^f = \left(\frac{1+\beta}{\beta}\right) \frac{(1+g_t)D_t}{Y^f - D_{t-1}} \tag{12}$$

The graph below shows that a reduction of the borrowing limit will shift the demand curve twice to the left during two consecutive periods. Full-employment equilibrium will ask for a lower real interest rate equal to r_t^f which can become negative.

Figure 1: Demand output, and full-employment real interest rate



3. Fiscal policy with distortionary taxation

In this chapter we introduce distortionary taxation into the model, derive expressions for demand and real interest rates, and analyze how those are affected by tax changes in this closed economy with borrowing constraints and a balanced budget. We will see how adequate fiscal policy may offset the recessionary effects of an increase of credit constraints. In Eggertson and Mehrotra's model taxes are lump-sum and paid by middle-age households. Firms don't pay taxes and household constraints are given by:

$$C_t^y = B_t^y - T_t^y \tag{13}$$

$$C_{t+1}^m = z_{t+1} + w_{t+1}L_{t+1} - (1+r_t)B_t^y - B_{t+1}^m - T_{t+1}^m \tag{14}$$

$$C_{t+2}^o = (1+r_{t+1})B_{t+1}^m - T_{t+2}^o \tag{15}$$

Where $T_t^y = T_t^o = 0$, and $T_t^m = T_t = G_t$ which is government spending per middle age household. Instead we introduce distortionary taxes, where consumption taxes for the young and old households are given by $T_t^{y,o} = \tau_t^c C_t^{y,o}$, and consumption and labor taxes for average middle aged households are given by $T_t^m = \tau_t^c C_t^m + \tau_t^l w_t \; L_t$. Firms pay taxes on labor costs: $T_t^{su} = \tau_t^{su} w_t \; L_t$. Euler equation is now given by:

$$1 + r_t = \frac{1}{\beta} \frac{(1 + \tau_{t+1}^c)}{(1 + \tau_t^c)} \frac{C_{t+1}^0}{C_t^m} \tag{16}$$

Combining the Euler equation with the constraint given by expression (15) we get $B_t^m = \frac{(1+\tau_{t+1}^c) C_{t+1}^o}{(1+r_t)} = \beta(1 + \tau_t^c) C_t^m$, or $C_t^m = \frac{B_t^m}{\beta(1+\tau_t^c)}$. From loan market equilibrium (5), and the borrowing constraint (6) we get expressions for household consumption:

$$C_t^y = \frac{1}{1 + \tau_t^c} \left(\frac{1}{1 + r_t} \right) D_t; \; C_t^m = \frac{1}{1 + \tau_t^c} \left(\frac{1}{\beta} \frac{1 + g_t}{1 + r_t} \right) D_t; \; C_t^o = \frac{1}{1 + \tau_t^c} (1 + g_{t-1}) D_{t-1}$$

Total consumption in period t is given by: $C_t^{Total} = N_{t+1} C_t^y + N_t \; C_t^m + N_{t-1} C_t^o$, from which the expression for total consumption per middle aged household is derived:

$$C_t \; = (1 + g_t) C_t^y + C_t^m + \frac{C_t^o}{(1 + g_t)} = \frac{1}{1 + \tau_t^c} \left[\left(\frac{1 + \beta}{\beta} \right) \frac{(1 + g_t) D_t}{1 + r_t} + D_{t-1} \right] \; \Leftrightarrow$$

$$(1 + \tau_t^c) C_t \; = \left(\frac{1 + \beta}{\beta} \right) \frac{(1 + g_t) D_t}{1 + r_t} + D_{t-1} \tag{17}$$

An expected result in this binding credit constrained closed economy, where equilibrium consumption is a negative function of consumption tax. Then, from (17), assuming a balanced budget, $T_t = G_t$, and using market clearing condition for goods, $Y_t - C_t = G_t$, expressions for demand and real interest rate are given by:

$$Y_t^d = C_t + G_t = \frac{1}{1 + \tau_t^c} \left[\left(\frac{1 + \beta}{\beta} \right) \frac{(1 + g_t) D_t}{1 + r_t} + D_{t-1} \right] + G_t \tag{18}$$

$$1 + r_t = \left(\frac{1 + \beta}{\beta} \right) \frac{(1 + g_t) D_t}{(1 + \tau_t^c)(Y_t - G_t) - D_{t-1}} \tag{19}$$

As expected in this constrained environment, demand expands for lower consumption taxes. And real interest rate increases for lower consumption tax, for constant output.

In appendix we derive alternative expressions for Y_t^d and r_t based on the middle-aged constraint (14):

$$Y_t^d = \left(\frac{1}{1-\alpha A_t}\right)\left[\left(\frac{1+\beta}{\beta}\right)\frac{(1+g_t)D_t}{1+r_t} + D_{t-1}\right] = \frac{1+\tau_t^c}{1-\alpha A_t} C_t, \text{with } A_t = \frac{\tau_t^{su}+\tau_t^l}{1+\tau_t^{su}} \quad (20)$$

$$1+r_t = \left(\frac{1+\beta}{\beta}\right)\frac{(1+g_t)D_t}{(1-\alpha A_t)Y_t - D_{t-1}} \quad (21)$$

Equivalence of aggregate demand expressions (18) and (20) under a balanced budget indicates that, with constant public spending, an increase in labor taxes gives room for reducing consumption taxes which will cause a demand expansion. This statement points to a relation between fiscal instruments and aggregate demand, illustrated by expression (22) derived from the above two alternative expressions for Y_t^d, together with the expression for total taxes per middle aged household:

$$T_t = \tau_t^c C_t + (\tau_t^{su} + \tau_t^l)\, w_t L_t = \tau_t^c C_t + \alpha A_t Y_t = \left(\frac{\tau_t^c + \alpha A_t}{1+\tau_t^c}\right) Y_t^d = \left[1 - \left(\frac{1-\alpha A_t}{1+\tau_t^c}\right)\right] Y_t^d =$$

$= Y_t^d - C_t = G_t$ when the budget is balanced. This equality is equivalent to:

$$\frac{G_t}{Y_t^d} = 1 - \left(\frac{1-\alpha A_t}{1+\tau_t^c}\right) \Leftrightarrow Y_t^d = G_t \frac{(\tau_t^c + 1)}{(\tau_t^c + \alpha A_t)} \Leftrightarrow A_t = \frac{1}{\alpha}\left[\frac{G_t - \tau_t^c\left(Y_t^d - G_t\right)}{Y_t^d}\right] \quad (22)$$

Any equality in expression (22) works like a "Taylor rule" for fiscal policy that ensures the budget is balanced on a period by period basis. We will call it the *fiscal rule*, which implies that at least one fiscal instrument is endogenously determined. For the time being we will assume this is the case for $A_t = \frac{\tau_t^{su}+\tau_t^l}{1+\tau_t^{su}}$. Moreover, labor supply is inelastic which, at full employment, makes taxes on labor non-distortionary.

As seen in previous chapter, full employment can be kept after a credit shock or a reduction of population growth as long as the real interest rate sticks to the *natural rate of interest*, now given by expression below, due to the distortionary taxation setting:

$$1+r_t^f = \left(\frac{1+\beta}{\beta}\right)\frac{(1+g_t)D_t}{(1+\tau_t^c)(Y^f - G_t) - D_{t-1}} = \left(\frac{1+\beta}{\beta}\right)\frac{(1+g_t)D_t}{(1-\alpha A_t)Y^f - D_{t-1}} \quad (23)$$

But as the *Natural Rate of Interest* decreases with a reduction of the borrowing limit, it may become negative and not reachable by the real interest rate if constrained by a zero lower bound for nominal interest rates together with a low inflation level. Full-employment equilibrium remains possible if the natural rate of interest increases back to an admissible real interest rate level, through a reduction of the consumption tax compensated by an increase of labor taxes in order to fulfill the *fiscal rule* given by expression (22). This would shift back to the right aggregate demand so that the intersection with full employment supply curve ($Y = Y^f$) moves up to an admissible real interest rate level so that a full employment equilibrium persists and a secular stagnation is avoided. The purpose of the next chapter is to analyze how to avoid a slump using fiscal and monetary policy.

4 Avoiding a Secular Stagnation

To avoid a liquidity trap, caused by a credit shock or a reduction of population growth, policy makers have a combination of two options: (i) To increase back with fiscal policy full-employment real interest rate r^f, if the real interest rate cannot be sufficiently reduced. (ii) Or alternatively, to unblock with monetary policy the required reduction of the real interest rate to the new lower level of the natural rate of interest r^f. Let's start by analyzing the first option, in a model with constant prices and flexible wages.

4.1. Avoiding a slump with fiscal policy

4.1.1. Constant prices and flexible wages

In face of a credit shock, with lowering borrowing limits, the way to maintain consumption level is to reduce consumption tax: If the economy is at full-employment in steady state 1 corresponding to a borrowing constraint D_1, where an exogenously determined real interest rate r_t is equal to the full employment real interest rate r_1^f, and there is a credit shock corresponding to a reduction of D_1 to D_2, then from expression

(23), all other parameters constant excepting taxes, full employment level is kept in steady state 2 if:

$$r_2^f = r_1^f = r_t \quad \Leftrightarrow \quad \frac{1+\tau_2^c}{1+\tau_1^c} = \frac{D_2}{D_1} = \frac{1-\alpha A_2}{1-\alpha A_1}, \quad \text{where } A_t = \frac{\tau_t^l + \tau_t^{su}}{1+\ \tau_t^{su}} \tag{24}$$

This states that a reduction of the borrowing limit can be compensated by a decrease in consumption taxes in order to keep constant the natural rate of interest. A reduction of the borrowing limit would shift the demand curve to the left, reducing the *natural rate of interest*, which could become lower than the lower admissible real interest rate level, causing a slump. But a sufficient reduction of consumption tax would neutralize the previous effect by moving back the demand curve to its previous position, thus increasing the *natural rate of interest* r_2^f back to its initial level r_1^f. Increasing one of the labor taxes, or both, is required to fulfill the *fiscal rule*. This process may be helpful in a model with nominal prices and nominal interest rates zero lower bound, when the *natural rate of interest* can become smaller than the real interest rate lower admissible level limit.

4.1.2. Introducing nominal price determination in the model, with flexible wages

From now on nominal prices are considered as in Eggertson and Mehrota's model, where nominal interest rate follows the Taylor rule. The household problem maximization constraints are given by:

$$P_t(1+\tau_t^c)C_t^y = P_t B_t \tag{25}$$

$$P_{t+1}(1+\tau_{t+1}^c)C_{t+1}^m = Z_{t+1} + W_{t+1}L_{t+1}\left(1-\tau_{t+1}^l\right) - (1+i_t)P_t B_t \ + P_{t+1}B_{t+1}^m \tag{26}$$

$$P_{t+2}(1+\tau_{t+2}^c)C_{t+2}^o = -(1+i_{t+1})P_{t+1}B_{t+1}^m \tag{27}$$

Exogenous borrowing limit: $(1+i_t)P_t B_t \ \leq P_{t+1}D_t$ (28)

Fisher equation is given by: $(1+i_t) = (1+r_t)\frac{P_{t+1}}{P_t}$ (29)

And the Taylor rule: $(1+\mathrm{i_t}) = \max\left\{1\ ,\ (1+i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right\}$, where $\phi_\pi > 1$ (30)

The Euler equation is now given by: $C_t^m = \frac{1}{\beta}E_t C_{t+1}^o\left(\frac{1+\tau_{t+1}^c}{1+\tau_t^c}\right)\frac{1}{(1+i_t)}\frac{P_{t+1}}{P_t}$ (31)

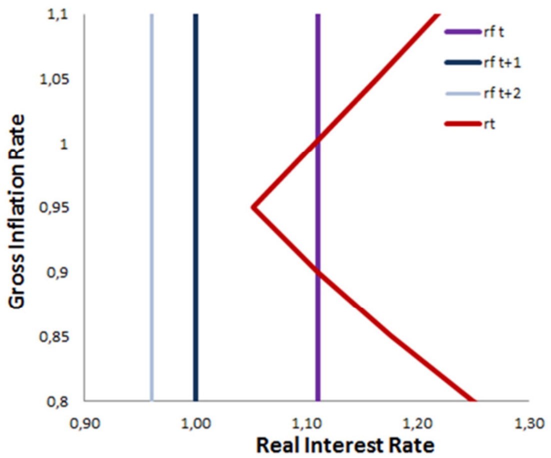
Factors in "t + 1" are canceled due to the loan market equilibrium and the borrowing limit for the young, and expressions for aggregate demand and real interest rates are the same as the ones derived in chapter 3 (see appendix).

Using the Taylor rule combined with the Fisher equation we get the following alternative expression for the real interest rate:

$$(1+r_t) = \frac{1+i_t}{\Pi_t} = max\left\{\frac{1}{\Pi_t}\ ,\quad \frac{1+i^*}{\Pi^*}\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi - 1}\right\} \geq \frac{(1+i^*)^{\frac{1}{\phi_\pi}}}{\Pi^*} = 1 + r^{kink} \quad (32)$$

If r_t^f becomes lower than r_t^{kink} (figure 2) after a credit shock then a full-employment equilibrium is not allowed, unless a sufficient consumption tax reduction (offset by an increase of other taxes) compensates for the decrease of the borrowing limit, as seen in previous chapter.

Figure 2: real interest rate function of inflation, and natural rate of interest



From expression (24), to keep r^f constant $\partial\tau_t^c = \frac{\partial D_t}{D_t}(1+\tau_t^c)$. Then a 10% reduction of the borrowing limit D, in an effort to keep a full-employment equilibrium, would require a reduction of more than 10 percentage points of the consumption tax, compensated by an increase of labor income tax by more than 10 percentage points. Although theoretically possible, trying to avoid a slump in such conditions using fiscal policy only, when government spending cannot increase, may be difficult to achieve. Monetary policy can then be a complement or an alternative, if available.

4.2. Avoiding a slump with monetary policy

Still with flexible wages…

To allow a full-employment equilibrium, an alternative to fiscal policy is to decrease the real interest rate lower bound r^{kink} to a level lower than the *natural rate of interest* r^f.

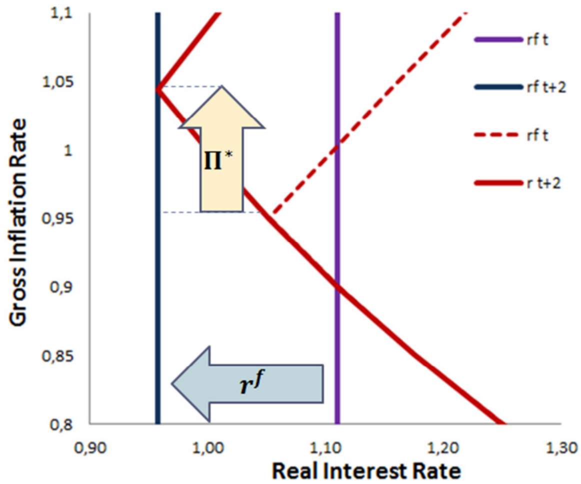
$r^{kink} = \frac{(1+i^*)^{\frac{1}{\phi_\pi}}}{\Pi^*} - 1$ is a function of monetary policy instruments, namely target inflation and target nominal interest rate. Since target nominal interest rate is positive because of ZLB, then r^{kink} is smaller than the target real interest rate $r^* = \frac{1+i^*}{\Pi^*} - 1$: Then r^* belongs to the set of admissible equilibrium real interest rates, as proven below:

$$1 + r^{kink} = \frac{(1+i^*)^{\frac{1}{\phi_\pi}}}{\Pi^*} = \frac{1+i^*}{\Pi^*}(1+i^*)^{\frac{1}{\phi_\pi}-1} \le (1+r^*) \quad \text{for } i^* \ge 0, \phi_\pi \ge 1 \quad (33)$$

For $i^* = 0$, $r^{kink} = r^*$, whereas a negative r^* implies a positive inflation target $1 \le \frac{1}{1+r^*} \le \Pi^*$. Furthermore, the Taylor rule for $\Pi_t \ge 1$ is equivalent to $\left(\frac{1+r_t}{1+r^*}\right) = \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi - 1}$. Then if r_t^* is set equal to r_t^f, then $\Pi_t = \Pi^*$ and $i_t = i^*$. This equality between instruments and variables can ensure monetary policy effectiveness for positive inflation: A full-employment equilibrium is achievable if target real interest rate is set smaller or equal than the natural rate of interest r^f, in a model with flexible wages.

What this means in practical terms is that, if fiscal policy agents cannot keep a natural rate of interest higher than r^{kink} , then the Central Bank should react to forces leading to a slump by adjusting monetary policy so that target real interest rate decreases to remain fully consistent with the new lower *natural rate of interest* r^f, by decreasing i^* and increasing Π^*, as seen in figure 3.

Figure 3: real interest rate function of inflation, and natural rate of interest



Deriving Aggregate Supply and equilibrium output when wages are sticky

In this model, when wages are sticky and the natural rate of interest becomes negative because of a credit shock or a decreasing population growth, then a unique determinate secular stagnation equilibrium becomes available. A timely and assertive reaction of policy agents is of primal importance to prevent the economy from diving into a slump. Sticky wages are introduced in the model as in Eggertson and Mehrotra (2014): Households will not accept working for a wage lower than a nominal wage norm $\widetilde{W}$ given by $\widetilde{W} = \gamma W_{t-1} + (1-\gamma)P_t w_t^{flex}$, whereas nominal wage will allways be greater or equal than the flexible labor full-employment nominal wage: $W_t = \max\ \{\widetilde{W}_t, P_t w_t^{flex}\}$, where $w_t = \frac{W_t}{P_t} = \frac{\alpha}{(1+\tau_t^{su})}\frac{Y_t}{L_t} = \frac{\alpha a_t^{\frac{1}{\alpha}} Y_t^{\frac{\alpha-1}{\alpha}}}{(1+\tau_t^{su})}$, and $w_t^{flex} = \frac{\alpha Y_f^{\frac{\alpha-1}{\alpha}}}{(1+\tau_t^{su})}$ for $a_t = 1$. The short term AS curve is given by:

$$Y_t^{\frac{\alpha-1}{\alpha}} = \max\left\{\frac{(1+\tau_t^{su})}{(1+\tau_{t-1}^{su})}\frac{\gamma}{\Pi_t}Y_{t-1}^{\frac{\alpha-1}{\alpha}} + (1-\gamma)Y_f^{\frac{\alpha-1}{\alpha}}, Y_f^{\frac{\alpha-1}{\alpha}}\right\} \tag{34}$$

In the long run, Aggregate Supply steady state is given by:

$$Y_{ss}^{AS\,high} = Y^f, \text{for } \Pi_t \geq 1; \quad \frac{\gamma}{\Pi} = 1-(1-\gamma)\left(\frac{Y^{AS\,low}}{Y_f}\right)^{\frac{1-\alpha}{\alpha}}, \text{for } \Pi_t < 1 \quad (35)$$

This expression is independent of τ^{su} and all other fiscal instruments. For negative inflation, supply is and upward sloping function of inflation.

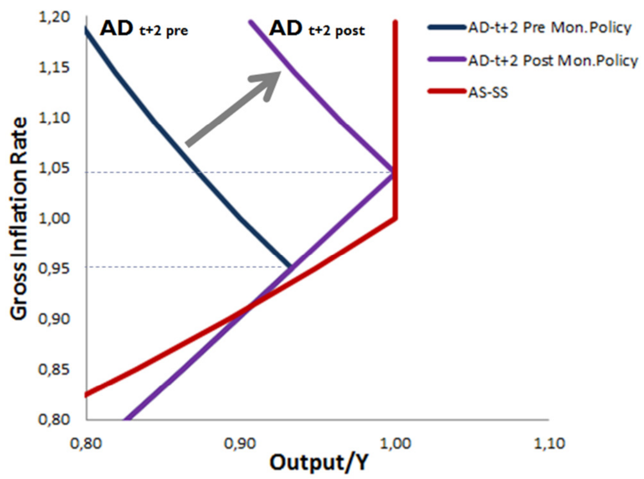
Moreover, we get the *aggregate demand* Y_t^d function of inflation by replacing the real interest rate expression (32) in the aggregate demand expression (18), given by:

$$Y_t^d = \begin{cases} Y_t^{d\,high} = \dfrac{1}{1+\tau_t^c}\left[\dfrac{(1+\beta)(1+g_t)D_t}{\beta}\dfrac{{\Pi_{kink}}^{\phi_\pi}}{{\Pi_t}^{\phi_\pi - 1}} + D_{t-1}\right] + G_t, \Pi_t > \Pi^{kink} & (36) \\ Y_t^{d\,low} = \dfrac{1}{1+\tau_t^c}\left[\dfrac{(1+\beta)(1+g_t)D_t}{\beta}\Pi_t + D_{t-1}\right] + G_t, \Pi_t \leq \Pi^{kink} & (37) \end{cases}$$

Where $\Pi^{kink} = \frac{\Pi^*}{(1+i^*)^{\frac{1}{\phi_\pi}}} = \frac{\Pi^{*\frac{\phi_\pi - 1}{\phi_\pi}}}{(1+r^*)^{\frac{1}{\phi_\pi}}}$.

As discussed in previous section, sustaining a full employment equilibrium when r^f becomes negative and fiscal policy cannot raise it back to a positive level, may require that monetary instruments stay consistent with monetary variables. (see figure 4 below).

. **Figure 4**: Avoiding a slump with monetary policy, and sticky wages



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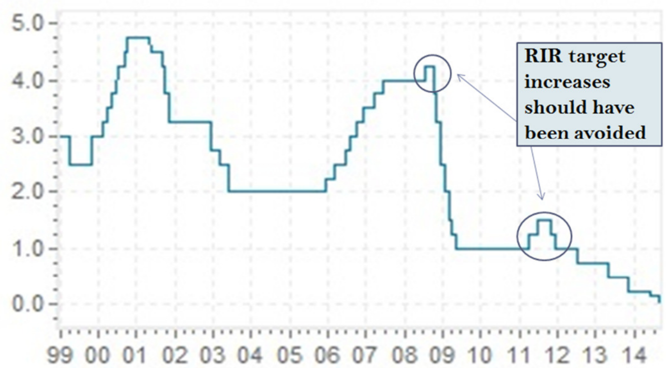
This means that $r^* = r^f < 0$ and $\Pi^* > 1$ because i^* is not negative. In that case $\Pi^{kink} > 1$. Moreover $\Pi^{kink} = \Pi^*$ for $i^* = 0$. For $\Pi = 1 < \Pi_{kink}$, $1 + r \quad = \frac{1}{\Pi} = 1 >$

$1 + r^f$ then $Y(\Pi = 1) < Y^f$ meaning that the lower AD segment crosses the vertical line $Y = Y^f$ at $\Pi^{AD\ flex} > 1$, when lower AS segment crosses the same vertical line at $\Pi = 1$. This is going to be important in the next chapter, besides being a crucial step to prove that there is a unique determinate secular stagnation equilibrium in the model.

From figure 4 we can infer that if r^f becomes negative, lack of assertiveness or consistency from fiscal and monetary policy agents may clear the full-employment equilibrium leaving the economy, even during a short period, only with a secular stagnation one, from where it is difficult to get out. This can have been the case in Europe when the ECB decided to increase the interest-rate targets in 2008 right after the Lehman Brothers crash, and in 2011 during the credit crash, which could have triggered and consolidated the creation of a stable and persistent liquidity trap in EU, where i^* can no longer be reduced and Π^* is not increasing for political matters. Leaving a slump is the purpose of the next chapter.

Figure 4: ECB interest-rate targets



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5 Leaving a Deflationary stable equilibrium

The purpose of the previous chapter was to discuss how to sustain the economy on a full employment equilibrium. In this chapter we will show how an economy can leave a liquidity trap: We will show that fiscal policy will play the most relevant role, that monetary policy is ineffective to clear from the model a secular stagnation, but may be a relevant tool in order to allow for a full employment equilibrium in the model to where the economy may migrate.

5.1. Eliminating a secular stagnation stable equilibrium

A secular stagnation is characterized by the intersection of the lower segments of AD and AS curves. None of those two expressions (35) and (37) depend on monetary policy instruments. Moreover the long term steady state expression for aggregate supply for negative inflation does not depend on fiscal instruments either. So the only way to clear a secular stagnation steady state is to ensure that lower AD intersects aggregate supply at full employment output, which corresponds to a zero inflation level for aggregate supply. As in previous chapter, let $\Pi^{AD\,flex} > 1$ be the gross inflation level where the line given by expression (37) corresponding to the lower AD segment intersects the vertical full employment line. To clear a secular stagnation steady state $\Pi^{AD\,flex}$ must change from $\Pi_1^{AD\,flex} > 1$ to $\Pi_2^{AD\,flex} = 1$, which requires a consumption tax change, derived from expression (23) and (24), given by (proof in appendix (A.6):

$$\Delta\tau^c = \frac{1+\tau_1^c}{\Pi_1^{AD\,flex} + \frac{1}{H}}\left(1 - \Pi_1^{AD\,flex}\right), \quad \text{where} \quad H = \frac{(1+\beta)(1+g\ \ )}{\beta} \qquad (37)$$

Because $\Pi_1^{AD\,flex} > 1$, the consumption tax change corresponds to a tax reduction that will increase the natural rate of interest sufficiently to clear-up the secular stagnation as in chapter (4.1). Labor taxes must increase to fulfill the *fiscal rule* (22), and monetary policy must ensure that demand gross inflation kink is greater than one to allow for a full-employment equilibrium in the model.

5.2. Migrating to a positive inflation equilibrium from a secular stagnation one

If the fiscal demand expansion described above cannot be fully implemented then it will not be possible to clear from the model a secular stagnation equilibrium in the long run.

But by acting over the short run aggregate supply given by expression (34), through and adequate change of the labor tax on firms τ^{su}, the liquidity trap equilibrium can be eliminated at least for one period:

During a secular stagnation steady state, both short term and long term lower AS segments intersect lower AD segment for $Y_t^{ss} < Y^f$ and $\Pi_t^{ss} < 1$. Similar to the previous section, to eliminate a slump equilibrium at least for one period, we need lower AD, and now lower short term AS segments to intersect at full employment output, for $Y_t^{ss} = Y^f$. Based on expression (34) short term aggregate supply intersects full employment vertical line at an inflation level $\Pi_t^{AS,f}$ given by:

$$\Pi_t^{AS,f}(\tau_t^{su}) = \frac{(1+\tau_t^{su})}{(1+\tau_{t-1}^{su})}\left(\frac{Y_{t-1}}{Y^f}\right)^{\frac{\alpha-1}{\alpha}} \tag{38}$$

We then need at least a shift of lower AS or of lower AD so that $\Pi_t^{AS,f} = \Pi_t^{AD,f}$, where $\Pi_t^{AD,f}$ is the intersection of AD lower segment with full employment for the period when the slump is supposed to be cleared. This requires a change of firm labor tax τ^{su} given by expression below, derived directly from expression (38):

$$\frac{(1+\tau_t^{su})}{(1+\tau_{t-1}^{su})} = \frac{\Pi_{ss}^{AD,f}}{\Pi_{ss}^{AS,f}}\left(\frac{\Pi_t^{AD,f}}{\Pi_{ss}^{AD,f}}\right) = \Leftrightarrow \Delta\tau_t^{su} = (1+\tau_{t-1}^{su})\left(\frac{\Pi_t^{AD,f}}{\Pi_{ss}^{AS,f}} - 1\right) \tag{39}$$

Where $\Pi_{ss}^{AD,f}$ and $\Pi_{ss}^{AS,f}$ are inflation levels corresponding to the intersections of lower AD and AS respectively with full employment line when the economy is in a secular stagnation equilibrium. Directly from expression (38), $\Pi_{ss}^{AS,f} = \left(\frac{Y^{ss}}{Y^f}\right)^{\frac{\alpha-1}{\alpha}} = y^{ss\frac{\alpha-1}{\alpha}} > 1$. $\Pi_{ss}^{AS,f}$ is high for low secular stagnation output levels, or more depressive slumps. Expression (39) can then also be written as:

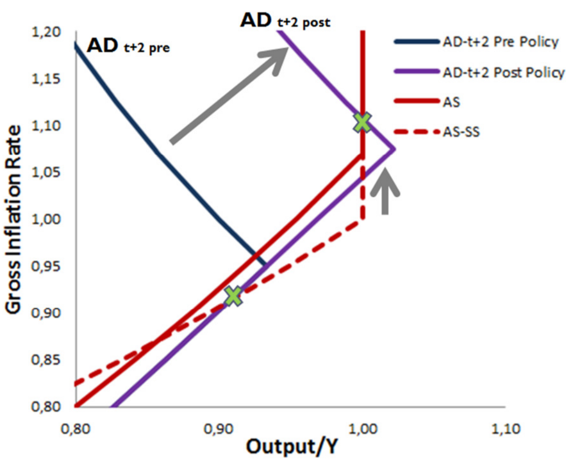
$$\frac{(1+\tau_t^{su})}{(1+\tau_{t-1}^{su})} = \Pi_t^{AD,f} y^{ss\frac{1-\alpha}{\alpha}} \Leftrightarrow \Delta\tau_t^{su} = (1+\tau_{t-1}^{su})\left(\Pi_t^{AD,f} y^{ss\frac{1-\alpha}{\alpha}} - 1\right) \tag{40}$$

If lower AD segment remains unchanged, then $\Pi_t^{AD,f} = \Pi_{ss}^{AD,f}$, and $\Delta\tau_t^{su}$ has positive or negative sign depending on the relation between $\Pi_{ss}^{AD,f}$ and $\Pi_{ss}^{AS,f} = y^{ss\frac{\alpha-1}{\alpha}}$.

(i) If $\Pi_{ss}^{AS,f} > \Pi_{ss}^{AD,f}$, or equivalently when $y^{ss} < \left(\Pi_{ss}^{AD,f}\right)^{\frac{\alpha}{1-\alpha}}$ when the slump is deeper, then $\Delta\tau_t^{su} < 0$ stating that a supply expansion driven by a reduction of labor tax on firms is necessary to eliminate the liquidity trap for one period. In that period the economy should move to the full-employment equilibrium that adequate monetary policy should ensure. Although the short term AS curve will return to its steady state shape on the following period, the economy might be able to sustain itself on the full-employment equilibrium. Alternatively, an adequate demand contraction through an increase of consumption tax, $\Delta\tau^c > 0$, would raise $\Pi^{AD,f}$ from $\Pi_{ss}^{AD,f}$ to $\Pi_t^{AD,f} > \Pi_{ss}^{AD,f}$, and could have the same effect of clearing the slump for one period, if monetary policy ensures that $Y_t^{kink} > Y^f$. But changing both taxes in opposite directions may be easier to implement than changing just one of them: Only increasing τ^c may help eliminate a slump in the short term but will deepen the available secular stagnation equilibrium in the model; moreover it may call for monetary policy intervention in order to ensure a full employment equilibrium to where the economy can migrate; monetary policy instruments might not be available. A reduction of τ^{su} without increasing τ^c might need to be too assertive, calling for an also too assertive increase of τ^l in order to fulfill the *fiscal rule* (22); although this would avoid the deepening of the steady state slump available in the model, the higher assertiveness of the changes in particular of labor income taxes could be politically difficult to sell. An adequate combination of a smaller τ^{su} reduction together with a smaller increase of τ^c could be politically more easy to implement, because the changes are smaller than if just one of those taxes is changed, and additionally τ^l could be kept constant.

(ii) Otherwise, if $\Pi_{ss}^{AS,f} < \Pi_{ss}^{AD,f}$, or equivalently when $y^{ss} > \Pi_{ss}^{AD,f\frac{\alpha}{1-\alpha}}$ for lighter slumps, a supply contraction through an increase of labor tax on firms τ^{su}, could clear the deflationary equilibrium for one period. This can be explained mentioning the paradox of toil, illustrated by Eggertson (2014), stating that an aggregate supply expansion can have contractionary effects when an economy is in a liquidity trap, by triggering deflationary pressures that raise the real interest rate $(1 + r = \frac{1}{\Pi})$ and further depressing demand. The key behind this is a lower AD segment positive slope, higher than the slope of the lower AS segment. In order to fulfill equation (22) and keep AD curve unchanged, income labor tax on households must be reduced. Alternatively a further demand expansion through a consumption tax reduction would have a similar effect, and could be, in that situation, a safe complement or alternative.

Figure 5: Using labor tax on firms



6. Introducing Capital and a Tax on Capital in the model

Capital is introduced in the model as in Eggertson and Mehrotra (2014), and our contribution is to add a tax on capital income τ^k . Although the introduction of capital in the model does not change significantly the shape of aggregate supply and aggregate demand curves from previous chapters, the presence of a tax on capital income for households τ_t^k can be a relevant fiscal policy instrument as alternative or a complement to consumption tax changes, in order to avoid, to clear, or migrate from a liquidity trap.

Aggregate demand: Constraints (26) and (27) of household problem with capital in real terms are given by:

$$(1+\tau_{t+1}^c)C_{t+1}^m = z_{t+1} + w_{t+1}L_{t+1}(1-\tau_{t+1}^l) + K_{t+1}(r_{t+1}^k(1-\tau_{t+1}^k)-1) - (1+r_t)B_t^y + B_{t+1}^m \quad (41)$$

$$(1+\tau_{t+2}^c)C_{t+2}^o = -(1+r_{t+1})B_{t+1}^m + K_{t+1}(1-\delta) \quad (42)$$

First order condition for K is given by: FOC K_{t+1}: $r_t^k = \frac{1}{(1-\tau_t^k)}\left(1-\frac{1-\delta}{1+r_t}\right)$ (43)

Firm problem: $z_t = \max_{L_t} Y_t - w_t L_t(1+\tau_t^{su}) - r_t^k P_t K_t$ s.t $Y_t = a_t {L_t}^{\alpha} {K_t}^{1-\alpha}$ (44)

Now $w_t = \frac{\alpha L_t^{\alpha-1}}{(1+\tau_t^{su})} = \frac{\alpha}{(1+\tau_t^{su})}\frac{Y_t}{L_t}$ and $r_t^k = (1-\alpha)A_t {L_t}^{\alpha} K_t^{-\alpha-1} = \frac{(1-\alpha)Y_t}{K_t}$.

Aggregate Demand has now the expressions below:

$$Y_t^d = \left(\frac{1}{1-\alpha A_t-(1-\alpha)B_t^l}\right)\left[\left(\frac{1+\beta}{\beta}\right)\frac{(1+g_t)D_t}{1+r_t} - D_{t-1}\right] \quad (45)$$

or $$Y_t^d = \left(\frac{1}{1+\tau_t^c-(1-\alpha)B_t^c}\right)\left[\left(\frac{1+\beta}{\beta}\right)\frac{(1+g_t)D_t}{1+r_t} - D_{t-1} + G_t(1+\tau_t^c)\right] \quad (46)$$

Where $B_t^l = \frac{1}{r_t^k}\left(\frac{1+\beta}{\beta}\right)\frac{(1-\delta)}{(r_t+\delta)} = \left(1-\tau_t^k\right)\left(\frac{1+\beta}{\beta}\right)\frac{(1-\delta)}{(r_t+\delta)}$, and $B_t^c = \frac{1}{r_t^k}\left(1+\tau_t^c\delta+\frac{1}{\beta}\frac{(1-\delta)}{(1+r_t)}\right)$.

From expression (43) we can observe that r_t^k decreases when τ^k is reduced, and from (45),(46) that a reduction of τ^k has an expansion effect on Y_t^d through a reduction of r^k.

Aggregate Supply: Expressions for aggregate supply are similar to the model without capital: $Y^s = Y^f$ for $\Pi_t \geq 1$, and $\frac{\gamma}{\Pi} = 1-(1-\gamma)\left(\frac{Y}{Y_f}\right)^{\frac{1-\alpha}{\alpha}}$ for $\Pi_t < 1$. The difference lies on the supply expression for positive inflation levels which is not constant, and expands with τ^k reductions:

$$Y_t^f = a_t \bar{L}^{\alpha} {K_t}^{1-\alpha} = \bar{L} a_t^{1/\alpha}\left(\frac{1-\alpha}{r_t^k}\right)^{\frac{1-\alpha}{\alpha}} = \bar{L} a_t^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)\left(1-\tau_t^k\right)}{1-\frac{1-\delta}{1+r_t}}\right]^{\frac{1-\alpha}{\alpha}} \quad (47)$$

Then the lower AS segment also shifts out with a reduction of τ^k, and the kink of AS curve is a negative function of capital tax, and is maximized at $\tau^k = 0$:

$$Y_t^{AS\,Kink} = \bar{L}a_t^{1/\alpha}\left[\frac{(1-\alpha)\left(1-\tau_t^k\right)}{\delta}\right]^{\frac{1-\alpha}{\alpha}} \leq \bar{L}a_t^{\frac{1}{\alpha}}\left[\frac{1-\alpha}{\delta}\right]^{\frac{1-\alpha}{\alpha}} \tag{48}$$

Secular stagnation equilibrium: A reduction of tax on capital expands both aggregate demand and aggregate supply. When the economy is in a steady state secular stagnation, a shift out of aggregate supply has deflationary effects (see paradox of toil in Eggertson 2014), and a shift out of aggregate demand has inflationary effects. The net effect of a τ_t^k reduction is an increase in inflation and output equilibrium levels (proof available), which makes τ_t^k an alternative or a complement to consumption tax changes to avoid, to clear, or to migrate from a liquidity trap. To clear a liquidity trap from the set of possible determinate equilibriums (see chapter 4) a decrease of τ^k can be used as an alternative to a decrease of consumption tax, or as complement. To migrate to a full employment equilibrium (chapter 5) changes of τ^k although having a similar effect than changes of τ^c of the same sign, should be managed more carefully as they trigger shifts of AS and AD in the same direction, and may attenuate the effect of τ^{su}changes. In any case, decreasing τ^k alone (compensated by increasing any of the labor taxes) will expand equilibrium output, and may mitigate, although not clear, a secular stagnation.

7. Conclusion

In this paper we show how a closed economy can avoid or leave a secular stagnation, using an adequate combination of monetary and fiscal instruments based on distortionary taxes over consumption, labor and capital, when increasing government spending and generating a budget deficit are not policy options available. In the model used, increasing borrowing constraints or decreasing population growth can generate negative real interest rates for full employment equilibriums. If target inflation cannot be raised enough such equilibriums may not be reached due to a binding zero lower bound on nominal interest rates. When recession pressures are mild, an adequate and timely fiscal policy expansion through reduction of consumption or capital taxes, compensated with increasing labor taxation, will shift out aggregate demand, raise back full employment real interest rate to an allowable level, and may clear a secular stagnation from the set of available determinate equilibriums. But when recession forces are intense, the level of assertiveness of fiscal demand expansion required to avoid a liquidity trap would call for a strong increase in income taxes in order to maintain a balanced budget, which could be hard to implement from a political standpoint. Politically acceptable fiscal policies may then not avoid the appearance of a new available secular stagnation equilibrium to where it is important to keep the economy away. Then, adequate, assertive and timely monetary policy is fundamental to sustain the economy on a full employment equilibrium, by reducing the target real interest rate to the corresponding required negative full employment level, through a sufficient increase of target inflation when the target nominal interest rate has reached the zero level. If the reaction of policy agents is timid the economy may dive into a persistent slump from where it is harder to get out, than to avoid getting in. Even if monetary policy ensures the existence of a full employment equilibrium in the model, only fiscal policy can ensure that the slump equilibrium is cleared at least for one period to force a migration to the full employment one. When the slump is strong, short term supply expansion by reducing τ^{su}, combined with a demand expansion by increasing τ^{c} , may clear the secular stagnation equilibrium for a while, forcing the migration to the full employment equilibrium available, where the economy should be able to sustain itself.

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APPENDIX

A.1. Household problem

$$\max_{C_t^y, C_{t+1}^m, C_{t+2}^o} E_t\{\log(C_t^y) + \beta\log(C_{t+1}^m) + \beta^2\log(C_{t+2}^o)\}, \text{ s.t.} \quad \text{(H.1)}$$

$$P_t(1+\tau_t^c)C_t^y = P_t B_t \quad \text{(H.2)}$$

$$P_{t+1}(1+\tau_{t+1}^c)C_{t+1}^m = Z_{t+1} + W_{t+1}L_{t+1}(1-\tau_{t+1}^l) - (1+i_t)P_t B_t + P_{t+1}B_{t+1}^m \quad \text{(H.3)}$$

$$P_{t+2}(1+\tau_{t+2}^c)C_{t+2}^o = -(1+i_{t+1})P_{t+1}B_{t+1}^m \quad \text{(H.4)}$$

And an exogenous borrowing limit: $(1+i_t)P_t B_t \leq P_{t+1}D_t$ (H.5)

Let's use the Fisher equation: $(1+i_t) = (1+r_t)\frac{P_{t+1}}{P_t}$ (M.1)

Replacing (H.2), (H.4), (H.5) and (M.1) in (H.3), and diving (H.3) by P_{t+1} we get:

$$(1+\tau_{t+1}^c)C_{t+1}^m + \frac{(1+\tau_{t+2}^c)C_{t+2}^o}{(1+r_{t+1})} = \frac{Z_{t+1}}{P_{t+1}} + w_{t+1}L_{t+1}(1-\tau_{t+1}^l) - D_t \quad \text{(H.3.1)}$$

From (H.1) s.t to (H.3.1) we obtain an Euler equation:

$$-B_{t+1}^m = \frac{(1+\tau_{t+2}^c)C_{t+2}^o}{(1+r_{t+1})} = \beta(1+\tau_{t+1}^c)C_{t+1}^m \quad \text{(H.6)}$$

And replacing (H.6) in (H.3.1) we get:

$$(1+\beta)(1+\tau_{t+1}^c)C_{t+1}^m = \frac{Z_{t+1}}{P_{t+1}} + w_{t+1}L_{t+1}(1-\tau_{t+1}^l) - D_t \quad \text{(H.3.2)}$$

A.2. Firm problem

$$Z_t = \max_{L_t} P_t Y_t - W_t L_t(1+\tau_t^{su}) \quad \text{s.t} \quad Y_{t=}a_t L_t{}^{\alpha} \quad \text{(F.1)}$$

Then: $w_t = \frac{W_t}{P_t} = \frac{\alpha a_t L_t^{\alpha-1}}{(1+\tau_t^{su})} = \frac{\alpha}{(1+\tau_t^{su})}\frac{Y_t}{L_t} = \frac{\alpha a_t^{\frac{1}{\alpha}} Y_t^{\frac{\alpha-1}{\alpha}}}{(1+\tau_t^{su})}$ (F.2)

A.3. Equilibrium in the bond market

$$N_t B_t^y = -N_{t-1}B_t^m \Leftrightarrow (1+g_t)B_t^y = -B_t^m \Leftrightarrow \frac{(1+g_t)}{(1+r_t)}D_t = -B_t^m \quad \text{(B.1)}$$

By replacing (H.6) in (B.1) we get: $\frac{(1+g_t)}{(1+r_t)}D_t = \beta(1+\tau_{t+1}^c)C_t^m$ (B.2)

A.4.Monetary Policy

Using the Taylor rule: $(1+i_t) = \max\{1\,,\ (1+i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\}$ (M.2)

Combining with the fisher equation:

$$(1+r_t) = \begin{cases}(1+r^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi-1} = \frac{1}{\Pi_t}\left(\frac{\Pi_t}{\Pi_{kink}}\right)^{\phi_\pi}, & \Pi_t > \Pi_{kink} \\ \frac{1}{\Pi_t}, & \Pi_t \leq \Pi_{kink}\end{cases} \quad \text{(M.3)}$$

A.5. Aggregate Demand

Using (F.1) and (F.2), (H.3.2) can be rewritten as:

$$(1+\beta)(1+\tau_t^c)C_t^m = Y_t(1-\alpha A_t) - D_t; \quad A_t = \frac{(\tau_t^l+\tau_t^{su})}{(1+\tau_t^{su})} \quad \text{(D.1)}$$

And now replacing (B.2) in (H.3.3) we finally get the expression for aggregate demand:

$$Y_t = \left(\frac{1}{1-\alpha A_t}\right)\left[\left(\frac{1+\beta}{\beta}\right)\frac{(1+g_t)D_t}{1+r_t} + D_{t-1}\right] \quad \text{(D.2)}$$

and for the real interest rate:

$$1+r_t = \left(\frac{1+\beta}{\beta}\right)\frac{(1+g_t)D_t}{(1-\alpha A_t)Y_t - D_{t-1}} \quad \text{(D.3)}$$

Replacing the real interest rate by (M.3):

$$\Pi_{kink} = \frac{\Pi^*}{(1+i^*)^{\frac{1}{\phi_\pi}}} \text{ ,and } Y_{kink} = \left(\frac{1}{1-\alpha A_t}\right)\left[\frac{(1+\beta)(1+g_t)D_t}{\beta}\Pi_{kink} + D_{t-1}\right] \quad (D.4)$$

And the expression of Aggregate Demand, $Y_t = Y(\Pi_t)$:

$$Y_t = \begin{cases} Y_t^{high} = \left(\frac{1}{1-\alpha A_t}\right)\left[\frac{(1+\beta)(1+g_t)D_t}{\beta}\frac{\Pi_{kink}{}^{\phi_\pi}}{\Pi_t{}^{\phi_\pi-1}} + D_{t-1}\right], \Pi_t > \Pi_{kink} & (D.5) \\ Y_t^{low} = \left(\frac{1}{1-\alpha A_t}\right)\left[\frac{(1+\beta)(1+g_t)D_t}{\beta}\Pi_t + D_{t-1}\right], \Pi_t \leq \Pi_{kink} & (D.6)\end{cases}$$

In steady state $D_t = D_{t-1} = D$

$$Y = \left(\frac{D}{1-\alpha A}\right)\left[\left(\frac{1+\beta}{\beta}\right)\frac{(1+g\ \ )}{1+r} + 1\right] \quad \text{(D.7)}$$

$$1+r = \left(\frac{1+\beta}{\beta}\right)\frac{(1+g\ \ )D}{(1-\alpha A\ \ )Y\ \ - D} \quad \text{(D.8)}$$

Full employment real interest rate, or *natural rate of interest*:

$$1 + r_f = \left(\frac{1+\beta}{\beta}\right)\frac{(1+g \quad)D}{(1-\alpha A \quad)Y_f - D} \tag{D.9}$$

If $1 + r^* = 1 + r_f$ then there is and equilibrium in the model where $\Pi_t = \Pi^*$. In particular if $\mathrm{i}^* = 0$ then $\Pi_t = \frac{1}{1+r_f} = \Pi^* = \Pi_{kink}$ and $\mathrm{Y}_{kink} = \mathrm{Y}_f$

A.5. Aggregate Supply

$$W_t = \max\{\widetilde{W}_t, P_t w_t^{flex}\} \text{ where } \widetilde{W} = \gamma W_{t-1} + (1-\gamma)P_t w_t^{flex} \tag{AS.1}$$

$$w_t = \max\{\widetilde{w}_t, w_t^{flex}\} \text{ where } \widetilde{w}_t = \gamma\frac{w_{t-1}}{\Pi_t} + (1-\gamma)w_t^{flex} \tag{AS.2}$$

$$\frac{\alpha a_t^{\frac{1}{\alpha}} Y_t^{\frac{\alpha-1}{\alpha}}}{(1+\tau_t^{su})} = \max\left\{\gamma\frac{\frac{\alpha a_{t-1}^{\frac{1}{\alpha}} Y_{t-1}^{\frac{\alpha-1}{\alpha}}}{(1+\tau_{t-1}^{su})}}{\Pi_t} + (1-\gamma)\frac{\alpha a_t^{\frac{1}{\alpha}} Y_t^{flex\frac{\alpha-1}{\alpha}}}{(1+\tau_{t-1}^{su})}, \frac{\alpha a_t^{\frac{1}{\alpha}} Y_t^{flex\frac{\alpha-1}{\alpha}}}{(1+\tau_{t-1}^{su})}\right\} \tag{AS.3}$$

Assuming for simplicity that A_t is constant in time,

$$\frac{Y_t^{\frac{\alpha-1}{\alpha}}}{(1+\tau_t^{su})} = \max\left\{\gamma\frac{\frac{Y_{t-1}^{\frac{\alpha-1}{\alpha}}}{(1+\tau_{t-1}^{su})}}{\Pi_t} + (1-\gamma)\frac{Y_f^{\frac{\alpha-1}{\alpha}}}{(1+\tau_t^{su})}, \frac{Y_f^{\frac{\alpha-1}{\alpha}}}{(1+\tau_t^{su})}\right\} \tag{AS.4}$$

$$Y_t^{\frac{\alpha-1}{\alpha}} = \max\left\{\frac{(1+\tau_t^{su})}{(1+\tau_{t-1}^{su})}\frac{\gamma}{\Pi_t}Y_{t-1}^{\frac{\alpha-1}{\alpha}} + (1-\gamma)Y_f^{\frac{\alpha-1}{\alpha}}, Y_f^{\frac{\alpha-1}{\alpha}}\right\} \tag{AS.5}$$

Then the lower part of the AS curve has the following expression for Π_t:

$$\frac{\gamma}{\Pi_t} = \left(\frac{1+\tau_{t-1}^{su}}{1+\tau_t^{su}}\right)\left[\left(\frac{Y_{t-1}}{Y_t}\right)^{\frac{1-\alpha}{\alpha}} - (1-\gamma)\left(\frac{Y_{t-1}}{Y_f}\right)^{\frac{1-\alpha}{\alpha}}\right] \tag{AS.6}$$

In steady state (with constant taxes) AS curve is independent of τ^{su}:

$$\frac{\gamma}{\Pi} = 1 - (1-\gamma)\left(\frac{Y}{Y_f}\right)^{\frac{1-\alpha}{\alpha}} \tag{AS.7}$$

A.6. Eliminating a Liquidity Trap with fiscal policy

In order to avoid an intersection of the lower part of AD curve with the lower part of AS curve in steady state it is sufficient to move AD curve so that it intersects the AS curve at $(Y_t,\ \Pi_t) = (Y_f, 1)$: $Y^d\left(\Pi_1^f, \tau_1^c\right) = Y^f = Y^d\left(\Pi_2^f, \tau_2^c\right) \underset{\Pi_2^f=1}{\Longleftrightarrow}$

$$\frac{D}{1+\tau_1^c}\left[\frac{(1+\beta)(1+g)}{\beta}\Pi_1^f + 1\right] + G \ = \frac{D}{1+\tau_2^c}\left[\frac{(1+\beta)(1+g)}{\beta}\Pi_2^f + 1\right] + G \ \underset{\Pi_2^f=1}{\Longleftrightarrow}$$

$$\frac{1+\tau_2^c}{1+\tau_1^c} = \frac{H+1}{H\Pi_1^f + 1}, H = \frac{(1+\beta)(1+g\ \ )}{\beta} \underset{\Pi_2^f=1}{\Longleftrightarrow} \Delta\tau^c \ = \frac{1+\tau_1^c}{\Pi_1^f + \frac{1}{H}}\left(1 - \Pi_1^f\right) \qquad (LT.1)$$

A.5. Migrating from a liquidity trap to a positive inflation equilibrium

From (AS.6), the lower part of the AS curve intersects $Y = Y_f$ at Π_t^f :

$$\Pi_t^f = \left(\frac{1+\tau_t^{su}}{1+\tau_{t-1}^{su}}\right)\left(\frac{Y_f}{Y_{t-1}}\right)^{\frac{1-\alpha}{\alpha}} \Leftrightarrow \left(\frac{1+\tau_t^{su}}{1+\tau_{t-1}^{su}}\right) = \ \Pi_t^f {y_{t-1}}^{\frac{1-\alpha}{\alpha}} \text{ where } y_{t-1} = \frac{Y_{t-1}}{Y_f}$$

This can be written like: $\Delta\tau_t^{su} = \tau_t^{su} - \tau_{t-1}^{su} = (1+\tau_{t-1}^{su})(\Pi_t^f {y_{t-1}}^{\frac{1-\alpha}{\alpha}} - 1)$

B. Derivation of the Model with Capital

B.1 Household problem

$$P_{t+1}(1+\tau_{t+1}^c)C_{t+1}^m = Z_{t+1} + W_{t+1}L_{t+1}\left(1-\tau_{t+1}^l\right) - P_{t+1}K_{t+1} + P_{t+1}K_{t+1}r_{t+1}^k(1 - \tau_{t+1}^k) - (1+i_t)P_tB_t \ + P_{t+1}B_{t+1}^m \qquad \text{(CH.3)}$$

$$P_{t+2}(1+\tau_{t+2}^c)C_{t+2}^o = -(1+i_{t+1})P_{t+1}B_{t+1}^m + P_{t+1}K_{t+1}(1-\delta) \qquad \text{(CH.4)}$$

$$(1+\tau_{t+1}^c)C_{t+1}^m + \frac{(1+\tau_{t+2}^c)C_{t+2}^o}{(1+r_{t+1})} = \frac{Z_{t+1}}{P_{t+1}} + w_{t+1}L_{t+1}\left(1-\tau_{t+1}^l\right) + K_{t+1}\left[-1 + r_{t+1}^k(1 - \tau_{t+1}^k) + \frac{(1-\delta)}{(1+r_{t+1})}\right] - D_t \qquad \text{(CH.3.1)}$$

FOCs revisited:

FOC C_{t+1}^m: $\frac{(1+\tau_{t+2}^c)C_{t+2}^o}{(1+r_{t+1})} = \beta(1+\tau_{t+1}^c)C_{t+1}^m$ (CH.6)

FOC K_{t+1}: $r_t^k = \frac{1}{(1-\tau_t^k)}\left(1 - \frac{1-\delta}{1+r_t}\right)$ (CH.7)

B.2. Firm problem

$$Z_t = \max_{L_t} P_t Y_t - W_t L_t (1 + \tau_t^{su}) - r_t^k P_t K_t \quad \text{s.t} \qquad Y_{t=} A_t {L_t}^{\alpha} {K_t}^{1-\alpha} \qquad \text{(CF.1)}$$

Then: $w_t = \frac{W_t}{P_t} = \frac{\alpha L_t^{\alpha-1}}{(1+\tau_t^{su})} = \frac{\alpha}{(1+\tau_t^{su})} \frac{Y_t}{L_t}$ (CF.2)

and: $r_t^k = (1-\alpha) A_t {L_t}^{\alpha} K_t^{-\alpha-1} = \frac{(1-\alpha)Y_t}{K_t}$ (CF.3)

Then (CH.3.1) can be rewritten as:

$$(1+\beta)(1+\tau_t^c) C_t^m = Y_t (1 - \alpha A_t) - K_t \left[1 - (1-\tau_t^k) r_t^k - \frac{(1-\delta)}{(1+r_t)} \right] - D_{t-1} \text{(CH. 3.2)}$$

Where $A_t = \frac{(\tau_t^l + \tau_t^{su})}{(1+\tau_t^{su})}$

B.3. Equilibrium in the bond market

$N_t B_t^y = -N_{t-1} B_t^m \Leftrightarrow (1+g_t) B_t^y = -B_t^m \Leftrightarrow \frac{(1+g_t)}{(1+r_t)} D_t \ = -B_t^m$ (CB.1)

From (CH.4): $-B_t^m = \frac{(1+\tau_{t+1}^c) C_{t+1}^o}{(1+r_t)} - K_t \frac{(1-\delta)}{(1+r_t)}$ (CB.2)

$\Leftrightarrow (1+\tau_{t+1}^c) C_{t+1}^o = (1+g_t) D_t \ + K_t (1-\delta)$ (CB.2.1)

Replacing (CH.6) in (CH.4): $-B_t^m = \beta (1+\tau_t^c) C_t^m - K_t \frac{(1-\delta)}{(1+r_t)}$ (CB.3)

By replacing (CB.1) in (CB.3) we get: $\frac{(1+g_t)}{(1+r_t)} D_t \ + K_t \frac{(1-\delta)}{(1+r_t)} = \beta (1+\tau_t^c) C_t^m$ (CB.4)

B.4. Aggregate Demand

Inserting (CB.4) in (CH3.2) we get:

$$\frac{(1+\beta)}{\beta} \left[\frac{(1+g_t)}{(1+r_t)} D_t \ + K_t \frac{(1-\delta)}{(1+r_t)} \right] = Y_t (1 - \alpha A_t) - K_t \left[1 - (1-\tau_t^k) r_t^k - \frac{(1-\delta)}{(1+r_t)} \right] - D_{t-1} \Leftrightarrow$$

$$\Leftrightarrow \ Y_t (1 - \alpha A_t) = D_{t-1} + \frac{(1+\beta)}{\beta} \frac{(1+g_t)}{(1+r_t)} D_t \ + K_t \left(1 + \frac{1}{\beta} \frac{(1-\delta)}{(1+r_t)} - (1-\tau_t^k) r_t^k \right)$$

And replacing: $K_t = \frac{(1-\alpha) Y_t}{r_t^k}$ we get:

$$Y_t = \left(\frac{1}{1 - \alpha A_t - (1-\alpha) B_t} \right) \left[\left(\frac{1+\beta}{\beta} \right) \frac{(1+g_t) D_t}{1+r_t} + D_{t-1} \right] \qquad \text{(CD. 1)}$$

$B_t = \frac{1}{r_t^k} \left(1 + \frac{1}{\beta} \frac{(1-\delta)}{(1+r_t)} - (1-\tau_t^k) r_t^k \right) = \frac{1}{r_t^k} \left(\frac{1+\beta}{\beta} \right) \frac{(1-\delta)}{(r_t+\delta)} = \left(1 - \tau_t^k \right) \left(\frac{1+\beta}{\beta} \right) \frac{(1-\delta)}{(r_t+\delta)}$ (CD.2)