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# Intraday Volatility Forecasting in High-Frequency Data Using Order Book Information

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# Abstract – Forecasting

This research conducts high-frequency intraday volatility forecasts on the Euro Stoxx 50 Future considering a multiplicative component GARCH framework, where the conditional volatility of high-frequency returns is decomposed into a daily, diurnal and stochastic intraday component. In contrast to extant research, in this work project a relatively long period of 423 trading days is covered corresponding to about 345.000 1-minute observations. To opt for a more practitioner-oriented approach we perform fixed window as well as rolling window forecasts. There is evidence that incorporating Limit Order Book information into the return series leads to superior forecasting results compared to the usage of simple trade returns. Nonetheless, the forecasting performance is time-varying and is often deteriorated by the seasonality of liquidity provision.


KEYWORDS: GARCH, volatility forecasting, high-frequency data, limit order book.

# 1. Introduction

In financial markets high-frequency trading plays a significant role in price discovery and liquidity provision according to recent literature (see e.g. Bouveret et al. (2014)). Jarnecic and Snape (2014) found that market makers provide small but stable liquidity on the lower levels around the best bid and ask price and earn the corresponding spread as a profit. Arbitrageurs use high-frequency algorithms to discover price inefficiencies across markets and securities and will exploit the inefficiency until it converges to its fundamental price again. The vast majority of recent literature covered the topics of price discovery and discussed if high-frequency trading contributes to it. Another stream of literature focused on the ability of intraday volatilities to model better and more accurate end of day volatility estimates that serve as an input for several risk applications. However, only little research has been done to uncover the predictability of spot (intraday) volatility. High-frequency trading strategies rely heavily on the expected future spot volatility, as an input parameter for algorithms to place limit orders or to schedule trades. The existing research on this topic is mainly focused on equities. However, we find that there is the need to extent the research to equity benchmark futures as they have far reaching applications in financial markets. They serve as a financial instrument to control for stock exposure in portfolio management and are used as a hedging instrument for option trading. As market participants only have to deposit a margin to trade futures, they are far less capital intensive compared to a replicated cash basket. Moreover, most of the current research covers only a relatively little time period (between 3 to 4 months of trading data) as historic intraday data is rarely available and computational expensive. Our data set covers data of almost two years of trading and therefore delivers robustness against seasonality effects and can also capture the effect of structural changes in trading sentiment.

As we are not only constraint to trading data, but also have information on the whole limit order book (LOB), we find clear evidence that returns derived from the order book have superior forecasting abilities compared to simple trade returns.

This paper is structured as follows. The first section provides a literature review on three different topics that are covered in our research. The first topic provides an overview on research that analysed the implications of dealing with high-frequency data. The second topic covers research that has been done on the informational content of the LOB. The last topic is about research focused on intraday volatility modelling. In Section 3, we discuss our data set, while providing general information about the Euro Stoxx 50 Future (FESX) market. In section 4 we introduce our model and the constructed prices from the LOB. In section 5, parameter estimation and property analysis are presented. Section 6 presents results from the forecasting exercise and is summarized in our conclusion.

# 2. Literature Review

### 2.1. High Frequency Sampling

The rise of high-frequency trading has flooded financial markets with large amounts of data recorded up to nanoseconds. Nonetheless, the majority of academic work suggests high-frequency data to be sampled at an arbitrary frequency of 5-minutes such as Anderson et al. (2001) and Liu et al. (2015). This results in much of the data being discarded from the analysis. The main reason for this low frequency sampling is due to the presence of market microstructure noise in high-frequency data. Microstructure noise refers to the bid-ask bounce, discreteness of price change in markets that are not decimalized, latency in representativeness of price changes and informational asymmetries among traders. Zang et al. (2005) argue that price series observed over a short time interval are mainly composed of shocks stemming from microstructure noise and reveal little about the true volatility of the price process. Assuming the amount of market microstructure noise remains constant at different frequencies, the volatility obtained by price series sampled at lower frequencies contains less microstructure noise. Therefore, these larger time intervals reveal more information about the true volatility of the price process. In more statistical terms, high-frequency price return series tend to experience a high degree of autocorrelation. This persistent memory is what leads to a highly biased estimation of the variance, when calculated as the sum of the squared returns as stated by Gatheral and Oomen (2010).

To correct for these microstructure effects Gatheral and Oomen (2010) suggest using, instead of transaction prices, volume weighted mid-quote prices, also called micro-prices. '*The micro-price, more familiar to practitioners, linearly weighs the bid and ask prices by the volume on the opposite side of the book and thus can be interpreted as the market clearing price when demand and supply curves are linear in price.*' (Gatheral and Oomen, 2010, p. 5) They show that micro-price return series suffer far less from autocorrelation than transaction price return series due to the reduction in the microstructure noise based on simulated data. This property makes them more suitable for sampling at higher frequencies. However, Stoikov (2017) argues that the micro-price, as calculated by Gatheral and Oomen (2010), has several shortcomings. The first one is that the order book receives updates every few nanoseconds, assuming a highly liquid market, which leads to continuously changing micro-prices. This may lead to noisy volatility estimations for micro-price series. Secondly, the micro-price lacks theoretical justification for being the 'fair' price of a specific asset, since the micro-price is not necessarily a martingale. Stoikov (2017) proposes a micro-price, which is constructed as a martingale, conditional on the information in the LOB, such as the bid-ask spread and the order book imbalance.

## 2.2. Limit Order Book Information

The before mentioned micro-prices incorporate information from the LOB. The LOB is basically a decentralized database, which was first proposed by the U.S. Securities and Exchange Commission (SEC) in the early 2000's. Since then its popularity surged and throughout the years it has become a central part of the global financial market structure. A LOB system allows its users to view and place orders at a number of price levels away from the best ask and bid price. For each price level the order book displays its price and its corresponding quantity. Market participants can either enter a market order which will be executed instantaneously at any given price, whereas a limit order sets the maximum (minimum) price someone is willing to buy (sell), but execution is not guaranteed. The question in current academic literature remains whether these different levels actually reveal any relevant price information beyond the first level.

Cao et al. (2004) hypothesize that limit orders after the best bid and ask price contribute to price discovery. The shape of the order book gives traders a useful overview of the current demand and supply in the market. Especially, the imbalance on the ask and bid side of the LOB indicates shifts in the supply and demand curves. Their empirical evidence suggests that the order book beyond its first step is moderately informative and the information share beyond the first level is around 22%, where the highest contribution stems from the fifth level up to the tenth level of the LOB.

Rock (1996), Angel (1997) and Harris (1998) argue in their theoretical LOB models that informed traders, who obtain short-lived private information, would prefer a market order to a limit order due to its immediate execution. This implying that traders mainly make use of market orders. In contrast, Anand et al. (2005) find empirical support for informed traders' use of limit orders. They examine the relative use of market orders versus limit orders by informed and liquidity traders during the day using detailed order and audit trail data from the NYSE for

144 stocks. In their research, institutional traders are classified as informed traders and individuals as uninformed traders. They find that informed traders actually use a combination of market order and limit orders, where market orders are preferred in the first half of the day and limit orders in the second half. Furthermore, limit orders placed by informed traders perform better than limit orders placed by uninformed traders.

### 2.3. Intraday Volatility

The rise in high-frequency trading has also driven interest in modelling the volatility of those high-frequency price return series. In other words, the modelling of intraday volatility. One of the main issues related to intraday volatility modelling is intraday seasonality. This relates to the U-shape that is often observed in the daily volatility pattern. This pattern can be explained by global trade activity, implying financial products that are continuously traded and is mainly due to the opening and closing hours of financial centres at different moments of the day. In the morning, around opening time, most market traders place their orders causing a subsequent increase in the volatility of that specific securities market. The following hours volatility decreases smoothly due to less activity in the market with the lowest activity normally observed during lunch time. The second spike is usually detected when another large financial centre starts trading, such as the American or European market. When the traders of that specific opening market start placing their orders is the moment when the second spike in the volatility occurs. This recurring pattern causes the return volatility to have a slow decay in autocorrelation coupled with a strong daily conditional heteroskedasticity (Anderson and Bollerslev, 1997).

In the literature there have been many attempts to resolve the issue of intraday seasonality sparked by diurnal trading activity patterns. Anderson and Bollerslev (1997) in their attempt to model the volatility of five-minute returns of exchange rates, build a multiplicative model of daily and diurnal volatility. In their paper the conditional variance is expressed as a product of daily and diurnal components. They estimate the diurnal pattern by a Fourier flexible functional form. Anderson and Bollerslev (1998) extend their previous model by adding a dummy variable which should be able to capture the effects of macroeconomic announcements on the volatility. This approach of capturing daily effects has generally been used in the literature. Nonetheless, Engle and Sokalska (2012) argue that adding a dummy variable associated with a particular announcement is not very practical, especially when modelling a large number of stocks. They argue that the majority of these macroeconomic announcements occur before markets open and that the consequent reaction of the market heavily depends on whether the news was genuinely expected or not. Furthermore, markets are more prone to shocks coming from asymmetric information among market participants. Engle and Sokalska (2012) propose a GARCH with a multiplicative component, which specifies the conditional variance to be the product of daily, diurnal, and stochastic intraday volatility. For the daily variance component, they make use of commercially available volatility forecasts, such as volatility forecasts derived from a multifactor risk model. The diurnal variance pattern is computed by dividing the variance of returns, by the daily variance forecast. Throughout the years the literature has mentioned several alternative ways to capture the diurnal pattern. Engle and Sokalska (2012), compared to Andersen and Bollerslev (1997), apply a more simplistic approach to calculate the diurnal pattern, which allows its daily shape to take on any form. The last step of their model is to normalize the stochastic component, the error term, by dividing it by the diurnal pattern and the daily volatility forecast. In their paper the model is used to forecast the volatility of 10-minute returns of 2,500 US stocks. Their research concludes that the addition of a new stochastic intraday component produces better volatility forecasts than the GARCH model with solely diurnal and daily components.

# 3. Market Environment, Data and Stylized Facts about the Limit Order Book

### 3.1. The Euro Stoxx 50 Market

The Euro Stoxx 50 Future (FESX) is a future contract on its underlying cash index, a market capitalization weighted stock index, comprising the 50 largest publicly traded companies within the Eurozone. The FESX Future has quarterly expirations, namely in March, June, September and December. Expiration day is the third Friday of the corresponding maturity month. If this is not a trading day, then it is the exchange day immediately preceding that day. The future is a cash settled instrument, meaning at expiration a seller or buyer receives/pays the difference between the initial trade price and the final settlement price. The tick size of a contract is 1 index point and is valued 10€/point. The minimum quote size for market makers is 10 contracts on the bid and ask side. The maximum spread is 1 index point. In a fast market environment, where market participants find eased quoting rules, the minimum quote size is reduced by 50% and spreads can increase by 100%. Fast markets are set by Eurex's market supervision in general before scheduled economic releases. Market makers have a minimum quote duration of 70% of the trading hours between 09:00 and 17:30 CET (on a monthly average) (Eurex Exchange - Matching Principles, 2018). Nevertheless, excluding the opening and closing auction, the FESX Futures are open for trade from 08:00 until 22:00 CET. A core element of the Eurex market model is the central order book (T7). During a trading day all orders and quotes are entered in this order book, except those entered via TES (Trade Entry Services). Those orders and quotes are sorted by price, type and entry time. Quotes and limit orders are sorted together. Market maker quotes are not specially considered. Equity futures follow the matching principle, better known as the price-time priority. This principle is applied to quotes and orders. When entering an order in the order book it receives a time stamp. By prioritizing orders with same price but earlier timestamp one or more transactions are generated if there are matching contrary orders. For the matching process, T7 treats orders and quotes identically. Therefore, in the following, the term “order” is generally applied to both orders and quotes. With 1,200,000 traded contracts on average a day in 2018 (Eurex Exchange - Trading Statistics, 2018), the FESX is one of the most liquid products of the Eurex Exchange.

### 3.2. Data

The sample period includes trading days from January 3, 2017 to September 28, 2018 resulting in 444 trading days for analysis. The research focuses on the actively traded future contract (front month). Taking the impact of rollovers into consideration, observations two days prior to an expiration date are excluded. Furthermore, the 12th of September was deleted from the analysis as the file contained errors.

This leads to a final data sample of 423 trading days. The order book data comprises every tick order with prices and sizes up to the 10th level for the bid and ask side including a timestamp, traceable up to nanoseconds. The trading data includes every trade with a timestamp, its executed price, traded volume and the side that initiated the trade (buy/sell). Intraday timespan are open market hours from 08:00 until 22:00 CET, excluding the opening and closing auction. The initial dataset (tick-by-tick) is 150 GB. Python was used to reconstruct the order book in a format such that it can be analysed for statistical purposes. For statistical analysis we used R.

We decided to subsample at 1-minute intervals, resulting in 344,449 observations during our sample period. Although we would like to analyse the data at higher frequencies the computational requirements are not met.

### 3.3. Stylized Facts about the Euro Stoxx Future Limit Order Book

In a LOB every market participant can enter his orders. Orders can be either sell (bid) orders or buy (ask) orders. As market participants do not necessarily want to buy or sell an asset at the current observed price, but somewhere close to this price they can enter limit orders. A trade will be executed once an order of the opposite direction is entered at the limit. As many participants enter such limit orders with the corresponding quantities, they are willing to buy or sell at a given price, the order book can be aggregated across price levels.

Figure 1 shows a schematic structure of a LOB. Imagine, someone wants to buy 200 contracts, but is not too concerned about price execution and therefore enters a market order. At a given point in time (ceteris paribus) the price level of 3,500 contains only 70 contracts to buy and thus the price will increase to 3,501 with still 130 (200 – 70) contracts to buy. As this level (3,501) only contains 60 contracts the price will jump one more level up to 3,502 and will remain at this level as the market order (200 contracts) is filled (50 contracts will remain at price 3,502).

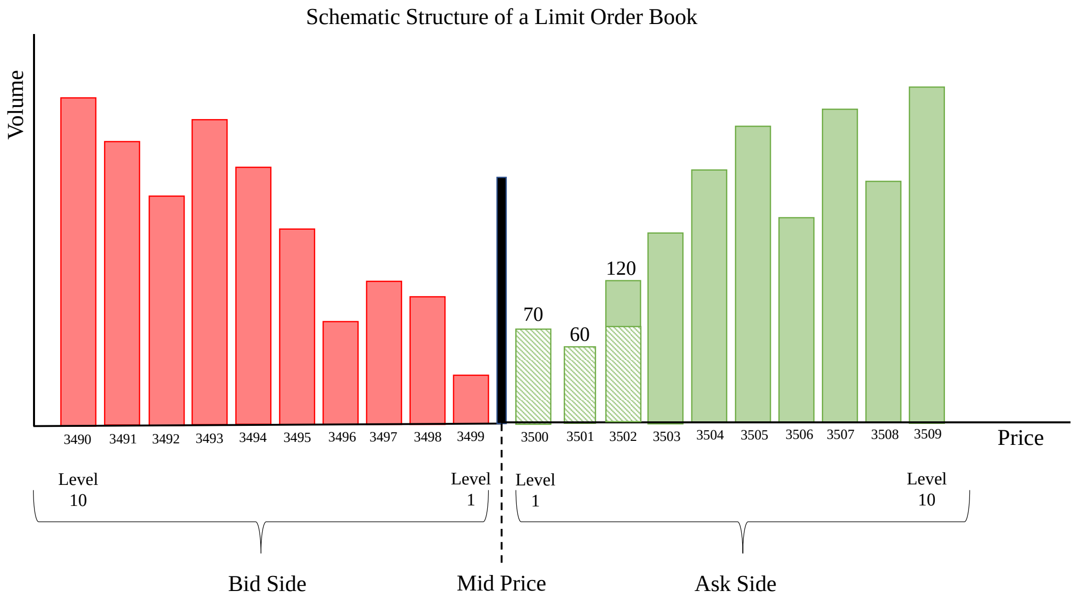


*Figure 1 – Schematic Illustration of a LOB*

If someone would have entered this order with a price limit of 3,501 only 130 contracts would have been traded (Level 1 and 2).

The FESX Future is a highly liquid market in many aspects. Over the sample period we find that spreads stayed at minimum tick (1 basis point) for 99.2%. Order book depth, defined as the cumulative volume of contracts across bid and ask levels displays intraday seasonality (Appendix Figure 1). In the morning hours of trading, market participants start to actively place limit orders and the order book gets filled. During the day you see an increase in order book volume, which is decreasing significantly around 17:30 when the cash market in Frankfurt closes. In the late evening hours, market participants start to cancel their remaining orders in the book leading to slow decrease in order book volume until Futures exchange closes. For descriptive statistics about the order book and trade data over our sample period see Appendix Table 1.

The order book is very sensitive to news impacts and the agreement upon a fundamental/fair price of the FESX Future at a given point in time (see Figure 2 for an illustration of the order book for two consecutive days). February 5, 2018 can be defined as a "normal" trading day, where at the best bid and ask level (in Figure 1 this is level 9 and level 10 respectively) most of the trading occurs, as characterized through a clearly shaped valley along the trading day. This occurs since market makers are active at these levels, contributing with stable, but small liquidity (volume). In the higher levels more liquidity can be found, as "hedgers" and "speculators" place their limit orders here. Hedgers tend to trade larger sizes to neutralize option delta or other offsetting positions. Speculators, in fact want to gain or reduce market exposure as they believe that markets are on the rise or declining. Both are concerned about price execution and therefore place limit orders instead of market orders. However, large sizes tend to be traded using the TES (Trade Service Functionality), where two or more market participants agree upon a price for a trade. Trades in the TES system do not appear in the LOB. In turbulent market times, the order book does not have this structure anymore. On February 6, European markets were hit by the "short vol-squeeze", caused by a sharp decline in the S&P500 and a spike in the VIX the evening before. The line chart in Figure 2 shows the realized spot volatility for the given days. One can clearly see, that volatility during February 6, 2018 exceeded the one observed during February 5, 2018 by far.

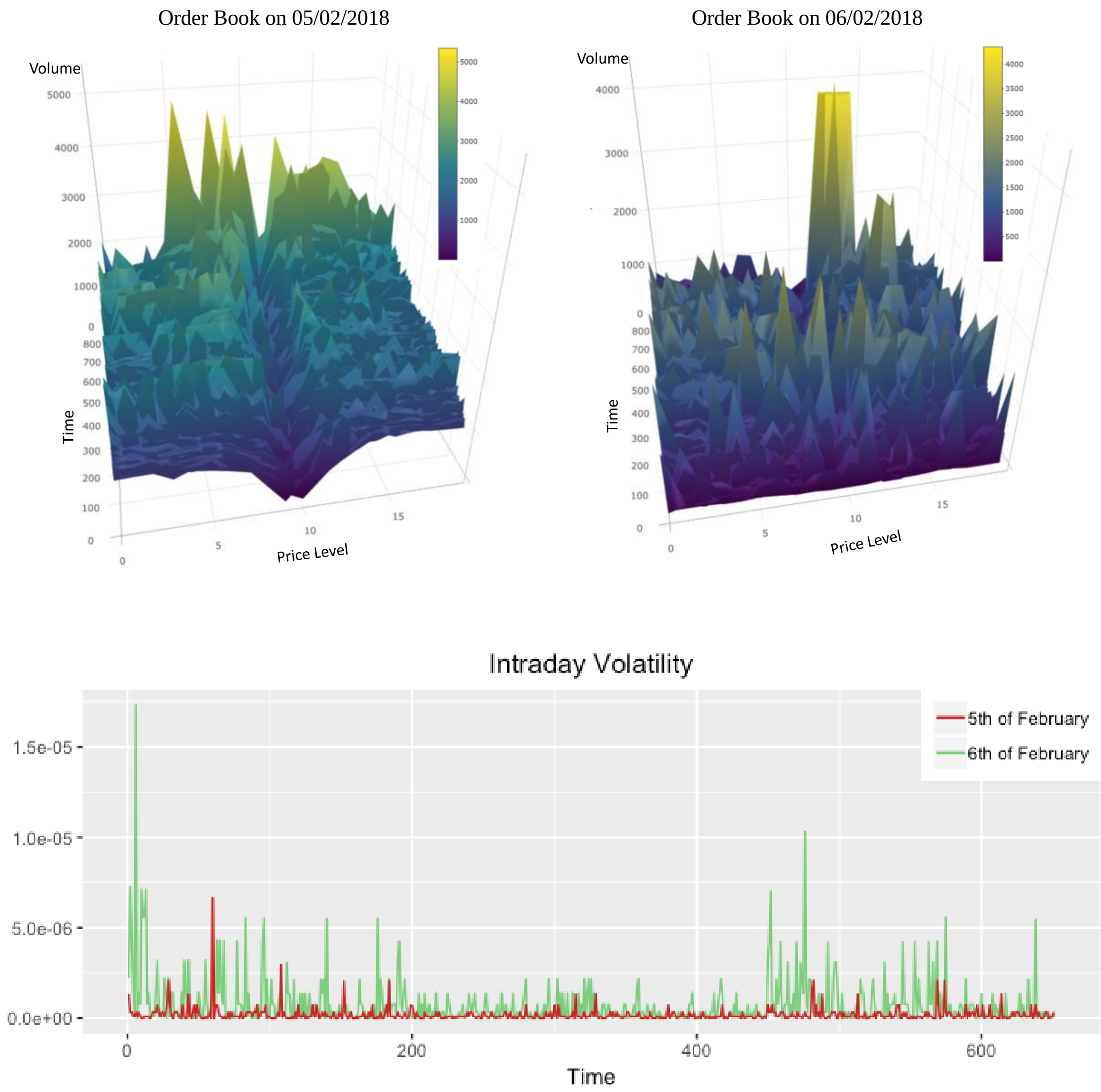


*Figure 2 – Intraday Order Book and Volatility for the 5th and 6th of February 2018*

As seen in Figure 2 the order book has random “volume” spikes concluding that market participants do not agree upon a fair price level. During such times, market makers and other high-frequency participants normally step out of the market, as they do not like excessive volatility (Easley et al., 2012).

# 4. Methodology

To estimate and forecast volatility in high-frequency data one needs to take into consideration several features of intraday returns, such as microstructure noise, the well-known intraday seasonality and the discreteness of the underlying price for FESX Futures, which has the minimum change of 1 index point by construction.

As previously discussed, recent literature (Liu et al., 2015) suggests to sub-sample intraday returns at a frequency between 5 to 10 minutes. In liquid markets, such as the FESX market, this would mean 99.7% of the observations (341 trades) would be lost for trade data on a randomly chosen day (20/02/2017 from 10:30 until 10:35), when sub-sampling at 5-minutes intervals. The loss is even larger when considering order book updates. Within the mentioned time interval there were 10,048 updates. Due to the nature of the FESX market (a lot of market makers, institutional traders and arbitrageurs) it would be naïve to believe that observations at higher frequencies do not contain any information about price formation in the market.

In the following section the model setup and the incorporated model assumptions to overcome the aforementioned features of high-frequency returns are explained in detail.

## 4.1. Notion

In the following, observation days are indexed by $t$ $(t = 1, \dots, T)$. Each observation day is subsampled into 1-minute intervals, where always the last available price for a particular bin was used. Intraday data is denoted as $i$ ( $i = 1, \dots, N$), i.e., a price for the FESX Future for a given day and time is expressed as $P_{t,i}$.

Continuous price returns are then calculated as,

$$r_{t,i} = \ln\left(\frac{P_{t,i}}{P_{t,i-1}}\right) \qquad \text{for } i \geq 1. \qquad (1.0)$$

The analysis follows the convention as in Engle and Sokalska (2012) who suggest to leave-out over-night returns, where implications will be discussed in detail later. Furthermore, for some time intervals there was no trade data available due to the fact that no trade was executed within a 1-minute interval. This occurred especially in the evening hours. For estimation and comparison those observations are deleted, leading to 344,449 1-minute bins during the sample period.

### 4.2. The Model

The paper follows closely the proposed multiplicative component (mcs)GARCH framework used in Engle and Sokalska (2012) with minor adjustments proposed by Ghalanos (2018), by decomposing the conditional variance of intraday returns as a product of stochastic intraday volatilities, and diurnal and daily components. The process of intraday returns can thus be expressed as:

$$r_{t,i} = \mu + \varepsilon_{t,i} \tag{2.0}$$

$$\varepsilon_{t,i} = \left(\sigma_{t,i} h_t s_i\right) z_{t,i}, \tag{2.1}$$

where

$\sigma_{t,i}$, is the stochastic intraday volatility;

$h_t$, is a proxy for the forecasted daily end of day volatility;

$s_i$, the diurnal pattern for each intraday interval;

$z_{t,i}$, is the i.i.d. (0,1) standardized innovation that follows a student-t distribution.

This paper finds that trade price returns as well as returns of the latent prices are leptokurtic and fat-tailed distributed (Appendix Figure 2). Thus, in estimation we assume a student-t distribution for the conditional distribution to try to capture most of these properties. In contrast to Gatheral and Oomen (2010), we do not find that any of the return series suffers from strong autocorrelation (Appendix Figure 3).

The daily forecast for $\sigma_t$ is derived from implied option volatilities on the FESX Future. The one day lagged VSTOXX Index, a benchmark index for implied option volatility on the FESX Future, thus serves as a forecast for the expected end of day volatility. As the VSTOXX is expressed in annualized terms this research uses market convention - the square root of 260 trading days - to come up with a daily volatility estimate. Busch et al. (2011) find for different asset classes that *'implied [option] volatility contains incremental information about future volatility'* (p. 1) and serves as an unbiased estimator for 2 out of 3 investigated asset classes, namely the FX and Stock market. If in our case the implied volatility on the FESX Future serves as an unbiased estimator for future realized volatility, and we assume the intraday returns to be serially uncorrelated, then the daily conditional variance is nothing else than the sum of the squared returns of each 1-minute interval.

Thus,

$$E\left(\sum_{i=1}^{N} \frac{r_{t,i}^2}{h_t}\right) = \lambda, \qquad (2.3)$$

where $\lambda$ is a fixed constant.

If overnight returns are included and the mentioned assumptions hold, $\lambda$ should equal to one. If the estimate is biased but constant over time, then $\lambda$ will be a value different from one. However, this will not affect the subsequent model. Using this parsimonious approach, daily forecasts over longer time horizons for the multiplicative component GARCH model are not necessary and one can work with shorter samples (Engle and Sokalska, 2012).

The diurnal component of the described process can be expressed as follows,

$$s_i = Med\left(\frac{\hat{\varepsilon}_{t,i}^2}{h_t^2}\right), \tag{2.4}$$

where $\hat{\varepsilon}_{t,i}$ is the actual residual of the estimation.

We thus obtain the normalized residuals by dividing the residuals by the diurnal and daily volatility, i.e.,

$$\bar{\varepsilon}_{t,i} = \frac{\hat{\varepsilon}_{t,i}}{h_t s_i} \tag{2.5}$$

which are then used to estimate the stochastic volatility component $\sigma_{t,i}^2$ following a plain GARCH(1,1) model, such as

$$\sigma_{t,i}^2 = \omega + \sum_{j=1}^{p} \alpha_j \bar{\varepsilon}^2{}_{t,i-j} + \sum_{j=1}^{q} \beta_j \sigma^2{}_{t,i-j}. \tag{2.6}$$

Deviating from Engle and Sokalska's (2012) approach, the conditional mean as well as the variance equation are jointly estimated. Moreover, this approach uses the median instead of the mean for the diurnal component as it is found to be more robust (Ghalanos, 2018).

We estimated different GARCH model specifications with different lags in $q$ and $p$. Nevertheless, depending on the latent price variable we find that a parsimonious specification i.e., (p=q=1) is generally enough, as the return series do not show large memory effects, beside outliers (Appendix Figure 3).

Figure 3 shows the volatility decomposition into the diurnal pattern, the stochastic volatility component and the daily end of day forecast from January, 3 2017 until February, 3 2017 for trade returns.

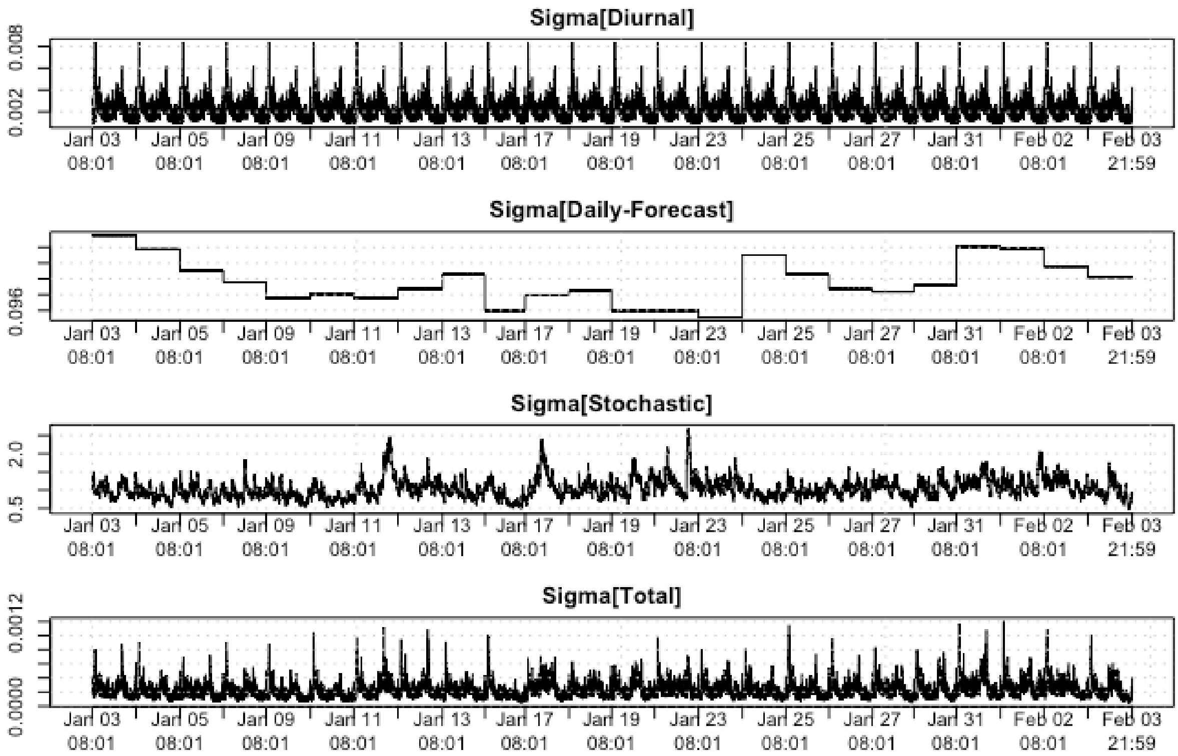


*Figure 3 – Decomposition of total volatility into the diurnal pattern, the daily forecast and the stochastic components for trade returns.*

### 4.3. Micro-Prices – Incorporating Limit Order Book Information

In recent literature a lot of research was done to uncover the information content of order book data, either by including liquidity measurements, such as order book depth and spreads to determine the variation in asset prices (see e.g. Malec, 2016, or Fuest and Mittnik, 2015). As most of the models need either forecasts of the estimated covariates or make use of a semi-parametric estimation for the state of the order book, it may result in latency problems for high-frequency strategies as computation time increases (Interview – Neetson, 2018). Furthermore, Malec (2016) finds that liquidity measurements seem to have a highly non-linear relationship with price fluctuation. Our research, confirms this, as we do not find liquidity measurements significant in a linear framework to explain the variance.

Micro-prices and derivations of it were currently investigated as a latent variable for asset prices, instead of using plain transaction prices or mid-prices. For example, Stoikov (2017) and Bonart and Lillo (2016) find that the order book imbalance contains strong predictive power for the next traded price. The effect of order book imbalance is most prevailing for large tick stocks and its effect is vanishing the smaller the tick size is. Nevertheless, the micro-price at level 1 can tend to be noisy, as market makers and arbitrageurs trade the spread at the first order book levels, known as pinging strategies. Thus, Hautsch and Huang (2012) conclude that this may not reflect a fundamental price at a given point in time.

Cao et al. (2009) report that most information is conveyed in the first level of an order book. Nevertheless, they found that imbalances in the order book across levels has significant prediction power for future short-term returns.

Therefore, we include the approach of Gatheral and Oomen (2010) to calculate micro-prices, while incorporating higher levels of the order book to show, (1) if in fact level 1 micro-prices are noisy, (2) higher levels contribute to forecasting ability of variation in short-term returns. And finally, (3) to show that coefficient estimation in a high-frequency framework is highly time-varying across different market periods. As far as we know, no one came up with the approach to include higher order book levels to compute micro-prices.

Thus, we construct the micro-price up to level $k$ $(k = 1, \dots, M)$ as:

$$MP_{t,i}^{(M)} = \frac{\sum_{k=1}^{M} v_{t,i}^{a(k)} p_{t,i}^{b(k)} + v_{t,i}^{b(k)} p_{t,i}^{a(k)}}{\sum_{k=1}^{M} v_{t,i}^{a(k)} + v_{t,i}^{b(k)}} \qquad (3.0)$$

where

$v_{t,i}^{a(k)}$ denotes the volume at each level for the ask side at a given time interval;

$v_{t,i}^{b(k)}$ denotes the volume at each level for the bid side at a given time interval;

$p_{t,i}^{a(k)}$ is the ask price at each level at a given time interval;

$p_{t,i}^{b(k)}$ is the bid price at each level at a given time interval.

Level selection for micro-prices is on an arbitrary basis and based on a best-practice approach. As shown in Appendix Figure 4 micro-prices do not heavily diverge from the current traded price. We do not undertake the analysis for mid-prices, as spreads wider than 1 tick occur rarely, even in stressed market periods compared to the overall sample size. Compared to trade returns, micro returns suffer more from the intraday seasonality the more levels are included as they bear two components. Seasonality in the volatility and additional induced seasonality by liquidity as shown in Appendix Figure 5. Compared to other financial markets we do not find the characteristic "U" or "L"-shape but more a "W"-shape, as the FESX-Futures has an opening and closing auction, as well is influenced during the day by the opening of the NYSE stock exchange around 15:30.

# 5. Parameter Estimation and Property Analysis

Based on the (mcs)GARCH model developed, we estimate the model for 80% of our sample (the remaining 20% are left for forecast evaluation) for different price returns. In the following, the estimated parameters of the model are briefly discussed, followed by an analysis of the residuals.

Table 1 summarises the estimated parameters from the (mcs)GARCH model (2.6). We find for all observed price returns that the conditional variance is highly persistent as the sum of $\alpha$ and $\beta$ is close to 1. By construction, the parameters of the GARCH models are weights and thus we find the constant $\omega$ of the GARCH equation close to 0. Interestingly, with trade returns we find the constant of the variance equation ($\omega$=0.0097) significant at the 1% nominal level with robust standard errors based on White's correction (Ghalanos, 2018). For micro-prices '$\omega$' is somewhat close to 0, but insignificant (none of the estimates is 0, but of the power of 1-e09). The same pattern holds for the constant $\mu$ of our mean equation, except for trade returns and micro returns (k=1-2), where we find a constant significantly different from 0.

As imposed by the GARCH framework a constant of ω = 0 is undesirable, as it would suggest that mean-variance in the long-run is not existent. Bollerslev (1986) states the condition of $\omega > 0,$ without further explanation on model implication if this condition is violated. However, Nelson (1992) states that this condition can be less restricted and allows for $\omega \geq 0$ in the GARCH framework. If ω is 0 and the condition α + β = 1 is satisfied, then the GARCH(1,1) process becomes an Exponential Weighted Moving Average (EWMA). Thus, one can write α = 1- β and obtain the EWMA (J.P Morgan/Reuters, 1996), using formula (2.6)

$$\sigma^2_{t,i|i-1} = (1-\beta)\bar{\varepsilon}^2{}_{t,i-1} + \beta\sigma^2{}_{t,i-1} \qquad (4.0)$$

In this case the decay factor of the EWMA process is not arbitrarily chosen but estimated. The forecast of an EWMA, is a martingale, meaning that the best forecast for one-step ahead is the current estimated value.

For higher-level micro-prices (k=1-10), we find that lagged innovations are found to be insignificant. This suggests a GARCH(0,q)-structure.

mcsGARCH(1,1) Paramater Estimation using different Price Returns

| | μ | ω | α | β | α+β |
|---|---|---|---|---|---|
| Trade Returns | 0.0000*** | 0.0097*** | 0.0289*** | 0.9615*** | 0.9900 |
| | (0.0000) | (0.0003) | (0.0004) | (0.0003) | |
| Micro Returns (k=1) | 0.0000 | 0.0000 | 0.0584*** | 0.9406*** | 0.9999 |
| | (0.0000) | (0.0000) | (0.0026) | (0.0030) | |
| Micro Returns (k=1-2) | 0.0000*** | 0.0000 | 0.0781*** | 0.9210*** | 0.9990 |
| | (0.0000) | (0.0000) | (0.0050) | (0.0060) | |
| Micro Returns (k=1-5) | 0.0000*** | 0.0000 | 0.1210*** | 0.8780*** | 0.9999 |
| | (0.0000) | (0.0000) | (0.02777) | (0.0370) | |
| Micro Returns (k=1-10) | 0.0000 | 0.0000 | 0.1390 | 0.8601*** | 0.9999 |
| | (0.0000) | (0.0000) | (0.1063) | (0.1411) | |

p-Value: *** Significance at 1%, ** Significane at 5%, * Significance at 10%, robust S.E. reported in brackets
Remarks: Reported Estimates in this table are not 0, but of the power of 1-e09.

*Table 1 – Coefficient estimation using 80% of the available sample*

If the variance can be fully explained by an GARCH(0,1) process, we thus express the variance at time $t, i$ as

$$\sigma_{t,i}^{2} = \omega + \beta\sigma_{t,i-1}^{2}. \tag{4.1}$$

From above's equation one can use iterative substitution and show that,

$$\sigma^{2} = \frac{\omega}{1-\beta} \tag{4.2}$$

concluding that whatever value the initial conditional variance assumes, after a long enough time horizon the conditional variance will converge to a level around $\frac{\omega}{1-\beta}$ implying unconditional homoscedasticity. This means, it will collide with the law of motion implying that (4.1) holds. In a special case one can reconcile both if $\omega$=0 and $\beta = 1$ for (4.1), where $\sigma_{t,i}^{2}$ will be a constant and equal to the unconditional variance $\sigma^2$ ($\sigma_{t,i}^{2} = \sigma^2$). Thus a GARCH(0,q)-structure would be redundant. This is in line with Bollerslev's (1986) stated condition, where p must be greater 0, whereas the q lag can be 0, implying an ARCH process.

We conclude, that micro-prices, which are in our case the clearing price of the order book (Gatheral and Oomen, 2010) appear to have a long-run stable volatility and it seems like new innovations become more a white noise process the more levels are included. This may be intuitively due to the nature of the FESX market, where the vast majority of trades occur at the lower levels (recall stylized facts about FESX). This means that orders and corresponding sizes near the mid-price are more frequently updated and thus including higher levels is vanishing/averages out the effect of liquidity shocks at lower levels, as larger trades tend to be traded via TES.

However, we find that micro returns tend to have better model specification properties, except for micro returns (k=1-2) when inspecting their residuals. Appendix Figure 6 shows the ACF plots of the standardized squared residuals for the different input returns under the (mcs)GARCH. From visual inspection it clearly follows that the lagged residuals of the micro-prices (Panel B, C and D) do not show any significant autocorrelation beside some random jumps, whereas standardized squared residuals in trade returns (Panel A) tend to be noisy. In fact, in panel C for standardized squared residuals on micro returns (k=1-2), we find a small but persistent negative memory effect. Moreover, including more levels, except for micro returns (k=2), results in more structured data as the jumps of the standardized squared residuals tend to decrease significantly. As the jumps for micro-prices occur randomly, we assume that they do not harm the overall model quality. This pattern also holds for lags larger than 120 (2hrs), for example up to the $800^{th}$ lag (1 trading day).

As we write this paper emphasizing a practitioner's view, we are more concerned with out-of-sample forecast accuracy rather than in-sample estimation. In the next section a forecast exercise is presented using a fixed and rolling window approach.

# 6. Forecasting Results

In the following section the forecasting results are presented. First, the fixed window forecast, where 80% of the sample (from 03/01/2017 – 08:01 to 24/05/2018 – 11:44) are used for estimation purpose and the remaining observations (until 28/09/2018 – 21:59) are left out for forecasting. Additionally, a rolling window forecast is conducted where 10% of the sample are used as the initial window size. To compare forecasting results we use two loss functions: the mean absolute error (MAE)

$$L_{1\{t,i\}} = \frac{\sum_{i=1}^{n} \left| \sigma_{t,i}^{2} - \widehat{\sigma_{t,i}^{2}} \right|}{n} \tag{7.0}$$

and the median of the squared errors

$$L_{2\{t,i\}} = med \left\{ \left( \sigma_{t,i}^{2} - \widehat{\sigma_{t,i}^{2}} \right)^{2} \right\}. \tag{7.1}$$

The forecasted values are compared to the squared returns of the trade returns and the micro return (k=1) series. In this research, squared returns are used as a proxy for realized spot volatility. Due to most trades occurring at the best bid and ask price, market participants, especially market makers and arbitrageurs, would be specifically interested in forecasting the volatility of micro-prices constructed of lower levels in the order book.

### 6.1 Fixed Window Forecast

The fixed window forecast is obtained by using a 1-step ahead forecast with a future innovation component. The future innovation component implies that after every forecast the error term is re-calculated by subtracting the forecasted return from the actual return in that certain time period. With this updated error term, the next 1-step ahead forecast is calculated, while coefficient estimates stay the same. The window size is, as stated before, 80% and the forecast results are compared to the 20% of the sample that was left out for comparison purposes.

| Fixed Window Volatility Forecast using different Price Returns | | | | |
|---|---|---|---|---|
| | Micro Return (k=1) Volatility | | Trade Returns Volatility | |
| | Mean Absolute Error | Median Squared Error | Mean Absolute Error | Median Squared Error |
| Trade Returns | 9.3572E-08 | 4.1761E-15 | 1.0141E-07 | 3.3522E-15 |
| Micro Return (k=1) | 7.4414E-08 | 1.2235E-15 | 9.7956E-08 | 2.5058E-15 |
| Micro Return (k=1-2) | 7.4555E-08 | 1.2158E-15 | 9.8107E-08 | 2.5112E-15 |
| Micro Return (k=1-5) | 8.0028E-08 | 1.6198E-15 | 1.0039E-07 | 2.5846E-15 |
| Micro Return (k=1-10) | 8.2902E-08 | 1.8506E-15 | 1.0187E-07 | 2.6457E-15 |

*Table 2 – Forecasting error for different returns series based on MAE and Median Squared Error*

The results are in favour of micro-prices including the first and second levels of the order book as the difference between the loss functions is marginal. One can see that the MAE in both cases is the lowest for micro returns (k=1). The median of the squared errors is the lowest for micro returns (k=1-2) when predicting micro returns (k=1) volatility and the lowest for micro returns (k=1) when forecasting trade return volatility. The worst performing return series is the trade returns series, which yields even worse forecasting results in predicting its own volatility than other return series. Except for the MAE of trade returns volatility, in which case the micro returns (k=1-10) underperforms the most.

## 6.2 Rolling Window Forecast

To build on a more practical view we also conduct a rolling window estimation over the entire sample. The rolling window procedure produces a recursive 15-step ahead forecast. After the forecast is completed, the window shifts 15 observations, simultaneously dropping the first 15 observations from the window and re-estimates the model. The window size of each estimation includes 10% of the data, which means 34,450 observations, so approximately 43 days. The rolling window forecast resulted in 310,050 forecasts per return series from 02/03/2017 – 10:50 to 28/09/2018 – 21:59. In order to correct for outliers, the forecasts are capped by replacing those values that lie outside the upper limit of 3 times the interquartile range (IQR) with the value of the 95$^{th}$ percentile.

| Rolling Window Volatility Forecast using different Price Returns | | | | |
|---|---|---|---|---|
| | Micro Return (k=1) Volatility | | Trade Returns Volatility | |
| | Mean Absolute Error | Median Squared Error | Mean Absolute Error | Median Squared Error |
| Trade Returns | 8.1545E-08 | 3.9562E-15 | 8.4802E-08 | 3.0385E-15 |
| Micro Return (k=1) | 6.7111E-08 | 1.5795E-15 | 8.3424E-08 | 2.5624E-15 |
| Micro Return (k=1-2) | 8.0156E-08 | 1.9353E-15 | 9.6147E-08 | 2.7534E-15 |
| Micro Return (k=1-5) | 1.0046E-07 | 3.8735E-15 | 1.0660E-07 | 3.8936E-15 |
| Micro Return (k=1-10) | 1.3339E-07 | 6.1874E-15 | 1.3762E-07 | 5.4458E-15 |

*Table 3 – Forecasting error for different returns series based on MAE and Median Squared Error*

Regarding both volatility forecasts and loss functions, we find that micro returns (k=1) clearly outperform the other price returns. One of the most interesting findings is that it performs better in forecasting trade return volatility even on a rolling window basis than the trade return series itself. Micro returns (k=1-2) perform second in most cases, except for the MAE when forecasting trade returns volatility. Whereas the results in the fixed window estimation between both micro returns were very close, the discrepancy between both return series increases when executing a rolling window forecast. On the other hand, the performance of trade return series enhances when using a rolling window forecast. Furthermore, we notice a larger deterioration in the performance of micro-price returns including higher levels of the order book. The micro returns (k=1-10) specifically underperform in forecasting the volatility of both micro returns (k=1) and trade returns.

### 6.3 Daily and Intraday Analysis of Forecasting Errors

For further analysis we focus on the forecasting errors on a daily and intraday basis. We specifically aim at identifying patterns in the median of the squared forecasting errors of the different return series. To gain more useful insights we also construct for the median value of the order book depth and trade activity on a intraday basis as shown in Appendix Figure 1.

Throughout the forecasting period the return series suffer the largest forecasting error on February 6, 2018, as shown in Figure 4. This, as already discussed before, was the infamous volatility squeeze. The fact that this spike occurs for all return series demonstrates the difficulty of forecasting volatility in times of switching volatility regimes.

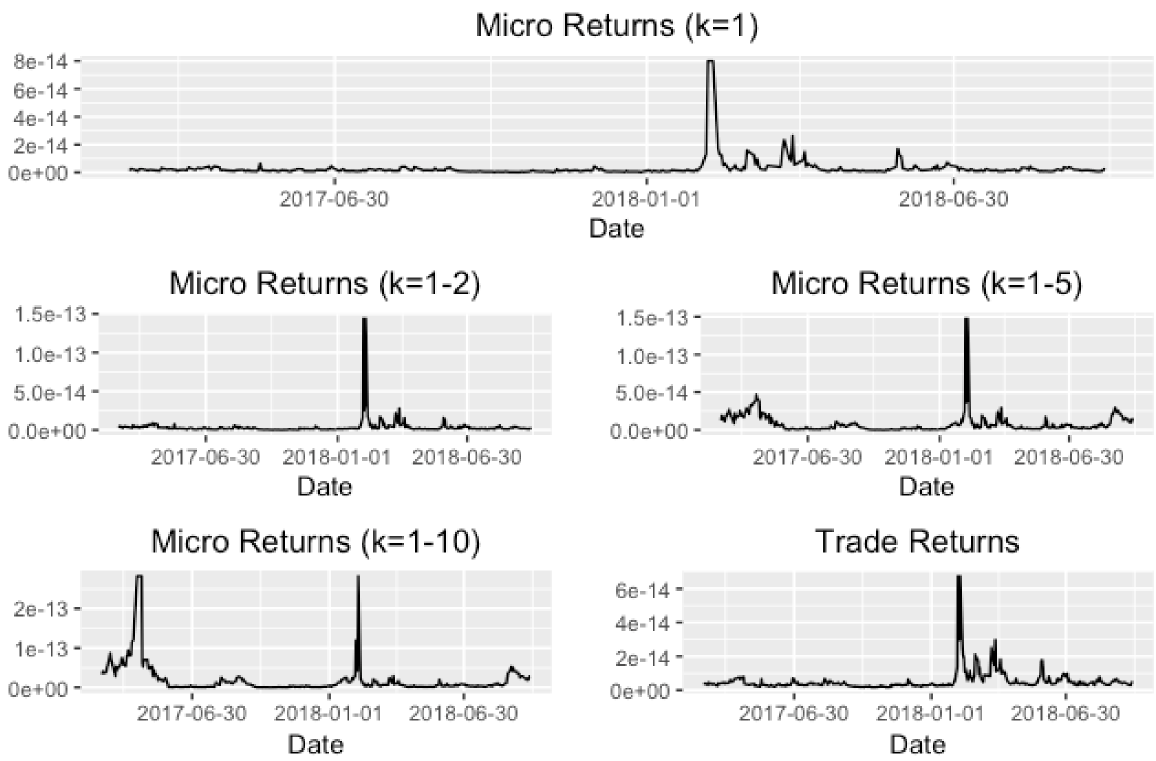


*Figure 4 – Median Squared Error on micro returns (k=1) realized volatility per day.*

The impact of this volatility squeeze is especially larger for micro-prices containing higher levels of the order book. This can be possibly explained by a surge in instability in the order book during more risky market times. For micro returns (k=1-10) there is also a peak to be found at the beginning of the graph, which is mainly flat for other return series. This peak perhaps indicates a lot of volatility that can only be detected in higher levels of the order book.

On an intraday basis the forecasting errors experience the intraday seasonality pattern, the so-called U-shape, which is also to be found for intraday volatility. As seen in Figure 5 at the beginning of the trading day, around 09:00, market participants start placing orders and executing trades, causing a minor peak in trade activity and depth. This is followed by an increase in volatility, which leads to higher forecasting errors.

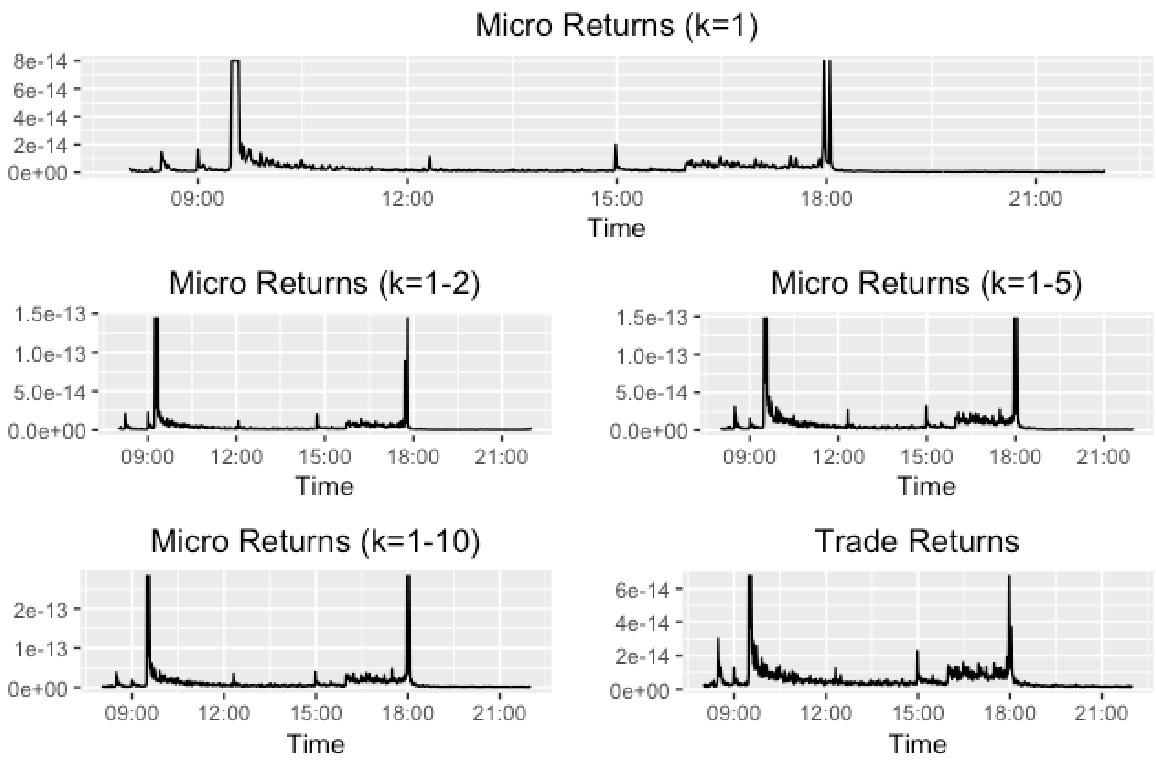


*Figure 5 – Intraday Median Squared Error on micro returns (k=1) realized volatility.*

When the cash market in Frankfurt (XETRA) closes, trade activity and depth experience their highest peak of the day. Once again, this is accompanied by a rise in volatility leading to higher forecasting errors. Furthermore, the largest forecasting errors are to be found for micro-prices including higher levels of the order book.

Additionally, we compute a forecast error ratio on an intraday and daily basis. The ratio is calculated as the forecast error of micro returns (k=1) divided by the forecasting error of each of the other return series. A ratio larger than 1 indicates that micro returns (k=1) performed worse than the other return series and vice versa.

From an intraday perspective the ratio for micro returns (k=1-5) and micro returns (k=1-10) is below 1 the entire day as seen in Figure 6.

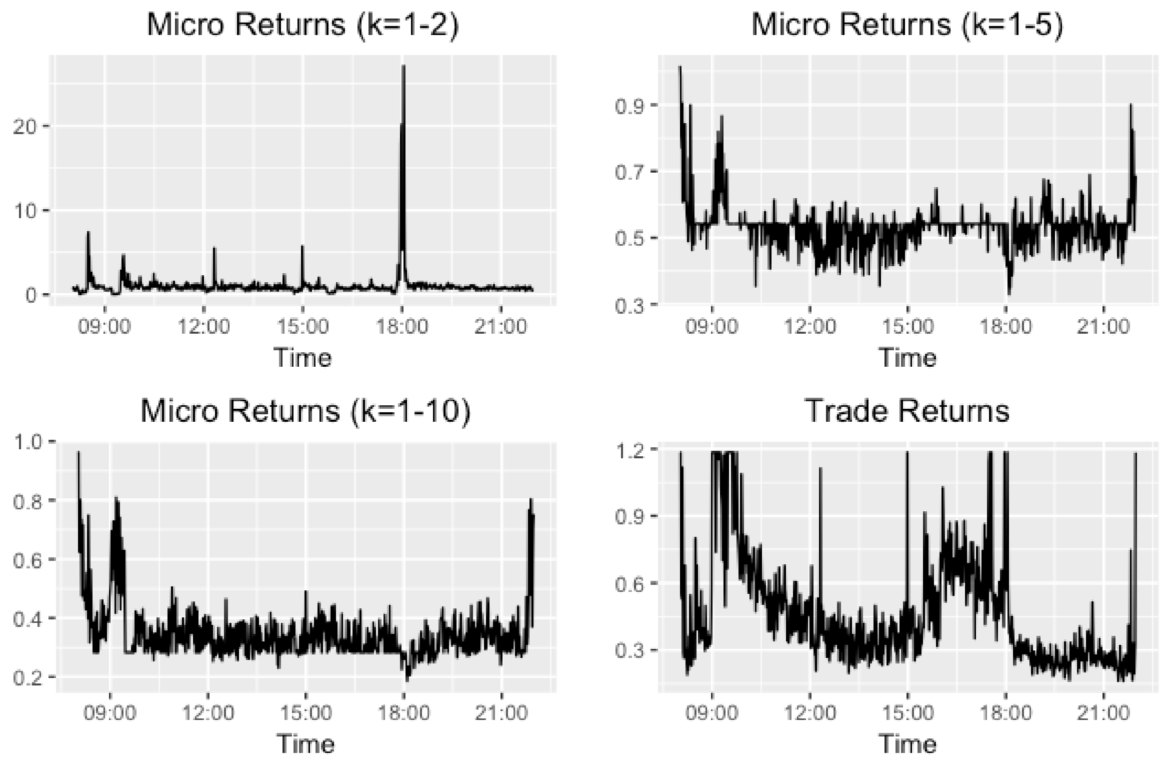


*Figure 6 – Intraday Median Squared Error ratio*

Although there are moments during the day, especially at the beginning and the end, at which the ratio severely increases and decreases.

For micro returns (k=1-2) one can see that there are many moments during the day at which micro returns (k=1-2) performs better. Around 18:00 especially a large spike in the ratio is detected. For the trade return series, a U-shaped pattern can be observed. The pattern can possibly be explained by the intraday seasonality of the LOB. Since the state of the LOB is incorporated into the micro-price, it has a larger impact on the micro returns (k=1) as opposed to trade returns. Relatively large adjustments in the order book depth are accompanied by a worsened performance of the micro returns (k=1) compared to trade returns. The highest spike to be seen shortly after 09:00, which is preceded by a jump in the order book depth, as can be seen in Appendix Figure 1. During the day the order book remains relatively stable, resulting in an enhanced performance for micro returns (k=1). When the depth starts to accelerate after 15:30, since the New York Stock Exchange is open for trading, the forecast error ratio surges again with peaks occurring before 18:00. After these peaks the ratio decreases due to stability promptly returning to the LOB.

If we assume that none of the return series suffers from intraday seasonality, we would expect that the median squared error ratio would stay constant over a day with random fluctuations around this level. The same is true, if we expect that all return series suffer from the same magnitude of intraday seasonality.

As seen in Figure 6 the intraday seasonality in the median ratio for higher-level micro returns is still prevailing but vanishes. We suggest since trade returns suffer far less from intraday seasonality (recall Appendix Figure 5) the diurnal component in the GARCH framework was able to better capture these effects, whereas the combined seasonality effects of return volatility and liquidity induced seasonality cannot be adequately captured by the diurnal component.

When observing the daily median forecast ratio, one can note a numerous amount of days on which the micro returns (k=1-2), micro returns (k=1-5) and micro returns (k=1-10) have a better daily performance than micro returns (k=1) as indicated by a ratio larger than 1 as shown in Figure 7.

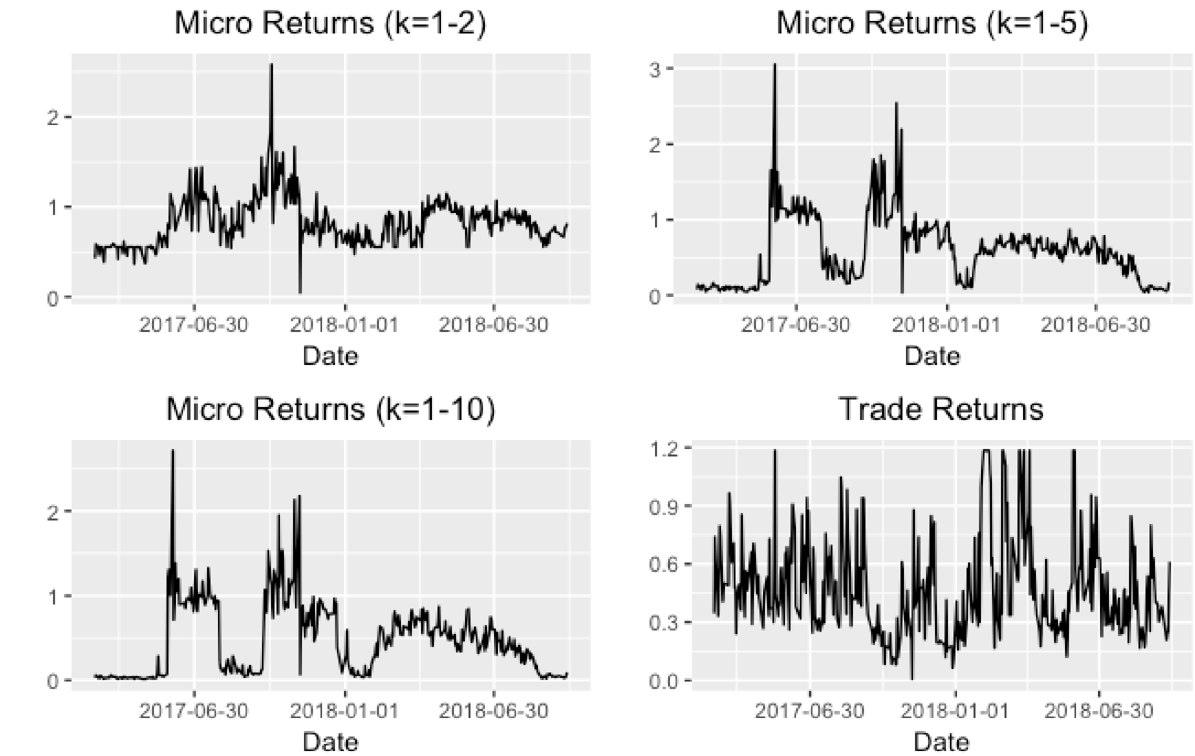


*Figure 7 – Median Squared Error ratio per day*

The majority of these days appear to be clustered and to follow a certain path. For the trade return series, the shocks in the daily forecast error ratio appear to be random and the magnitude of the ratio is smaller than those of the other return series.

# 7. Conclusion

This research used 1-minute Euro Stoxx data under the multiplicative component GARCH framework to provide high frequency forecasts of spot volatility. This study finds that LOB information has strong predictive power to forecast short-term variation in trade and micro returns. Especially, micro returns at the lower levels have superior forecasting power compared to simple trade returns or higher-level micro returns. However, we find clustered periods where micro returns incorporating higher order book levels deliver same or even better forecasts.

The often-discussed prevalence of intraday seasonality in high-frequency data seems to have negative effects on the forecasting ability. However, we find that the (mcs)GARCH using trade returns is better able to capture these effects as the magnitude of the seasonal pattern is smaller compared to micro returns. We suggest that this is due to the fact that micro returns suffer from a two-fold seasonality, the intraday volatility seasonality and an additional liquidity induced seasonality component.

There is not a lot of research conducted on high-frequency volatility forecasting and most of the research either focused on the stock or foreign exchange market. To our knowledge, no research has been conducted on equity benchmark futures. Therefore, this paper provides valuable insights for academia and market participants that are involved in high-frequency trading.

It would be interesting in future research to focus on finding patterns and explanations under which circumstances the incorporation of higher order book levels help to enhance forecasting ability. Additionally, exploring the relationship between the intraday seasonality pattern of volatility and liquidity would also be an interesting path of research.

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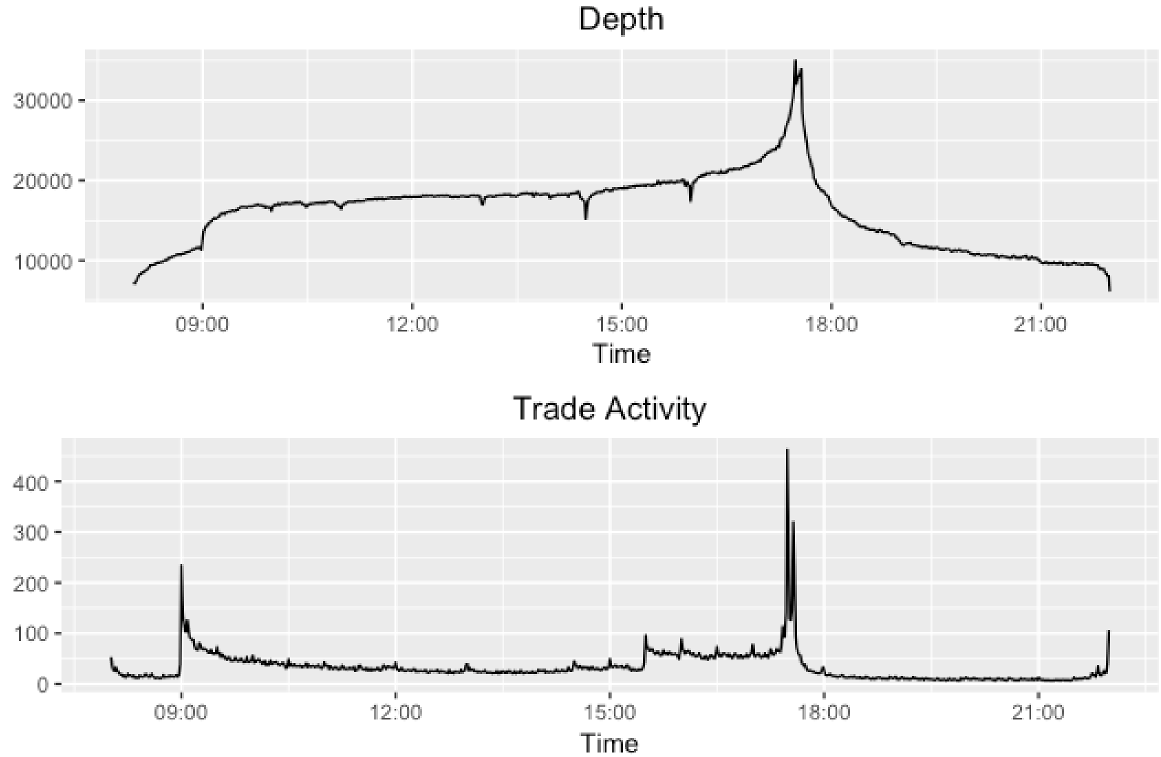
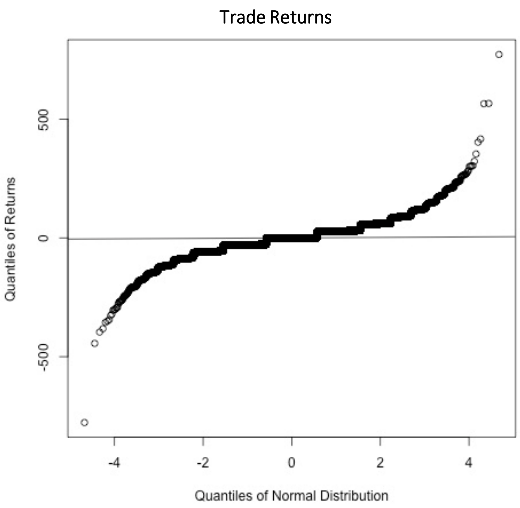
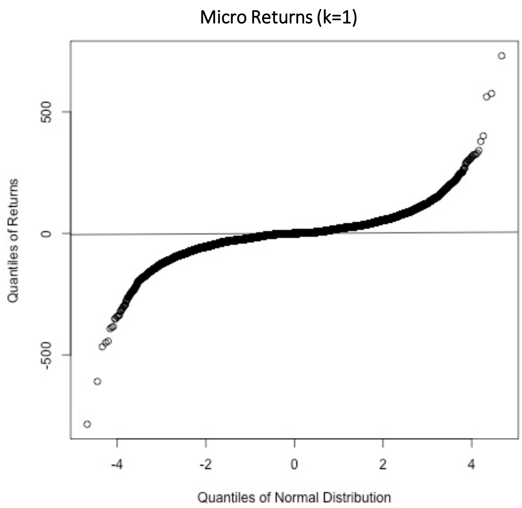
# Appendix



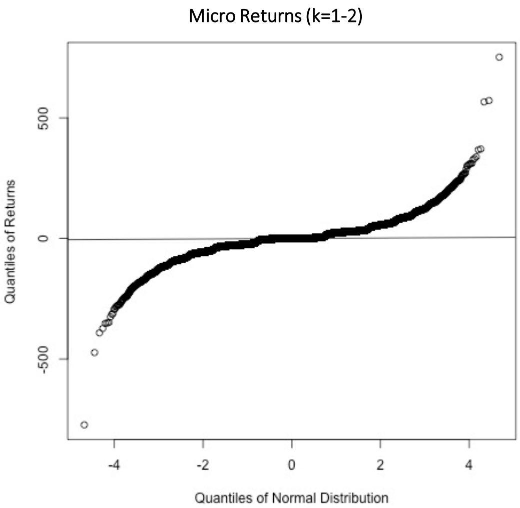

*Appendix Figure 1 – Seasonality pattern in order book depth and trade activity. Shown is the median depth and median cumulative number of trades for each 1-minute interval over the sample period.*



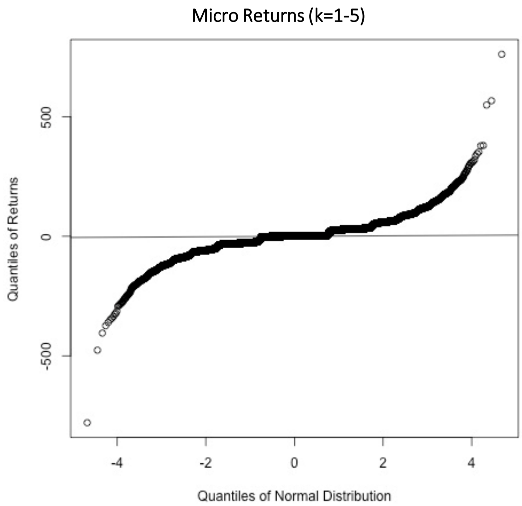
Panel A: QQ Plot on trade returns



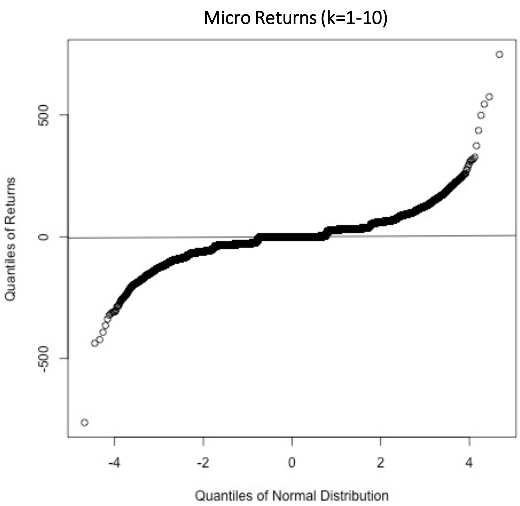
Panel B: QQ Plot on micro returns (k=1)



Panel C: QQ Plot on micro returns (k=1-2)



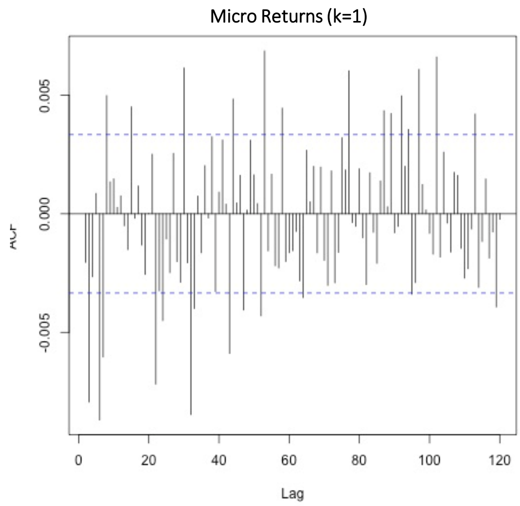
Panel D: QQ Plot on micro returns (k=1-5)



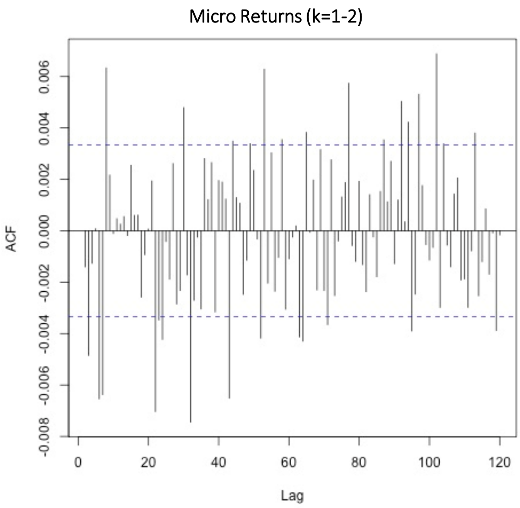
Panel E: QQ Plot on micro returns (k=1-10)

*Appendix Figure 2 – QQ-Plot for different returns*

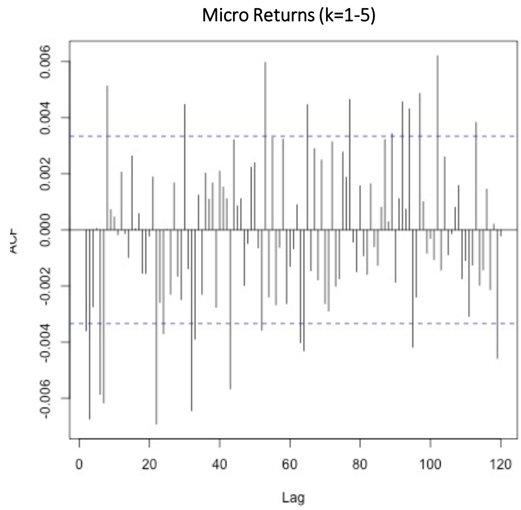

Panel A: ACF Plot - trade returns



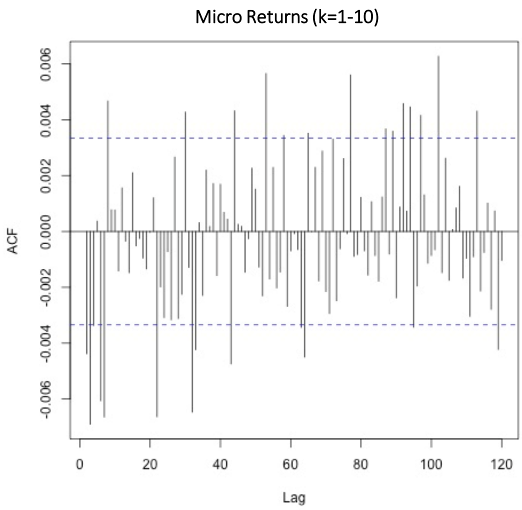

Panel B: ACF Plot - micro returns (k=1)




Panel C: ACF Plot - micro returns (k=1-2)

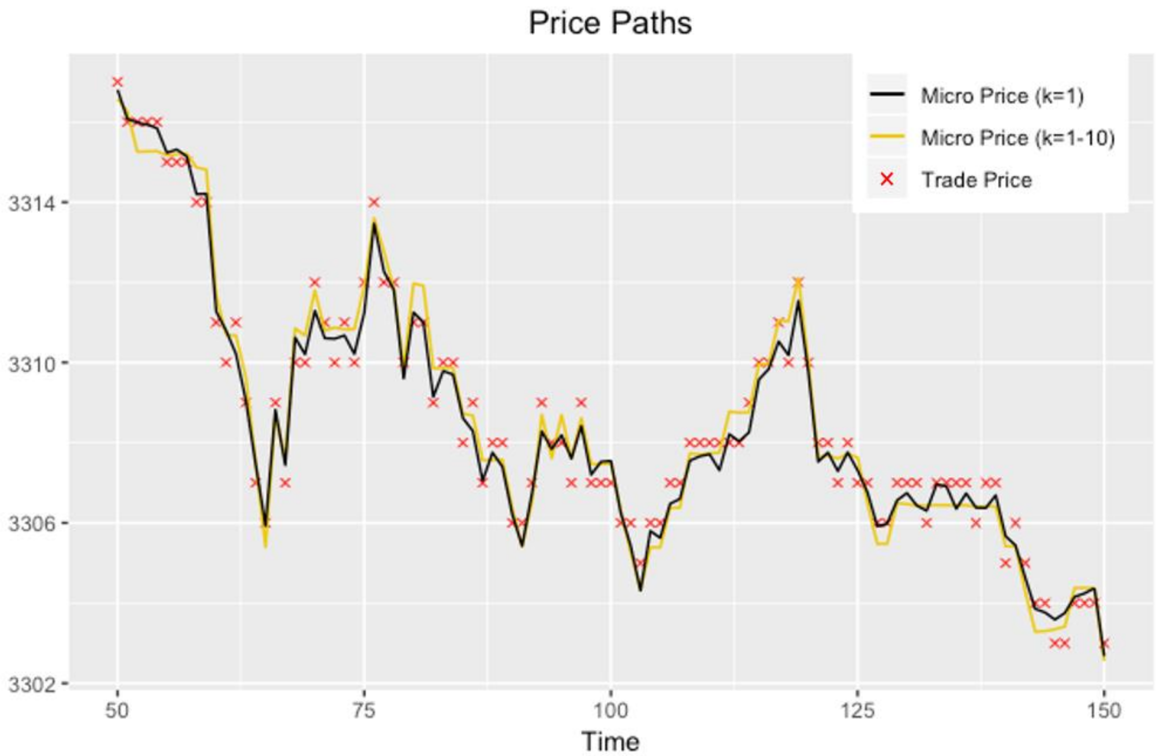



Panel D: ACF Plot - micro returns (k=1-5)

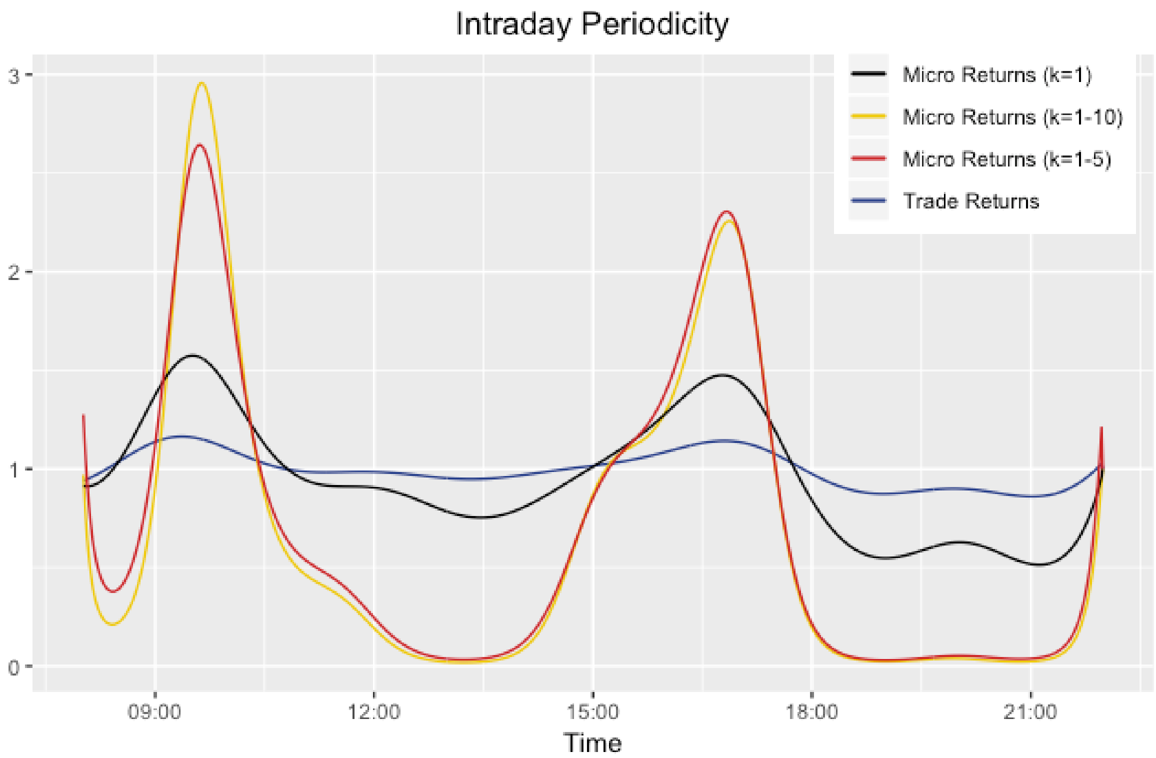



Panel E: ACF Plot - micro returns (k=1-10)

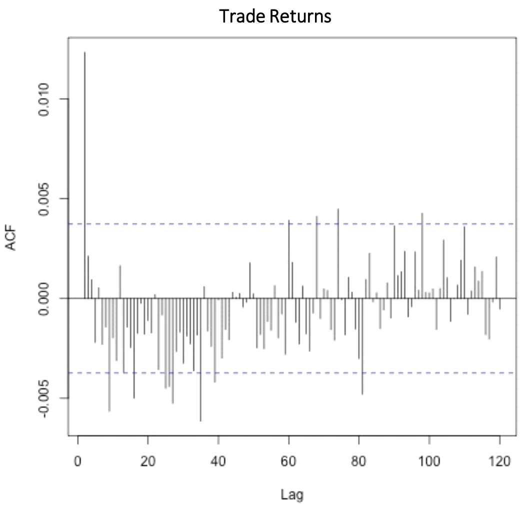
*Appendix Figure 3 – ACF-Plot for different returns*



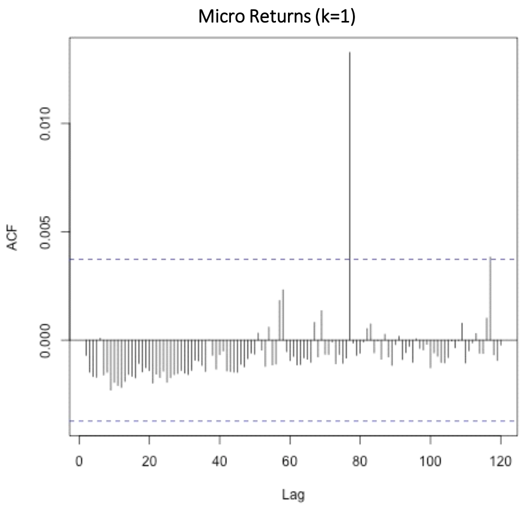

*Appendix Figure 4 – Different price series for 2.5 hours of data*



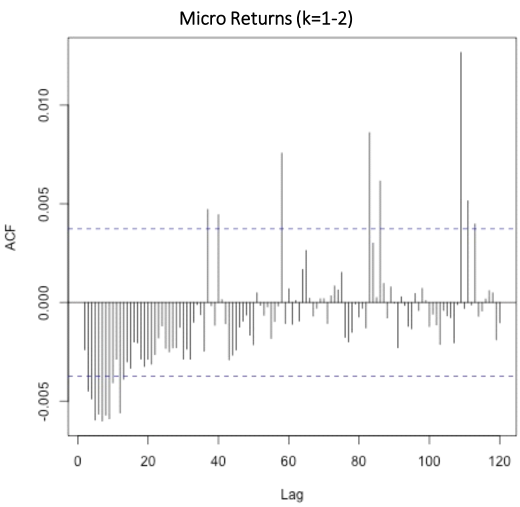

*Appendix Figure 5 – Intraday periodicity for different returns*



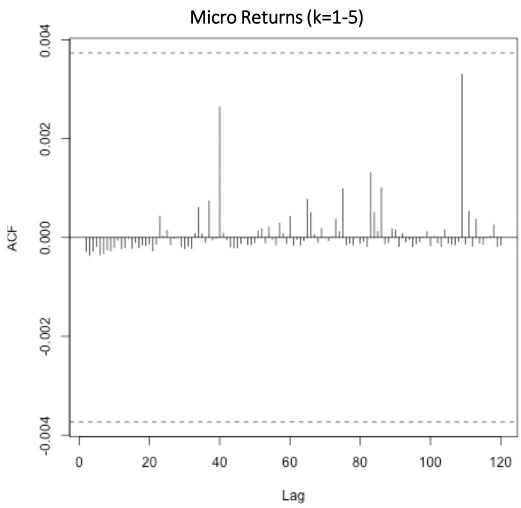

Panel A: ACF Plot - standardized squared residuals on trade returns



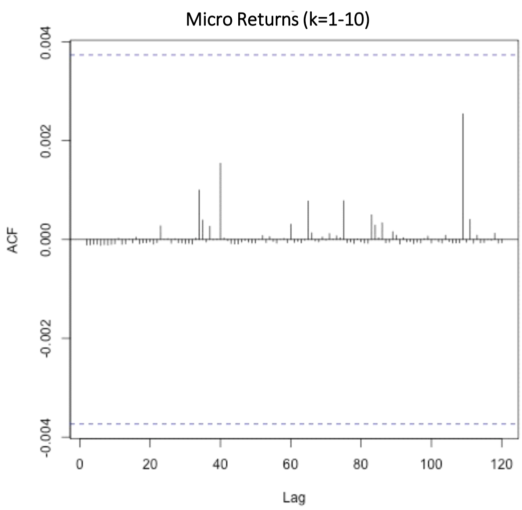

Panel B: ACF Plot - standardized squared residuals on micro returns (k=1)




Panel C: ACF Plot - standardized squared residuals on micro returns (k=1-2)




Panel D: ACF Plot - standardized squared residuals on micro returns (k=1-5)




Panel E: ACF Plot - standardized squared residuals on micro returns (k=1-10)

*Appendix Figure 6 – ACF on standardized squared residuals*

| Descriptive Statistics - Trade + Order Book Data | | | | | |
|---|---|---|---|---|---|
| | Trade Activity | Mean Volume | Sell Ratio | Spread | Depth |
| Min | 1 | 1 | 0 | -35 | 472 |
| 1st Quartile | 12 | 8.9 | 0.3333 | 1 | 12503 |
| Median | 28 | 16.92 | 0.5 | 1 | 16429 |
| 3rd Quartile | 56 | 27.05 | 0.6562 | 1 | 19468 |
| Max | 1837 | 772 | 1 | 3 | 56626 |
| Mean | 43.32 | 20.46 | 0.4952 | 0.9223 | 16335 |

*Appendix Table 1 – Descriptive trade and LOB statistics based on 1-minute intervals. Trade Activity is based on the number of occurred trades within an interval, mean volume is based on the mean traded volume within a minute, sell-ratio is based on the number of trades that were initiated by a sell order divided by the total number of trades (trade activity) that occurred within a 1-minute interval, spread is defined as the difference between best ask and best bid price and depth is defined as the cumulative volume of all order book levels (bid and ask).*