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This paper explores some implications for stabilization policy of the existence of political or institutional limits on the use of control variables. The importance of some non-linear autoregressive processes for modeling government behaviour in the face of such constraints is highlighted. Normative implications for constitutional design are explored by comparing the relative efficiency of a balanced budget versus a spending limit rule.

1. INTRODUCTION

Observation of stabilization policies conducted in contemporary economies suggests that economic authorities very often find themselves severely constrained in the use of their control variables. Perhaps one of the most widely known circumstances where such restrictions are felt refers to the downward inflexibility of government expenditure or its growth rate that emerges when, as happened recently in many western economies, there is an explicit desire to cut in the size of public spending either for allocation or stabilization purposes. The downward rigidities arise in general as a consequence of political and institutional factors. In the United States, for example, it is well-known the difficulties faced by the Reagan Administration in reducing spending in

* Department of Economics, New University of Lisbon, I have benefited from discussions of this subject with José D. Coelho, João I. Pinheiro, B. Murteira and C. Wyplosz. I am specially indebted to David A. Jones for a clarification of a number of critical issues. Errors are of my own responsibility.

social security programs when the proposed cuts severely endanger previously acquired benefits, the so-called 'entitlements'.

In the present paper we intend to explore in a simple analytical framework some potential consequences for stabilization policy of the existence of such kind of restrictions affecting the use of policy instruments. Specifically, we attempt to examine within a stochastic framework some implications for aggregate demand management of rigidities in the growth rate of money creation as a result of political or institutional factors. In this context we shall try to highlight the importance of some non-linear autoregressive processes for the analysis of that problem. In section 2 we describe the basic model, investigate the question of stability posed by that rigidity and analyse some of its steady-state effects. Section 3 is devoted to the study of the occupancy times spent by the economy under the two alternative regimes emerging from such institutional framework and their consequences for the average growth rate of money. In section 4 some normative implications for constitutional design are derived from the analytical apparatus explored in earlier sections.

2. THE MODEL

We envisage an economy essentially subject to aggregate demand disturbances. The growth rate of nominal demand y_t is governed by the following simple first-order autoregressive model

$$y_t = \alpha + \lambda y_{t-1} + \beta g_t + u_t \quad (1)$$

where $\lambda, 0 < \lambda < 1$, accounts for positive serial correlation in nominal demand, a fact observed in many economies ¹. This demand-side persistence may be rationalized via the adjustments in velocity following changes in money growth rates. An increase, say, in money growth leads initially to a movement of opposite direction in velocity. As inflation comes to increase the general level of interest rates, velocity will subsequently increase ². The control variable of economic authorities, g_t , reflects money growth originating in the deficit of the stabilization branch. We assume therefore that public expenditures (taxes) of this branch, consisting essentially of positive (negative) net transfers to the private sector, are reflected on a one-to-one basis in money creation (destruction) at a growth rate g_t ³. The parameter α denotes other exogeneously given sources of nominal demand growth, including perhaps the effects of the deficits of the other budget branches or of, say, external imbalances. The economy is subject to nominal shocks, u_t being a Gaussian white noise $n(0, \sigma_u^2)$.

Assume that demand management authorities are interested in minimizing the mean squared error $E \left[(y_t - y^*)^2 \right]$ where y^* is some target

1 - See, for instance, Nelson and Plosser (1982).

2 - See for instance, Basevi, Blanchard, Buiter et al. (1983), Gordon (1982).

3 - See Musgrave (1959).

level for y_t ⁴. To keep the exposition simple assume further that full employment prevails in the labor market perhaps as a result of full indexation in this nominally disturbed economy⁵. The criterion function will thus reflect the desire to minimize the variability of inflation around some optimal rate⁶.

We suppose further that the authorities are subject to the following feedback policy rule:

$$g_t = g_0 + g_1 y_{t-1}, \quad (2)$$

where g_0 and g_1 are parameters to be appropriately chosen. Now, substituting (2) into (1) gives

$$y_t = (\alpha + \beta g_0) + (\lambda + \beta g_1) y_{t-1} + u_t. \quad (3)$$

Therefore, the steady-state variance of y around its mean is given by

$$\sigma_y^2 = \sigma_u^2 / [1 - (\lambda + \beta g_1)^2] \quad (4)$$

Minimizing (4) with respect to g_1 gives

$$g_1 = -\lambda/\beta \quad (5)$$

4 - Nominal income targeting has been advocated by some authors as preferable to a simple money rule since it may account for velocity adjustments. See Basevi, Blanchard, Buiter et al. (1983).

5 - As suggested by Fischer (1977) wage indexation makes good sense in an economy predominantly disturbed by demand shocks.

6 - Discussion about the optimal inflation rate has been conducted among others by Friedman (1969) and Phelps (1973). Emphasis on the welfare losses due to variability of inflation can be found, for instance, in Friedman (1977) and Lucas (1981).

From (3) the steady-state mean of y is given by

$$\bar{y} = (\alpha + \beta g_0) + (\lambda + \beta g_1) \bar{y}. \quad (6)$$

By substituting (5) into (6) and making $\bar{y} = y^*$ we readily obtain the appropriate value for g_0 , namely

$$g_0 = (y^* - \alpha) / \beta \quad (7)$$

Using (5) and (7) into (3) finally gives the stochastic process governing y when the stabilization policy described by (2), (5) and (7) is followed

$$y_t = y^* + u_t, \quad (8)$$

that is, the fiscal authorities are able to set y equal to its target level except for an irreducible noise. Note the 'leaning-against-the-wind' characteristic of the optimal fiscal-monetary policy rule as clearly indicated by the negative coefficient g_1 .

Assume now that due to political and/or institutional reasons fiscal authorities are not able in any circumstances to cut public spending below certain exogeneously given limits which imply

$$g_t \geq L, \quad \forall t. \quad (9)$$

Assume further that g_t will keep being governed by (2) as long as $g_t \geq L$.⁷ What are the stochastic implications for the y_t process of this different set of constraints?

7 - It can be shown that it is still optimal for the government to follow rule (2) when the constraint $g_t \geq L$ is not binding.

We first inquire about the stationarity of the new process. From (2) and (9) it follows that if

$$y_{t-1} \leq (y^* - \beta L - \alpha) \lambda^{-1} \quad (10)$$

we have g_t following the rule (2), call it Regime 1, in which case y_t will be given by (8). If, instead (10) is not satisfied, g_t will be set equal to the lower limit L , call it Regime 2, and y_t will be then governed by

$$y_t = \alpha + \lambda y_{t-1} + \beta L + u_t \quad (11)$$

Adding and subtracting y^* to the right hand side of (11), multiplying and dividing the constant term thus obtained by λ and making $B = (y^* - \beta L - \alpha) \lambda^{-1}$ will allow us to write the complete model in the following way

$$y_t = \psi(y_{t-1}) + u_t \quad (12)$$

where

$$\psi(y_{t-1}) = \begin{cases} y^* & \text{if } y_{t-1} \leq B \quad (\text{Regime 1}) \\ \lambda y_{t-1} - \lambda B + y^* & \text{if } y_{t-1} > B \quad (\text{Regime 2}) \end{cases} \quad (13.a)$$

$$(13.b)$$

This is a nonlinear autoregressive process of the threshold type ⁸.

Tweedie (1975) has shown that if the input distribution is absolutely

8 - For a discussion of nonlinear autoregressive processes see D.A. Jones (1978). The study of threshold autoregressive models (TAR) has been done by H. Tong (1978).

continuous, a sufficient condition for stationarity is the existence of constants $\epsilon, \gamma > 0$, such that

$$E \{ |\psi(x+u)| - |\psi(x)| \} \leq -\epsilon, \quad (|x| > \gamma). \quad (14)$$

This condition is satisfied. In fact, in view of the semi-linear characteristic of $\psi(x)$, then for any finite B ,

$$\lim_{|x| \rightarrow \infty} \{ E |\psi(x+u)| - |\psi(x)| \} = 0, \quad (15)$$

and since $0 \leq \psi'(x) < 1$, for $x \in (-\infty, B) \cup (B, +\infty)$, then

$$\lim_{|x| \rightarrow \infty} \{ |\psi(x)| - |x| \} = -\infty \quad (16)$$

Therefore, by choosing γ sufficiently large, one shall be able to guarantee that (14) is verified for any given finite value of B or equivalently, L . The restriction on money growth, whatever its magnitude does not therefore affect the stationarity of the economy. We proceed now by investigating the effect of the institutional restriction on the steady-state mean and variance of aggregate demand. Following Jones (1978),

it is possible through formal power-series expansions to represent the non-linear stochastic process y_t as an infinite sum of stationary stochastic processes $y^{(i)}$.

$$y_t = y_t^{(0)} + \mu y_t^{(1)} + 1/2 \mu^2 y_t^{(2)} + \dots \quad (17)$$

where μ is a parameter and the component processes are given by:

$$y_t^{(0)} = a + b y_{t-1}^{(0)} + u_t \quad (18.a)$$

$$y_t^{(1)} = b y_{t-1}^{(0)} + \Lambda(y_{t-1}^{(0)}) \quad (18.b)$$

$$y_t^{(2)} = b y_{t-1}^{(2)} + 2 \Lambda'(y_{t-1}^{(0)}) y_{t-1}^{(1)} \quad (18.c)$$

...

In these processes a and b are arbitrary but fixed constants, $\Lambda(x)$ is given by:

$$\Lambda(x) = \psi(x) - bx - a \quad (19)$$

and $\Lambda'(x)$ denotes the first-order derivative of $\Lambda(x)$. For our purposes, and in order to get tractable expressions below we assume $\mu=1$, $b = 0$, and set $a = y^*$. Therefore, the y process can be described by

$$y_t = y^* + u_t + \Lambda(y_{t-1}), \quad (20)$$

where

$$\Lambda(x) = \begin{cases} 0 & x \leq B \\ \lambda(x-B) & x > B \end{cases} \quad (21.a)$$

$$(21.b)$$

Clearly, since $\Lambda(x) > 0$, then $E[\Lambda(x)] > 0$ and therefore the steady-state mean of the process, $\bar{y}$, will satisfy

$$\bar{y} > y^*. \quad (22)$$

This shows that an inflationary bias is imparted into the economy, an expected conclusion in view of the fact that when the economy becomes too heated fiscal authorities are no longer able to countervail demand to a sufficient extent.

An approximation for $\bar{y}$ can be obtained if we truncate the formal expansion (17) by taking, say, only the first three terms ⁹ and

9 - In view of the truncation it should be clear that the goodness of approximation will depend on the choice of the μ , a and b parameters.

take its expected value so as to give:

$$\hat{y} = E(y^* + u) + E\{\Lambda(y^* + u)\} \{1 + E[\Lambda'(y^* + u)]\} \quad (23)$$

It can be easily shown (see Appendix) that:

$$\delta \hat{y} / \delta \lambda > 0, \quad \delta \hat{y} / \delta L > 0 \quad (24)$$

so that the bias is positively related to the degree of persistence of the uncontrolled economy, and to the magnitude of the lower bound on g_t .

The steady-state variance of aggregate demand can in turn be approximated by:

$$\sigma_{\hat{y}}^2 = \sigma_u^2 + \text{var} [\Lambda(y^* + u)], \quad (25)$$

Inspection of (25) clearly shows that the variance of the system has the variance of the noise as a lower bound and it is easy to show (see Appendix) that:

$$\delta \sigma_{\hat{y}}^2 / \delta \lambda > 0, \quad \delta \sigma_{\hat{y}}^2 / \delta L > 0, \quad (26)$$

thus indicating a direction of response of aggregate demand variability to changes in the parameters λ , L , essentially of the same pattern as that of the mean value.

If we interpret L as reflecting some irreversible effect of past public spending and still assume that the objective function of stabilization authorities is $\max W = -E[(y - y^*)^2]$ it is possible now to compute an approximate value to the marginal cost of such type of irreversibility by using:

$$\delta W / \delta L \simeq -\delta \sigma_{\hat{y}}^2 / \delta L - 2(\hat{y} - y^*) \delta \hat{y} / \delta L, \quad (27)$$

and the expressions developed in the Appendix. This is the relevant stabilization cost to be balanced against marginal benefits possibly derived on other grounds.

3 . OCCUPANCY TIMES AND AVERAGE PUBLIC SPENDING

It is of interest to compare the occupancy time of Regime 2 in this model, call it $\bar{\theta}$, with θ , the fraction of total time aggregate demand y exceeds B in the original unrestricted model. For such purpose take y_t as described by (20). Since $\Lambda(x) \geq 0$ and $\Pr[\Lambda(x) > 0] > 0$, and in view of the independence of u_t and y_{t-1} , we have

$$\Pr[y^* + u_t + \Lambda(y_{t-1}) > B] > \Pr[y^* + u_t > B],$$

that is,

$$\bar{\theta} > \theta \quad (28)$$

This perhaps is not a surprising result in view of the fact that as soon as g_t reach the lower bound L , fiscal-monetary policy is no longer able to eliminate the underlying persistence of aggregate demand growth. Therefore y will comparatively spend more time above B than under the original economy where persistence is fully removed from the system.

At this point one may ask about the effect of the institutional lower limit L on the average money growth rate, call it $\bar{g}$, and therefore one average public spending of the stabilization branch. This inquiry seems to be justified in view of the result just obtained. In other words, it can be asked whether the increase in time spent by the economy in Regime 2 where the institutional restriction is active ($g_t = L$)

might offset the direct effect of the limit itself in eliminating low realizations of g_t , so as to generate on average a smaller value for g .

It is easy to see that the stochastic process governing g can be described as:

$$g_t = \begin{cases} = g_0 + g_1 y_{t-1} & y_{t-1} \leq B \\ = g_0 + g_1 B = L & y_{t-1} > B \end{cases} \quad (29.a)$$

$$(29.b)$$

Denoting by $h(y)$ the steady-state density function of y given the institutional parameter B , the steady-state mean of g will be described by

$$\bar{g} = g_0 + g_1 B + g_1 \int_{-\infty}^B (y - B) h(y) dy \quad (30)$$

From (13) the mean of y can be represented by:

$$\bar{y} = y^* + \lambda \int_B^{\infty} (y - B) h(y) dy, \quad (31)$$

Now,

$$\bar{y} - B = \int_{-\infty}^B (y - B) h(y) dy + \int_B^{\infty} (y - B) h(y) dy, \quad (32)$$

and using (31) and (32) into (30) finally gives

$$\bar{g} = g_0 + g_1 y^* + g_1 (1 - \lambda) \lambda^{-1} (y^* - \bar{y}) \quad (33)$$

which proves that average public spending goes up if $\bar{y}$ exceeds y^* , which was shown before to be true.

4. CONSTITUTIONAL DESIGN

So far the discussion has proceeded at the positive level by inquiring about predictable effects of existing institutional rigidities.

The stochastic model developed earlier is able, however, to give certain additional insights into some recent normative constitutional controversies.

Assume as before that the choice of g is made by the stabilization authorities which again are interested in minimizing the mean-squared error $E [(y-y^*)^2]$.

Suppose now, however, that at the constitutional level of decision-making it is recognized that the choice of g produces side allocative effects over and above the stabilization ones. This might result from assuming that the tax-transfer policies associated with g are all distortionary. Politicians and Bureaucrats in charge of the stabilization function, acting with some autonomy vis-a-vis other budgetary functions, might not have these indirect detrimental effects fully reflected in their own cost-reward structure. Internalization of these spillovers is called for in such circumstances. Assume this is obtainable via constitutional discipline. Specifically suppose that the relevant index of social welfare is described at the constitutional stage by

$$W = - \{ E [(y - y^*)^2] + v (\bar{g})^2 \} \quad (34)$$

where the last term attempts to measure the allocative cost. We suppose the latter is proportional to the square of the steady-state growth rate $\bar{g}$ to allow for presumably rising marginal efficiency costs of government size. To keep the exposition simple, we further abstract now from the possible downward rigidity of g earlier discussed, by assuming that proper selection of flexible government spending projects is undertaken.

Now, constitutional designers in an attempt to maximize W are

faced with the choice among three possible constitutional rules. The first, which we term the 'unrestricted rule' (UR) allows the stabilization authorities to select g without any further constraint besides the one implicit in the use of our previous linear feedback rule (2). This rule will thus correspond to a basic Keynesian stance where there is typically no concern for allocative side effects. The second rule, call it the 'balanced budget rule' (BBR), requires the budget to be balanced in the steady-state, although allowing for inperiod imbalances¹⁰. Finally, the third rule, call it the 'spending limit rule' (SLR) imposes an upper positive limit $L > 0$ on money growth which finances the stabilization deficit¹¹.

We commence by comparing the relative welfare performance of the first two rules. If an UR is operating, we know from (2) and the initial optimization problem that:

$$\bar{g} = g_0 + g_1 y^* \quad (35)$$

Using the optimal values for g_0 and g_1 previously determined one gets the maximum social welfare attainable under UR, namely:

$$W^*_{UR} = -\{\sigma_u^2 + v[\frac{(1 - \lambda)y^* - \alpha}{\beta}]^2\} \quad (36)$$

10 - This rule together with competing budget-balanced alternatives are discussed in a constitutional perspective by Buchanan and Wagner (1977).

11 - This type of rule may approximate what M. Friedman (1979) had in mind when he asserted: "The proponents of a constitutional amendment seek a balanced budget not for its own sake but in order to halt inflation, and to reverse the steady increase in the fraction of our income being spent by Washington on our behalf. A far better way to achieve those objectives is to enact new limitations on Federal spending."

where a positive allocative cost becomes apparent.

Consider next the case where a BBR is in force. From (1) and setting $\bar{g} = 0$ one readily obtains

$$\bar{y} = \frac{\alpha}{1-\lambda}, \quad (37)$$

so that:

$$W_{BBR}^* = - \left\{ \sigma_u^2 + \left(\frac{\alpha}{1-\lambda} - y^* \right)^2 \right\} \quad (38)$$

and stabilization cost emerges above the irreducible noise component. Now, from (36) and (38) we can see that if there is no systematic bias in exogenous demand growth so that $y^* = \alpha(1-\lambda)^{-1}$, social welfare will be identical under the two rules. In such circumstances, government will be able to reduce the variability of aggregate spending to its minimum level σ_u^2 while obtaining at the same time maximum allocative efficiency. If, however, there is a systematic bias (say downward) in uncontrolled spending, that is, if $y^* < \alpha(1-\lambda)^{-1}$, a conflict emerges between allocative and stabilization objectives. Maximization of social welfare will require some compromise and will in general be achieved by allowing for some positive steady-state deficit, $\bar{g}^*$. This can be easily obtained by differentiating (34) so as to give:

$$\bar{g}^* = \frac{\beta^{-1} [(1-\lambda) y^* - \alpha]}{1 + v \left(\frac{1-\lambda}{\beta} \right)^2} > 0 \quad (39)$$

As Figure 1 illustrates $\bar{g}^*$ will be somewhere between the values obtained under the two rules:

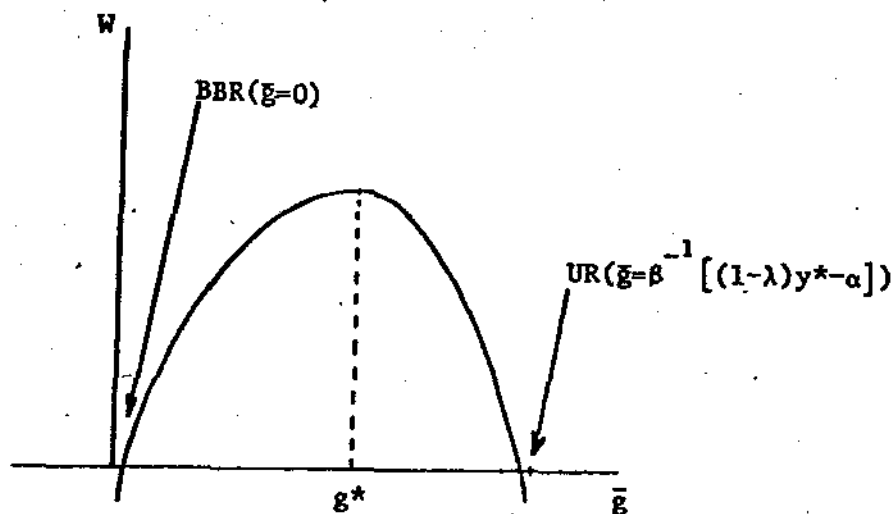


Figure 1

As can be seen from (39) the greater the weight v placed on allocation considerations the closer the optimal rate is to the BBR solution.

Consider now the SLR. It is convenient to decompose the mean squared error (MSE_{SLR}) so as to have social welfare given by

$$W = -\{\sigma_y^2 + (\bar{y} - y^*)^2 + v(\bar{g})^2\} \quad (40)$$

Differentiating the first two terms with respect to $\bar{g}$ gives

$$\partial(MSE_{SLR}) / \partial \bar{g} = \delta \sigma_y^2 / \delta L(\delta L / \delta \bar{g}) + 2(\bar{y} - y^*) \delta \bar{y} / \delta \bar{g} \quad (41)$$

The similarity between the stochastic framework under the present rule and the one analysed in sections (1)-(3), except for having now an upper bound on L , allow us to use (33) so as to obtain.

$$\delta(\text{MSE}_{\text{SLR}})/\delta\bar{g} = \delta\sigma_y^2 / \delta L(\delta L/\delta\bar{g}) + 2 [(\alpha+\beta\bar{g})(1-\lambda)^{-1} - y^*] \beta(1-\lambda)^{-1} \quad (42)$$

Going back to our earlier discussion of UR and BBR and taking the MSE as a function of some steady-state required deficit $\bar{g}$, it can be readily shown that

$$\delta(\text{MSE})/\delta\bar{g} = 2 [(\alpha+\beta\bar{g})(1-\lambda)^{-1} - y^*] \beta(1-\lambda)^{-1} \quad (43)$$

which is just equal to the second term in (42).

At the same time, from (26), (24) and (33), we know mutatis mutandi that the first term is negative. This indicates that for any required value of the steady-state $\bar{g}$, the trade-off between minimizing MSE and $\bar{g}$ has worsened under the SLR. Therefore, the SLR proves to be a comparatively inefficient constitutional rule¹². Figure 2 where we measure $\bar{g}^2$ and MSE on the horizontal and vertical axis respectively, to obtain linear indifference curves, illustrates the case.

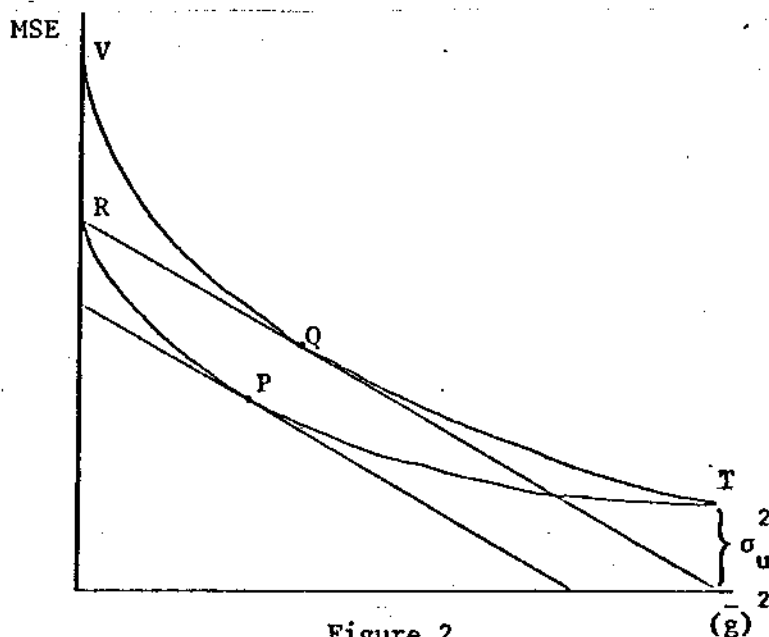


Figure 2

12 - If the welfare index includes the variance of g as an additional argument the above conclusion is no longer warranted since it can be shown that the variance of g varies positively with L .

The TPR curve describes the opportunity locus when the magnitude of the required steady-state deficit $\bar{g}$ varies. The TQV curve in turn describes opportunities under the SLR when L varies parametrically. Point T corresponds to the optimal solution under UR while R indicates the result of BBR. Point P shows the improvement in welfare that can be obtained through some compromise between these two rules as pointed out before. Point Q corresponds to the maximum welfare that can be derived from SLR through some optimal choice of L . As is clearly seen, the SLR opportunity locus is everywhere dominated by the RPT line. It can also be noted that a steady-state balanced budget can also be obtained through SLR as indicated by point V.

5. SUMMARY AND CONCLUSIONS

We attempted to analyse in the present paper some stochastic implications of having a limited control variable in a simple first-order auto-regressive model governing the growth rate of nominal aggregate demand. By modelling government behaviour under such constraints, an attempt was made to derive an approximate measure for the marginal cost of injecting rigidities into the system (irreversible effects of public expenditures). We used also the model to derive some normative implications for constitutional design. With a simple welfare index which penalizes the mean squared error of aggregate demand growth about some target value and the steady-state size of the stabilization deficit we were able to show that a constitutional rule requiring an upper limit on the growth rate of money is relatively inefficient. An alternative rule requiring some optimal steady-state growth rate emerges as a preferred solution.

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APPENDIX

We show first that

$$\hat{y} = y^* + \lambda \int_B^{\infty} (z-B) f(z) dz \{1 + \lambda [1 - \phi(\frac{B-y^*}{\sigma_u})]\}. \quad (A1)$$

Start by restating (23) as

$$\hat{y} = E(z) + E\{\Lambda(z)\} \{1 + E[\Lambda'(z)]\}, \quad (A2)$$

where $z \sim n(y^*, \sigma_u^2)$. By definition $E(z) = y^*$, and

$$E\{\Lambda(z)\} = \int_B^{\infty} \lambda (z-B) f(z) dz. \quad (A3)$$

Also,

$$\begin{aligned} E\{\Lambda'(z)\} &= \frac{\partial}{\partial s} E\{\Lambda(z+s)\}_{s=0} \\ &= \frac{\partial}{\partial s} \left[\int_B^{\infty} \lambda (z+s-B) f(z) dz \right]_{s=0} \\ &= \lambda \int_B^{\infty} f(z) dz = \lambda \left[1 - \phi\left(\frac{B-y^*}{\sigma_u}\right) \right] \end{aligned} \quad (A4)$$

Expression (A1) is immediately obtained by using (A3) and (A4) into (A2).

The sign of $\partial \hat{y} / \partial \lambda$ is readily obtained by differentiation of (A1) to give

$$\partial \hat{y} / \partial \lambda = \int_B^{\infty} (z-B) f(z) dz \{1 + 2\lambda [1 - \phi]\} \quad (A5)$$

where ϕ is the standard normal cumulative distribution evaluated at $(B-y^*)/\sigma_u$. (A5) is clearly positive.

By straightforward differentiation of (A1) and since $\partial B / \partial L = -\beta/\lambda$ we get

$$\partial \hat{y} / \partial L = \beta \left\{ \int_B^{\infty} f(z) dz [1 + \lambda (1 - \phi)] - \frac{\lambda}{\sigma_u} \phi \right\} \quad (A6)$$

where ϕ is the standard normal density evaluated at $(B-y^*)/\sigma_u$. (A6) is clearly positive.

The response of the variance of $\hat{y}$, $\sigma_{\hat{y}}^2$, to changes in λ , L and σ_u is readily obtained. From (26) one obtains

$$\sigma_{\hat{y}}^2 = \sigma_u^2 + \lambda^2 \left\{ \int_B^\infty (z-B)^2 f(z) dz + \left[\int_B^\infty (z-B) f(z) dz \right]^2 \right\} \quad (A7)$$

We note, by simple inspection, that $\partial \sigma_{\hat{y}}^2 / \partial \lambda > 0$. On the other hand, straightforward differentiation with respect to B , and using $\partial B / \partial L = -\beta / \lambda$ gives

$$\partial \sigma_{\hat{y}}^2 / \partial L = 2 \beta \lambda \int_B^\infty (z-B) f(z) dz \left[1 + \int_B^\infty f(z) dz \right] > 0. \quad (A8)$$