

A Work Project, presented as part of the requirements for the Award of a Master's degree in
Finance from the Nova School of Business and Economics.

**MACHINE LEARNING IN FINANCIAL FORECASTING: PREDICTING EQUITY
VOLATILITY AND ASSESSING PORTFOLIO STRATEGIES**

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Abstract

In this directed research, we look to capitalize on a tree gradient boosting model for stock volatility prediction. Through an extensive literature review we build on past research focusing on a wide range of volatility drivers and integrate them as model features. We propose different rebalancing techniques to the market portfolio according to our volatility predictions and assess their viability. Finally, we arrive at an approach that offers a robust framework for equity volatility forecasting and propose portfolio constructions that can further advance the current understanding on the use of volatility for fund managers.

Keywords: Volatility Timing, Supervised Learning, XGBoost, Portfolio Rebalancing

Acknowledgements

To Beatriz, my parents, grandparents, and siblings.

Link to Algorithm

All relevant code and analysis presented throughout this report is stored on Github.com. Accessible through the following [LINK](#).

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1. Introduction

In recent years, the financial industry has witnessed a remarkable transformation driven by advancements in financial technology. Among the key areas of focus, quantitative finance has increasingly turned its attention to harnessing the power of Machine Learning (ML) technologies for understanding price movements and patterns. This has led to a surge of interest in developing ML-based models that outperform traditional human fund managers. Traditionally, financial practitioners relied heavily on their expertise to predict market movements. However, as financial markets have grown more complex and dynamic, these traditional methods presented limitations in capturing the intricacies of market behaviour. Academics and investment professionals have long recognized that human biases in the world of fundamental money management can often lead to suboptimal investment decisions (Shiller 1999). The emergence of rule-based systems (including ML) has presented a promising alternative to human error by making decisions solely based on data.

In this research we explore the potential of ML for predicting volatility at the stock-level. Through our literature review we identify decision tree gradient boosting as the ideal model to explore. Macroeconomic, market and firm data are gathered to provide the developed model with features representing potential volatility drivers. A GARCH model is developed to predict market-level volatility, and its predictions are used as a feature of the model forecasting stock-level volatility. Additionally, the accuracy of forecasting equity volatility is compared to that of forecasting market volatility. We build portfolios based on approaches that explore volatility, such as volatility-timing and risk-parity, and tweak them to propose two new constructions that look to leverage on our volatility forecasts. We conclude that portfolios seeking to leverage volatility estimates are often better served using historical volatility rather than a better approximation of future volatility and propose reasons for this. Finally, we show that a stock-level approach to volatility timing may outperform the original portfolio-level proposal.

In the subsequent chapters, we will detail the implementation of our research objective, evaluate the algorithm's performance, and conduct a comprehensive investigation of its validity in different market conditions. The ultimate goal is to provide a compelling case for the integration of ML methodologies in the decision-making process of managers regarding volatility, enhancing the potential for generating superior risk-adjusted returns.

2. Theoretical Background

The second chapter performs a brief literature review of volatility in financial markets and portfolio construction, followed by a contextualization into Machine Learning applications relevant for this proposal. After that, we analyse the current state of research in related works, and finally derive the precise research objective for this project.

2.1. Volatility and Portfolio Construction

Volatility is a fundamental aspect of financial assets, representing the degree of variation in an asset's price or returns over a given period. It is a pivotal metric for several reasons: first, it is used as a proxy to quantify the risk inherent to a given asset, with higher volatility indicating greater uncertainty; second, volatility is central to pricing models for financial derivatives, with models like the Black-Scholes model (1973) relying on volatility estimates; and third, it underpins various investment strategies, with traders often leveraging volatility to capitalize on market inefficiencies.

The first relevant exploration of the relationship between risk and return was developed by Sharpe (1964), in which the author presents the basis for the CAPM that depicts a linear relationship between expected return and systematic risk. Contrary to this traditional notion, Moreira and Muir's (2017) research presented in "Volatility Managed Portfolios" finds a weak relationship between portfolio returns and volatility. The authors demonstrate that actively managing portfolio volatility can mitigate losses during market downturns and capitalize on

stable periods, thus yielding enhanced risk-adjusted returns. This idea also goes against the notion that recessions and market crises are favourable times to invest in assets at a discount, as the volatility-managed portfolios formed by the authors reduce risk-taking during these times and prefer to weather-out the storm, returning to the market only when volatility recedes.

In the context of portfolio construction based on volatility, it is also important to mention the concept of risk parity (Hurst, Johnson, and Ooi 2010), that proposes the diversification of investments within a portfolio based on the risk contribution of each asset. In a risk parity portfolio, each asset class is allocated in such a way that it contributes equally to the overall portfolio risk. This means that riskier assets receive smaller weights, and less volatile assets receive a larger weight, resulting in a more balanced risk profile.

2.2. Volatility Drivers

Various factors can contribute to the volatility of financial assets, ranging from economic indicators to idiosyncratic metrics. This section explores the key drivers of equity volatility.

Macroeconomic variables directly impact volatility, by representing broad conditions that affect capital markets (Engle, Ghysels, and Sohn 2013). Government bond yields are important indicators, playing a pivotal role in shaping investor sentiment and asset allocation strategies. Generally, when bond yields experience an upswing, they often bring about higher discount rates, which can lead to a shift towards traditionally safer quality assets (Baker and Wurgler 2012). Macroeconomic indicators related to economic performance also hold significant sway, for example, an increasing level of government debt can instil concerns regarding fiscal sustainability, prompting market uncertainty (Michaelides et al. 2012).

Financial ratios offer valuable insights into equity idiosyncratic performance. Metrics like the Book/Market (BM) ratio, Shiller's Cyclically Adjusted P/E Ratio (CAPE) (Campbell and Shiller 2001), and Dividend Yields (Fama and French 1988) are fundamental to assess individual

performance for different firms, and directly affect stock price movements. The BM ratio, often regarded as the value factor, compares a firm's book value against its market value. Higher BM ratios tend to be associated with companies more susceptible to market fluctuations (Fama and French 1992). Shiller's CAPE ratio, which spans a 10-year earnings period, provides a long-term perspective on valuations. Elevated CAPE ratios may signify overvaluation, increasing the likelihood of price corrections. Financial ratios collectively offer a comprehensive view of how companies are not only influenced by economic and market dynamics, but also by their own idiosyncratic risks.

Not only are equity prices impacted by the aforementioned factors, but they can also be affected by their own past performance, as documented by Jegadeesh and Titman (1993). The authors illustrate that stocks with superior recent performances tend to outperform their underperforming peers.

Equity volatility is influenced by a complex interplay of macroeconomic variables and idiosyncratic factors. Understanding these drivers is essential for investors to make informed decisions and manage risk effectively.

2.3. Extreme Gradient Boosting and Hyperparameterization

Within the volatility forecasting framework, Gradient Boosting has been one of the most explored ML approaches and has been shown to improve performance relative to traditional models, such as GARCH (Mittnik, Robinzonov, and Spindler 2015). Extreme Gradient Boosting (XGBoost) was introduced by Chen and Guestrin (2016) as an efficient and scalable machine learning algorithm designed for gradient boosting trees. It aims to address challenges in existing gradient boosting methods by combining the strengths of tree-based models and gradient boosting whilst mitigating their individual weaknesses. XGBoost operates within the framework of tree boosting by combining weak learners (shallow decision trees) to create a

robust predictive model. It employs a level-wise growth strategy resulting in less overfitting and more robust models when compared to other gradient boosted trees methods.

In the realm of ML, hyper-parameterization is a critical process for optimizing model performance. Hyperparameters represent external configurations that guide the learning process, including learning rates and other architecture-related parameters (Yang and Shami 2020). These settings are predetermined and significantly impact a model's performance. The sheer number of potential hyperparameter combinations poses a challenge for manual exploration, leading to the uprising of hyperparameter tuning, that aims to systematically search for the optimal configuration.

The Tree-structured Parzen Estimator (TPE) is a probabilistic model-based optimization algorithm that employs Bayesian optimization principles and is widely used in hyperparameter optimization (Watanabe 2023). The algorithm assumes a predefined search space that defines possible values for each hyperparameter and tries to capture the relationship between hyperparameter values and the objective function. The algorithm maintains two distributions: one for configurations leading to better outcomes (exploitation) and another for configurations to explore further (exploration). The distributions are repeatedly adjusted to assign higher probabilities to hyperparameter configurations that lead to better outcomes, directing the search towards promising regions.

2.4. Opportunities and Limitations of Supervised Learning for Volatility Prediction

While supervised learning models such as XGBoost offer promising opportunities, they also come with certain limitations that must be considered when applying them to the task of volatility prediction. On the opportunity's column, we may point to the data-driven approach, as ML models analyse historical data, capturing complex patterns and relationships that may not be apparent to the naked eye, and thus avoiding human bias. Another positive relates to

feature engineering, as models are capable of selecting the most relevant features in order to solve a given problem, guiding investors towards relevant data sources and reducing noise.

As for the limitations: financial markets exhibit non-stationarity, meaning that their statistical properties change over time. Supervised learning models assume stationarity, and this mismatch can lead to poor predictions in rapidly changing market conditions (Tsay 2005). Data quality is another concern, since prediction accuracy heavily depends on it. Financial data can be noisy, contain missing values, and suffer from outliers, which can impact performance. It is also important to mention model overfitting, as ML models are prone to overfitting when they capture noise in the training data rather than genuine patterns (Cawley and Talbot 2010).

2.5. Related Work

Volatility forecasting has been a widely researched topic in quantitative finance throughout the field's history. Much of the early implementations are based on simple auto-regressive times series models (Poon and Granger 2003). While these models provided basic insights into asset volatility, they often failed to capture the intricate dynamics of financial markets, especially during periods of extreme volatility (Engle 1982). Whilst traditional time series operated under the assumption of constant variance, in the early 1980's emerge the Conditional Heteroskedasticity models with Engle proposing the ARCH model, a process that allowed for conditional variance to change with time as a function of past errors (Engle 1982). The model is considered appropriate when assuming that the error variance in a time series follows an autoregressive (AR) process. This opened the door for Bollerslev (1986) to introduce the GARCH model, which follows the same logic as ARCH processes, except the model assumes that the error variance follows an autoregressive moving average (ARMA) process. Further developments into time series models have been made, where the use of algorithms such as

SVMs has shown capability to improve upon the performance of traditional GARCH models (Gestel et al. 2001) in the prediction of underlying volatility.

With the rise of ML in finance, investors' approaches began to shift towards the integration of features that could aid forecasting besides historical prices. To tackle volatility forecasting several approaches have been undertaken and with different aims. Among the main examples, it is important to refer the work of Chatzis et al. (2018), that explores multiple ML techniques across various markets, to predict the probability of a financial crisis in two different time frames (1 day and 20 days). Audrino & Colangelo (2010) explore a semi-parametric model to predict the implied volatility surface of S&P500 options. Byun (2016) develops a model where dispersion in returns across different stocks is modelled jointly with aggregate volatility. He finds that the use of cross-sectional dispersion can improve parameter estimates of a GARCH process for aggregate volatility to generate better forecasts both in sample and out of sample. Zhu et al. (2023) consider the problem of forecasting realized volatility using high-frequency data. They employ panel data analysis as inputs to six machine-learning methods to improve forecasting accuracy in the short term. The results show that the panel-data-based machine learning method outperforms traditional methods such as GARCH. Finally, the closer research to the one proposed is developed by Mittnik et al. (2015), as the authors use financial ratios and macro variables as inputs to a gradient boosting model to forecast volatility of the S&P 500 index, showing that their model provides better estimates than classical GARCH models.

2.6. Research Objective

In this research we propose a decision tree gradient boosting approach for predicting equity-level volatility on a monthly basis. The model aims to leverage a comprehensive set of features, encompassing macroeconomic variables, financial ratios and historical performance.

Incorporating macroeconomic variables into the predictive model serves to integrate broader economic trends. Simultaneously, the inclusion of financial ratios at both industry and firm levels aims to capture idiosyncratic conditions affecting stock-level performance. Furthermore, we incorporate GARCH predictions for overall index volatility, along with the historical beta of a firm relative to the index, adding a nuanced layer of market sensitivity to the forecasting framework. Past equity and market index performances are also inputted as model features.

We look to find whether the attained volatility predictions can aid investor decisions and provide added-value strategies. Based on Moreira and Muir's (2017) original proposal for portfolio construction, we propose a similar construction at the equity-level to find whether equity volatility timing outperforms the portfolio level approach used by the authors. We build another portfolio based on the idea of risk-parity, where we look to overweight on predicted low volatility assets and find if there is an advantage in rebalancing an unlevered portfolio according to our predictions. In evaluating the effectiveness of the developed model and portfolio strategies, this research seeks to provide insights into their performance relative to traditional methods. Comparisons will be drawn, and the implications of the findings for investors will be discussed. This work aspires to contribute not only to the academic understanding of volatility prediction but also to offer tangible strategies for fund managers.

3. Model Implementation

This section documents the implementation of the proposed model. It starts with a brief review of the dataset construction and features attained, followed by the presentation of data processing techniques before detailing the implementation of the actual volatility forecasts.

3.1. Data Description

To build upon the proposed approach, there are different types of data required that must be gathered (according to the volatility drivers explored in 2.3.). The following are the topics for

which data was extracted: stock information on a monthly and daily basis, firm financial ratios, industry financial ratios and macroeconomic data. Except for the macroeconomic data, all other data is gathered from the S&P Global Market Intelligence Data through Wharton Research Data Services (WRDS). In the following sub-chapters, we describe the gathered data and its handling (a scheme of the created structure is presented in Exhibit 1).

3.1.1. Stock Returns Monthly

As a starting point, we create an index that replicates overall market performance. As a proxy for the market, we use the top 500 stocks by market capitalization. Monthly data was collected for all available individual stocks and the selection was narrowed down to American companies listed on the primary stock exchanges in the United States. It is important to keep in mind the concept of formation and holding months when creating this index. The formation month corresponds to the month where we construct a portfolio of the top 500 stocks by market capitalization. As for the holding month, it is the month where we actually hold these 500 stocks (this is important so as to not fall for survivorship bias (Malkiel 1995)). The top 500 stocks are selected as part of the index, considering the Daniel-Moskowitz (2016) restrictions.

3.1.2. Stock Returns Daily

To calculate monthly stock realized volatility, we need daily returns. This is attained by subsetting all daily stock information to match the index constituents and the months where they are one of the top 500 stocks. Using daily price data, we attain both the formation and holding month realized volatility according to Equation 1.

Equation 1 - Monthly Volatility Formula.

$$\sigma_{\text{Monthly}} = \sigma_{\text{Daily}} \times \sqrt{n}$$

Where:

σ_{Monthly} = Monthly Volatility

σ_{Daily} = Standard Deviation of Daily Returns

n = Considered number of trading days in a month

3.1.3. Firm Financial Ratios

Another explored aspect when studying firm-level volatility was the endogenous financial situation of each firm. As mentioned in 2.2., each firm possesses risks inherent to its line of business that are often reflected in its financial statements. To address this, financial ratios were extracted that match the company identifiers for each firm in the index. It is important to mention that the ratios were matched using the date when data became public and the formation month (to not commit look-ahead bias). Missing values were replaced by the median of the corresponding industry ratio.

3.1.4. Industry Financial Ratios

WRDS contains data on financial ratios aggregated by industry, for our project we collected the median ratio by industry. This serves as a proxy to compare the performance of a given firm in relation to its own sector of activity. It allows the model to comprehend if a company is performing worse or better than its peers. To implement this, we gather both information regarding the industry median, and a quotient between the firm's ratio and the industry median. The historical variations of the industry's financial ratios were also obtained and added as features (firm-level would be highly unpractical since the composition of the index is constantly changing). Variations were obtained for the 1-, 3- and 6-months periods.

3.1.5. Macroeconomic Data

The Macroeconomic data used in this project was directly obtained from the FRED API made available by the St. Louis Fed (2023). The indicators collected include Unemployment, GDP, Government Debt, Inflation, Dollar Exchange Rates, Commodity prices and Government Bond yields. To this information, it was added the historical spot price of Gold (public data made available by the WGC (World Gold Council, 2023)). The only processing required to handle

this data was the transformation of weekly Jobless Claims, for which we took the monthly average. For all variables, variations were obtained for the 1-, 3- and 6-months periods.

3.1.6. GARCH and Past Performance

As proposed in 2.6., we look to implement GARCH predictions of index volatility as a model feature. To do this, we gather daily value-weighted returns of all stocks present in the index we created. Following, we apply a GARCH(1,1) model to the index daily returns and estimate volatilities for the following 22 days. We attain the mean of daily volatilities and use Equation 1 to transform daily volatility into monthly (Figure 1 presents GARCH performance). We also look to explore the correlation (β) of stocks relative to the market index. Using the index daily returns, and daily returns of each stock, we calculate the correlation between the two, and for each month-equity pair, Beta is the correlation between daily returns up until that month. Past performance is also considered, as we integrate into the model the past month's return, volatility and Sharpe Ratio both for the index and for the individual stocks (Sharpe Ratio is calculated using the monthly risk-free rate obtained from the Fama and French public data).

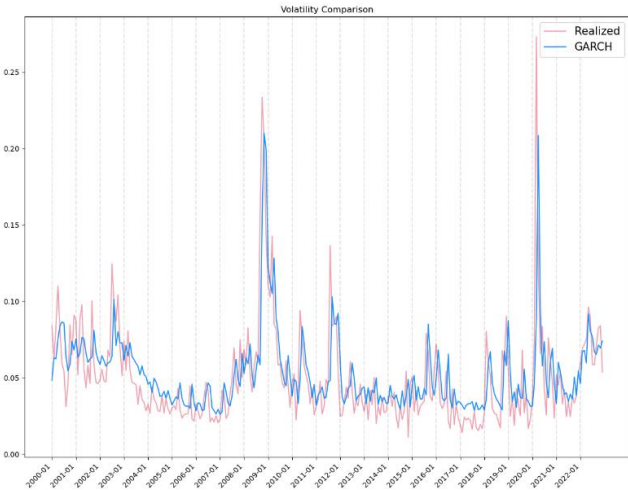


Figure 1 - GARCH(1,1) Performance against Realized Volatility.

Table 1 presents a list of all features that are used as inputs to the XGBoost model.

Table 1 - Model Features.

* - Historical variations were obtained for the 1-, 3-, and 6-month periods and added as features.

** - Quotients against corresponding industry ratios were also added as features.

Type	Variable	Category	Type	Variable	Category
Macroeconomic Data *	Unemployment Rate	Labour Market	Financial Ratios	Industry Return Equal-Weighted	-
	Initial Jobless Claims	Labour Market	Industry Level *	Industry Return Value-Weighted	-
	GDP Growth Rate	GDP		Book/Market	Valuation
	Debt to GDP	Government Debt		Shillers Cyclically Adjusted P/E Ratio	Valuation
	CPI	Inflation		Dividend Yield	Valuation
	PCE	Inflation		Price-to-Earnings, excl. Extraordinary Items	Valuation
	Core PCE	Inflation		Price/Book	Valuation
	EUR/USD	Dollar Exchange Rates		Return on Assets	Profitability
	USD/GBP	Dollar Exchange Rates		Return on Equity	Profitability
	USD/JPY	Dollar Exchange Rates		Total Debt / Total Assets	Solvency
	Crude Oil WTI	Commodity Prices		Current Liabilities/Total Liabilities	Financial Soundness
	Crude Oil Brent	Commodity Prices		Total Debt / EBITDA	Financial Soundness
	Gold	Commodity Prices	Short-Term Debt/Total Debt	Financial Soundness	
	Copper	Commodity Prices	Cash Conversion Cycle (Days)	Liquidity	
	Wheat	Commodity Prices	Cash Ratio	Liquidity	
	3-month yield	Government Bond Yields	Current Ratio	Liquidity	
	1-year yield	Government Bond Yields	Quick Ratio (Acid Test)	Liquidity	
	2-year yield	Government Bond Yields	Performance / Others	GARCH (1,1) Index Volatility Forecasts	Volatility
	5-year yield	Government Bond Yields		Asset Past Month Volatility	Volatility
	10-year yield	Government Bond Yields		Index Past Month Volatility	Volatility
	20-year yield	Government Bond Yields		Asset Past Month Return	Return
	30-year yield	Government Bond Yields		Index Past Month Return	Return
	10y2y yield spread	Government Bond Yields		Asset Past Month Sharpe Ratio	Sharpe Ratio
10y3m yield spread	Government Bond Yields	Index Past Month Sharpe Ratio		Sharpe Ratio	
Financial Ratios Firm Level **	Book/Market	Valuation		Past Month Risk-Free Rate	Risk-Free
	Shillers Cyclically Adjusted P/E Ratio	Valuation		Asset Past Month Weight on VW Market Portf.	-
	Price-to-Earnings, excl. Extraordinary Items	Valuation		Industry	-
	Price/Book	Valuation			
	Return on Assets	Profitability			
	Return on Equity	Profitability			
	Total Debt / Total Assets	Solvency			
Total Debt / EBITDA	Financial Soundness				

3.2. Assumptions

Based on the relationship between returns (at time t) and past-month volatility (at $t-1$) documented by Moreira and Muir (2017) for the market and factor portfolios, we investigate whether the same relationship can be found at the stock level. In the original paper, the relationship is observed for the entire market portfolio. To replicate at the asset level, we will consider two cases: dividing the volatilities into buckets considering the entirety of the data (possibly grouping entire months into one bucket); and considering each specific month to form the buckets (divide the 500 stocks into 5 buckets each month, to differentiate volatilities relative to the current month). Our analysis (Figure 2 and 3) demonstrates that the Moreira and Muir findings hold up for the first case proposed, the one that is most similar to the original proposal. As for the second case, there are traces of an unbalanced relationship but the difference between buckets weakens. According to our findings we will explore the construction of portfolios that look to overweight on low volatility assets and underweight highly volatile assets. For

comparison we build two portfolios for each situation: one based on volatility in t-1, and another based on our predicted volatility for t.

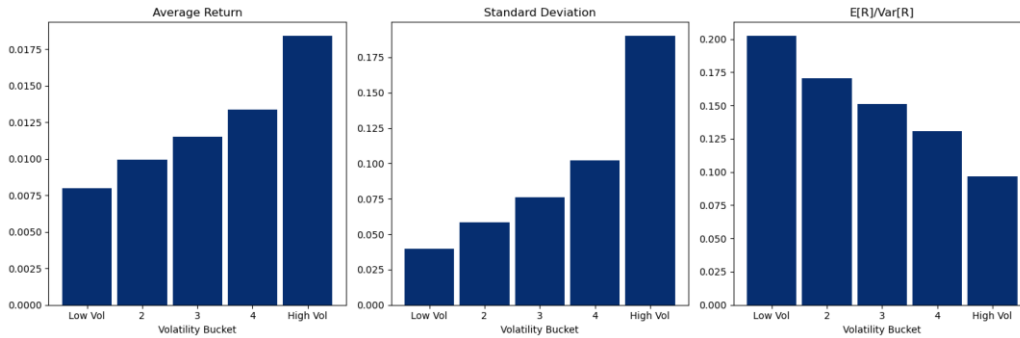


Figure 2 – Volatility buckets considering entire data.

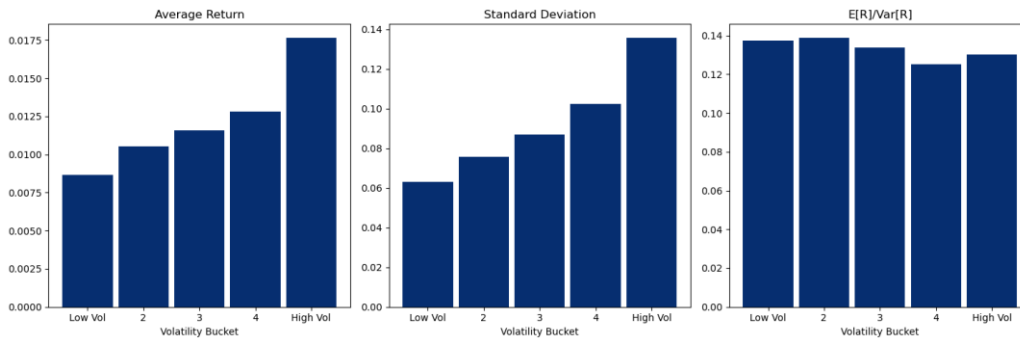


Figure 3 - Volatility buckets by month.

3.3. Pre-Processing

Within the pre-processing of collected data, we consider three main problems: the distribution of the numeric variables and their outliers, the normalization of numerical variables, and the encoding of categorical variables. As for the encoding, the One Hot Encoding method is applied, as there is no underlying order for the different classes. For outlier treatment, we explore Tukey’s range test for replacement (Tukey 1977). Finally, for normalization, we explore the effectiveness of three scalers: MinMax, Standard and Robust (Equation 2). Numerical variables preprocessing was tuned as hyper-parameters within the model pipeline.

Equation 2 - Applied Scalers Formulas.

MinMax:

$$x_{scaled} = \frac{x - x_{min}}{x_{max} - x_{min}}$$

Standard:

$$x_{scaled} = \frac{x - mean(x)}{\sigma_x}$$

Robust:

$$x_{scaled} = \frac{x - median(x)}{x_{75} - x_{25}}$$

3.4. Model Construction

The model constructed aims to predict volatility for a given time frame after the training period. As new information is constantly being made available for investors, the new information must also constantly be incorporated into the model when moving through time. For these reasons, we built an algorithm that re-trains the XGBoost model on new data once every three months and re-calculates the optimal hyper-parameters every three years, using TPE optimization. The pseudo-code representing the constructed algorithm is presented in Figure 4.

```
function RunRollingModel (start date, last date, parameterization dates)
  predictions ← empty list
  current date ← start date
  while current date ≠ last date do
    train_set ← all data until current date
    test_set ← 3-month period after current date
    if current date is parameterization date do
      parameters_set ← train_set
      run tpe hyperparameterization using cv 5-fold split of parameters_set
      parameters ← optimal parameters found by tpe hyperparameterization
      update pipeline with new optimal parameters
    fit pipeline to train data
    predicted_test_data ← predict test data with fitted pipeline
    predictions.append(predicted_test_data)
    current date ← current date + 3 months
  return predictions
```

Figure 4 - Pseudocode of the developed function to run rolling model.

3.5. Portfolios Constructed

Volatility forecasts are used to determine the future risk attributed to a given stock for the considered period. As observed in 3.2., stock returns are often unrelated to their volatility, and therefore, by lowering the overall portfolio volatility, we can look to augment the risk-return trade-off. To explore this disconnection, we construct the following portfolios:

1. Moreira and Muir (MM) Portfolio - Levered portfolio, based on the weight for the risky asset obtained using the Moreira and Muir (2017) method with the GARCH(1,1) predicted variance acting as the proxy for conditional variance. The objective of this construction is to benchmark the performance of forecasting market volatility against stock-level applications.

2. Replication Portfolio – Levered portfolio, weights for individual assets are attained by adapting the Moreira and Muir (2017) method to the stock level, adjusting the Value-Weighted weights of the market portfolio to the forecasted assets volatility (Equation 3). With this construction we explore whether asset-level volatility timing can benefit investors.

Equation 3 - Construction of Replication Portfolio.

$$w_{A,t+1} = \frac{c}{\hat{\sigma}_A^2} \times w_{A,t}^{VW}$$

Where:

$w_{A,t+1}$ = Asset A weight at t+1

$\hat{\sigma}_A^2$ = Proxy for the Asset A Variance

$w_{A,t}^{VW}$ = Value-Weighted weight of Asset A at time t

c = Constant that controls the average exposure of the strategy

3. Inversed Portfolio - Unlevered, 100% invested in risky assets portfolio. Portfolio weights are based on the inversion of the variance for a given asset: $\frac{1}{\sigma^2}$ (Exhibit 10). This portfolio aims to draw the idea from risk-parity (actual risk-parity would require information on the covariances between assets, which is out of the scope of this project), rebalancing portfolios according to asset volatility in a simple manner. With this portfolio we look to find whether there is an unlevered advantage to holding safer assets (similar to the one found for long-short portfolios by Blitz and Vliet (2007)).

As for comparison benchmarks we explore the following: Equal-Weighted index portfolio, Value-Weighted index portfolio, and similar portfolio constructions to the ones described above using past month volatility (volatility proxy used by Moreira and Muir (2017)).

4. Evaluation

In this chapter, we explore both the performance of the model in predicting asset volatility, as well as the performance of the forecast-based constructed portfolios. The performance of the attained forecasts is compared against the use of past volatility, considering different metrics and time frames. Portfolio's performances are benchmarked employing diversified metrics.

Later in our analysis, we test performance robustness considering different time periods and restrictions.

4.1. Evaluation of the Model

To evaluate how the model constructed in 3.4. performs in forecasting stock-level volatility we explore multiple regression measures to compare the predicted values against the actual realized volatility. As a benchmark for performance, we also obtain regression statistics for the use of previous month's volatility against the actual volatility values. Empirical evidence shows that volatility is highly persistent over short time frames, and therefore past volatility can be considered a good benchmark. As detailed in Table 1, we incorporate past volatility as a model feature, therefore we expect model performance to be at least as good as the benchmark.

Our model estimates obtain a Root Mean Squared Error (RMSE) of 0.0459 improving from a value of 0.0515 when compared to previous volatility. A similar improvement is also found when it comes to the R-Squared, where we upgrade from 0.204 to 0.369. Although the performance shows improvement from the simple use of the previous month's volatility, we can derive that equity volatility is not easily estimated, even when using state-of-the-art methodologies. Compared to the performance of the GARCH(1,1) model constructed in 3.1.6. for index volatility predictions, we find that the less-sophisticated model for index-level has a much better performance than the more complex model for equity-level. Equity-level volatility being less predictable than index-level can be justified by its smaller persistence and the effect of idiosyncratic events that are uncaptured by lagged data (this is consistent with Table 2 comparison of previous volatility performance as a proxy for both.). Nevertheless, we still find that there is some predictive power to the use of lagged data and ML.

To extend our understanding of the variations in performance upon different market conditions, we explore time variations for the proposed measures (Figure 5 and Exhibit 12). Our

exploration points to superior model performance in times of low market turbulence (based on RMSE), more precisely in the period between the Sovereign Debt crisis and the Covid-19 pandemic. This can be justified by the fact that crisis periods tend to present higher variations in volatility and therefore become less predictable (as observed in Figure 1).

Table 2 - Quality of Volatility Predictions for Stock-Level (above) and Market-Level (below).

	R-Squared	RMSE	Mean Squared Log Error	Max Error
Predicted Volatility	0.3686	0.0459	0.0015	1.3771
Previous Month Volatility	0.2045	0.0515	0.0018	2.8317

	R-Squared	RMSE	Mean Squared Log Error	Max Error
GARCH (1,1)	0.4770	0.0232	0.0004	0.1846
Previous Month Volatility	0.3072	0.0267	0.0006	0.1998

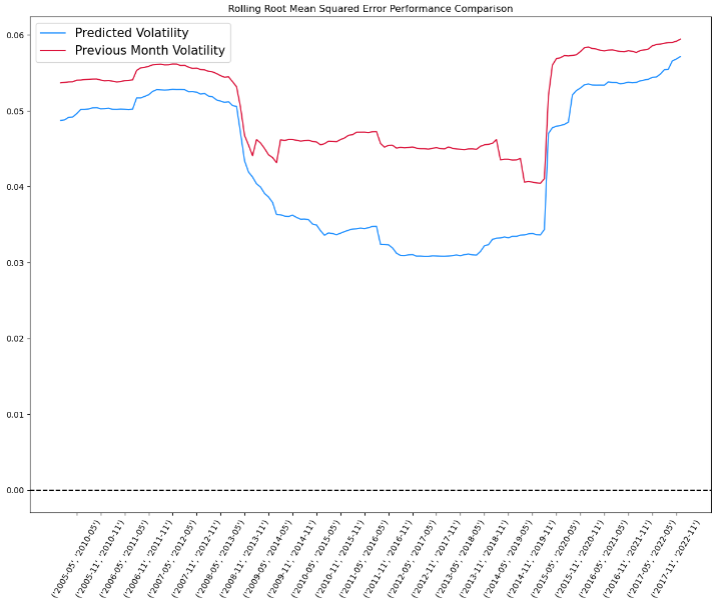


Figure 5 - RMSE Rolling Comparison for Predicted and Previous Volatility (Stock-Level forecasts).

4.2. Portfolio Performance

As mentioned in 3.5., portfolios were constructed based on our volatility predictions. To assess their performance we evaluate traditional metrics such as annualized excess returns, volatility, Sharpe Ratio and Sortino Ratio (Table 3). For the MM Portfolios, we find that using GARCH volatility forecasts clearly improves upon the approach using the proxy proposed in the original paper (Sharpe: 0.93 vs. 0.78). The Replication portfolio lags behind the original method (MM) using our predicted volatilities (Sharpe: 0.88), nevertheless, it is still able to beat the Value-Weighted Index Portfolio (Sharpe: 0.86). Inverse Portfolio results are also discouraging for the

use of our forecasts, with the use of predicted volatility (Sharpe: 0.81) not beating the Value-Weighted Index Sharpe Ratio. Nonetheless, we still find some evidence in favour of rebalancing based on individual stock volatility: for both the Replication and the Inverse Portfolio, the use of previous volatility clearly beats the market. It is also important to notice that the use of previous volatility in our two stock-level portfolios outperforms the original proposal of application at the portfolio level (Sharpe: Rep. 1.02 vs. Inv. 0.93 vs. MM 0.78).

Table 3 - Portfolio performance comparison.

	Annualized Return	Annualized Volatility	Sharpe Ratio	Downside Deviation	Sortino Ratio
EW Portfolio	13.07%	16.26%	0.804	12.17%	1.074
VW Portfolio	12.76%	14.81%	0.861	10.23%	1.247
Predicted MM Portfolio	13.83%	14.81%	0.934	11.07%	1.250
Previous MM Portfolio	11.51%	14.81%	0.777	13.72%	0.839
Replication Predicted Portfolio	13.03%	14.81%	0.879	11.23%	1.160
Replication Previous Portfolio	15.05%	14.81%	1.016	9.87%	1.524
Inv. Predicted Portfolio	11.47%	14.24%	0.805	10.72%	1.070
Inv. Previous Portfolio	11.20%	12.02%	0.932	8.89%	1.261

Focusing on the Sortino Ratio, as a better risk management measure, it is possible to observe that only the Replication construction with previous month’s volatility clearly improves upon the holding of the Value-Weighted market portfolio. The main divergence in comparison to the Sharpe Ratio analysis is the vanishing of the difference between both the MM portfolio using GARCH and the Inverse portfolio using previous volatility, compared to the Value-Weighted Market Portfolio.

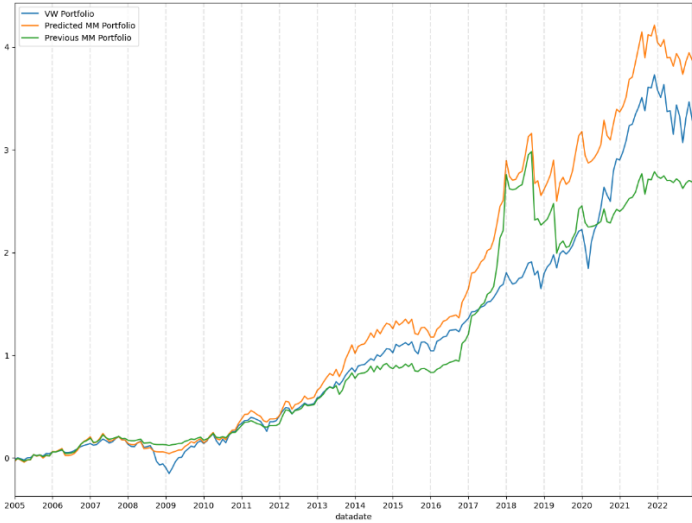


Figure 6 - Performance comparison of MM portfolios using the forecast and the proposed proxy.

To extend our performance understanding, we plotted different comparisons of cumulative returns (Exhibit 14). In this analysis we notice that Moreira and Muir's original proposal using past volatility (Figure 6) significantly trails market performance after 2017 (the year the authors published their paper). Results are further discussed in section 5.2..

4.3. Factor Benchmarking

To extend our understanding on whether the developed strategies can add value, we benchmark our constructed portfolios against factor portfolios, made famous by Fama and French (1992). For this, factors data was attained from the Kenneth French (2023) publicly available repository. We look to two principal metrics to guide our evaluation: Alpha (α) and Information Ratio (IR). To start, we benchmark our portfolios only against the Market Portfolio, using our constructed Value-Weighted Index (Exhibit 15). We only find a statistically significant (at the 5% confidence level) alpha for the Replication Portfolio using previous volatility (at the 10% confidence level, the MM GARCH alpha is also significant). As for Information Ratios, we find two portfolios that display promising values: the Replication Portfolio using previous volatility (0.55) and the MM Portfolio using GARCH forecasts (0.43). The Inverse Predicted Portfolio presents a negative Information Ratio, reflecting an underperformance in relation to the market.

Additionally, we benchmark the Portfolios against the Size and Value factors (in line with what is done in the Moreira and Muir paper) and find that exposure to these factors does not justify the significant alpha found for the Replication Previous Portfolio (Table 4). Within the MM constructions, the use of forecasts is negatively exposed to the value factor (low B/M stocks). Both Replication portfolios are negatively exposed to the Size factor (larger cap stocks). Our forecast-based Inversed Portfolio is exposed to both the Size and Value factors (smaller corporations with higher Book-to-Market ratios). Finally, we also control for the influence of the Betting-against-Beta factor (Frazzini and Pedersen 2014), often associated with low-

volatility stocks, and find some exposure for the Replication Previous Portfolio ($\beta = 16\%$), although alpha is still significant at the 10% confidence level and IR remains above 0.4 (Exhibit 17).

Table 4 - Portfolio regression against Fama-French 3 Factor Model.

	R-Squared	Alpha		Mkt-RF		SMB		HML		IR
		Coef	P-Value	Coef	P-Value	Coef	P-Value	Coef	P-Value	
Predicted MM Portfolio	0.581	0.003	0.098	0.775	0.000	-0.026	0.751	-0.131	0.032	0.408
Previous MM Portfolio	0.325	0.003	0.176	0.584	0.000	-0.057	0.584	-0.079	0.304	0.333
Replication Predicted Portfolio	0.644	0.002	0.253	0.829	0.000	-0.170	0.026	-0.058	0.298	0.281
Replication Previous Portfolio	0.610	0.004	0.039	0.813	0.000	-0.217	0.007	-0.086	0.142	0.510
Inv. Predicted Portfolio	0.956	0.000	0.795	0.914	0.000	0.092	0.000	0.076	0.000	-0.064
Inv. Previous Portfolio	0.904	0.001	0.095	0.758	0.000	0.045	0.158	0.043	0.065	0.411

4.4. Investment Periods

To compare performance over time, we check for alpha persistence for our portfolios using rolling 5-year periods (Exhibit 18). For forecast portfolios we find that the MM construction persistently beats our stock-level approaches, this may be justified by the difference in the quality of forecasts mentioned in 4.1.. For the use of previous volatility (Figure 7), we find that alpha is consistently superior for our Replication portfolio in comparison to the MM original construction. Nevertheless, deterioration is noticed in the period after the pandemic for both. All portfolios attain their largest alphas in the period involving the Great Financial Crisis and the Sovereign Debt Crisis. This can be seen as natural, as those are the moments that these constructions look to explore, by deleveraging during stress periods. It would also be expected that alphas would rise during the Covid distress period, but the difference is not as considerable as one might have expected. This is in line with Figure 1 and 5, where predicted volatility presents a worse performance during the Covid volatility spike, than during the financial crises, and can therefore be a justification as to why portfolios did not de-lever as much as they should. We also find that, while levered portfolios show an increasing alpha during the Covid distress, the Inverse portfolios' performances worsen. This finding can be explained by the nature of the different situations: whilst during the financial crisis volatility presented larger variations for

sectors that were more exposed to the episode (financials as the clear example), during the Covid crisis, the nature of the impact was more egalitarian for the different companies in the index, and therefore the advantage for going towards safer companies disappears.

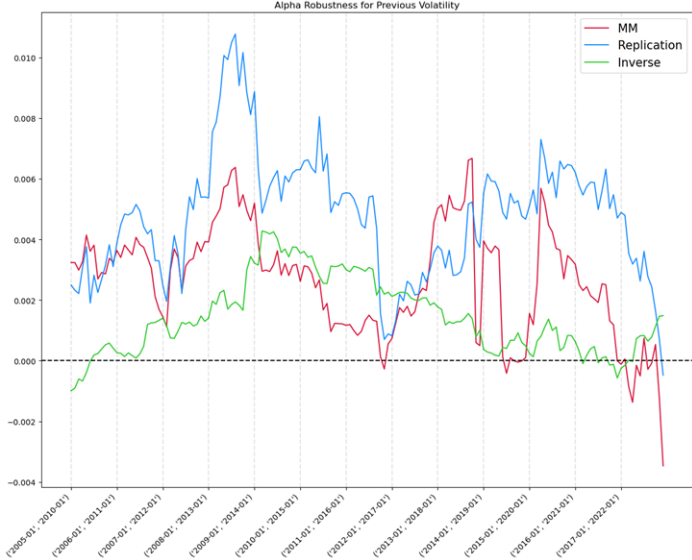


Figure 7 - Alpha persistence for Previous Volatility Portfolios.

4.5. Weight Constraints

To further extend our comprehension we apply weight constraints to test authentic portfolio performances in a realistic setting. For stock-level portfolios we look into restrictions on minimum weights for a given asset: very small allocations are often made impossible by conditions such as market liquidity. To levered portfolios, we apply restrictions to condition portfolios into complying with realistic leverages for a fund manager (0%, 50% and 100%).

Whereas the stock-level portfolios present nearly no relevant change in performance, the levered portfolios do (Exhibit 19-20). For both the MM and the Replication portfolios constructed with our forecasts, we find that limits to leverage benefit portfolio performance, increasing both Sharpe and Sortino ratios. For the use of previous volatility, we find conflicting results: for the MM portfolios, there are benefits to limiting leverage, whilst for the Replication portfolio it hurts performance. This can pose a limitation for the construction of the portfolio

for which we found significant alpha, and a justification as to why the market has not yet corrected this possible anomaly. Finally, we tested whether the portfolios based on our forecasts making use of leverage restrictions can consistently beat the market. We find that alpha is still not statistically significant, whilst Information Ratios are yet to surpass the 0.4 mark.

5. Conclusions

In this chapter, we summarize our project's scope, discuss findings, raise limitations, and propose areas for further research.

5.1. Project Summary

In this project we aimed to construct a Supervised Learning model to predict stock-level volatility and use the attained forecasts to improve portfolio rebalancing. As a baseline for our research, we provided a theoretical background on research involving volatility and the applied ML techniques. We identified the opportunity of applying a decision tree gradient boosting model, incorporating multiple identified volatility drivers, including GARCH index-volatility predictions as a vehicle to add value to previous research. Furthermore, we propose three strategies for portfolio construction based on the attained forecasts, and benchmark those same portfolios. Finally, we evaluate performance robustness through time and considering weight restrictions.

5.2. Discussion of Findings

Regarding the constructed model, we establish that our algorithm provides an improvement compared to the use of previous month volatility, and that this improvement is constant through the evaluated period. Even though performance beats the proposed proxy, we still find that volatility is highly unpredictable at the stock level, with our model only being able to explain 37% of the total variance in volatility. Although our model can attain better predictions than the proxy, these do not directly translate into superior performance for constructed portfolios.

The use of our volatility forecasts never beats the use of previous volatility in any of the stock-level portfolio constructions. We postulate the following reason as to why this occurs: previous month's volatility is publicly available information and is likely incorporated into the decision making process of fund managers when re-balancing their portfolios (in line with the discovery of the low-volatility anomaly (Blitz and Vliet 2007)), leading to a self-reinforcement cycle that makes previous month volatility have an impact on market performance, whereas a closer approximation of the realized volatility does not.

We find that the Replication Portfolio using past volatility beats the market consistently, finding an Alpha of 0.5% and outperforms both the MM original proposal and the MM Portfolio using GARCH. This happens despite the Inverse portfolio not finding a significant alpha, meaning that the basis for the exploration of a low-volatility anomaly may have disappeared, but its combination with the MM approach might still be profitable. A reason as to why this occurs is pointed out in 4.5. where we find that the application of realistic leverage restrictions for most managers would destroy the performance differential of the strategy. Asness, Frazzini and Pedersen (2012) show that constrained investors tend to hold riskier assets distorting the risk-return trade-off, opening the opportunity for strategies increasing leverage on low-volatility assets. The Betting-against-Beta factor (Frazzini and Pedersen 2014) also explores a similar train of thought, betting on low-beta assets that are often overlooked by constrained investors. Although the Replication portfolio's performance is exposed to the BAB factor, as we saw in 4.3., there is still some indication of overperformance. Therefore, our proposal might offer a different way to explore a similar market inefficiency.

Observing the originally proposed construction, we find that the MM portfolio using past volatility has observed some deterioration since the publication of the study (08/2017). This is noticeable both in Figure 6 and in the rolling alpha plot mentioned in 4.4., where we may be in the presence of alpha deterioration. In our view, there are two plausible justifications for this

case. First, believing that we observed an anomaly up until 2017, we posit that investors exploring similar strategies may have contributed to the dissipation of said anomaly. Secondly, a change of regime in Financial Markets can affect the performance of the strategy making it ineffective under new conditions, this is in line with the concept of non-stationarity mentioned in 2.4.. Some signs of this possible change have been detected in some asset classes and relationships after the pandemic, such as the relationships between stocks and bonds (Brixton et al. 2023), and between the US Dollar and Oil Spot Prices (Thiagarajan et al. 2023).

5.3. Limitations and suggestions for further research

In our view, there are two main limitations within this research. The first is caused by the lag between information being made publicly available and affecting the following months' volatility. Most of the time in an efficient market, we would expect information to be immediately incorporated into the price, and therefore, its use for the following months vanishes (the use of analyst forecasts for our proposed volatility drivers could be used to try and circumvent this limitation). The second limitation relates to the computational power available that did not allow for hyper-parameterization using the full extent of training data.

Regarding further research, the finding of a lack of relationship between returns and volatility at the stock level for US companies from 2005 onwards (a short period when compared to most research) may be further explored within different markets and time frames for a better understanding of persistence in the cross-section. In our opinion, the main interrogation mark within this research is the detection that model forecasts outperform the proxy, yet portfolios constructed based on the proxy outperform those based on forecasts. Further research may dive into the causes for this, and we suggest commencing with the exploration of the hypothesis proposed in 5.2. of self-feeding loops coming from fund managers' use of past volatility.

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Appendix

Exhibit 1

Representation of created structure for data collection and treatment.

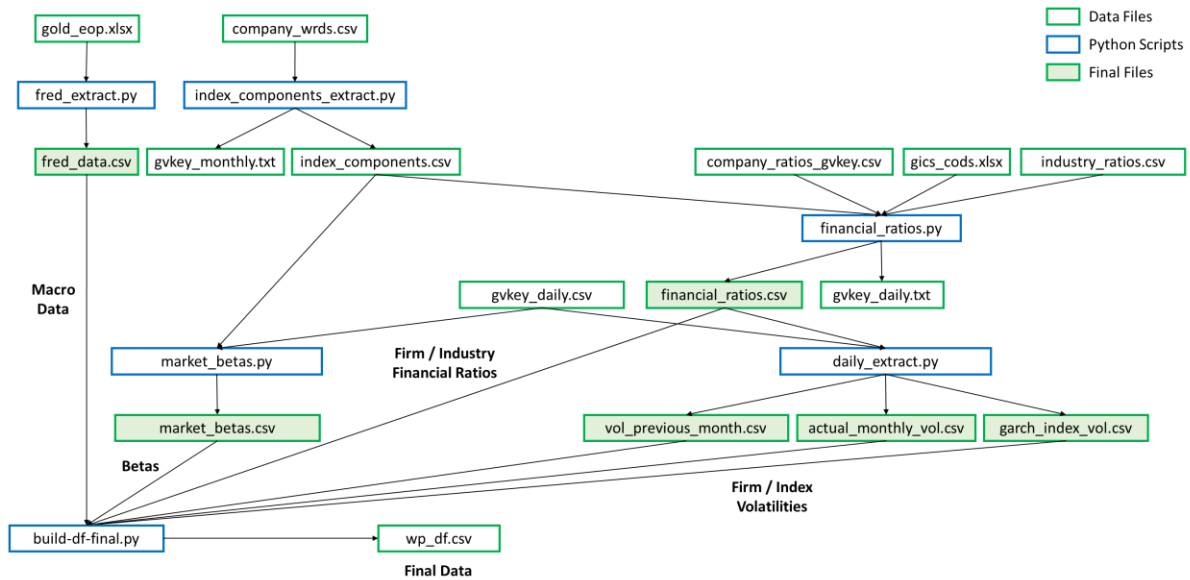


Exhibit 2

Conditions for Index integration.

Variable	Selected	Description
Issue Type Code	Common, Ordinary	Identifies the type of security an issue is.
Stock Exchange Code	NYSE, Nasdaq	Identifies the major exchange on which the company's stock is traded.
Current ISO Country Code	USA	Identifies the country in which the company is incorporated.
Primary Joiner Flag	Primary Issue	Identifies the issue with the highest market presence.

Exhibit 3

Formula to calculate monthly volatility.

$$\sigma_{Monthly} = \sigma_{Daily} \times \sqrt{n}$$

Where:

$\sigma_{Monthly}$ = Monthly Volatility

σ_{Daily} = Standard Deviation of Daily Returns

n = Number of days in a month

Exhibit 4

Financial ratios collected that integrate the constructed model as features.

Level	Abbrev.	Variable	Category
Firm Level	BM	Book/Market	Valuation
	CAPEI	Shillers Cyclically Adjusted P/E Ratio	Valuation
	PE_EXI	Price-to-Earnings, excl. Extraordinary Items	Valuation
	PTB	Price/Book	Valuation
	ROA	Return on Assets	Profitability
	ROE	Return on Equity	Profitability
	Debt_Assets	Total Debt / Total Assets	Solvency
	Debt_Ebitda	Total Debt / EBITDA	Financial Soundness
Industry Level	Indret_EW	Industry Return Equal-Weighted	-
	Indret_VW	Industry Return Value-Weighted	-
	BM	Book/Market	Valuation
	CAPEI	Shillers Cyclically Adjusted P/E Ratio	Valuation
	Divyield	Dividend Yield	Valuation
	PE_EXI	Price-to-Earnings, excl. Extraordinary Items	Valuation
	PTB	Price/Book	Valuation
	ROA	Return on Assets	Profitability
	ROE	Return on Equity	Profitability
	Debt_Assets	Total Debt / Total Assets	Solvency
	Curr_Debt	Current Liabilities/Total Liabilities	Financial Soundness
	Debt_Ebitda	Total Debt / EBITDA	Financial Soundness
	Short_Debt	Short-Term Debt/Total Debt	Financial Soundness
	Cash_Conversion	Cash Conversion Cycle (Days)	Liquidity
	Cash_Ratio	Cash Ratio	Liquidity
	Curr_Ratio	Current Ratio	Liquidity
Quick_Ratio	Quick Ratio (Acid Test)	Liquidity	

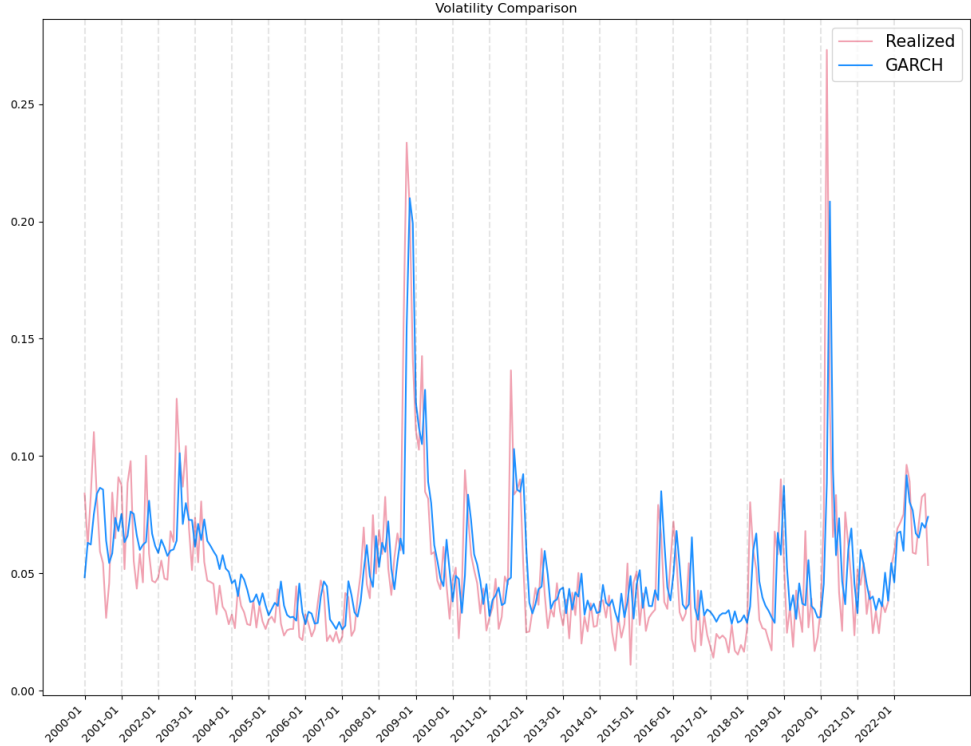
Exhibit 5

Macroeconomic data that integrates the constructed model as features.

Category	Definition
Labour Market	Unemployment Rate
	Initial Jobless Claims
GDP	GDP Growth Rate
Government Debt	Debt to GDP
Inflation	CPI
	PCE
	Core PCE
Dollar Exchange Rates	EUR/USD
	USD/GBP
	USD/JPY
Commodity Prices	Crude Oil WTI
	Crude Oil Brent
	Gold
	Copper
	Wheat
Government Bond Yields	3-month yield
	1-year yield
	2-year yield
	5-year yield
	10-year yield
	20-year yield
	30-year yield
	10y2y yield spread
	10y3m yield spread

Exhibit 6

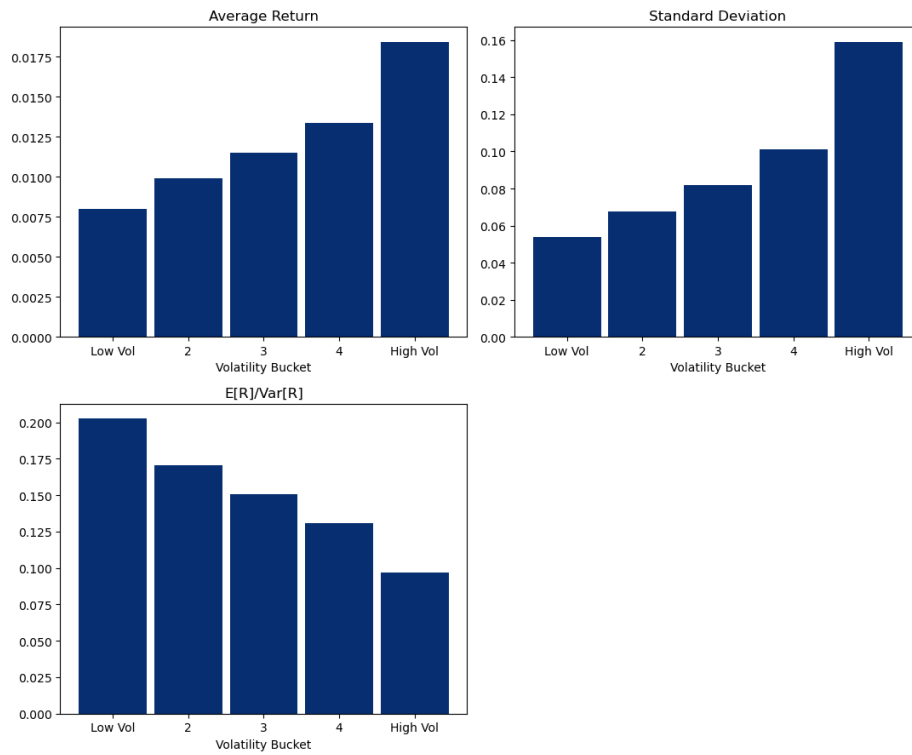
Comparison of GARCH(1,1) predictions against actual realized volatility.



	R-Squared	Root Mean Squared Error	Mean Squared Log Error	Max Error
GARCH (1,1)	0.4770	0.0232	0.0004	0.1846
Previous Month Volatility	0.3072	0.0267	0.0006	0.1998

Exhibit 7

Case 1 – Volatility buckets with entire data – Previous Volatility (t-1) and Return (t).



Case 2 – Volatility buckets per month – Previous Volatility (t-1) and Return (t).

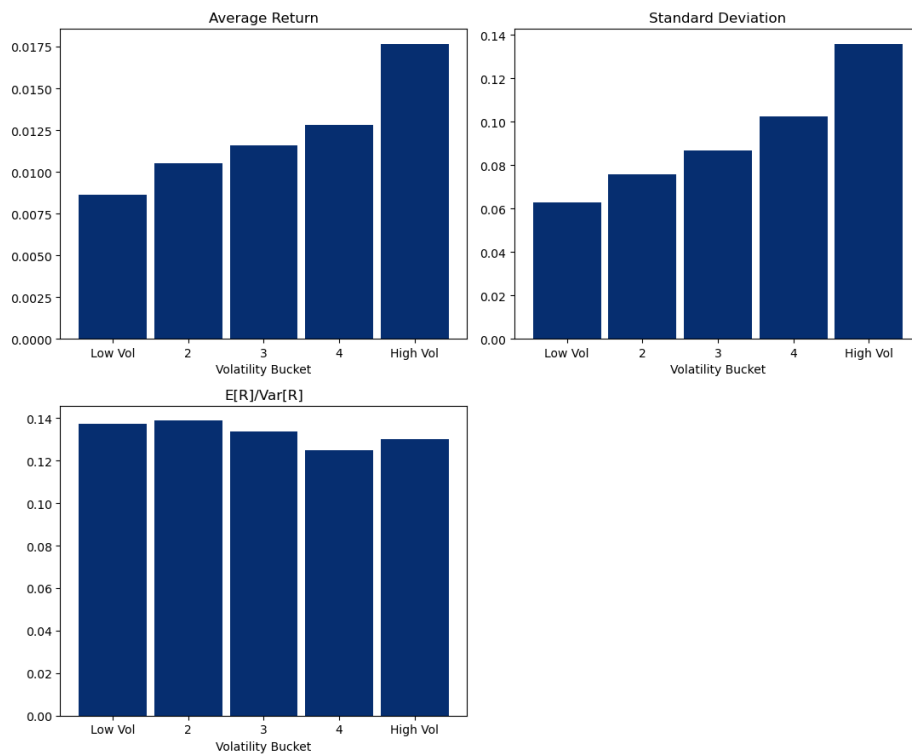


Exhibit 8

Moreira and Muir original proposal for portfolio construction.

Managed Portfolio:

$$f_{t+1}^{\sigma} = \frac{c}{\hat{\sigma}_t^2(f)} f_{t+1}$$

Where:

f_{t+1} = Buy-and-Hold Portfolio Excess Returns

$\hat{\sigma}_t^2(f)$ = Proxy for the Portfolio Conditional Variance

c = Constant that controls the average exposure of the strategy

Exhibit 9

Replication Portfolio construction.

Asset weight:

$$w_{A,t+1} = \frac{c}{\hat{\sigma}_A^2} \times w_{A,t}^{VW}$$

Where:

$w_{A,t+1}$ = Asset A weight at t+1

$\hat{\sigma}_A^2$ = Proxy for the Asset A Variance

$w_{A,t}^{VW}$ = Value-Weighted weight of Asset A at time t

c = Constant that controls the average exposure of the strategy

Exhibit 10

Construction of Inverted Volatility Portfolios.

Consider for a portfolio of two assets (A and B) the following:

$$\sigma_A = 5\% \rightarrow \sigma_A^2 = 0.0025$$

$$\sigma_B = 10\% \rightarrow \sigma_B^2 = 0.01$$

Unadjusted Weights (uw) = 1/Asset Variance

$$uw_A = \frac{1}{\sigma_A^2} = \frac{1}{0.0025} = 400$$

$$uw_B = \frac{1}{\sigma_B^2} = \frac{1}{0.01} = 100$$

Asset Weight = Asset Unadjusted Weight / Sum of Unadjusted Weights

$$w_A = \frac{uw_A}{\sum_x uw_x} = \frac{400}{500} = 80\%$$

$$w_B = \frac{uw_B}{\sum_x uw_x} = \frac{100}{500} = 20\%$$

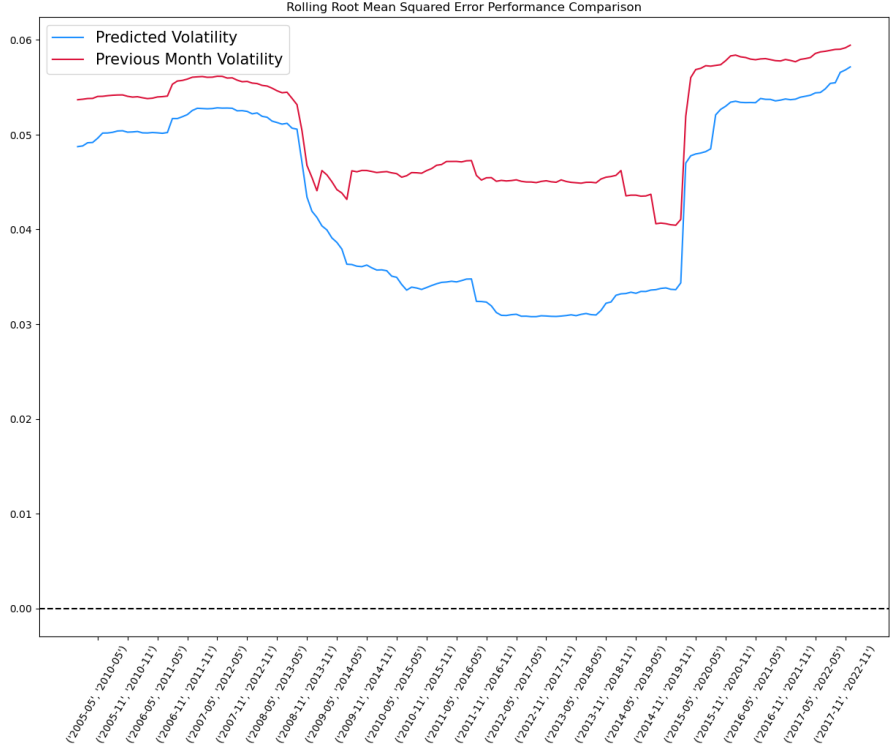
Exhibit 11

Comparison of performance for equity volatility forecasts and the proxy (previous month volatility).

	R-Squared	Root Mean Squared Error	Mean Squared Log Error	Max Error
Predicted Volatility	0.3686	0.0459	0.0015	1.3771
Previous Month Volatility	0.2045	0.0515	0.0018	2.8317

Exhibit 12

Rolling RMSE of equity volatility forecasts and the proxy (previous month volatility).



Rolling R-Squared of equity volatility forecasts and the proxy (previous month volatility).

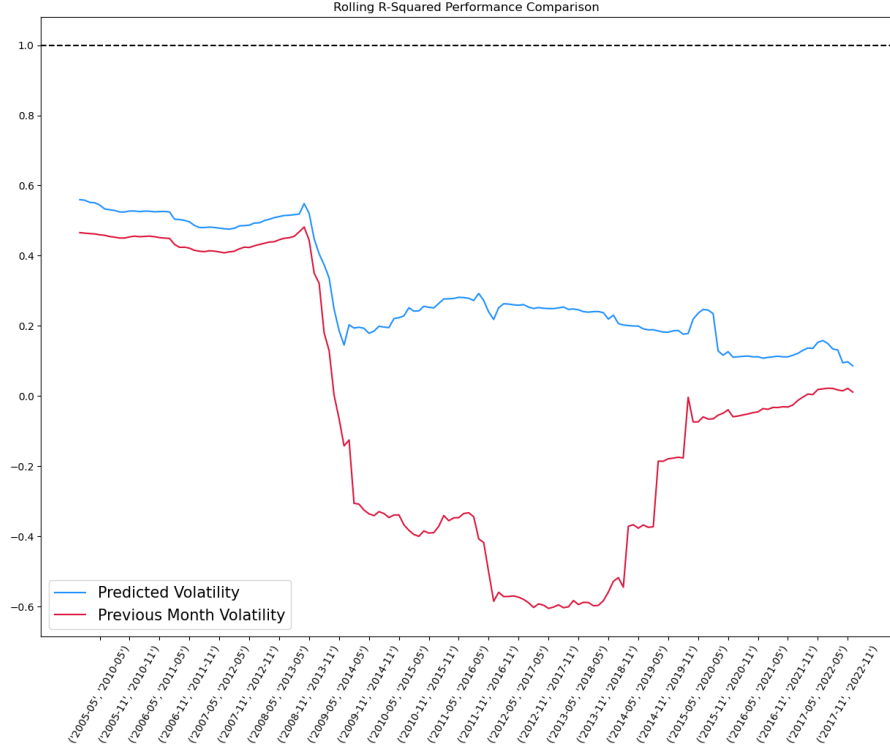


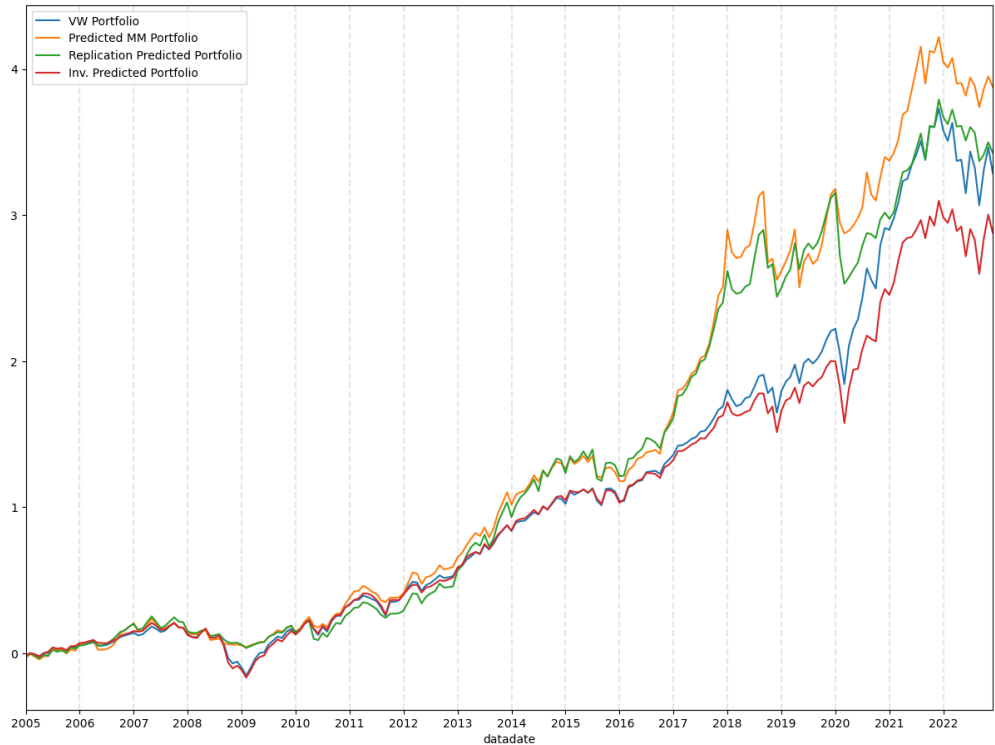
Exhibit 13

Metric comparison of constructed portfolios and benchmarks.

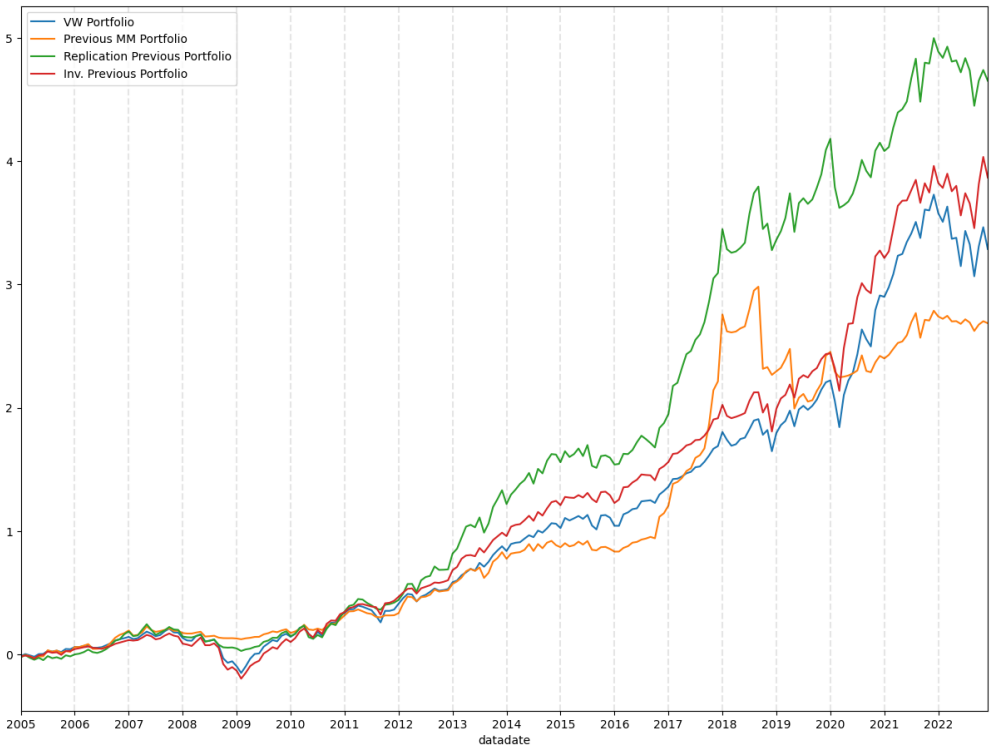
	Annualized Return	Annualized Volatility	Sharpe Ratio	Downside Deviation	Sortino Ratio
EW Portfolio	13.07%	16.26%	0.804	12.17%	1.074
VW Portfolio	12.76%	14.81%	0.861	10.23%	1.247
Predicted MM Portfolio	13.83%	14.81%	0.934	11.07%	1.250
Previous MM Portfolio	11.51%	14.81%	0.777	13.72%	0.839
Replication Predicted Portfolio	13.03%	14.81%	0.879	11.23%	1.160
Replication Previous Portfolio	15.05%	14.81%	1.016	9.87%	1.524
Inv. Predicted Portfolio	11.47%	14.24%	0.805	10.72%	1.070
Inv. Previous Portfolio	11.20%	12.02%	0.932	8.89%	1.261

Exhibit 14

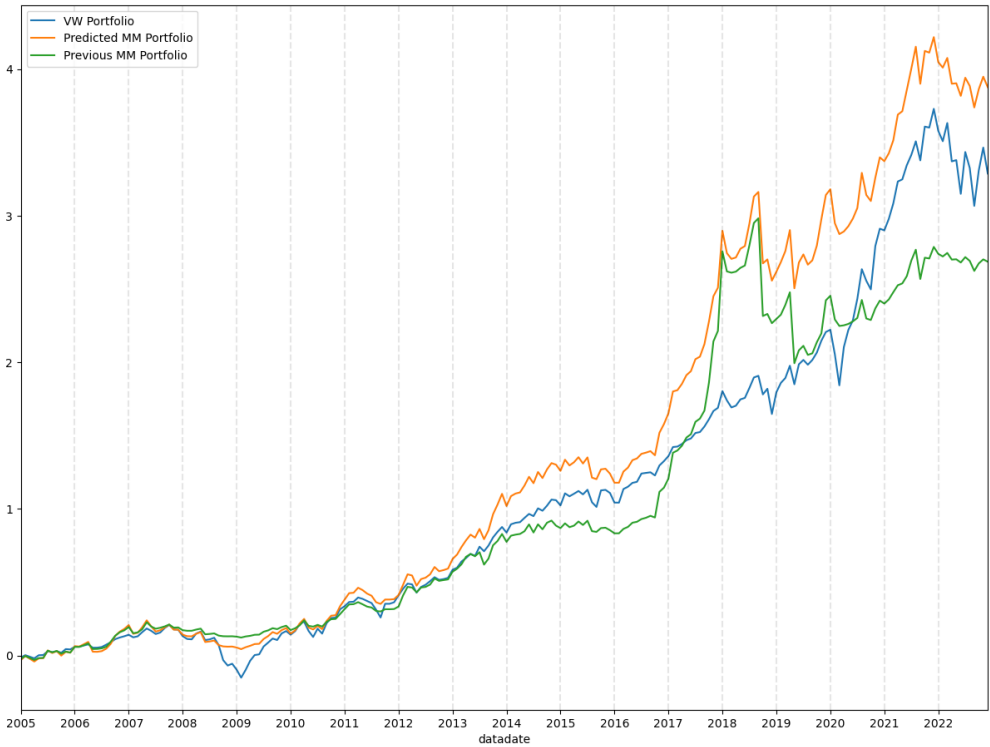
Cumulative Returns comparison of Forecast-based Portfolios (targeting 10% volatility).



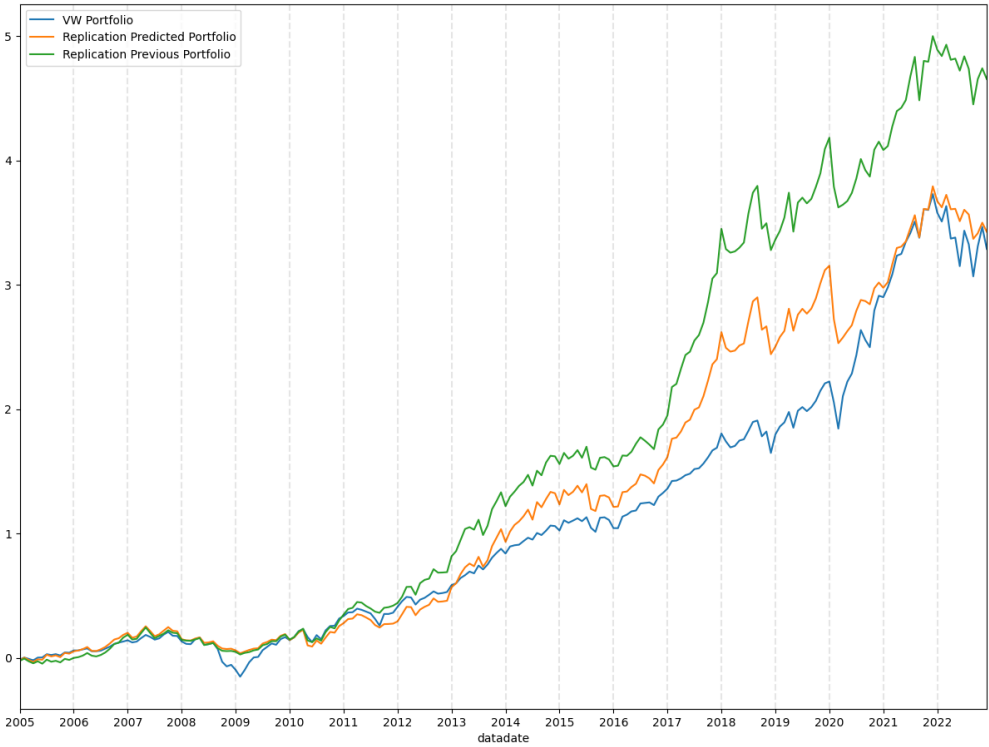
Cumulative Returns comparison of previous volatility portfolios (targeting 10% volatility).



Cumulative Returns comparison of MM Portfolios (targeting 10% volatility).



Cumulative Returns comparison of Replication Portfolios (targeting 10% volatility).



Cumulative Returns comparison of Inverse Portfolios (targeting 10% volatility).

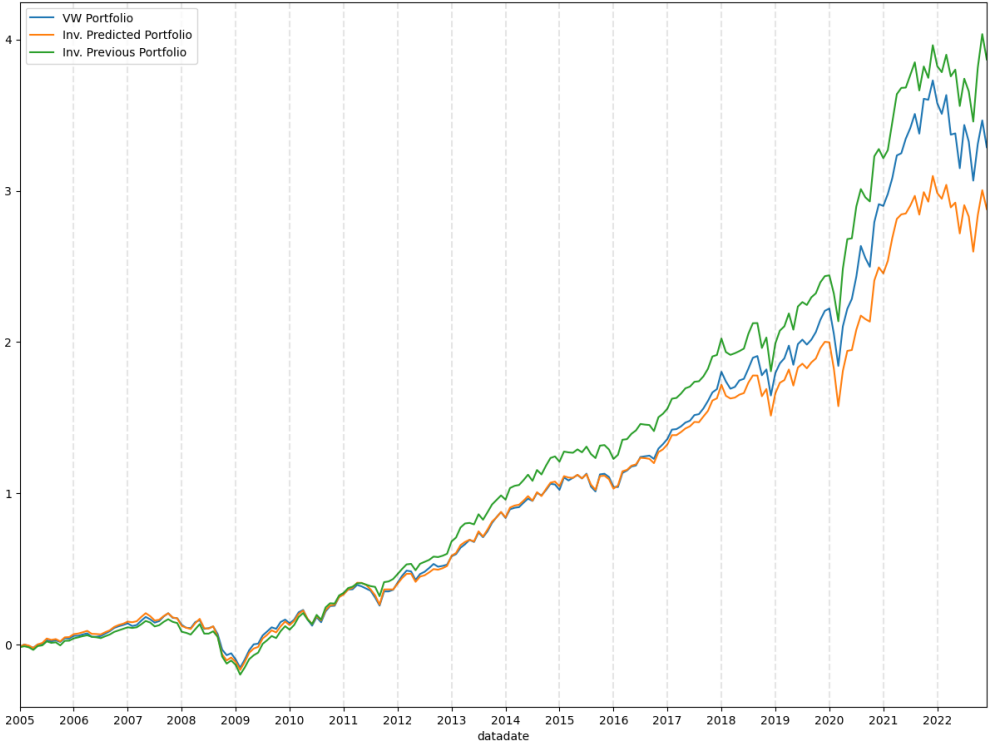


Exhibit 15

Portfolio regression against Market Portfolio.

	R-Squared	Alpha		Mkt-RF		IR
		Coef	P-Value	Coef	P-Value	
Predicted MM Portfolio	0.572	0.003	0.078	0.756	0.000	0.431
Previous MM Portfolio	0.321	0.004	0.151	0.566	0.000	0.351
Replication Predicted Portfolio	0.634	0.002	0.190	0.796	0.000	0.320
Replication Previous Portfolio	0.592	0.004	0.024	0.769	0.000	0.553
Inv. Predicted Portfolio	0.949	0.000	0.539	0.937	0.000	-0.150
Inv. Previous Portfolio	0.902	0.001	0.135	0.770	0.000	0.365

Exhibit 16

Portfolio regression against Fama-French 3 Factor Model.

	R-Squared	Alpha		Mkt-RF		SMB		HML		IR
		Coef	P-Value	Coef	P-Value	Coef	P-Value	Coef	P-Value	
Predicted MM Portfolio	0.581	0.003	0.098	0.775	0.000	-0.026	0.751	-0.131	0.032	0.408
Previous MM Portfolio	0.325	0.003	0.176	0.584	0.000	-0.057	0.584	-0.079	0.304	0.333
Replication Predicted Portfolio	0.644	0.002	0.253	0.829	0.000	-0.170	0.026	-0.058	0.298	0.281
Replication Previous Portfolio	0.610	0.004	0.039	0.813	0.000	-0.217	0.007	-0.086	0.142	0.510
Inv. Predicted Portfolio	0.956	0.000	0.795	0.914	0.000	0.092	0.000	0.076	0.000	-0.064
Inv. Previous Portfolio	0.904	0.001	0.095	0.758	0.000	0.045	0.158	0.043	0.065	0.411

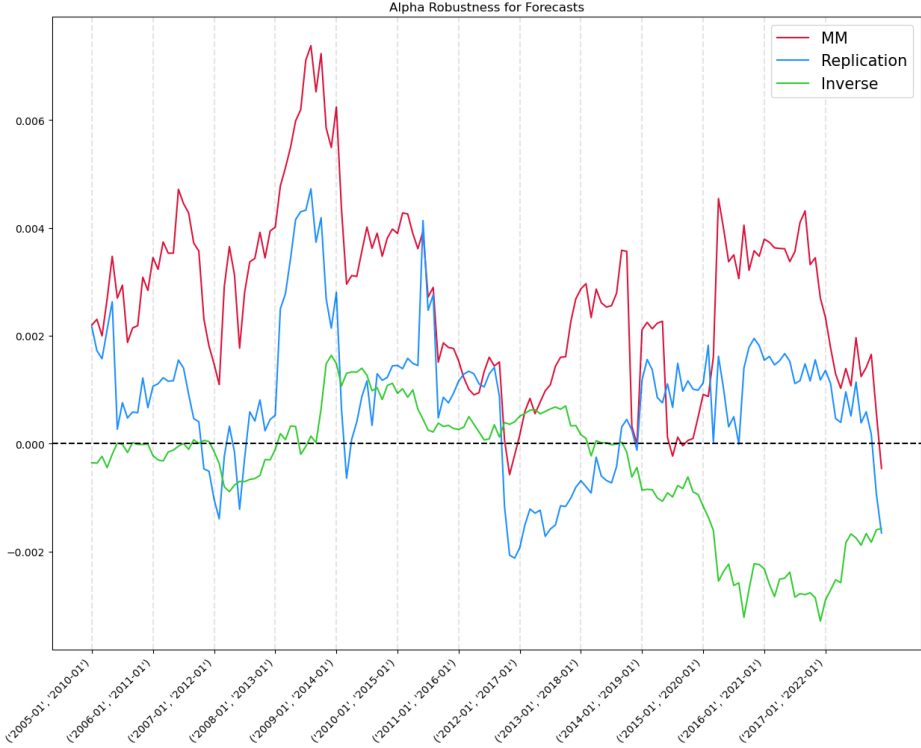
Exhibit 17

Portfolio regression against Fama-French 3 Factor Model + BAB Factor.

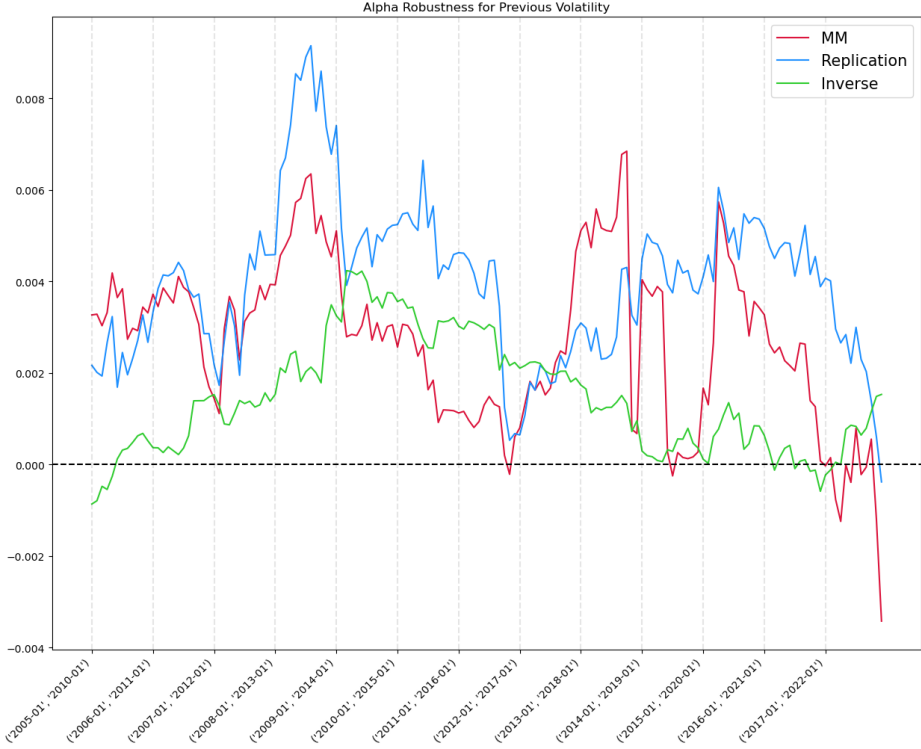
	R-Squared	Alpha		Mkt-RF		SMB		HML		BAB		IR
		Coef	P-Value	Coef	P-Value	Coef	P-Value	Coef	P-Value	Coef	P-Value	
Predicted MM Portfolio	0.582	0.003	0.115	0.775	0.000	-0.024	0.770	-0.127	0.040	0.025	0.723	0.394
Previous MM Portfolio	0.325	0.003	0.190	0.584	0.000	-0.056	0.593	-0.077	0.324	0.014	0.880	0.327
Replication Predicted Portfolio	0.654	0.001	0.441	0.827	0.000	-0.158	0.036	-0.035	0.527	0.156	0.016	0.192
Replication Previous Portfolio	0.620	0.003	0.088	0.811	0.000	-0.205	0.010	-0.062	0.287	0.160	0.018	0.426
Inv. Predicted Portfolio	0.959	-0.001	0.362	0.913	0.000	0.099	0.000	0.089	0.000	0.089	0.000	-0.227
Inv. Previous Portfolio	0.906	0.001	0.171	0.758	0.000	0.049	0.123	0.051	0.032	0.051	0.059	0.342

Exhibit 18

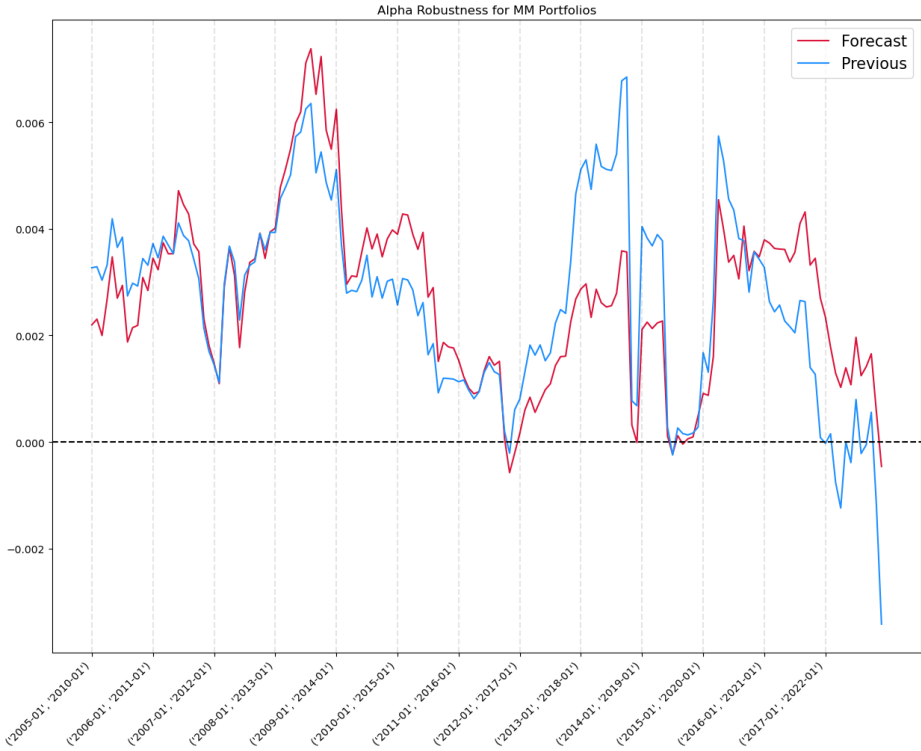
Alpha persistence for Forecast-based Portfolios.



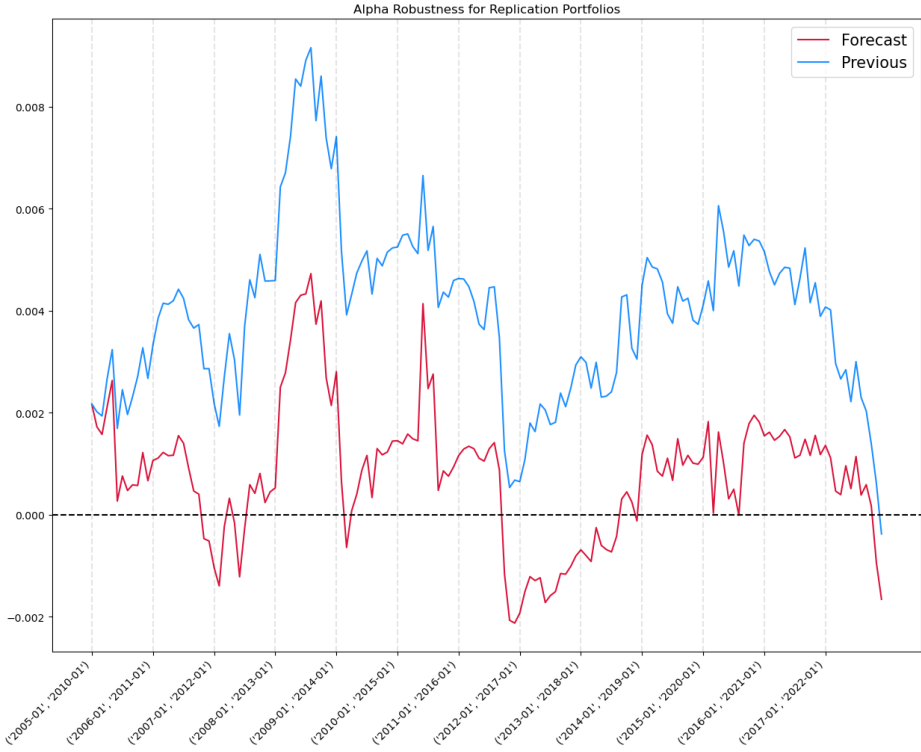
Alpha persistence for previous volatility portfolios.



Alpha persistence for MM Portfolios.



Alpha persistence for Replication Portfolios.



Alpha persistence for Inverse Portfolios.

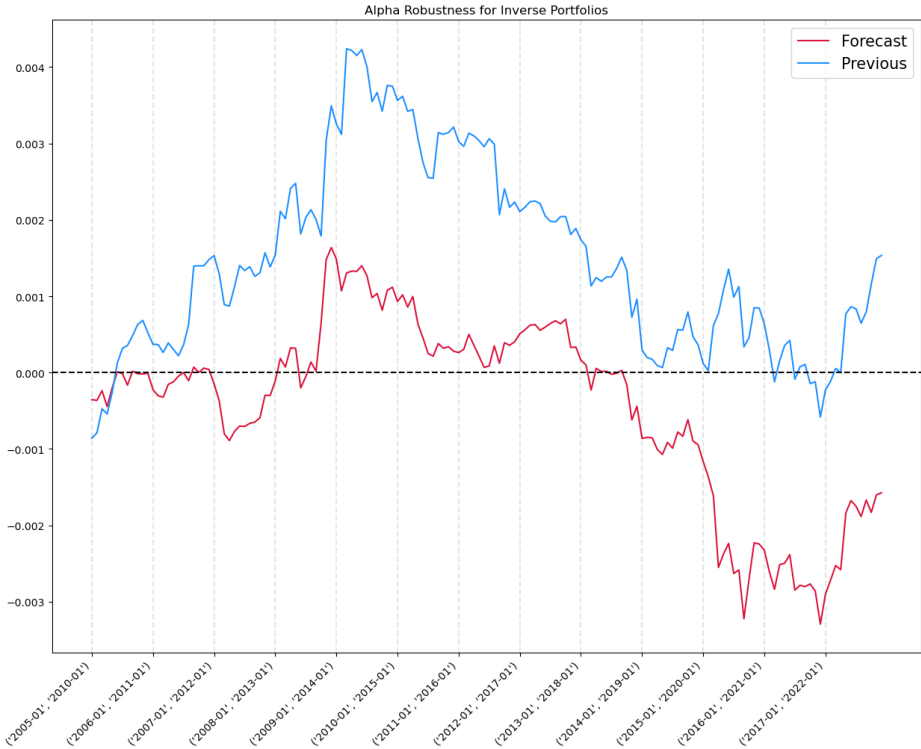


Exhibit 19

Metric comparison of minimum-weight portfolios.

	Annualized Return	Annualized Volatility	Sharpe Ratio	Downside Deviation	Sortino Ratio
Cons. Inv. Predicted Portfolio	11.12%	14.01%	0.794	10.54%	1.054
Inv. Predicted Portfolio	11.47%	14.24%	0.805	10.72%	1.070
Cons. Rep. Predicted Portfolio	12.71%	14.62%	0.869	11.14%	1.140
Replication Predicted Portfolio	13.03%	14.81%	0.879	11.23%	1.160

Exhibit 20

Metric comparison of leverage-restricted portfolios.

	Annualized Return	Annualized Volatility	Sharpe Ratio	Downside Deviation	Sortino Ratio
Cons. 1 Predicted MM Portfolio	9.70%	10.20%	0.951	6.80%	1.426
Cons. 1.5 Predicted MM Portfolio	11.88%	12.33%	0.964	8.37%	1.419
Cons. 2 Predicted MM Portfolio	13.38%	13.76%	0.972	9.63%	1.389
Predicted MM Portfolio	13.83%	14.81%	0.934	11.07%	1.250
Cons. 1 Predicted Rep. Portfolio	10.05%	11.12%	0.904	7.76%	1.294
Cons. 1.5 Predicted Rep. Portfolio	12.82%	13.92%	0.921	9.82%	1.306
Cons. 2 Predicted Rep. Portfolio	14.00%	15.51%	0.902	11.51%	1.217
Replication Predicted Portfolio	13.03%	14.81%	0.879	11.23%	1.160

	Annualized Return	Annualized Volatility	Sharpe Ratio	Downside Deviation	Sortino Ratio
Cons. 1 Previous MM Portfolio	7.42%	7.68%	0.967	5.62%	1.320
Cons. 1.5 Previous MM Portfolio	9.01%	9.56%	0.942	7.35%	1.226
Cons. 2 Previous MM Portfolio	10.18%	11.05%	0.922	8.98%	1.134
Previous MM Portfolio	11.51%	14.81%	0.777	13.72%	0.839
Cons. 1 Previous Rep. Portfolio	9.95%	11.40%	0.872	7.81%	1.274
Cons. 1.5 Previous Rep. Portfolio	12.78%	14.53%	0.880	10.24%	1.248
Cons. 2 Previous Rep. Portfolio	14.30%	16.60%	0.862	12.08%	1.184
Replication Previous Portfolio	15.05%	14.81%	1.016	9.87%	1.524

Exhibit 21

Leverage-restricted Portfolio regression against Market Factor.

	R-Squared	Alpha		Mkt-RF		IR
		Coef	P-Value	Coef	P-Value	
Cons. 1 Predicted Rep. Portfolio	0.814	0.001	0.229	0.677	0.000	0.294
Cons. 1.5 Predicted Rep. Portfolio	0.734	0.002	0.145	0.805	0.000	0.356
Replication Predicted Portfolio	0.634	0.002	0.190	0.796	0.000	0.320