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Forecasting Volatility and Value at Risk of an Islamic Tangency Portfolio

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Abstract

Academic literature arrives at diverse conclusions about the volatility forecasting accuracy of GARCH and EWMA models. Most studies analyse conventional equities, not focusing on shariah-compliant investing and the Islamic community. In this study, GARCH and EWMA models under different distributional assumptions were used to evaluate the one-step-ahead volatility and VaR forecasting accuracy for an Islamic Tangency Portfolio. Analysis confirms findings by Ding & Meade(2010) and shows that EWMA also outperforms GARCH(1,1) models for a sharia-compliant portfolio under short selling restrictions, while indicating the lowest failure rate of actual losses exceeding predicted VaR estimates.

Keywords: Islamic Portfolio, Volatility Forecasting, Backtesting

1. Introduction

Latest turbulences at the international financial markets encouraged many investors to overthink their risk management strategy, searching for alternative asset classes. Shariah compliant investing has been constantly growing at 28% per year over the past 17 years as they are perceived to be less volatile and the wealth of the Muslim population experiences steady growth (Hayat,2015). Induced shariah law (Shariah supervisory board) requires eligible stocks (companies) to have revenue of less than 5% in industries such as Alcohol, Tobacco, Pork etc. However, more influential are financial criteria which require debt to market capitalization, accounts receivables to market cap., sum of cash to market cap ratios to be less than 33% on a trailing 24 months basis (S&P Global, 2018). Owing to this relatively new trend in finance, most academic literature applies volatility forecasting and model appraisals to conventional stocks not considering the emerging importance of the need of empirical evidence to the Islamic financial community. Rizwan and Khursheed (2018) stress this issue and together with Mohammed, Bakar and Ariff (2018) are one of the few researchers applying volatility forecasting principles to shariah compliant stocks and indices for Pakistan and Malaysia, respectively. This thesis will take a comparable approach by analysing GARCH and EWMA forecasting ability for a newly created Islamic Tangency Portfolio under shariah compliant short selling conditions. In particular, this study examines how different distributional assumptions in the shock term of a GARCH model impact the reliability of Value at Risk forecasts, compared to EWMA estimates for a shariah compliant tangency portfolio under short selling restrictions and the German DAX. Reliability of VaR forecasts will be assessed using statistical backtesting procedures as well as error functions. Therefore, I will develop four models in order to forecast volatility and consequently VaR estimates for the returns of the Islamic tangency portfolio as well as the DAX. Lastly, this study examines the independence of Value at risk forecast failures of the Islamic portfolio to examine if failures in VaR forecasting (Actual loss > VaR estimate) are independently distributed or cluster on consecutive days over the out-of-sample period. This will add to the increasing need of the Islamic investors community to implement adequate risk management structures and obtain empirical evidence on which method to implement for one-step ahead volatility forecasting given an Islamic Portfolio. The following chapter will outline some of the key findings of related empirical studies and will elaborate this thesis` research question as well as its origin based on a study conducted by Ding and Meade (2010) as well as Tse and Tsung (1992). The third methodology section will explain key concepts applied in this study. The fourth section will present the empirical analysis and outlines respective results. Finally, the conclusion will summarize major findings, elaborates limitations and gives ideas for future research to be conducted.

2. Literature Review

Cont (2000) in its widely cited academic papers, defines the so called "*stylized*" facts about empirical properties of asset returns. Those conditions are major insights, that this thesis will use as a foundation for later analyses. Cont (2000) elaborates that asset returns often exhibit heavy tails (unconditional tails). However, more importantly he states that even after using a GARCH model, residuals often exhibit heavy tails (conditional heavy tails). It is elaborated that studying asset price behaviour implicitly assumes "stationarity", which means that some statistical properties need to be constant over time. Consequently, an assumption will be taken that returns at any point in time $r(t_1,T)$ exhibit the same joint distributions as at $r(t_k,T)$ (Cont, 2001). A study by Angelidis, Benos and Degiannakis (2003) estimated a series of ARCH models and found leptokurtic distributions to be most accurate when estimating VaR forecasts one-step-ahead. They furthermore found results, that the mean process specification does not significantly alter the results considering the calculation of value at risk forecasts. Herein, they combined classical GARCH models with autoregressive processes of different orders and analysed the impact on VaR estimates. Shamiri and Isa (2009) analysed the Malaysian stock markets volatility and found the distributional assumption within the error term to most affect performance rather than implementing more complex GARCH model types. Mohammed, Bakar and Ariff (2018) again analysed the volatility behaviour of the Malaysian Stock market, however with a clear focus on Islamic stocks. They analysed the time period from January 2009 until October 2016 of the FTSE-Bursa Malaysia Emas Shariah Index. Implementing an EGARCH and GARCH model, they found the EGARCH to slightly outperform the GARCH model. However, they found similar results than Shamiri and Isa (2009) that the distribution of the error term shows a significantly bigger impact on the forecasting accuracy than implementing a different model. Rizwan and Khursheed (2018) analysed the Karachi stock exchange Meezan index (KMI-30) which represents the Islamic stock index of Pakistan. They found the GARCH (1,1) as well as the ARIMA (2,1) to be most suitable for the Pakistani Islamic index. It is evident that most studies focus on the application of more complex GARCH and E-GARCH models when it comes to evaluating forecasting accuracy. This study also considers the EWMA method as part of the determination of forecasting accuracy. In this regard, Minkah (2007) showed that GARCH models deliver more accurate out sample results over short timeframes. However, the in-sample results proves the EWMA to be most accurate based on the RMSE function. Nonetheless, GARCH and EWMA deliver close forecasts. Lee, Nguyen and Ry (2017) investigated the differences between developed and emerging markets and conducted analysis on volatility forecasting for the Indonesian, Malaysian, Japanese and Hong Kong stock markets. In their study they analysed a sample from 1998-2015, where 2010-2015 denotes the out-of-sample period. The researchers showed that it is not always the most complex model yielding the best estimates. They proved this statement by finding that the simple EWMA model is the most accurate for forecasting volatility of the Hong Kong stock market in their out-of-sample dataset. The RMSE is calculated with 0.016 for the EWMA and 0.0542 for the GARCH Model. The researchers used a lambda of 0.94 for daily returns. Tse and Tsung (1992) challenge the established findings by Akgiray (1989) that GARCH models deliver superior forecasting accuracy compared to simpler historical approaches. Tse and Tsung (1992) analysed five Singaporean indices from various industries from 1975 to 1988 using daily returns. They created 4 data periods to create sub-samples for robustness checks. They applied 25 day forecasting intervals with updating model parameters. Pursuing this robust approach, enabled the researchers to show that EWMA clearly outperforms based on RMSE statistics. Furthermore, Tse and Tsung (1992) state, that in times of excess volatility, EWMA still outperforms the GARCH (1,1) approach. GARCH (1,1) models were however assumed to be reacting faster to those high volatility environments. Ding and Meade (2010) take a comprehensive approach towards assessing the accuracy of GARCH, EWMA and Stochastic Volatility (SV). They separate their analysis into different categories such as FX Rates, Equity indices, Equities and commodities. The researchers state that applying GARCH or SV type models significantly improves forecasting accuracy. Nevertheless, the researchers furthermore elaborate, that: "[…] if the objective [of the analysis] is to achieve forecasting accuracy, then forecasting volatility using EWMA is a reliable policy that will only come unstuck if the series exhibits SV with a high volatility of volatility." Ding and Meade (2010) found that FX rates, equity indices and equities are best forecasted using the EWMA approach and commodities for more than three months horizon using GARCH. Herein, they considered data from 1st January 2001 until the 29th December 2006. Based on the findings by Tse & Tsung (1992) as well as Ding & Meade (2010) the question arises if the same result would occur when analysing a less volatile Islamic Tangency Portfolio created according to shariah induced short selling restrictions. In this regard, this thesis will first estimate respective GARCH and EWMA parameters, conduct respective statistical tests, to finally based on volatility and forecasting estimates assess, whether an Islamic investor should use EWMA to get one-step ahead forecasted volatility estimates.

3. Methodology

The following chapter will elaborate on the major concepts and tests applied in this study. Emphasis has been put on different statistical tests and criteria to determine a suitable model such as GARCH (1,1) and EWMA for an Islamic investor, to forecast one-step-ahead volatility and finally Value at Risk of the Islamic Tangency Portfolio and DAX.

3.1 Data - Creation of an Islamic Tangency and 1/N Portfolio

The underlying return data starting from 07.11.2013 to 07.11.2018 has been mainly gathered from Bloomberg and Standard and Poor's website. The in-sample period is defined from 07.11.2013 to 07.11.2017 and the out-of-sample period from 08.11.2017-07.11.2018. This timeframe has been chosen due to availability as well as liquidity conditions of the individual indices. Consequently, in order to come up with the tangency portfolio that an Islamic investor should be invested in, 23 shariah indices have been analysed in order to create the optimal risky portfolio. All countries' indices that follow shariah conform principles as well as S&P shariah indices have been considered with the exception of those yielding negative expected returns. Owing to short selling limitations induced by shariah law, only four shariah compliant indices have been calculated to be included in the optimal risky portfolio. Using variance/covariance matrix, individual tangency weights have been calculated under the given conditions. Those are the Shariah Egypt Price Index, S&P 500 Shariah Index TR, S&P Bangladesh BMI Shariah and S&P BSE 500 SHARIAH with 5.713%, 24.773%, 33.074%, 36.441% in weights, respectively. The newly generated data series will be compared to the time series of the German DAX as well as the famous 1/N Index developed by DeMiguel, Garlappi and Uppal (2009). The Islamic Portfolio shows significant excess kurtosis, indicating heavy tails in the returns. However, the 1/N Index exhibits an even higher kurtosis which is mainly caused by investing 1/12 (1/N; where n = number of assets incl.) of the portfolio into the APAC index which is not included in the Islamic Portfolio. All series exhibit negative skewness showing slightly left skewed distributions. Appendix 1 compares the minimum variance, as well as tangency portfolio considering expected returns, Sharpe ratio, standard deviation and variance estimates under short selling as well as non-short selling conditions. The 1/N index shows a smaller Sharpe ratio of -0.194 to -0.097 for the tangency approach out of sample, see Appendix 2. In contrast to the study conducted by DeMiguel, Garlappi and Uppal (2009), this paper includes 12 shariah assets that have a positive expected return. In order to obtain the full diversification effect found by the researcher, the Islamic portfolio would have to include more assets. Consequently, the tangency approach allows the investor to avoid additional volatility within the portfolio. Therefore, all analysis regarding model development and volatility forecasting were conducted using the Tangency portfolio and German DAX. Relying on daily data shows that the average of returns is very close to zero in all cases. The following table outlines the major characteristics of the three data series that have been examined:

	Average:	St.Deviation:	Skew:	Kurtosis:	Median:	Min	Max	1st Quart.	3rd Quart
1/N INDEX	0.00029	0.005011	-0.87	5.28	0.00058	-0.0366	0.02048	-0.0021	0.0032
ISLAMIC	0.00044	0.004687	-0.47	4.15	0.00057	-0.03683	0.02232	-0.00226	0.0032
DAX	0.00037	0.011761	-0.38	2.41	0.00090	-0.0707	0.04852	-0.0055	0.0062

Table 1. Characteristics of Dax and Islamic Portfolio return series

3.2 Modelling and Forecasting Volatility - An approach to the GARCH (1,1)

The GARCH (Generalized Autoregressive Conditional Hetereoscedasticity) model in this study follows the principles developed by Bollerslev (1986) on the generalization of the ARCH processes which have been developed by Engle (1982). The most discussed variation of the GARCH model is the GARCH (1,1) which got its name from a one-step lagged time period in the squared innovations component as well as a one-step lagged volatility component. Brooks (2008) describes the popularity of GARCH due to its "parsimonious" nature, that consequently avoids overfitting the model based on the underlying data. The lagged volatility term allows the consideration of *"volatility clustering"* as first described by Mandelbrot(1963): "[...] large changes [in volatility] tend to be followed by large changes-of either sign-and small changes tend to be followed by small changes". Large values in σ_{t-1}^2 will therefore directly impact the forecasted value for σ_t^2. The GARCH model for the volatility equation in this thesis is defined as:

$$\sigma_t^2 = \alpha_0 + \underbrace{\sum_{i=1}^{p} \alpha_i a_{t-1}^2}_{\text{ARCH Component}} + \underbrace{\sum_{j=1}^{q} \beta_j \sigma_{t-j}^2}_{\text{GARCH Component}} \quad (1)$$

The ARCH component includes the innovation/shock term α_t which is defined as $\sigma_t \times \epsilon_t = \alpha_t$. Where σ_t is representing the conditional volatility at the respective point in time t. The parameter ϵ_t is a representation of the standardized residuals which are expected to follow $[\epsilon t]$~i.i.d with $E[\epsilon t]=0$ a zero mean and $VAR[\epsilon t]=1$ a variance of 1. During the analysis in Chapter 4, the innovation term ϵ_t will be assumed to follow a normal , student t and generalized error distribution (GED) in order to evaluate the different accuracies between the developed models. Köksal (2009) found the t-distribution in the innovation term to be the most accurate one. Normality tests and goodness of fit tests will allow to compare the accuracies of different distributional assumptions of the Islamic portfolio to the data used in comparable studies. The GARCH model will run under the following mean equation: $r_t = \mu + \alpha_t$; $\alpha_t = \sigma_t \times \epsilon_t$. Where, μ_t represents the conditional mean. The conditional mean is defined as $\mu_t = E[r_t | F_{t-1}]$, where Lindberg (2016) defined F_{t-1} as $F_{t-1} = \{ r_1, \ldots r_{t-1} \}$. Therefore reflecting the information available at time t_{-1}.The return r_t is calculated by $\ln(P_t) - \ln(P_{t-1})$. In which P is the price of the security at the respective day. It is furthermore assumed that the conditional variance is defined as $\sigma_t^2 = Var[r_t|F_{t-1}] = E[(r_t - \mu_t)^2|F_{t-1}]$. Lindberg (2016) states, that $\mu_t=0$ is a common assumption when dealing with daily return data. Köksal (2009) found that in many cases a constant mean assumption increases the accuracy of the underlying model compared to the assumption taken by Lindberg (2016). Although the daily returns are indeed very close to zero, this study's' assumption can be written as follows $\mu_t=\mu_0$. A constant mean has been considered throughout this work. In order to full fill the conditions incorporated within the defined GARCH model, the autocorrelation $Cov[rt,r_{t-1}]=0$ between two consecutive time periods of returns is also assumed to be zero.

3.3 GARCH (1,1) Parameter and Unconditional Variance Estimations

Allocating weights to the respective ARCH and GARCH term requires the specification of certain conditions. This specification is essential as it requires $\alpha + \beta < 1$. Furthermore, $\alpha + \beta = 1$ would imply non-stationarity in the variance. In the scope of a stationary GARCH(1,1) and all the above conditions fulfilled, the forecasted variance estimate will converge towards its long term mean the further the forecasting horizon is extended. Wennström (2014) also states that α and β need to be >0 in order to meet the non-negativity constraint. Using this information, one can calculate the long-run variance (unconditional variance) by taking the square root out of the proposed weights $\sqrt{(\alpha 0/(1 - \alpha - \beta))}$. This is necessary as α_0 is a function of γ multiplied by the LRV (Long-run variance). Brooks (2008) stresses that the closer $\alpha + \beta$ get towards one, the longer shocks will persistent and therefore influence future forecasts. The weight assigned to the first term determines how fast the estimate converges back to its long-run mean after experiencing a shock. The parameters of the GARCH (1,1) have been estimated using Maximum Likelihood Estimation (MLE) with respect to the individual parameters. This has been conducted using the NUM XL statistical add-in for Excel.

3.4 Exponentially Weighted Moving Average (EWMA)

The Risk metrics EWMA originally developed by JP Morgan, allocates exponentially decreasing weights to σ^2_{t-j}. Therefore putting different weights on the past days variances. The recursive form in order to calculate the variance one-step ahead is defined as follows:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2 \quad (2)$$

The value of Lambda is usually set to 0.94 as developed by JP Morgan. However, Bollen (2014) challenges this general assumption and shows that accuracy of λ can be significantly improved by minimizing the RMSE error function with respect to λ. Optimization of Lambda gives the following parameters for: Portfolio: 0.98; DAX: 0.98 considering daily data in the analysis.

3.5 Model Determination and testing for Autocorrelation

Choosing a model that fits the underlying data series most accurately, is essential when coming up with reliable volatility forecasts. The following sections will elaborate the major statistical tests that will been performed to decide on the model used to forecast volatilities and consequently VaR. Autocorrelation test of the time series will be conducted using the Ljung Box Test . This test analyses whether the data is overall independently distributed and therefore not exhibits serial autocorrelation. Wennström (2014) defines the statistical test as follows:

$$Q = n(n+2)\sum_{j=1}^{h}\frac{\hat{p}^2(j)}{n-j} \tag{3}$$

In this case n denotes the number of observations, $\hat{p}$ denotes the sample autocorrelation at the given point in time j, where h denotes the overall number of lags tested. Q is defined as a chi-squared distribution $\chi^2_{(h)}$ with h degrees of freedom. Consequently, as the H_0 hypothesis states that observations are free from serial autocorrelation, $Q < \chi^2_{1-\alpha}(h)$ must be true. $\chi^2_{1-\alpha}(h)$, where h reflects the degrees of freedom, denotes the 1 - α region considering a chi-squared distribution. In addition to the Ljung Box test, the sample autocorrelation function will be plotted in order to visualize possible autocorrelations at the individual lag level.

3.6 Measuring the Goodness of Fit

Quantifying the accuracy of the models has been done by applying the Root Mean Squared Error (RMSE), Mean absolute error (MAE), the Akaike information criterion (AIC), Bayesian information criterion (BIC) as well as the log likelihood function for the fitted model. The Root Mean Squared Error is defined by Lim & Sek (2013) as follows:

$$RMSE = \sqrt{\sum_{t}^{n}\frac{e_t^2}{n}} \tag{4}$$

Where e_t is defined as being the difference between y_t the actual observation and ŷ head representing the fitted values. Applying this condition is a common procedure in evaluating the performance between actual and predicted values. It will be used to detect the error of GARCH models under different distributional assumptions, as well as to assess the deviation in the forecasted Value at Risk estimates. Furthermore, the MAE has also been considered to measure forecasting accuracy. It follows the same formula, however not taking the square root as done within RMSE calculation. Both measures have been chosen as they consider large errors significantly stronger than small errors, which is especially desirable when it comes to the Value at Risk estimates. ARCH/GARCH literature often uses the Aikon Information Criterion (AIC) to measure the performance of the estimated model and its errors. Kosapattarapim, Lin and McCrae (2011) analyse the accuracy of different distributional assumptions by fitting their model with respect to the AIC. However, they also outline that the model that fits the best is not necessarily the most accurate one regarding forecasting errors. This study examines different model´s AIC, BIC, log likelihood function and the results of error statistics such as RMSE. BIC is a common measurement, as it penalizes the introduction of additional parameters within the model. Lindberg (2016) describes this as selecting "parsimonious" models. Both AIC & BIC use the log-likelihood function as their main calculation basis. The log-likelihood function has been maximised to find the parameter estimate that best describe the underlying data.

3.7 Measuring / Backtesting Value at Risk (VaR) Forecasting accuracy

Lastly, being able to assess the Value at Risk forecasting performance, requires to implement some statistical backtesting analysis. This thesis will use Kupiec's unconditional coverage test for failure examination and Christoffersen‘s Independence test of failures to examine if the applied models create statistically significant failures on consecutive days. Kupiec's POF test (LR) (proportion of failure) uses mainly the number of exceptions caused by the forecasted Value at Risk compared to actual losses. The number of exceptions is calculated by determining in how many cases the actual losses exceeds the loss forecasted by the VaR estimate. The observed value will then be compared with its critical value. H_0 will be rejected in case the proposed LR estimation exceeds the χ^2 for the tested significance level. Haas (2001) sets up the H_0 hypothesis of the Kupiec POF test as follows: $H_0 = p = \widehat{p} = \frac{n}{T}$. Where p represents the proportion of failure and $\hat{p}$ the observed failure rate. This is then set equal to the number of exceptions x divided by T (total number of observations). Holton (2014) shows that the likelihood estimation developed by Kupiec can be used to create non-rejection intervals by solving the following estimation (5) defined by Haas (2001) for the critical x values (following an asymptotic chi-squared distribution with one degree of freedom). This will later be applied, to show the individual rejection intervals for the assumed timeframe of the past 125 and 252 days.

$$LR_{POF} = -2\ ln\left(\frac{p^x(1-p)^{n-x}}{\tilde{p}^x(1-\tilde{p})^{n-x}}\right) \qquad (5)$$

This measure ensures that the VaR forecasts do not over- or underestimate the actual given level of risk (Appendix 9). Owing to shariah law, Islamic investors are especially aware of risk associated with their investments. In that sense, finding a suitable VaR measurement based on the forecasted volatilities, requires to test the independence of occurred loss exceedance. Academic literature mostly uses the Christoffersen independence test of observed exceedance. This statistic uses the framework of Kupiec's unconditional test, but furthermore tests the condition of failure at t_{+1} given no failure at t, or failure at t or vice versa. Nieppola (2009) shows the test statistic as follows being asymptotically chi-squared distributed (1 degree of freedom):

$$LR_{ind} = -2\ ln\left(\frac{(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi_0^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1^{n_{11}}}\right) \qquad (6)$$

Where n_{00} defines no failure given no failure on the previous day, n_{10} defines failure on previous day but no failure on t, n_{11} defines failure at t given failure at t_{-1} and n_{01} defines failure at t given no failure at t_{-1}. Furthermore, π_i defines the probability of observing an exceedance given condition i. Probabilities are calculated as follows: $\pi_0=n_{01}/(n_{00}+n_{01})$; $\pi_1=n_{11}+(n_{10}+n_{11})$;

$\pi = n_{01}+n_{11}(n_{00}+n_{01+}n_{10}+n_{11})$. When accepting H_0, π_0 and π_1 should be equal (Nieppola, 2009).

4. Empirical Data Analysis and Interpretation

Deciding on which model to apply is subject to certain conditions within the data set. In order to be able to apply the aforementioned GARCH (1,1) the data is required to show certain characteristics. The first test performed was the Jarque – Bera normality test and furthermore QQ-Plots on both data series (DAX/ISLAMIC) in order to visualize the respective distribution. Appendix 3 shows the respective distribution and output charts. The p-value of the Jarque-Bera test for both data series is equal to zero. Rejecting the null hypothesis H_0 shows, that the data is not normally distributed. Looking at the QQ-Plots indicates that this is mainly caused by heavy tails that are a common phenomenon when analysing financial data series. The second step is evaluating the autocorrelation of the data series, as this is required to be not significant in this case. This has been done by running the white noise test in the form of a Ljung Box test as explained in the previous chapter. Engle (1982) proposes that testing up to the 15^{th} lag is a reasonable assumption. Conducting the test up to the 15^{th} lag yields a p-value of 3.10 % for the portfolio and 2.04% for the German DAX. We are accepting the null hypothesis of not having serial autocorrelation at a 1% level for both series. Although at $\alpha = 0.05$ the H_0 could not have been accepted, this study uses a constant μ and does not model the mean using an ARMA model or comparables. The Augmented Dickey-Fuller test for Stationary rejects H_0 of the presence of a unit root (non-stationary) for both series at p-value of 0.1%, clearly indicating that the stationary conditions are fulfilled for both assets. The Ljung Box test has also been applied on the squared returns in order to test for the existence of ARCH Effects to be captured by the GARCH model. This follows the concept that time series that exhibit conditional heteroscedasticity (speaking autocorrelation) in the squared return series, show the so called ARCH Effect. This effect is a representation of volatility clustering as outlined earlier in this paper. Appendix 13 & 14 show the volatility clustering of the portfolio, as well as the asset returns with their overall historical development over time. The Ljung Box test on the squared return series yields a p-value of 0% for both indices at all tested 15 lags, which means that we have to reject the null hypothesis of "No-Arch effect" and proved the strong existence of conditional heteroscedasticity. The correlogram of the autocorrelation function in Appendix 4 visualizes the existence of a significant ARCH Effect present in the data of both Dax and Portfolio series. The upper or lower limit under a 1% significance level is also indicated in the respective correlogram. Evaluating the aforementioned statistics clearly indicate, that implementing a GARCH model seems to be the method of choice. Three GARCH models have been developed based on the underlying data. Herein, normal, student t and generalized error distribution have been assumed for the innovation/shock term. Appendix 5 shows in detail the estimated parameters for each GARCH (1,1) model using Maximum Likelihood Estimation (MLE). Looking at the estimated coefficients shows, that in comparison the DAX experiences higher persistency of shocks than the portfolio. Consequently, for the latter, more weight has been assigned on γ causing a faster mean reversion process towards the implied unconditional variances as calculated in Appendix 7 per index and model. The Islamic Portfolio exhibits significantly lower unconditional volatility compared to the DAX estimate (~0.46 and ~1.12%, respectively). Furthermore, it becomes obvious that the β estimate for the DAX is always close to 90% putting the majority of the weight on t_{-1} variance (σ^2_{t-j}). Angelidis, Benos and Degiannakis (2003) found similar parameter estimates for the DAX 30 for the earlier timeframe of 1987 to 2002 (β =0.83, 0.88,0.86; α= 0.13, 0.10, 0.11 for Normal, T, GED distribution, respectively). Islamic portfolio exhibits similar characteristics, however allocating slightly more weights to α (towards the α^2_{t-j} term). Incorporating around 7.5 degrees of freedom demonstrates the heavy tails captured by the t-distribution. It is highly interesting to see that the shape parameters v of 1.41 and 1.24 of the GED distribution calls for a mixture of Laplace (v=1) and normal (v=2) distribution for both samples. However, it tends more towards Laplace to account for the heavy tails. Evaluating the goodness of fit of the estimated models, the AIC, BIC and LLF statistics have been calculated for each distributional assumption. Appendix 8 shows an overview about the conducted calculations. Fitting the GARCH(1,1) to the Islamic tangency portfolio returns shows, that the GED-distrib. marginally exhibits the highest LLF over student t with the lowest AIC & BIC criterion indicating the best fit in-sample. Nevertheless, the t-distribution values are marginally different signalling an equal fit. This is in line with the findings by Angelidis, Benos and Degiannakis (2003) that showed that assumed leptokurtic distributions such as the student-t distribution yield better fits then assuming normal distributions. Köksal (2009) tested various models and demonstrated that the t-distribution was the best performing distribution during the analysis. Furthermore, align with the findings of this study, he elaborated that there are no significant differences between the three tested distributions. DAX statistics suggests the GED distribution to be the best fit for the underlying data with an LLF value 3.177.58 and AIC of -6347.16. As stated before, Kosapattarapim, Lin and McCrae (2011) found that best fit does not always imply best forecasting ability. This hypothesis will later be confirmed when analysing forecasting errors. Wennström (2014) achieves similar results when comparing the MSE of his in-sample models with out-of-sample performance. It is evident the normal distribution seems to be the worst fit for both data series. Conducting the Ljung-Box test on the models (Islamic & DAX) standardized residuals shows, that the ARCH effect has been adequately captured by the models. P-values at all lags >1% significance level, always indicating the effectiveness of the model and the acceptance of H_0 at all lags. Therefore, no significant autocorrelation is left in the squared standardised residuals. The standardized residuals do not exhibit statistically significant differences from the assumed mean of 0 and a standard deviation of 1 after standardization with p - values > 0.01 at all distributions for both data series. Portfolio analysis further shows, that the standardized residuals still exhibit *significant* excess skew for the student t distribution only. Evaluating the QQ-Plots of the standardized residuals shows, that the excess skewness that is left for the Portfolio of -012, -0.23, -0.18 (Normal, Student t, GED, respectively) can be mainly attributed towards a few isolated outliers in the sample. Skewness for the DAX is not significantly different from zero. Small amount of excess kurtosis is calculated with 0.29 & 0.13 for both assets above the targets t-distribution kurtosis (Portfolio and Dax, respectively).

4.1 In-Sample Model Performance

EWMA and GARCH models with respective distributions have now been applied to obtain the one-step ahead volatility forecast for the in-sample period starting on the 07.11.2013 until the 07.11.2017. The model is using the estimated set of parameters and always considers the actual realized volatility of t-1 to forecast volatility at time t based on the estimated parameters. The obtained values have then been compared to the actual, realized volatility at the given day. Calculating the realized daily volatility is a complex issue being constantly discussed in academic literature. Köksal (2009) calculates 5-minute intraday variances to finally achieve a realized daily volatility estimate. Nevertheless, this procedure would require extensive data availability for the analysed assets. In case of the Islamic portfolio it is not possible to obtain such data. Wennström (2014) proposes the High-Low proxy method developed by Parkinson (1980) using the intraday range between the highest and lowest price. Again, this method requires extensive data not being available for the created asset. Poon (2008) elaborates that squared daily returns deliver less accurate results than using high-frequency data, however have been used in many studies that could not access significant high-frequency data sets. Consequently, this paper follows the concept of Lindberg (2016) of taking the squared log returns as a proxy although being a noisy estimate. In contrast to the estimated Goodness of fit statistics, the t-distribution shows the highest RMSE within the sample period (0.0041) for the Portfolio. This opposes our first assumption based on AIC and LLF. It proves that the statement regarding forecast ability made by Kosapattarapim, Lin and McCrae (2011) also holds within this study. The RMSE is the lowest for the normal followed by the GED distribution. (0.0039 & 0.0040 respectively). Surprisingly, the EWMA analysis presents the lowest error statistic for the Islamic portfolio with 0.0032. This confirms the results found by Minkah (2007), who found the EWMA to be the best estimation method throughout all five indices that were tested in-sample. This raises the assumption that the EWMA will also be the one with the least failures in Kupiec's unconditional coverage test for the out of sample analysis. This hypothesis will be tested later in this paper. Although rejecting the H_0 for the Normality test for the DAX, the RMSE and MSE both indicate that the best GARCH fit is achieved assuming a normal distribution (0.0102). The T-Distrb. for the DAX shows marginally higher results for both parameters (0.0103). Again, the EWMA approach delivers the lowest estimate of 0.00783 for the DAX.

4.2 Out-of Sample Model Performance

The developed models have now been applied on the out-of sample data from 08.11.2017 - 07.11.2017 which is equivalent to 252 trading days. Goodness of fit for the portfolio out-of sample period indicates, that the student t model that has been previously estimated, delivers a better fit than the GED-distribution out-of sample. RMSE analysis demonstrates the outperformances of EWMA with results of 0.0029, 0.0035,0.0036, 0.0036 for EWMA, Norm, T-Dist and GED, respectively (Appendix 11). The same is observed for the DAX in which the t-distribution shows a marginally better fit than GED distribution out-of sample (-1647.28 to -1642.86, respectively). Although forecasting errors being slightly higher for the out-of sample period, it becomes obvious that for the DAX the ranking of the models with respect to RMSE calculation did not change. Again, EWMA with 0.0056 performs the best compared to 0.0073, 0.0074, 0.0074 (Norm, T-dist, GED respectively). Having found the EWMA to deliver the smallest forecasting error for both assets sounds appealing as a recommendation for the Islamic investor in order to get the most accurate forecast. However, the significance level for VaR forecasting failures should not be neglected. The smallest forecasting errors might infer steady violations of VaR forecasts by actual losses exceeding the predicted ones.

4.3 Out-of-Sample Performance / Forecasting and Backtesting Value at Risk

VaR estimates have been calculated by considering the estimated volatilities of each model, multiplying it with the respective distributional assumption for the 5% quantile, times the assumed portfolio value of 100,000 Euro of the investor. Conducting Kupiecs unconditional coverage test as well as Christoffersen's independence test allows, to backtest the gathered VaR estimates at different significance levels for the out-of sample results. Rejection intervals being calculated in Appendix 9. Following Hypothesis is tested: EWMA will show the best estimates for VaR as it shows the best forecasting accuracy in out-of sample testing. The Islamic Portfolio VaR Backtesting results indicate (see Appendix 6.), that both EWMA and GARCH-Norm are significant at 95% confidence for 125 and 252 days (EWMA also at 97.5%). T & GED distribution fail to be significant at 95% for 252 trading days only, caused by 1 exception more than the threshold. Nevertheless, VaR estimates out-of sample are only significant at a 90% level. Failures constantly being higher than expected. Consequently, models are underestimating risk. Where, the EWMA shows less failures than the GARCH models almost being significant at 95% conf. for the 125 days test (Portfolio). Islamic stocks often exhibit less volatile behaviours caused by shariah induced investing restrictions (eg. sector restrictions; less than 33% of debt to market cap). A possible explanation would be: The EWMA reacts significantly slower to changes in volatility as inferred by the 0.98 lambda. The GARCH (1,1) however undergoes a complex estimation procedure implying more room for estimation errors. This is a key insight as usually complex models are regarded as being more efficient. The finding is confirmed by EWMA showing the lowest RMSE between actual loss and VaR estimates, outperforming for the Islamic Portfolio compared to Normal, t-dist and GED distribution (Appendix 10). Minkah (2007) encountered similar results and elaborates that complex models using more parameters often create higher estimation errors leading to consistently poor out-of sample results. The different interpretation of EWMA being most accurate becomes obvious at the DAX. Here

EWMA produces least failures in-sample, however most failures out-of sample. In contrast to the Portfolio, the lowest RMSE might infer steady VaR estimate exceedance and not the most accurate estimate for the DAX anymore. 252 days test interval with 38 exceptions rejects the EWMA forecast even at a 90% confidence level for the more volatile DAX out-of sample. This confirms the theory of EWMA outperforming for the less volatile Islamic Portfolio. Noticeably, in both assets the calculated VaR estimates fail to perform at 95% significance level out-of sample, only being significant considering the 90% confidence interval (Appendix 6). Nevertheless, as the non-rejection interval ends at 12 failures for 95% confidence for 125 days, out of sample Islamic EWMA performance fails marginally to be significant at 95% (13 failures). Calculating Christoffersen's independence test statistic and comparing it to the critical value of 3.84 (95% conf. quantile of chi-squared distribution, 1df) shows that all GARCH models for both the last 125/252 day intervals for both assets are accepted (test statistics <1 for all assets, Appendix 12). Summarizing, they do not produce statistically relevant failures on two consecutive days. EWMA produces a test statistic of 0.34 & 3.14 for 125/252 days for the Islamic returns, respectively. The test statistics are lower than 3.84, indicating the model's acceptence at a 95% confidence. Consequently, the EWMA model shows a solid performance to be applied by an Islamic investor, to get an adequate estimation for next day's volatility.

4.4 Limitations and Future Research

The results are subject to certain assumptions. The most popular GARCH (1,1) does not account for the scientifically proven asymmetric effect of volatility behaviour. Herein, negative news have higher impact on volatility than positive news. Future research might apply EGARCH models to account for the asymmetrical behaviour of volatility. Additionally, skewed versions of t- and GED distributions as proposed by (Grek,2014) might be used to replicate this study. Furthermore, the realized volatility assumption of taking squared log returns is a noisy approximation. Alternative methods, as outlined in the respective section, might be applied as far as high-frequency data will be made available as required.

5. Conclusion and Recommendation

Estimated GARCH Parameters for the Islamic Portfolio show, that the GED-distribution was calculated with the highest goodness of fit statistics in-sample. Nevertheless, goodness of fit changes out-of sample where the t-distribution achieves marginally higher statistics. Analysis shows, that the EWMA produces the most accurate one-step ahead volatility forecasts (RMSE) for the Islamic portfolio, by at the same time performing at a lowest failure rate (actual loss > predicted VaR). It becomes evident, that this is a special characteristic of a shariah compliant portfolio, as EWMA produces significantly more failures than GARCH models (GARCH T-Dist. with the least failures) for the DAX out-of sample (Kupiec POF). The RMSE statistic for EWMA is the lowest when comparing actual & predicted losses. Failures were calculated to be statistically independent for both assets. Christoffersen Independence test for the EWMA of the Islamic Portfolio over the 125/252 days (Cr. Value 0.34/3.14<3.84) indicating acceptance of the model. Possible reason might be the less volatile and more stable development of shariah-compliant stocks due to their inherent risk nature (<33% of debt to market cap; sector restrictions). In-sample analysis shows that the EWMA produces the least forecasting errors which confirms the findings of Minkah (2007). This study has shown, that less complex methods such as EWMA can produce better out-of sample one-step ahead forecast results, than complex GARCH models. Regarding the statement of Ding and Meade (2010) elaborated in the literature review, that EWMA accuracy "[…] will only come unstuck if the series exhibits SV with a high volatility of volatility", this study confirms the findings by Ding & Meade (2010) and Tse & Tung (1992). EWMA proved to be most accurate for the lower volatility Islamic Portfolio, compared to a higher volatility DAX. Therefore, an investor holding a tangency portfolio of Islamic stock indices with shariah compliant short selling restrictions, should use the EWMA approach to get adequate next day volatility forecasts.

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APPENDICES

Minimum Variance Port. incl short selling	
Expected Return	7.343%
Variance	0.341%
Standard Deviation	5.842%
Sharpe Ratio	1.086

Tangency Port. incl. short selling	
Expected Return	24.183%
Variance	1.248%
Standard Deviation	11.169%
Sharpe Ratio	2.076

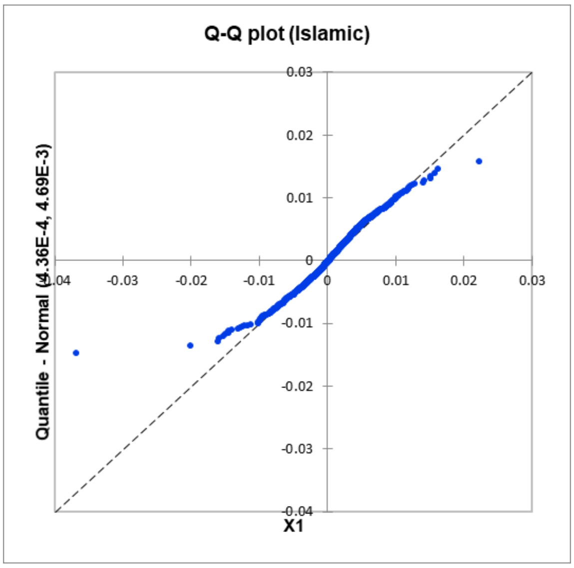
Minimum Variance Port. without short selling	
Expected Return	6.897%
Variance	0.353%
Standard Deviation	5.940%
Sharpe Ratio	0.993

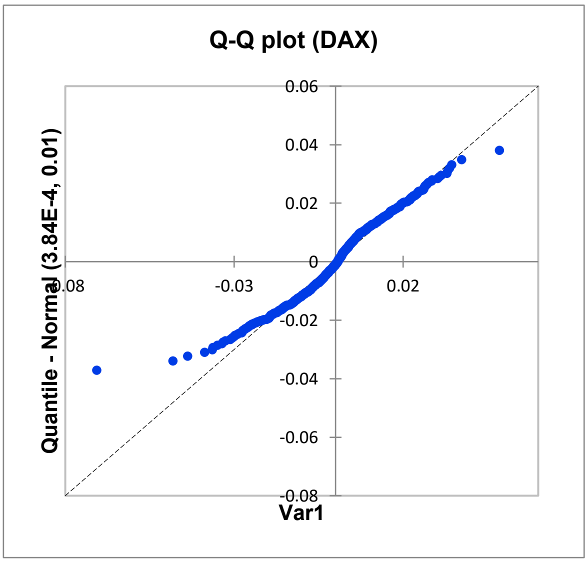
Tangency Port. without short selling	
Expected Return	13.859%
Variance	0.554%
Standard Deviation	7.440%
Sharpe Ratio	1.728

Appendix 1. Comparison of short selling vs non-short selling conditions

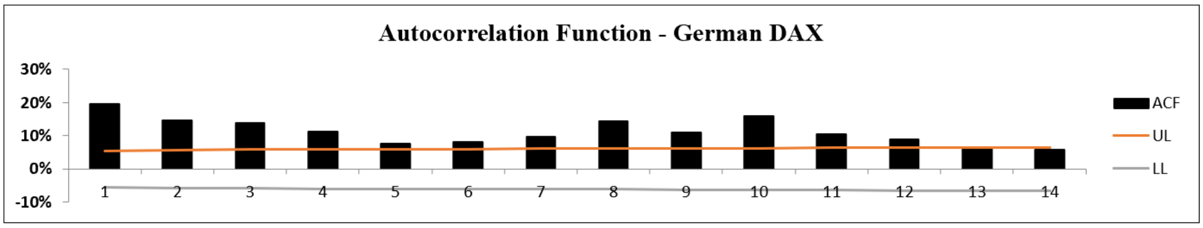
1/N Index vs Tangency Portfolio - Out of Sample			
1/N Index		**Tangency Portfolio**	
Expected Return	-0.577%	Expected Return	0.301%
Variance	0.659%	Variance	0.521%
Standard Deviation	8.118%	Standard Deviation	7.215%
Sharpe Ratio	-0.194	Sharpe Ratio	-0.097

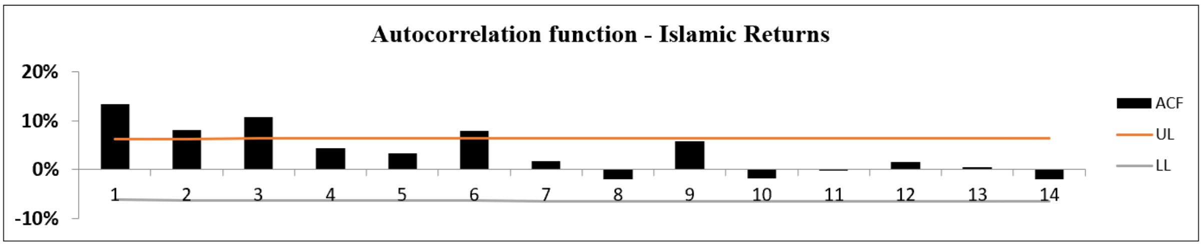
Appendix 2. Out of Sample Portfolio construction, 1/N Index vs Tangency Approach





Appendix 3. QQ-Plots of Islamic Portfolio and Dax Returns





Appendix 4. Correlogram - Exhibition of ARCH Effects (autocorrelation in squared returns) in Islamic Portfolio and German DAX

Islamic Portfolio

GARCH(1,1)	
Param	Value
μ	0.0006631
α0	0.0000031
α1	0.1328268
β1	0.7282753

GARCH(1,1) & t-dist(v)	
Param	Value
μ	0.0005694
α0	0.0000016
α1	0.0773836
β1	0.8506466
ν	7.590

GARCH(1,1) & GED(v)	
Param	Value
μ	0.0005916
α0	0.0000021
α1	0.0991507
β1	0.8065405
ν	1.41

German DAX

GARCH(1,1)	
Param	Value
μ	0.0007941
α0	0.0000023
α1	0.0826718
β1	0.8990837

GARCH(1,1) & t-dist(v)	
Param	Value
μ	0.0005801
α0	0.0000014
α1	0.0850549
β1	0.9025051
ν	6.979

GARCH(1,1) & GED(v)	
Param	Value
μ	0.0008994
α0	0.0000017
α1	0.0895957
β1	0.8975000
ν	1.24

Appendix 5. Estimated GARCH parameters under different distributional assumptions

OUTOF SAMPLE
No. of Actual Loss Exceedance of Daily VaR Estimate ISLM

Days	*EWMA*	*GARCH_Norm*	*GARCH_T-DIST*	*GARCH_GED*
125	13	17	15	17
252	24	29	29	32

OUTOF SAMPLE
No. of Actual Loss Exceedance of Daily VaR Estimate DAX

Days	*EWMA*	*GARCH_Norm*	*GARCH_T-DIST*	*GARCH_GED*
125	21	18	15	18
252	38	33	29	32

INSAMPLE
No. of Actual Loss Exceedance of Daily VaR Estimate ISLM

Days	*EWMA*	*GARCH_Norm*	*GARCH_T-DIST*	*GARCH_GED*
125	7	10	10	11
252	12	19	22	22

INSAMPLE
No. of Actual Loss Exceedance of Daily VaR Estimate DAX

Days	*EWMA*	*GARCH_Norm*	*GARCH_T-DIST*	*GARCH_GED*
125	10	16	16	16
252	17	32	33	33

Appendix 6. No. of actual loss exceedance of forecasted VaR estimates (95% confidence)

Implied unconditional variance estimates			
	Norm	**T dist**	**GED**
Dax	1.124%	1.049%	1.140%
Islamic	0.472%	0.466%	0.467%

Appendix 7. Unconditional Variance Estimates based on estimated GARCH parameters

Islamic Portfolio			
	Norm	**T-Dist**	**GED**
LLF	3993.35	4011.24	4011.95
AIC	-7980.70	-8014.49	-8015.89
BIC	-7965.969	-7994.839	-7996.246

German DAX			
	Norm	**T-Dist**	**GED**
LLF	3147.17	3170.47	3177.58
AIC	-6288.34	-6332.94	-6347.16
BIC	-6273.59	6313.27	-6327.49

Appendix 8. Goodness of fit statistics for DAX and Islamic Portfolio

		Rejection Interval per Days	
Critical value	***Confidence Level***	***125d***	***252d***
6.635	0.99	0<X<5	0<X<8
5.024	0.975	0<X<8	2<X<12
3.841	0.95	2<X<12	6<X<21
2.706	0.9	6<X<20	17<X<34

Appendix 9. Calculation of non-rejection interval of Kupiec POF-test (Proportion of failure)

IN-SAMPLE

RMSE

Days	***EWMA***	***GARCH_Norm***	***GARCH_T-DIST***	***GARCH_GED***
125	0.00652504	0.007322692	0.00794445	0.007324053
252	0.00701248	0.007896596	0.008610355	0.007929092

Out-of Sample

RMSE

Days	***EWMA***	***GARCH_Norm***	***GARCH_T-DIST***	***GARCH_GED***
125	0.00743891	0.008741059	0.00964539	0.008847514
252	0.00772191	0.008858995	0.009773902	0.008963564

Appendix 10. RMSE of Actual Loss vs predicted VaR forecast

Islamic Port. RMSE, Realized vs Predicted volatility

	EWMA	Norm	T-Dist	GED
In Sample	0.003270	0.003994	0.004124	0.004077
Out Sample	0.002906	0.003522	0.003644	0.003600

German DAX RMSE, Realized vs Predicted volatility

	EWMA	Norm	T-Dist	GED
In Sample	0.007834	0.010290	0.010345	0.010328
Out Sample	0.005677	0.007375	0.007429	0.007412

Appendix 11. RMSE of Realized vs Predicted volatility

Islamic Portfolio

125 days	EWMA	NORM	T-DIST	GED
π0	0.09821	0.13889	0.12727	0.13889
π1	0.15385	0.11765	0.06667	0.11765
π	0.10400	0.13600	0.12000	0.13600
LR(Ind)	0.34790	0.05834	0.52612	0.05834

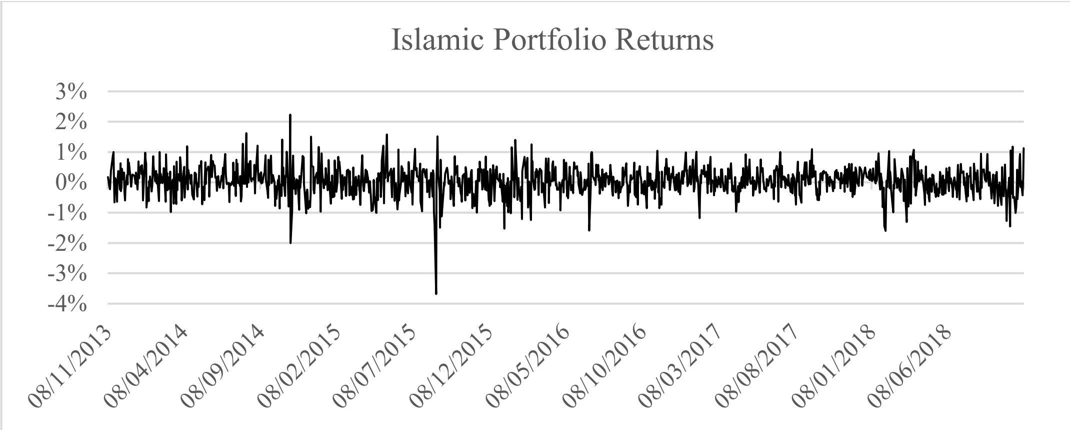
252 days	EWMA	NORM	T-DIST	GED
π0	0.08333	0.11659	0.12108	0.13182
π1	0.20833	0.10345	0.06897	0.09375
π	0.09524	0.11508	0.11508	0.12698
LR(Ind)	3.14334	0.04471	0.77264	0.39226

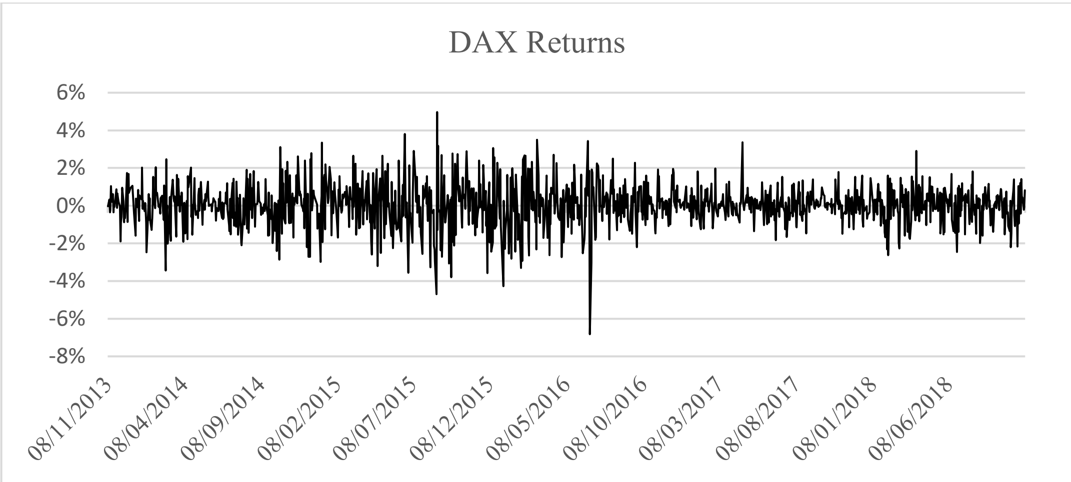
German DAX

125 days	EWMA	NORM	T-DIST	GED
π0	0.16346	0.14953	0.12727	0.14953
π1	0.19048	0.11111	0.06667	0.11111
π	0.16800	0.14400	0.12000	0.14400
LR(Ind)	0.08881	0.19562	0.52612	0.19562

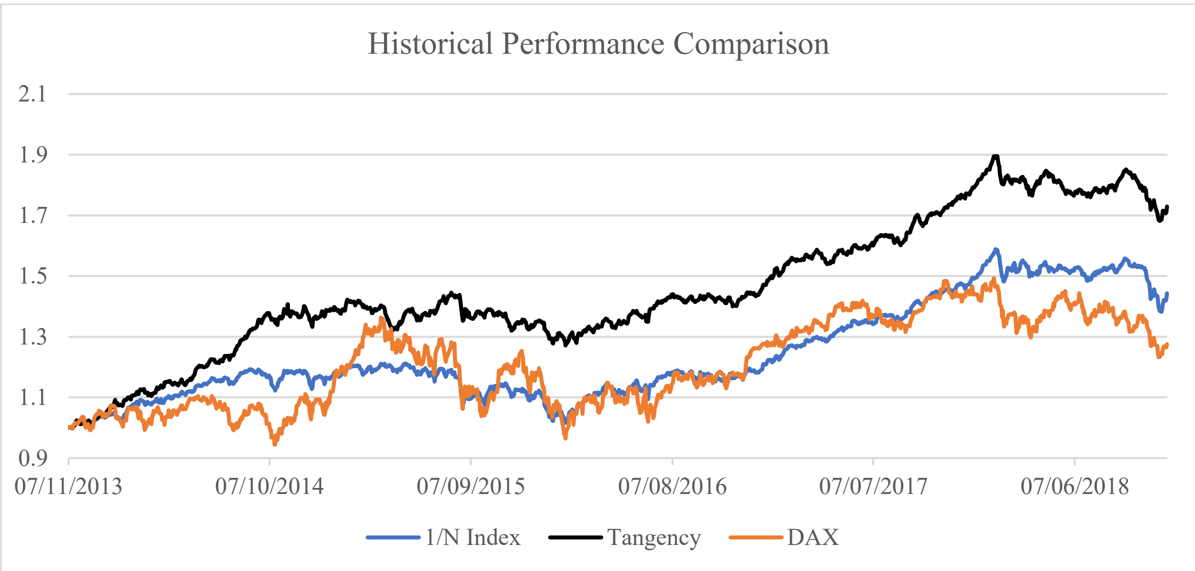
252 days	EWMA	NORM	T-DIST	GED
π0	0.14019	0.13699	0.12108	0.13182
π1	0.21053	0.09091	0.06897	0.09375
π	0.15079	0.13095	0.11508	0.12698
LR(Ind)	1.15480	0.58216	0.77264	0.39226

Appendix 12. Christoffersen's Test for Independence of VaR forecasting failures





Appendix 13. Returns Islamic Portfolio and DAX



Appendix 14. Historical Return series of German Dax, Islamic Tangency Portfolio and 1/N Index approach (Cumulative Investment of 1 Euro over time)