

Approval Rules for Sequential Horizontal Mergers

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Abstract

Merger approval decisions lie at the heart of competition policies. Farrell and Shapiro (1990) presented a model stating safe harbor rules for merger approval. However, in the presence of sequential mergers, computation of the sufficient external effect criterion for each merger may not be possible as for the second merger it will depend on the equilibrium emerging after the first merger. If the mergers are close enough in time, the second merger must be evaluated without the knowledge of the equilibrium point after the first merger. Two alternatives are proposed: joint merger evaluation and independent merger evaluation. The decision errors (too much approval or rejection) are identified for each of the alternative rules. It is shown that joint merger evaluations generate too much rejections and independent evaluations originate too much approval of mergers.

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1 Introduction

Merger approval lies at the heart of competition policies and a fair amount of literature has been devoted to the theory and practice of mergers. In particular, Farrell and Shapiro (1990) have laid down a model of mergers intended to be useful for policy purposes. Their main assumption regarding merger policy is that internal gains to the merging entities are difficult to assess. Therefore, they propose safe harbor rules based on external effects to the merger. Internal effects are presumably positive, otherwise firms would not merge. Hence, a sufficient condition for a welfare increasing merger is a positive effect over all other market participants.

The application of the safe harbor rule usually requires knowledge about market shares and demand elasticity (for usual parametrizations of demand and cost functions). However, when more than one merger occur in the same market almost simultaneously, decisions regarding different mergers must be made within a short time period. The economic analysis of mergers must then be modified accordingly.

In particular, a sequence of mergers makes unavailable the market shares associated to the equilibrium situation after the first merger. It depends on the internal gains of the first merger, which are by assumption not known by the merger authority. Thus, the information needed for application of the criterion proposed by Farrell and Shapiro is not available. Merger evaluators are not able to compute the interim situation. Two natural possibilities are then the joint evaluation of both mergers and the evaluation of each merger *per se*. The problem is how to evaluate a new merger proposal when just a short lapse of time since the last merger has occurred.

The aim of the paper is to investigate the effects of sequential decisions on merger policy, not presenting a theory of sequential mergers.¹ Moreover, firms and the antitrust authority are not able to anticipate whether subsequent mergers will occur.

We make the crucial assumption of myopic behavior of merger authorities with respect to the appearance of further mergers. This assumption is the natural extension of the idea that merger authorities cannot observe the internal gains of a merger. Forecasting of future mergers would have to be based on the knowledge of gains from the merger, that is, of internal gains.

The "decision errors" involved in a sequential use of the "external effect" criterion of Farrell and Shapiro (1990) for sequential merger approval are also investigated.

The lack of information on interim market shares results in a sufficient condition con-

¹For a model in that vein, see Nilssen and Sorgard (1995).

siderably more restrictive than the sufficient conditions that would be in place if market equilibrium after the first merger was observed. Independent evaluation of each merger is, on the other hand, too lenient with mergers.

The paper is organized in the following way. Section 2 briefly reviews the model of Farrell and Shapiro (1990). Next, Section 3 presents the analysis of simple rules for sequential merger evaluation. Section 4 reports the application of the rules to a sequence of two mergers in the Portuguese banking industry. Finally, Section 5 concludes.

2 The Farrell–Shapiro model

Since we depart from the work of Farrell and Shapiro (1990), a brief presentation of their model is warranted. The model of Farrell and Shapiro (1990) considers a welfare measure given by

$$W = \int_0^Q P(\bar{Q})d\bar{Q} - \sum_i c_i(q_i) \quad (1)$$

where $P(Q)$ is the inverse demand function and c_i is the cost function of firm i . A merger is then seen as a sequence of infinitesimal mergers. Under Cournot behavior and some conditions upon demand and cost functions, Farrell and Shapiro show that a condition for a positive external effect of a merger (sufficient to give approval to the merger) has the form:

$$\sum_{i \in I} s_i = s_I < \sum_{i \in O} \lambda_i s_i \quad (2)$$

where s_i is market share of firm i , I denotes the set of firms participating in the merger (insiders). Similarly, the set O denotes the set of firms not participating in the merger (outsiders). Finally, $\lambda_i = (P' - q_i P'') / (P - c_i'')$. The value λ_i is closely related to the slope of the reaction function ($dq_i = -\lambda_i / (1 + \lambda_i) dQ_{-i}$).² If marginal cost is non-decreasing and demand is sufficiently convex ($P'' > -qC''/q_i$) then $0 < \lambda_i \leq 1$. For linear demand and cost functions, $\lambda_i = 1$.

3 The external criterion for sequential mergers

Suppose now that two mergers in the same industry are put forward. The second merger is presented to authorities for approval after the decision on the first merger has been delivered. Economic authorities are not able to predict whether another merger will be proposed after the decision on the first merger.³

²The conditions for the global effect to have the same sign of the marginal effect are stated in Farrell and Shapiro (1990, p. 116): $P'' \geq 0, P''' \geq 0, c'' \geq 0, c_i''' \leq 0$.

³This is consistent with the economic authority not knowing internal gains to the mergers.

The application of the Farrell-Shapiro criterion (condition (1)) to two successive mergers results in the following. A sufficient condition for the approval of the first merger is

$$s_{I_1}^0 < \sum_{i \in O_1} \lambda_i^0 s_i^0 \quad (3)$$

where superscript 0 denotes the initial situation before any merger, superscript 1 will denote the equilibrium point after the first merger and before the second merger – the intermediate equilibrium. I_j denotes the set of firms participating in merger j and the set O_j the set of firms not participating in merger j . The following notation is used to describe market shares of merging firms: $s_{I_j}^t = \sum_{i \in I_j} s_i^t$, $j = 1, 2$, $t = 0, 1$. Sequential application of condition (1) means that the second merger evaluation will have as starting point the equilibrium resulting from the first merger, given that such merger is approved (otherwise, the initial situation is the reference point). The condition is

$$s_{I_2}^1 < \sum_{i \in O_2} \lambda_i^1 s_i^1 \quad (4)$$

For simplicity of exposition we assume that mergers are mutually exclusive. The decision rule is:

Rule 1

1. Approve first merger if satisfies the Farrell-Shapiro criterion;
2. If the first merger was approved, approve second merger if satisfies the Farrel-Shapiro criterion, where evaluation is made at the equilibrium point resulting from the previous merger.
3. If the first merger was rejected, approve second merger if the Farrell-Shapiro criterion is satisfied.

The interesting case occurs when the first merger is approved and market equilibrium is not yet restored. The decision on the second merger involves less knowledge about market conditions than the first decision.

Lack of knowledge on internal gains of a merger are essential to the analysis. In a standard Cournot oligopoly model, the order of merger is irrelevant. The comparison to be performed is between the final equilibrium (with both mergers) and the initial equilibrium (no merger) for the decision of approving both mergers or none. However, from a policy perspective, it is often the case that merger proposals (and approval decisions) are done sequentially and under more limited knowledge.

If the merger authority knows that two mergers are to be proposed, the choice to be made is to pick one of the following outcomes: (i) no merger is allowed; (ii) one merger is allowed, the other is rejected; and (iii) both mergers are allowed. Knowledge of the internal gains would allow straightforward computation of a defined objective function of the antitrust authority. The issue, however, is that such knowledge on the internal gains of mergers is not available.

The sequential nature of decision making in merger approval procedures may reveal (indirectly) information on the first merger internal gains, by means of equilibrium market shares after the first merger. In the absence of such information (interim equilibrium market shares) the problem under sequential decision-making includes a commitment of the economic authority to the first-merger decision. For mergers put forward in close sequence in time, the equilibrium market shares after the first merger are not observable, and the observed market shares are meaningless, as no dynamics of adjustment are specified). Thus, we retain the assumption that the only 'good' information available are the pre-merger market shares.

Since the merger authority cannot foresee future mergers, a sequential Farrell-Shapiro type of rule is the optimal one to follow. When firms anticipate the likely effects of decisions to merger upon future mergers, Rule 1 needs not to be a sufficient rule. The first merger proposed may yield a negative internal gain, contradicting the merger authority's assumption that a merger if proposed must have a positive internal effect. Nevertheless, at the point of evaluation of the first merger, the merger authority does not anticipate whether a future merger will occur or not. Hence, the best guess is that the internal effect of the first merger is positive.

Suppose a second merger is notified to the antitrust authority, after the first merger. At this point in time, the decision on the first merger is 'sunk' and the second merger should be evaluated *per se*.

Sequential mergers that are sufficiently spaced in time will allow for the computation of both conditions. However, there will be circumstances where mergers are sufficiently close in time to render such independent evaluation possible, as the intermediate equilibrium is not observed. So, this is a problem faced by merger authorities.

For an authority following a Farrell-Shapiro type of criterion, two possibilities, which do not rely on knowledge over the intermediate market equilibrium, are then at hand. First, treat both mergers jointly and impose on the second merger a sufficient rule for a positive external effect to *both* mergers. This case will be called joint merger evaluation.

The second possibility is to treat each merger independently, using the initial situation as

the benchmark. This is termed independent merger evaluation. We deal now with each case in turn.

3.1 Joint merger evaluation

Assume the first merger was approved. A possibility for antitrust authorities under the limited knowledge assumption imposed is to perform a joint evaluation of mergers in the moment of approval of the second merger. That is, to look at the external effect to both mergers, and approve only if the effect is positive.

Note this constitutes a choice rule for sequential mergers which implicitly assigns to the second merger the difference between the first-merger external effect and the joint external effect.

The joint evaluation of the two mergers generates the sufficient condition stated in the next proposition.⁴

Proposition 1 *A sufficient condition for both mergers to originate a welfare gain is:*

$$\sum_{i \in I_1} s_i^0 + \sum_{i \in I_2} s_i^0 \leq \sum_{i \in (O_1 \cap O_2)} s_i^0 \lambda_i^0 \quad (5)$$

Since the merger authority does not know the internal gains of a merger, it is also unable to anticipate whether another merger will occur or not. The authority rule based on this sufficient condition has the following form:

Rule 2

1. For the first merger, use the Farrell-Shapiro condition.
2. If the first merger was approved, the second merger is approved only if the joint evaluation external effect is positive.
3. If the first merger was rejected, use the Farrell-Shapiro criterion to evaluate the second merger.

Comparison with the case where the intermediate equilibrium point is observed reveals that potential decision errors (according to the sufficient criterion and in top of 'sufficiency mistakes') occur in the second merger decision only. The next proposition shows that the

⁴All proofs are presented in appendix.

decision error works against second merger approval, stating that Rule 2 implies a more conservative policy than Rule 1. By more conservative policy we mean that under the sequential rule there will be approved mergers that are rejected if evaluated jointly with the first merger.

Proposition 2 *The joint merger evaluation rule (Rule 2) is more restrictive in the approval of the second merger than the sequential rule (Rule 1).*

This proposition shows that the joint external effect sufficient condition would reject mergers that are to be approved under the sequential rule. Under myopic behavior of agents this means that policy errors will be in the same direction: second mergers are rejected too often.

The more conservative merger policy in Rule 2 can be easily seen by noting that the joint evaluation criterion disregards the positive welfare cross effect between the two new entities (as one set of firms contracts output, firms participating in the other merger expand production, which increases the positive external effects). This provides the source of over-rejection of second mergers.

3.2 Independent merger evaluation

The second natural possibility at hand for merger authorities is to treat each merger independently. The benchmark equilibrium for approval decisions is the initial one. The rule followed is

Rule 3

1. Approve first merger if

$$s_{I_1}^0 < \sum_{i \in O_1} \lambda_i^0 s_i^0 \quad (6)$$

2. Approve second merger if

$$s_{I_2}^0 < \sum_{i \in O_2} \lambda_i^0 s_i^0 \quad (7)$$

The main difference to Rule 1 is that the initial equilibrium is used for evaluation of the second merger instead of the intermediate equilibrium.

Straightforward comparisons are not possible, since Rule 3 assumes that no merger has occurred before and Rule 1 includes in the set of outsiders to the second merger the firm resulting from the first merger. Rule 3 implicitly assumes a behavior for firms participating in the first merger that clearly does not match their actual response (as they have merged). The appeal of the rule is to rely only on observable market shares. The difference between

the threshold in Rule 1 and Rule 3 depends on the internal gains to the first merger, which are unknown.

If we are willing to impose further assumptions, some special cases allow for a more precise characterization. For a constant λ , Rule 1 and Rule 3 imply the same decision threshold for insiders' market share for the combined market share of merging entities.⁵

Proposition 3 *Under the assumption of constant λ , Rule 3 and Rule 1 for the approval of sequential mergers have the same critical value $\lambda/(1 + \lambda)$.*

The threshold value is therefore the same for both rules. Since the market share of insiders to the second merger increases after the first merger in a Cournot oligopoly, usage of initial market shares for the application of the independent valuation criterion generates a more permissive rule in the approval of second mergers than the sequential rule.

The decision bias is the opposite direction of the one in Rule 2. Comparison of the two alternative rules is presented in the next proposition.

Proposition 4 *Approval of the second merger under Rule 2 (joint evaluation) is more restrictive than under Rule 3 (independent valuation).*

For a market share of insiders to the second merger, s_{I_2} , in the interval between thresholds for those two rules, the joint merger evaluation rule rejects mergers that are allowed to proceed under the independent evaluation case. This means Rule 3 is more permissive than Rule 2.

In the linear oligopoly case,⁶ a stronger result can be shown.

Proposition 5 *Suppose two mergers (mutually exclusive) are proposed in sequence. In the linear oligopoly model, and according to the external effect criterion, one of mergers is always approved.*

The specified rules leave out of the *safe heaven* several cases. For example, take the case where each merger would be accepted if no other merger existed, but both mergers are rejected. The issue then is which merger to approve. This case cannot be addressed by rules based on the infinitesimal merger concept. The endpoints of each merger must be compared. Therefore an important class of sequential merger situations is ruled out in the specified rules.

This fact reinforces the limitations of the approach. Nevertheless, since sufficiency is retained, clearance under the rules specified constitute the right decision. Rejections may, on the other hand, give room for challenge to the decision.

⁵A constant value of $\lambda = 1$ results, for instance, in a linear oligopoly.

⁶Linear demand and cost functions, as in Salant, Switzer and Reynolds (1983).

Table 1: Market Shares

	LOANS	DEPOSITS
Caixa Geral de Depósitos (CGD)	21,35	22,56
Banco Português do Atlântico (BPA)	11,55	11,24
Banco Totta & Açores (BTA)	9,58	8,92
Banco Espírito Santo (BES)	7,35	6,72
Banco Comercial Português (BCP)	6,9	7,07
Banco de Fomento e Exterior (BFE)	5,1	5,04
Banco Pinto & Sotto Mayor (BPSM)	3,46	5,52
Banco Nacional Ultramarino (BNU)	3,86	3,54
Banco Comercial de Macau (BCM)	4,06	3,81
Banco Português de Investimentos (BPI)	2,63	3,62
Banco Borges & Irmão (BBI)	2,23	3,28
União de Bancos Portugueses (UBP)	2,82	2,99
Banco Fonecas & Burnay (BFB)	1,83	3,04
Crédito Predial Português (CPP)	3,32	2,71
Montepio Geral (MG)	2,17	1,62
Banco Comércio e Indústria (BCI)	1,49	0,80
Deutsche Bank de Investimento (DBI)	0,54	0,92
Banco Internacional do Funchal (BANIF)	1,20	1,00
Credit Lyonnais Portugal (CL)	0,65	0,63
Barclays Bank (BARCL)	0,55	0,61
Banco Internacional de Crédito (BIC)	1,23	0,68
Banco Bilbao Vizcaya (Portugal) (BBV)	0,69	0,59
ABN Amro (ABN)	0,47	0,48
Banco Mello (BMELLO)	0,67	0,38
CISF - Banco de Investimento (CISF)	0,47	0,48
Banco de Investimento Imobiliário (BII)	0,84	0,27
Sample Total	97,01	98,52

4 An application

The problem highlighted in this paper can be illustrated with the recent experience of the Portuguese banking sector.

Within a brief period of time, two important acquisitions occurred. The first one was Government induced, the other took place in the market.⁷ The first acquisition has created an economic group including the following Portuguese banks: BTA, CPP and BPSM.⁸ The second one was a takeover of BCP over BPA, which creates an economic group constituted by BCP, BII, CISF, BPA and BCM. Another bank controlled by BPA, UBP is now under control of a smaller partner of BCP in the takeover bid, BMELLO.

⁷Although a previous attempt of takeover has been held back by the Government.

⁸For the meaning of acronyms, see Table 1.

Table 2: Market shares of merging firms

	Loans	Deposits
$s_{f_1}^0$ (= CPP + BTA + BPSM)	16.36%	17.15%
$s_{f_2}^0$ (=BCP + BPA + BII + BCM + CISF)	23.72%	22.57%
$s_{f_1}^0 + s_{f_2}^0$	40.08%	39.57%

Application of the rules presented earlier still requires two steps. First, banks arguably operate in two distinct markets, loans and deposits.⁹ Rules for merger approval must then be extended to a multimarket context. Only in the case of approval in both markets can we guarantee sufficiency of the rule (again, remember that the analysis is based on the use of safe harbor rules).

If the acquisitions were to be blocked according to data for one market but not in the other market, then full effects must be computed in order to assess the trade-off between positive effects in one market and negative effects in the other market.

The second step is to specify demand and cost functions (more precisely, the value of λ_i).¹⁰ In this respect, two alternative assumptions are considered. The first one is linear demand and cost functions, rendering $\lambda_i = 1$. The second one is constant-elasticity demand and constant costs, giving $\lambda_i = 1 - s_i(1 + 1/\epsilon)$.¹¹ Market shares according to loans and deposits on June 1994 are presented in table 1. Computation of market shares of merging banks can be found in Table 2.

For linear functions, both mergers should be cleared in a Cournot oligopoly model, as the critical threshold for the joint evaluation rule is 50%. Independent valuation of mergers naturally corroborates this decision. The linearity assumption can be seen as first-order approximation to the underlying true functions. Given the magnitude of the merging parties, this is probably not a good characterization.

Resorting to the other set of assumptions, the constant-elasticity demand and linear cost functions, table 3 presents the minimum value of demand elasticity that guarantees the sufficiency criterion for joint approval of mergers.

Thus, for elasticities greater than the critical value computed, one should approve both mergers. Although no estimate of the market elasticity of demand of loans is available, critical values under independent valuation are low and suggest that each merger seen in isolation

⁹See Pinho (1994) for a theoretical discussion. Barros and Leite (1995) present some empirical evidence supporting this view.

¹⁰In a more accurate way, one should talk about demand for loans and supply of deposits. Deposits supply works in a symmetric way. To adapt the analysis to this interpretation is straightforward.

¹¹This case does not respect the condition $P''' \geq 0$, but it is possible to show that has the desired characteristics. See Farrell and Shapiro (1990: p.119).

Table 3: Critical elasticities

	Loans	Deposits
Joint Evaluation	0.42	0.45
Independent Evaluation		
first merger	0.13	0.15
second merger	0.15	0.16

should be approved.

As to the deposits market, some order of magnitude of the elasticity can be obtained from Pinho (1995). The estimated supply elasticity of deposits is approximately 0.144. This means that under independent evaluation each merger is rejected, although we are deciding on the margin. On the other, joint evaluation clearly rejects the decision to approve both mergers.

Thus, this approach reveals that the sequence of mergers should have been treated more carefully in its competitive effects, as it does not pass the sufficiency criteria. Of course, this does not imply immediate rejection of the second merger.

A final remark on the example is in order. The implicit assumption of homogeneous products is probably too strong. Earlier evidence (Barros, 1994; Barros and Leite, 1995) suggest that spatial, localized, competition through branches is an important element in banking competition. The figures presented serve mainly illustrative purposes.

A more collusive outcome than Nash behaviour to start with favors the approval of mergers, as some of the anticompetitive effects are already in place. However, tacit agreements between banks are usually facilitated by a smaller number of players. The analysis developed does not provide any measurement of this ability to sustain tacit collusion, but this possibility is to be taken into account in decisions regarding mergers.

5 Conclusion

Merger evaluation received an important contribution from the work of Farrell and Shapiro (1990). However, in the presence of sequential mergers, computation of the sufficient external-effect criterion for each merger may not be possible as for the second merger it will depend on the equilibrium emerging after the first merger. Two alternative rules relying on observed market shares were investigated: joint merger evaluation and independent merger evaluation.

The starting assumption of the analysis was that the antitrust authority has no information regarding the size of the internal effect of the merger. The very same assumption prevents the calculation of the bias sign (except for the special case of linear oligopolies). The application of the simultaneous merger rule yields a more demanding sufficient condition than the one

necessary for the approval of the first merger.

The divergence between the sequential application of the one merger rule and the joint evaluation merger rule results because the latter criterion foregoes the cross effects between the two merging entities, which are positive. In the joint evaluation there is no account for the effect from one merged entity over the other. In a sequential merger rule both effects are included.¹² Hence, the simultaneous merger criterion works against the approval of the mergers.

Joint evaluation means that the internal gains disregarded in the analysis are bigger. Also, the external effect, at the margin, will be smaller as fewer firms are included in it.

As to independent merger evaluation, no comparison with the sequential application of Farrell and Shapiro rule can be made. Ambiguity of the bias is not resolved by resorting to particular functional forms (that is, for explicit computation of λ_i). This is so because comparisons need the knowledge of the intermediate equilibrium, which in turn is determined by the internal gains to the merger. The initial position for the second merger evaluation contains information about the internal effects of the first merger (as the absolute change that gives rise to market shares prior to the second merger matters).

The rules presented for approval of sequential mergers were based on myopic perceptions of both firms and the merger authority. The myopic assumption on the behaviour of antitrust authorities can be discussed. One may argue that economic authorities do have some idea about possible future mergers when they decide on a merger case. The logical step in economic modelling would be to assign a probability distribution over the set of all possible mergers in the industry. This seems, however, to ask for very strong information requirements for the economic authority.¹³

A simple 'rule of thumb', and probably one that corresponds to certain cases in reality, is to say that each merger is evaluated on its own. Here, a somewhat more sophisticated alternative was explored.

The analysis of sequential merger rules when firms and the merger authority have perfect foresight over future mergers raises more delicate problems, as strategic motives for merger permeate the analysis.¹⁴ Such interaction should be pursued in future research.

¹²It is nevertheless always required that firms behave like Cournot competitors.

¹³In the limit, the merger authority should forecast until the end of times all the mergers that are going to be proposed. The same is true for firms.

¹⁴See Nilssen and Sorgard (1995) for a general analysis of such motivations for merger.

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Appendix

Proof of Proposition 1 Welfare measure is defined as:

$$W = \int_0^Q P(\bar{Q})d\bar{Q} - \sum_i c_i(q_i) \quad (8)$$

Both new firms will have an incentive to reduce their quantity (in Cournot oligopoly). Total quantity in the market decreases.¹⁵ The external effect to both mergers is given by

$$dW - d\Pi_{I_1} - d\Pi_{I_2} = \frac{\partial P}{\partial Q} Q \left(\sum_{i \in O_1 \cap O_2} \lambda_i s_i - (s_{I_1} + s_{I_2}) \right) dQ \quad (9)$$

This external effect to both mergers ignores the cross-effects between mergers, as firms participating in one merger are outsiders to the other merger. In our setting, these cross-effects are positive, and the external effect to both mergers is clearly a sufficient criterion.

A positive joint external effect of an infinitesimal merger is given by

$$\sum_{i \in O_1 \cap O_2} \lambda_i s_i - (s_{I_1} + s_{I_2}) > 0 \quad (10)$$

as $dQ < 0$ and $\partial P/\partial Q < 0$. By definition $s_{I_j} = \sum_{i \in I_j} s_i$, $j = 1, 2$ in the initial infinitesimal merger. Under the conditions stated in footnote 2 the sign of the infinitesimal merger does not change in the integration path for computation of the total merger effect. Thus, this is a sufficient condition for the total welfare change to be positive. Note that the gain from participating in the first merger must be positive. The profit after the two mergers of firms participating in the first merger must be higher than either the profit they have if the second merger is approved or the status quo if the second merger is not approved. Since firms participating in the first merger are outsiders to the second merger, they profit from it in a quantity-reducing Cournot oligopoly. Thus, in either case, first merger internal gains must be positive. ■

Proof of Proposition 2 The joint merger evaluation rule requires

$$s_{I_2}^0 < \sum_{i \in O_1 \cap O_2} \lambda_i^0 s_i^0 - s_{I_1}^0 \quad (11)$$

The sequential rule requires, under complete information on the interim equilibrium,

$$s_{I_2}^1 < \sum_{i \in O_2} \lambda_i^1 s_i^1 \quad (12)$$

¹⁵Unless substantial cost savings result from the merger. See Farrell and Shapiro (1990) on this.

Under the sufficient conditions for the discrete external effect to have the same sign of the external effect of an infinitesimal merger, it results $d(\lambda_i s_i)/dQ < 0$ for firms outsiders to the merger. The last condition can then be transformed in a sufficient condition:

$$s_{I_2}^1 < \sum_{i \in O_1 \cap O_2} \lambda_i^0 s_i^0 \quad (13)$$

or

$$s_{I_2}^0 < \sum_{i \in O_1 \cap O_2} \lambda_i^0 s_i^0 - \Delta s_{I_2} \quad (14)$$

Since market shares sum one,

$$\Delta s_{I_1} + \Delta s_{I_2} + \sum_{i \in O_1 \cap O_2} \Delta s_i = 0.$$

Also, outsiders increase their equilibrium market share ($\Delta s_i > 0, i \notin I_1$) and the entity resulting from the first merger still operates in the market ($-\Delta s_{I_1} < s_{I_1}^0$). Therefore,

$$\Delta s_{I_2} < s_{I_1}^0.$$

Hence, the sufficient condition for approval of the second merger under the joint merger evaluation rule is more conservative than a condition stronger than the sufficient criterion in the sequential rule. ■

Proof of Proposition 3 Under a constant value for λ , Rule 3 can be written, for the second merger, as

$$s_{I_2}^0 < \lambda \left(\sum_{i \in (O_1 \cap O_2)} s_i^0 + s_{I_1}^0 \right) \quad (15)$$

and Rule 1 as

$$s_{I_2}^1 < \lambda \left(\sum_{i \in (O_1 \cap O_2)} s_i^1 + s_{I_1}^1 \right) \quad (16)$$

Since $\sum_i s_i^j = 1, j = 0, 1$, the critical threshold for merger approval in both cases is that the participating firms in merger 2 have a combined market share lower than $\lambda/(1 + \lambda)$. ■

Proof of Proposition 4 First note that both rules are based on the same period market shares. We can write the threshold market share for each rule as

$$\bar{s}_{2|R_2}^0 = \sum_{i \in (O_1 \cap O_2)} \lambda_i^0 s_i^0 - \sum_{i \in I_1} s_i^0$$

$$\bar{s}_2^0|_{R_3} = \sum_{i \in (O_1 \cap O_2)} \lambda_i^0 s_i^0 + \sum_{i \in I_1} \lambda_i^0 s_i^0$$

From direct inspection is clear that $\bar{s}_2^0|_{R_2} < \bar{s}_2^0|_{R_3}$. ■

Proof of Proposition 5 For both mergers to be rejected it must be the case that

$$s_{I_1} > \sum_{i \in O_1} s_i \text{ or } s_{I_1} > 1/2 \quad (17)$$

$$s_{I_2} > \sum_{i \in O_2} s_i \text{ or } s_{I_2} > 1/2 \quad (18)$$

That is, none of the sufficient conditions a la Farrell-Shapiro is satisfied. Since $I_1 \cap I_2 = \emptyset$, then

$$\sum_{i \in I_1} s_i + \sum_{i \in I_2} s_i \leq 1 \quad (19)$$

However, to have both mergers rejected, $\sum_{i \in I_1} s_i + \sum_{i \in I_2} s_i > 1$. Therefore, on the basis of the external effect, one of the mergers must be approved. ■