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Hunting with Two Bullets: Moral Hazard with a Second Chance

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Abstract

I study the moral hazard problem where an agent can create an extra instance of effort and potentially improve bad realizations of the outcome before the principal observes it. The agent cannot hide the outcome of his effort, but just the way he achieved it. Findings are that both, principal and agent, value the option of improving the outcome in case of a bad realization if doing so is cheap. I also find that contracted effort is not always decreasing in its cost. Finally, if the creation of the extra instance can cause a punishment for the principal, and if that punishment is sufficiently big, the principal will avoid writing contracts that incentive effort only on the extra chance.

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1 Introduction

In real life we can think of many situations in which principals face agents who, while unable to hide the outcome, can hide the way it was achieved (for example,

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when and how much effort was exerted). The agent may have ways to achieve the outcome that are completely unknown to the principal when offering the contract.

In this paper, I introduce a moral hazard model in which the agent takes sequential decisions. In the beginning, the principal offers a contract, which is contingent on the final outcome. Later the agent exerts effort, observes the outcome of his effort, and in case the outcome is bad, can decide to exert further effort in an attempt to obtain a better outcome. A key piece of the model is that effort is cumulative, that is, the probability of success is higher the more effort has been exerted in total.

This setup allows us to consider different situations. For example, the agent might at first exert low levels of effort, with the hope of having a good outcome, and knowing that in case this *bet* goes wrong, he will have another chance to work hard and increase the chances of delivering. Consider the example of a honey dealer buying from cheap low reputation suppliers. If he has enough time until delivery, he might gamble with the cheap suppliers, and buy from more expensive and reliable suppliers only if he received low-quality products from the cheap ones. This decreases the chances for the principal observing a good outcome, when compared to going to the good suppliers from the beginning, as the agent has a higher probability of success, and higher chances to fix a unlikely bad outcome.

The extra chance is, however, not necessarily a bad thing for the principal. Indeed, it might even reduce the cost of effort, by introducing the option of gambling at first. In this article, I show that the principal, under certain conditions, designs contracts that make the agent gamble even in scenarios without moral hazard. However, moral hazard increases the agency costs significantly compared to the standard agency model (it is easier for the agent to hide what his actions are because of his larger set of options). I show that these facts create non-convexities and non-monotonicities in the implementation of effort as a function of its cost.

Finally, I study the case where the extra chance represents an undesired activity, which can trigger later a punishment to the principal if caught. I show that if the penalty is big enough, the principal will never contract a strategy involving gambling (no effort and later trying to fix an adverse outcome). As the agent does not suffer the consequences other than the cost of the extra chance,[1] the agency costs increase substantially in the regions where gambling is the agent's best response, and therefore no effort is contracted in a broader set of parameters than in the case without the punishment.

In Section 2, I review the literature that relates to the problem presented in this

[1] "The Justice Department has lost the will and ability to prosecute top corporate executives. They focus on settlements with corporations for money...", Jessie Eisinger in an interview with Knowledge@Wharton on August 2017 - http://knowledge.wharton.upenn.edu/article/why-wrongdoing-executives-are-rarely-prosecuted/.

paper. In Section 3, I present the baseline model with its efficiency implications. In Section 4, I describe how a cost for the principal, of having used the second chance, affects the implementation of strategies involving the use of that extra chance with and without moral hazard. Finally, I conclude in Section 5.

2 Related Literature

The traditional framework used in moral hazard consists on an agent that has to perform a task for a principal, and the principal cannot observe the effort exerted by the agent. For that reason, the principal sets a payment schedule contingent on the outcome. Examples of this can be found for example in Bolton and Dewatripont (2005), Laffont and Martimort (2002), and Salanié (2005). The model introduced in this paper, incorporates a second chance to exert effort by the agent, before the principal observes the outcome.

The literature so far has a set of different branches to which this model can relate. In particular, Holmstrom and Milgrom (1987) introduced a pure moral hazard model in which the agent has to exert effort a number of times before the principal compensates him. In their work the authors' model has the following timeline: in each of the N periods of the game the agent has to exert effort, and after each of those periods there will be an instant realization that cannot be obscured by the agent. At the end of the N periods, the principal will be able to observe the whole history of realizations and then proceed to compensate the agent. They conclude that the optimal contract involves aggregation of realizations and linear compensation on this aggregated performance along with constant effort from the agent. While they were pointing to show that not always it is necessary to use all the information to reach optimal compensation schemes and that sometimes simple functions, as observed in the real world, turn out to be optimal solutions for the principal, I focus more on the behavior of the agent trying to exploit the fact of having more than one period to achieve a final output, and I do not provide more information to the principal than a single outcome. Besides, in my framework effort is cumulative, and the agent can stop working after the first period if desired.

Another one is multitasking (Holmstrom and Milgrom, 1991), as the two actions are different (as effort is cumulative) and both go in the direction of increasing the principal's utility. Even though the complementarity of both actions is a similarity with the model presented in this article, the timing is quite different. As effort is cumulative in my model, the first time the agent exerts effort impacts on the productivity of effort in the extra chance; however, the effort exerted during this second chance does not affect the productivity of the first one.

Zhao (2008) provides a model in which two contracting parties might be unaware about their own or the counterpart's strategy set. This was extended by articles like Auster (2013) and von Thadden and Zhao (2014), although in these situations, usually the agent is unaware of some of his options, and the principal decides to reveal — or not — information about those to the agent through the contract. The model of this paper considers a similar framework, however it departs from the main stream by considering that: the principal might be unaware of the agent's strategies, the principal makes a take-it-or-leave-it contract, and therefore the agent has no way to pass information to the principal, and finally, I explore the possibility of the principal removing some of the agent's strategies by using deadlines.

Varas (2017) provides a model to explain why contracts exhibit low turnover rates and deferred compensation. In his setup, managers can shorten the time they take to carry out a project by sacrificing quality. The principal, therefore, delays compensation to the future, so the quality of the project is revealed. This framework is related to the one introduced in this paper in the sense that the agent can exert actions that can create costs for the principal in the future. However, Varas' setup is intended for longer principal-agent relationships, in which termination is fundamental in the stationary contracts. In this article, I focus on a short-term relationship between a principal and an agent. This can be applied to suppliers, contractors, etc.

There is another branch of the literature that studies fraud using counterfeit signals, but with a very important difference: the literature considers the problem as pure adverse selection or instead as moral hazard followed by adverse selection, in which the agent can choose the signal to present to the principal about a previous realization which in turn can depend on some effort measure. In this literature, we find for example Maggi and Rodríguez-Clare (1995) which propose a model in which the agent must reveal to the principal its true type. Crocker and Morgan (1998), and Crocker and Slemrod (2007) incorporate a first stage in which the agent must indeed exert an effort level, and later he can decide to reveal or not the outcome (that is, reveal the true outcome or a false one).

Clausen (2013) introduces a novel concept that extends the previous models incorporating the fact that the agent can decide to obscure the true outcome from the principal, but he cannot control what it is going to be the signal that indeed the principal observes. Clausen considers a model in which the agent exerts effort once, and later there are successive accurate signals about the outcome realizations that are privately observed by the agent each time before the principal. The agent can then decide to counterfeit each signal for a better one deceiving the principal. The type of situations that can be represented by this model are different from the ones I try to describe. Indeed, Clausen mentions internet advertising click fraud, in which companies exaggerate the reported clicks for internet advertising, or security companies that hide breaches, to obscure the fact of having failed in their mission. A crucial difference with respect to the model I present in this work is that the agent, instead of sending a counterfeit signal to the principal, can exert effort and *de facto* improve the outcome. He cannot deceive the principal by obscuring the outcome.

Finally, the *gambling for resurrection* literature (Calveras, Ganuza, and Hauk, 2004, Thaler and Johnson, 1990) might also seem to be close to this work. Nevertheless, there is a key difference. While the model presented in this paper focuses on the effects of having the possibility of improving a poor outcome after the first effort was exerted, the gambling for resurrection focuses on the risk attitudes of agents given a previous event. The classical example is what Thaler and Johnson (1990) call the break-even effect, as when agents with previous losses will take risky opportunities to recover even if those carry even more risk. I am more interested in how the possibility of additional effort can change the agent's behavior from the beginning and how this affects the incentive scheme design.

3 The Model

The classical moral hazard models in the literature considers a principal that makes a take-it-or-leave-it offer of a contract to an agent. This contract establishes a payment schedule, from the principal to the agent, conditional on publicly observed outcomes. The outcomes are stochastic, but their distribution is influenced by the amount of effort the agent has exerted. This effort is not observable, and that is precisely what creates the moral hazard problem. I extend this classical model by giving the agent the opportunity to improve a bad outcome after he has already exerted some effort, but before it is observed by the principal.

Principal and agent are both assumed to be risk neutral. I assume further that the agent is cash constrained. First principal and agent sign the contract, establishing payments $w \in \{w_l, w_h\}$ contingent on the observed output $y \in \{y_l, y_h\}$. The effort the agent can exert is denoted by $e \in \{0, 1\}$, which impacts the probability of obtaining a high output y_h. This probability depends on the amount of effort exerted in the present period and the past. Let (e_1, e_2) be the agent's strategy that works in the following way: the agent in the first period will exert e_1 and later will observe an interim accurate signal $\hat{y} \in \{y_l, y_h\}$. If $\hat{y} = y_h$, the agent will not exert more effort whatsoever (as the outcome cannot be improved) and the publicly observed outcome is $y = \hat{y} = y_h$. However, if $\hat{y} = y_l$ then what happens next depends on the agent's choice of e_2. If $e_2 = 0$, there is no second lottery and the previous outcome is maintained. The principal observes $y = y_l$ and pays w_l.[2] If $e_2 = 1$ the outcome y is drawn from a lottery that assigns a higher probability of occurrence to y_h than in the previous period, because the total amount of exerted effort has increased, and then capturing this model's feature that effort is considered cumulative. A contract under this setup is a wage schedule that induces the agent to choose a particular effort strategy. Without any loss of generality and for the sake of simplicity I will assume from now on that $y_l = 0$.

The utility function of the principal is $u_p = y_i - w_i$, $i = h, l$, while the utility function of the agent is $u_a = w_i - e_1 - \beta e_2$, $i = h, l$, where $\beta > 0$ represents the cost of exerting effort *to improve upon the already realized outcome*. Let the agent's reservation utility be $\overline{u} = 0$. Furthermore, $w_0 \geq 0$, as the agent was assumed to be cash constrained.

Effort influences the probability of having a good outcome. This probability may take the values p_0, p_1, and p_2 defined as:

- $p_0 > 0$ is the probability of success when no effort was exerted in the past, nor the present.
- $p_1 > p_0$ is the probability of success when effort was exerted only once.
- $p_2 = 1$ is the probability of success when effort is exerted now and in the past.

Note that the subscripts indicate how many times the agent has exerted effort at that time. It is also worth noting that the probability of having a good outcome after the first period can only be p_0 or p_1, while in the second lottery it can be p_1 or p_2. Letting $p_2 = 1$ allows focusing on the significance of p_0 and p_1. Finally, p_0 is the probability of success without exerting any effort, and as that it helps to measure how much the effort can add to the outcome.

The complexity of the task is captured by p_1. If p_1 is high, for a given p_0, then exerting effort once is enough to have the goal achieved, and therefore it represents a simple task, but if p_1 is very low, then exerting effort once is most likely not to be enough to achieve the desired goal, representing a complex task.

The model gives the agent the alternative to delay effort if convenient. If the agent observes a bad interim outcome, exerting effort to fix that outcome implies the same probability of success as having exerted effort at the beginning (p_1). Moreover, when choosing the strategy, the probability of success of $(0, 1)$ is strictly higher when compared to $(1, 0)$. Note that the agent would delay effort, not because of impatience

[2] I assume a second lottery only if effort is exerted in the second period, while for the first one, no effort still has a positive probability of success. This assumption brings tremendous gains in simplicity and parsimony to the model. I have verified that giving the agent a free draw when $e_2 = 0$ does not change the main conclusions of the model.

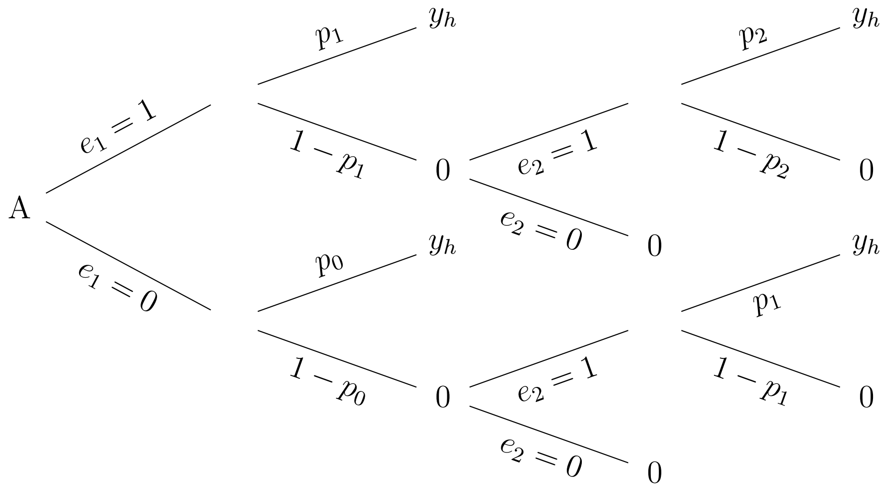

Figure 1: Agent's decision tree.

(which is not modeled), but because of the value of the option of exerting the effort in the future. In this model, delaying effort increases the chances of success compared to exerting effort only in the first period. Of course this option is valuable as long as its cost (β) is not too high. An illustration of the decisions the agent can make and their impact on the outcome can be observed in Figure 1.

In the following subsections, I find the contracts under full information, with an unaware principal with moral hazard, and with an aware principal also in the presence of moral hazard. Later I present a discussion on the inefficiencies created by the information asymmetry.

Full Information

Without any information asymmetry, the maximization problem is equivalent to the one the agent would solve if owning the project. Note that removing all information asymmetry implies the principal is aware of the existence of both chances the agent can use to exert effort. As such, she considers the direct trade-off between the expected outcome and the cost of each strategy, allowing us to disregard the wage schedule. Following this setup has many advantages. Firstly it allows identifying the welfare maximizing strategies for the parameters of the problem. Secondly, it allows to set up very explicitly the best response for the agent in the problem with moral hazard. The maximization problem under full information is:

$$\max_{(e_1,e_2)} p_{e_1} y_h - e_1 + e_2(1 - p_{e_1})(p_{e_1+1} y_h - \beta) \tag{1}$$

Lemma 1. *There exist $\underline{\beta}_1 < \overline{\beta}_1$ such that with full information:*

1. $\forall\ \beta > \underline{\beta}_1$, $(0,1)$ *is never contracted.*

2. $\forall\ \beta < \overline{\beta}_1$, $(1,0)$ *is never contracted.*

Proof. In Appendix A, I find the strategies that are implemented for different levels of y_h, given β. It can be established that $\underline{\beta}_1 = p_1/[(1-p_0) - p_1(1-p_1)]$ and $\overline{\beta}_1 = 1/(p_1 - p_0)$ are such that the lemma is satisfied. Note that $\underline{\beta}_1$ can be higher than 1, if $p_1 > (3-\sqrt{5})/2$ and $p_0 > (1-p_1)^2$. $\square$

Lemma 1 shows that, if the cost of exerting effort in the extra chance is too high, it is never optimal to use it. In the same direction, if the cost of this extra chance is low, and if it is worth to exert some effort, this chance is to be used. Moreover, if this extra chance is very cheap, effort is going to be implemented in the form of $(0,1)$ even for very low values of y_h. Another interesting implication of Lemma 1 is that as $p_1 - p_0 \to 0$, or in words, either the task is so hard that needs effort twice, or the task is easy enough that effort (once or twice) adds little, $\underline{\beta}_1 \to p_1/(1-p_1)^2$ while $\overline{\beta}_1 \to \infty$, so while $(0,1)$ is still going to be implemented, $(1,0)$ is going to be implemented only for an infinitely large cost of creating the second chance.

Lemma 1 considerably facilitates the computation of the contract with full information, as it reduces the strategies to consider within intervals of β. The solution to the problem stated in (1) is represented in Proposition 1.

Proposition 1. *The contract under perfect information is given by:*

1. *For* $\beta < \underline{\beta}_1$,
   (a) $(0,0)$ *for* $y_h \leq \frac{\beta}{p_1}$.
   (b) $(0,1)$ *for* $\frac{\beta}{p_1} \leq y_h \leq \frac{1-\beta(p_1-p_0)}{(1-p_0)(1-p_1)}$.
   (c) $(1,1)$ *otherwise.*
2. *For* $\underline{\beta}_1 < \beta < \overline{\beta}_1$,
   (a) $(0,0)$ *for* $y_h \leq \frac{1+(1-p_1)\beta}{1-p_0}$.
   (b) $(1,1)$ *otherwise.*
3. *For* $\overline{\beta}_1 < \beta$.
   (a) $(0,0)$ *for* $y_h \leq \frac{1}{p_1-p_0}$.
   (b) $(1,0)$ *for* $\frac{1}{p_1-p_0} \leq y_h \leq \beta$.
   (c) $(1,1)$ *otherwise.*

The contract under full information is represented in Figure 2. Note that it results from the possibility of delaying effort that at least some effort is contracted even for very low levels of y_h. This is not only true because close to the origin the future effort is cheap, as the fact that even for values of β greater than 1 this is still the case, but because of the value of the option of delaying effort.

At higher levels of y_h we observe that for low levels of β the strategy involving exploiting the option of exerting future effort is preferred, and as β increases then $(1, 1)$ becomes optimal. This is because even though for those levels of β exerting the option is still profitable, as β is now higher, the agent tries to diminish the probability of having to use that option, and he achieves that by exerting effort in the first period. If y_h is high enough, by increasing β, we only disregard the use of the option (as it will never give positive expected profits), while if y_h is lower, then from $(1, 1)$ by increasing β, the optimal strategy will be to exert no effort at all. This is another very graphical way to observe the value of the option of delaying effort, as for a fixed y_h the strategy $(1, 1)$ changes to $(0, 0)$ instead of moving to $(1, 0)$ as β increases.

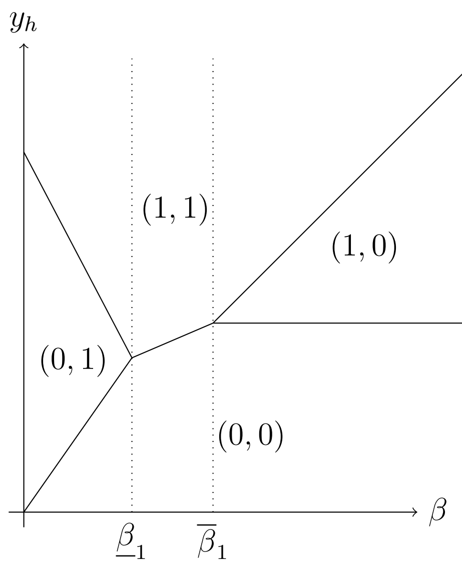

Figure 2: Contract under perfect information.

Finally, we can observe that while e_2 does not increase in the cost of the second effort (β), we also observe that, for some levels of y_h, e_1 is not monotonic in β. It is surprising that e_1 is exerted optimally at all for levels below $y_h = 1/(p_1 - p_0)$. To pin down the cause of this, recall the previous discussion: setting $e_1 = 1$ affects e_2 in two ways. First, it makes it less likely that the second chance is going to be used, and second, it increases the expected revenue for doing so, by increasing the chance of success of $e_2 = 1$ from $p_1 < 1$ to $p_2 = 1$. When increasing β, and y_h is such that the strategy moves from $(1, 1)$ to $(0, 0)$, the increasing cost of the second chance has destroyed the benefits of this complementarity, leading to the optimality of no effort at all.

Moral Hazard and Unaware Principal

In this subsection, I portray the situation in which the principal does not know about the possibility of the extra chance.

This unaware principal will propose the classical textbook moral hazard contract, with $w_h = 1/(p_1 - p_0)$ for a good observed outcome, and $w_l = 0$ otherwise.

We can use the optimal strategies under full information to describe the best response of the agent, by considering $w_h = 1/(p_1 - p_0)$. The principal will offer this wage level if and only if $y_h \geq p_1/(p_1 - p_0)^2$. Replacing w_h in the agent's best response, we obtain the strategies followed by the agent. These depend on the value of β. Note that while $(1, 0)$ and $(0, 0)$ are the strategies the principal expects, $(0, 1)$ and $(1, 1)$ strategies the principal does not expect, as she is unaware of them.

It can be shown (Appendix A) that, for $w_h = 1/(p_1 - p_0)$, the agent will never choose $(0, 0)$. Moreover, it can be shown that there exists thresholds on β such that the agent will choose $(0, 1)$, $(1, 1)$ or $(1, 0)$ within different intervals for β. Each of these strategies will lead to different levels of outcome for the principal:

Strategy	Exp. outcome for the principal	Exp. outcome by unaware principal
$(0, 1)$	$[p_0 + p_1(1 - p_0)]\left(y_h - \frac{1}{p_1 - p_0}\right)$	$p_1\left(y_h - \frac{1}{p_1 - p_0}\right)$
$(1, 1)$	$y_h - \frac{1}{p_1 - p_0}$	$p_1\left(y_h - \frac{1}{p_1 - p_0}\right)$
$(1, 0)$	$p_1\left(y_h - \frac{1}{p_1 - p_0}\right)$	$p_1\left(y_h - \frac{1}{p_1 - p_0}\right)$

Table 1: Principal's expected outcome.

In Table 1, we observe that for each of these scenarios, the principal is at least as good as she expect to be in the traditional model without the extra chance. However, that is not all. The agent also benefits from this, as he obtains larger rents which are created by the extra chance. More specifically, the principal could have induced the same strategies with a much lower wage, keeping a higher share of the outcome.

It can be shown that the agent's gains are always positive, given his optimal chosen strategy for the parameters involved (probabilities of success and future cost of effort, β).

Strategy	w_h - unaware principal	Optimal w_h	Agent's gains
$(0,1)$	$\frac{1}{p_1-p_0}$	$\frac{\beta}{p_1}$	$\frac{1}{p_1-p_0} - \frac{\beta}{p_1}$
$(1,0)$	$\frac{1}{p_1-p_0}$	$\frac{1}{p_1-p_0}$	0
$(1,1)^*$	$\frac{1}{p_1-p_0}$	$\frac{1-\beta(p_1-p_0)}{(1-p_0)(1-p_1)}$	$\frac{1}{p_1-p_0} - \frac{1-\beta(p_1-p_0)}{(1-p_0)(1-p_1)}$
$(1,1)^{**}$	$\frac{1}{p_1-p_0}$	$\frac{1+(1-p_1)\beta}{1-p_0}$	$\frac{1}{p_1-p_0} - \frac{1+(1-p_1)\beta}{1-p_0}$

Table 2: Rents distribution with unaware principal. $*$ when $\beta < \overline{\beta}_1$, and $**$ when $\beta \geq \overline{\beta}_1$.

Moral Hazard

Under moral hazard, I consider the traditional participation constraint and the incentive compatibility constraint ensuring that the agent accepts to sign the contract and chooses the desired strategy. The cost for the principal is represented by the wages w_h and w_l for the good and bad outcome respectively. Recall that the agent is assumed to be cash constrained, and therefore I set immediately $w_l = 0$. The principal's maximization problem when facing asymmetric information is represented by:

$$\begin{aligned} \max_{w_h, e_1, e_2} \quad & p_{e_1}(y_h - w_h) + e_2(1-p_{e_1})p_{e_1+1}(y_h - w_h) \qquad (2) \\ \text{s.t.} \quad & p_{e_1}w_h - e_1 + e_2(1-p_{e_1})(p_{e_1+1}w_h - \beta) \geq 0 \\ & (e_1, e_2) \in \arg\max_{(\hat{e}_1, \hat{e}_2)} p_{\hat{e}_1}w_h - \hat{e}_1 + \hat{e}_2(1-p_{\hat{e}_1})(p_{\hat{e}_1+1}w_h - \beta) \end{aligned}$$

The incentive compatibility is given by the solution to the problem with full information. The only difference is that it is necessary to replace y_h with w_h, as the agent is now getting only w_h instead of the whole outcome. This gives immediately the optimal incentive compatible wages for a given β and outcome distribution. As such $\underline{\beta}_1$ and $\overline{\beta}_1$ from Lemma 1 are also critical points for the case with moral hazard, as for the principal, it is impossible to implement a contract involving $(1,0)$ or $(0,1)$ between those parameters, no matter the wage. Having the incentive compatible wage for each strategy, and having the implementable contracts as a function of β, it is just a matter of comparing expected profits of implementing each strategy, leading to the first result in the presence of moral hazard, expressed in Lemma 2.

Lemma 2. *There exist $\underline{\beta}_2$ $(< \underline{\beta}_1)$ and $\overline{\beta}_2$ $(> \overline{\beta}_1)$ such that under asymmetric information:*

1. *$\forall\ \beta > \underline{\beta}_2$, $(0,1)$ is never implemented.*

2. $\forall\, \beta < \overline{\beta}_2$, $(1,0)$ *is never implemented.*

Proof. Comparing the expected profits of implementing each strategy, given the wages are incentive compatible, it happens that there exist y_h such that $(0,1)$ is optimal only if

$$\beta < \underline{\beta}_2 = \frac{p_1^2}{p_0 + p_1 - (1-p_1)[p_1^2 + p_0^2 - p_0^2 p_1] - p_0 p_1 (3 - p_1)},$$

while there exists y_h such that $(1,0)$ is optimal only if

$$\beta > \overline{\beta}_2 = \frac{p_1(1-p_0)}{(p_1 - p_0)^2}.$$

It can be shown after some algebra that $\underline{\beta}_2 < \underline{\beta}_1$ since $p_1 p_0 (1-p_0)(1-p_1)^2 > 0$ and $\overline{\beta}_1 < \overline{\beta}_2$ since $0 < p_0 < p_1 < 1$. □

An important implication of Lemma 2 is that the contracts implementing $(0,1)$ and $(1,0)$ have more restrictive requirements over β than when compared to the case with perfect information. An important remark is that, even though the β_2s found in Lemma 2 play the same role that the β_1s played in the case without moral hazard, the β_1 are still very important for the case with asymmetric information. They now affect directly the incentive compatibility constraint, as they define the regions for which the agent will never play $(1,0)$ or $(0,1)$, no matter w_h. The contract under moral hazard is described for five different intervals for β as stated in Proposition 2.

Proposition 2. *Under asymmetric information, the contract with a cash constrained agent is given by:*

1. *For* $\beta < \underline{\beta}_2$,
   - *(a)* $(0,0)$ *for* $y_h \leq \beta\left(\frac{p_0 + p_1(1-p_0)}{(1-p_0)p_1^2}\right)$.
   - *(b)* $(0,1)$ *for* $\beta\left(\frac{p_0 + p_1(1-p_0)}{(1-p_0)p_1^2}\right) \leq y_h$,
     and $y_h \leq \frac{1}{[(1-p_0)(1-p_1)]^2} - \beta\left[\frac{p_1 - p_0}{[(1-p_0)(1-p_1)]^2} + \frac{p_0 + p_1(1-p_0)}{p_1(1-p_0)(1-p_1)}\right]$
   - *(c)* $(1,1)$ *otherwise.*
2. *For* $\underline{\beta}_2 < \beta < \underline{\beta}_1$,
   - *(a)* $(0,0)$ *for* $y_h \leq \frac{1 - \beta(p_1 - p_0)}{(1-p_0)^2(1-p_1)}$.
   - *(b)* $(1,1)$ *otherwise.*
3. *For* $\underline{\beta}_1 < \beta < \overline{\beta}_1$,
   - *(a)* $(0,0)$ *for* $y_h \leq \frac{1 + (1-p_1)\beta}{(1-p_0)^2}$.

(b) $(1,1)$ *otherwise.*

4. *For* $\overline{\beta}_1 < \beta < \overline{\beta}_2$,

   (a) $(0,0)$ *for* $y_h \leq \frac{\beta}{1-p_0}$.

   (b) $(1,1)$ *otherwise.*

5. *For* $\overline{\beta}_2 < \beta$.

   (a) $(0,0)$ *for* $y_h \leq \frac{p_1}{(p_1-p_0)^2}$.

   (b) $(1,0)$ *for* $\frac{p_1}{(p_1-p_0)^2} \leq y_h \leq \frac{\beta(p_1-p_0)-p_1}{(1-p_1)(p_1-p_0)}$.

   (c) $(1,1)$ *otherwise.*

Focusing the attention on the first interval for β, we observe how $(0,1)$ becomes optimal for a broader set of parameter values than compared to the case with full information. Note that the denominator in the interval is quite small, leading to a very big intercept on the frontier between $(0,1)$ and $(1,1)$ as optimal contracts. Looking at Figure 3 there is a very interesting fringe of y_h about the middle. There are some values of y_h for which, by increasing β, we have the following transition: starts with $(0,1)$, then moves to $(0,0)$, followed by $(1,1)$ to finally come back to $(0,0)$.

Proposition 3. *With information asymmetries there exist y_h such that e_2 is not monotonically decreasing in β.*

Proof. The proof follows directly Proposition 2 and Lemma 2. Let

$$y_h \in \Upsilon := \left(\frac{1-\underline{\beta}_2(p_1-p_0)}{(1-p_0)^2(1-p_1)}, \frac{1-\underline{\beta}_1(p_1-p_0)}{(1-p_0)^2(1-p_1)} \right)$$

The set Υ is nonempty since $\underline{\beta}_2 < \underline{\beta}_1$. From Proposition 2, it can be seen that between $\underline{\beta}_2$ and $\underline{\beta}_1$ the optimal contract will change from $(0,0)$ to $(1,1)$, as the slope of the frontier between both is decreasing in β. It can be observed as well, that for β increasing above $\underline{\beta}_1$ the optimal contract will move from $(1,1)$ to a contract for which e_2 is zero, as the frontier between both has always a positive slope.

□

The set of $(0,0)$ is not convex, which follows directly from Proposition 2 where the slope of the frontier of $(0,0)$ changes from positive to negative and later to positive again. At very low levels of β, $(0,1)$ is implemented following the same logic it had in the case with full information. The second chance, being so cheap, makes it preferable to bet all in taking the risk and later try to fix any bad outcome.

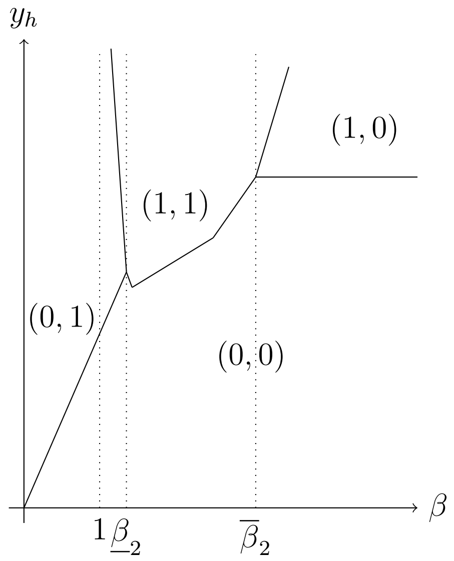

Figure 3: Contract with moral hazard.

As β gets closer to $\underline{\beta}_2$ (for the same level of y_h) the extra chance becomes more costly, so it would be better to avoid it. The logic behind the solution with full information to achieve that was to implement $(1, 1)$ instead, in order to decrease the probability of using the extra chance. However, as β is still low, the agent would prefer to deviate to gamble with $(0, 1)$, and therefore the incentives to make him stick to $(1, 1)$ become too high. In this situation, the principal decides to implement $(0, 0)$. As β increases further, approaching now $\underline{\beta}_1$, the principal knows that the incentives for the agent to deviate from $(1, 1)$ decrease, diminishing the agency costs, so she will start implementing $(1, 1)$ again. The lower y_h inside the fringe, the closer the β needs to be to $\underline{\beta}_1$. In fact in the limit, it will coincide and will implement $(1, 1)$ just in $\beta = \underline{\beta}_1$. When β is above $\underline{\beta}_1$, the logic of the model follows the case with full information. $(1, 1)$ is implemented instead $(0, 1)$ as β becomes larger, to decrease the possibility of paying the cost of the extra chance, and as β increases even further, the principal will implement strategies with $e_2 = 0$, and setting $e_1 = 1$ or $e_1 = 0$ depending on the level of y_h. While, as expected, introducing agency costs in the model changes the optimal solution for the principal, the information rents around $\underline{\beta}_1$ create the non-convexity in the model. Note that to the left of $\underline{\beta}_1$ the agent has the option to deviate between three contracts, while to the right of $\underline{\beta}_1$ he endogenously will never choose $(0, 1)$, so the principal does not require to provide incentives to prevent that deviation.

As it was the case under full information, we observe that e_1 is not monotonic on β for some values of y_h either. This follows the same rationale previously discussed in the sense that, increasing β, the benefits of the complementarity between e_1 and e_2 are offset by the higher cost, requiring higher values of y_h to justify this strategy.

The other comparisons that can be made between the cases with full or imperfect information are in line with what would be expected in the traditional models of moral hazard and are illustrated in Figure 4. First, we observe that $(0,0)$ expands against all the other contracts. Besides the change in convexity of the set of $(0,0)$ discussed previously for the contract with imperfect information, the change in the extension of the sets implementing $(0,1)$ and $(1,0)$ is clear. In particular, the change in the slope of the frontier between $(0,1)$ and $(1,1)$ makes it very hard (require very high values of y_h) to implement $(1,1)$ when β falls below $\underline{\beta}_2$. It is remarkable that $\underline{\beta}_2$ is not required to be below 1 for this to happen, so this case happens even when the creation of this extra chance is relatively more costly than exerting effort on the first chance.

The conclusions of Lemma 2 can also be observed in Figure 4. Note how larger the interval of $[\underline{\beta}, \overline{\beta}]$, for which only $(0,0)$ or $(1,1)$ contracts are implemented, is in when compared to the case with full information. This happens because without full information, it is easier for the agent to deviate from one of those contracts to the other, or from $(1,1)$ to $(1,0)$, and therefore the principal will prefer to implement contracts that give him better information about what was done by the agent. When β is very high (above $\overline{\beta}_2$), the principal can incentivate the agent to not deviate from his strategy, starting to implement $(1,0)$ again. Basically to the left of $\underline{\beta}_2$ the principal is resigned not to implement $(1,1)$ when it would be optimal.

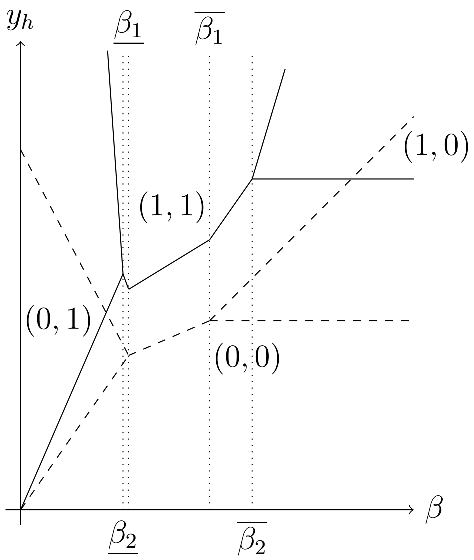

Figure 4: Contracts with full (dashed) and imperfect (solid) information.

A final conclusion that can be made from the comparison between the contracts with and without moral hazard is that the space in which e_2 is used decreases considerably because of the expansion of $(0,1)$ to regions for which $(1,1)$ was optimal. Also by the change in the slope of the frontier between $(1,0)$ and $(1,1)$ from the case with full to imperfect information, it is clear that not only $(0,0)$ is to be implemented for a broader set of parameters, but $(1,0)$ as well, although for higher values of y_h and β.

The study of the scenario with moral hazard shows that, when the agent has a low cost for creating the extra instance, it is very costly for the principal to dissuade him from shirking in the beginning with the hope of fixing a bad outcome in future scenarios.

Although I assumed that the principal is not concerned with the way in which the task was achieved (or not), but only if it was, this does not rule out potential costs for her. Indeed, having more strategies to evaluate, creates higher agency costs, as according to the model, by making harder for the principal to infer the strategy chosen by the agent. Furthermore, we observe that when the cost of creating the extra chance for the agent is low, it is too expensive for the principal to incentivize the strategy that would be the optimal with full information, and she simply implements the $(0,1)$ strategy for a wide set of values of y_h.

4 Externalities

In this section, I assume that the principal cares about how the outcome was achieved. Some of the analogies previously made with the strategies that the agent can follow imply risky financial activities or dubious process manipulation. Even further, creating the extra chance can be sometimes illegal (forcing workers to work extra hours when it is not allowed), or incurring in practices that might be censored by a regulator or the industry.

These costs might be the expected punishment the principal faces if caught by the regulator. Consider the recent case in the automobile industry, in which many brands modified the computers in their motors to pass the emission regulation tests. If the people in charge knew that software like this could be produced, that has a high probability of succeeding to passing the test, at a very cheap cost (in the model this would be very high p_1 with a very low β) they have strong incentives to take their chances with the software. If the expected cost of the transgression is low, then the principal might contract strategies in which gambling is encouraged, obtaining higher profits: if β is low, the required compensation is also low, and therefore the principal obtains a bigger surplus.

In what follows, I will assume that exerting e_2, if detected by the regulator, triggers a punishment to the principal, such that its expected cost, if e_2 was exerted, is ξ. The immediate effect we can depict is that the strategies involving the second effort will see their expected return decreased, and therefore the strategies without

e_2 will be optimal for a wider set of parameters.

The introduction of an externality cost, under perfect information, that is if the principal could indeed choose the optimal level of effort to maximize aggregated surplus, has a similar effect than facing a massive cost of e_2, or β. The detailed derivation of this version of the model is left for Appendix B.1. Figure 5 contains the main implications of introducing an externality in the case without moral hazard.

To start with, the strategy involving shirking and exerting effort when facing a bad outcome becomes less optimal than when there is no externality. Exerting effort in both periods becomes preferred over $(0,1)$, but it is nevertheless implemented less than in the original model, losing against $(0,0)$ and $(1,0)$. As the shape of the plot remains more or less equal, we obtain the first result of introducing the externality. The variables $\underline{\beta}$ and $\overline{\beta}$ are shifted to the left along with the plot, exactly in the value of ξ. This implies that the strategy $(0,1)$ might end up being completely wiped off in the absence of moral hazard for some value of ξ high enough, this threshold is stated in the following proposition:

Proposition 4. *With full information, if the cost* ξ *exceeds* $p_1/[(1-p_0)-p_1(1-p_1)]$ *then* $(0,1)$ *is never optimal.*

Proof. In Appendix B.1 can be observed that $(0,1)$ is implemented only for values of β between 0 and $p_1/[(1-p_0)-p_1(1-p_1)]-\xi$. As $\beta \geq 0$ then for $\xi > p_1/[(1-p_0)-p_1(1-p_1)]$ the set for which $(0,1)$ results optimal is empty. □

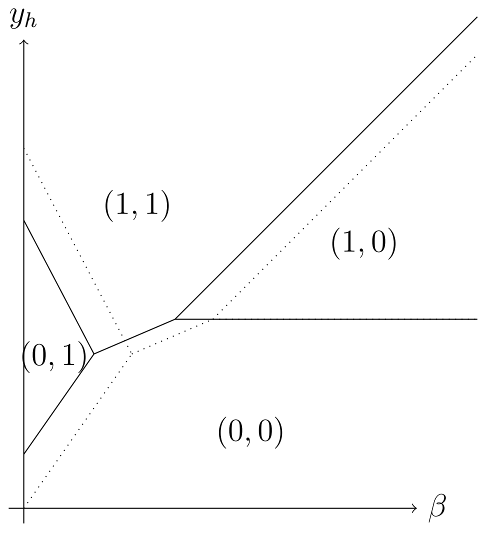

Figure 5: Contract with full information and externality.

Now when considering moral hazard, I assume as usual that the principal cannot observe the agent's actions. Again, the introduction of the externality should carry similar consequences of those that higher values of β would carry.

The detailed solution of this problem can be found in Appendix B.2. As expected, now the strategies $(0, 1)$ and $(1, 1)$ have higher potential cost for the principal, and therefore they turn out to be optimal in a smaller parameter space when compared to the problem without the cost of ξ. The effect on the model with moral hazard is not identical to the one that happens in the version with perfect information though. Now, because of the agency problem, the plot does not move to the left as it happened before. Note that the values of $\underline{\beta}_1$ and $\overline{\beta}_1$ are not affected at all.

A similarity, though, with the full information case is that there is a value for ξ that makes $(0, 1)$ never optimal.

Proposition 5. *If the cost ξ is higher or equal than*

$$\frac{p_1}{(1-p_0)(1-p_1)[(1-p_0)-p_1(1-p_1)]}$$

then $(0, 1)$ is never optimal.

Proof. In Appendix B.2 it can be observed that $(0, 1)$ is implemented only for values of beta such that

$$\beta \quad \in \quad \left[0, \frac{p_1^2+[p_1(1-p_0)(1-p_1)\{(1-p_1)p_1-(1-p_0)\}]\xi}{[p_0+(1-p_0)p_1](1-p_0)(1-p_1)+(p_1-p_0)p_1^2}\right]$$

As $\beta \geq 0$ then for $\xi \geq p_1/\{(1-p_0)(1-p_1)[(1-p_0)-p_1(1-p_1)]\}$ the set for which $(0, 1)$ is optimal is empty. □

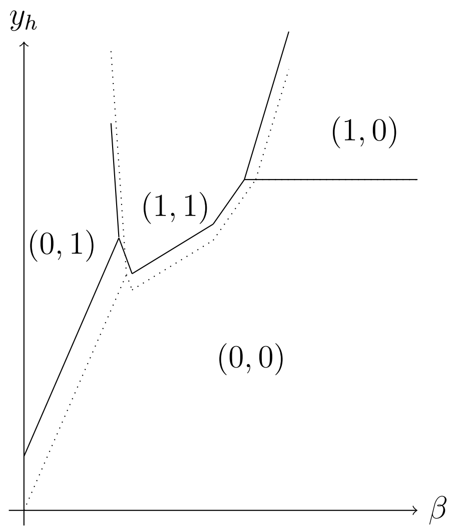

Figure 6: Contract under moral hazard and externality.

The strategy $(1, 1)$ can never be ruled out for a finite externality cost, because even if $\xi > 1/(p_1 - p_0)$, which would make $\overline{\beta}_1$ negative, there can always be found a

y_h big enough such that it will be optimal to exert effort in both opportunities. As a consequence, if the regulator would like to prevent second effort in any scenario, it would need, as expected, to relate the punishment (ξ) to y_h, such that the principal avoids implementing strategies with $(-, 1)$ at all. Furthermore, this would make the principal to implement shorter deadlines more often.

The agency costs are aggravated, which is reflected in Figure 6. In the region where the non-convexity of $(0, 0)$ occurs, with the externality these costs are higher. As the principal is bearing the costs ξ and not the agent, the principal contracts strategies involving $(0, 0)$ more aggressively to the left of $\underline{\beta}_1$.

5 Conclusions

I propose a model of moral hazard in which the agent has more than one chance to exert effort before revealing the output to the principal. The agent, after exerting some effort, observes an interim signal and then decides whether to make more effort to improve an adverse outcome or to deliver the outcome immediately.

Both agent and principal value this extra chance. I show how this situation creates extra rents that are split between principal and agent, when the principal offers the classical contract without the second chance, to an agent that can actually create this extra chance. Moreover, I find that when the principal knows about the extra chance, and there is no information asymmetry, both agent and principal value the possibility of fixing a bad outcome as an option, by being able to shirk at the beginning, if convenient, and later trying to fix a potentially adverse outcome.

However, once the information asymmetries are introduced, implementing a never shirking contract, and when the cost of creating the additional instance is low, happens for very high values of positive outcomes only. One of the main findings of the model is that with moral hazard the effort in the extra chance is not necessarily decreasing on its cost, existing values of output for which an increase in the cost of the second chance might increase the effort contracted in that second chance.

Finally, I study what happens if the extra chance can bring later consequences, and therefore costs, to the principal. This is done in order to adapt the model to situations in which this extra chance represents the use of illegal techniques to deliver the outcome as promised. I find that this possible cost for the principal causes that the strategies that do not involve the use of this extra chance become optimal for a wider set of parameters. Moreover, the principal will write contracts that incentive effort in the first period where with full information would have contracted effort only in the second chance, with the hopes of decreasing the probability of the agent using the second chance.

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Appendix

A Baseline Model

A.1 Perfect Information

In this appendix I solve the problem with perfect information. For simplicity and without loss of generality I assume $e_h = 1$. For the same reason, I solve the problem assuming the agent owns the firm, or conversely, the principal exerts the effort. Because of this assumption, the maximization problem lacks considerations about the wage schedule. The maximization problem is:

$$\max_{(e_1,e_2)} p(e_1)y_h - e_1 + e_2[1 - p(e_1)]\{p(e_1, 1)y_h - \beta\}$$

There is no need to compare all the strategies for a given pair of β and y_h. For example, if $e_1 = 1$, we know that if $y_h \geq \beta$, then the agent will always exert effort for e_2, and therefore the strategy $(1, 0)$ will never be followed.[3] Conversely, if $y_h < \beta$ the agent will never exert effort in his second chance, and therefore the strategy $(1, 1)$ should never be considered. We can say even more, as $y_h < \beta \Rightarrow y_h < \frac{\beta}{p_1}$, we can also discard $(0, 1)$ from the possibilities.[4] With this information, we can define a correspondence, relating values of β to strategies to be implemented for some value of y_h.

The agent will stick with a strategy involving $e_2 \neq 0$ if, after observing $\hat{y} = 0$, the return of exerting effort again is positive. In practical terms $(0, 1)$ will be optimal after observing $\hat{y} = 0$ if and only if $p_1 y_h > \beta 1$. In the same way $(1, 1)$ will be optimal after observing $\hat{y} = 0$ if and only if $y_h > \beta 1$.

From these two constraints, we obtain two critical values of y_h: $\frac{\beta}{p_1}$ and β. These critical points can be described in the diagram in Figure 7.

[3]Note that if $y_h \geq \beta$, given $\hat{y} = 0$ it is always optimal to exert $e_2 = 1$, as the revenues (y_h) are higher that the cost of that effort.

[4]Note that if $y_h < \beta$ then, given e_1, the expected revenues of $e_2 = 1$ are $p_1 y_h$, which is lower than β.

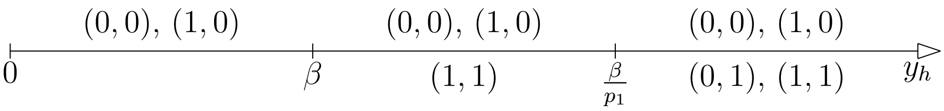

Figure 7: Strategies to be considered, for some level of y_h, given β.

Figure 7 shows which strategies are profitable as a function of y_h given β. We can say more though. If it is rational to exert effort in the second period, given effort in the first period (which implies that the probability of success is now 1), then not exerting effort in the second period given effort in the first period will never be played. In the same way it can be argued that, if it is rational to exert effort in the second period, given no effort in the first period (which implies that the probability of success is now p_1), then it is not rational to exert no effort in the first period without exerting effort in the second period. The new graphical description is in Figure 8.

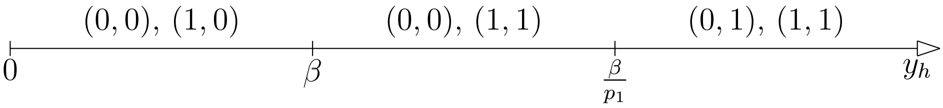

Figure 8: Strategies that can be implemented for different levels of y_h.

Comparing the profits between each pair, we can find the intervals that are relevant.

1. For the first interval (0, 0) versus ($\geq$) (1, 0),

$$\begin{aligned} p_0 y_h &\geq p_1 y_h - 1 \\ y_h &\leq \frac{1}{p_1 - p_0} \end{aligned}$$

2. For the second interval (0, 0) versus ($\geq$) (1, 1),

$$\begin{aligned} p_0 y_h &\geq y_h - 1 - (1 - p_1)\beta \\ \frac{1 + (1 - p_1)\beta}{1 - p_0} &\geq y_h \end{aligned}$$

3. For the final interval (0, 1) versus ($\geq$) (1, 1),

$$
\begin{aligned}
p_0 y_h + (1-p_0)[p_1 y_h - \beta] &\geq y_h - 1 - (1-p_1)\beta \\
1 - (p_1 - p_0)\beta &\geq y_h(1-p_0)(1-p_1) \\
\frac{1-\beta(p_1-p_0)}{(1-p_0)(1-p_1)} &\geq y_h
\end{aligned}
$$

The next step is to check which of the strategy sets is empty.

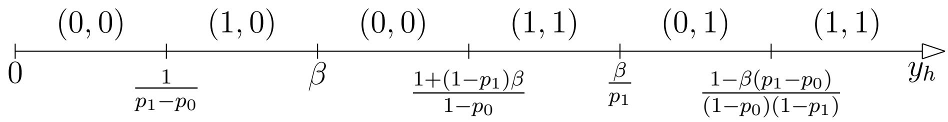

Figure 9: Strategies that can be implemented for different levels of y_h.

Name the intervals from left to right as A, B, C, etc.
It is clear that set A is nonempty. For B to be nonempty, it is necessary that:

$$\frac{1}{p_1 - p_0} \leq \beta$$

For C to be nonempty, it is necessary:

$$
\begin{aligned}
\beta &\leq \frac{1+(1-p_1)\beta}{1-p_0} \\
(1-p_0)\beta &\leq 1 + (1-p_1)\beta \\
\beta &\leq \frac{1}{p_1 - p_0}
\end{aligned}
$$

From here we get that B and C are mutually exclusive sets. Now for D to be nonempty,

$$
\begin{aligned}
\frac{1+(1-p_1)\beta}{1-p_0} &\leq \frac{\beta}{p_1} \\
p_1 + p_1(1-p_1)\beta &\leq (1-p_0)\beta \\
p_1 &\leq \beta[(1-p_0) - p_1(1-p_1)]
\end{aligned}
$$

The only way this could hold is if $(1-p_0) - p_1(1-p_1) > 0$, which happens when $p_0 < 1 - p_1 + p_1^2$. As $p_0 < p_1$ by assumption, the last inequality always holds, and therefore the condition for D being non empty is:

$$\frac{p_1}{(1-p_0) - p_1(1-p_1)} \leq \beta$$

For E to be nonempty:

$$\frac{\beta}{p_1} \leq \frac{1-\beta(p_1-p_0)}{(1-p_0)(1-p_1)}$$
$$\beta \leq \frac{p_1}{(1-p_0)-p_1(1-p_1)}$$

So again, E and D are mutually exclusive. One more comparison I want to show is that indeed or B or C is always empty.

$$\frac{1}{p_1-p_0} \leq \frac{1+(1-p_1)\beta}{1-p_0}$$
$$p_0(p_1-1) \leq (1-p_1)^2$$

Which always holds. Finally, analyzing the relationship between these two important thresholds for β,

$$\frac{1}{p_1-p_0} > \frac{p_1}{(1-p_0)-p_1(1-p_1)}$$
$$1-p_0-p_1+p_1^2 > p_1^2-p_1p_0$$
$$1 > p_0(1-p_1)+p_1$$

Which always holds, for being the right-hand side a convex combination of something strictly lower than 1, and 1.

For a given effort level e_h, the optimal contracts with moral hazard would exclude the combination A-C-E-F but consider only A-B-E-F, A-B-D-F, and A-C-D-F.

In Figure 2 it is represented the contract with full information. As can be seen in the derivation, this was done as if the principal and the agent were the same single person. Also the two key betas have been labeled as $\underline{\beta_1} = \frac{p_1}{(1-p_0)-p_1(1-p_1)}$ and $\overline{\beta_1} = \frac{1}{p_1-p_0}$.

A.2 Moral Hazard and Unaware Principal

When the principal is unaware of the possibilities of the agent, she will offer a contract with $w_h = \frac{1}{p_1-p_0}$ as the wage for a successful project, and $w_0 = 0$ otherwise.

In the agent's best response, this is exactly the division between $(0,0)$ and $(1,0)$, and as the contour of the $(0,0)$ region is increasing, it happens that, for $w_h = 1/(p_1-p_0)$ the agent never chooses $(0,0)$, independently of the value of β.

Moreover, by equalizing the frontier between $(0,1)$ and $(1,1)$ when $\beta < \underline{\beta}_1$, we obtain the intervals for β for which the agent will exert $(0,1)$, $(1,1)$ and $(1,0)$ respectively.

$$\frac{1}{p_1 - p_0} = \frac{1-\beta(p_1-p_0)}{(1-p_0)(1-p_1)}$$
$$\tilde{\beta} = \frac{(p_1-p_0)-(1-p_0)(1-p_1)}{(p_1-p_0)^2}$$

If $\beta < \tilde{\beta}$, the agent's strategy is $(0,1)$, if $\beta \geq \tilde{\beta}$, the agent's strategy is $(1,1)$, and if $\beta > \underline{\beta_1}$ the agent chooses $(1,0)$.

A.3 Moral Hazard

For the contract under moral hazard what was considered interim rationality constraint in the case with perfect information, is now an additional incentive compatibility. First, it is necessary that the agent accepts a wage that will make him choose the strategy at $t = 0$ as decided the principal, but later it is further necessary that after the first realization he sticks with that strategy. Again this would imply that $w_h \geq \beta e_h$ for (e_h, e_h) and $w_h \geq \frac{\beta e_h}{p_1}$ for $(0, e_h)$, while the opposite should be true for $(e_h, 0)$ and $(0, 0)$ respectively.

The maximization problem for the principal is now:

$$\begin{aligned}
\max_{w_h,(e_1,e_2)} \quad & p(e_1)\{y_h - w_h\} + e_2[1-p(e_1)]p(e_1,1)\{y_h - w_h\} \\
\text{s.t.} \quad & p(e_1)w_h - e_1 + e_2[1-p(e_1)]\{p(e_1,1)w_h - \beta\} \geq 0 \\
& (e_1, e_2) \in \arg\max_{(\hat{e}_1,\hat{e}_2)} p(\hat{e}_1)w_h - \hat{e}_1 + \hat{e}_2[1-p(\hat{e}_1)]\{p(\hat{e}_1,1)w_h - \beta\}
\end{aligned}$$

The first constraint is the participation constraint that will make the agent to sign the contract at $t = 0$. The second constraint is going to make the agent to choose the strategy (e_1, e_2) that the principal desires.

The minimum necessary wage for each strategy, which satisfies all the constraints is given by the solution to the problem under perfect information, recalling though that the agent keeps only w_h and not y_h. The expected profits for each strategy for the principal are:

We then can separate the three cases, each one of them represented in a column. The most complex is the first column while the simplest is the second column.

1. $\beta \leq \frac{p_1}{(1-p_0)-p_1(1-p_1)}$

   (a) $(0,0)$ versus $(0,1)$,

$$p_0 y_h \geq \left(y_h - \frac{\beta}{p_1}\right)(p_0 + (1-p_0)p_1)$$
$$\beta\left(\frac{p_0 + p_1(1-p_0)}{(1-p_0)p_1^2}\right) \geq y_h$$

	$\beta \leq \frac{p_1}{(1-p_0)-p_1(1-p_1)}$	$\frac{p_1}{(1-p_0)-p_1(1-p_1)} \leq \beta \leq \frac{1}{p_1-p_0}$	$\frac{1}{p_1-p_0} \leq \beta$
$(0,0)$	$p_0 y_h$	$p_0 y_h$	$p_0 y_h$
$(1,0)$	-	-	$p_1\left(y_h - \frac{1}{p_1-p_0}\right)$
$(0,1)$	$\left(y_h - \frac{\beta}{p_1}\right)[p_0 + (1-p_0)p_1]$	-	-
$(1,1)$	$y_h - \frac{1-\beta(p_1-p_0)}{(1-p_0)(1-p_1)}$	$y_h - \frac{1+(1-p_1)\beta}{1-p_0}$	$y_h - \beta$

Table 3: Expected profits for principal for each feasible strategy according to β, with moral hazard.

(b) $(0,1)$ versus $(1,1)$,

$$\left(y_h - \frac{\beta}{p_1}\right)[p_0 + (1-p_0)p_1] \geq y_h - \frac{1-\beta(p_1-p_0)}{(1-p_0)(1-p_1)}$$

$$y_h \leq \frac{1}{[(1-p_0)(1-p_1)]^2} - \ldots$$
$$\ldots - \frac{\beta}{(1-p_0)(1-p_1)}\left[\frac{p_1-p_0}{(1-p_0)(1-p_1)} + \frac{p_0 + p_1(1-p_0)}{p_1}\right]$$

(c) $(0,0)$ versus $(1,1)$,

$$p_0 y_h \geq y_h - \frac{1-\beta(p_1-p_0)}{(1-p_0)(1-p_1)}$$
$$y_h \leq \frac{1}{(1-p_0)^2(1-p_1)} - \frac{\beta(p_1-p_0)}{(1-p_0)^2(1-p_1)}$$

The intersection of the constraints is at

$$\underline{\beta} = \frac{p_1^2}{-p_0^2(p_1-1)^2 + p_0(p_1^2 - 3p_1 + 1) + p_1(p_1^2 - p_1 + 1)}$$

2. $\frac{p_1}{(1-p_0)-p_1(1-p_1)} \leq \beta \leq \frac{1}{p_1-p_0}$

This is achieved by comparing the two implementable contracts that are at hand:

$$p_0 y_h \geq y_h - \frac{1+(1-p_1)\beta}{1-p_0}$$
$$y_h \leq \frac{1+(1-p_1)\beta}{(1-p_0)^2}$$

3. $\frac{1}{p_1-p_0} \leq \beta$

(a) $(0,0)$ versus $(1,0)$,

$$p_0 y_h \geq p_1 \left(y_h - \frac{1}{p_1 - p_0} \right)$$
$$y_h \leq \frac{p_1}{(p_1 - p_0)^2}$$

(b) $(1,0)$ versus $(1,1)$,

$$p_1 \left(y_h - \frac{1}{p_1 - p_0} \right) \geq y_h - \beta$$
$$y_h \leq \frac{\beta}{1-p_1} - \frac{p_1}{(1-p_1)(p_1-p_0)}$$

(c) $(0,0)$ versus $(1,1)$,

$$p_0 y_h \geq y_h - \beta$$
$$\frac{\beta}{1-p_0} \geq y_h$$

The intersection of the constraints is at,

$$\overline{\beta} = \frac{p_1(1-p_0)}{(p_1-p_0)^2}$$

A.3.1 With a Cash Unconstrained Agent

In this appendix, I will develop the model with moral hazard, but assuming that the agent is no longer cash constrained.

The maximization problem for the principal is now:

$$\max_{(w_h, w_0),(e_1,e_2)} \quad -w_0 + p(e_1)\{y_h - (w_h - w_0)\} + e_2[1-p(e_1)]\{p(e_1,1)[y_h - (w_h - w_0)]\}$$
$$\text{s.t.} \quad w_0 - e_1 + p(e_1)(w_h - w_0) + e_2[1-p(e_1)]\{p(e_1,1)(w_h - w_0) - \beta\} \geq 0$$
$$(e_1, e_2) \in \arg\max_{(\hat{e}_1,\hat{e}_2)} w_0 - \hat{e}_1 + p(\hat{e}_1)(w_h - w_0) + \hat{e}_2[1-p(\hat{e}_1)]\{p(\hat{e}_1,1)(w_h - w_0) - \beta\}$$

From where we can replace the participation constraint in the objective function and the incentive compatibility constraint. We obtain the following maximization problem:

$$\max_{(e_1,e_2)} - e_1 + p(e_1)y_h + e_2(1-p(e_1))[p(e_1,1)y_h - \beta]$$

Replacing the participation constraint in the first incentive compatibility cancels all the terms, once the optimal must be in the arg max and therefore they should coincide. As w_h and w_0, the principal will choose the effort levels such that the objective function is satisfied, and therefore the effort choice will coincide with the one in the case with full information. He then can adjust the w_h and w_0 to satisfy all the constraints, in particular, the second incentive compatibility.

B Externality

B.1 Full information

In this appendix we repeat the process followed for the baseline contract with full information, but including the externality effect ξ. It will be clear that including this externality will imply a shift to the left on the frontiers between strategies. For space-saving reasons, I will omit several steps that are already clarified in Appendix A.

The agent/principal will stick with a strategy involving $e_2 \neq 0$ if after observing $\hat{y} = 0$ the return of exerting effort (again) is positive. In practical terms, the strategy $(0, e_h)$ will be optimal after observing $\hat{y} = 0$, if and only if $p_1 y_h > \beta e_h + \xi$. In the same way (e_h, e_h) will be optimal after observing $\hat{y} = 0$ if and only if $y_h > \beta e_h + \xi$.

From these two constraints, we obtain two critical values of y_h: $\frac{\beta+\xi}{p_1}$ and $\beta + \xi$. These critical points can be described in the diagram in figure 10.

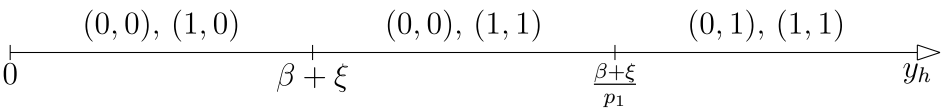

Figure 10: Strategies that can be implemented for different levels of y_h.

Comparing the profits between each pair, we can find the intervals that are relevant.

1. For the first interval $(0, 0)$ versus $(\geq)$ $(1, 0)$,

$$y_h \leq \frac{1}{p_1 - p_0}$$

2. For the second interval $(0, 0)$ versus $(\geq)$ $(1, 1)$,

$$\frac{1 + (1 - p_1)(\beta + \xi)}{1 - p_0} \geq y_h$$

3. For the final interval $(0, 1)$ versus $(\geq)$ $(1, 1)$,

$$\frac{1 - (\beta + \xi)(p_1 - p_0)}{(1 - p_0)(1 - p_1)} \geq y_h$$

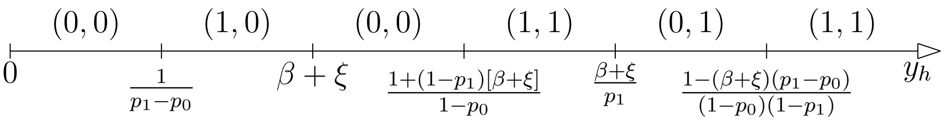

Figure 11: Strategies that can be implemented for different levels of y_h.

The next step is to check if any of the strategy sets is empty.

Name the intervals from left to right as A, B, C, etc.

It is clear that set A is nonempty. For B to be nonempty, it is necessary that:

$$\frac{1}{p_1 - p_0} \leq \beta + \xi$$

For C to be nonempty it is necessary:

$$\beta + \xi \leq \frac{1}{p_1 - p_0}$$

From here we get that B and C are mutually exclusive sets. Now for D to be nonempty,

$$\frac{p_1}{(1-p_0) - p_1(1-p_1)} \leq (\beta + \xi)$$

For E to be non-empty:

$$\beta + \xi \leq \frac{p_1}{(1-p_0) - p_1(1-p_1)}$$

So again, E and D are mutually exclusive.

For a given effort level e_h, the optimal contracts with moral hazard would exclude the combination A-C-E-F, but consider only A-B-E-F, A-B-D-F, and A-C-D-F.

In Figure 5 it is represented the contract with full information and the externality. As can be seen in the derivation, this was done as if the principal and the agent were the same single person. The two key betas have been labeled as $\underline{\beta_1^E} = \frac{p_1}{(1-p_0)-p_1(1-p_1)} - \xi$ and $\overline{\beta_1^E} = \frac{1}{p_1-p_0} - \xi$. The superscript E means variables considering the externality.

B.2 Moral Hazard

The maximization problem for the principal is:

$$\max_{w_h,(e_1,e_2)} \quad p(e_1)\{y_h - w_h\} + e_2[1-p(e_1)]\{p(e_1,1)(y_h - w_h) - \xi\}$$
$$\text{s.t.} \quad p(e_1)w_h - e_1 + e_2[1-p(e_1)]\{p(e_1,1)w_h - \beta\} \geq 0$$
$$(e_1,e_2) \in \arg\max_{(\hat{e}_1,\hat{e}_2)} p(\hat{e}_1)w_h - \hat{e}_1 + \hat{e}_2[1-p(\hat{e}_1)]\{p(\hat{e}_1,1)w_h - \beta\}$$

The first constraint is the participation constraint that will make the agent to sign the contract at $t = 0$. The second constraint is going to make the agent to choose the strategy (e_1, e_2) that the principal desires. Finally, we will also require interim rationality.

The minimum necessary wage for each strategy, which satisfies all the constraints is given by the solution to the problem under full information, recalling though that the agent keeps only w_h and not y_h. The expected profits for each strategy for the principal are:

	$\beta \leq \frac{p_1}{(1-p_0)-p_1(1-p_1)}$	$\frac{p_1}{(1-p_0)-p_1(1-p_1)} \leq \beta \leq \frac{1}{p_1-p_0}$	$\frac{1}{p_1-p_0} \leq \beta$
$(0,0)$	$p_0 y_h$	$p_0 y_h$	$p_0 y_h$
$(1,0)$	-	-	$p_1\left(y_h - \frac{1}{p_1-p_0}\right)$
$(0,1)$	$\left(y_h - \frac{\beta}{p_1}\right)[p_0 + (1-p_0)p_1] - (1-p_0)\xi$	-	-
$(1,1)$	$y_h - \frac{1-\beta(p_1-p_0)}{(1-p_0)(1-p_1)} - (1-p_1)\xi$	$y_h - \frac{1+(1-p_1)\beta}{1-p_0} - (1-p_1)\xi$	$y_h - \beta - (1-p_1)\xi$

Table 4: Expected profits for principal for each feasible strategy according to β, with moral hazard.

We then can separate the three cases, each one of them represented in a column. The most complex is the first column while the simplest is the second column.

1. $\beta \leq \frac{p_1}{(1-p_0)-p_1(1-p_1)}$

   (a) $(0,0)$ versus $(0,1)$,

$$p_0 y_h \geq \left(y_h - \frac{\beta}{p_1}\right)(p_0 + (1-p_0)p_1) - (1-p_0)\xi$$
$$y_h \leq \beta\left(\frac{p_0 + p_1(1-p_0)}{(1-p_0)p_1^2}\right) + \frac{\xi}{p_1}$$

   (b) $(0,1)$ versus $(1,1)$,

$$\left(y_h - \frac{\beta}{p_1}\right)[p_0 + (1-p_0)p_1] - (1-p_0)\xi \geq y_h - \frac{1-\beta(p_1-p_0)}{(1-p_0)(1-p_1)} - (1-p_1)\xi$$

$$
\begin{aligned}
y_h \leq & \frac{1}{[(1-p_0)(1-p_1)]^2} - \ldots \\
& - \frac{\beta}{(1-p_0)(1-p_1)} \left[\frac{p_1 - p_0}{(1-p_0)(1-p_1)} + \frac{p_0 + p_1(1-p_0)}{p_1} \right] - \ldots \\
& - \frac{p_1 - p_0}{(1-p_0)(1-p_1)} \xi
\end{aligned}
$$

(c) $(0,0)$ versus $(1,1)$,

$$
\begin{aligned}
p_0 y_h &\geq y_h - \frac{1 - \beta(p_1 - p_0)}{(1-p_0)(1-p_1)} - (1-p_1)\xi \\
y_h &\leq \frac{1}{(1-p_0)^2(1-p_1)} - \frac{\beta(p_1-p_0)}{(1-p_0)^2(1-p_1)} + \frac{1-p_1}{1-p_0}\xi
\end{aligned}
$$

The intersection of the constraints is at

$$
\underset{\sim}{\beta_2} = \frac{p_1^2 + [p_1(1-p_0)(1-p_1)\{(1-p_1)p_1 - (1-p_0)\}]\xi}{[p_0 + (1-p_0)p_1](1-p_0)(1-p_1) + (p_1 - p_0)p_1^2}
$$

2. $\frac{p_1}{(1-p_0)-p_1(1-p_1)} \leq \beta \leq \frac{1}{p_1 - p_0}$

   This is achieved by comparing the two implementable contracts that are at hand:

$$
\begin{aligned}
p_0 y_h &\geq y_h - \frac{1 + (1-p_1)\beta}{1-p_0} - (1-p_1)\xi \\
y_h &\leq \frac{1 + (1-p_1)\beta}{(1-p_0)^2} + \frac{1-p_1}{1-p_0}\xi
\end{aligned}
$$

3. $\frac{1}{p_1 - p_0} \leq \beta$

   (a) $(0,0)$ versus $(1,0)$,

$$
\begin{aligned}
p_0 y_h &\geq p_1 \left(y_h - \frac{1}{p_1 - p_0} \right) \\
y_h &\leq \frac{p_1}{(p_1 - p_0)^2}
\end{aligned}
$$

   (b) $(1,0)$ versus $(1,1)$,

$$
\begin{aligned}
p_1 \left(y_h - \frac{1}{p_1 - p_0} \right) &\geq y_h - \beta - (1-p_1)\xi \\
y_h &\leq \frac{\beta}{1-p_1} - \frac{p_1}{(1-p_1)(p_1-p_0)} + \xi
\end{aligned}
$$

(c) $(0,0)$ versus $(1,1)$,

$$p_0 y_h \geq y_h - \beta - (1-p_1)\xi$$
$$\frac{\beta}{1-p_0} + \frac{1-p_1}{1-p_0}\xi \geq y_h$$

The intersection of the constraints is at,

$$\tilde{\beta}_2 = \frac{p_1(1-p_0)}{(p_1-p_0)^2} - (1-p_1)\xi$$

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