A Work Project, presented as part of the requirements for the Award of a Master Degree in Finance from the NOVA – School of Business and Economics.

VOLATILITY SURFACE AND MARKET PRICE UNCERTAINTY

Elia Lahouiri 31780

A Project carried out on the Master in Finance Program, under the supervision of:

Professor Paulo M.M. Rodrigues

23rd May 2018

ABSTRACT:

Volatility surface is a major factor in the valuation of several instruments. The models behind it are many. In this work project, we are going to discuss the major stochastic models used in practice, the hypotheses of these models, how to construct them and their main drawbacks. These drawbacks lead sometimes to valuation uncertainty in the market which is an important risk factor in different fields in finance. In this work we are going to focus on risk management. The aim of this analysis is to understand the motivation under which the European Bank Authority (EBA) creates a prudent valuation framework and the risk management solution to assess this risk.

Key words: Volatility surface, Risk management, Price uncertainty, Regulation.

1. Introduction:

Volatility is a central topic in finance and financial mathematics. The ability to forecast volatility is of fundamental importance and it is widely used in several areas. In this work project, we are going to analyze how to build a volatility surface and where to use it. Moreover, we are going to focus our attention on the volatility surface as a risk factor, in particular in the field of risk management. Looking at risk management, it is possible to detect a connection between the valuation uncertainty caused by the implied volatility forecast and prudent value, it is the sum between the fair value and any value adjustment necessary (art.34 CRR 575/2013[1]) requested to banks in Europe. Prudent valuation results from the necessity to consider in the valuation of a certain instrument the additional source of uncertainty that is not considered in the simple fair value. A prudent value is a safer valuation of the instrument, in case of uncertainty on the market price.

From the idea that the fair value is not a safe enough measure, prudent valuation becomes a regulatory framework, to guarantee a higher certainty in the trade book of the banks. The idea of prudent valuation is strongly linked to the efficient market hypothesis (EMH). In the case of completeness of the market, prudent valuation would not have to exist, because of the law of one price. But in reality, a single instrument, may have different prices, although with small differences. This can create problems in the assessment of the risk, from a risk management point of view. This concept, in the case of this work, is linked to implied volatility surfaces (IVS) and to the different models used. Even though from our analysis we are going to claim that the market is incomplete, it does not mean that it is possible to have an arbitrage profit. In fact, we are in the super- and sub-replication bounds hypotheses; which corresponds to an interval of prices where the market is still incomplete but the difference in price is not big

[1] CRR 575/2013 is the Capital Requirement Regulation.

enough to make profits a possibility. This is the theoretical explanation of the need for prudent valuation and why regulation in this sense is particularly reasonable.

2. Volatility surface

The volatility surface is constructed from all options prices of a certain stock, given maturity and moneyness. This corresponds to a curve formed by plotting the implied volatility derived from the market's option price, the strike price and maturity. The result is the implied volatility of a given asset at different moneyness and maturity combinations.

2.1. Stylized facts

The typical features of stock returns are:

Leptokurtosis

One of the fundamental features of market volatility are the heavy tails and sharp peaks. It means that the probability of extreme events occurring is higher than in a normal distribution. Thus, the presence of fat tails needs to be taken into account in the construction of every stochastic model.

Volatility clustering

Volatility presents a certain momentum, so a variation today is usually followed by a similar variation in the following days.

Persistence

Usually, any shock that impacts volatility needs a long horizon to be completely absorbed. Moreover, this process is very smooth.

Asymmetry

When there is a negative change in the market, volatility increases and when the change is positive volatility decreases but at a lower rate. It means that volatility is more sensitive to negative variations than to positive ones. This behavior is linked to two different theories:

1. Leverage effect: this hypothesis says that the asymmetric behavior of volatility is caused by the leverage D/E ratio (debt to equity ratio, i.e., the ratio between the debt value and the equity value of a company). In fact, if the price decreases, the equity value drops and consequently the ratio increases, increasing in this way the risk of the firm and volatility.
2. Volatility feedback effect: this hypothesis considers that the volatility is priced, so in this case, the required rate of return increases when prices decrease; causing in this way a different reaction to positive or negative price variations.

2.2. Stochastic volatility model

We consider that volatility follows a stochastic process. With this approach, it is easier to satisfy the stylized facts previously described, and it is widely used in finance. In specific, we consider that

$$\varepsilon_t = \sigma_t z_t, \quad \sigma_t^2 = \exp(h_t), \quad t = 1, \dots, T$$

where h_t is some stochastic process. One possibility, used also by Knight and Satchell (2007), is to use an AR(1). In this case, the process is described as:

$$h_t = \alpha + \beta h_{t-1} + \eta_t, \quad \eta_t \sim NID(0, \sigma_\eta^2).$$

Thus, although volatility is a latent variable that is not observed directly it can be estimated from historical data.

The Heston Model

The Heston model is widely used in practice and is based on the correlation between the volatility and the spot price of the underlying asset. It considers that volatility follows an Ornestein-Uhlenbeck (OU) process (described in detail below) and that the spot price is a Wiener process. The model is very common because of the simplicity in calibration and the good empirical results, even though it is not as realistic as other models. Focusing on the market approximation capacity, Gatherals (2006) obtained good results in general but highlighted major problems in longer maturities when fitting the smile of the surface.

SABR (stochastic alpha beta rho):

For valuation purpose, the number of possible models applicable is significantly large. A model commonly used in practice is the SABR, that seems to be empirically superior to the Heston model. In this model the asset price and the volatility are correlated, so variation in volatility implies variation in the asset price and vice versa. To consider the correlation between the volatility and the asset, the model requests that (Hagan, Kumar, Lesniewski, Woodward (2002)):

$$d\hat{F} = \hat{a}\,\hat{F}^{\beta} dW_1$$

$$d\hat{a} = v\hat{a}\,dW_2$$

where F is the forward on the asset considered, a is the volatility, and W_1 and W_2 are two correlated Brownian motions that describe the movement of the asset's price and volatility, respectively.

This model fits the skewness and the smile of the volatility. Back-testing the results on the implied volatility surface of the market suggests interesting results but only for short maturities because of the no mean-reverting properties; see Gatheral (2006).

This model is mainly used to mark and manage the implied volatility surface. As highlighted by Jackel and Kahhl (2007), this model has a major issue in the convergence of one of the variables, future prices, that can lead to wrong forecasting.

2.3.Model implementation

We have analyzed some fundamental tools and methods to model volatility, now it seems appropriate to explain more in detail how to implement a model to explain some common processes across stochastic models. We are going to follow the method of Wang, Chen, Tao and Zhang (2007).

Using the options present in the market with different maturities and moneyness, it is possible to construct directly a surface, using a quadratic method of interpolation (see Figure 1).

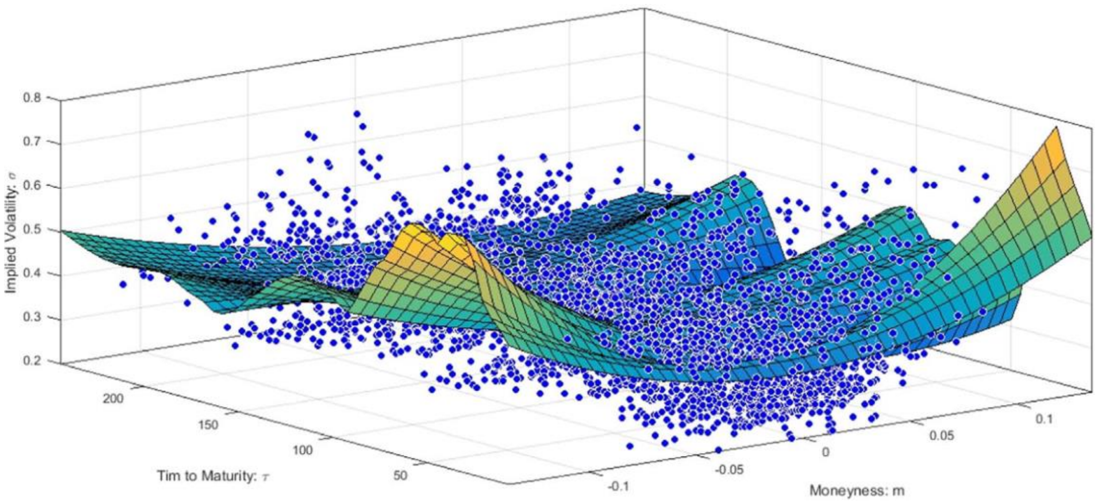


Figure 1:Volatility surface interpolated by a local quadratic method on Shanghai 50ETF

The model for forecasting volatility is defined by two equations:

$$\hat{\sigma}(t, m, \tau) = g[t, m, \tau, y_1(t), \dots y_p(t)], \quad (1)$$

$$dy_i(t) = \alpha_i dt + \gamma_i d\omega_i(t), \quad i=1, 2, \dots, p \quad (2)$$

where $y_1, \dots, y_p$ are latent factors, τ is time to maturity, and t is the time index.

Equation (1) is the volatility at time t and equation (2) corresponds to the process of latent factors that are central to the shape of the implied volatility surface. In the second equation, α_i is a drift parameter, γ_i is a volatility parameter and $d\omega_i(t)$ is an increment of a Brownian motion that put a stochastic fluctuation inside the model. The first equation is the most important one because it explains the relationship between the volatility and the market factors. The equation (2) considers the fluctuations of the latent factors that change the shape of the volatility surface.

The model is built in 3 steps. First a cross-sectional analysis between the implied volatility and the latent factors is considered. Second, how the variations of the latent factors impact the implied volatility is determined. Third, a model is created that considers both the cross-sectional and the dynamic variations.

Step-one: Cross-sectional analysis

This can be seen as a cross-sectional model of IVS:

$$\sigma(m, \tau) = \beta_1 + \beta_2 m + \beta_3 m_2 + \beta_4 \tau + \beta_5 m\tau + \varepsilon \qquad (3)$$

where β_2 and β_3 are the first and second derivatives in term of moneyness, it means that they represent the slope and curvature; β_4 is the first derivative in term of maturity; β_5 is a factor that connects moneyness, maturity and volatility; and ε is an error term.

This functional form is due to the sticky delta rule (the skewness of volatility remains unchanged with moneyness), applied to the implied volatility surface. From this idea, the value of σ is just a simple Taylor expansion.

Time series

The cross-sectional analysis highlights the main variables that affect volatility, but it is important to consider the dynamics over time which can change the value of the betas and the value of the implied volatility surface.

The OU[2] process is used for the estimation of the betas. This process is mean-reverting and Markovian[3], properties that fit well with the volatility surface.

The model proposed for estimation of the slope parameters is:

$$d\beta_i(t) = k_i\big(\theta_i - \beta_i(t)\big)dt + \gamma_i d\omega_i(t)^4 \qquad (4)$$

where k_i is the rate at which it is necessary to modify the model, the so-called revision rate; θ_i is the average of the factors and γ_i their variations.

Dynamic system

From the two previous sections, it is possible to create a dynamic system that takes into consideration the cross-sectional analysis and the time series dynamics of the latent factors. The last equation of the system accounts for the correlation between the latent factors. In specific,

$$\begin{cases} ln\sigma(m,\tau) = \beta_1 + \beta_2 m + \beta_3 m^2 + \beta_4 m_\tau + \varepsilon & (5) \\ d\beta_i(t) = k_i\big(\theta_i - \beta_i(t)\big)dt + \gamma_i d\omega_i(t) & (6) \\ d\omega_i(t)\omega_j(t) = \rho_{ij}dt \qquad i,j = 1, \dots 5 & (7) \end{cases}$$

[2] OU process: Ornestein_Uhlenbeck process. It is a Gaussian-Markov process, homogenous and stationary. Moreover, it is mean -reverting.

[3] Markovian: it is a process that depends just on the previous event.

[4] Approximated solution of the estimated beta $\beta_i(t) = \theta\big(1 - e^{-k\Delta t}\big) + e^{-k\Delta t}\beta(t-1) + \gamma\sqrt{\frac{1}{2k}(1 - e^{-2k\Delta t})z(t)}$

From the dynamic system in (5) – (6), it is possible to create a state space[5] model, such as,

$$\begin{cases} z(t) = H(t)x(t) + \varepsilon(t) & (8) \\ x(t) = c + Ax(t-1) + \xi(t) & (9) \end{cases}$$

Equations (8) and (9) are a state space representation of the behaviour of the volatility. This system is properly defined in equations (10) and (11) below.

Estimation

The previous process is the basis to estimate the factors. The equation given below in (10) is the measurement equation, that is the equation of estimated values. The first matrix corresponds to the factor values at time t, observed in the market. The sample consists of q observations, and the betas are the coefficients of the latent factors. In other words,

$$\begin{bmatrix} \ln \sigma_t(m_1\tau_1) \\ \vdots \\ \ln \sigma_t(m_q\tau_q) \end{bmatrix} = \begin{bmatrix} 1 & m_1 m_1^2 & \tau_1 m_1 \tau_1 \\ \vdots & \vdots \ \vdots & \vdots \ \vdots \\ 1 & m_q m_q^2 & \tau_q m_q \tau_q \end{bmatrix} \begin{bmatrix} \beta_1(t) \\ \vdots \\ \beta_q(t) \end{bmatrix} + \begin{bmatrix} \varepsilon_1(t) \\ \vdots \\ \varepsilon_q(t) \end{bmatrix} \quad (10)$$

State equation

The OU process derived has some interesting properties that describe well the behavior of the volatility surface, i.e.,

$$\begin{bmatrix} \beta_1(t) \\ \vdots \\ \beta_5(t) \end{bmatrix} = \begin{bmatrix} \theta_1(1-e^{-k_1\Delta t}) \\ \vdots \\ \theta_5(1-e^{-k_5\Delta t}) \end{bmatrix} + \begin{bmatrix} e^{-k_1\Delta t} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-k_5\Delta t} \end{bmatrix} \begin{bmatrix} \beta_1(t-1) \\ \vdots \\ \beta_5(t-1) \end{bmatrix} + \begin{bmatrix} \xi_1(t) \\ \vdots \\ \xi_5(t) \end{bmatrix} \quad (11)$$

$$\xi(t_i) | F_{t_{i-1}} \sim N(0, Q) \quad (12)$$

$$Q = \left[\frac{\gamma_i \gamma_j \rho_{ij}}{k_i + k_j} (1 - e^{-(k_i+k_j)\Delta t}) \right] \quad with\ i,j = 1, \ldots, 5 \quad (13)$$

[5] State space: it is a system of possible values of the variables, in its case it represents the possible values of the factors, latent or not.

The empirical evidence obtained from the model is useful for the central part of the volatility surface, in particular for at the money (ATM) and medium maturity options. For the external part, the results are less brilliant and the error of the model tends to increase. The results in-sample are satisfactory over the whole surface, and the major issues emerge out-of-sample.

Calibration

The main issue of all the stochastic models is calibration. It is not easy to perfectly or approximately fit the market data and moreover make the model arbitrage-free. The main arbitrage to avoid is Butterfly arbitrage, calendar effects and maintain the valuation inside an arbitrage-free bound. The bid and ask for each maturity are not continuous so it is necessary to interpolate them. It may create problems in the calibration and it is not obvious how to avoid arbitrage.

3. Model results

From all the previous models there are some fundamental facts to consider. The movement of the volatility as a random process, the parameterization of these processes from an historical perspective, based on the implied volatility and the calibration to avoid arbitrage. But it is also evident that the consequent valuation of the instruments is not unique, but in a certain interval, and it can change. All the papers considered present a good theoretical and practical explanation of the existence of prudent valuation. For instance, as noted by Gatheral (2006), using a simple Black-Sholes model, with implied volatility, means changing continuously this variable, to have a perfect hedging, so the stochastic model can be useful in this context. Moreover, for pricing exotic derivatives it is also more efficient to use a stochastic model, because of the possible mistakes of the implied volatility model but some derivatives may change the valuation outcome with regard to the model used. In the next section of the work project, we are going to analyze how the different volatility surfaces are linked, on a market-data basis; and how the use of a single surface can be a good proxy to take into account the uncertainty of an entire portfolio. We are going to follow the practical standard to calculate the additional value adjustment of the market price uncertainty (AVA MPU). Moreover, this analysis, commonly used to reduce the number of risk factors (in our case volatility) can highlight the legislator ratio on valuation problems and how this theoretical idea has an impact on legislation and on risk management practice.

4. Empirical analysis:

The purpose of this section is to apply the theoretical framework described in the previous sections to an euro-based equity portfolio. The underlying composition is Eni, AXA, Bayer, BMW, Enel, Generali, Telefonica, SPMIB and SPMIB Bank. In our case, the risk factor taken into account is the volatility surface, in particular, the volatility of the single assets, at different maturity and moneyness. The risk factors are represented by a matrix.

In this analysis, we are focusing only on some of the tradable nodes that we have considered to be more representative. In particular, we have chosen three moneyness, 0.8, 1 and 1.2, to cover the in-the-money, at-the-money, and out-the-money scenarios, and five maturities: 365, 730, 1095, 1460, 1825 days. Thus, we can describe the tradable nodes as:

0.8	0.8	0.8	0.8	0.8	1	1	1	1	1	1.2	1.2	1.2	1.2	1.2
365	730	1095	1460	1825	365	730	1095	1460	1825	365	730	1095	1460	1825

The tradable notes taken into account are 15 and they are based on market data, as required by the European Bank Authority Regulatory Technical Standards (EBA RTS) regulation on the prudent valuation, when they are reliable and available.

Following a sensitivity approach, the additional value adjustment market price uncertainty (AVA MPU) is equal to,

$$APV\ A_{ij}^{MPU}(t) = 0.5\left|\frac{\partial FV_i(t)}{\partial u_j}(u_j - \widehat{u_j})\right|$$

where FV is the fair value, u is the valuation input and $\hat{u}$ is its prudent value.

To calculate the AVA MPU it is necessary to:

1) Calculate the sensitivity to the pillars chosen as tradable nodes;

2) Calculate the uncertainty level $(u_j - \widehat{u_j})$;

3) Perform the Profit&Losses (P&L) variance test required by EBA for all the risk factors and tradable nodes in a 100 days horizon.

We are going to focus on the first step, that is closer to the theoretical model of the volatility surface. In our analysis, the market data initiate from 11/08/2017 and from 11/08/2003, for the 5-year analysis, to 29/12/2017. The data is provided by Intesa Sanpaolo.

4.1. Identification of the risk factor drivers

We want to identify which volatility surface can explain the movement of the others, that we have already considered as risk factors for our portfolio. The main idea is to simplify the number of risk factors that are affecting the value of the portfolio, finding one factor that can represent well the others. The identification of a single volatility surface can lead to a reduction of the capital caused by the reduced sensitivity of the AVA MPU. The preliminary analysis, that we want to perform, is a correlation analysis between the risk factors described in the previous section and the candidate risk driver. In this case, we have chosen the EUROSTOXX50 volatility as a driver. The correlation used in this work is the Pearson correlation:

$$\rho_{XY} = \frac{cov(X, Y)}{\sigma_X \sigma_Y}$$

where X and Y are two random variables.

In our specific case, we took X and Y as the vectors of volatilities, with a certain moneyness and maturity, and we have calculated the correlation between the EUROSTOXX50 and the other assets, both in term of volatility and variation in volatility. The first correlation implemented is based on a sample of 100 days (for robustness of the test a larger sample of 5 years is also considered). Moreover, we have calculated the correlations between different pillars, focusing in the subsequent analysis on the vector with the same moneyness/maturity.

Table 1 below is an example of Eni's correlation analysis, the same analysis has been done for all stocks. In Table 1, we have calculated the correlation between EUROSTOXX50 volatility with moneyness 0.8, 1 and 1.2 at all maturities, with the 15 pillars of Eni.

Table 1: Eni's correlation analysis of simple volatility

ENI	0,8					1					1,2				
Eurostoxx(0,8)	365	730	1095	1460	1825	365	730	1095	1460	1825	365	730	1095	1460	1825
365	0,977	0,982	0,981	0,980	0,979	0,964	0,976	0,978	0,979	0,978	0,963	0,971	0,976	0,977	0,976
730	0,982	0,985	0,983	0,980	0,978	0,971	0,979	0,978	0,978	0,976	0,969	0,974	0,974	0,974	0,972
1095	0,983	0,986	0,984	0,982	0,980	0,973	0,980	0,980	0,979	0,978	0,971	0,976	0,976	0,976	0,974
1460	0,983	0,986	0,985	0,984	0,983	0,972	0,980	0,981	0,982	0,981	0,971	0,976	0,978	0,979	0,978
1825	0,981	0,985	0,985	0,985	0,985	0,971	0,980	0,982	0,983	0,983	0,969	0,975	0,979	0,981	0,981

Eurostoxx(1)	365	730	1095	1460	1825	365	730	1095	1460	1825	365	730	1095	1460	1825
365	0,967	0,972	0,973	0,974	0,973	0,953	0,968	0,972	0,974	0,974	0,953	0,962	0,971	0,974	0,973
730	0,982	0,986	0,985	0,983	0,981	0,970	0,980	0,981	0,981	0,980	0,969	0,975	0,978	0,979	0,978
1095	0,982	0,986	0,984	0,982	0,981	0,972	0,980	0,981	0,980	0,979	0,971	0,976	0,978	0,978	0,976
1460	0,982	0,986	0,985	0,984	0,983	0,972	0,981	0,982	0,982	0,982	0,971	0,976	0,979	0,980	0,979
1825	0,982	0,985	0,986	0,985	0,985	0,972	0,981	0,983	0,984	0,983	0,970	0,977	0,980	0,982	0,981

Eurostoxx(1,2)	365	730	1095	1460	1825	365	730	1095	1460	1825	365	730	1095	1460	1825
365	0,950	0,958	0,962	0,964	0,963	0,937	0,956	0,962	0,965	0,964	0,939	0,952	0,963	0,966	0,966
730	0,979	0,984	0,985	0,984	0,982	0,968	0,979	0,981	0,982	0,981	0,967	0,975	0,979	0,980	0,979
1095	0,980	0,985	0,983	0,981	0,979	0,970	0,979	0,979	0,979	0,978	0,969	0,975	0,977	0,977	0,975
1460	0,981	0,985	0,984	0,983	0,982	0,971	0,980	0,981	0,981	0,980	0,970	0,976	0,979	0,979	0,978
1825	0,982	0,986	0,986	0,985	0,985	0,973	0,981	0,983	0,984	0,983	0,971	0,978	0,981	0,982	0,981

A similar analysis has been done with the variation of the volatility, which is defined as the difference between the volatility at time t and the volatility at time t-1, i.e.,

$$V = volatility_t - volatility_{t-1}$$

Table 2: Eni's correlation analysis of volatility variation

ENI variation	0,8					1					1,2				
Eurostoxx(0,8)	365	730	1095	1460	1825	365	730	1095	1460	1825	365	730	1095	1460	1825
365	0,996	0,996	0,995	0,994	0,994	0,994	0,992	0,993	0,993	0,993	0,993	0,993	0,993	0,993	0,993
730	0,996	0,997	0,996	0,996	0,995	0,996	0,994	0,994	0,994	0,994	0,994	0,995	0,994	0,994	0,994
1095	0,997	0,997	0,997	0,996	0,996	0,996	0,994	0,995	0,995	0,995	0,995	0,995	0,995	0,995	0,995
1460	0,997	0,998	0,997	0,996	0,996	0,996	0,994	0,995	0,995	0,995	0,995	0,996	0,995	0,995	0,995
1825	0,997	0,998	0,997	0,997	0,997	0,996	0,994	0,995	0,995	0,995	0,995	0,996	0,995	0,995	0,995

Eurostoxx(1)	365	730	1095	1460	1825	365	730	1095	1460	1825	365	730	1095	1460	1825
365	0,994	0,996	0,995	0,994	0,994	0,994	0,992	0,993	0,993	0,993	0,993	0,993	0,993	0,993	0,993
730	0,996	0,997	0,996	0,995	0,995	0,995	0,994	0,994	0,994	0,994	0,994	0,995	0,994	0,994	0,994
1095	0,997	0,997	0,997	0,996	0,996	0,996	0,994	0,995	0,995	0,995	0,995	0,995	0,995	0,995	0,995
1460	0,997	0,998	0,997	0,996	0,996	0,996	0,994	0,995	0,995	0,995	0,995	0,996	0,995	0,995	0,995
1825	0,997	0,998	0,997	0,997	0,997	0,996	0,994	0,995	0,995	0,995	0,995	0,996	0,995	0,995	0,995

Eurostoxx(1,2)	365	730	1095	1460	1825	365	730	1095	1460	1825	365	730	1095	1460	1825
365	0,994	0,995	0,994	0,994	0,994	0,993	0,993	0,993	0,993	0,993	0,992	0,993	0,992	0,992	0,992
730	0,996	0,997	0,996	0,995	0,995	0,995	0,994	0,994	0,994	0,994	0,994	0,994	0,994	0,994	0,994
1095	0,996	0,997	0,997	0,996	0,996	0,996	0,994	0,995	0,995	0,995	0,995	0,995	0,995	0,995	0,995
1460	0,996	0,997	0,997	0,996	0,996	0,996	0,994	0,995	0,995	0,995	0,995	0,996	0,995	0,995	0,995
1825	0,996	0,998	0,997	0,997	0,997	0,996	0,994	0,995	0,995	0,995	0,995	0,996	0,995	0,995	0,996

This analysis has been done for all the stocks present in our portfolio. But the one that appears to be more interesting is the one with the same moneyness and maturity. Moreover, looking at the complete analysis it is evident that the correlation between the EUROSTOXX50 and the other assets is very high, confirming the idea of using EUROSTOXX50's volatility as a driver for all others. If we compare the results of the volatility and the variations, one of the variations is even higher. These results highlight the strong relationship not only in terms of volatility but also in term of variation in volatilities, over time. It is interesting to have good results also in a small sample of 100 days, a typical sample required by the EBA for the P&L variance analysis. To better understand the whole portfolio, we have constructed some tables for each asset, to show the minimum, maximum and mean values of correlation between curves with the same moneyness and maturity. This synthetic explanation highlights that the correlation in general moves from a min of 0.93 for BMW, with 1.2 moneyness, to 0.998. According to this preliminary analysis it seems necessary to enforce this hypothesis with other tests.

Table 3: Correlations -100 days horizon

ENI			
Moneyness	mean	min	max
0,8	0,9830	0,9770	0,9854
1	0,9759	0,9529	0,9834
1,2	0,9703	0,9391	0,9815

ENI variation			
Moneyness	mean	min	max
0,8	0,9964	0,9955	0,9970
1	0,9947	0,9937	0,9954
1,2	0,9945	0,9923	0,9955

ENEL			
Moneyness	mean	min	max
0,8	0,9800	0,9745	0,9897
1	0,9752	0,9547	0,9897
1,2	0,9724	0,9473	0,9898

ENEL variation			
Moneyness	mean	min	max
0,8	0,9977	0,9968	0,9983
1	0,9972	0,9958	0,9979
1,2	0,9968	0,9955	0,9973

AXA			
Moneyness	mean	min	max
0,8	0,9872	0,9836	0,9908
1	0,9896	0,9836	0,9948
1,2	0,9848	0,9746	0,9939

AXA variation			
Moneyness	mean	min	max
0,8	0,9984	0,9968	0,9991
1	0,9980	0,9957	0,9989
1,2	0,9978	0,9957	0,9987

GENERALI			
Moneyness	mean	min	max
0,8	0,9944	0,9901	0,9966
1	0,9946	0,9834	0,9981
1,2	0,9943	0,9857	0,9981

GENERALI variation			
Moneyness	mean	min	max
0,8	0,9987	0,9971	0,9993
1	0,9986	0,9963	0,9994
1,2	0,9986	0,9969	0,9995

TELEFONICA			
Moneyness	mean	min	max
0,8	0,9875	0,9792	0,9935
1	0,9901	0,9810	0,9947
1,2	0,9872	0,9722	0,9932

TELEFONICA variation			
Moneyness	mean	min	max
0,8	0,9980	0,9944	0,9991
1	0,9978	0,9929	0,9992
1,2	0,9977	0,9926	0,9992

BAYER			
Moneyness	mean	min	max
0,8	0,9913	0,9858	0,9933
1	0,9895	0,9729	0,9954
1,2	0,9852	0,9586	0,9947

BAYER variation			
Moneyness	mean	min	max
0,8	0,9973	0,9939	0,9983
1	0,9970	0,9927	0,9984
1,2	0,9966	0,9914	0,9983

BMW			
Moneyness	mean	min	max
0,8	0,9759	0,9653	0,9833
1	0,9727	0,9650	0,9804
1,2	0,9590	0,9307	0,9677

BMW variation			
Moneyness	mean	min	max
0,8	0,9976	0,9955	0,9984
1	0,9975	0,9951	0,9984
1,2	0,9969	0,9946	0,9980

BANK			
Moneyness	mean	min	max
0,8	0,9720	0,9641	0,9774
1	0,9777	0,9733	0,9858
1,2	0,9790	0,9699	0,9936

BANK variation			
Moneyness	mean	min	max
0,8	0,9990	0,9982	0,9994
1	0,9989	0,9978	0,9994
1,2	0,9988	0,9973	0,9994

SPMIB			
Moneyness	mean	min	max
0,8	0,9890	0,9820	0,9934
1	0,9905	0,9804	0,9965
1,2	0,9917	0,9837	0,9973

SPMIB variation			
Moneyness	mean	min	max
0,8	0,9992	0,9984	0,9996
1	0,9991	0,9979	0,9995
1,2	0,9987	0,9965	0,9995

To increase the reliability of our analysis we have taken a sample of 5 years, to better understand the relationships between the different curves. Replicating the same process already described above, in this case, the results are worse than before, even though they are still satisfactory, and they confirm the idea of using EUROSTOXX50 as driver curve. But with the 5-year sample, it is clear that the use of the simple volatility is not the best choice, because of the lower correlation in this larger sample. On the other hand, the variation in daily volatility maintains a high correlation, reason why we decided to continue the analysis focusing on this variable.

Table 4: Correlations - 5 years horizon

ENI			
Moneyness	mean	min	max
0,8	0,7162	0,5818	0,8932
1	0,6846	0,5898	0,8342
1,2	0,6997	0,6386	0,8022

ENI variation			
Moneyness	mean	min	max
0,8	0,9409	0,9251	0,9503
1	0,9330	0,9191	0,9430
1,2	0,9087	0,8756	0,9241

ENEL			
Moneyness	mean	min	max
0,8	0,7716	0,7663	0,7785
1	0,7889	0,7708	0,8136
1,2	0,7403	0,7028	0,7873

ENEL variation			
Moneyness	mean	min	max
0,8	0,9492	0,9408	0,9539
1	0,9430	0,9177	0,9512
1,2	0,9316	0,8963	0,9441

AXA			
Moneyness	mean	min	max
0,8	0,7309	0,7139	0,7533
1	0,6304	0,6167	0,6569
1,2	0,5411	0,5066	0,5906

AXA variation			
Moneyness	mean	min	max
0,8	0,9109	0,7593	0,9562
1	0,8826	0,6976	0,9405
1,2	0,8495	0,6399	0,9181

GENERALI			
Moneyness	mean	min	max
0,8	0,7340	0,6962	0,7621
1	0,6297	0,5967	0,6436
1,2	0,5679	0,5196	0,6156

GENERALI variation			
Moneyness	mean	min	max
0,8	0,9404	0,9274	0,9554
1	0,9315	0,9129	0,9495
1,2	0,9159	0,8796	0,9391

TELEFONICA			
Moneyness	mean	min	max
0,8	0,6231	0,5263	0,7588
1	0,5448	0,4631	0,6877
1,2	0,5633	0,4993	0,6758

TELEFONICA variation			
Moneyness	mean	min	max
0,8	0,9200	0,7702	0,9614
1	0,9115	0,7461	0,9577
1,2	0,9026	0,7205	0,9530

BAYER			
Moneyness	mean	min	max
0,8	0,9511	0,9080	0,9762
1	0,9360	0,8851	0,9604
1,2	0,8800	0,8163	0,9116

BAYER variation			
Moneyness	mean	min	max
0,8	0,9757	0,9591	0,9831
1	0,9698	0,9433	0,9809
1,2	0,9602	0,9141	0,9780

BMW			
Moneyness	mean	min	max
0,8	0,7735	0,6841	0,8905
1	0,7574	0,6710	0,8493
1,2	0,7448	0,6674	0,7835

BMW variation			
Moneyness	mean	min	max
0,8	0,6450	0,6244	0,6558
1	0,6307	0,6121	0,6410
1,2	0,6180	0,6008	0,6260

BANK			
Moneyness	mean	min	max
0,8	0,7975	0,7647	0,8320
1	0,7266	0,6992	0,7771
1,2	0,6789	0,6356	0,7454

BANK variation			
Moneyness	mean	min	max
0,8	0,9689	0,9473	0,9770
1	0,9624	0,9285	0,9747
1,2	0,9532	0,9024	0,9709

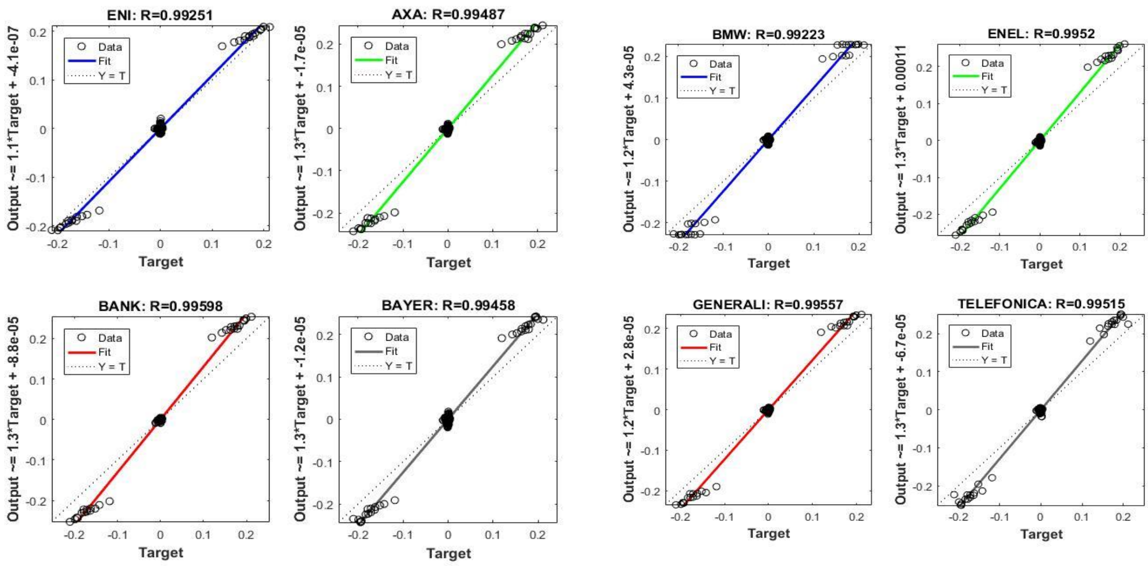
SPMIB			
Moneyness	mean	min	max
0,8	0,8244	0,7734	0,9042
1	0,7535	0,6865	0,8695
1,2	0,7113	0,6185	0,8407

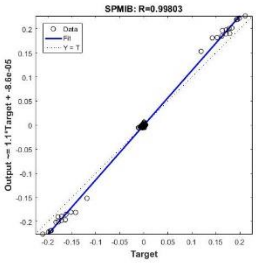
SPMIB variation			
Moneyness	mean	min	max
0,8	0,9782	0,9620	0,9845
1	0,9740	0,9498	0,9825
1,2	0,9677	0,9282	0,9809

4.2. Regressions

It is now necessary to continue the analysis with different tools. A good possibility to understand the relationship between the curves is to build linear regression. According to the previous analysis, it seems appropriate to continue with the variations in volatility, because of the higher correlation coefficients. The results of the regressions are interesting, and confirm the idea of using EUROSTOXX50 as a driver.

Table 5: Linear regression between volatility variations of EUROSTOXX50 and stocks





These graphs are the regression adjustments made for every asset with the EUROSTOXX50, all in variation terms. The results are very good, with a coefficient of determination (R^2) higher than 0.99 for all regressions and betas close to one. The R^2 highlights that the regression fits very well, and the capacity of EUROSTOXX50 volatility variation to explain the volatility variation of the other assets is very good. Moreover, betas close to one mean that the relation between variation in the index and variation in the stock volatilities are almost 1:1. These regressions are estimated with a matrix of all the maturity/moneyness combinations.

To better understand the relationship between curves with same maturity/moneyness we have run a regression just for this sample. The results are of the same quality as before.

Table 6: Coefficient of determination results (R^2)

Moneyness	Maturity	ENI	Bank	AXA	BAYER	BMW	ENEL	GENERALI	TELEFONICA	SPMIB
	365	0,9770	0,9908	0,9741	0,9858	0,9818	0,9745	0,9901	0,9792	0,9820
	730	0,9854	0,9836	0,9641	0,9933	0,9833	0,9897	0,9946	0,9879	0,9917
0,8	**1095**	0,9840	0,9883	0,9698	0,9932	0,9782	0,9828	0,9952	0,9845	0,9886
	1460	0,9838	0,9875	0,9748	0,9932	0,9710	0,9783	0,9966	0,9923	0,9895
	1825	0,9850	0,9858	0,9774	0,9907	0,9653	0,9748	0,9957	0,9935	0,9934
	365	0,9529	0,9913	0,9858	0,9729	0,9735	0,9547	0,9834	0,9810	0,9804
	730	0,9799	0,9948	0,9781	0,9952	0,9804	0,9897	0,9964	0,9926	0,9965
1	**1095**	0,9808	0,9918	0,9733	0,9954	0,9749	0,9818	0,9979	0,9881	0,9898
	1460	0,9823	0,9864	0,9739	0,9932	0,9696	0,9766	0,9981	0,9944	0,9919
	1825	0,9834	0,9836	0,9771	0,9910	0,9650	0,9733	0,9970	0,9947	0,9942
	365	0,9391	0,9746	0,9936	0,9586	0,9307	0,9473	0,9857	0,9722	0,9837
	730	0,9752	0,9939	0,9856	0,9933	0,9638	0,9898	0,9929	0,9913	0,9973
1,2	**1095**	0,9767	0,9915	0,9733	0,9947	0,9677	0,9785	0,9972	0,9863	0,9888
	1460	0,9791	0,9836	0,9699	0,9909	0,9661	0,9743	0,9981	0,9931	0,9935
	1825	0,9815	0,9804	0,9727	0,9887	0,9669	0,9719	0,9974	0,9932	0,9951

The value of the R^2 for all the assets, couple maturity/moneyness, is very high and close to one. This confirms the previous hypothesis of a good representativeness of EUROSTOXX50 volatility variation.

Moreover, if we look at the beta estimates of all single regressions, the maximum value is 1.69 and the minimum is 0.98. It confirms the result of a close 1:1 relation between the curves with same moneyness/maturity.

Table 7: Beta estimates

Moneyness	Maturity	ENI	Bank	AXA	BAYER	BMW	ENEL	GENERALI	TELEFONICA	SPMIB
	365	0,9868	1,1530	1,1986	1,1110	1,0733	1,2247	1,1099	1,0590	1,0705
	730	1,0500	1,1861	1,2452	1,1880	1,1488	1,2658	1,1705	1,2200	1,1141
0,8	**1095**	1,0501	1,2287	1,2668	1,2429	1,1817	1,2814	1,1954	1,2806	1,1330
	1460	1,0526	1,2364	1,2601	1,2535	1,1810	1,2630	1,1881	1,3014	1,1318
	1825	1,0507	1,2226	1,2478	1,2469	1,1725	1,2428	1,1750	1,2883	1,1257
	365	1,1783	1,3793	1,4438	1,3289	1,4844	1,4228	1,3172	1,2846	1,1864
	730	1,1598	1,3370	1,3855	1,3048	1,3979	1,3684	1,2998	1,3772	1,1988
1	**1095**	1,1069	1,3067	1,3316	1,2780	1,3276	1,3203	1,2555	1,3436	1,1516
	1460	1,0910	1,2612	1,2947	1,2541	1,2830	1,2867	1,2180	1,3238	1,1286
	1825	1,0735	1,2196	1,2624	1,2266	1,2473	1,2565	1,1833	1,2848	1,1004
	365	1,4143	1,6678	1,6948	1,6019	1,6199	1,6447	1,5932	1,5046	1,2781
	730	1,2436	1,4554	1,4913	1,4033	1,3979	1,4527	1,4337	1,5011	1,2688
1,2	**1095**	1,1183	1,3245	1,3544	1,2748	1,2444	1,3086	1,2696	1,3503	1,1393
	1460	1,0937	1,2427	1,3043	1,2280	1,1747	1,2726	1,2087	1,3084	1,1040
	1825	1,0623	1,1845	1,2472	1,1858	1,1338	1,2391	1,1581	1,2542	1,0619

From this analysis, it seems clear that the historical movements of the EUROSTOXX50 are able to describe the movements of the other curves. The analysis is not sufficient to assess the risk, but it is a measure of the possibility to reduce the number of risk factors affecting an instrument, considering only EUROSTOXX50. From this preliminary analysis, it could be interesting to quantitively assess the result of the AVA MPU based on all the risk factors and only on the index. Regulation requires to implement a P&L variance test to assess the capacity of the reduced number of risk factor to maintain a prudent enough valuation. In this work project we decided to focus more on the reasons of prudent valuation and the possibility of considering a reduced number of risk factors for this purpose.

It is important to highlight that some of the stocks taken into consideration are part of EUROSTOXX50, which can be problematic in the regression. However, we believe that it has a minimum impact on the results, since these stocks are a very small percentage of the total volume of EUROSTOXX50. In fact, Eni is the biggest one in terms of volume and it corresponds to just 1.53% of the index. Moreover, considering Generali, SPMIB and SPMIB Banks, not included in the index, they present good results, confirming our conjecture.

5. Conclusions

The analysis of the literature on some major models on volatility and the consequent empirical evidences highlight the fact that in practice it is possible to use different models and that they can have problems in fitting reality. This issue can create difficulties in the valuation and consequently a level of uncertainty for some instruments. In the work project, we have tried to explain the motivation of this uncertainty and its effects on the legislation, going from the theoretical motivation, through the main models and their drawbacks, to the data. Focusing on the data, strong correlation between market volatility and the volatility of several assets is observed, basing the analysis on the market implied volatility. This strong correlation is a fundamental fact to manage more easily the market price uncertainty and its impact on the capital of the banks. Even thought, the price of several instruments can be different between valuations, it is still possible to assess the risk increasing the fair value on a certain range and according to our analysis, not using all the volatilities as risk factors for the portfolio but a reduced one, formed just by the surface of the index. We have focused the analysis on European stocks but it is possible to increase the sample and replicate the analysis for different geographical areas. Of course, the analysis is not complete, because of the absence of a capital quantification linked to the approach. Anyway, the main aim of the work project has been to describe why a prudent valuation framework is necessary on a theoretical basis and explain how in practice it is possible to detect strong links between volatility surfaces and use them to asses efficiently the risk.

Bibliography:

Decision, C. (2004). (Text with EEA relevance). *Euratom*, *2001*(May), 20–30. http://doi.org/http://eur-lex.europa.eu/pri/en/oj/dat/2003/l_285/l_28520031101en00330037.pdf

The, S., & May, N. (2012). Predictable Dynamics in the S & P 500 Index Options Implied Volatility Surface Author (s): Sílvia Gonçalves and Massimo Guidolin Predictable Dynamics in the S & P 500 Index Options Implied Volatility Surface *, *79*(3), 1591–1635.

Wang, J., Chen, S., Tao, Q., & Zhang, T. (2017). Modelling the implied volatility surface based on Shanghai 50ETF options. *Economic Modelling*, *64*(April), 295–301. http://doi.org/10.1016/j.econmod.2017.04.009

Gautam, Kumar. (2011). Cluster Based Factor Model. EDHEC Business School.

Kivilo, Erik. (2014). Prudent Valuation & Model Risk Quantification. Lund University.

Homescu, C. (2011). Implied Volatility Surface: Construction Methodologies and Characteristics, 1–40. https://arxiv.org/pdf/1107.1834

Crisóstomo, R. (2014). An Analysis of the Heston Stochastic Volatility Model: Implementation and Calibration using Matlab List of Contents, (December).

European Banking Authority. (2013). EBA FINAL draft regulatory technical standards, *94*(December), 61. Retrieved from http://www.eba.europa.eu/documents/10180/526386/EBA-RTS-2013-11+(On+identified+staff).pdf

AIFIRM. (2016). Prudent Valuation Guidelines and Sound Practices March 2016, (March).

Characteristics, S., & Factors, P. R. (n.d.). Security Characteristics and Pervasive Risk Factors.

Baker, W. (2016). Bridging the Gap Between Threat Intelligence and Risk Management, (November).

Gatheral, J., & Jacquier, A. (2014). Arbitrage-free SVI volatility surfaces. *Quantitative Finance*, *14*(1), 59–71. http://doi.org/10.1080/14697688.2013.819986

Chen, S., Zhou, Z., & Li, S. (2016). An efficient estimate and forecast of the implied volatility surface: A nonlinear Kalman filter approach. *Economic Modelling*, *58*, 655–664. http://doi.org/10.1016/j.econmod.2016.06.003

Academic, B., Trial, C., Nova, U., An, D. E. L., Analysis, M. R., Financial, T., & Account, I. (2018). III.5 Portfolio Mapping.

Cai, F., Le-Khac, N.-A., & Kechadi, T. (2016). Clustering Approaches for Financial Data Analysis: a Survey. *International Conference on Data Mining (DMIN 2012)*. Retrieved from http://arxiv.org/abs/1609.08520

Cremon, A. (2016). Prudent Valuation and Market Price Uncertainty. Bocconi University.

Cox, J. C., & Ross, S. A. (1976). The valuation of options for alternative stochastic processes. *Journal of Financial Economics*, *3*(1–2), 145–166. http://doi.org/10.1016/0304-405X(76)90023-4

Heston, S. L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *Review of Financial Studies*, *6*(2), 327–343. http://doi.org/10.1093/rfs/6.2.327

Heston, S. L. (2006). *The Volatility Surface: A Practitioner's Guide*. (I. John Wiley &Sons, Ed.).

Carr, P., & Wu, L. (2011). A New Simple Approach for Constructing Implied Volatility Surfaces. *New York*, (212), 1–23. Retrieved from http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1701685

Peter, J. (2008). Hyp Hyp Hooray. *Amro*, 1–18.

Hagan, P. S., Kumar, D., Lesniewski, A. S., & Woodward, D. E. (2002). Managing smile risk. *Wilmott Magazine*, *m*, 84–108. Retrieved from http://www.math.columbia.edu/~lrb/sabrAll.pdf

Volatility - Sticky strike vs Sticky delta. (n.d.). Retrieved from http://deltaquants.com/volatility-sticky-strike-vs-sticky-delta

Knight, J., Satchell S. (2006). Forecasting Volatility in the Financial Markets,