

A Work Project, presented as part of the requirements for the Award of an International Master's degree in Finance from the Nova School of Business and Economics.

Price impact for large orders

Sami Mohammed BENAZZOUZ

Work project carried out under the supervision of:

Nicholas H Hirshey

Karim Berrahil

Marvin Roche

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## Abstract

This study examines the relationship between Kyle's lambda and the square root law in financial markets. We investigate how these concepts affect the phenomenon of price impact and discuss their implications for trading strategies. Our findings suggest that a thorough understanding of Kyle's lambda and the square root law can provide valuable insights into price impact and lead to more profitable trading decisions.

## Keywords

Market microstructure, algorithmic trading, Kyle's lambda, square root law, price impact, financial markets, trading strategies, market volatility, market participants, transaction costs, slippage.

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## I. Introduction:

The fluctuations in stock prices are a result of the actions of the people and organizations that make up the market. These market participants typically act in their own best interests, but human behavior is often unpredictable. As a result, studying financial markets can be a fascinating and enlightening journey. Despite the many complexities and uncertainties of the market, the behavior of market participants is driven by a few basic principles. By understanding these principles and how they interact, we can gain insight into the forces that drive stock prices up and down.

It is generally accepted that the price of a stock is determined by the balance between the supply of and demand for that stock. While this may be the most straightforward explanation for how stock prices are determined, it is not the whole story. We know that buy orders tend to push stock prices up, while sell orders tend to push them down, but we don't always know by how much or for how long. This uncertainty leads us to believe that an aggressive order will have a greater impact on a stock's price than a normal or ordinary order (small quantity). And what makes prices efficient is the way they absorb and incorporate the information contained in these orders. In other words, prices reflect the collective wisdom of the market, as encoded in the orders placed by market participants.

The use of algorithms for trading has grown significantly in recent years, with approximately 80% of market transactions now being executed by algorithms. Hedge funds are constantly searching for new strategies and edges in order to find and exploit any remaining "alpha" in the market. However, this alpha is not without limitations. When backtesting a trading

strategy, it is important to make it as realistic as possible by incorporating factors such as transaction costs, slippage, and price impact. Failure to do so can lead to unrealistic expectations and potentially disastrous results when the strategy is put into practice in the real world.

Price impact is not as well-known as it might be because it only comes into play for large orders, which are big enough to significantly affect the price of a stock. In order to avoid the potential negative effects of price impact, traders and trading algorithms often divide their orders into smaller, more manageable parts, which are then executed over a period of time using a variety of different order types, such as market orders and limit orders. This allows them to avoid the pitfalls of executing large, single orders that can have a significant impact on the market.

One of the best ways to study the behavior of price impact is by examining the Limit Order Book (LOB), which is a record of all outstanding buy and sell orders for a particular stock. The LOB is particularly useful for this purpose because its activity is restricted to a predefined grid of prices and volumes (Tick and Lot size). When a matching buy and sell order is received, it is immediately executed and removed from the LOB. However, when a large order is received, it can have a significant impact on the LOB, removing a large number of limit orders and causing the price of the stock to shift from its initial state.

## **II. Problem identification and Objectives:**

The problem that we are trying to address in this study is the lack of understanding and knowledge about the phenomenon of price impact. Our objective is to gain a better understanding of how price impact works, and how it can be studied and measured. By doing

so, we hope to improve our ability to predict and manage the effects of large orders on stock prices, and ultimately to make more informed and profitable trading decisions.

If you have ever placed a market order, you may have noticed that the order is not always filled at the price you expected. This discrepancy between the anticipated and actual price is known as "slippage." Slippage occurs when the market conditions at the time the order is placed are different from the conditions that were anticipated when the order was placed. This can lead to the order being filled at a different price than the one that was originally anticipated. Slippage is a common occurrence in financial markets, and it can have significant implications for traders and investors. It is therefore important to understand and account for the potential effects of slippage when making trading decisions.

Slippage can be caused by a variety of factors, such as market volatility, trading volume, and the size of the order being placed. These factors can affect the amount of slippage that occurs, and in turn, they can also affect the price impact of a large order. In order to understand how price impact works, we need to study the behavior of these factors and how they interact. By doing so, we can gain a better understanding of the expected behavior of price impact, and we may be able to identify the best times to execute large orders in order to minimize their impact on the market.

### **III. Volume and Volatility intraday behavior:**

One of the key factors that can affect the intraday behavior of price impact is the volume of trading that occurs over the course of a single trading day. The volume of trading can have a significant impact on the behavior of stock prices, and it is therefore important to understand the patterns that can be observed in trading volume over the course of a day.

In this study, we are focusing on the intraday behavior of trading volume and market volatility. Previous research has investigated the patterns that can be observed in these factors over the course of a single trading day, and we believe that these patterns can provide valuable insight into the intraday behavior of price impact. By analyzing the relationship between volume, volatility, and price impact, we hope to gain a better understanding of how these factors interact and to develop more accurate predictions of the effects of large orders on stock prices.

### 1. Volume intraday behavior:

In order to thoroughly examine the intraday behavior of volume in financial markets, we have adopted a rigorous methodology in our analysis. This includes dividing the trading day into 1-minute intervals and excluding overnight periods, in order to ensure the precision and accuracy of our results. In order to comprehensively examine the intraday behavior of volatility in financial markets, we have utilized a large dataset consisting of the historical prices of all stocks listed on the NASDAQ from 2005 to 2022. In order to manage the computational demands of our analysis, we have divided the stocks into batches of 10. This approach allows us to conduct our analysis efficiently, without encountering memory issues.

In order to make the volumes comparable across different stocks and days, we have employed min-max normalization (formula below). By grouping the data by hour and minute, we are able to compute the average volume across all days and gain insights into the patterns and trends in volume throughout the trading day.

$$x_{scaled} = \frac{x - x_{min}}{x_{max} - x_{min}}$$

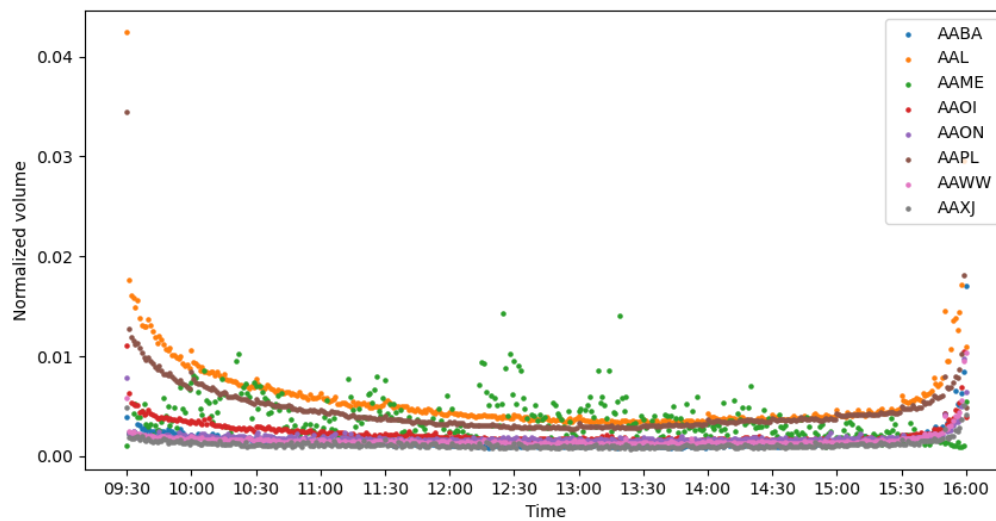


Figure 1 - Volume intraday behavior [batch:0]

The graphical representation in our analysis reveals a pronounced U-shaped pattern in volume, which can be explained by the incorporation of non-trading period information into stock prices at the opening of the trading day. This phenomenon has been observed in previous studies and underscores the importance of considering the effects of those periods on market activity.

## 2. Volatility intraday behavior:

One aspect of our analysis focuses on the intraday behavior of volatility in financial markets. Given the nature of the data utilized in our study, we have employed the Garman-Klass volatility estimator, noted as  $\sigma_{GK}$ , which takes into account both the opening and closing prices. This approach is particularly relevant, as markets tend to be most active during the opening and closing of a trading session. Therefore, the Garman-Klass estimator is well-suited for our purposes and provides more accurate results.

$$\sigma_{GK} = \sqrt{\frac{1}{2T} \sum_{t=1}^T \ln\left(\frac{h_t}{l_t}\right)^2 - \frac{2\ln(2) - 1}{T} \ln\left(\frac{c_t}{o_t}\right)}$$

Where:

- $T$ : Number of records in the sample period
- $o_t$ : Open price at  $t$
- $h_t$ : High price at  $t$
- $l_t$ : Low price at  $t$
- $c_t$ : Close price at  $t$

We have utilized the same methodology as in our analysis of volume, and we have observed that volatility exhibits a similar behavior to volume. This result is expected, as the two metrics are known to be correlated. (See Figures 2)

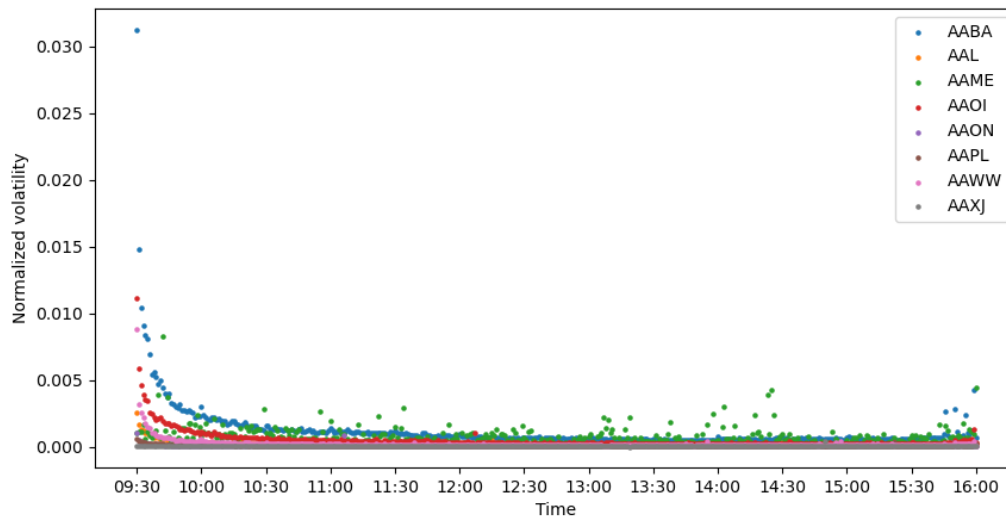


Figure 2 - Volatility intraday behavior [batch:0]

Based on our analysis, we have found that volume exhibits a U-shaped pattern whereas volatility exhibits an inversed J-shaped pattern over the course of a trading day. This suggests that the price impact of financial markets may also exhibit a similar pattern, with the greatest impact occurring at the opening. Our findings provide evidence for the importance of considering the time of day when examining the price impact of financial markets.

## IV. Kyle's lambda:

Kyle's lambda, proposed by Kyle (1985), is a statistical measure of price impact that attempts to answer the question of how much of an asset can be traded without moving the market. The formula for Kyle's lambda predicts a linear relationship between the quantity of an asset traded and the resulting price change. This approach has been widely used in finance as a way to measure the market impact of trading activity.

The formula for Kyle's lambda, noted as  $\lambda$ , is as follows:

$$\lambda = \frac{|\Delta Price_t|}{Volume_t}$$

Where:

- $|\Delta Price_t|$ : Absolute price change
- $Volume_t$  : Volume traded over that period

In order to estimate the value of lambda using Kyle's formula, we ran a regression analysis using 17 years of intraday data for each stock in our universe. Our dependent variable was the absolute difference between the high and the low of 1-minute candles, and our independent variable was the volume traded during the same 1-minute period. This allowed us to examine the relationship between volume and price change and estimate the value of lambda for each stock. By analyzing this relationship over a long time period, we were able to gain a better

understanding of the market impact of trading activity.

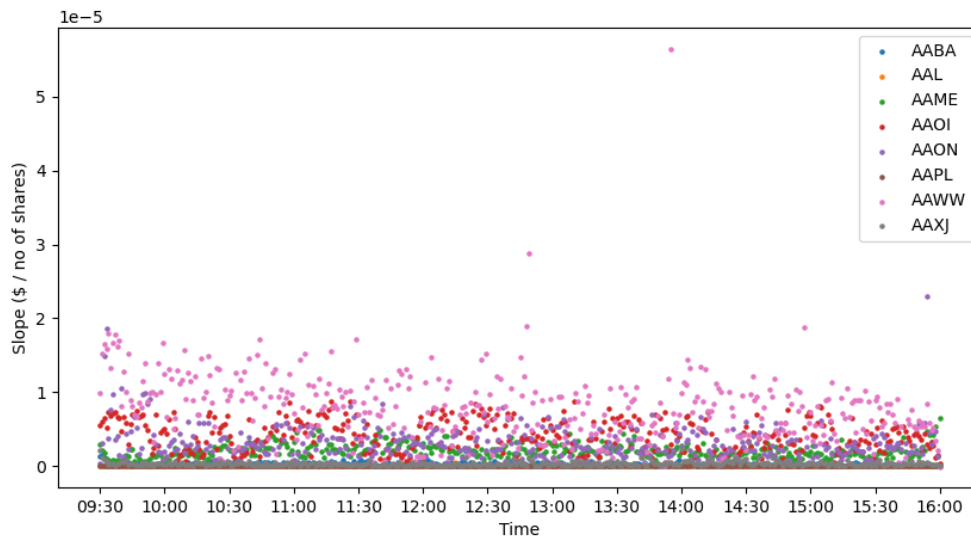


Figure 3 - Slope intraday behavior (Kyle's model) [batch:0]

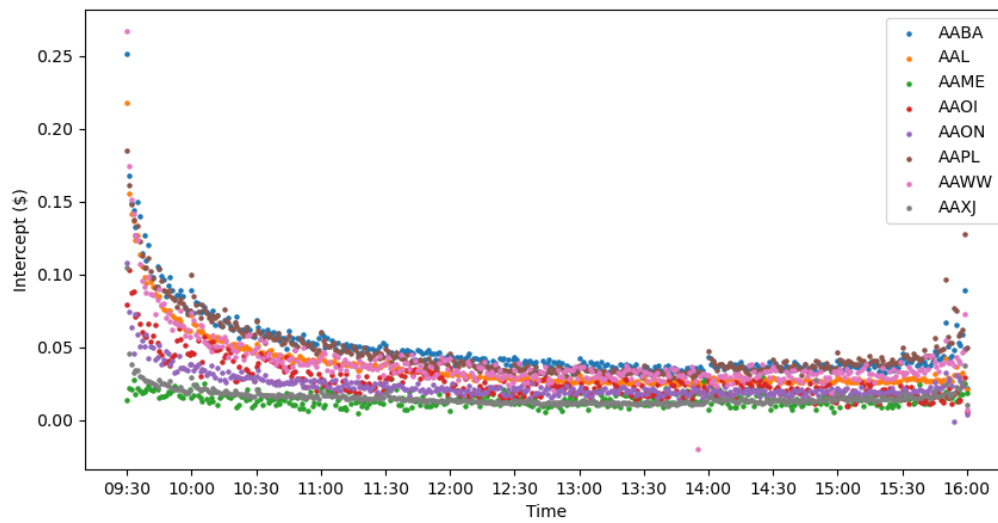


Figure 4 - Intercept intraday behavior (Kyle's model) [batch:0]

As depicted in Figure 3, the intercept exhibits an inverted J-shaped behavior, similar to the volatility. This suggests that when executing a trade, there will be an initial market impact that is not dependent on the quantity traded. The exact form and behavior of the intercept may vary

depending on the market conditions and other factors, but this general relationship is commonly observed. By understanding this relationship, we can better analyze when it is optimal to enter or exit a position.

Figure 4 indicates that the slope exhibits a random behavior and does not exhibit any discernible pattern throughout the day. However, the slope remains negligible relative to the intercept below a certain threshold of the quantity traded.

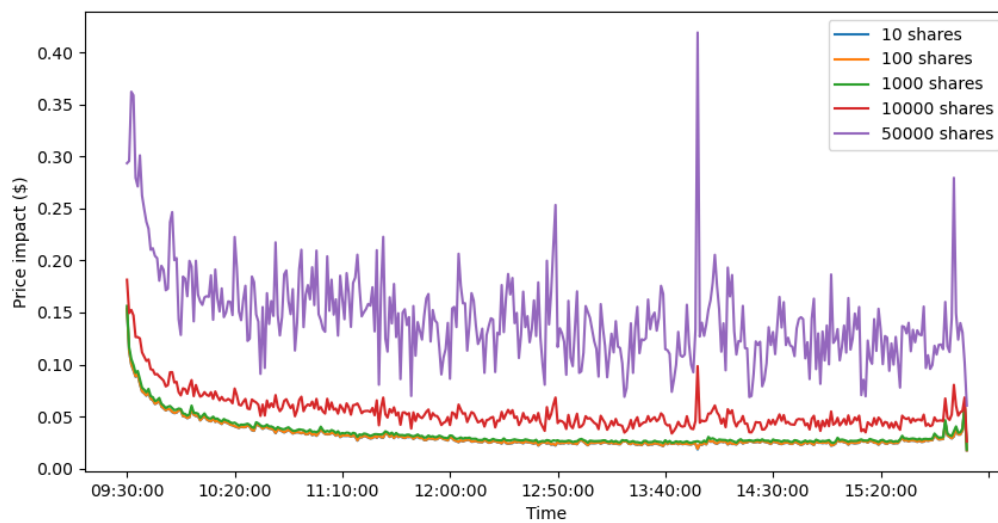


Figure 5 - Simulation of the execution (Kyle's model) [batch:0]

Figure 5 demonstrates that the inverted J-shaped market impact persists below a certain threshold. In this particular batch, the shape remains unchanged until around 50,000 shares.

One major limitation of Kyle's lambda is that it assumes a linear relationship between the change in price and the quantity traded. This means that, according to the model, a 10% increase in the quantity traded would result in a 10% increase in the market impact. However, this relationship may not hold in all cases, and the actual impact of trading activity on price may not be accurately captured by this model. Therefore, it is important to consider the

limitations of Kyle's lambda when using it to measure the price impact of financial markets.

## V. Square root law:

As financial markets continue to evolve, the ability to accurately measure the impact of trading activity on price has become increasingly important. Alternative methods, such as the square-root method, have been used to assess market impact.

The square root law is not only empirically robust, but it also appears to arise from the nature of the order book near the boundary between buying and selling. As one approaches the "current price", the density of the order book rapidly decreases.

The formula for the square-root impact method is as follows:

$$\Delta P = c \sigma \sqrt{\frac{Q}{V}}$$

Where:

- $\Delta P$ : Price change
- $\sigma$  : The daily volatility
- $V$  : The daily volume
- $c$  : a constant

To further investigate the relationship between the price impact and the quantity traded, we used regression analysis to estimate the value of the constant in the square root law formula. By using the same data as in our previous model, we were able to accurately determine the value of the constant and to gain a deeper understanding of the underlying relationship between these variables.

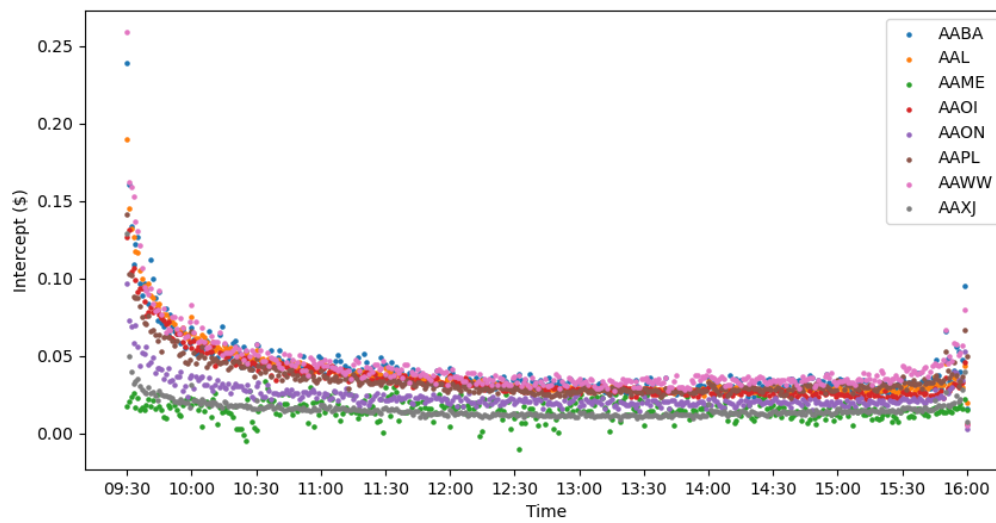


Figure 6 - Intercept intraday behavior (Square root model) [batch:0]

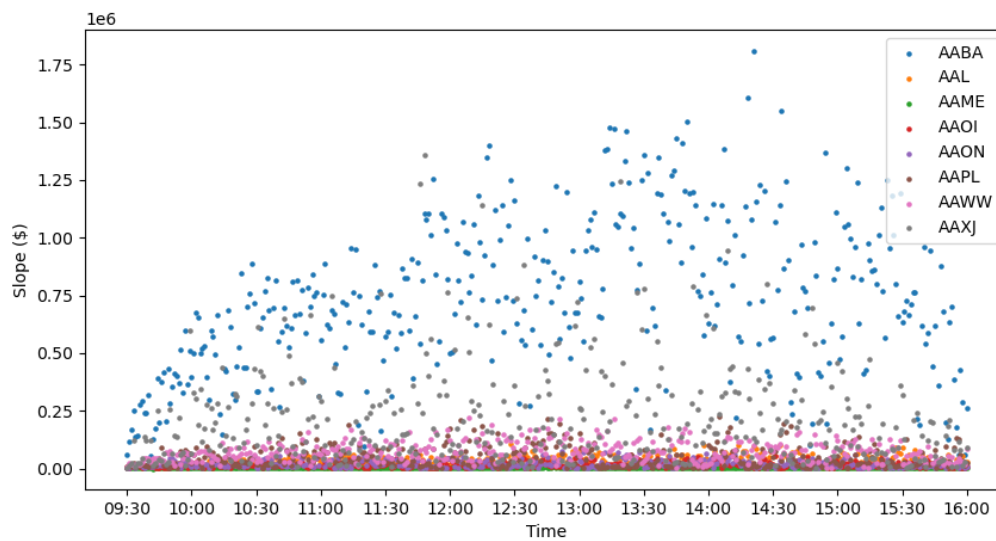


Figure 7 - Slope intraday behavior (Square root model) [batch:0]

As shown in Figure 6, both models exhibit the same behavior. However, Figure 7 reveals that the slope of the regression line in our model is smaller than that of Kyle's model. Additionally, the slope in our model appears to behave randomly, with no clear pattern.

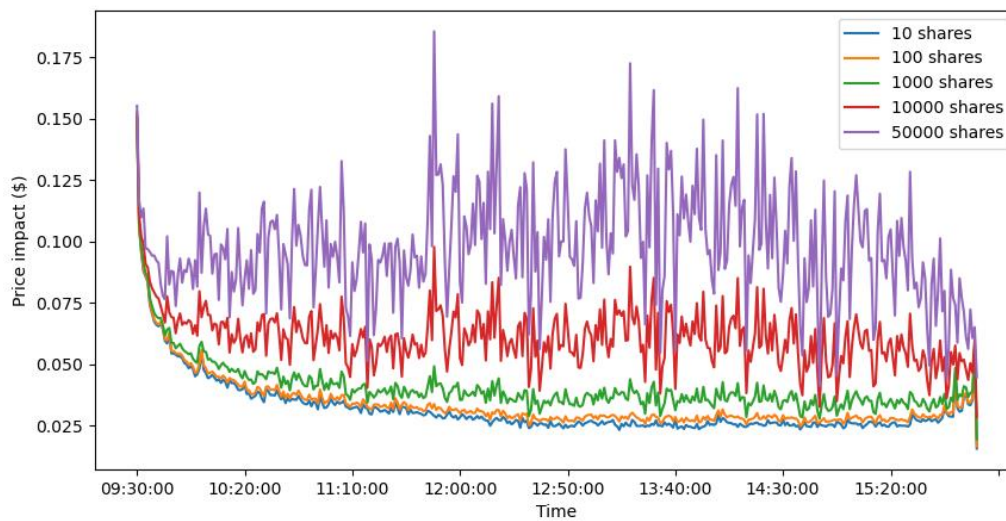


Figure 8 - Simulation of the execution (Square root model) [batch:0]

Figure 8 shows that the inverted J-shaped market impact persists for this model also below a certain threshold. This threshold is lower than the one observed in Kyle's lambda. In our model, the price impact becomes noisy around the execution of 10,000 shares.

## VI. Discussion and conclusion:

In this study, we worked with a vast amount of data, consisting of approximately 3368 stocks over a period of 17 years of 1-minute data. This resulted in a total of 6,059,975,040 lines of data, which equates to around 36 billion data points since we utilized open-high-low-close prices and volume data, in addition to timestamps. In order to optimize our memory usage, we decided to divide the stocks into manageable batches, which allowed us to conduct a total of 2,829,120 regressions (3368 stocks x 7 hours x 60 minutes x 2 regressions).

Kyle's model and the square root model are both important tools for predicting the market impact of executing a trade. Each model offers unique insights and considerations that can

help market participants in the execution of their trades. Kyle's model, focuses on the relationship between the size of the trade and the impact it has on the price of the underlying asset. On the other hand, the square root model takes into account the liquidity of the market and the volatility of the asset and adds non-linearity, providing a more nuanced view of the potential market impact.

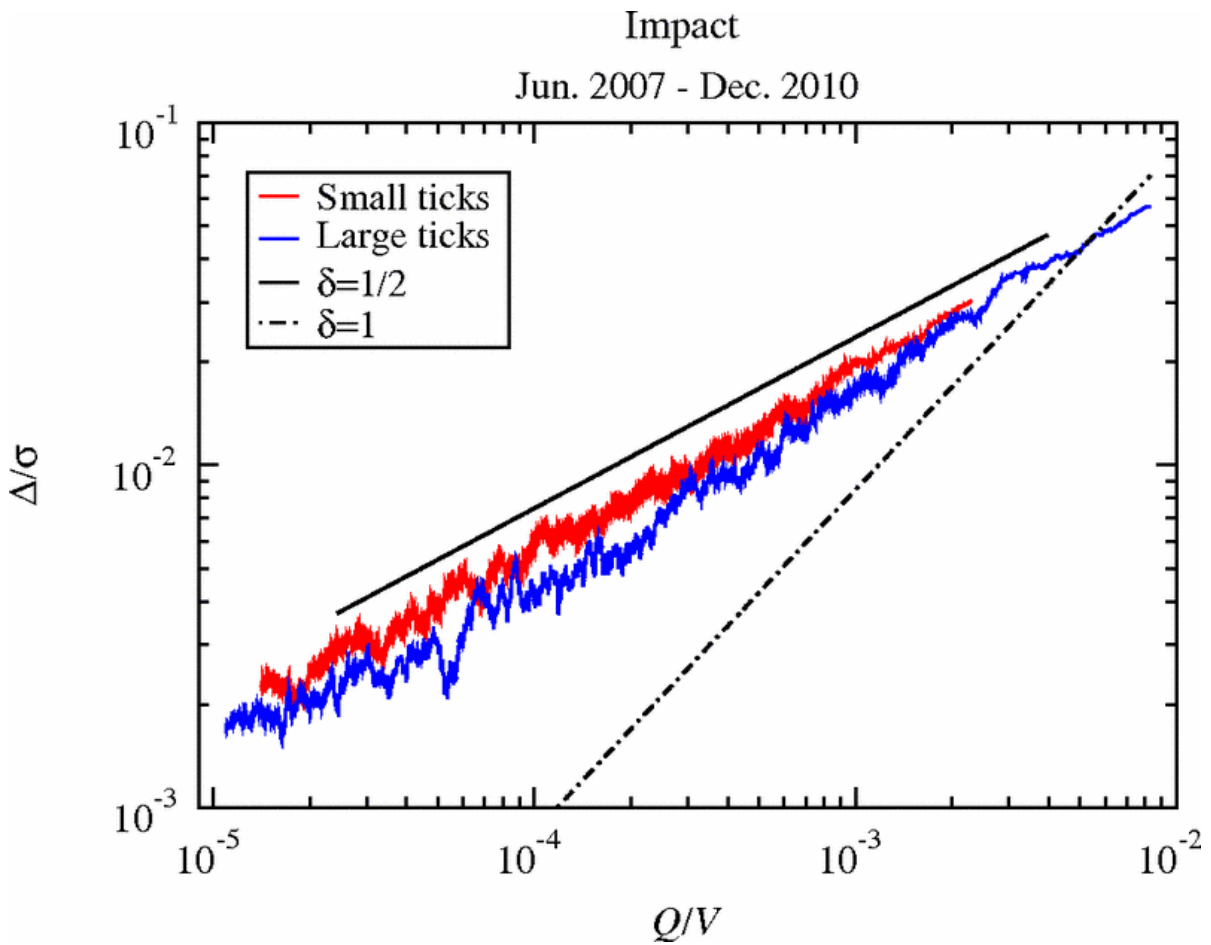


Figure 9 - The impact of large orders for CFM proprietary trades on futures markets

The figure above (from Toth et al.) demonstrates the results of an analysis conducted using a database of nearly 500,000 orders to measure the market impact on the futures market between June 2007 and December 2010. The line of slope 0.5 corresponds to a square-root impact,

while the line of slope 1 corresponds to a linear impact. This analysis provides us with both an upper bound and a lower bound for the real market impact, allowing us to better assess the maximum and minimum consequences of executing a trade.

## References:

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## Appendix:

Batch 0	Batch 1	Batch 2	Batch 3	Batch 4
AABA	AAXN	ABTX	ACHN	ACRX
AAL	ABAC	ABUS	ACHV	ACSF
AAME	ABCB	ACAD	ACIA	ACST
AAOI	ABCD	ACBI	ACIU	ACT
AAON	ABDC	ACER	ACIW	ACTG
AAPL	ABEO	ACET	ACLS	ACWI
AAWW	ABEOW	ACGL	ACMR	ACWX
-	ABIL	ACGLO	ACNB	ACXM
-	ABIO	ACGLP	ACOR	ADAP
-	ABMD	ACHC	ACRS	ADBE

Figure 10 - members of the 5 first batches

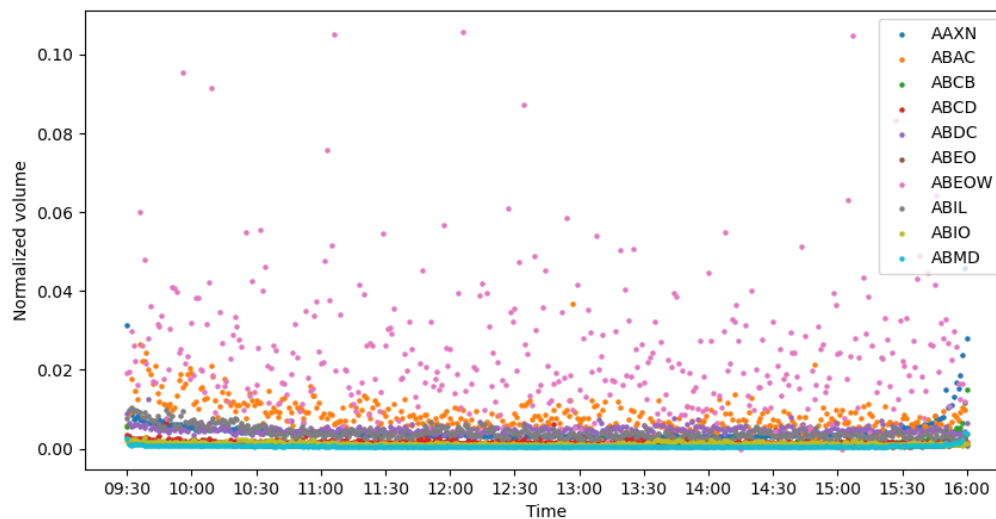


Figure 11 - Volume intraday behavior [batch:1]

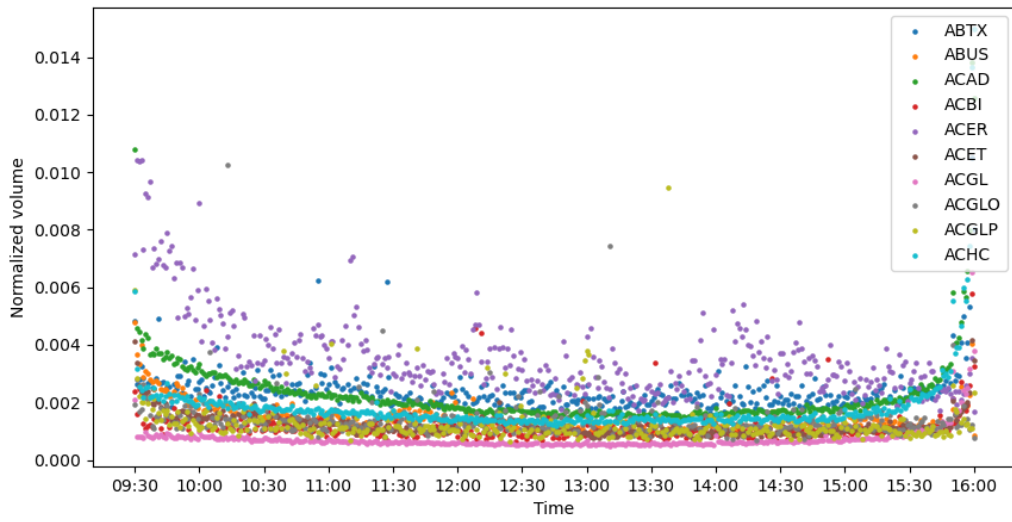


Figure 12 - Volume intraday behavior [batch:2]

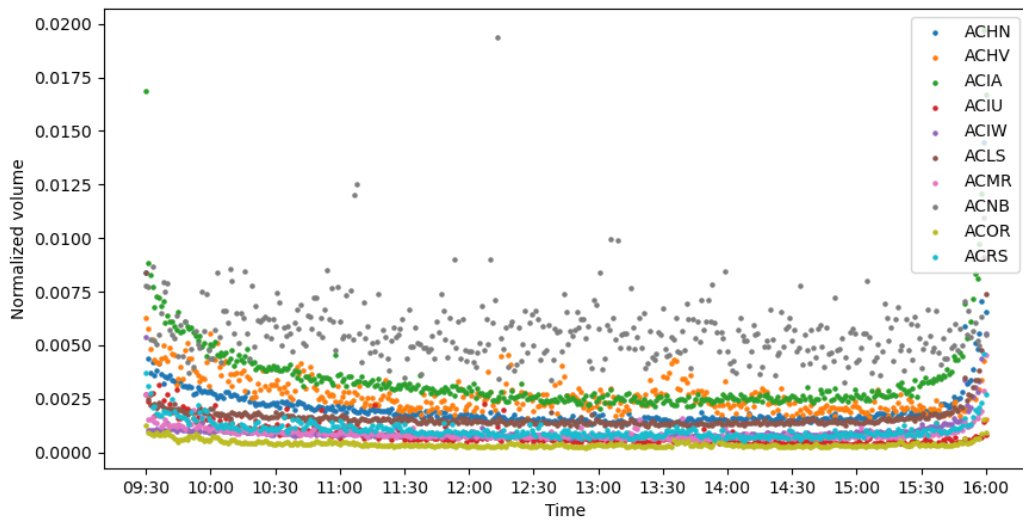


Figure 13 - Volume intraday behavior [batch:3]

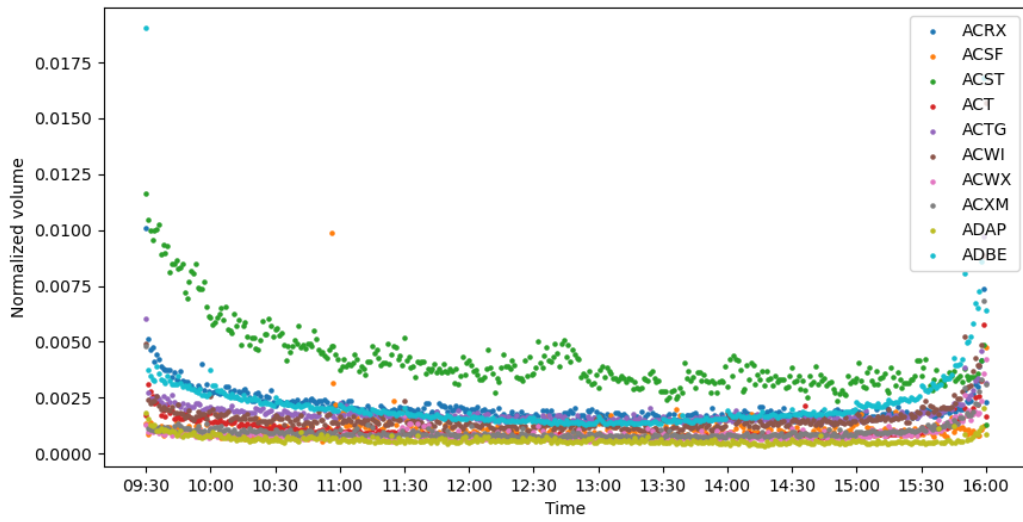


Figure 14 - Volume intraday behavior [batch:4]

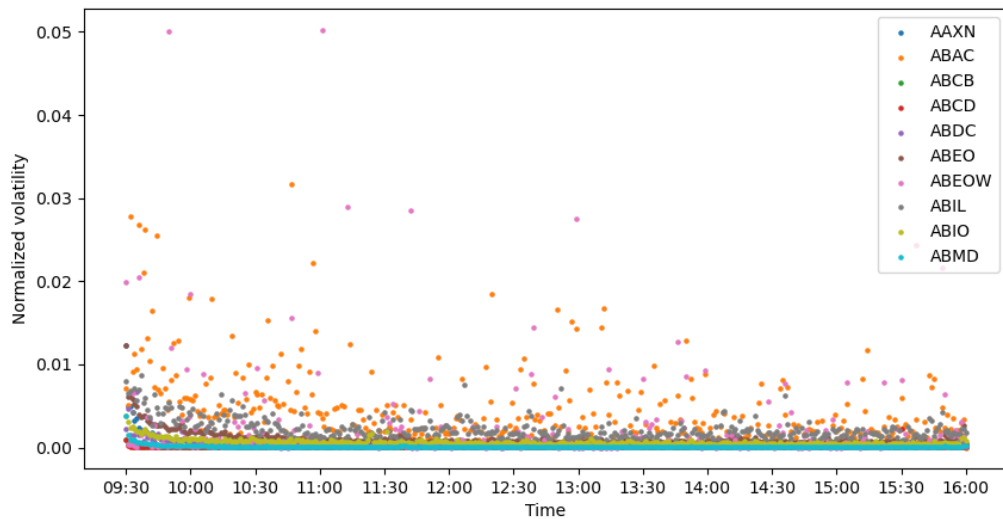


Figure 15 - Volatility intraday behavior [batch:1]

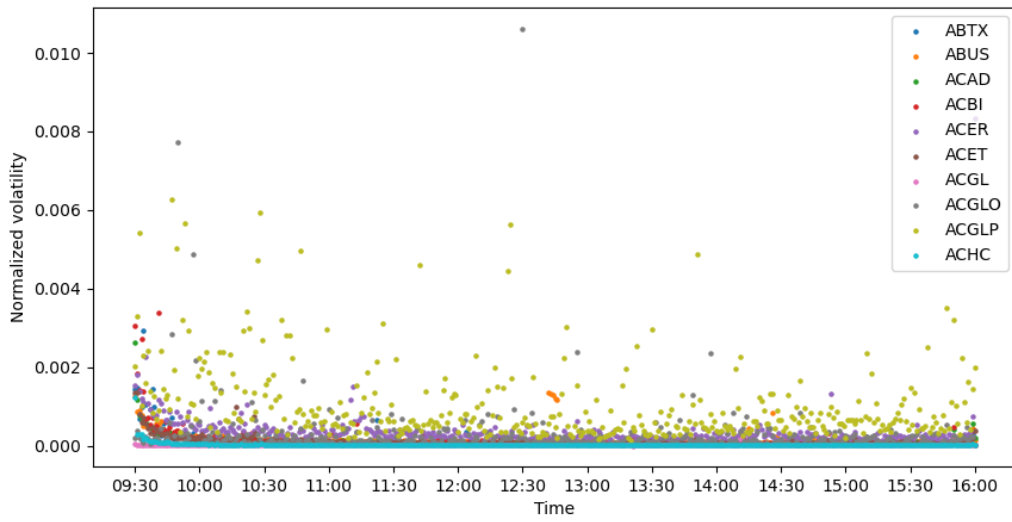


Figure 16 - Volatility intraday behavior [batch:2]

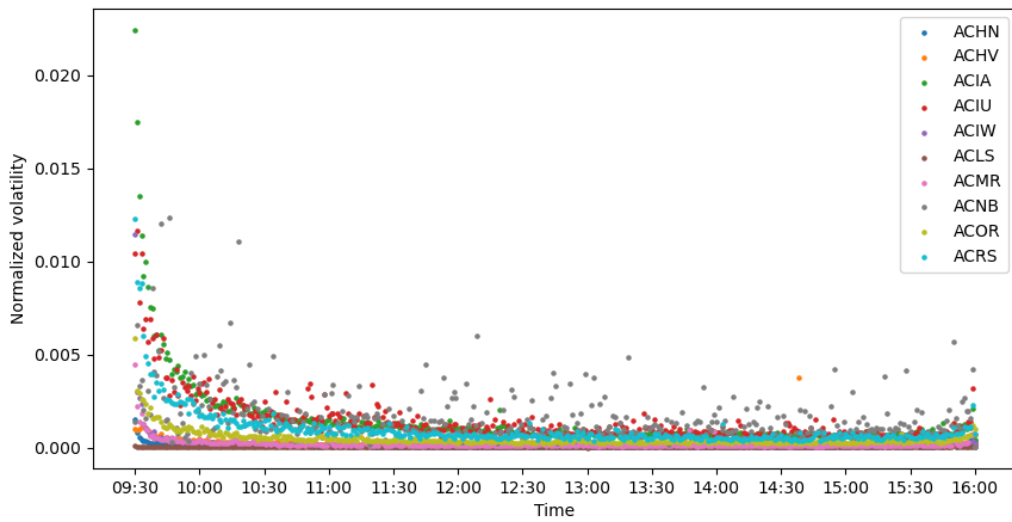


Figure 17 - Volatility intraday behavior [batch:3]

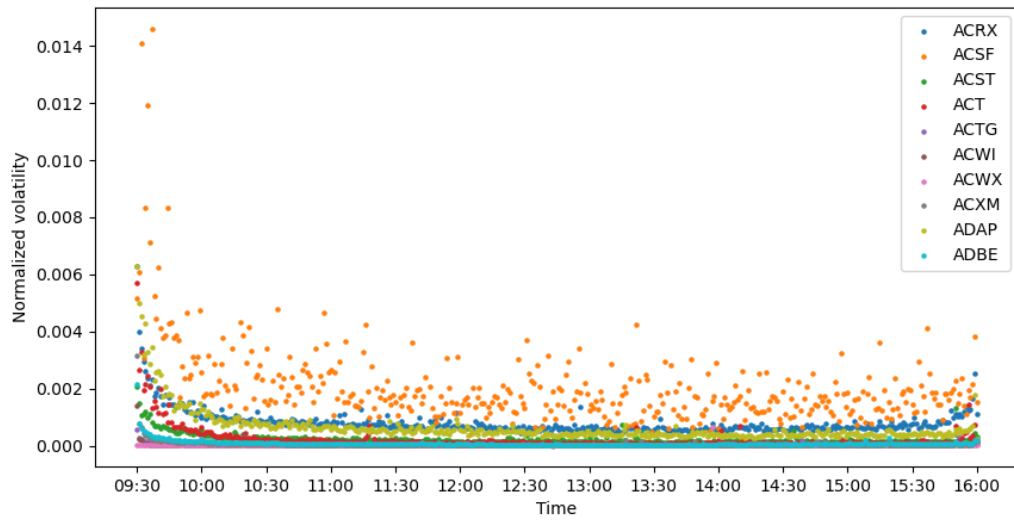


Figure 18 - Volatility intraday behavior [batch:4]

		count	mean	std	min	25%	50%	75%	max
lambda	AABA	391.0	2.748830e-07	2.172773e-07	-1.007566e-08	1.149523e-07	2.257069e-07	3.909988e-07	1.567205e-06
pval	AABA	391.0	2.796663e-02	9.214392e-02	1.874385e-26	1.382065e-09	1.572635e-05	3.254248e-03	6.212765e-01
cst	AABA	391.0	4.899654e-02	2.293531e-02	2.206275e-02	3.670209e-02	4.083736e-02	5.200129e-02	2.514800e-01
cst_pval	AABA	391.0	1.138089e-21	2.250425e-20	9.056621e-179	5.250304e-120	6.226140e-108	2.882657e-95	4.449928e-19
lambda	AAL	391.0	1.001619e-07	3.662936e-08	8.360507e-09	8.223583e-08	9.803091e-08	1.168534e-07	5.160095e-07
pval	AAL	391.0	2.310804e-04	4.473049e-03	2.739045e-229	6.345631e-51	1.519382e-40	5.355877e-31	8.843917e-02
cst	AAL	391.0	3.732320e-02	2.083477e-02	1.697329e-02	2.679710e-02	2.884395e-02	3.941753e-02	2.180108e-01
cst_pval	AAL	391.0	5.053336e-42	9.980472e-41	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	1.973515e-39
lambda	AAME	391.0	2.112303e-06	9.049290e-07	4.968402e-07	1.501513e-06	2.007261e-06	2.520776e-06	6.471431e-06
pval	AAME	391.0	6.530292e-05	7.994701e-04	8.954992e-118	1.220128e-36	1.122223e-24	7.075678e-15	1.505006e-02
cst	AAME	391.0	1.426679e-02	4.209094e-03	4.391654e-03	1.111753e-02	1.401442e-02	1.692015e-02	2.985830e-02
cst_pval	AAME	391.0	5.127469e-03	2.976485e-02	3.266235e-35	3.504247e-05	2.302969e-04	1.439746e-03	5.276575e-01
lambda	AAOI	391.0	4.207623e-06	2.073835e-06	2.872213e-07	2.502966e-06	4.128932e-06	5.888592e-06	8.819961e-06
pval	AAOI	391.0	1.537101e-08	2.938262e-07	2.649941e-281	1.907011e-119	5.018684e-81	8.276339e-45	5.807040e-06
cst	AAOI	391.0	2.679617e-02	1.308081e-02	5.602514e-03	1.856408e-02	2.339102e-02	3.042251e-02	1.028591e-01
cst_pval	AAOI	391.0	3.688824e-07	7.294151e-06	1.041138e-171	2.757059e-77	5.849245e-61	2.013215e-45	1.442325e-04
lambda	AAON	391.0	3.070921e-06	2.333387e-06	2.827189e-08	1.714130e-06	2.628656e-06	3.997058e-06	2.301611e-05
pval	AAON	391.0	2.933699e-02	1.054662e-01	2.064013e-289	9.015525e-10	1.254839e-05	2.340540e-03	8.318576e-01
cst	AAON	391.0	2.381255e-02	9.536938e-03	-1.033077e-03	1.894583e-02	2.089470e-02	2.539758e-02	1.077762e-01
cst_pval	AAON	391.0	1.475159e-03	2.916933e-02	6.016370e-272	1.988989e-54	4.982930e-42	1.318247e-32	5.767861e-01
lambda	AAPL	391.0	-8.852745e-09	2.842734e-09	-1.633850e-08	-1.099362e-08	-8.546493e-09	-6.883396e-09	3.158900e-09
pval	AAPL	391.0	2.697918e-06	3.983138e-05	9.317168e-101	1.264574e-44	5.142048e-30	3.504694e-20	7.235520e-04
cst	AAPL	391.0	4.762136e-02	2.140489e-02	2.886632e-02	3.469729e-02	3.995758e-02	5.141334e-02	1.852240e-01
cst_pval	AAPL	391.0	1.707548e-71	3.357341e-70	0.000000e+00	4.339897e-300	4.872535e-276	2.233076e-240	6.638700e-69
lambda	AAWW	391.0	8.021320e-06	4.766052e-06	-3.050833e-08	5.136811e-06	8.025894e-06	1.022604e-05	5.648941e-05
pval	AAWW	391.0	1.902434e-03	2.624404e-02	0.000000e+00	1.701636e-86	1.624883e-64	1.281092e-39	4.051822e-01
cst	AAWW	391.0	3.984686e-02	2.146359e-02	-1.955702e-02	3.041621e-02	3.419532e-02	4.073732e-02	2.673666e-01
cst_pval	AAWW	391.0	5.045734e-17	7.557675e-16	0.000000e+00	7.336815e-200	6.800039e-163	1.095366e-137	1.385662e-14
lambda	AAXJ	391.0	3.551987e-07	2.863127e-07	1.307307e-08	1.534845e-07	2.800881e-07	4.762984e-07	2.373850e-06
pval	AAXJ	391.0	3.829626e-03	3.244429e-02	2.239172e-82	1.246609e-24	4.543384e-16	2.437416e-09	5.139836e-01
cst	AAXJ	391.0	1.556510e-02	6.470262e-03	9.512526e-03	1.236654e-02	1.401146e-02	1.663581e-02	1.046359e-01
cst_pval	AAXJ	391.0	8.506318e-05	1.681276e-03	0.000000e+00	2.869425e-250	8.635982e-206	2.953078e-164	3.324512e-02

Figure 19 - Kyle results [batch:0]

		count	mean	std	min	25%	50%	75%	max
lambda	AAXN	391.0	1.117404e-05	0.000004	5.455726e-08	8.160307e-08	1.144794e-05	1.434798e-05	0.000028
pval	AAXN	391.0	5.156419e-04	0.008888	1.049793e-111	3.456099e-37	9.069821e-30	1.002986e-21	0.174138
cst	AAXN	391.0	4.842103e-02	0.028650	4.795817e-03	3.275103e-02	3.857826e-02	5.361324e-02	0.231093
cst_pval	AAXN	391.0	3.186616e-06	0.000055	3.889376e-78	2.057919e-45	2.257529e-35	1.514405e-27	0.001074
lambda	ABAC	391.0	4.630322e-06	0.000004	-1.466022e-06	2.145973e-06	3.641868e-06	6.205002e-06	0.000023
pval	ABAC	390.0	1.418466e-01	0.247142	2.282324e-24	4.839634e-05	1.012749e-02	1.454045e-01	0.979698
cst	ABAC	391.0	4.604496e-03	0.005437	-2.475737e-02	1.773230e-03	4.607839e-03	7.896547e-03	0.022724
cst_pval	ABAC	390.0	3.264099e-01	0.279261	3.027946e-07	9.176394e-02	2.485248e-01	4.995922e-01	0.996025
lambda	ABCB	391.0	6.471285e-06	0.000004	7.502615e-08	3.439913e-06	6.207769e-06	8.602688e-06	0.000027
pval	ABCB	391.0	5.731341e-04	0.010191	7.349037e-317	8.698419e-45	7.687148e-31	7.675149e-17	0.201306
cst	ABCB	391.0	2.092342e-02	0.011190	6.330355e-03	1.544518e-02	1.807441e-02	2.197917e-02	0.108355
cst_pval	ABCB	391.0	7.648243e-07	0.000015	0.000000e+00	8.719163e-90	1.748312e-70	4.038297e-50	0.000299
lambda	ABCD	391.0	3.457209e-06	0.000003	-7.879276e-08	1.527199e-06	3.149528e-06	4.848249e-06	0.000016
pval	ABCD	391.0	5.504070e-02	0.176688	4.188058e-63	7.492425e-12	1.068678e-06	2.243403e-03	0.997316
cst	ABCD	391.0	8.046696e-03	0.004113	-4.067283e-03	5.586309e-03	7.470649e-03	9.650489e-03	0.031055
cst_pval	ABCD	391.0	2.108708e-02	0.097124	8.270340e-87	2.457203e-09	3.177043e-06	3.355294e-04	0.970572
lambda	ABDC	391.0	5.873070e-06	0.000004	5.626103e-08	3.499667e-06	5.353823e-06	7.436715e-06	0.000031
pval	ABDC	391.0	2.509905e-02	0.093532	1.261454e-29	1.227174e-07	3.647862e-05	4.562069e-03	0.768575
cst	ABDC	391.0	9.721728e-03	0.004675	-1.601067e-02	6.800509e-03	9.204237e-03	1.172583e-02	0.027399
cst_pval	ABDC	391.0	3.272129e-02	0.108841	4.055985e-67	1.580536e-05	6.136295e-04	9.241272e-03	0.997991
lambda	ABEO	391.0	3.628295e-05	0.000050	-8.769797e-06	4.738308e-06	1.991634e-05	4.944863e-05	0.000474
pval	ABEO	391.0	4.053630e-01	0.339179	4.010940e-46	6.760614e-02	3.615229e-01	7.069293e-01	0.998494
cst	ABEO	391.0	3.940592e-01	0.127218	1.066776e-01	3.206219e-01	3.452538e-01	4.310819e-01	1.171871
cst_pval	ABEO	391.0	1.071652e-07	0.000002	2.797999e-183	6.229906e-89	1.784028e-59	3.057242e-53	0.000042
lambda	ABEOW	390.0	2.451365e-05	0.000105	-7.275735e-05	-3.370974e-07	5.652298e-06	2.209669e-05	0.001607
pval	ABEOW	350.0	4.383997e-01	0.339474	3.424739e-09	6.304578e-02	4.412894e-01	7.570055e-01	0.985600
cst	ABEOW	391.0	6.137169e-03	0.032396	-3.300000e-01	-2.070710e-03	2.307747e-03	1.263479e-02	0.265923
cst_pval	ABEOW	351.0	4.781555e-01	0.262329	2.268101e-04	2.744769e-01	4.696261e-01	6.861959e-01	0.991556
lambda	ABIL	391.0	2.773151e-06	0.000001	4.282065e-08	1.808803e-06	2.568712e-06	3.466170e-06	0.000014
pval	ABIL	391.0	8.595569e-02	0.172030	1.140449e-22	5.738666e-05	6.852091e-03	7.914678e-02	0.986600
cst	ABIL	391.0	4.057125e-02	0.015934	6.221265e-03	2.915472e-02	3.760281e-02	4.962716e-02	0.101702
cst_pval	ABIL	391.0	7.833389e-03	0.025987	2.521610e-18	3.355434e-05	6.528469e-04	3.781292e-03	0.364877
lambda	ABIO	391.0	-5.725589e-05	0.000053	-2.684644e-04	-7.915273e-05	-3.925719e-05	-1.829983e-05	-0.000003
pval	ABIO	391.0	4.388402e-01	0.191093	2.546274e-04	3.032439e-01	4.510137e-01	5.919649e-01	0.825817
cst	ABIO	391.0	2.414394e+00	0.727129	1.314314e+00	1.952027e+00	2.267054e+00	2.732718e+00	6.867658
cst_pval	ABIO	391.0	1.117616e-04	0.000884	5.099605e-53	1.626130e-09	1.221412e-07	4.636566e-06	0.012926
lambda	ABMD	391.0	1.900277e-05	0.000010	1.429361e-07	1.225465e-05	1.781225e-05	2.450573e-05	0.000075
pval	ABMD	391.0	1.124303e-03	0.021456	1.800014e-144	3.711658e-41	1.699588e-30	2.853320e-21	0.424254
cst	ABMD	391.0	9.500149e-02	0.050021	1.687025e-02	6.860578e-02	7.901586e-02	1.009650e-01	0.476467
cst_pval	ABMD	391.0	1.302050e-07	0.000003	5.893550e-193	6.039625e-90	6.765512e-72	2.975535e-58	0.000051

Figure 20 - Kyle results [batch:1]

		count	mean	std	min	25%	50%	75%	max
lambda	AABA	391.0	7.281137e+05	3.336188e+05	3.512335e+04	4.991195e+05	7.114338e+05	9.550127e+05	1.811560e+06
pval	AABA	391.0	9.957916e-18	1.969050e-16	3.132317e-215	4.884627e-131	2.634057e-113	5.616280e-96	3.893545e-15
cst	AABA	391.0	4.050666e-02	2.008881e-02	1.595000e-02	3.025796e-02	3.417338e-02	4.332238e-02	2.389621e-01
pval_cst	AABA	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	AABA	391.0	1.554358e-19	3.073544e-18	4.376013e-194	4.970453e-157	2.939599e-146	8.311230e-139	6.077540e-17
lambda	AAL	391.0	3.523536e+04	2.025604e+04	3.349750e+03	1.899411e+04	3.198703e+04	4.808661e+04	1.041234e+05
pval	AAL	391.0	2.396054e-68	3.359379e-67	0.000000e+00	7.530524e-190	1.106799e-158	1.283453e-132	5.270111e-66
cst	AAL	391.0	3.853965e-02	1.887158e-02	1.950995e-02	2.845863e-02	3.085873e-02	4.028092e-02	1.899960e-01
pval_cst	AAL	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	AAL	391.0	4.352273e-104	8.606062e-103	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	1.701739e-101
lambda	AAME	391.0	6.515643e+02	8.311229e+02	6.911966e+00	1.640488e+02	3.676448e+02	8.192573e+02	6.279862e+03
pval	AAME	391.0	5.654468e-03	3.375956e-02	1.037008e-102	9.828185e-12	3.230605e-07	2.058732e-04	5.110798e-01
cst	AAME	391.0	1.561507e-02	6.070314e-03	-9.794589e-03	1.238282e-02	1.562306e-02	1.922910e-02	3.352809e-02
pval_cst	AAME	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	AAME	391.0	7.823413e-02	1.697534e-01	3.060887e-46	6.385900e-04	9.893175e-03	5.774145e-02	9.386398e-01
lambda	AAOI	391.0	1.821643e+04	1.300429e+04	1.617699e+02	8.081612e+03	1.560935e+04	2.608521e+04	7.252305e+04
pval	AAOI	391.0	3.790803e-04	3.268029e-03	0.000000e+00	5.642843e-43	2.257109e-30	1.839046e-18	4.168385e-02
cst	AAOI	391.0	3.546724e-02	1.573693e-02	6.102852e-03	2.651565e-02	2.964128e-02	3.804039e-02	1.313134e-01
pval_cst	AAOI	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	AAOI	391.0	3.345957e-08	6.616202e-07	6.575482e-307	8.681668e-111	1.908617e-96	7.258785e-84	1.308269e-05
lambda	AAON	391.0	2.025637e+04	1.652013e+04	2.952837e+01	7.785554e+03	1.523018e+04	3.045333e+04	8.848158e+04
pval	AAON	391.0	3.218018e-05	3.747052e-04	0.000000e+00	2.804655e-50	2.987014e-27	7.633048e-14	6.814904e-03
cst	AAON	391.0	2.427035e-02	8.879580e-03	2.875408e-03	1.982895e-02	2.146290e-02	2.532333e-02	9.707941e-02
pval_cst	AAON	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	AAON	391.0	2.248597e-08	4.446312e-07	0.000000e+00	3.626727e-68	3.965749e-56	2.326670e-48	8.792013e-06
lambda	AAPL	391.0	4.966620e+04	4.589069e+04	4.418584e+02	1.335299e+04	3.305639e+04	7.628297e+04	2.253988e+05
pval	AAPL	391.0	5.355554e-04	7.211377e-03	0.000000e+00	1.191785e-83	4.539835e-30	1.993915e-13	1.068919e-01
cst	AAPL	391.0	3.510820e-02	1.302502e-02	2.308077e-02	2.797660e-02	3.099656e-02	3.643442e-02	1.415717e-01
pval_cst	AAPL	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	AAPL	391.0	4.221199e-114	7.555757e-113	2.092056e-308	6.815826e-273	4.448285e-249	7.982108e-217	1.485483e-111
lambda	AAWW	391.0	7.249313e+04	4.236803e+04	2.675223e+03	4.019796e+04	6.552906e+04	9.919004e+04	2.202662e+05
pval	AAWW	391.0	3.362718e-37	6.649344e-36	0.000000e+00	1.825201e-230	1.055828e-190	5.894458e-142	1.314823e-34
cst	AAWW	391.0	4.341852e-02	2.111480e-02	5.001584e-03	3.371415e-02	3.696345e-02	4.504622e-02	2.596089e-01
pval_cst	AAWW	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	AAWW	391.0	1.671936e-16	3.306039e-15	0.000000e+00	5.023077e-292	2.133824e-258	3.307361e-227	6.537270e-14
lambda	AAXJ	391.0	2.137947e+05	2.106539e+05	6.528141e+02	6.598058e+04	1.535980e+05	2.911395e+05	1.359276e+06
pval	AAXJ	391.0	8.799372e-49	1.604878e-47	0.000000e+00	1.517721e-206	2.054473e-150	2.828194e-108	3.161902e-46
cst	AAXJ	391.0	1.552779e-02	7.496774e-03	7.583072e-03	1.220210e-02	1.408520e-02	1.643819e-02	1.293805e-01
pval_cst	AAXJ	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	AAXJ	391.0	7.403372e-09	1.463922e-07	0.000000e+00	1.315581e-310	1.023479e-259	1.617096e-228	2.894719e-06

Figure 21 - Square root law results [batch:0]

		count	mean	std	min	25%	50%	75%	max
lambda	AAXN	391.0	2.439302e+05	1.274417e+05	9.504187e+03	1.431804e+05	2.408890e+05	3.290415e+05	6.459484e+05
pval	AAXN	391.0	1.547858e-12	3.059986e-11	0.000000e+00	2.089740e-126	6.988087e-107	5.254844e-85	6.050893e-10
cst	AAXN	391.0	5.447906e-02	3.198219e-02	1.708508e-03	3.871921e-02	4.451717e-02	5.742940e-02	3.270554e-01
pval_cst	AAXN	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	AAXN	391.0	1.067208e-05	2.110267e-04	2.041022e-147	1.217003e-84	5.315842e-78	9.520805e-69	4.172783e-03
lambda	ABAC	391.0	5.287858e+02	6.930002e+02	0.000000e+00	1.585244e+02	3.084787e+02	6.827208e+02	6.613844e+03
pval	ABAC	390.0	2.691917e-03	1.158021e-02	2.999139e-40	8.071797e-13	4.882840e-08	3.023533e-05	1.119139e-01
cst	ABAC	391.0	6.535957e-03	3.999117e-03	0.000000e+00	3.598222e-03	5.827584e-03	8.872173e-03	2.491384e-02
pval_cst	ABAC	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	ABAC	390.0	6.792801e-02	1.039533e-01	3.601292e-15	8.418063e-03	2.822721e-02	8.761614e-02	8.720705e-01
lambda	ABCB	391.0	6.794723e+03	6.224676e+03	2.598583e+02	2.394077e+03	4.794240e+03	9.225771e+03	3.909504e+04
pval	ABCB	391.0	3.159885e-13	3.971198e-12	0.000000e+00	5.245555e-68	4.398932e-45	3.850779e-30	6.909298e-11
cst	ABCB	391.0	2.562498e-02	1.175621e-02	4.835589e-03	1.977788e-02	2.159848e-02	2.839839e-02	1.119408e-01
pval_cst	ABCB	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	ABCB	391.0	2.445191e-14	4.835053e-13	0.000000e+00	7.887185e-133	8.677864e-118	6.805513e-106	9.580898e-12
lambda	ABCD	391.0	1.013149e+03	1.011814e+03	1.787120e+01	2.850258e+02	6.705792e+02	1.439610e+03	5.594786e+03
pval	ABCD	391.0	5.541130e-04	4.914287e-03	0.000000e+00	1.467983e-19	4.238170e-12	1.746593e-07	8.915803e-02
cst	ABCD	391.0	1.014170e-02	3.008677e-03	1.114809e-03	8.437898e-03	9.637037e-03	1.104405e-02	2.522775e-02
pval_cst	ABCD	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	ABCD	391.0	9.795982e-04	9.462367e-03	1.963647e-209	3.097891e-15	2.814478e-13	3.782386e-11	1.403896e-01
lambda	ABDC	391.0	9.920026e+03	7.858988e+03	3.818618e+02	5.517895e+03	8.532046e+03	1.305054e+04	1.162424e+05
pval	ABDC	391.0	4.455437e-10	8.558211e-09	0.000000e+00	1.236543e-38	5.954600e-30	7.843494e-23	1.691467e-07
cst	ABDC	391.0	1.046747e-02	3.699105e-03	1.238049e-19	8.133287e-03	9.735387e-03	1.201388e-02	3.516734e-02
pval_cst	ABDC	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	ABDC	391.0	7.893257e-04	1.558399e-02	1.517086e-115	3.939541e-11	2.498942e-09	1.384399e-07	3.077600e-01
lambda	ABEO	391.0	3.478929e+04	2.861732e+04	1.368800e+02	1.395811e+04	2.925911e+04	4.868849e+04	1.878581e+05
pval	ABEO	391.0	7.707452e-03	4.923671e-02	1.652177e-275	1.083597e-28	5.138009e-17	1.041553e-09	5.693898e-01
cst	ABEO	391.0	3.872794e-01	1.154974e-01	1.033952e-01	2.971964e-01	3.241781e-01	4.002350e-01	1.058844e+00
pval_cst	ABEO	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	ABEO	391.0	5.317293e-08	1.051427e-07	2.103391e-200	2.120822e-88	2.092231e-58	7.665450e-52	2.079091e-06
lambda	ABEOW	390.0	1.799381e+04	1.162731e+05	-4.898971e+01	1.001510e+02	5.145075e+02	4.005467e+03	1.470260e+08
pval	ABEOW	350.0	8.033381e-02	1.942528e-01	1.861733e-173	2.517047e-13	3.272392e-05	2.047358e-02	9.812549e-01
cst	ABEOW	391.0	6.581821e-03	1.929046e-02	-5.971581e-03	0.000000e+00	1.684703e-03	7.153812e-03	3.215000e-01
pval_cst	ABEOW	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	ABEOW	350.0	3.774672e-01	2.330953e-01	1.011956e-02	1.960324e-01	3.539311e-01	4.950772e-01	9.898915e-01
lambda	ABIL	391.0	2.414987e+03	2.600137e+03	9.505243e+00	7.475503e+02	1.588771e+03	3.290118e+03	2.351074e+04
pval	ABIL	391.0	2.335525e-02	9.846608e-02	9.057935e-132	6.256143e-16	2.384980e-08	1.968779e-04	8.174999e-01
cst	ABIL	391.0	3.368037e-02	1.326387e-02	-8.515479e-03	2.413831e-02	3.150471e-02	4.344485e-02	7.654236e-02
pval_cst	ABIL	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	ABIL	391.0	2.453378e-02	9.011287e-02	4.965812e-14	2.065941e-05	4.254784e-04	4.936599e-03	7.393078e-01
lambda	ABIO	391.0	1.419875e+05	1.599889e+05	-8.083425e+02	3.847880e+04	8.584528e+04	2.102915e+05	1.397437e+08
pval	ABIO	391.0	4.933389e-02	1.450079e-01	0.000000e+00	1.758297e-18	2.598817e-07	5.572279e-03	9.890590e-01
cst	ABIO	391.0	1.835477e+00	5.864216e-01	-5.077534e-01	1.488821e+00	1.766077e+00	2.091444e+00	4.299221e+00
pval_cst	ABIO	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	ABIO	391.0	1.179823e-02	7.400452e-02	1.153245e-45	9.025707e-08	8.982468e-08	3.858349e-04	8.280321e-01
lambda	ABMD	391.0	5.768953e+04	5.419882e+04	7.024204e+02	1.890989e+04	4.197983e+04	8.700719e+04	3.516404e+05
pval	ABMD	391.0	1.553045e-03	8.343271e-03	0.000000e+00	8.283109e-23	8.897879e-11	8.461557e-06	1.141442e-01
cst	ABMD	391.0	1.172395e-01	5.711691e-02	1.174084e-02	8.584833e-02	9.452940e-02	1.287359e-01	5.789810e-01
pval_cst	ABMD	391.0	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
cst_pval	ABMD	391.0	1.171282e-07	2.318020e-06	0.000000e+00	5.670718e-139	1.505839e-123	4.726955e-110	4.579634e-05

Figure 22 - Square root law results [batch:1]