Robustness analysis of deteriorating reinforced concrete slabs

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Summary

Corrosion of reinforcement bars in concrete structures is the most significant deterioration mechanism in these structures. Corrosion is extremely difficult to predict and, consequently, can be regarded as an unpredictable event. Following this, robustness assessment methods can be employed to define the susceptibility of a structure to corrosion.

In this work, robustness is measured in terms of the remaining safety of a deteriorated structure. The proposed methodology is illustrated by means of a reinforced concrete (RC) slab subjected to dead and live loads. The performance of the corroded slab is evaluated using non-linear analysis. The reliability index is adopted to assess the safety of the deteriorated structure. To compute the reliability index a strategy combining the First Order Reliability Method (FORM) and the Response Surface Method (RSM) is used.

Keywords: Robustness; Corrosion; Reinforced Concrete; Reliability; Safety; Performance; Damage; FORM .

1. Introduction

Structural Robustness is an emergent concept in civil engineering, especially due to the tragic consequences of several failures caused by localized damage. The ability of modern structures to sustain localized damage was questioned, as was the ability of existing codes to guarantee safety under unpredictable events.

The aging of existing infra-structures has increased the interest in assessing the effects of deterioration mechanism, in particular, corrosion. Due to the difficulty in predicting the time evolution of corrosion, this event can be taken as unpredictable, and robustness assessment can be used to compare the susceptibility of structures to deterioration.

Several attempts to define and assess robustness have been proposed [1-6]. In this paper the definition proposed by Cavaco et al. [6] has been adopted. According to the authors, robustness can be defined as a structural property that measures the degree of structural performance loss after damage occurrence. Several performance indicators may be considered related to service or ultimate limit states, depending on the purpose of the robustness evaluation. Similarly several damage scenarios can be regarded depending on the events the structure is subjected and its structural type.

Although robustness concept seems to be more oriented towards the analysis of structures subjected to extreme events and consequences, the fact is that robustness can also be very useful in the context of more probable scenarios such as deterioration. Reinforcement corrosion is one of the major causes of concrete structures deterioration, leading to a relevant decrease on structural safety in relation to the design safety levels. Having this in mind, the adopted definition for robustness, is applied considering damage as the reinforcement corrosion level and performance as the reliability index.

2. Robustness Assessment

In order to assess robustness, as defined by [6], it is necessary to consider a performance indicator and to evaluate its vulnerability to damage. In this paper the performance indicator adopted is the reliability index, β, related to ultimate limit state due to excessive structural load. In order to evaluate the impact of corrosion on the reliability index, damage is defined in terms of the reinforcement corrosion level, X_p, measured in terms of weight loss percentage:

$$X_p = 1 - \frac{A_s^{eff}}{A_s} \quad (1)$$

where A_s represent the original reinforcement area and A_s^{eff} denote the remaining uncorroded reinforcement area of the deteriorated structure.

An evaluation of the influence of reinforcement corrosion on the reliability index, β, can give a clear indication of the impact of corrosion on the structural performance. The reliability index of a structure considered as robust should not exhibit a significant variation with corrosion progression. In order to assess robustness, [6] recommends the estimation, for a specific structure, of the evolution of the reliability index as a function of the corrosion rate, $\beta(X_p)$. The next step is to normalize this function in relation to the reliability index of the uncorroded structure $\beta(X_p=0)$. Finally, robustness can be computed through the evaluation of the area bellow the normalized curve for corrosion levels varying from *0%* to *100%*:

$$R_d = \int_{X_p=0\%}^{X_p=100\%} \frac{\beta(x)}{\beta(0)} dx \quad (2)$$

where R_d is the robustness index varying from 0 to 1. Robustness values close to zero mean that reinforcement corrosion has a huge impact on system reliability. When robustness is close to 1 the structure sustains corrosion without a relevant reduction in safety.

3. Corrosion Evaluation

Reinforcement corrosion leads to structural deterioration and consequently to load carrying capacity decrease. Several mechanisms are responsible for safety reduction such as reinforcement area reduction, concrete cracking, deterioration due to expansion around reinforcement bars and bond strength deterioration between reinforcement and concrete. Accordinglto [7] bond strength decreases rapidly with corrosion. For advanced corrosion states, bond strength tends to be negligible and reinforcement slips freely inside the concrete core. In these situations the evaluation of section resistance must take this effect into consideration.

In this paper only the effect of longitudinal reinforcement corrosion on cross section bending resistant moment is studied.

The resistant bending moment of a RC structure is determined by establishing equilibrium conditions between sectional applied forces and resultant tensions on both reinforcement and concrete. Non-linear behaviour was assumed for both materials. For concrete, a parabolic constitutive relation was adopted. A tensile stress limit, f_t, equal to 1/10 of the concrete compression resistance, f_c, is considered. For reinforcement a simple elastic-plastic model is adopted.

In order to take into account the adhesion loss between reinforcement and concrete, the slipping-fibre model as proposed by [8] was adopted. According to this approach reinforcement and its bond can be simulated with two serial springs working together (Figure 1). The slipping-fibre strain, ε^f, is given by the sum of the reinforcement strain, ε^d, and the interface strain, ε^i. Since the components work as a series system, both reinforcement and interface are under de same system stress σ^f, i.e., $\sigma^f = \sigma^d = \sigma^i$.

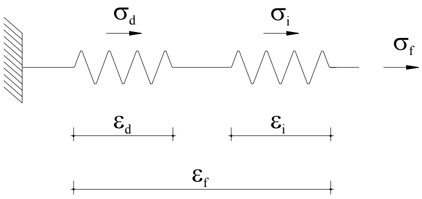

Fig. 1: Slipping-fibre model

Considering for the interface, a simple elastic-plastic constitutive model, the slipping-fibre constitutive behaviour results also elastic-plastic, as shown in Figure 2. The slipping-fibre stress limit σ^f_{lim} is given by the limit of its components:

$$\sigma^f_{\lim} = min(\sigma^d_{\lim}, \sigma^i_{\lim}) \quad (4)$$

The slipping-fibre stiffness is given by equation 5:

$$E^f = \frac{1}{\dfrac{1}{E^d} + \dfrac{1}{E^i}} \quad (5)$$

Since there is no available relevant research about the characterization of bond stiffness, it is assumed a rigid bond, i.e., $E^i \rightarrow \infty$. Regard that if this hypothesis is combined with the hypothesis of the steel yielding stress, σ^d_{lim}, being higher than the bond strength, σ^i_{lim}, the slipping-fibre model reproduces only the reinforcement behaviour. In this paper, structural reliability related to bending ultimate limited state is analysed. Consequently the simplified hypothesis of considering a rigid bond does not introduce relevant perturbation on the achieved results. The impact of bond strength in the structure reliability was analysed by considering a set of different values, equal and higher than σ^d_{lim}.

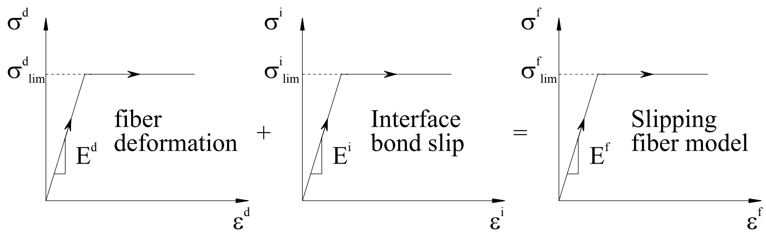

Fig. 2: Slipping-fibre model

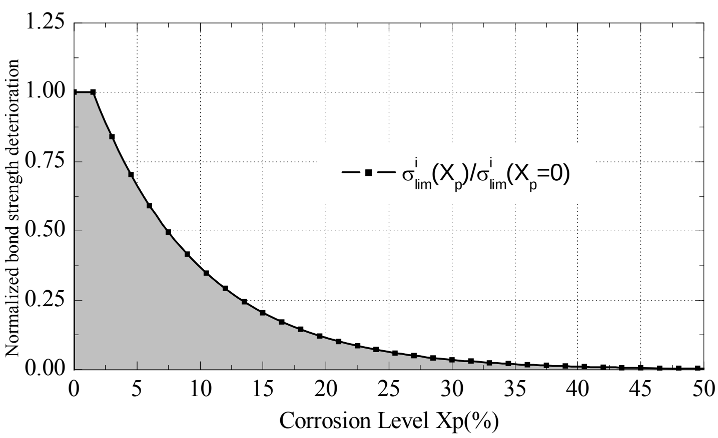

Fig. 3: Bond strength deterioration

In order to account for the bond strength the M-Pull model proposed by [9] is adopted. This model gives the bond strength deterioration as a function of the corrosion level measured in term of weight lost percentage (equation (6)):

$$\frac{\sigma^{i}_{\lim}(X_p)}{\sigma^{i}_{\lim}(X_p=0)}=\begin{cases}1.0 & if \quad X_p \leq 1.5\% \\ 1.192 \cdot e^{-0.117X_p} & if \quad X_p > 1.5\%\end{cases} \quad (6)$$

This model was developed based on the available experimental data and is represented in Figure 3.

4. Reliability Analysis

4.1 Case Study

As discussed previously this paper presents a methodology to evaluate the impact of corrosion in the robustness of a reinforced concrete structures. Robustness indicator is based in the reliability index of the uncorroded and corroded states. The example considered to illustrate the proposed methodology is a simply supported reinforced concrete slab adapted from [10] (Figure 4). The RC slab has a 5m span and a cross-sectional depth of 0.20m and it is subjected to dead load, *g*, and long-term and short-term loads, *q*, resulting from office areas usage. All the variables relating to the problem are probabilistically modelled as synthesized in Table 1:

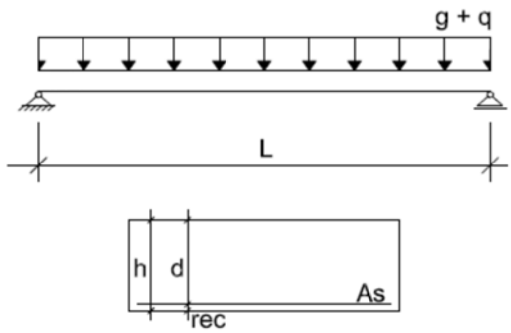

Fig. 4: Reinforced concrete slab

Table 1: Probabilistic models

Basic Variable	Symbol	Distr. type	Dimension	Mean	Std. Deviation
Concrete compressive strength	f_c	lognormal	MPa	30	5
Steel yield stress	f_y	lognormal	MPa	560	30
Slab span	L	determ.	m	5	-
Reinf. Area	A_s	determ.	cm^2	-	-
Slab depth	h	normal	m	0.2	0.005
Concrete cover	rec	gamma	m	0.3	0.005
Concrete density	γ	normal	kN/m^3	25	0.75
Long-term live load	q_{lt}	gamma	kN/m^2	0.5	0.75
Short-term live load	q_{st}	exponential	kN/m^2	0.2	0.2
Resistance uncertainty	θ_R	lognormal	-	1.1	0.077
Load-effect uncertainty	θ_E	lognormal	-	1	0.2

Regard that both slab span and reinforcement area are considered deterministic. In the last case several reinforcement ratios are analysed. Bond strength, $\sigma^{i}_{\lim}$,is assumed equal to steel strength just for the uncorroded state. For corrosion states bond strength is given by the M-pull model [9]. Further details on the variables probabilistic models can be founded in [10].

4.2 FORM - First Order Reliablity Method

The first order reliability method is adopted to compute the reliability index used in equation (2). In this paper only the ultimate limit state related to bending is considered. To compute the flexural capacity of the slab, a non-linear analysis of the cross section is performed following the methodology proposed in section 3. The limit state function is defined in term of resistant and applied bending moments at slab's mid span:

$$G = M_R(f_c; f_y; h; rec; X_p) \times \theta_R - \frac{L^2 \cdot (\gamma \cdot h + q_{lt} + q_{st})}{8} \times \theta_E \quad kNm/m \quad (7)$$

where $M_R(f_c;f_y;h;rec;X_p)$ is the resistant bending moment of the slab obtained through the non-linear analysis. Since, to perform FORM, the partial derivatives of the limit state function are needed, it is necessary to replace M_R with a quadratic polynomial surface of the type:

$$M_R(f_c; f_y; h; rec; X_p) \approx a_0 + \sum_{i=1}^{N} a_i x_i + \sum_{i<j}^{N} a_{ij} x_i x_j + \sum_{i=1}^{N} a_{ij} x_i^2 + \ldots \quad kNm/m \quad (8)$$

For the present second-order model several coefficients a_i need to be estimated in a total number of:

$$1 + n + \frac{n(n+1)}{2} \quad (9)$$

where n is the number of predictor variables. In this case M_R depends on 5 parameters. Since X_p is considered constant within each FORM cycle, there is no need to include it as a predictor variable. Thus, it is necessary at least a set of 15 support points $p_i(f_{ci};\ f_{yi};\ h_i;\ rec_i)$, including respective bending moment M_{Ri} in order to compute the polynomial coefficients. Three values for each variable were defined and combined between them leading to 3^4=81 support points. Despite this number being much higher than the minimum (15), the consequence, as computational time is not an issue, is only a redundant design with increased precision on results. The three values defined for each variable were the design point d_p and the left and right neighbourhood at a distance of one standard deviation. The methodology adopted to compute the reliability index, β, for the deteriorated slab subjected to a corrosion level X_p, is as described below:

Step 1: definition of the corrosion level X_p and estimation of the reliability index, $\beta_0(X_p)$ and the respective design point $d_{p0}(X_p)$;

Step 2: definition of the 81 support point and the non-linear analysis in order to compute the respective resistant bending moment;

Step 3: definition of the polynomial surface adjusted to the bending moment;

Step 4: FORM analysis is performed to calculate the design point $d_p(X_p)$ and the respective reliability index $\beta(X_p)$;

Step 5: the reliability index obtained is compared with the value estimated in STEP 1. If the difference exceed the adopted tolerance of 0.005 than STEP 2 to 4 are repeated until convergence is achieved.

This procedure is repeated several times for reinforcement corrosion levels varying from 0% to 100% in order to obtain the function $\beta=\beta(X_p)$.

5. Results

In Figure 5 the reliability index of the uncorroded (X_p=0) reinforced concrete slab is presented as a function of the reinforcement ratio. The reliability index varies from around 2.5 to 4.9 for an interval of reinforcement ratios from 0.2% to 0.5%. These results are in accordance with the results presented in [10] for the same conditions. Figure 5 shows the influence of corrosion on structural safety. It can be observed that, for the first corrosion stages (X_p<4%), slabs with different reinforcement ratios are equally affected, since the curve $\beta(A_s)$ suffers a simple translation. For corrosion levels higher than 4% the reliability index gradually tends to stabilize around 1.5. For 0.2% and 0.5% reinforcement ratios, the reliability index tends to 1.5, for corrosion levels of 5% and 12 % respectively. Since the lower bound is the same, independently of the reinforcement ratio, it can be stated that corrosion has a greater impact on highly reinforced slabs. This can also be observed in Figure 6 where the reliability index is plotted against the corrosion level. Figure 6 shows clearly that independently of the reinforcement ratio the reliability index of highly deteriorated slabs is around 1.5. The reliability index reaches this value when steel bars completely lose the adherence to concrete and the bending resistant moment is given by the steel and concrete working independently. Comparing the curves shapes in Figure 6 with curve shape in Figure 3 it is possible to understand the importance of bond strength deterioration in reliability decreasing with corrosion. In fact, bond strength deterioration is the most important factor causing load capacity decreasing.

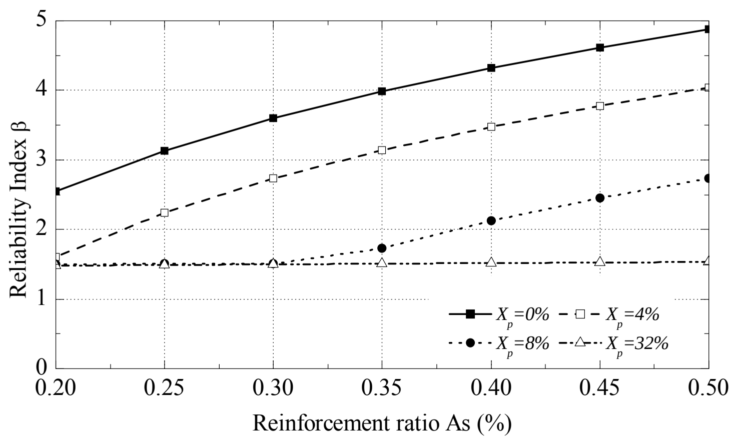

Fig. 5: Reliability Index as function of the reinforcement ratio

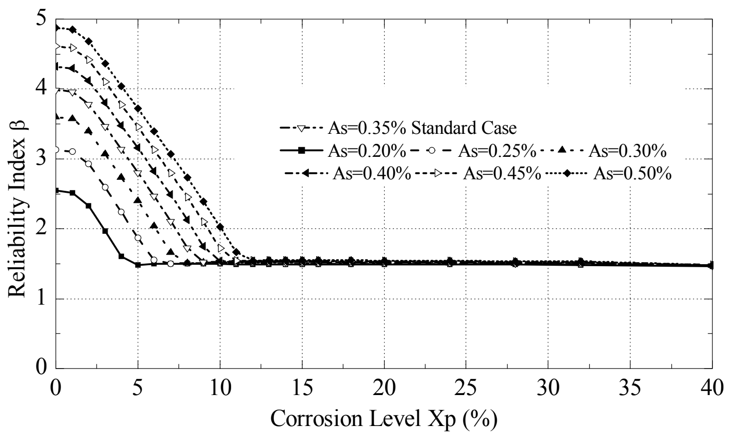

Fig. 6: Reliability Index as function of the corrosion level

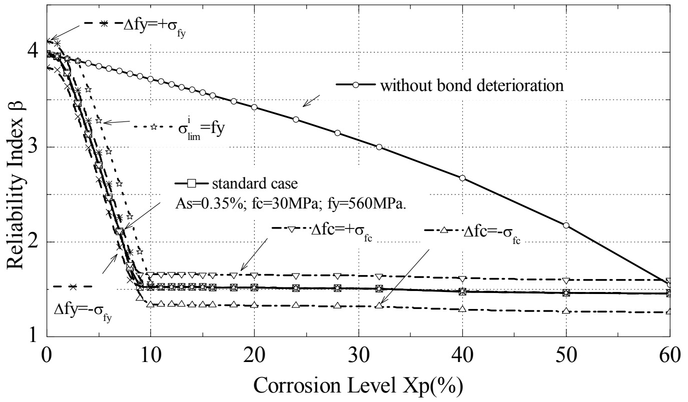

Fig. 7: Influence of bond, concrete and steel strength in the reliability index variation due to corrosion.

Figure 7 shows the comparison between the reliability index considering or not the bond strength deterioration. Considering only a reduction of the reinforcement effective area, the reliability index decreases almost linearly, from the initial value of 4 to the residual value of 1.5, in this case for a corrosion level of about 60%. Figure 7 shows also the impact of increasing or reducing concrete and reinforcement strengths. This is done by considering concrete and steel strengths values deviated one standard deviation from the values presented in Table 1. Results show that concrete strength plays an important role especially in the residual reliability due to variation in the tension strength. This is not absolutely true since the proposed model does not consider concrete deterioration due to cracking. On the other hand the influence of reinforcement strength is only noticeable in the first corrosion stages. For advance corrosion level it is irrelevant since reinforcement, independently of its strength grade, completely lose adherence to concrete. In Figure 7 results obtained considering higher bond strength (σ^{i}_{lim}=1.2f_y) are also presented. The scenario of having σ^{i}_{lim} lower than the f_y=σ^{d}_{lim} is not considered because it is assumed that reinforcement is well designed and the necessary anchorage lengths were respected. Results show an intermediate situation between considering or not the influence of bond deterioration.

For a better perception of the impact of corrosion on the structure reliability, the vertical axis in Figure 7 is normalized as shown in Figure 8. Although more reinforced slabs reveal a higher reliability index the fact is that it decreases more with corrosion. On the other hand the residual reliability is smaller. For instance, for a 0.2% reinforcement ratio the residual reliability index is about 55% of the original. For a 0.5% reinforcement ratio this value represents only 30%.

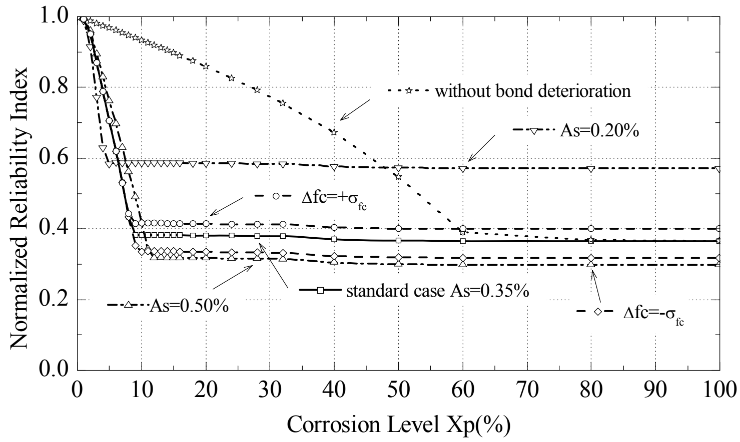

Fig. 8: Normalized reliability index for robustness assessment

Table 2: Robustness Assessment

Case Study	R_d
As=0.20%	59%
As=0.35%	40%
As=0.50%	35%
As=0.35%; Δf_c=-σ_{fc}	36%
As=0.35%; Δf_c=+σ_{fc}	43%
As=0.35%; without bond deterioration	60%

The robustness indicator R_d presented in equation (2), measures the impact of corrosion based on a single indicator, providing a clear overview of these phenomena. The calculation of R_d consists on assessing the area bellow the curves in Figure 8. This value represents an average percentage of the structure reliability index when subjected to generalized corrosion levels varying from 0% to 100%. Table 2 shows the robustness values for the different situations presented in Figure 8. Robustness of the standard case (A_s=0.35%) is 40%. As expected decreasing (A_s=0.20%) and increasing (A_s=0.50%) reinforcement area results in an increase (R_d=59%) and decrease of the robustness (R_d =35%), respectively. The influence of concrete strength in the reliability index residual value is captured by R_d. A negative and positive variation of one standard deviation of the concrete strength, from the standard case, results in R_d =36% and R_d =43%, respectively. However, as explained previously, the adopted model overestimates this effect. Finally, the influence of bond strength deterioration explains the difference between the standard case, R_d =40% and the same case without considering this effect, R_d =60%.

6. Conclusions

In this paper a strategy to evaluate the impact of corrosion on the robustness of reinforced concrete structures is presented. To assess robustness the index proposed by Cavaco et al. [6] is adopted. This index is defined based on structure reliability. To calculate the reliability index of the corroded structure a nonlinear analysis of the cross section coupled with FORM is used. Results show that structure reliability is significantly affected by reinforcement corrosion. The most important factor causing load capacity decreasing is bond strength deterioration. Independently of the reinforcement ratio the residual reliability tends to lower bound. This bound respects to the resistance of the concrete section. At this stage null adhesion exists between steel and concrete. Corrosion affects less the less reinforced structures. This can be measures in terms of the robustness indicators. Despite less reinforced slab reveals a lower safety level it is more robust since that safety level is less affected by corrosion.

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