A Work Project presented as part of the requirements for the Award of a Master's degree in Economics from the Nova School of Business and Economics.

EXUBERANCE, BUBBLES AND BUBBLE CONTAGION IN THE PORTUGUESE HOUSING MARKET: A PERSPECTIVE FROM DISAGGREGATE BANK EVALUATIONS

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21-05-2020

Abstract:

Bubbles in housing markets can impact the macroeconomy via wealth effects and the banking system via credit backed by overvalued collateral. Housing bubbles also reduce affordability of shelter as a basic need. This paper finds bubbles/exuberance in the Portuguese housing markets using suitable empirical methods. Evidence for bubble contagion from Lisbon and Porto towards surrounding housing markets is provided. Alternative explanations for the findings are discussed. Further, the work sheds light on the link between housing bubbles and the banking system; and provides possible policy suggestions addressing the current bubble behaviour.

Key words: Portuguese housing market, Housing bubbles, Exuberant behaviour, Bubble contagion

This work used infrastructure and resources funded by Fundação para a Ciência e a Tecnologia (UID/ECO/00124/2013, UID/ECO/00124/2019 and Social Sciences DataLab, Project 22209), POR Lisboa (LISBOA-01-0145-FEDER-007722 and Social Sciences DataLab, Project 22209) and POR Norte (Social Sciences DataLab, Project 22209).

1. Introduction

According to Brunnermeier (2008) "Bubbles are associated with dramatic asset price increases followed by a collapse. Bubbles arise if the price exceeds the asset's fundamental value. This can occur, if investors hold the asset because they believe that they can sell it at a higher price [to] some investor even though the asset's price exceeds its fundamental value." (New Palgrave Dictionary of Economics 2008). This definition comprises three dimensions often found in existing descriptions of asset price bubbles: The asset trades above its fundamental value, investors might act speculatively, a strong price increase and a bust.

Studying house price bubbles is important for several reasons. First, asset price bubbles generate wealth effects leading to higher consumption during the boom and lower consumption during the bust phase. This behaviour stems from the permanent income hypothesis, relating current consumption stronger to the discounted value of expected income than to current income because households prefer a smooth consumption pattern (Friedmann, 1957). The discounted value of expected income does strongly depend on changes in wealth and through this channel, changes of asset prices translate into the macroeconomy. Changes in wealth can account for much of the observed consumption fluctuations, (Carroll, Otsuka and Slacalek, 2011). Wealth effects from changes in house prices might be particularly important because housing accounts for much of the households' investment portfolio (Englund et al., 2002). Changes in housing wealth can translate into higher consumption than a similar increase in financial wealth (Carroll, Otsuka and Slacalek, 2011).

Second, asset price bubbles might be associated with a substantial increase in borrowing for capital accumulation due to the positive expectations generated by high asset prices. Further, as assets can serve as collateral, rising asset prices during a bubble can lead to an increase of debt, creating a link to the banking sector. Banks' balance sheets look safe as long as asset prices are high and back up the accumulated debt until the bust. Banks might react with tighter credit conditions, leading to a credit crunch that impacts economic activity (Bean, 2004).

Third, the fact, that the supply of housing is not fixed can give rise to further welfare losses associated to bubbles in housing (Glaeser, Gyourko, Saiz, 2008). Strong positive house price changes signal profitable projects in construction, potentially leading to oversupply. An example are ghost cities in Spain. Following Spain's housing bubble in the years before 2007, construction overshot housing demand, leading to unoccupied and unused housing.

Fourth, besides a loss for investors buying the asset at a bubble price and selling after the bust, in housing, the social side is important to be considered as well. A strong increase in the price for housing means that the basic need for shelter becomes less affordable.

Studying the Portuguese housing market and examining a possible bubble is attractive as country specific empirical literature on housing bubbles (except for the U.S.) is less extensive. After the sovereign debt crisis, roughly ending in 2014, Portuguese house prices experienced a strong growth up until present. Several public press publications call for a Portuguese real estate or housing bubble. In the Portuguese case, the above-mentioned affordability aspect of house price growth appears to be prominent.

The objective of this paper is to answer the following questions: Do Portuguese house prices deviate from their fundamental determinants? Do bank evaluations at the local level present bubble-like developments and are they contagious? The two questions are related in the sense that housing markets are driven by local determinants. Therefore, a local level analysis can give insights on the origins of a nationwide bubble and if bubbles do migrate. The usage of bank evaluation data at the disaggregate level helps to shed more light on the above-mentioned channel for a bubble's impact on bank risk.

This work contributes to existing literature in several aspects. Firstly, using quantile regressions to assess a possible divergence between house prices and their fundamental value in Portugal, current exuberance (starting from 2017 Quarter 4 onwards) is found as house price growth persists. Secondly, using disaggregate bank evaluation data for 24 Portuguese municipalities, recursively estimated right tailed Dickey-Fuller tests are used to detect bubble behaviour. The evidence reflects the aggregate picture of current exuberance. Thirdly, possible contagious effects of local level bubbles in Portugal are examined. Based on a time varying contagion coefficient, proposed by Greenaway-Mcgrevy and Phillips (2016), this work provides evidence that the bubbles in Lisbon and Porto may have been contagious for their surrounding areas. Especially the local level analysis and the analysis of contagion contribute to research on Portugal specific literature.

The remainder of this work is structured as follows. Section 2 briefly talks about related literature on housing bubbles and bubble contagion, section 3 and 4 describe the methodologies and the data used, section 5 analyses the Portuguese housing market and provides results and discussion. Section 6 concludes.

2. Housing Bubbles - Related Literature

Literature on housing bubbles is extensive. In the following review, the focus lies on rational bubbles in housing and on bubble contagion. Works that incorporate well plausible special features of the housing market will not be discussed in detail (see e.g. Head, Lloyd-Ellis and Sun, 2013 for a search model, for the role of easy credit see e.g. Mian and Sufi, 2009, Stein, 1995, Glaeser and Nathanson, 2014 and for the role of housing supply and construction costs, see e.g. Poterba, 1984, Glaeser et al., 2008). Neither presented in detail are models departing from the assumption of rationality (see e.g. Case and Shiller (2003) for evidence of optimistic expectations, further see Piazzesi and Schneider (2009), Guren (2014) for models with limited rationality). For a study, explicitly examining the Portuguese housing market and its drivers after 2007, see e.g. Lourenço and Rodrigues (2017).

2.1. Rational Speculative Bubbles in Housing

Following Cuthbertson (1996), a model with risk neutral, homogeneous agents, rational expectations, informational efficiency (no information asymmetries) and a required constant real rate of return $E_t R_{t+i} = r$ is assumed.

In this model, the price of a house is given by the Euler Equation

$$P_t = \delta(E_t P_{t+1} + E_t D_{t+1}) \quad (1)$$

with $\delta = 1/(1+r)$. E_t is the expectations operator, P_t is the house price in period t and D_t is the rent generated by the house in period t.

With the assumption of rational expectations, (1) can be solved by forward substitution,

$$P_t = P_t^f = \sum_{i=1}^{\infty} \delta^i \, E_t D_{t+i}. \quad (2)$$

Assuming the transversality condition $\lim_{i \to \infty} \delta^i \, E_t D_{t+i} = 0$, a unique solution and therefore a unique fundamental house price can be found, P_t^f.

In their essence, user cost models (such as Poterba, 1984, 1992 or Glaeser and Shapiro, 2002) are similar to (2) with D_t generally denoting the benefits of owning a house (Glaeser and Nathanson, 2014). When trying to measure the benefits of owning a home, D_t is usually associated with rental income (Himmelberg, Mayer and Sinai, 2005) or the benefits of living in a particular area, such as income possibilities and amenities, (Glaeser, Gyourko and Morales and Nathanson, 2014, Head, Lloyd-Ellis and Sun, 2014).

The mathematical foundation of an asset price bubble is that there is another solution to (1) which, apart from the fundamental value, P_t^f, contains a bubble term, B_t, such that

$$P_t = \sum_{i=1}^{\infty} \delta^i E_t D_{t+i} + B_t = P_t^f + B_t \tag{3}$$

The properties of B_t can be restricted to $E_t B_{t+m} = B_t/\delta^m$ (see appendix A for an intuitive demonstration). This restricts the bubble's behaviour over time to a martingale, meaning that the existence of a bubble term in the house price equation still satisfies the efficient market hypothesis. For $B_t \neq 0$ the Euler equation is satisfied but the transversality condition is not, because the bubble term is arbitrary.

The basic model can be extended by making the bubble stochastic; see e.g. Evans (1991) for strictly positive bubbles that grow with a certain probability each period and almost surely collapse over time. Addressing the exogeneity of the bubble models, following Froot and Obstfeld (1991), authors create a link between rental income and the bubble term. For example, Nneji, Brooks and Ward, (2013) find evidence of these intrinsic bubbles in the U.S. before 2000. For a more complete overview of different bubble models and how they might be tested econometrically, see e.g. Gürkaynak (2008).

2.2. Bubble Contagion and Bubble Spillovers in Housing

Looking at the contagion between bubbles, one might loosely formulate the situation as a migration of a bubble from one market to another (Gomez-Gonzalez et al. 2017, Deng et al. 2017).

Teng, Chang and Chen (2017) employ a state space model on data from the Taipei city centre and suburbs, finding a diffusion of bubble prices from the centre to suburbs. After diffusion, bubbles in suburbs are found to be larger than in the centre. DeFusco et al. (2012) finds that contagion in the US market played a role during the years before 2007, while not finding evidence for contagion in the burst after 2007.

Martinez-Garcia and Grossmann (2018) model how risk spread shocks can cause exuberance in house prices even in the absence of a bubble term in the house price equation. In their framework, the channel through which shocks in financial markets are spilled over to the housing market is a time varying discount factor in the house price equation, equation (3). Using logit and probit models, they provide evidence that financial market spillovers can cause explosive behaviour in house prices. This contrasts with purely speculative rational bubbles and confirms the implications of their stylized model.

Analysing contagious bubble behaviour between national housing markets, Greenaway-Mcgrevy and Phillips (2016) look at house prices in New Zealand metropolitan areas. To examine the timing of bubble behaviour in metropolitan areas, the study develops a time varying contagion coefficient (the methodology is described further in section *3.3.*). The authors find evidence for Auckland city centre leading the bubble phase and being contagious for its metropolitan area as well as for further metropolitan areas in the country.

Two other works use the method of Greenaway-Mcgrevy and Phillips (2016) to explain financial market spillovers to housing markets. Hu and Oxley (2018) used the contagion coefficient to provide evidence that in Japan, the 1980-1990 asset bubble migrated to the housing market. Deng et al. (2017) concludes that there is evidence that the asset price bubble in China migrated to the housing market between 2005 and 2010.

Gomez-Gonzalez et al. (2017) use the Greenaway-Mcgrevy and Phillips (2016) contagion coefficient considering 20 OECD countries. They find evidence for the US bubble being contagious for a range of countries while not finding evidence of the UK bubble being contagious.

3. Methodology

3.1. Multivariate approach to detect exuberance - Quantile Regression

As ordinary least squares (OLS) regressions are used to estimate the conditional mean of a variable, quantile regressions (QR) are used to estimate conditional quantiles of the variable of interest. QR, first developed by Koenker and Bassett (1978), capture different impacts of predictors depending on the point in the distribution of the dependent variable.

The framework to describe asset price exuberance based on QR was implemented by Machado and Sousa (2006). In their case, stock price developments are analysed conditional on a set of macroeconomic variables. Gerdesmeier, Lenarčič and Roffia (2012) and Lourenço and Rodrigues (2015) apply the method to house prices in OECD countries. In the QR framework, exuberant behaviour is evident when the observed asset price moves in the upper tail of the modelled distribution of the predicted asset price, conditional on the set of regressors (Machado and Sousa, 2006). Furthermore, following the authors' arguments, exuberance can only be determined, if there is an estimate of what exuberant house prices are. The measure for extraordinary high prices should be conditional on the macroeconomic environment, instead of the past behaviour of house prices. Gerdesmeier, Lenarčič and Roffia (2012) mention that especially phases of boom (bubble/exuberance) and bust are known to be non-linear which can be captured in the QR framework.

In practice, one might estimate QR at the median, a high quantile and a low quantile of house prices conditional on some fundamental variables. The high and low QR predictions can be interpreted as an upper and lower bound, determining the tails of the distribution. When the observed house price moves into these tails, exuberant/bubble behaviour or undervaluation can be detected conditional on the selected macroeconomic drivers.

3.2. Univariate testing for Bubble behaviour

To detect bubble behaviour in a time series, Phillips, Shi and Yu (2015a, b) develop the General Augmented Dickey Fuller test (GSADF).The aim of the GSADF is to test episodes of mildly explosive behaviour in a time series. Mildly explosive behaviour is defined by Phillips and Magdalinos (2007a) as

$y_t = \delta_t y_{t-1} + \epsilon_t$, $\epsilon_t \sim i.i.d.(0, \sigma^2)$ in which $\delta_t = 1 + \frac{c}{T^\alpha}$, $\alpha \in (0,1)$ and T is the sample size. When $c > 0$ the root is explosive and approaches unity (random walk) as $T \rightarrow \infty$.

The GSADF for testing mildly explosive behaviour uses recursively estimated right tailed Augmented Dickey Fuller (ADF) tests. Normalizing the sample on the interval $[0,1]$, r_1, r_2 are defined as the start and end of the subsample being tested such that $0 \leq r_1 < r_2 \leq 1$. The null hypothesis of a unit root versus the alternative hypothesis of a mildly explosive root is tested with the following recursive regression:

$$\Delta y_t = a_{r_1,r_2} + \beta_{r_1,r_2} y_{t-1} + \sum_{j=1}^{k} \psi_{r_1,r_2}^{j} \Delta y_{t-j} + \epsilon_t , \epsilon_t \sim i.i.d.\left(0, \sigma_{r_1,r_2}^2\right) \qquad (4)$$

where y_t is the time series of interest, Δ is the first difference operator, a_{r_1,r_2} is the intercept, β_{r_1,r_2} is the autoregressive coefficient, ψ_{r_1,r_2}^{j} for $j = 1, \ldots, k$ are the coefficients of the lagged first differences denoted as Δy_{t-j}, for $j = 1, \ldots, k$. ϵ_t is an $i.i.d$ error term with zero mean and variance σ_{r_1,r_2}^2. The subscripts r_1, r_2 are the subsample start and end points respectively for which the regression is estimated.

Using $r_1 = 0$ and $r_2 = 1$ gives the right tailed ADF test on the full sample. Using the right tailed ADF on the full sample to detect bubbles, the power is harmed by collapsing bubbles (i.e. the bubble does not only build up in the sample but also collapses). The test for the whole sample might therefore not detect the bubble even though bubble behaviour is present in one or more subsamples of the time series. Addressing this issue, Phillips, Wu and Yu (2011) use a recursive estimation of the ADF, beginning with the smallest feasible window, starting from $r_1 = 0$. Keeping the starting point fixed, the ADF is performed with r_2 increasing one observation at a time, until $r_2 = 1$. Phillips, Wu and Yu (2011) then define the test statistic, Sup ADF (SADF) as the supremum value of the set of ADF test statistics yielded by the recursive estimation. The test is more powerful in detecting mildly explosive behaviour than the ADF over the full sample if a bubble not only builds up in the sample but also collapses within the sample, (Homm and Breitung, 2012).

Phillips, Shi and Yu (2015a & b) show that the SADF is less powerful in case multiple periodically collapsing bubbles exist in the sample and introduce the General Sup ADF (GSADF). The test uses the expanding window structure of the SADF while not keeping $r_1 = 0$ fixed but allowing r_1 to be flexible. This leads to a set of ADF tests for all feasible subsamples. For the test statistic and limit distribution of the GSADF test see Appendix B.

The way in which the GSADF is constructed, makes it more powerful for samples with reoccurring and collapsing bubbles than the ADF over the whole sample or the SADF over the forward expanding window.

3.3. Measuring Bubble Contagion

As discussed in section *2.2.*, bubbles might be substance to contagion in the sense that a bubble in one market migrates to another. To formally describe this effect over time, Greenaway-Mcgrevy and Phillips (2016) develop a time varying contagion coefficient.

In a first step, the method estimates autoregressions similar to equation (4) with a fixed window, leading to sequences of autoregressive coefficients, $\{\hat{\beta}_{i,s}\}_{s=S}^{T}$, i indexing regions, S being the length of the fixed window, and s being the end of the subsample, $s = S, \dots, T$. The autoregressive coefficients are obtained by ordinary least squares.

With the estimated sequences, the following regression is estimated

$$\hat{\beta}_{j,s} = \delta_{1j} + \delta_{2j}\left(\frac{s}{T-S+1}\right)\hat{\beta}_{core,s-d} + \epsilon_t, s = S, \ldots, T \tag{5}$$

where j denotes some region and $core$ denotes the potential contagious region from which a bubble spreads. d is a lag parameter for the lagged contagion effect from $core$ to j. δ_{2j} is the time varying contagion coefficient.

The contagion coefficient is estimated by local level kernel regression with

$$\delta_{2j}(r; h, d) = \frac{\sum_{s=S}^{T} K_{hs}(r)\tilde{\beta}_{j,s}\tilde{\beta}_{core,s-d}}{\sum_{s=S}^{T} K_{hs}(r)\tilde{\beta}_{core,s-d}^{2}}, \text{ and } \tilde{\beta}_{j,s} \coloneqq \hat{\beta}_{j,s} - \frac{1}{T-S+1}\sum_{s=S}^{T}\hat{\beta}_{j,s}\ .$$

$K_{hs}(r) = \frac{1}{h}K\left(\frac{s/T-r}{h}\right)$ where $K(.)$ is the Gaussian Kernel $K(.) = (2\pi)^{-1/2}e^{-1/2(.)^2}$ and h is the bandwidth parameter. For further details, see Greenaway-Mcgrevy and Phillips (2016).

The framework lets the contagion parameter evolve smoothly over time (see Greenaway-Mcgrevy and Phillips, 2016). One can track the change in the contagion coefficient during bubble periods. The method helps to visualize the shape of the contagious impact over time in one region and allows for a comparison between regions.

4. Data

4.1. Used data series

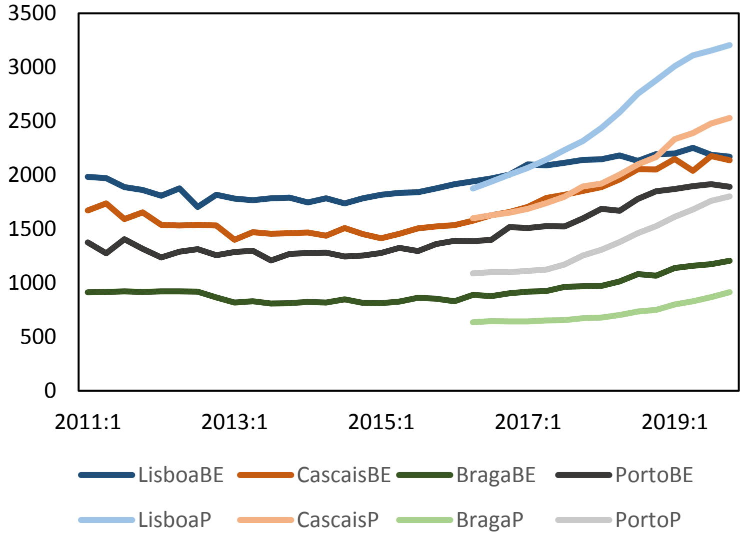
Quarterly data for the price to rent ratio, price to income ratio, rent price index, disposable income, GDP, private consumption deflator, population, real money market rate, unemployment rate, labour force, real mortgage rate, real gross fixed capital formation (GFCF) housing are obtained from OECD, Banco de Portugal and European Central Bank. Quarterly house prices are also taken from OECD and correspond to hedonic price data of newly built and existing dwellings sales (adjusted for quality measures of housing: square meter price, size of dwelling, location of dwelling). Real series are defined as nominal data divided by the private consumption deflator. The series span from 1988:Q1–2019:Q3.

Monthly bank evaluation data (2011:M1–2020:M2) for 24 municipalities was obtained from Instituto Nacional de Estatístico (INE), based on the institution's survey on bank evaluation on housing.

4.2. Bank Appraisals vs Market prices

As bank appraisal data (used interchangeable with bank evaluation) is used in the disaggregate analysis due to constraints in the availability of market price data, it is important to have a closer look at the differences in appraisals and market prices. Appraisals provide a safety mechanism for banks, as the minimum of the appraisal value and the purchase price is pledged as collateral. The value pledged as collateral is also used for the calculation of the Loan-to-Value (LTV) as a key metric for risk measurement in lending. In case of bubbles, LTVs are biased downwards, skewing banks risk profiles.

Figure 1: Bank Appraisals vs. Market Prices



Y Axis is in €/m², light colours are market prices (P), median dwelling sales in €/m², dark colours are average values of Bank evaluations in €/m² (BE).

Source: INE.

Nakamura (2010) argues that appraisals are systematically upward biased (above the market price), due to a conflict of interest as banks hire the appraiser.

Figure 1 plots average bank evaluations against median sales values for some important Portuguese housing markets. For Lisbon, average appraisals are substantially smaller than median sales prices towards the end of the series. The picture for Cascais is similar but less strong. This might be explained by banks' precautionary behaviour towards the strong increase in market prices. For Braga, appraisals are above market prices throughout the series. Similarly, appraisals for Porto are higher than market prices but come closer together towards the end.

To thoroughly examine appraisal bias, micro data shall be considered such as in Cho and Megbolugbe (1996).

5. Empirical Results and Discussion

5.1. Aggregate Analysis: Exuberance

To analyse Portuguese house prices at the aggregate level, it is opted for the quantile regression approach, described in section *3.1*. The literature on determinants for house prices in the long run is extensive (as partly described in section *2.*). Looking at the demand side of housing, typical fundamental variables used include: Economic activity, income, demographic variables such as population or labour force, employment, mortgage rate and other interest rates and the development of substitutive investments (e.g. stock market returns). Usually housing supply is considered to be fixed in the short run but flexible in the long run. Supply side variables include building permits, construction costs, and GFCF housing (Glaeser, Gyourko and Saiz, 2008).

Given the length of the sample and available data the model is chosen as follows. To capture a main driver of housing demand, disposable income per capita is included. If income rises, housing becomes more affordable and thus, demand for housing will rise, pushing prices upwards. The second variable included, is a three-month money market rate, capturing effects of the availability of credit to finance housing (by affecting the mortgage rate for households and borrowing rates for institutional real estate investors). To capture demographic effects that influence housing demand, the labour force is included as well.

As we are looking at long run relationships, the supply of housing is included. GFCF housing is used as supply variable. The variable captures investment in housing, providing a good proxy for the change in housing supply. Thus, when GFCF rises, the supply of housing shall rise, putting downward pressure on house prices.

The model is given by,

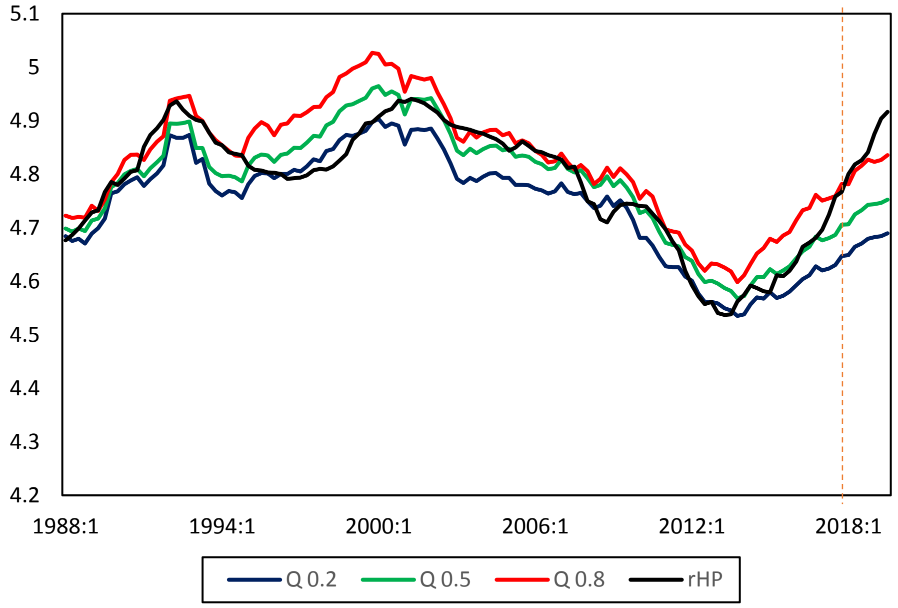
$$rHP_t(\tau|F_t) = \alpha_0(\tau) + \alpha_1(\tau)rdic_t + \alpha_2(\tau)rmmi_t + \alpha_3(\tau)rgfcf_t + \alpha_4(\tau)labour_t \quad (6)$$

where rHP_t is the natural logarithm of the real house price index in period t, $rdic_t$ is the natural logarithm of real disposable income per capita in period t, $rmmi_t$ corresponds to a real 3 month money market rate in period t, $gfcf_t$ is the natural logarithm of the GFCF housing in period t and $labour_t$ is the natural logarithm of labour force in period t. τ is the quantile for which the model is estimated.

As the variables used in (6) are nonstationary (confirmed by unit root tests), it has to be tested for quantile cointegration. Following Xiao (2009), testing quantile cointegration by performing a cusum test on the residuals of (6), the null of cointegration cannot be rejected (1.7470 for $\tau = 0.3$, 0.5950 for $\tau = 0.5$, 1.6380 for $\tau = 0.7$, smaller than the critical value 1.82). 0.3, 0.5 and 0.7 are used for the quantile cointegration test as the cusum type test can get too sensitive towards the tails of the distribution that contain less information. See Appendix C for specifics of the quantile cointegration test and previously conducted unit root tests. Thus (6) can be seen as a quantile specific long run relationship.

Figure 2 is showing the estimation results of the model, plotting the predicted 0.2, 0.5 and 0.8 quantiles and the natural logarithm of the observed real house price index over time. 0.2 and 0.8 are chosen as limits due to the length of data. One can see that the estimated quantiles, together, can explain a lot of the variation in real house prices as the observed house prices are between the estimated 0.2 and 0.8 quantiles for most of the period. Towards the end of the series, more or less from 2016:Q2 onwards, house prices grow faster than the estimated quantiles.

Figure 2: House prices and estimated fundamental quantiles



Y Axis denotes natural logarithm of real house prices. predicted quantiles and observed house prices are plotted, blue is the 0.2 quantile, green is the median, red is the 0.8 quantile, black is the observed series. The dashed orange line marks the start of current exuberant behaviour.

Source: OECD, ECB, Banco de Portugal and author's calculations.

The persistence in faster growth leads to observed house prices moving further in the upper tail of the estimated distribution. In 2017:Q4 house prices cross the 0.8 quantile, marked by the dashed orange line. Given the threshold, house prices are considered exuberant from 2017:Q4. The result clearly is conditional on the chosen macroeconomic fundamentals. Different model specifications were examined (including GDP per capita, unemployment, mortgage rate, real rents) and are reported in Appendix D. The qualitative result, that house prices are exuberant in the end of the series does hold for these specifications. Still, factors remain, that might drive house prices above the fundamental benchmark set by macroeconomic variables (such as investment in tourism accommodation, foreign direct investment, regulations and other observed or unobserved economic variables, section *5.4.* will further discuss this issue).

5.2. Disaggregate Analysis: Bubble behaviour

Housing markets are known to be highly affected by local level variables. The price of housing in Lisbon is likely to move differently from the price of housing in Leiria. Thus, detecting exuberant behaviour when treating Portugal as one housing market, local differences are interesting to explore.

To capture these differences, we use bank appraisal data for 24 Portuguese, mainly urban, municipalities from January 2011 to February 2020. Due to a lack of time series for determinants at the local level, we apply the univariate GSADF test for mildly explosive behaviour (in this part used interchangeably with bubble behaviour), described in section *3.2.*, to the series. The test is consistent with the rational bubble introduced in section *2.1.*, as the martingale characteristics of asset price bubbles are captured (Greenaway-Mcgrevy and Phillips, 2016). A drawback, when using the univariate approach is, that explosive behaviour in the series can be caused by explosiveness in the underlying determinants (discussed in section *5.4.*). When considering a model of rational asset price bubbles with a constant discount factor, the determinant is expected rental income. Sustained explosive growth in expected rental income would mean periods of consecutive good news of which each information is better than the previous news which might lack credibility (Greenaway-Mcgrevy and Phillips, 2016). The test is conservative in the sense that prolonged but not explosive growth of prices will not qualify as a bubble.

The GSADF test was performed with an initial window of 24 month and using three lags, supported by the Bayesian Information Criterion. Critical values are derived by Monte Carlo simulation. The test results are reported in *table 1*.

Table 1: GSADF Test Results for Bank Evaluation Data for Portuguese Municipalities

GSADF Test Statistics			
Portugal	3.878***		
Almada	2.359***	Maia	2.760***
Amadora	2.882***	Matosinhos	4.084***
Barcelos	1.620**	Odivelas	2.316***
Braga	2.270***	Oeiras	1.966**
Cascais	3.026***	Porto	2.295***
Coimbra	2.103**	Santa Maria da Feira	0.995
Funchal	1.907**	Seixal	2.802***
Gondomar	3.647***	Setúbal	2.192***
Guimarães	2.110**	Sintra	4.574***
Leiria	1.926**	Vila Franca de Xira	2.174**
Lisboa	2.723***	Vila Nova de Famalicão	3.023***
Loures	2.655***	Vila Nova de Gaia	3.248***

, ** and * denote significance at 10, 5 and 1 percent level. All results are for 3 autoregressive lags. Critical values are obtained by Monte Carlo Simulation.*

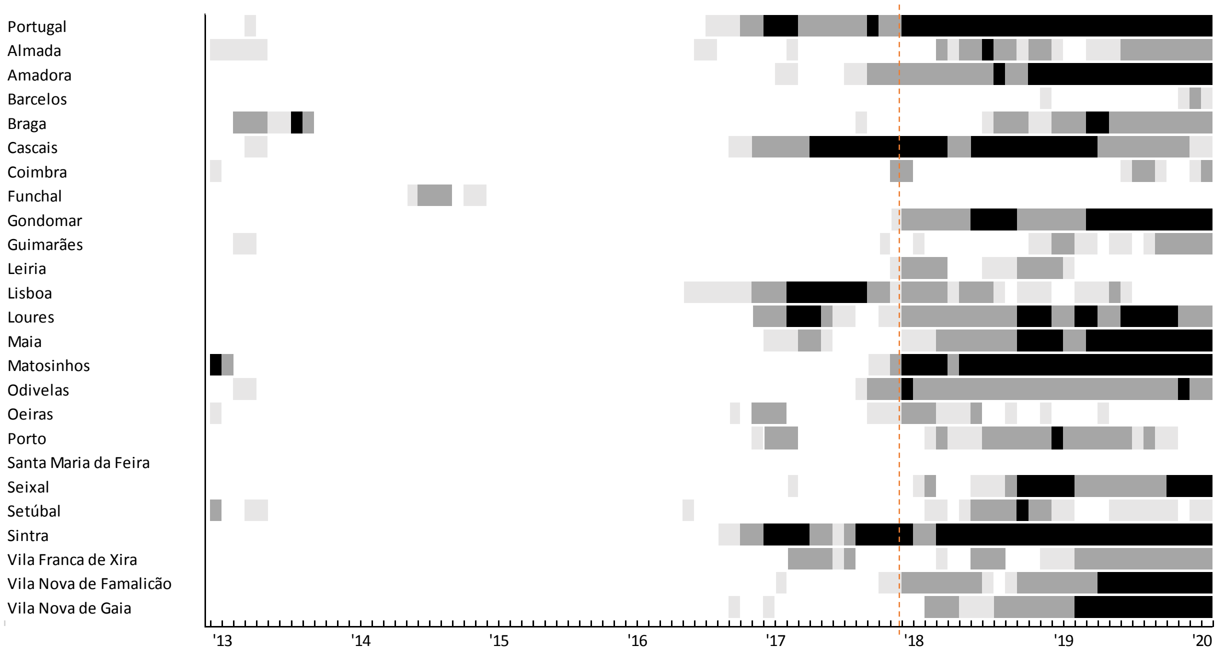
Source: INE and author's calculations.

We notice that evidence for bubble behaviour is found in close to all regions under analysis, solely for Santa Maria da Feira, the null hypothesis of a unit root is not rejected in favour of the presence of mildly explosive behaviour in the sample.

To get a more systematic picture of the occurring episodes of bubble behaviour, *figure 3* plots the timely course of bubble episodes in the 24 municipalities at usual confidence levels.

At first sight, bubble behaviour seems to be correlated across municipalities. Before 2016, evidence for bubble behaviour is rather sporadic in the sample. Beginning in mid-2016, first evidence for the subsequently prolonged and widespread bubble behaviour is recorded for Lisbon at the 10% level. By December 2016, Lisbon, Cascais, Loures, Porto and Oeiras show evidence for bubble behaviour at the 5% level, Sintra at the 1% level. Moving in time, bubble behaviour is detected across more cities, in December 2017, 11 of the 24 cities under consideration are evident for bubbles at the 5% level. Towards the end of the sample, the highest numbers of bubbles are detected (18 out of 24 municipalities).

Figure 3: Evidence for bubbles in Portuguese municipalities against time



Areas represent the rejection of the null of the GSADF test in favour of mildly explosive behaviour at the 10% (light grey), 5% (grey) and 1% (black) confidence level. The dashed orange line indicates the start of exuberant behaviour found in 5.1.

Source: INE and author's calculations.

This is in accordance with the aggregate analysis in section *5.1.*, in which exuberant behaviour is detected starting in 2017:Q4, represented by the dashed orange line in *figure 3*. Interestingly, despite Lisbon leading the bubble cycle, the Lisbon bubble seems to have ended in July 2019. Also, the bubble in Porto is not evident towards the end of the considered period (November 2019), while there is strong evidence for bubble behaviour in a wide range of municipalities. It has to be recalled that bank evaluations are used for this analysis and do differ from market prices, as in section *4.2*. Thus, some results in *figure 3* might reflect banks' precautionary behaviour (e.g. Lisbon, Cascais, Porto).

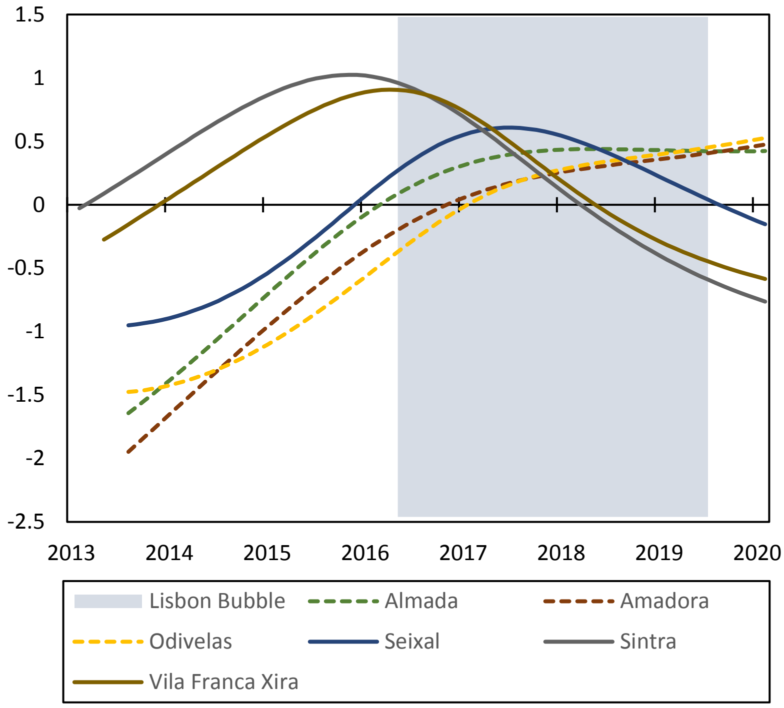
5.3. Bubble Contagion

The results of the previous section point towards possible bubble contagion as bubbles are correlated and some municipalities present bubble behaviour earlier than others. To analyse bubble contagion, the contagion coefficient proposed by Mcgrevy and Phillips (2016) and presented in section *3.3.* is used. The contagious response before, during and after bubble periods is tracked.

Lisbon and Porto are chosen as core markets because the two municipalities present the centres of the country's largest metropolitan areas. In case of Lisbon, the bubble also starts earlier than in any other municipality. The lag order for municipality j possibly behaving contagious to the core bubble is determined by the months between first evidence for bubble behaviour in the core and first bubble behaviour in the tested municipality. In case more than 6 months are between the initiation of bubbles, it is opted for 6 lags. For Lisbon as a core market, Almada, Amadora, Cascais, Loures, Odivelas, Oeiras, Seixal, Setubal, Sintra and Vila Franca da Xira were tested towards possible contagious reactions due to their geographic proximity. Regression (5) was estimated using local level kernel regression with local constant fitting. To estimate the autoregressive coefficients for the regression, a fixed window of 24 months was used. Results are presented for municipalities with strongest responses in *figure 4*.

The contagion coefficients for Sintra, Vila Franca de Xira and Seixal present an inverted U shape. The contagious effect from Lisbon to Sintra and Vila Franca de Xira builds up before the bubble in Lisbon is evident and are at the maximum at the start of Lisbon's bubble period (Vila Franca de Xira) and 4 months before (Sintra). The contagious effect decreases during the bubble period before becoming adverse around April 2018. The contagious behaviour for Seixal becomes positive 4 months before Lisbon's bubble period. It reaches a maximum roughly one year after the start of Lisbon's bubble and decreases to zero one month after the Lisbon bubble ends. The contagious response for Almada becomes positive about one month before the Lisbon bubble, responses for Amadora and Odivelas become positive about 7 months after the initiation of the Lisbon bubble.

Figure 4: Time varying contagion coefficient from Lisbon housing market for other municipalities in geographic proximity

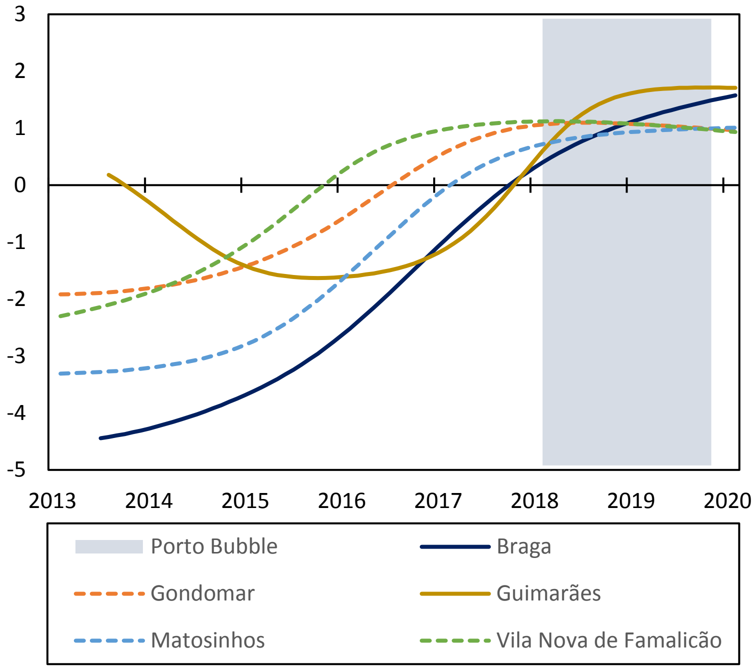


The shaded area indicates the Lisbon bubble period evident in section 5.2. Lines are the plotted time varying contagion coefficients δ_{2j} for the respective municipalities.

Source: INE and author's calculations.

In a similar way, Porto is analysed as a core region having a contagious impact on its surrounding municipalities, displayed in *figure 5*. All municipalities present some sort of S shape in their responses. Effects for Gondomar, Matosinhos and Vila Nova de Famalicão become positive before the Porto bubble and are rather flat during the Bubble Period. Braga and Guimarães both turn positive roughly 2 months before the Porto bubble and increase during the bubble period. It shall be noted that the Porto bubble does not lead the bubble periods for Gondomar, Matosinhos and Vila Nova de Famalicão, thus for these municipalities the response to the Porto bubble can rather be interpreted as a reinforcing effect.

Figure 5: Time varying contagion coefficient from Porto housing market for other municipalities in geographic proximity



The shaded area indicates the Porto bubble period evident in 5.2. Lines are the plotted time varying contagion coefficients δ_{2j} *for the respective municipalities.*

Source: INE and author's calculations.

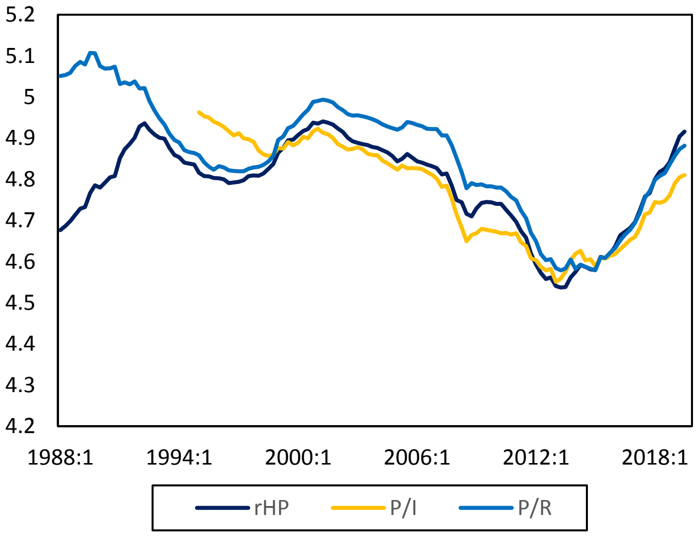
In general, the results indicate bubble contagion because responses are positive during bubble periods. Nevertheless, the results are rather descriptive, tracking how the municipalities' autoregressive coefficients are related to the respective core market autoregressive coefficients in time, through the kernel regression specification. To better explain bubble contagion and commonalities and differences in response functions above, further possible driver variables on the local level would need to be included.

5.4. Discussion

As previously mentioned, the detected exuberance in section *5.1.* and the detected bubble behaviour in section *5.2.* can be caused by other variables, not only by purely speculative behaviour (as in *2.*). In this section, we discuss different drivers for house prices, that were not possible to include in formal methods.

Common variables considered in the literature are income and rent, capturing affordability, the choice between renting and buying and the return on housing investment. As can be seen in *figure 6*, the price to rent and price to income ratios are rising similarly to house prices from roughly 2014. Thus, at the aggregate level, it seems that a rather small part of the increase in real house prices can be explained by rising income or rising rents (also confirmed by model 3 in appendix D, shown in Figure D.3 in which exuberance is evident despite including income and rent data).

Figure 6: Real house prices, price-to-rent and price-to-income ratio for Portugal



Plotted are the natural logarithms of real house price index(rHP) and natural logarithms of Price-to-rent ratio (P/R) and Price-to-income ratio (P/I), 2015=100.

Source: OECD.

Apart from the demand variables mentioned in section *5.1.*, an important driver can be tourism and investment in tourism accommodation, especially in the Portuguese case. Due to online platforms fostering access to clients and management for short-term rental, housing investments to accommodate tourists became more attractive for a wider range of investors. This not only increases demand on the housing market, but also willingness to pay from an investment point of view, since short-term letting to tourists (if occupied enough) will lead to a substantially higher return than permanent letting to the local population. The tie between local income and return on housing is cut in this case. The argument can be supported to some extent by the explosive growth in the number of firms that are associated to "furnished accommodation for tourists" from 2013 and 2014 onwards, see *table 2*. The data ends before the detected exuberance and bubble behaviour but it does give a hint towards the importance of investment in tourism accommodation.

Table 2: Number of Firms associated to furnished short-stay accommodation for tourists

	2009	2010	2011	2012	2013	2014	2015	2016
Shortstay accomodation	2112	2578	2,784	2,990	3,333	5,228	11,917	17,196
Furnished accomodation for tourists	340	504	572	665	906	2,374	8,303	13,122
Other shortstay accomodation	1772	2074	2,212	2,325	2,427	2,854	3,614	4,074

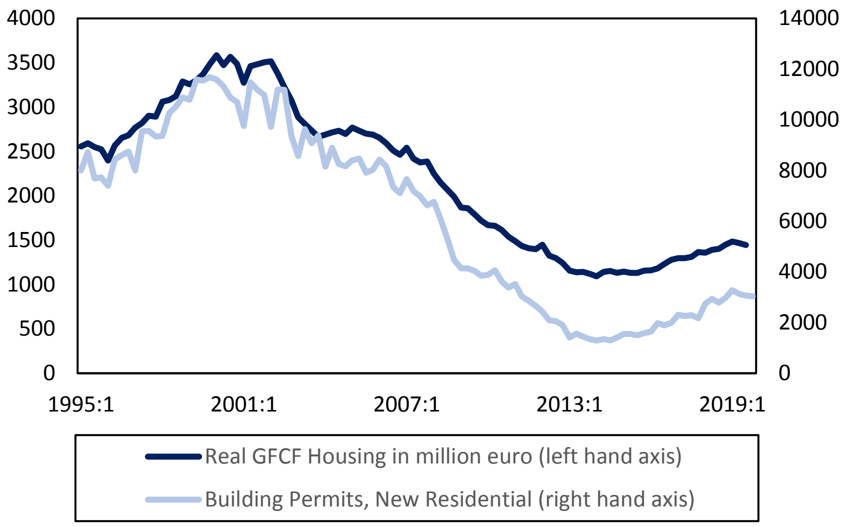
Source: Banco de Portugal, IES.

Further arguments on drivers for house prices include foreign direct investment in housing. Following the Golden Residence Permit Programme, introduced in October 2012, foreign direct investment in housing likely accelerated. The regulation allows residence after investing more than 500.000€ in property. The number of resident permits given increased from 464 in 2013 to 1526 in 2014. In 2018 the number was 1332 and in 2019 1160 (Serviço de Estrangeiros e Fronteiras, 2020).

Another argument that can be thought of is taste. Portugal and Lisbon won several awards, e.g. Portugal as "World's Leading Destination" in 2018 and 2019 and Lisbon as "World's Leading City Destination 2018" (World Travel awards, 2020). This might cause broad based attraction for people choosing Portugal as their home for a medium period of time (such as exchange students and remote workers that are not captured in migration or labour force statistics). Options for medium term rentals to this group of people again offer higher returns on housing investment.

These factors present further demand for housing. Housing supply is only growing slowly, from 2014 after a prolonged decline, leaving upward pressure on prices (see *figure 6*).

Figure 7: Housing supply



Source: Banco de Portugal, INE.

Together, the presented arguments can explain at least some of the detected exuberant and bubble behaviour. Due to data restrictions it is not possible to quantify the impact. Further disaggregate data would be crucial to explain local differences in bubble behaviour. However, it seems the arguments leave space for rational speculation as a possible explanation.

Why can housing bubbles be contagious? The channels for contagion cannot be explored in detail in this work but a narrative for the findings is still interesting to provide. The reason for contagion might be, that when speculation begins in one place, say Lisbon, prices start to increase rapidly, leading first investors to search for alternatives and invest in other property markets, where they expect the next housing bubble. Prices start to rise in these regions, attracting more investors and might create bubble behaviour. Geographic proximity is an important channel, since bubble behaviour in centres is likely to push people to the suburbs, pushing prices up in those regions. Even though bubble contagion can be described and analysed after the occurrence, a general prediction for bubble contagion patterns goes against the fundamental rule that bubbles cannot be predicted (martingale behaviour).

5.5. Policy Implications

On a general level, the presented analysis points towards policies addressing housing bubbles that are present in bank evaluations in many Portuguese municipalities. Further, the presented evidence for bubble contagion indicates quick and direct response for inflationary bubbles in one region, since otherwise the inflating bubble might well be contagious causing bubbles in more regions.

When it comes to concrete measures regarding bubbles, economic literature does not provide a defined toolbox to address the problem.

In the case of housing, authors call for the supply side. The argument is that if supply can adjust quicker because of e.g. less bureaucracy in building permits, housing bubbles will not be as strong. Glaeser, Gyourko and Saiz (2008) point towards possible welfare losses due to overbuilding during housing bubbles.

Given the evidence of bubble behaviour in bank evaluations, one feasible suggestion is a more prudent property valuation, such that current LTV's will better reflect bank's risk. To restrict lending, the down payment requirement could be adjusted, such that LTV is advised to be less than currently 0.9. Another option to restrict lending is higher required equity for banks. A deterioration of LTVs based on bubble valuations can itself cause defaults by causing higher interest payments for borrowers, underlining the importance of adequately valued LTVs.

Further, policies regarding home ownership should take into account that the asset that many households take on a lot of debt for, does present bubble behaviour. The large amounts of credit backed by an asset which value and valuation (bank evaluation) is substance to bubble behaviour has to be considered and communicated as a risk.

The lack of further suitable policy answers to bubbles and discussions that usually focus on how bubbles are impossible to be addressed by policies, points towards the importance of future research.

6. Conclusion

The paper presents an analysis of Portuguese house price behaviour. For the aggregate data we conduct a quantile regression analysis. Conditional on the chosen macroeconomic determinants the results indicate exuberant behaviour starting in 2017:Q4.

To get a better picture of the exuberant behaviour detected at the aggregate level we turned to disaggregate data on bank evaluations of 24 Portuguese municipalities. To detect local level bubble behaviour in bank evaluations, the General Supremum Augmented Dickey Fuller test of Phillips, Shi and Yu (2015a & b) is used. Broad based bubble behaviour is evident. Timely differences in the bubble development can be noticed. To explore the question if the bubbles in Lisbon and Porto were contagious for their surrounding municipalities, we use the contagion coefficient proposed by Greenaway-Mcgrevy and Phillips (2016). The results indicate that Lisbon and Porto seem to have a contagious effect on surrounding housing markets.

Clearly, exuberant behaviour in prices can be caused by a range of reasons apart from speculative bubbles. Together, local and foreign investment associated to tourism accommodation, foreign direct investment and shifts in taste can explain some but supposedly not all explosiveness/exuberance in Portuguese house prices.

Bubble behaviour in bank evaluations for housing has strong implications for banks' risk. LTV's can be biased downward causing a too high volume in credit and a skewed risk profile.

A bust of the housing bubble could as well cause defaults by rising mortgage payments as they are based on LTV.

Policy implications shall address the current bubbles detected in bank evaluations and take into account that bubbles can be contagious. Banks and appraisers might be advised to value property more prudently.

Future research on Portuguese local level housing markets is important to better understand the country's housing market dynamics (as they are known to be driven locally) even though data is less available than in other countries. Future research on how and why appraisals differ from market values in Portugal is a further interesting task. Also, a framework to analyse channels through which bubbles can be contagious is interesting to research. On a further level, it is important to research and improve techniques to detect bubbles and as well to conduct research on feasible policy responses.

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Appendix

Appendix A: Restricting Rational Bubbles to a martingale

Following Cuthbertson (1995) the bubbles behaviour can be restricted by assuming that (3) is a solution to (1).

Leading (3) one period ahead, to time $t+1$, gives the expression

$E_t P_{t+1} = E_t(\delta E_{t+1} D_{t+2} + \delta^2 E_{t+2} D_{t+3} + \cdots + B_{t+1})$,

And using the law of iterated expectations, $E_t(E_{t+1} D_{t+j})= E_{t+1} D_{t+j}$,

the expression becomes:

$$E_t P_{t+1} = (\delta E_{t+1} D_{t+2} + \delta^2 E_{t+2} D_{t+3} + \cdots + B_{t+1}). \quad \text{(A.1).}$$

Equation (1) contains $\delta(E_t P_{t+1} + E_t D_{t+1})$. Using (A.1), it is given by

$\delta(E_t D_{t+1} + E_t P_{t+1}) = \delta E_t D_{t+1} + (\delta E_{t+1} D_{t+2} + \delta^2 E_{t+2} D_{t+3} + \cdots + \delta E_t B_{t+1})$ and

using the definition of P_t^f one arrives at $\delta(E_t D_{t+1} + E_t P_{t+1}) = P_t^f + \delta E_t B_{t+1}$.

Using (1), the left-hand side becomes P_t and one arrives at

$$P_t = P_t^f + \delta E_t B_{t+1} \quad \text{(A.2)}$$

Since (3) and (A.2) are only both solutions to (1), if equated, leading to

$$B_t = \delta E_t B_{t+1} \text{ or } E_t B_{t+1} = B_t/\delta = (1+r)B_t \quad \text{(A.3)}$$

If (a3) holds, (3) and (A.2) are the same and both satisfy (1).

More generally (A.3) provides with $E_t B_{t+m} = B_t/\delta^m$.

This restricts the bubble’s behaviour over time to a martingale, meaning that the best forecast of the bubble’s future values only depends on its current value, given all available information.

Appendix B: General Supremum Augmented Dickey Fuller test (GSADF)

The GSADF test statistic is given by

$$GSADF(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} \left\{ \sup_{r_2 \in [r_0, r_1]} ADF_{r_1}^{r_2} \right\} \tag{B.1}$$

in which r_0 is the smallest feasible window chosen. Under the null hypothesis, the limit distribution of $GSADF(r_0)$ is

$$\sup_{r_1 \in [0, r_2 - r_0],\, r_2 \in [r_0, r_1]} \left\{ \frac{\frac{1}{2} r_w [W(r_2)^2 - W(r_1)^2 - r_w] - \int_{r_1}^{r_2} W(r)dr[W(r_2) - W(r_1)]}{r_w^{1/2} \left\{ r_w \int_{r_1}^{r_2} W(r)^2 dr - \left[\int_{r_1}^{r_2} W(r)dr \right]^2 \right\}^{1/2}} \right\} \tag{B.2}$$

where W is a standard Wiener process and $r_w = r_2 - r_1$ is the size of the window.

If $GSADF(r_0)$ is larger than the critical value from the limit distribution, the null hypothesis of a unit root is rejected against the alternative hypothesis of mildly explosive behaviour.

Appendix C: Quantile cointegration

To interpret (6) as a long run relationship for each quantile, after confirming nonstationarity, it needs to be tested for quantile cointegration.

Following Xiao (2009), quantile cointegration can be tested as follows. Looking at $\psi_\tau(u) = \tau - I(u < 0)$ and the residual from the estimated quantile regression:

$\varepsilon_{t\tau} = y_t - Q_{yt}(\tau|\mathcal{F}_t) = y_t - \Theta(\tau)'Z_t = \varepsilon_t - F_\varepsilon^{-1}(\tau)$, one has $Q_{\varepsilon_{t\tau}}(\tau) = 0$, where $Q_{\varepsilon_{t\tau}}(\tau)$ is the τ-th quantile of $\varepsilon_{t\tau}$ and $E\psi_\tau(\varepsilon_{t\tau}) = 0$. Xiao (2009) proposes to test for cointegration by testing the stability of the process of $\varepsilon_{t\tau}$. Considering the partial sum process such as

$Y_n(r) = \frac{1}{\omega_\psi^* \sqrt{n}} \sum_{j=1}^{[nr]} \psi_t(\varepsilon_{j\tau})$, where ${\omega_\psi^*}^2$denotes the long run variance of $\psi_\tau(\varepsilon_{j\tau})$, under appropriate assumptions, that process follows an invariance principle and weakly converges to a standard Brownian motion $W(r)$. Choosing a continuous functional $h(.)$ measuring the fluctuation of $Y_n(r)$, a robust test for cointegration can be examined using $h(Y_n(r))$. With the continuous mapping theorem and under regularity conditions and the null hypothesis of cointegration $h(Y_n(r)) \Rightarrow h(W(r))$.

Under the alternative hypothesis, the statistic diverges to ∞.

If a quantile specific cointegrating relationship is present, the residuals of the quantile regression only present fluctuations around a long-term equilibrium, reflected by a stable process of $\varepsilon_{t\tau}$.

Previously conducted unit root tests to confirm non-stationarity of the variables in (6) are presented in *table C.1*.

Table C.1: Test results of ADFGLS and Phillips Perron

	ADFGLS test				Phillips Perron test		
Variable	test statistic				test statistic		
						Z(t)	Z(rho)
rHP	-1.148					-1.301	-3.595
rgdpc	-1.209					-2.942	-5.954
rdic	-1.181					-3.087	-6.930
rmmi	-2.540					-2.651	-13.122
rgfcf	-1.505					-0.523	-0.777
labour	-0.957					-0.862	-1.463
rmrti	-1.700					-2.327	-11.003
rrent	0.602					-3.147	-7.723
unemp	-2.008					-1.140	-2.564
Confidence level	10%	5%	1%		10%	5%	1%
Critical values	-2.64	-2.93	-3.46	Z(t)	-3.14	-3.44	-4.03
				Z(rho)	-17.58	-20.80	-27.57

$rdic$, $rmmi$, $rgfcf$, $labour$ are defined as in *5.1*. $rgdpc$ corresponds to the logarithm of real GDP per capita, $rmrti$ is the real mortgage rate, $rrent$ is the logarithm of real rent index and $unemp$ is the unemployment rate. The ADFGLS test uses the Perron-Qu method and it is tested down from 12 lags. A trend is always included except for rHP (the asymptotic p-value of the ADFGLS without including a trend for rHP is 0.229 and the critical values for the Phillips Perron test when not including a trend are -2.578, 2.888 and -3.501 for the 10%, 5% and 1% level respectively).

As the null of a unit root is not rejected for all series (except for $rrent$ at the 10% level using Phillips Perron Z(t)), the series are nonstationary and qualify for testing quantile cointegration.

Source: OECD, ECB, Banco de Portugal and author's calculations.

Appendix D: Additional figures for the different quantile regression specifications

To check the robustness of the model in *5.1*, following four models were considered as well, shown in table D.

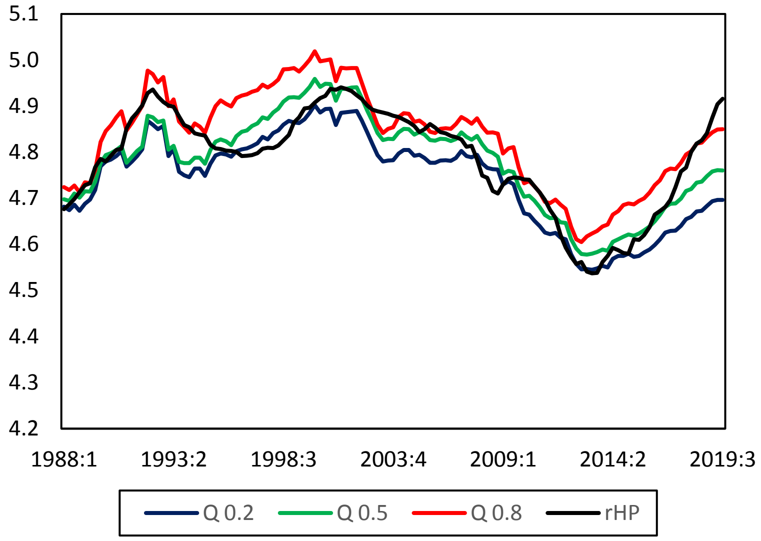
Table D: Further Model Specifications for exuberance analysis

Model	Explanatory Variables			
1	rgdpc	rmmi	rgfcf	labour
2	rdic	rmrti	rgfcf	labour
3	rdic	rmmi	rgfcf	rrent
4	rdic	rmmi	rgfcf	unemp

$rdic$, $rmmi$, $rgfcf$, $labour$ are defined as in *5.1*. $rgdpc$ corresponds to the logarithm of real GDP per capita, $rmrti$ is the real mortgage rate, $rrent$ is the logarithm of real rent index and $unemp$ is the unemployment rate.

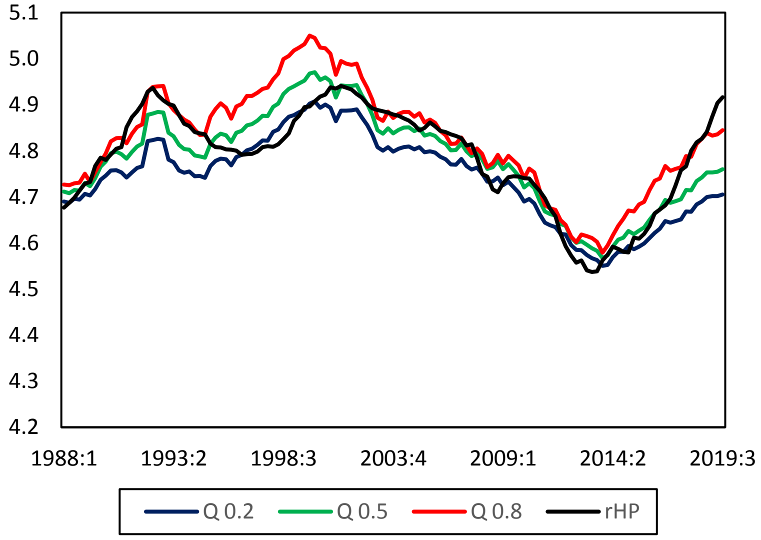
The results for estimating model 1-4 are shown in *figures* D.1-D.4.

Figure D.1: Estimation Results for Model 1



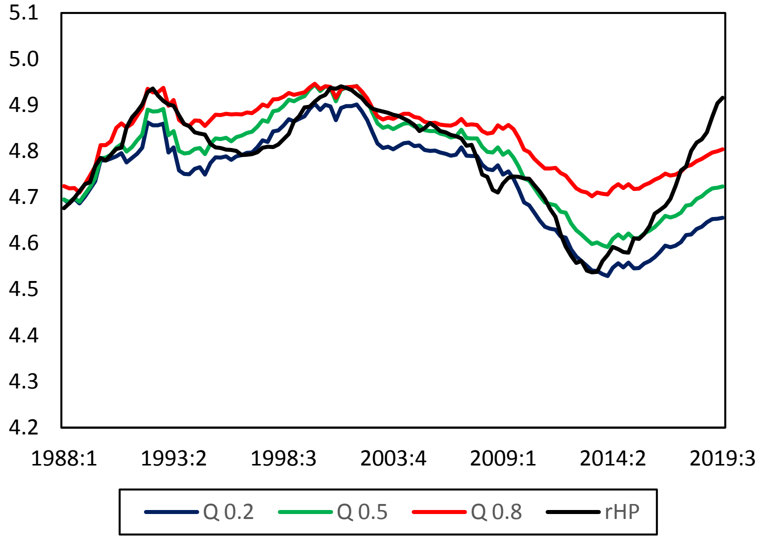
Source: OECD, ECB, Banco de Portugal and author's calculations.

Figure D.2: Estimation Results for Model 2



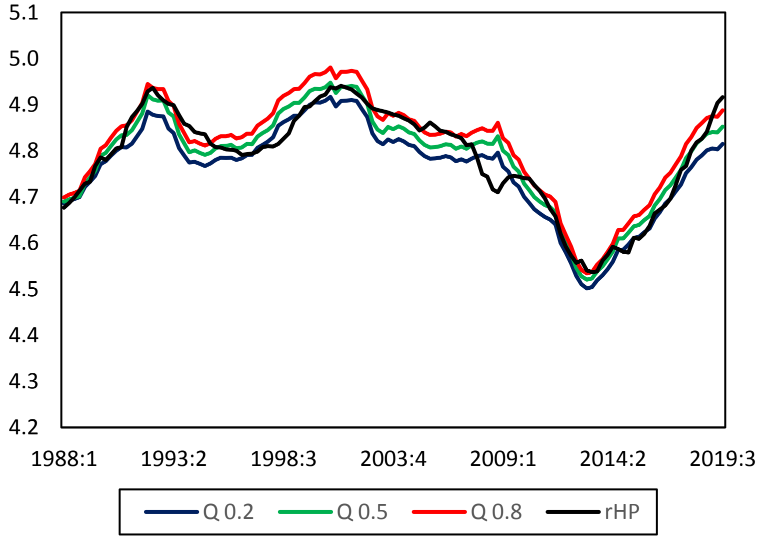
Source: OECD, ECB, Banco de Portugal and author's calculations.

Figure D.3: Estimation Results for Model 3



Source: OECD, ECB, Banco de Portugal and author's calculations.

Figure D.4: Estimation Results for Model 4



Source: OECD, ECB, Banco de Portugal and author's calculations.