Characterization of TPACK in a GeoGebra professional development programme

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In this work, we investigate the different domains of knowledge mobilized by in-service teachers, with reference to TPACK, in the use of GeoGebra in a professional development programme with middle and secondary school teachers, in Portugal. We established performance indicators within the scope of technological, pedagogical and content knowledge, which we evaluate through content analysis of the activities produced by teachers during the programme. We present examples of the tasks proposed in the programme for the development of TPACK and identify aspects that demonstrate the mobilization of TPACK by teachers, in solving the proposed tasks and in the construction of an original task, aimed at pedagogical practice. We conclude that teachers can mobilize this type of knowledge, fundamental for the integration of technology, namely GeoGebra, in an educational context, within the scope of Mathematics teaching.

Keywords: Professional development, Educational Technology, TPACK, GeoGebra.

Introduction

Teaching with technology is not an easy task. Teachers' competencies such as knowledge are a decisive factor, but there are few studies investigating Professional Development Programs (PDP) for teaching mathematics with technology (Thurm & Barzel, 2022). In this paper, we aim to investigate the different domains of the TPACK model (Koehler et al., 2013) regarding knowledge developed and mobilized by teachers using GeoGebra to implement tasks in the context of a PDP. Research will be based on the analysis of the tasks proposed to trainee teachers to learn mathematics and the work produced by them in the PDP in the framework of TPACK.

With this work, we want to answer the following question: What domains of knowledge within the TPACK framework are mobilized by in-service teachers when designing tasks for learning mathematics with GeoGebra during a PDP?

In Portugal, GeoGebra is one of the main technological resources introduced in PDP for basic, middle and secondary teachers, as it integrates the Portuguese curriculum at these levels of education (DGE, 2021, 2023). It is therefore crucial that teachers acquire and mobilize adequate knowledge so that they have confidence and competence in applying the proposed approaches (Murphy et al., 2015) to transform the prescribed curriculum into a curriculum in action (Gimeno, 2000).

This work is part of a larger project in which we study the integration of digital technology by teachers into pedagogical practices, where we have already seen that teachers consider that they had knowledge (and self-confidence) to use technology in teaching (Pimenta et al., 2022). However, they not always place technology on students' desks as a learning instrument, so there is a need for more research to understand how these technologies may become widespread instruments for student learning (Pimenta et al., 2023). Thus, characterizing the different domains of the TPACK in teachers' knowledge may provide more understanding in this matter.

Theoretical background

In the fundamental literature that supports this study, we considered two areas: the use of GeoGebra in an educational context and the TPACK model to frame teachers' knowledge of how to teach using technology.

GeoGebra in an educational context

The use of GeoGebra in an educational context is widely recommended in the literature. GeoGebra was designed, as Hohenwarter (2013) describes, to combine dynamic geometry with the functionalities of computer algebra systems, for learning and teaching mathematics. It is a rich resource that can contribute to improving pedagogical practices and more meaningful learning (Zengin, 2019; Weinhandl et al., 2020; Lyublinskaya & Cabral , 2023). For example, evaluation by García et al. (2020) demonstrated that students positively value the proposed tasks and the use of GeoGebra to develop them, which facilitated understanding and deepening geometric concepts. Zulnaidi and Zamri (2017) showed that students using GeoGebra, compared to a control group subject to methods they designate as conventional, have greater conceptual and procedural mathematical knowledge. Hernández et al. (2020) in an investigation within the scope of initial teacher training, noted that the use of technology, and particularly GeoGebra, can be crucial to clarify mathematical aspects and to support the understanding of the mathematics to be taught.

In the new Mathematics curricula for basic and secondary education in Portugal, there is explicit reference to the use of resources such as GeoGebra. In secondary education (DGE, 2021) the “Systematic use of technology” is presented as one of the key ideas of essential learning, to “Encourage the exploration of ideas and concepts, integrating technology (…)”. More specifically, “strategic teaching actions” are listed and involve dynamic geometry. In the 3rd cycle of basic education (7th to 9th years) (DGE, 2021), in the section “How to promote the learning of Mathematics”, regarding resources/technology, it is mentioned that «The tools Technological technologies must be considered as unavoidable and powerful resources for teaching and learning Mathematics. (…) All students must be able to freely access (…) software for statistical processing, geometry, functions (…).”. Dynamic geometry environments are explicitly mentioned (for example, “Perform constructions in dynamic geometry environments a that mobilize geometric places, regular polygons, relationships between angles and isometries (…).”.

TPACK

Regarding the development of teachers' knowledge for efficient teaching of Mathematics with technology, we have as a reference the Technological, Pedagogical and Content Knowledge (TPACK) model, as introduced by Mishra and Koehler (2006). The acronym TPACK stands also for Total Package and, according to Mishra and Koehler (2008), it better reflects the interdependence of the intervening knowledge domains, as represented in Figure 1.

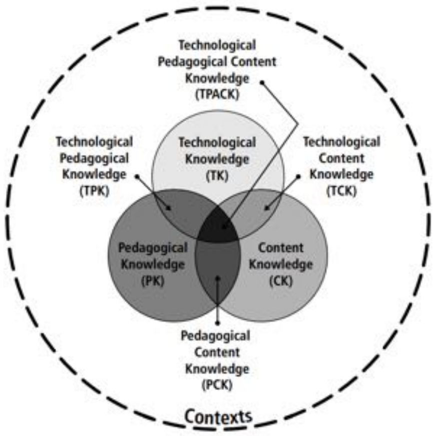

Figure 1 – TPACK model (Koehler et al. , 2013)

Mishra and Koehler (2006) introduce TPACK as a theoretical framework that contextualizes the intersection between knowledge in the domains of pedagogy, content and technology. Koehler et al. (2013) describe the various components of TPACK: TK (technological knowledge), always changing; TCK (technological and content knowledge), understanding the ways in which technology and content influence each other; TPK (pedagogical and technological knowledge), understanding how teaching and learning can be enhanced with the use of certain technologies; finally, TPACK, an emerging form of knowledge that goes beyond content, pedagogy and technology: it is an understanding that emerges from the interactions between these three dimensions. Integrating these knowledges into the educational context, teachers mobilize TPACK whenever they teach. As characterized by Koehler et al. (2013), TPACK is the support of a "truly meaningful and deeply qualified teaching with technology (p.16)", which goes beyond the three types of knowledge individually.

Regarding the evaluation of teachers' TPACK, we consider Niess et al. (2009) who, specifically regarding TPACK for teaching mathematics, describe a model that consists of several phases of cognitive development: recognition, acceptance, adaptation, exploration and advanced. This work proposes how to identify what stage a mathematics teacher is in by observing their activities. Bowers and Stephens (2011) present an instrument for distinguishing five domains of knowledge (CK, TK, TCK, TPK and TPACK). Based on work projects proposed by the participants (generally teachers or students in initial teacher training), in which they present proposals for learning activities, using an dynamic geometry environments, and based on the answers to a self-assessment questionnaire, the researchers characterized the knowledge of practitioners in the five domains mentioned. Harris et al. (2015) state that, in general, TPACK assessment can have three scopes: self-assessment, observation of behaviour and analysis of teachers' productions (such as lesson plans, for example). In 2018, Akyuz, in a work carried out within the scope of initial training for Mathematics teachers, uses an assessment instrument for the seven TPACK knowledge domains, based on three performance descriptors for each model, applied in the analysis of planning, activities and the applications produced by future teachers.

Regarding the type of activities to be proposed to teachers for developing TPACK, we had as a reference Harris et al. (2014), that offer TPACK-based "activity types" to the professional development approaches.

Methodology

In this research, using a qualitative methodology (content analysis), with an interpretative paradigm (Cohen et al., 2018) we analysed the development and mobilization of teachers' TPACK within the framework of a training course implemented with 25 middle and secondary school math teachers, all with excellent grades at the end of the course. This 25-hour course (12 synchronous + 13 asynchronous, in e-learning), entitled “GeoGebra applications”, was implemented in 2020/2021, at the Portuguese Mathematics Society (https://www.spm.pt/). Teachers' participation in this PDP and in the study was voluntary and their anonymity was guaranteed.

To characterize the teachers' TPACK mobilized and developed when carrying out tasks, we defined three performance descriptors for analysing the work produced by teachers during the course:

- TK – to use the right tools correctly, including non-trivial and dynamic procedures;
- CK – to fulfil the task objectives from a mathematical point of view (without scientific errors) involving meaningful mathematical to allow students explore the topic;
- PK – to include tasks for students with the aim of active learning, including asking for solutions, generalizations or conjectures.

To define these indicators, we start from previous work in this field. For example, Akyuz (2018) proposes three descriptors for each of the domains CK, PK, TK, PCK, TCK, TPK and TPACK, but we aimed to consider a smaller number of indicators, so that they are clearly understandable by trainees who, as teachers, will have to mobilize and integrate this knowledge into their teaching practice. In the indicators we present, we wanted to summarize and group some of the ideas proposed in Bowers and Stephens (2011) “Components of TPACK (…)” (p.293). Then, we consider the intersections of the three indicators presented. For example, if TK and CK are mobilized simultaneously, we consider that TCK is mobilized; if the three indicators are mobilized, we consider that TPACK has been mobilized. We begin by presenting and characterizing the tasks proposed to the trainees, intentionally designed to promote the development of teachers' knowledge to teach mathematics. Then we analyse teachers' productions resulting from these proposals and the tasks they autonomously created to implement with students, for learning mathematics using GeoGebra, to illustrate the knowledge mobilized in the construction of the tasks.

Data presentation and analysis

In the PDP, to improve the teachers’ TPACK in the use of GeoGebra, we propose a set of tasks, in the areas of Geometry and Functions, in which teachers should mobilize and develop TPACK, in its three domains: TK, CK and PK. At the end of the course, each teacher had to create an original task, which involved dynamic constructions in GeoGebra, to propose to students, so that they use GeoGebra in a Mathematics learning activity.

Tasks proposed in the PDP

We present below some examples of the initially proposed tasks. In Geometry: «Create a dynamic construction that illustrates the Pythagorean Theorem. Include dynamic text that illustrates the corollary of the theorem. Publish at geogebra.org»; «Construct the 9-point circle for a triangle. Study the relationship between the radius of the 9-point circle and the radius of the circumscribed circle.» In Functions: «Create a model, using sliders, that illustrates the influence of the parameters a, h and k, on the parabolas relating to the quadratic functions, with analytical expression in the form $f(x) =$

$a(x - h) + k$»; «Verify geometrically in GeoGebra that $(\cos x)' = -\sin x$, using command input, the tangent line and slope tools, the locus tool and the activate trace mode of an object.»

These tasks were designed by the trainers, based on their experience and qualifications, but also using literature, such as the work of Harris et al. (2014), that characterize learning activities to develop and apply TPACK, namely, "Producing an artifact", "designing an exhibition" and "creating a game".

Teachers' productions

The following are constructions made by the trainees, which put into practice the statements proposed by the trainer. In the activities shown in Figure 2, we can identify the mobilization of TCK, given that, on the one hand, TK is mobilized, building, for example, boxes to show and hide objects, and dynamic text, in addition to dynamic construction, and, on the other, it is mobilized the CK, since there is total mathematical correctness in the constructions. In the nine-points circle construction the trainee included a question to be answered based on the exploration of the application – "In any triangle, the radius of the 9 points circle is…. of the triangle circumradius" – that shows an implicit pedagogical concern to the exploration of the activity by students, mobilizing PK. So, in an integrated way, we consider this as examples of TPACK mobilization.

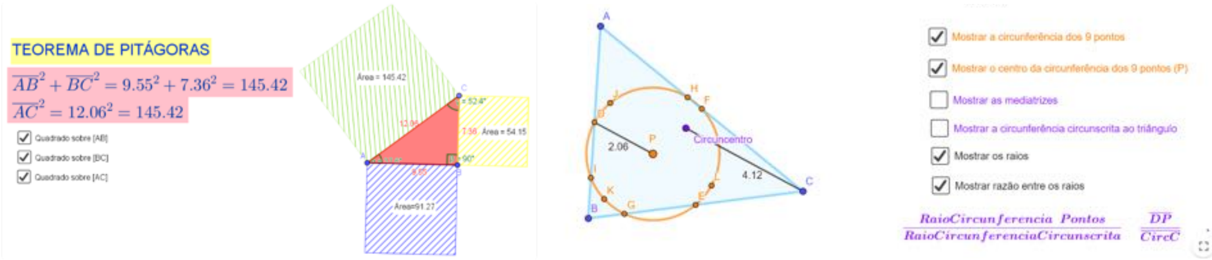

Figure 2: Pythagorean Theorem / Nine points circle – teachers' productions, mobilizing TPACK

In Figure 3, a construction is shown in which teachers mobilize TCK completing the trigonometry task proposed by the trainer. TK – applying non-trivial procedures, knowing how to use command input, locus tool and the trace mode of an object; CK – knowing and applying the definition of derivative at a point.

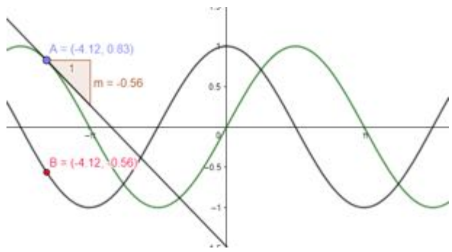

Figure 3: $(\cos x)' = -\sin x$ construction, mobilizing TCK

Regarding original works by trainees, as shown in Figure 4, with greater weight in our analysis, since they did not originate from any script, other than the general recommendation for them to construct a task for classroom application, mobilizing the knowledge developed in PDP, we verified in most of the works presented that teachers were able to build activities fulfilling this recommendation, mobilizing TPACK: TK, as they use the right tools correctly (including non-trivial procedures, such as sliders, boxes to show and hide objects and dynamic text); CK, achieving the objectives of the task from a mathematical point of view (without scientific errors) involving meaningful mathematics to allow students to explore the topic (as the ratio between perimeters and areas of similar figures); PK, including tasks for students, with the aim of active learning, including asking for solutions, generalizations or conjectures (as in the circumcentre example, where the teacher creates a to assign to the students, which includes the proposal to determine, in the context of problem solving, using the available tools, the equidistant point of three given points, to introduce the mathematical topic with an exploratory activity.)

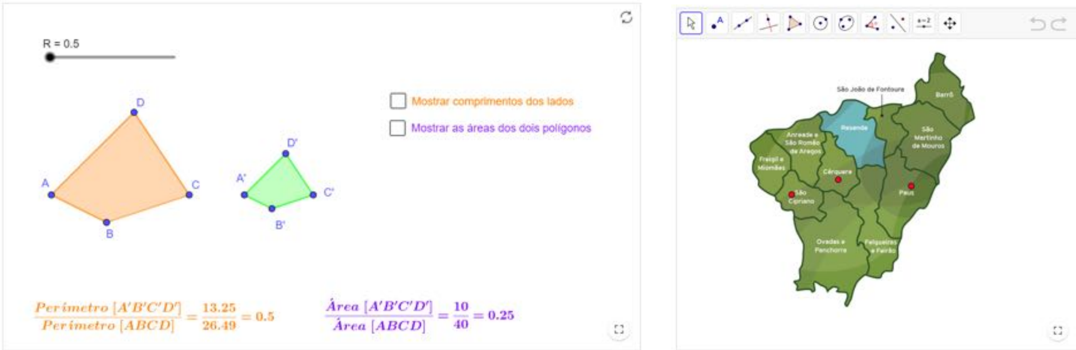

Figure 4: Figures similarity and circumcentre construction, mobilizing TPACK

More examples of this are the works that can be accessed through the following hyperlinks (authorized by their authors): https://www.geogebra.org/m/bpan4wf6; https://www.geogebra.org/m/dexapmhf; https://www.geogebra.org/m/y48pqxmv .

Discussion and final considerations

We verified that teachers mobilised different domains of TPACK (Koehler et al., 2013), namely Content Knowledge (CK), Pedagogical Knowledge (PK) and Technological Knowledge (TK), including the integration of these domains, PCK, TCK and TPK, resulting in the TPACK. Based on Bowers and Stephens (2011) work, we concretized examples of tasks to be proposed to trainees, in a PDP environment, for the development of the TPACK domains. Based on the descriptors proposed by Akyuz (2018), we proposed three simplified indicators (CK, TK and PK) so that they can be clearly understood and applied not only by researchers, but also by trainers or trainees within the scope of PDP and, in general, by teachers who must now be well aware of this type of pedagogical practice, using technology such as GeoGebra, as prescribed in the curriculum (DGE, 2021, 2023). In this way, we were able to exemplify, based on the productions of the trainee teachers (figures 2 to 4), whether based on initially proposed tasks or based on their final individual original works, in which context the three main domains of TPACK are mobilized in the development of these activities. For example, the use of boxes to show and hide objects or dynamic text is an indicator of TK mobilization; a construction that illustrates meaningful mathematics correctly, from a scientific point of view, that allows the student to learn within the scope of this mathematical topic, is a CK mobilization indicator; including questions that allow students to draw conclusions from exploring the application is an PK mobilization indicator. Finally, we conclude that teachers are able to mobilize knowledge within the scope of TPACK to build activities for learning mathematics using GeoGebra. This can help to meet the learning objectives set out in the curriculum (DGE, 2021, 2023). These results seem to be in line with teachers who considered that they had knowledge and self-confidence to use technology in teaching (Pimenta et al., 2022). However, it does not clarify why teachers do not promote these technologies for student learning as identified in Pimenta et al. (2023). Thus, there is a need to develop more research on this matter.

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