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# **INTEREST RATES AND FISCAL CONSOLIDATION PROGRAMS**

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## Abstract

We claim that changes in the interest rate impose impacts on multipliers of fiscal consolidations programs. We use the database and extend the regression in Alesina et al. (2015a) with interest rates to check the relationship with fiscal multipliers. Then, we adapt the overlapping generations model with incomplete markets used in Brinca et al. (2021) and fix the wage and interest rate. Using that adaptation of the model, we assess the impact of a change in the interest rates of three calibrated European economies. We then test the mechanism and compare the impacts based on countries' different Gini coefficients and percentage of agents constrained. For both analysis we conclude that there is a *negative* relationship between the interest rate and fiscal consolidation multipliers under a fixed wage and interest rate environment. We also conclude that countries with higher percentage of agents constrained are more resistant to changes on interest rates.

*Keywords:* Macroeconomics, Fiscal Consolidation, Interest Rates, Fiscal Multipliers, Wealth Inequality

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# 1 INTRODUCTION

Fiscal consolidation became a huge topic when the world got involved in the huge global financial crisis of 2008. Many countries, namely European countries, increased their spending and public debt during that period, but then the sovereign debt crisis hit Europe in 2010. The European response was austerity or increased taxation to cope with the large debt contracted in the previous years. With that came the large fiscal consolidation plans. What once was a necessity during the economic crisis of 2008 might be once again needed to cope with the impacts of the Covid-19 pandemic. Fears of inflation might imply a rise on interest rates by the central banks to stop that trend. To face the lockdown measures' consequences, many countries had to resort in debt issuance so as to allow for extra spending with little-to-no production during the peak of the pandemic. Putting these two together, the pressure for interest rates increase to control inflation and the possible need of another round of fiscal consolidation, what would be the impact of higher interest rates on these consolidation plans?

Some literature seeks to evaluate the impact of fiscal consolidation on the monetary policy practice and not the other way around, i.e., the impact of the interest rate level on the impact multipliers of fiscal consolidation. According to [Molnár \(2012\)](#) in an OECD report, “monetary conditions (...) have a bearing on fiscal consolidation, though the literature exploring this is scant and inconclusive” before adding that “large interest [rates] differential [to Germany’s] may, inter alia, be a signal of the need to consolidate”. Further into the report they neglect this possible impact saying that interest rates and inflation do not seem to matter much on the determination of the size of a consolidation. However, they leave the possibility for inflation to affect the intensity of the consolidation episode. The relationship of inflation and interest rates might signal the necessity of further research on interest rate influence. [Molnár \(2012\)](#) gives the possibility that “rising long-term interest rates are associated with more intense, very large consolidations”. Also, they state that “declining interest rates help stabilising debt through reducing debt servicing costs and cushioning the contractionary impact of consolidation”, some evidence that we can test by checking the multiplier effect of interest rate changes.

Another studies support the possibility for interest rate impacts on fiscal consolidation. According to [J. Erceg and Lindé \(2012\)](#), there is indeed a connection between interest rate cuts

and the lower adverse impact on aggregate demand due to that same interest decrease. This is supported in [Ahrend, Catte, and Price \(2006\)](#). The paper concludes that lower interest rates have a contribution on offsetting contractionary implications of fiscal tightening on aggregate demand and that lower interest rates would ease fiscal consolidation since they create conditions for these potential negative impacts to be weaker. In [Brinca et al. \(2021\)](#) there was a short experiment of an open economy with fixed wages and interest rates. Based on the possibility that interest rates create, indeed, conditions to offset or to worsen the negative impacts of fiscal consolidation plans, and on the said experiment, we decided to test some conclusions regarding interest rates impacts on such plans under the same wage and interest rate environment.

We are not entirely interested in the magnitude of the fiscal multipliers, but more specifically on the change that an increase/decrease of the interest rate from the calibrated model has on the fiscal multipliers. What follows will be a more qualitative approach to the matter, instead of giving the main focus of the analysis to quantitative measures.

[Alesina et al. \(2015a\)](#) suggests that fiscal consolidation plans are not one-time shocks, but rather multi-year plans that extend from before the actual consolidation takes place and for some periods after that consolidation was put into practice. That posts itself as the backbone of the first step of our analysis and motivation. We use the database of [Alesina et al. \(2015a\)](#) to assess if interest rates have a significant role when interacted with that consolidation variables. Using the regression proposed in [Alesina et al. \(2015a\)](#) augmented with interest rate, we conclude that interest rates affect fiscal consolidation multipliers.

Afterwards, we proceed to the assessment of our main question using a model based on the overlapping generations model with incomplete markets in [Brinca et al. \(2021\)](#). We then adjust it to an environment with wages and interest rates constant. We calibrate three countries to analyse throughout this paper: Germany as our benchmark economy, Portugal, and the Netherlands. We use the calibration in the original paper, and compute the implicit wage and interest rate. After that, we take advantage of having now the interest rates and wage as exogenous parameters and maintain the wage rate constant, while changing the interest rate to evaluate the impact of that change on our fiscal multipliers.

Our conclusions go along with the experiment of an open economy made in [Brinca et al.](#)

(2021), as we see similar fiscal multipliers to the ones reported on that experiment. However, the addition to that paper is the changing interest rate. What we concluded was that higher interest rate levels affect negatively the fiscal multipliers' values for both types of consolidation, be it via government spending or labor income tax. When comparing to the economy at the starting point, we can clearly see the multipliers to become more negative to higher levels of the interest rate and to approach zero as we decrease the interest rate. This is more evident for consolidations made through increasing in labor income taxes, since consolidations via government spending show itself to be very close to zero, only being relevant at the fifth decimal case. The more negative multipliers are caused by the larger effort necessary to decrease debt by 10%, as it is the experiment, when facing higher levels of interest. Interest on debt repayments increases, so the rise on labor income tax needs to be larger to cover for that additional interest payment. A higher taxation will imply that the disposable income of agents is reduced. The labor supply retracts, since we expect that future income will be higher, as lump-sum transfers – that are set to clear the government budget balance – increase. However, this future income effect is small, since it is almost only driven by the lump-sum transfers. The other effect, the substitution effect, on the other hand, is the one that dominates. The fact that labor income tax decreases disposable income and makes agents to supply less labor does not imply that wages respond to that drop. In a constant wage environment, that response is non-existent, enhancing the substitution effect for agents.

The analysis connected to inequality was based on the Gini coefficient and the percentage of agents constrained. [Brinca et al. \(2021\)](#) states that the intertemporal elasticity of substitution of labor is increasing in wealth, which means that low-earners will react less to future income changes and to current price changes. We use that to predict the impact of changes in interest rates on multipliers. We could not conclude anything regarding the relationship between the Gini coefficient and this impact of interest rates, since we see no pattern on this matter, but we can when we compare them with the percentage of agents constrained. We see that the impact of interest rate changes will be stronger on fiscal multipliers of countries with more percentage of agents constrained than the economies with lower percentages. Portugal is the one with the highest value and is the one with the multipliers closer to zero. Germany is the opposite. The

explanation behind this is the intertemporal elasticity of substitution of labor: countries with more low-earners will react less to changes in future income. Portugal has more low-earners, so the substitution effect of labor is lower, giving a lower drop of labor supply, GDP per capita and, thus, on the fiscal multipliers.

The paper is structured in 4 sections, being the Introduction and Literature Review the first. On section 2 we go through the analysis of the regression from the database in [Alesina et al. \(2015a\)](#) extended to include interest rates; Section 3 is divided in three subsections – subsection 3.1 describes the overlapping generations model similar to the one in [Brinca et al. \(2021\)](#); subsection 3.2 describes the calibration process and subsection 3.3 goes along the results. Section 4 concludes the results and debates some possible extensions of this analysis.

## **2 REGRESSION IN ALESINA ET AL. (2015a)**

In this section it is documented the relationship significance that poses itself as the cornerstone of this thesis' question. In this analysis, we extend the exercises in [Alesina et al. \(2015a\)](#), that studies recent impacts of fiscal consolidation shocks, and adapt to test for impacts of a change in the interest rate in the multipliers of fiscal consolidation shocks.

As said, we use the database available in [Alesina et al. \(2015a\)](#), which consists of an annual data set on fiscal consolidation events in 12 European economies between the years of 1978 and 2014.

[Alesina et al. \(2015a\)](#) takes advantage of the methodology in [Alesina et al. \(2015b\)](#), which states that fiscal adjustments are not single-year corrections on expenditure and/or taxation, but multi-year plans instead, that have prolonged interactions with an economy's variables throughout a period of time. Moreover, [Alesina et al. \(2015b\)](#) adds that fiscal consolidations are implemented either unexpectedly or are known in advance. Henceforth, not adjusting and excluding the relationship between the unanticipated and anticipated nature of those shocks can lead to biased results and incorrect analysis.

By using the dataset in [Alesina et al. \(2015a\)](#) that collects that same fiscal consolidation shocks for the said period, we estimated the following equation in order to identify how these

multipliers of fiscal plans can change given a change in the interest rate:

$$\Delta Y_{i,t} = \alpha + \beta_1 \cdot e_{i,t}^u + \beta_2 \cdot e_{i,t}^a + \gamma \cdot ir_{i,t} + \iota_1(ir_{i,t} \cdot e_{i,t}^u) + \iota_2(ir_{i,t} \cdot e_{i,t}^a), \quad (1)$$

where  $\Delta Y_{i,t}$  is the logarithm of the first difference of GDP in economy  $i$  in year  $t$ ,  $e_{i,t}^u$  is an unanticipated fiscal consolidation shock,  $e_{i,t}^a$  is an anticipated fiscal consolidation shock and  $ir_{i,t}$  is the short-term interest rate. The interaction terms  $\iota_1(ir_{i,t} \cdot e_{i,t}^u)$  and  $\iota_2(ir_{i,t} \cdot e_{i,t}^a)$  will give us an insight on the signal in which way a change in the interest rate will impact the fiscal consolidation plans multipliers, given the presence of the levels of  $e_{i,t}^u$ ,  $e_{i,t}^a$  and  $ir_{i,t}$ . The interpretation of these parameters is dependent on the fact that the dependent variable is in logarithmic terms. Finally, the regression was done with country and year fixed effects and there were used robust standard errors.

Table 1: Regression with Interest Rates on Data From Alesina et al. (2015a)

Coefficients	(1) Regression
$\beta_1$	-0.014* (0.007)
$\beta_2$	-0.007** (0.003)
$\gamma$	0.423* (0.205)
$\iota_1$	0.339 (0.437)
$\iota_2$	0.319* (0.152)
Constant	0.022** (0.007)
Observations	120
Number of countries	12
$R^2$	0.186

Robust standard errors in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

With the results of the regression on Table 1 we can deduce some conclusions. The negative sign on the coefficients of the fiscal consolidation variables tell us that the higher the consolidation, the lower will be the multiplier towards GDP growth rate. This conclusion goes along

with [Brinca et al. \(2021\)](#), the paper which the model of this thesis is based on and that we will go through deeper, further in the next section.

On what regards the variable of interest for this work, we have the interactions between the interest rate and the fiscal consolidations. Firstly, going through the interest rate at level terms, we have a positive coefficient. The intuition is that a higher level of interest rate will induce a higher level of growth rate. Regarding the interactions, both have positive coefficients, but must be analysed with attention. Given the negative nature of the coefficients of the fiscal consolidation variables, the fact that the interaction between the interest rate and these variables gives a positive coefficient will mean that an increased interest rate will affect negatively fiscal multipliers' values.

The results of this empirical analysis suggest that the interest rate might be a relevant variable to pay attention to when interpreting the outcomes of fiscal consolidations. Furthermore, they suggest that the relationship between interest rate and the fiscal multipliers is a negative one. This goes along to the findings in [Ahrend, Catte, and Price \(2006\)](#) and in [J. Erceg and Lindé \(2012\)](#). Both papers state that lower interest rates, or interest rate cuts, seem to offset the negative impact on aggregate demand when conducting fiscal consolidation. If the results on this regression imply that higher interest rates will affect negatively the fiscal multipliers' values, the opposite stands as true as well.

On the next section we will go through the model in [Brinca et al. \(2021\)](#), describing it, how it was calibrated and going through the results.

### **3 MODEL IN BRINCA ET AL. (2021)**

In this section we describe the model used to evaluate the impact of the response to fiscal consolidation shocks in different countries. The model is identical to that of [Brinca et al. \(2021\)](#), a standard overlapping generation model with heterogenous agents and incomplete markets. In subsection [3.1](#) we describe the model, in subsection [3.2](#) we go through the calibration and in subsection [3.3](#) we analyse the results.

## 3.1 Model

### 3.1.1 Technology

A representative firm produces one sole good, output, and follows a Cobb-Douglas production function:

$$Y_t(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}, \quad (2)$$

with  $K_t$  as the capital input in period  $t$  and  $L_t$  the number of efficient units of labor force used in production in the same period  $t$ . Capital follows the standard law of motion:

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad (3)$$

in which  $\delta$  is the annual depreciation rate of capital stock and  $I_t$  is the gross investment in period  $t$ . Each period the firm chooses  $L_t$  and  $K_t$  with the goal of maximization of profits:

$$\max_{L_t, K_t} \Pi_t = Y_t - [w_t L_t + (r_t + \delta)K_t]. \quad (4)$$

In a competitive equilibrium, the factor prices,  $w_t$  and  $r_t$  will be equal to the marginal product of labor and capital, respectively:

$$w_t = \frac{\partial Y_t}{\partial L_t} = (1 - \alpha) \left( \frac{K_t}{L_t} \right)^\alpha, \quad (5)$$

$$r_t = \frac{\partial Y_t}{\partial K_t} = \alpha \left( \frac{K_t}{L_t} \right)^{1-\alpha} - \delta. \quad (6)$$

In this case, as we are solving for the impact of a change in interest rate, we will assume and use the case of an open economy with mobility of capital where the wage and interest rates are set exogenously in the international market. For this analysis, the interest rate and wages will be fixed. Both variables are initially set to the endogenous values that result by solving the original model. Afterwards, we maintain the wage constant and proceed to change the interest rate as a parameter of the model to conduct our analysis.

### 3.1.2 Demographics

There are  $J$  overlapping generations of households that live for a finite amount of time in this economy. Households enter the labor market at the age of 20 and retire at the age of 65. We define  $j$  as the household's age. The age-dependent probability of dying that retired households face is defined by  $\pi_j$  and at the age of 100 that probability is equal to 1, which is the same as saying that they face certain death. Periods are defined as one year, so each agent has 45 periods of active work life. Also, there is no population growth *i.e.* population size is fixed. We normalize the size of each age-cohort to 1. Using  $w(j) = 1 - \pi(j)$  to refer the age-dependent probability of surviving, applying the law of large numbers, the mass of retired agents of age  $j \geq 65$  still alive at period  $t$  is equal to  $\Omega_j = \prod_{q=65}^{q=J-1} \omega(q)$ .

There is heterogeneity of households in respect to asset holdings, idiosyncratic productivity,  $u$  permanent ability, and the subjective discount factor which can be, with the same probability, one of the following three values  $\beta \in \{\beta_1, \beta_2, \beta_3\}$  and is constant throughout each household's lifetime. Regarding the permanent ability,  $a$  it is assumed to be the same during the households' active work life, realized at birth, meaning that at the age of 20, when entering the labor market, they already have a given level of productivity. Every period they decide how much to save,  $k'$ , to consume,  $c$ , and how many hours they will work,  $n$ . When households retire, they stop producing and, so, they do not supply any labor, stopping that part of the decision-making. However, they receive a social security payment,  $\Psi_t$ . There are no annuity markets, so that a fraction of households leave unintended bequests that will then be redistributed to the currently-living households through lump-sum transfers. The per-household bequest is represented by  $\Gamma$ . Finally, the utility of the retired households is increasing in the bequest they leave for the ones who stay alive after they die.

### 3.1.3 Labor Income

Permanent ability,  $a$  – with  $a \sim N(0, \sigma_a)$  –, age,  $j$ , and a household-specific productivity shock,  $u_t$ , will determine each households' endowment of efficiency units for any given period.

The  $u_t$  follows an AR(1) process:

$$u_{t+1} = \rho u_t + \varepsilon_{t+1}, \quad \varepsilon \sim N(0, \sigma_\varepsilon^2). \quad (7)$$

An individual's wage will be given by:

$$w(j, a, u) = w e^{\gamma_1 j + \gamma_2 j^2 + \gamma_3 j^3 + a + u}, \quad (8)$$

where  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3$  are the age profile of labor efficiency units, and  $w$  is the wage per each of those units, that is determined in the competitive labor market. Once more, the model used on our analysis departs from this original model and then sets wage as constant.

### 3.1.4 Preferences

The decision-making by the household is based upon a monetary utility function,  $U(c, n)$ , that depends on hours worked and consumption,  $n \in [0, 1]$ , adding to a pure public good,  $G$  that is provided by the government, and is defined by:

$$U(c, n, G) = \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{n^{1+\eta}}{1+\eta} + \log(G), \quad (9)$$

where  $\sigma$  is the risk-aversion parameter,  $\chi$  is the parameter regarding the disutility of work, and  $\eta$  the inverse Frisch elasticity. Households that retire, as said, benefit, in utility terms, from the bequest they leave when dying to the ones that stay alive:

$$D(k) = \varphi \log(k). \quad (10)$$

### 3.1.5 Government

There are two parts for the Government action: the social security and fiscal policy actions. The social security system is balanced, as government taxes employees and the employer at rates  $\tau_{ss}$  and  $\tilde{\tau}_{ss}$ , respectively, and pays benefits,  $\Psi_t$ , to retirees. The government must get funds for the expenditure on pure public good,  $G_t$ , interest payments on national debt,  $rB_t$ , and a lump-

sum redistribution,  $g_t$ . These funds come from the taxation on consumption, capital income, and labor income. The first two have flat tax rates,  $\tau_c$  and  $\tau_k$ , respectively. The latter, taxation on labor income, is a non-linear function of income and using the functional form suggested in [Benabou \(2002\)](#):

$$\tau_l(y) = 1 - \theta_0 y^{-\theta_1}, \quad (11)$$

where  $y$  is the pre-tax labor income and  $\tau_l(y)$  the average tax rate given a level of pre-tax labor income,  $y$ .  $\theta_0$  and  $\theta_1$  define the level and the progressivity of taxation, respectively.

We assume that the debt-to-output ratio,  $B_y = \frac{B_t}{Y_t}$ , does not vary over time, being also the case with the constant revenue-to-output and expenditure-to-output ratios, which means that there is not creations of new debt. If we have the government's revenue from the taxes collected on labor, capital, and consumption denoted by  $R_t$  and the government's revenue from social security taxes denoted by  $R_t^{ss}$ , we have that the government's budget constraint is the following:

$$g \left( 45 + \sum_{j \geq 65} \Omega_j \right) = R - G - rB, \quad (12)$$

$$\Psi \left( \sum_{j \geq 65} \Omega_j \right) = R^{ss}. \quad (13)$$

### 3.1.6 Recursive Formulation of the Household Problem

At any given time a household is characterized by the vector  $(k, \beta, a, u, j)$ . The household must solve its optimization problem choosing the amount of consumption,  $c$ , facing a borrowing limit of  $b$  future asset holdings,  $k'$ , and work hours,  $n$ , that will be optimal for them. Hence, they must solve their optimization problem that is formulated as:

$$V(k, \beta, a, u, j) = \max_{c, k', n} [U(c, n, G) + \beta E_u [V(k', \beta, a, u, j + 1)]]$$

*s.t.* :

$$c(1 + \tau_c) + k' = (k + \Gamma)(1 + r(1 - \tau_k)) + g + Y^L \quad (14)$$

$$Y^L = \frac{n\omega(j, a, u)}{1 + \tilde{\tau}_{ss}} \left( 1 - \tau_{ss} - \tau_l \left( \frac{n\omega(j, a, u)}{1 + \tilde{\tau}_{ss}} \right) \right)$$

$$n \in ]0, 1], \quad k' \geq -b, \quad c > 0.$$

In this case,  $Y^L$  is the labor income net of social security and labor income taxation. The social security taxes paid by the employee and the employer are, respectively,  $\tau_{ss}$  and  $\tilde{\tau}_{ss}$ . For retired households, their optimization problem is defined differently, having the probability of dying  $\pi(j)$  and the utility from leaving a bequest  $D(k')$ . So we formulate it as follows:

$$\begin{aligned}
V(k, \beta, j) &= \max_{c, k'} [U(c, n, G) + \beta(1 - \pi(j))V(k', \beta, j + 1) + \pi(j)D(k')] \\
&\text{s.t. :} \\
c(1 + \tau_c) + k' &= (k + \Gamma)(1 + r(1 - \tau_k)) + g + \Psi. \\
k' &\geq 0, \quad c > 0.
\end{aligned} \tag{15}$$

### 3.1.7 Stationary Recursive Competitive Equilibrium

The measure of households with the corresponding characteristics is defined as  $\Phi(k, \beta, a, u, j)$ . The stationary recursive competitive equilibrium is defined as follows:

1. Having the factor prices and the initial conditions, the consumer's solve their optimization problem by the value function  $V(k, \beta, a, u, j)$  and the policy functions  $c(k, \beta, a, u, j)$ ,  $k'(k, \beta, a, u, j)$ , and  $n(k, \beta, a, u, j)$ .

2. Markets clear:

$$\begin{aligned}
K + B &= \int kd\Phi, \\
L &= \int n(k, \beta, a, u, j)d\Phi, \\
\int cd\Phi + \delta K + G &= K^\alpha L^{1-\alpha}.
\end{aligned}$$

3. The factor prices satisfy:

$$\begin{aligned}
w &= (1 - \alpha) \left( \frac{K}{L} \right)^\alpha, \\
r &= \alpha \left( \frac{L}{K} \right)^{1-\alpha} - \delta.
\end{aligned}$$

4. The Government budget balances:

$$g \int d\Phi + G + rB = \int \left( \tau_k r(k + \Gamma) + \tau_c c + n\tau_l \left( \frac{nw(a, u, j)}{1 + \tilde{\tau}_{ss}} \right) \right) d\Phi.$$

5. The social security system balances:

$$\psi \int_{j \geq 65} d\Psi = \frac{\tilde{\tau}_{ss} + \tau_{ss}}{1 + \tilde{\tau}_{ss}} \left( \int_{j \geq 65} nwd\Psi \right).$$

6. The assets of the deceased are uniformly distributed among the living:

$$\Gamma \int \omega(j) d\Psi = \int (-\omega(j)) k d\Psi.$$

### 3.1.8 Fiscal Experiment and Transition

The fiscal experiment analyzed mirrors the one made by [Brinca et al. \(2021\)](#). There is an initial steady-state as a starting point. Then, all of a sudden, without any previous warnign nor announcement, the government decides to reduce public debt,  $B$  by 10% of GDP, throughout 50 periods. This debt-reduction is made through two different policies: by either increasing labor income tax,  $\tau_l$  by 0.1% of the steady-state GDP every period for each and every agent, or by reducing government spending,  $G$ , of 0.2% of the steady-state GDP every period. After 50 periods of consolidation, there is a return to initial level of the changing variable, be it the government spending or the labor income tax. There is the assumption that, in order to converge to the new steady-state equilibrium with the lower debt-to-GDP ratio, it takes the economy another 50 periods. The lump-sum transfer,  $g$ , is set to clear the government budget.

In appendix, Section [6.1](#), there is a definition of a transition equilibrium after the fiscal experiment. The main difference comparing to the steady-state is that the dynamic-programming problem of the households requires another state variable, which is time,  $t$ , that captures all the changes in policy and price varibables relevant in this maximization problem. We base in [Krusell and Smith \(1998\)](#) the way to get to the numerical solution of the model. This method guesses paths for all the variables that will depend on time and then solves the problem backwards after updating the guess.

### 3.1.9 Definition of the Fiscal Multiplier

In the fiscal experiment, the definition given to fiscal multiplier is being the net present value of the sum of all changes in output divided by the same period changes in the fiscal instrument (government spending or labor income tax). Therefore, the impact multiplier is the ratio of the change in output to the change in the fiscal instrument when the initial shock is realized:

$$\text{impact multiplier} = \frac{\Delta Y_0}{\Delta I_0}, \text{ with } I \in \{G, R\}. \quad (16)$$

where  $\Delta Y_0$  is the change of output from period 0 to period 1, and  $\Delta I_0$  is the change in government spending from period 0 to period 1 if  $I=G$ , or the change in government revenue between the same period if  $I=R$ . During a consolidation based on government spending,  $G$ ,  $\tau_l$  and  $g$  are kept constant; while throughout a consolidation based on labor income taxes,  $\tau_l$ ,  $G$  and  $g$  remain unchanged.

$$\text{cumulative multiplier } I(T) = \frac{\sum_{t=0}^{t=T} \left( \prod_{s=0}^{s=T} \frac{1}{1+r_s} \right) \Delta Y_t}{\sum_{t=0}^{t=T} \left( \prod_{s=0}^{s=T} \frac{1}{1+r_s} \right) \Delta I_t}, \text{ with } I \in \{G, R\}. \quad (17)$$

having  $\Delta Y_t$  as the change in output in period 0 to period  $t$  and  $\Delta I_t$  the change in government spending from period 0 to period  $t$  if  $I = G$ , or the change in government revenue between the same period if  $I = R$ .

## 3.2 Calibration

The calibration of the model in Section 3.1 is made following the same procedure in [Brinca et al. \(2021\)](#), but calibrated for liquid wealth as suggested in [Bernardino \(2019\)](#). It is calibrated to match the moments of the German, Portuguese and the Dutch economies. There are some parameters that can be calibrated outside of the model due to the existence of direct empirical counterparts. However, other parameters do not have these, which implies the necessity of being calibrated in some other way. This endogenous calibration is done using a Simulated Method of Moments (SMM) approach.

### 3.2.1 Wages

To estimate the life cycle profile of wages, we use data from the Luxembourg Income Study (LIS) and run the following regression for each of the countries in the sample data:

$$\ln(w_i) = \ln(w) + \gamma_1 j + \gamma_2 j^2 + \gamma_3 j^3 + \epsilon_i, \quad (18)$$

with  $j$  as the age of individual  $i$ , and  $w$  as the wage rate.

The variance of the ability,  $\sigma_a$ , is assumed to be equal across all countries and set equal to the average of the European countries in [Brinca et al. \(2021\)](#). Regarding the parameter for the persistence of idiosyncratic shock,  $\rho$ , it was also set as the same for each country. It was set to  $\rho = 0.335$  as it is in [Brinca et al. \(2021\)](#), that used the U.S. data from the Panel Study of Income Dynamics (PSID). The variance of the previous parameter,  $\sigma_\epsilon$ , is then calibrated endogenously such that there is a match between it and the variance of log wages in the data.

### 3.2.2 Preferences and the Borrowing Limit

A large debate in the literature exists surrounding the appropriate value for the Frisch elasticity of labor supply,  $\eta$ . We set it to 1, the same way some recent studies have done it, like the cases of [Guner et al. \(2016\)](#) or [Trabandt and Uhlig \(2011\)](#). The parameter  $\chi$ , that determines the disutility of hours work,  $b$  that is the borrowing limit, the discount factors  $\beta_1, \beta_2, \beta_3$ , and  $\varphi$ , the benefit from leaving bequests, are calibrated in such way that the model output matches the data's.

### 3.2.3 Taxes and Social Security

We apply the labor income tax function of equation (11), proposed by [Benabou \(2002\)](#). We use the U.S. labor income tax data provided by the OECD to estimate the parameters  $\theta_0$  and  $\theta_1$  for different family types. To obtain a tax function for the single individual in our model, we take a weighted average of  $\theta_0$  and  $\theta_1$ , where the weights are each family's type share of the population.

The social security rates for the employer,  $\tilde{\tau}_{ss}$ , and for the employee,  $\tau_{ss}$ , were established as

equal to the average tax rates between 2001 and 2007 for each of the countries. From [Trabandt and Uhlig \(2011\)](#), we took the consumption and capital tax rates,  $\tau_c$  and  $\tau_k$ , respectively. Table A1, in the appendix Section 6.2 summarizes these parameters for each country.

### 3.2.4 *Parameters Calibrated Endogenously*

Some of the parameters needed do not have any direct empirical counterparts, as said above, so they must be calibrated endogenously. The parameters are the following:  $\varphi, \beta_1, \beta_2, \beta_3, b, \chi$  and  $\sigma_\epsilon$ . For their calibration, we use the simulated method of moments, minimizing the following loss function:

$$L(\varphi, \beta_1, \beta_2, \beta_3, b, \chi, \sigma_\epsilon) = \|M_m - M_d\| \quad (19)$$

where  $M_m$  and  $M_d$  are the model and data moments, respectively. Having seven parameters for calibration, we must have seven data moments in order to have an exactly identified system. The data moments chosen align with [Brinca et al. \(2021\)](#) and are: the mean asset position held by the households with 75 to 80-years old relative to the mean wealth in the economy, from the Luxembourg Wealth Study (LWS); the three quartiles of the cumulative liquid wealth distribution taken from the Household Finance and Consumption Survey (HFCS); the variance of log wages, from the Luxembourg Income Study (LIS); the average of hours worked in a year, extracted from the OECD Economic Outlook; and the capital to output ratio,  $\frac{K}{Y}$ , taken from the Penn World Table 8.0. The calibration of the target moments is made with an average error margin of 0.32%. Table A2 summarizes the parameters that are constant to every country, Table A3 the calibrated parameters for each country and Table A4 the target quartiles and the model generated quartiles.

## 3.3 Results

The main proposition that we are defending is that the interest rates are also to be taken into account when assessing the effectiveness of fiscal consolidation. For that reason, we evaluated the impact of a change in the interest rate on the impact multipliers for a benchmark economy, that matches the Germany one. We latter assess also this impact on two other economies: Portugal and the Netherlands, both calibrated the same way that it was done for Germany.

These two additional economies were selected given the values of their Gini coefficient for wealth inequality. We consider Portugal with the highest Gini coefficient (0.727), Germany with an intermediate one (0.661), and the Netherlands with the lowest Gini coefficient (0.598) out of the sample in [Bernardino \(2019\)](#). We decided to incur in complementary analysis for the relationship between the interest rate and inequality.

We first present how the model [Brinca et al. \(2021\)](#) was adjusted to assess the impact of interest rates on fiscal multipliers (in [3.3.1](#)) and after that we explain the results of the dynamics between the two variables (in [3.3.2](#)). We then proceed with a robustness analysis by checking the results with the Gini coefficient of the three countries and the percentage of agents constrained, and how these characteristics change or not with the same experiment of changing the interest rate (in [3.3.3](#)). Finally, we give an economic intuition behind all of this dynamics (in [3.3.4](#)).

### **3.3.1 *Model Preset***

We depart from the benchmark economy of each of the countries, calibrated to match the moments of each economy, as stated in [Section 3.2](#). This is our starting point. For each of the countries we start the analysis with the interest given endogenously: the German interest rate at 4.95%, the Portuguese at rate 4.21%, and the Dutch at 5.68%. Using that same modelling, we proceed to include the interest rate and the wage rate as a parameter of the model, chosen exogenously instead of endogenously as it was previously done in the original model. The advantage of this set-up is that we can easily alter the interest rate, maintaining all the other parameters as the same values calibrated before-hand, and assess the change in the fiscal multiplier due to the change in the interest rate.

The initial and final steady-states of the economies are assumed to have exactly the same parameters throughout the transition, except for the debt-to-GDP ratio, which is decreased by 0.1% annually, as predicted in the experiment described in the [Model](#) section. As such, the economies will transition in a open economy with mobility of capital and constant wages and interest rates.

For each of the three countries we proceed with an increase of the interest rate until 8%

and, then, a decrease of the interest rate until 1%. The purpose of this is to evaluate 8 different environments: we will have each economy in an environment where they are facing an interest rate from 0.95% to 7.95% for Germany, 1.22% to 8.22% for Portugal, and 1.69% to 8.69% for the Netherlands, increasing marginally by 1% in each country. The logic of not using precisely the same interest rates for each country – e.g., using 1%, 2%, etc. for each of them – is that we depart from each country’s calibration model and the starting interest rate is the one given endogenously by that calibrated model. Also, for this analysis, we are not entirely interested at the magnitude of the multiplier, but rather the change that an increase/decrease of the interest rate from the calibrated model has on that multiplier. That said, the evaluation will focus on the signal of that change, that is, if a multiplier is further negative or further positive when applying a change in the interest rate.

With that said, we will now proceed with the in-depth analysis of Germany’s economy, the direction of its multipliers when changing the interest rate, analysing both consolidations via government spending,  $G$ , and via labor tax,  $\tau_l$ , the labor supply impact, the GDP per capita counterpart, and the lump-sum associated with that shock. Afterwards, we will confirm the assumption proceeding the same way for the Portuguese and the Dutch economies.

### **3.3.2 *Interest Rate and Fiscal Multipliers Dynamics***

The test for Germany gave us results aligned with the data results made in Section 2: higher interest rates implied more negative fiscal multipliers. Figure 1 and Table 2 summarize the results of the change in interest rate impact on fiscal multipliers. The exercise was made for a consolidation via government expenditure,  $G$ , and via labor taxes,  $\tau_l$ . Both multipliers had the expected behaviour. As the interest rate increases, the more negative the multiplier is. This happens for both types of consolidation. On Figure 1, the vertical line of interest rate equal to the endogenous model (4.95%) shows how the multipliers go further away from zero in negative terms at the right of that line and approach zero as we go to the left of that same line.

On what regards the multipliers for a consolidation via government spending, the effect of an increase of the interest rate was significantly small, as the size of the multipliers was nearly zero. All of them were only different from zero from the fifth decimal point onwards. This

Table 2: Interest Rate and Fiscal Multipliers of Consolidation via  $\tau_l$

Interest Rate	Germany	Interest Rate	Portugal	Interest Rate	Netherlands
0.95%	-1.789	1.22%	-0.737	1.69%	-1.364
1.95%	-1.901	2.22%	-0.804	2.69%	-1.454
2.95%	-2.061	3.22%	-0.902	3.69%	-1.548
3.95%	-2.272	4.22%	-1.033	4.69%	-1.704
4.95%	-2.584	5.22%	-1.232	5.69%	-1.933
5.95%	-3.089	6.22%	-1.533	6.69%	-2.314
6.95%	-3.953	7.22%	-2.058	7.69%	-3.062
7.95%	-6.575	8.22%	-3.468	8.69%	-4.856

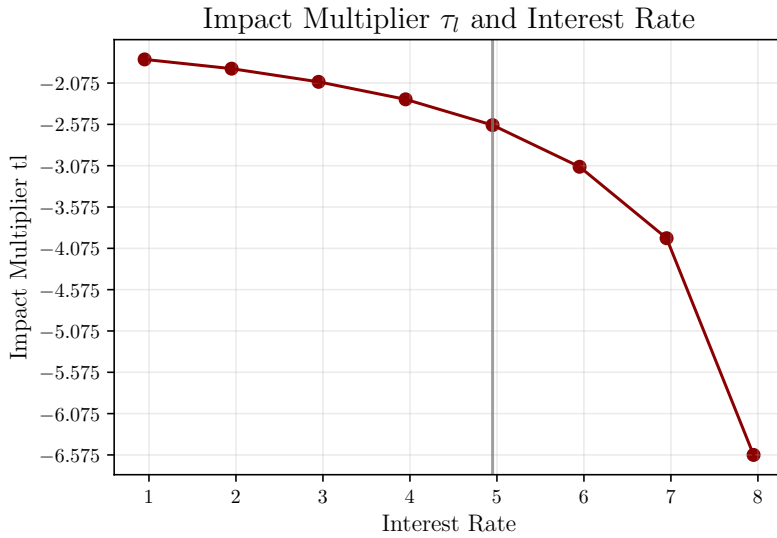


Figure 1: Interest Rate and Multipliers via  $\tau_l$  of Germany

goes along with the open economy test made in [Brinca et al. \(2021\)](#). Despite not changing the interest rate, the case of an open economy with fixed wages and interest rates implied a close-to-zero multiplier for consolidations via government spending, which are confirmed by our experiment. Also, the values of the multipliers we got for consolidations via labor taxes are aligned with that experiment on [Brinca et al. \(2021\)](#), something that gives us more confidence on the robustness of our results. For both of these reasons, we decided to deepen the analysis to the results of a consolidation via labor tax.

Another conclusion was that the increase in the interest rate, which implied more negative multipliers via labor tax, generated a retraction on the supply of labor. Figure 2 plots the directions of the labor supply when exposing them with the impact multipliers and the interest rate: an increase on the interest rate, which will generate more negative multipliers, creating lower

labor supply; the less negative are the multipliers, something that happens as the interest rate decreases, the higher is the labor supply. We can also see in Figure F1, in the appendix Section 6.2, that this same dynamic happens when plotting GDP per capita with the same variables: the higher the interest rate, the lower will be GDP per capita; the less negative is the multiplier, which happens when interest rates are lower, the higher will be GDP per capita.

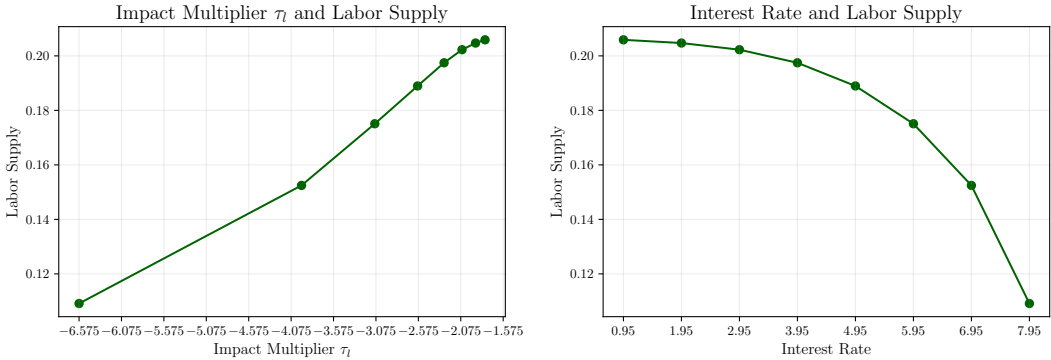


Figure 2: Labor Supply with Impact Multipliers  $\tau_l$  and Interest Rate

The decrease in labor supply seems to be compensated by lump-sum transfers and to follow the same direction of GDP per capita, as we can see in Figure F2 – the case of Germany – and confirm that this is a general pattern in Figure 3 – with the three countries in the same graph.

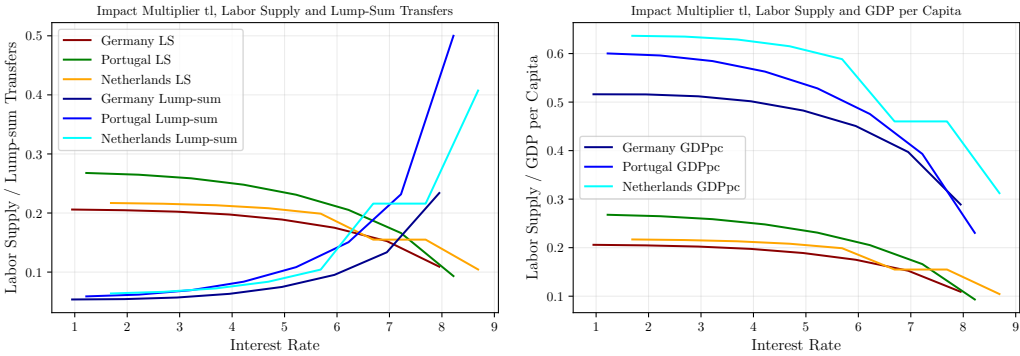


Figure 3: Labor Supply, Lump-sum and GDP per capita for the three countries

An interesting result was that as the interest rate increases, the marginal impact on the multiplier increases as well. Figure 4 shows this very well. The bars are the change in the multiplier from an increase of 1% in the interest rate. As interest rate grows, an equal 1% increase in the interest rate will generate a more negative impact on the multipliers of consolidating via labor tax.

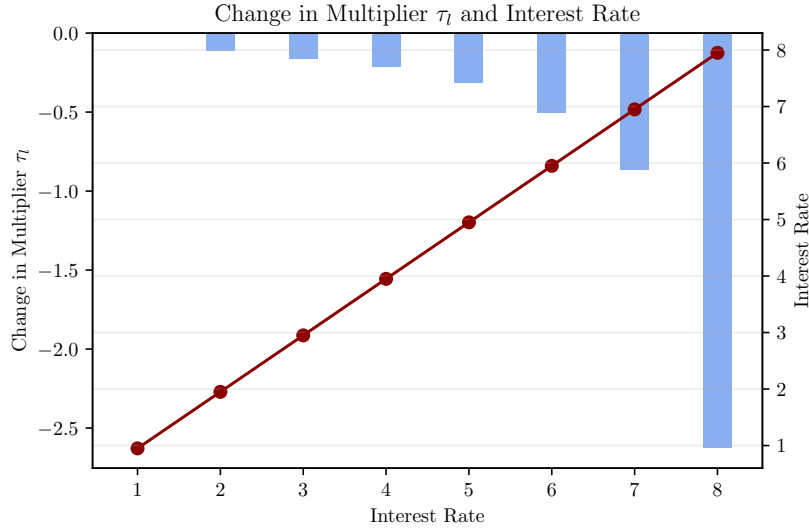


Figure 4: Change of Multiplier  $\tau_l$  by Increases of Interest Rate of Germany

### 3.3.3 Robustness Analysis and Inequality

At first, the objective of this analysis was to assess the impact of changes in interest rate on the multipliers of fiscal consolidations for a benchmark economy, Germany. However, to enhance the robustness of the results, we decided to make the same experiment for Portugal and the Netherlands. We confirmed that both countries presented the same trend than that of Germany, which gives us some confidence on the results. The trend of the three countries can be seen in Figure 5, panel (a). Table 3 has the values for the percentage of agents constrained and the Gini coefficient for each one of the three countries

Table 3: Percentage of Agents Constrained and Gini Coefficient of the three countries

Country	Agents Constrained (%)	Gini Coefficient
Portugal	0.1985	0.5986
Netherlands	0.0685	0.7265
Germany	0.0383	0.6612

But more than that, the inclusion of these specific countries was the value of their Gini coefficients. Having one country from one side of inequality, one from the other extreme and one in the middle was the reason that got us to choose Portugal and the Netherlands to serve as comparison to our base economy. The intention of the complementary analysis was to have some insight on whether country's Gini coefficient would imply a different, causal and predictable, impact on the multipliers. As such, maintaining all else equal, we plotted in Figure 5, panel (b)

the change in the multipliers for a given interest rate for each of the three countries. To find a pattern we should expect that that change would be mild for Germany and more/less intense for one of the other countries. The fact that we do not find a pattern on this experiment does not give us the possibility to get any conclusion regarding Gini coefficients and the marginal impact on multipliers of a change in interest rate. However, we also have the values for the percentage of agents constrained, which can give us another insight. What we see is that Portugal, the country with the highest percentage of agents constrained has the multipliers closest to zero, following by the Netherlands – which have a percentage of agents constrained between the three countries – and, finally, Germany – the country with the lowest percentage of agents constrained. This does not tell us that there is no relationship between the impact of fiscal consolidations and the Gini coefficient, but that the percentage of agents constrained plays a part on those.

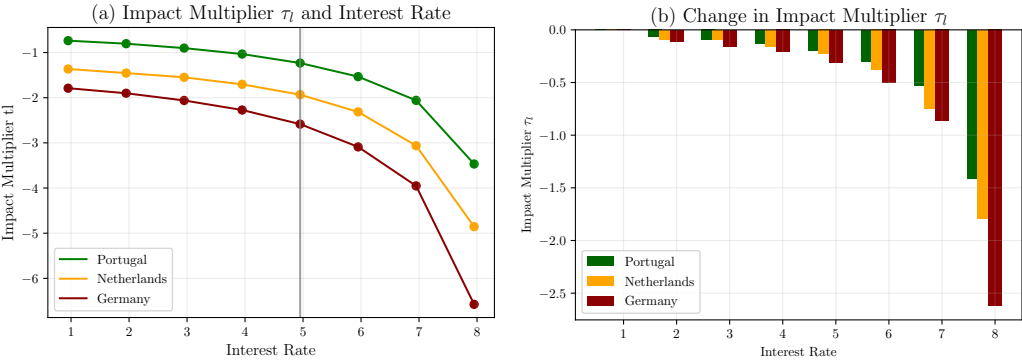


Figure 5: (a) Interest Rate and Impact Multipliers  $\tau_l$ ; (b) Change in Impact Multipliers  $\tau_l$

### 3.3.4 Economic Intuition

After going through the results of our experiment, we will now try to give an economic intuition behind the dynamics of the change in interest rate and the impact it has in the economy. Remember that we are in an economy where the interest rates and wages are constant throughout the consolidation, which will give us different intuition than if the wage could adjust to the labor supply shock.

The story starts with the change in interest rate. Let us assume a positive shock on interest rate, that is, an increase in this variable, and focus on the case of a consolidation via labor tax. The increased interest rate is implying larger interest payments on debt, which makes it harder to pay than in the world with a lower interest rate. The effort of debt payment is going to be

larger. The fact that this effort is larger will imply that there must be a larger increase in the labor tax than before to ensure that the same amount of debt is going to be paid. Remember that all other parameters are maintained equal, so the 10% decrease in debt is the same, but now with a larger part for interest payment.

The effect of this on labor supply will be, as said, negative. The increase in labor income tax will make households with lower current income. Also, knowing that after consolidation there will be a return to the same level of labor income tax, agents will prefer to work less today given the fact that the disposable income that they receive now is going to be lower than before. Hence, there will be a negative labor supply effect. The fact that wages are constant will imply that they will not adjust to the labor supply decrease, which will diminish the income effect of the transition, but the current substitution effect is stronger now, as current wage will not increase as a consequence of the drop in labor supply. What we see, on the other hand, is the rise in lump-sum transfers, and that is the only source of income effect. Given that the government balance must balance, and that lump-sum transfers,  $g_t$ , are the variable that ensures the balanced budget binds, agents will receive higher lump-sum transfers given a higher interest rate, since the interest debt payments and labor income tax are now larger than in the world with lower interest rates. Therefore, to ensure the government budget balance equation must hold, the source to ensure that that happens must increase.

We must emphasize that the case here in study is not the assumption that at the beginning of the consolidation we had a lower interest rate and at the moment of the consolidation the interest rate is higher, but rather the comparison of the world with a given interest rate throughout the consolidation and another world with a higher interest rate throughout the consolidation. This must be taken into account to make sure the intuition follows a sensible reasoning.

The logic behind the drop in GDP per capita is easy to follow, and almost explained by the intuition given before. If the interest rate is higher and the debt repayment is done with higher effort, there will be lower labor supply without higher wages for the agents, and the ones that continue to work will receive lower disposable income, the multiplier will be negative and imply a decrease in GDP. The number of agents maintained equal gives the last piece of information needed to understand why GDP per capita decreases.

Finally, our test on inequality was short, but connected the Gini coefficient and percentage of agents constrained. We can see how a country with more constrained agents – a country with more low-earning agents – has the multipliers closer to zero and how the marginal increase in interest rate creates lower impact on the multipliers – that is, they become more negative with lower magnitude. The intuition is that constrained agents do not have the access to as large savings or borrowing capacity as the unconstrained agents. To smooth their consumption, they will need to continue working, instead of having the capacity of waiting until labor income taxes become the same as before. Hence, by continuing to work, the labor supply drop will be lower and the drop in GDP will be lower as well. The more constrained agents there is, the more resistant to increases in the interest rate the country will be when conducting fiscal consolidations via  $\tau_l$ .

## 4 CONCLUSION

This thesis analyses the impact of a change of interest rate on the impact multipliers of a fiscal consolidation plan via government spending and labor income taxation. We also take a look at how does the marginal change in interest rate affects the change of the fiscal multipliers from one environment to the other and how can we connect inequality to this results.

The first step was to extend the regression in [Alesina et al. \(2015a\)](#) with interest rates and see if there would be any sign of a relationship between the interest rates and the impact multipliers. We see that it exists and that the relationship was negative, with higher interest rates affecting fiscal multipliers' negatively. We then built on [Brinca et al. \(2021\)](#) overlapping generations model with incomplete markets and calibrated our three economies to match each countries' economic moments. We then adapted the model to our reality and put interest rate and wage as constant parameters of our model to better assess the impact of a change on the interest rate.

With that set-up, we saw that moving from the starting economy of each country to an economy with a higher constant interest rate throughout the consolidation process would give us multipliers further away from zero in negative terms for both consolidations via government spending and labor tax. The opposite was also true: from the starting point to an economy

with lower interest rates, the multipliers approached zero instead. This results aligned with the ones of the extended regression analysed in Section 2. Therefore, we can conclude that in an environment of constant wages and interest rates, higher interest rates will imply a negative effect on fiscal multipliers' values. We also saw that the multipliers from a consolidation via government spending are only different from zero starting on the fifth decimal point, which gives us an insight that they are close to zero. Another aspect of the analysis was that as interest rates become higher, the marginal impact on the multipliers will also be stronger. This happens for the three countries, in different proportions.

These proportions might be caused by the inequality of each country. We could not conclude any relationship between the Gini coefficient and the size of the impact (or marginal impact) on the multipliers, but could connect the percentage of agents constrained. The result was that the higher that percentage, the lower would be the impact (and the marginal impact) of interest rate changes.

To sum up, this paper goes through two strategies to check the impact of interest rates on fiscal multipliers. Firstly, it extends the regression in [Alesina et al. \(2015a\)](#) and, secondly, it adapts the model in [Brinca et al. \(2021\)](#). We also incur in a robustness analysis of the results from the adapted model, checking if countries with different Gini coefficients and percentage of agents constrained behave in the same way and, if differently, why that happens. The results of both analysis are aligned and we conclude that higher interest rates make fiscal multipliers' values to go for more negative territory. Finally, we see that the higher the percentage of agents constrained, the less impact an increase/decrease in the interest rate will have on fiscal multipliers.

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## 6 Appendix

### 6.1 Definition of a Transition Equilibrium after the Unanticipated Fiscal Consolidation Shock

<sup>1</sup> The definition of a recursive competitive equilibrium throughout the transition between steady states is as follows:

Having the initial capital stock as given, the starting point of the distribution of initial taxes and households, respectively  $K_0, \phi_0$  and  $\{\tau_l, \tau_{ss}, \tilde{\tau}_{ss}, \tau_c, \tau_k\}_{t=1}^{t=0}$ , a competitive equilibrium is a sequence of individual functions for the household,  $\{V_t, c_t, k'_t, n_t\}_{t=1}^{t=\infty}$ , of production plans for the firm,  $\{K_t, L_t\}_{t=1}^{t=\infty}$ , factor prices,  $\{r_t, w_t\}$ , government transfer  $\{g_t, \Psi_t, G_t\}_{t=1}^{t=\infty}$ , government debt,  $\{B_t\}_{t=1}^{t=\infty}$ , inheritance from the dead,  $\{\Gamma_t\}_{t=1}^{t=\infty}$ , and of measures  $\{\Phi\}_{t=1}^{t=\infty}$ , such that for all  $t$ :

(1) Given the factor prices and the initial conditions the consumers' optimization problem is solved by the value function  $V(k, \beta, \alpha, u, j)$ , and  $n(k, \beta, \alpha, u, j)$ .

(2) Markets clear:

$$\begin{aligned} K_{t+1} + B_t &= \int k_t d\Phi_t \\ L_t &= \int (n_t(k_t, \beta, \alpha, u, j)) d\Phi_t \\ \int c_t d\Phi_t + K_{t+1} + G_t &= (1 - \delta)K_t + K_t^\alpha L_t^{1-\alpha} \end{aligned}$$

(3) The factor prices satisfy:

$$\begin{aligned} w_t &= (1 - \alpha) \left( \frac{K_t}{L_t} \right)^\alpha \\ r_t &= \alpha \left( \frac{K_t}{L_t} \right)^{\alpha-1} - \delta \end{aligned}$$

(4) The government budget balances:

$$g_t \int d\Phi_t + G_t + r_t B_t = \int \left( \tau_t r_t (k_t + \Gamma_t) + \tau_c c_t + n_t \tau_t \left( \frac{n_t w_t(\alpha, u, j)}{1 + \tilde{\tau}_{ss}} \right) \right) d\Phi_t + (B_{t+1} - B_t).$$

<sup>1</sup>This appendix was borrowed from [Brinca et al. \(2021\)](#)

(5) The social security system balances:

$$\Psi_t \int_{j \geq 65} d\Phi_t = \int (1 - \omega(j)) k_t d\Phi_t.$$

(6) The assets of the dead are uniformly distributed among the living:

$$\Gamma_t \int \omega(j) d\Phi_t = \int (1 - \omega(j)) k_t d\Phi_t.$$

(7) Aggregate law of motion:

$$\Phi_{t+1} = \Upsilon_t(\Phi_t).$$

## 6.2 Tables and Figures

Table A1: Parameters Calibrated Exogenously

Country	$\theta_0$	$\theta_1$	$\tau_{ss}$	$\tilde{\tau}_{ss}$	$\tau_c$	$\tau_k$	$\gamma_1$	$\gamma_2$	$\gamma_3$
Germany	0.881	0.221	0.210	0.206	0.155	0.233	0.176	-0.003	2.3e-05
Netherlands	0.938	0.254	0.200	0.102	0.194	0.293	0.307	-0.007	4.9e-05
Portugal	0.937	0.136	0.110	0.238	0.194	0.293	0.172	-0.004	2.6e-05

Table A2: Parameters Constant to all Countries

$\alpha$	$\delta$	$\rho$	$\sigma_a$	$\sigma$	$\eta$
0.33	0.06	0.335	0.423	1.2	1

Table A3: Parameters Calibrated Endogenously Estimated by Simulated Method of Moments

Country	$\beta_1$	$\beta_2$	$\beta_3$	$b$	$\chi$	$\varphi$	$\sigma_\epsilon$	$r$	$w$
Germany	0.9650	0.9953	0.9650	0.003	16.42	3.81	0.4386	4.9489%	1.1536
Netherlands	0.9680	0.9856	0.9579	-0.022	14.72	2.99	0.2625	5.6894%	1.1170
Portugal	0.8965	0.9921	0.8900	-0.030	11.62	6.70	0.3810	4.2192%	1.1935

Table A4: Target and Model Quartiles

Country	Target Quartiles			Model Quartiles		
	Q1	Q2	Q3	Q1	Q2	Q3
Germany	0.0063	0.0544	0.2234	0.0066	0.0536	0.2233
Netherlands	0.0106	0.0812	0.3119	0.0100	0.0851	0.3091
Portugal	0.0039	0.0283	0.1399	0.0034	0.0223	0.1458

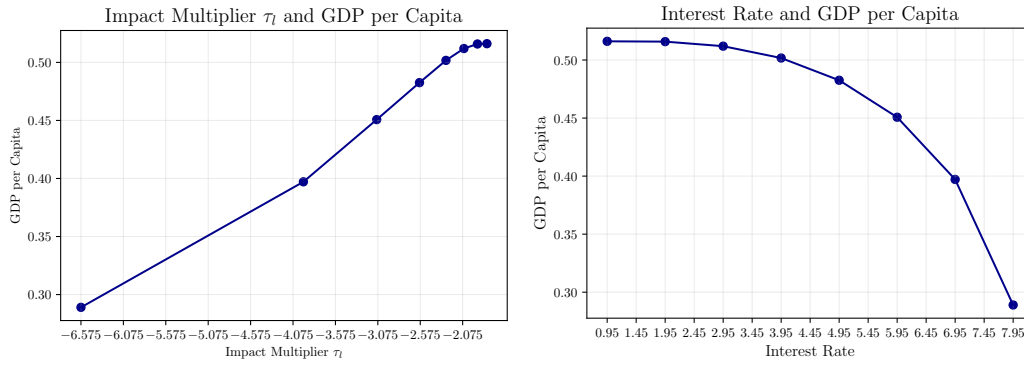


Figure F1: GDP per Capita with Impact Multipliers  $\tau_l$  and Interest Rate for Germany

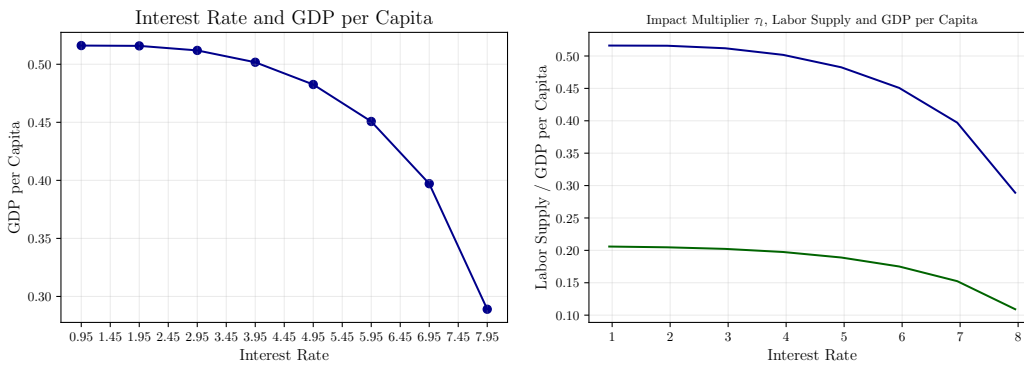


Figure F2: Labor Supply, Lump-sum and GDP per Capita for Germany