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Geomorphic dam-break flows. Part I: conceptual model

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This paper presents a one-dimensional conceptual model for simulating geomorphic dam-break flows. The model is based on conservation laws drawn from continuum mixture theory that are integrated over the flow depth, assuming that the flow is composed of two transport layers. Closure equations were derived from the review and reanalysis of previous studies on granular flow, debris flow and sheet flow. The sediment transport is modelled assuming capacity regime. The closure equation coefficients are estimated based on a large set of experiments available in the literature. The validity of the model is discussed by computing the most relevant dimensionless parameters. The model is expected to simulate adequately the friction at the wave front and the sheet flow throughout the wave profile. Ultimately, although the problem incorporates complex phenomena, the model can be easily implemented by practising engineers for simulating one-dimensional geomorphic dam-break flows. The implementation and the verification of the adequacy of the model for simulating these flows are presented in a companion paper.

NOTATION

a	coefficient that depends on the type of sediment
b	coefficient that depends on the type of sediment
C_s	sediment volumetric concentration
d_s	sediment mean diameter (m)
e_b	erosion rate (m/s)
g	gravity acceleration vector (m^2/s)
h	flow depth (m)
k	bed permeability (m^2)
N_{Bag}	Bagnold number
N_{Dar}	Darcy number
N_{fric}	friction number
N_{mass}	mass number
N_{Rey}	Reynolds number of sediments
N_{Sav}	Savage number
p	bed porosity
R	dimensionless Chezy's friction coefficient
S_f	energy slope
s	sediment specific gravity
$sgn()$	sign function that takes the value 1 if the variable is positive and -1 if the variable is negative
t	time (s)
u	mean mixture velocity vector (m/s)
u_*	friction velocity (m/s)

w_*	dimensionless sediment settling velocity
w_s	sediment settling velocity (m/s)
x, y, z	spatial coordinates (m)
z	elevation (m)
z_c	equivalent bed elevation including the sediment storage in the water column (m)
z_{ws}	water surface elevation (m)
α	power of proportionality equation between u_c and z
β	dimensionless coefficient related to the type of sediment
θ	Shields parameter
ν	water kinematic viscosity at $20^\circ\text{C} = 10^{-6} \text{ m}^2/\text{s}$
ρ	mixture mass density (kg/m^3)
σ	stress tensor (N/m^2)
σ_{bc}	normal bed stress (N/m^2)
σ'_{bc}	effective normal bed stress (N/m^2)
$\sigma_{(ij)}$	stress in the normal plane to i and in the j direction (N/m^2)
τ_{bc}	bottom shear stress (N/m^2)
∇	mathematical operator $[\partial/\partial x \partial/\partial y \partial/\partial z]^T$
ϕ_s	sediment static friction angle (rad)

Subscripts or superscripts

above	refers to values above the immobile bed surface
b	immobile bed
below	refers to values below the immobile bed surface
c	sheet-flow layer
f	fluid or liquid constituent
s	solid or sediment constituent
w	clear water layer

1. INTRODUCTION

During the past two decades, dam-break flows have been modelled by many researchers and engineers. Initially, the flow was treated as a purely hydrodynamic problem. Recent observations and records from real accidents (Brooks and Lawrence, 1999) have shown that dam-break flows can interact strongly with the mobile bed (Figure 1). Several experimental studies (Capart and Young, 1998; Leal, 2005; Leal *et al.*, 2002; Spinewine and Zech, 2007) highlight the existence of a transport layer with high concentration of sediments within the flow. The thickness of this layer can vary from small values in the upstream reach of the wave profile to the entire water depth near the wave-front. Due to the high shear rate and corresponding high concentration of sediments, the flow can be classified as debris flow or sheet flow, where stresses due to



grain collisions become relevant. Although some geomorphic dam-break flow models have been developed in the past, most of them rely on conceptual models incorporating closure equations obtained for low shear rates (Brufau *et al.*, 2000; Caleffi *et al.*, 2003; Cao *et al.*, 2004). Since the pioneering study of Bagnold (1954), it has been well known that, at high shear rate, the grain collisional and frictional effects and the liquid-grain interactions, that are negligible at low shear rate, can influence the flow behaviour (Iverson, 1997). Therefore, it is important to develop conceptual models based on closure equations incorporating these effects. Within this framework, the advances in granular flows using gas kinetic theory (Chou, 2000; Jenkins and Hanes, 1998; Pouliquen and Forterre, 2002; Savage, 1984; Savage and Hutter, 1989; Savage and Jeffrey, 1981) and some results obtained in debris flow and sheet flow (Asano, 1995; Nnadi and Wilson, 1992; Pugh and Wilson, 1999; Sumer *et al.*, 1996; Takahashi, 1991; Wan and Wang, 1994; Wilson, 1987) constitute a good basis to build up conceptual models. A conceptual model for geomorphic dam-break flows was previously presented by other authors, based on transport layers incorporating some results previously obtained in granular flows (Fraccarollo and Capart, 2002; Jenkins and Hanes, 1998). As those authors were interested in achieving an analytical solution they adopted some important simplifications. In the present paper, a more complete

conceptual model is developed based on recent studies (Ferreira, 2005; Leal, 2005). The model incorporates, in a simple way, some of the most relevant conclusions that can be drawn from granular flow, debris flow and sheet-flow studies. In the companion paper (Leal *et al.*, 2010), the results of the model are compared with experimental data.

2. CONSERVATION LAWS

Assuming that no external mass transfer occurs, the mass and momentum conservation of fluid (liquid) and solid (sediment) mixture can be written as (Iverson, 1997)

$$1 \quad \frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{u}) = 0$$

$$2 \quad \begin{aligned} \frac{\partial(\rho \mathbf{u})}{\partial t} + \rho_f(1 - C_s)\mathbf{u}_f \nabla(\mathbf{u}_f) \\ + \rho_s C_s \mathbf{u}_s \nabla(\mathbf{u}_s) = \nabla(\boldsymbol{\sigma}_f + \boldsymbol{\sigma}_s) + \rho \mathbf{g} \end{aligned}$$

where subscripts f and s stand for fluid and solid constituents, respectively, $\rho = \rho_f(1 - C_s) + \rho_s C_s$ is the mixture mass density, C_s is the sediment volumetric concentration, $\mathbf{u} = [\rho_f(1 - C_s)\mathbf{u}_f + \rho_s C_s \mathbf{u}_s] / \rho$ is the mixture mean velocity vector, t is time, ∇ is the mathematical operator $[\partial/\partial x \ \partial/\partial y \ \partial/\partial z]^T$, x , y and z are the spatial coordinates, $\boldsymbol{\sigma}$ is the stress tensor and \mathbf{g} is the gravity acceleration vector.

As the bottom can be subjected to erosion and deposition, Equations 1 and 2 are not sufficient to describe the flow and the mass conservation of the solid constituent must be added

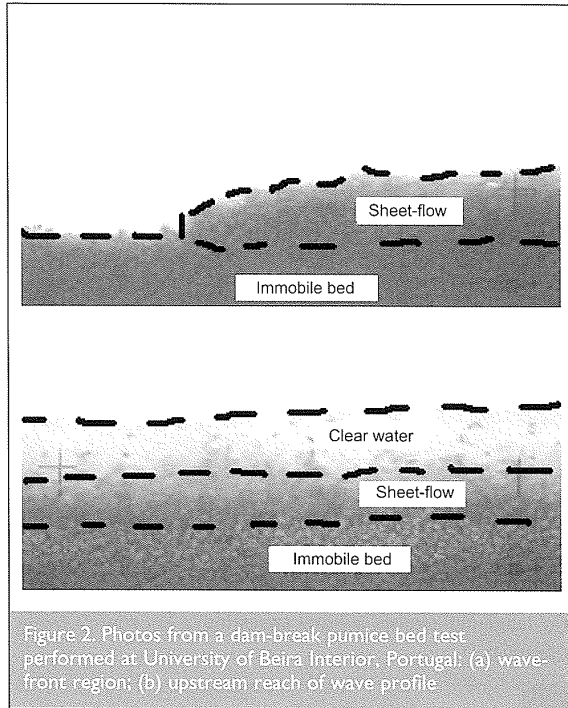
$$3 \quad \frac{\partial}{\partial t}(\rho_s C_s) + \nabla(\rho_s C_s \mathbf{u}_s) = 0$$

3. TRANSPORT LAYERS (DEPTH AVERAGING)

Experimental observations of geomorphic dam-break flows show different sediment transport magnitude along the wave profile (Capart and Young, 1998; Leal, 2005; Spinewine and Zech, 2007). In the wave-front the entire flow depth is occupied with a debris or sheet-flow (Figure 2(a)), while upstream the flow presents two distinct transport layers: one upper clear water layer and one lower sheet-flow layer (Figure 2(b)). Based on this observed flow configuration and assuming hydrostatic pressure distribution, a one-dimensional (1D) conceptual transport layers model (Figure 3) was proposed in a previous study (Leal, 2005). In this figure, subscripts w, c and b stand for clear water layer, sheet-flow layer and immobile bed, respectively, h is the flow depth, z the elevation and p is the bed porosity.

The depth average flow depth, mean velocity, volumetric sediment concentration and mass density are given, respectively, by

$$4 \quad h = h_c + h_w$$



$$5 \quad u = (u_c h_c + u_w h_w) / h$$

$$6 \quad C = (C_c u_c h_c) / (u h)$$

$$7 \quad \rho = \rho_w [1 + (s - 1) C]$$

where $s = \rho_s / \rho_w$ is the sediment specific gravity.

The depth integration of Equations 1 to 3 in each transport layer, under the hypotheses of (a) 1D flow, (b) small bottom slope, (c) negligible local and convective accelerations normal to the direction of flow propagation, (d) equal velocity of sediment and fluid in the sheet-flow layer, renders (Ferreira, 2005)

$$8 \quad \frac{\partial z_{ws}}{\partial t} + \frac{\partial}{\partial x} (hu) = 0$$

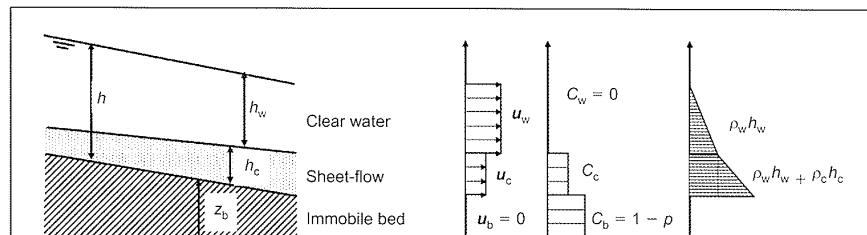


Figure 3. Conceptual transport layers model

$$9 \quad \frac{\partial(\rho u h)}{\partial t} + \frac{\partial}{\partial x} (\rho_w u_w^2 h_w + \rho_c u_c^2 h_c) = -g(\rho_w h_w + \rho_c h_c) \frac{\partial z_b}{\partial x} - \int_{z_b}^{z_{ws}} \left(\frac{\partial \sigma_{(xx)}}{\partial x} + \frac{\partial \sigma_{(yy)}}{\partial y} + \frac{\partial \sigma_{(zz)}}{\partial z} \right) dz$$

$$10 \quad \frac{\partial z_c}{\partial t} + \frac{\partial}{\partial x} (Chu) = 0$$

where $z_{ws} = h + z_b$ is the water surface elevation, $z_c = (1 - p)z_b + C_c h_c$ is an equivalent bed elevation that includes the sediment storage in the water column and $\sigma_{(ij)}$ is the stress in the plane normal to i , in the direction j .

4. STRESS TENSOR

The stress tensor will account for the following stress-generating processes (Iverson, 1997): (a) grain collisions, (b) friction between grains, (c) fluid viscosity, (d) turbulent fluctuations, (e) interaction between fluid and grains. Processes (b), (c) and (e) are responsible for viscous or friction stresses, whereas processes (a) and (d) generate inertial stresses. The magnitude and importance of each of the five processes will be evaluated later.

The depth integration of momentum Equation 2, between z and z_s , under the hypotheses mentioned before, renders the normal stress

$$11 \quad \sigma_{(zz)} = \begin{cases} \rho_w g(z_s - z) & \text{if } z \geq z_b + h_c \\ \rho_w g[h_w + (1 - C_c)(h_c + z_b - z)] + \rho_s g C_c (h_c + z_b - z) & \text{if } z_b \leq z < z_b + h_c \end{cases}$$

For the clear water layer, Equation 11 means that the normal stress is caused by the weight of the fluid and that turbulent stresses are neglected. For the sheet-flow layer, the first term of Equation 11 represents the normal stress due to the weight of the fluid, whereas the second stands for the normal stress originated by sediment grain collisions that transfer the weight to the bottom. The normal stresses due to friction between grains are neglected. The stresses due to the interaction between fluid and grains are also omitted since the mixture is

considered to be fully fluidised and therefore interstitial pressures are insignificant. Whenever the mixture behaves like a liquid, the normal stresses are isotropic, namely $\sigma_{(xx)} = \sigma_{(yy)} = \sigma_{(zz)}$ (Pouliquen and Forterre, 2002). The hydrostatic pressure assumption implies normal stresses isotropy for the liquid $\sigma_f^{(xx)} = \sigma_f^{(yy)} = \sigma_f^{(zz)}$ and

when grain collisions dominate upon frictional interactions the grains normal stresses are also isotropic $\sigma_{s(xx)} = \sigma_{s(yy)} = \sigma_{s(zz)}$, because grain collisions are isotropic as long as the elasticity of the grains is small (Chou, 2000). Using Equation 11, one can compute the integral of normal stress included in Equation 9, namely

$$12 \quad \int_{z_b}^{z_s} \frac{\partial(\sigma_{s(xx)})}{\partial x} dz = \int_{z_b}^{z_s} \frac{\partial(\sigma_{s(zz)})}{\partial x} dz = \frac{\partial}{\partial x} \left[\frac{1}{2} g(\rho_w h_w^2 + 2\rho_w h_w h_c + \rho_c h_c^2) \right]$$

The bottom shear stress, τ_{bc} , can be obtained by analysing the bottom of the sheet-flow layer, where grains collisions dominate over viscous effects. Assuming that tangential stresses are proportional to $(du/dz)^2$ (Bagnold's inertial regime), which can be extrapolated from gas kinetic theory applied to granular flows with interstitial liquid (Jenkins and Hanes, 1998), the bottom shear stress, τ_{bc} , is given by a Chézy-type equation (Fraccarollo and Capart, 2002). Recently, the following relation was proposed (Ferreira, 2005)

$$13 \quad \tau_{bc} = \text{sgn}(u) \rho_w R u^2$$

where R is the dimensionless Chezy's friction coefficient and $\text{sgn}()$ is the sign function that takes the value 1 if the variable is positive and -1 if the variable is negative. Equation 13 allows the usual definition of friction velocity

$$u_* = \sqrt{\tau_{bc}/\rho_w} = \sqrt{g h S_f}$$

where $S_f = (R u^2)/g h$ is the energy slope. This definition is important since the substitution in Equation 13 yields the proportionality between u and u_* , corroborating debris-flow observations (Takahashi, 1991).

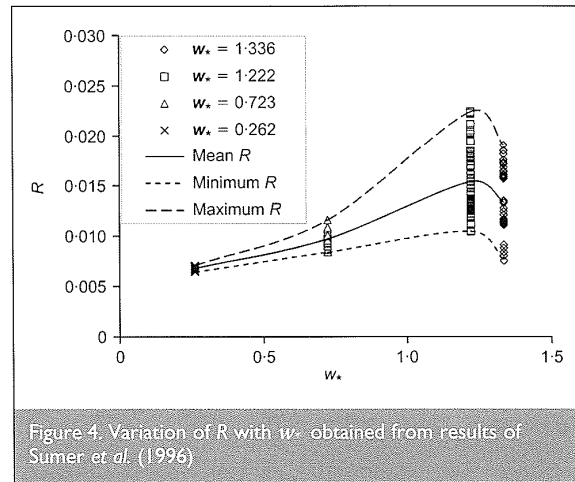
The sheet-flow results highlight a variation of the friction coefficient with the dimensionless sediment settling velocity

$$w_* = w_s / \sqrt{g(s-1)d_s}$$

where w_s is the sediment settling velocity and d_s is the sediment mean diameter (Sumer *et al.*, 1996). This variable can be computed using, for example, the general expression (Jiménez and Madsen, 2003)

$$14 \quad \frac{1}{w_*} = a + \frac{4\nu}{d_s} \frac{b}{\sqrt{(s-1)d_s}}$$

where a and b are coefficients that depend on the type of sediment, and $\nu = 10^{-6} \text{ m}^2/\text{s}$ is the water kinematic viscosity at 20°C. Figure 4 presents the variation of the friction coefficient, R , with w_* , obtained by reanalysing the experimental data of Sumer *et al.* (1996). It can be concluded that R varies for constant w_* . For each w_* , a mean value of R will be assumed;



this assumption is discussed in a companion paper by the authors (Leal *et al.*, 2010).

The tangential stresses integral appearing in Equation 9 can be written as

$$15 \quad \int_{z_b}^{z_s} \left(\frac{\partial \sigma_{s(xx)}}{\partial z} \right) dz = \tau_{bc} = \rho_w R u^2$$

Finally, and in accordance with the proposed 1D approach, the variation of the tangential stresses in direction y is considered to be negligible

$$16 \quad \int_{z_b}^{z_s} \left(\frac{\partial \sigma_{s(xx)}}{\partial y} \right) dz = 0$$

5. CLOSURE EQUATIONS

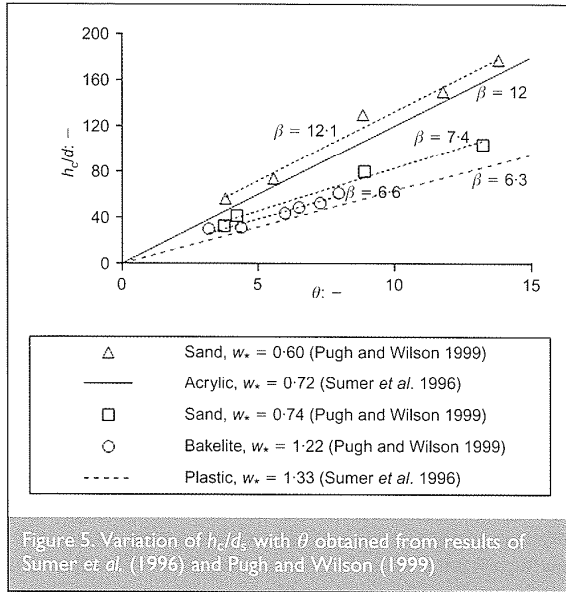
Several authors have proposed relations for estimating the height of the sheet-flow layer h_c . Based on Bagnold's 1954 work, one can obtain theoretically $h_c/d_s \approx 10\theta$, where

$$\theta = \tau_{bc}/[\rho_w g(s-1)d_s] = u_*^2/[g(s-1)d_s]$$

is the Shields parameter (Wilson, 1987). The revision of Daniel's (1965) results showed $h_c/d_s \approx 7.5\theta$ (Nnadi and Wilson, 1992). Sheet-flow visual observations led to the relation $h_c/d_s \approx 6.3\theta$ for the larger sediments; using the measurements of sediment concentration profiles, the following relation $h_c/d_s \approx 12\theta$ was also derived for the smaller sediments (Sumer *et al.*, 1996). The application of gas kinetic theory to a mixture of grains and water revealed that h_c/d_s varies linearly with θ (Jenkins and Hanes, 1998). This behaviour was also observed experimentally (Pugh and Wilson, 1999). Taking all these results into consideration, it can be concluded that

$$17 \quad \frac{h_c}{d_s} = \beta \theta$$

where β is a dimensionless coefficient related to the type of



sediment that depends on w_* (Sumer *et al.*, 1996). Figure 5 presents the values of β obtained by reanalysing the previous results on sheet-flow (Pugh and Wilson, 1999; Sumer *et al.*, 1996). For a given type of sediment, the value of β can be estimated from Figure 5 knowing w_* .

For dry granular flows, it is consensual that the velocity profile is linear (Savage, 1984; Savage and Hutter, 1989; Savage and Jeffrey, 1981). For mixtures of liquid and grains, most authors agree that the velocity in the sheet-flow layer should be proportional to z^α . The value of power α is not consensual. Some authors suggested $\alpha = 3/2$ (Asano, 1995; Takahashi, 1991), whereas others proposed $\alpha = 3/4$ (Jenkins and Hanes, 1998; Sumer *et al.*, 1996; Wan and Wang, 1994). As Sumer *et al.* (1996) performed a large set of tests using different sediments and their results were theoretically confirmed (Jenkins and Hanes, 1998), the equation proposed by those authors for the local velocity is adopted

$$18 \quad \frac{u_c(z)}{u_*} = 2.5\theta^{-3/4} \left(\frac{z}{d_s} \right)^{3/4}$$

Replacing the result of the depth integration of Equation 18 over the sheet-flow layer into Equation 17 and dividing it by the height of that layer one obtains the depth-averaged velocity within the sheet-flow layer

$$19 \quad u_c = \frac{10}{7} \beta^{3/4} \sqrt{g(s-1)d_s\theta}$$

Assuming that a small thickness layer exists between the bottom of the sheet-flow layer and the top of the immobile bed, where frictional effects are dominant, the integration of Equation 9 renders (Ferreira, 2005)

$$20 \quad e_b = \frac{\tau_{bc}^{above} - \tau_{bc}^{below}}{\rho_c(u^{above} - u^{below})}$$

where e_b is the erosion rate and superscripts 'above' and 'below' refer to values above and below the interface between the sheet-flow layer and the immobile bed. Equation 20 is similar to the equation obtained by other authors assuming a discontinuity in the shear stress at the bed surface (Fraccarollo and Capart, 2002). τ_{bc}^{below} is a pure frictional stress and therefore it can be computed using Coulumb's law (Lambe and Whitman, 1979)

$$21 \quad \tau_{bc}^{below} = \sigma'_{bc} \tan(\phi_s)$$

where ϕ_s is the sediment static friction angle and $\sigma'_{bc} = \sigma_{bc} - \rho_w g h = \rho_w g (s-1) C_c h_c$ is the effective normal bed stress (at $z = z_b$) and σ_{bc} is the normal bed stress. This normal stress is obtained by subtracting the interstitial liquid hydrostatic pressure in Equation 11. τ_{bc}^{above} is a pure collisional stress; it can be computed with Equation 13. Assuming equilibrium sediment transport – that is, $e_b = 0$ – and replacing Equations 13 and 21 into Equation 20 one obtains

$$22 \quad C_c = \frac{R u^2}{g(s-1)h_c \tan(\phi_s)} = \frac{d_s \theta}{h_c \tan(\phi_s)} = \frac{1}{\beta \tan(\phi_s)}$$

From Equation 22, it can be concluded that the sediment concentration in the sheet-flow layer is constant and depends only on the type of sediments. Some authors have used this assumption to model geomorphic dam-break flows (Fraccarollo and Capart, 2002; Spinewine and Zech, 2007).

Taking into consideration the closure Equations 17, 19 and 22 it can be easily concluded that the sediment transport rate $C_c u_c h_c$ is proportional to $\theta^{3/2}$ and therefore proportional to u^3 . This result has been reported in several studies on sediment transport under unsteady conditions (Asano, 1995; Ribberink and Al-Salem, 1990). As $u_c \leq u$ and $C_c \leq C_b = 1 - p$, the closure Equations 19 and 22 yield the following limits

$$23 \quad R \leq \frac{49}{10\beta^{3/2}}$$

$$24 \quad \beta \geq \frac{1}{(1-p)\tan(\phi_s)}$$

6. VALIDITY OF THE MODEL FOR GEOMORPHIC DAM BREAK FLOWS

To verify the validity of some of the hypotheses adopted in the derivation of the stress tensor and closure equations, it is important to estimate the value of some dimensionless parameters that characterise debris flow and others that characterise fluvial flows. As the clear water layer has no sediment, the parameters will only be analysed in the sheet-flow layer. The parameter that represents the ratio between collisional stresses and grain friction stresses is the Savage number (Iverson, 1997)

$$25 \quad N_{Sav} = \frac{\rho_s d_s^2 (du_c/dz)^2}{(\rho_s - \rho_w) g C_c (h_c - z) \tan(\phi_s)}$$

The ratio between collisional stresses and liquid viscous stresses is the Bagnold number (Bagnold, 1954)

$$N_{\text{Bag}} = \frac{C_c \rho_s d_s^2 (du_c/dz)}{(1 - C_c) \mu}$$

where $\mu = 0.0001$ Pa s is the liquid dynamic viscosity. The ratio between the collisional stresses and the turbulent stresses is the mass number (Iverson, 1997)

$$N_{\text{mass}} = \frac{C_c \rho_s}{(1 - C_c) \rho_w}$$

The ratio between stresses originated by liquid-grain interactions and collisional stresses is the Darcy number (Iverson, 1997)

$$N_{\text{Dar}} = \frac{\mu}{C_c \rho_s k (du_c/dz)}$$

where k is the bed permeability. The ratio between turbulent stresses and liquid viscosity stresses is the Reynolds number of sediment

$$N_{\text{Rey}} = \frac{N_{\text{Bag}}}{N_{\text{mass}}} = \frac{\rho_w d_s^2 (du_c/dz)}{\mu}$$

The ratio between friction stresses and liquid viscous stresses is the friction number (Iverson, 1997)

$$N_{\text{fric}} = \frac{N_{\text{Bag}}}{N_{\text{Sav}}} = \frac{(\rho_s - \rho_w) g C_c^2 h_c d_s \tan(\phi_s)}{(1 - C_c) \mu (du_c/dz)}$$

One of the most important parameters in fluvial hydraulics is the Shields parameter θ that has been defined before and represents the ratio between the total bottom shear stress and the submerged weight of sediment particles. Another important parameter is the suspension parameter which represents the relation between the sediment settling velocity and the friction velocity and was defined by Batchelor (1965) as w_s/u_* (Sumer *et al.*, 1996).

To compute the values of the dimensionless parameters defined above it is necessary to establish the sediment properties. Table 1 presents those properties and the related coefficients for the two sediment types used in previous tests performed by the authors: natural sand and pumice (Leal *et al.*, 2010).

Type of sediment	d_s : mm	s	w_s	R	β	$\tan(\phi_s)$	k : m ²	p
Sand	0.77	2.65	0.99	0.007	7.0	0.49	10 ⁻⁹	0.4
Pumice	1.22	1.40	1.25	0.010	6.6	0.53	10 ⁻⁹	0.4

Table 1. Sediment properties and related coefficients

Table 2 presents the values of the dimensionless parameters computed with the sediment properties presented in Table 1, assuming that the gradient $du_c/dz \approx (u_c)_{z=z_c}/h_c$, where $(u_c)_{z=z_c} = 2.5 \sqrt{g(s-1)} (d_s \theta)^{-1/4} h_c^{3/4}$ is the velocity at the top of the sheet-flow layer given by Equation 18. To evaluate the frictional and collisional effects, two values of the elevation were used: one near the bottom ($z = z_b = 0$ m), where frictional effects should be relevant, and the other at the middle of the sheet-flow layer ($z = h_c/2$), where the collisional effects should dominate. In a dam-break wave, the Shields parameter θ varies from small values upstream to high values near the wave-front; therefore, two typical values were used: one small, $\theta = 0.8$, and another high, $\theta = 8.0$ (Leal, 2005).

Analysing the values presented in Table 2, one can conclude that, for high values of θ , collisional stresses dominate since $N_{\text{Sav}} > 0.1$ (Savage and Hutter, 1989), except near the bottom, where that parameter presents a small value. This confirms the adequacy of considering a purely collisional sheet-flow layer and the adoption of Coulomb's frictional law in the bottom. For the smaller value of θ , the frictional stresses are irrelevant, even near the bottom, and therefore the use of Coulomb's law may be inadequate. If $N_{\text{Bag}} > \approx 200$, the collisional stresses overcome the liquid viscous stresses (Bagnold, 1954). The N_{Bag} values reported in Table 2 are close to that limit, which means that viscous stresses can have some relevance. Nevertheless, near the bottom, those values are higher and therefore viscous stresses can be discarded. At the bottom, the values of N_{mass} are close to the values presented by Iverson (1997) for debris flow ($N_{\text{mass}} = 4$), meaning that, in this region, the grains inertia is more important than fluid inertia. The values of N_{Dar} are lower than the ones presented by Iverson (1997) (N_{Dar} ranging from 600 to 6×10^7), indicating that it is correct to neglect the stresses originated by liquid-grain interactions. As $N_{\text{Rey}} > 1$, the flow does not have a pure viscous behaviour and should exhibit turbulent fluctuations. The differences between the values of N_{fric} at the bottom and at the middle of the sheet-flow layer highlight the decrease of friction effects over viscous effects with the increase of the elevation. For lower values of θ , the suspension parameter satisfies the criteria of non-suspension, $w_s/u_* > 0.8$; this supports the assumption of a clear water layer and the non-consideration of turbulent effects in sediment suspension. For higher values of θ , typically near the wave front, that criterion is not verified and therefore it is not rigorously correct to discard the turbulent effects on sediment suspension. Summarising, the use of Coulomb's law seems more adequate for the wave-front region than for the upstream reach of the wave profile. The option of not considering turbulent effects on sediment suspension seems more realistic in the upstream reach than in the wave-front region. For the whole wave profile, it seems reasonable to neglect stresses originated by liquid-grain interactions.

	Sand				Pumice			
	Middle of sheet-flow layer		Near the bottom		Middle of sheet-flow layer		Near the bottom	
θ	0.8	8	0.8	8	0.8	8	0.8	8
h_c : m	0.004	0.043	0.004	0.043	0.006	0.064	0.006	0.064
z : m	0.002	0.022	0.000	0.000	0.003	0.032	0.000	0.000
C_c		0.29		0.6		0.31		0.6
du_c/dz : (s^{-1})	249	79	249	79	99	31	99	31
N_{sav}	19.56	0.20	4.75	0.05	9.84	0.10	2.54	0.03
N_{bag}	161	51	587	186	92	29	309	98
N_{mass}		1.09		3.98		0.63		2.10
N_{dar}	5	16	3	8	23	74	12	38
N_{key}	148	47	148	47	147	47	147	47
N_{fric}	8	260	123	3905	9	296	122	3853
w_s/u_*	0.99	0.31	0.99	0.31	1.23	0.39	1.23	0.39

Table 2. Dimensionless parameters

7. CONCLUSIONS

A two transport layers conceptual model for simulating geomorphic dam-break flows is presented. The use of results from granular flow, debris flow and sheet flow studies allowed the establishment of physically based closure equations. The parameters in these equations are dependent on the sediment properties and can be estimated easily, by reanalysing existing sheet-flow results (Sumer *et al.*, 1996).

Based on the analysis of the relevant dimensionless parameters, the model is expected to simulate adequately the friction at the wave front and the sheet flow throughout the wave profile. Nevertheless some limitations can be pointed out, namely the equilibrium sediment transport assumption, that can fail to simulate short-term morphological bed changes, and the assumption of a constant friction coefficient that can lead to wrong prediction of the wave-front position and height.

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