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Education and Inequality: A TANK Framework

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Abstract

Once considered to be the engine of upward mobility, the educational system has been criticized as a perpetuator of inequality. This paper revisits the discussion initiated in Stiglitz (1973), in the context of a Two-Agent New Keynesian (TANK) framework augmented, with human capital and innate ability uncertainty of lower income households. While the model predicts a decrease in inequality, measured by labour income gap, if government invests in poorer schooling districts, the outcome is less clear if it subsidizes higher education. It will depend on the quality of the latter. As it increases, the poorer households lower their beliefs of being admitted, rendering the schooling system more fitting to reproduce income inequality. Insofar as ability uncertainty decreases with one´s educational achievements, the model calls for higher government spending on mandatory education of lower income agent.

Keywords: Education, Inequality, Human capital, Fiscal policy and taxation, Uncertainty

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1 Introduction

"Although it has been argued that the educational system is not the cause of inequality, the difficult question remains: is it possible [...] to use the educational system to promote greater equality?"

(Stiglitz, 1973, p. 145)

Income inequality has been a rising concern in developed countries, particularly since the 1950s. Economists have blamed it on globalization (Bourguignon, 2015; Kremer and Maskin, 2006; Milanovic, 2016), lack of redistributive fiscal policies (Piketty, 2014; Lustig, 2018) and skill-biased technological change (Autor et al., 2003; Katz and Autor, 1999), among others. An important root that may be amplifying the latter is the educational system. Once viewed as the central pillar of upward mobility, it has been criticized as a perpetuator of inequality and more so since the COVID-19 crisis.

Such discussion can be traced back at least to Stiglitz (1973), the foundation of this work. The author´s arguments are twofold. First, public spending on elementary and secondary education varies across school districts. To the extent that those better-off attend the high and the poorer attend low expenditure schools, then system may perpetuate inequality. On the other hand, choosing to attend university is a risky decision. Generally, it is a long haul until most of returns are materialized and these rest upon achievement of the degree, meaning individuals cannot insure against such risk. Insofar as poorer households have higher risk and are less informed about their talent, they shall demand less education than richer ones. Hence, there is room for government intervention.

Even so, most of macro modelling has focused on a single forward looking representative agent, hereby denominated as unconstrained, who smooths consumption to respect the Euler path. Only following the influential paper by Mankiw and Campbell (1989), where authors estimated a fraction of 0.4-0.5 of US population who instead consumed according to its income, did economists begin incorporating a second type of agent, the spender.[1] Allied to the theoretical framework first laid out by Kydland and Prescott (1982) and several Keynesian features, such as price rigidities as well as

1. Also referred to as hand-to-mouth, rule-of-thumb, etc.

monopolistic competition, a revolutionary model birthed: the Two-Agent New Keynesian (TANK). The latter has emerged as a core tool for macroeconomic modelling and policymaking.

Since the discussion carried in Stiglitz (1973) relies on certain philosophical presumptions, the core aim of this paper is to obtain more palpable results by studying it in light of a TANK model, extended for human capital.[2] In particular, it seeks to answer the aforementioned question. Human capital accumulation builds on Blankenau and Simpson (2004),[3] though augmented with innate ability uncertainty of spender households, following the lines of Costa and Maestri (2007). The immediate implication of the latter is a lower demand for education by such agent, *ceteris paribus*, rendering too little investment on human capital. Hence, its law of motion is defined by the education effort and/or level of public education expenditures, as well as productivity of each. Whether it is a combination of both depends on the scenario under examination, as the model distinguishes between public investment on elementary/secondary and higher education. Additionally, it is implicitly assumed each agent attends the former in the district he/she resides, consistent with education legislation.

If state education funds are channelled towards lower income schooling districts, then inequality, measured by labour income gap, unmistakably decreases. The outcome is less straightforward if government invests on universities, for these are freely accessible to every household. Instead, it will depend on the quality of higher education. If it is such that spender households no longer believe it is attainable, then schooling system perpetuates inequality. Insofar as ability uncertainty decreases with one´s educational achievements (Höhne and Zander, 2019b; Marsh and O´Mara, 2008), then these findings call for higher public investment on primary and secondary education of spender households.

The secondary aim is to contribute to the ongoing literature exploring the link between government

2. According to (World Bank, 2019, p. 50), human capital is defined as "*the knowledge, skills, and health that people invest in and accumulate throughout their lives (...) and this is key to ending extreme poverty and creating more inclusive societies*."

3. Blankenau and Simpson (2004) develop an overlapping generations model of growth to explore the link between public education expenditures and human capital, thus distinguishing between direct and indirect effects. The former depends on the productivity of state schooling investment, while the latter varies mainly with level of spending and method of finance. The authors find an increase in consumption tax to be preferred, in line with Voyvoda and Yeldan (2000). However, this finding is at odds with Annabi et al. (2011) and Dissou et al. (2016), which advocate a reallocation of public expenditures and rise in lump-sum taxation, respectively.

spending on education and human capital (Annabi et al., 2011; Blankenau and Simpson, 2004; Dissou et al., 2016; Voyvoda and Yeldan, 2000). While studies at the micro level document a significant relationship,[4] the evidence is less clear at the macro level due to stronger interplay between direct (DE) and indirect effects (IE) on schooling effort.[5] Hence, the model dissects the two. The former induces agents to study with magnitude conditional on the level of fiscal spending as well as quality of state schooling institutions. On the other hand, the latter deters agents by increasing schooling opportunity cost. In line with the literature, the size of IE chiefly depend on financing method of government expenditures and work via two channels: labour and credit market. Under reasonable calibration, the model predicts an increase in aggregate human capital regardless of the education cycle invested in, consistent with micro evidence. This result is dampened (amplified) if government expenditures are financed via debt (taxation), as it strengthens (weakens) general equilibrium effects.

Related Literature. The literature supporting the role of schooling institutions in perpetuating inequality is not new. By shaping how students define themselves and the consequent institutional track chosen, the educational system can act as a relevant source of the access gap to academic and career opportunities (Bourdieu and Passerson, 1977; Croizet et al., 2019; Tilly, 1999).

Research has evidenced relationship between uncertainty and schooling demand (Cundiff et al., 2013; Woodcock et al., 2012; Walton et al., 2015; Höhne and Zander, 2019a). Albeit less extensive, some studies have explored the roots of uncertainty (Höhne and Zander, 2019b; Walton and Cohen, 2007). In particular, Höhne and Zander (2019b) focus on one´s perception of its academic aptitude, which is linked to students´ achievements (Marsh and O´Mara, 2008; Valentine et al., 2004).

Greiner (2008) studies the impact of public education expenditures on stock of human capital under alternative methods of finance. The author demonstrates that an irresponsible fiscal policy, whereby government debt is not stabilized, has negative repercussions on human capital. On the other hand, in a model with uninsurable risks to human capital accumulation, Gottardi et al. (2015) find

4. See, for example, Card and Krueger (1992).

5. Levine and Renelt (1992) do not find a robust correlation between public education expenditures and economic growth. On the contrary, Barro and Sala-i-Martin (2004) support a positive relationship. In a review of empirical studies, Lin (1994) corroborates with the latter in the short-run and with the former in the intermediate-run (25 years).

desirable to issue debt whenever government expenditures are small enough. However, if fiscal authorities must have a balanced budget every period, a labour income tax is preferred for it reduces risk. This paper contributes to the literature by resorting to a dynamic stochastic general equilibrium (DSGE) framework and differentiating human capital accumulation across agents.

Outline. The rest of the paper is structured as follows. Section 2 presents the main features of the model. The key results shall be derived in Section 3, from which a potential answer to Stiglitz (1973) will follow. These are then discussed in Section 4 from an efficiency perspective. Section 5 concludes.

2 A Two-Agent New Keynesian Model

The economy consists of two types of households: The unconstrained and the spender. The crucial distinction between the two lies in the ability of the former to smooth consumption over time. Hence, the model allows for physical capital accumulation, a form of savings by the unconstrained agent. The latter, on the other hand, consumes according to his labour income, where wages are set by unions on behalf of workers and hours worked are demand-determined. In addition, human capital investment decision is endogenized and influenced by public education expenditures.

On the other hand, the production side consists of a perfectly competitive firm producing a single final good whose inputs are provided by a continuum of monopolitiscally competitive firms producing differentiated goods, using effective labour and physical capital. The latter face price rigidities as in Calvo (1983). The policy block of the model includes a government spending on public education by raising (lump-sum) taxes and debt, and a central bank setting the nominal interest rate.

The model builds on Galí, López-Sallido and Valles (2007; GLV (2007) henceforth), although extended for nominal wage rigidities, along the lines of Colciago (2011).[6] The main novelty of this paper is the presence of human capital accumulation.

6. The motivation lies in the high volatility of schooling effort decision under a flexible wage setting environment.

2.1 Households

The economy is populated by a unit mass of infinitely lived households, indexed by $i \in [0, 1]$, who live in a cashless economy.[7] It consists two types of assets: bonds and capital (physical and human), and two groups of agents: Unconstrained (U) and Spender (S), where latter has fraction λ of the population.

The preferences of households are characterized by a Constant Relative Risk-Aversion (CRRA) utility function. It is increasing in consumption C_t^i but decreasing in working hours N_t,

$$\mathcal{U}(C_t^i, N_t) = \frac{(C_t^i)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{1}$$

where the i index in N_t is dropped for every household is assumed to work the same amount of hours. The parameter σ measures the inverse intertemporal elasticity of substitution and φ is the inverse Frisch elasticity. Throughout, it is imposed $\varphi > 0$, such that the labour supply positively sloped. For tractability purposes, both are common across households.

Unconstrained Households

The unconstrained agent discounts future utility (1) by factor $\beta \in [0, 1]$. It will seek to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^U, N_t) \tag{2}$$

subject to a sequence of flow budget constraints,[8]

$$P_t C_t^U + P_t I_t^U + R_t^{-1} B_{t+1}^U = W_t N_t H_t^U + P_t D_t + R_t^k K_t^U + B_t^U - P_t T_t \tag{3}$$

7. The only role of money is as unit of account.

8. Although not introduced explicitly, in the background the No-Ponzi game and transversality condition are imposed.

as well as physical and human capital accumulation

$$K_{t+1}^U = (1-\delta_k)K_t^U + \phi\left(\frac{I_t^U}{K_t^U}\right)K_t^U \tag{4}$$

$$H_{t+1}^U = (1-\delta_h)H_t^U + \left(X_t^U\right)^{v_u}(H_t^U)^{1-v_u} \tag{5}$$

where B_t denotes nominal risk-free bond holdings carried over from period $t-1$ and paying one unit of the numéraire in period t. The agent´s hourly nominal wage is given by W_t, while dividends are denoted by D_t and T_t represents real taxes. Physical capital K_t^U is solely owned by agent U who supplies it to firms at the nominal rental price, rendering him an income of $R_t^k K_t^U$. It evolves according to (4), following GLV (2007), where $\phi\left(\frac{I_t^U}{K_t^U}\right)K_t^U$ is the physical capital adjustment cost. Throughout it is assumed $\phi' > 0$, $\phi'' \leq 0$.

Furthermore, human capital is endogenized and accumulates according to effort (or time) devoted to education, X_t^U. Its law of motion is defined by (5) where $v_u \in [0,1]$ captures the productivity of education effort and its decreasing marginal returns. Furthermore, the depreciation rate of capital assets is defined by $\delta_k, \delta_h \in [0,1]$.

The optimality conditions read as,

$$B_{t+1}^U : 1 = R_t E_t\{\Lambda_{t,t+1}\} \tag{6}$$

$$K_{t+1} : P_t Q_{k,t} = E_t\left\{\Lambda_{t,t+1}\left[R_{t+1}^k + P_{t+1}Q_{k,t+1}\left((1-\delta_k) + \phi_{t+1} - \left(\frac{I_{t+1}^U}{K_{t+1}^U}\right){\phi'}_{t+1}\right)\right]\right\} \tag{7}$$

$$H_{t+1}^U : P_t Q_{h,t}^U = E_t\left\{\Lambda_{t,t+1}\left[W_{t+1}N_{t+1} + P_{t+1}Q_{h,t+1}^U\left((1-\delta_h) + (1-v_u)(X_{t+1}^U)^{v_u}(H_{t+1}^U)^{-v_u}\right)\right]\right\} \tag{8}$$

$$I_t : Q_{k,t} = \frac{1}{\phi'\left(\frac{I_t^U}{K_t^U}\right)} \tag{9}$$

$$X_t^U : P_t Q_{h,t}^U = \frac{W_t H_t^U}{v_u (X_t^U)^{v_u-1}(H_t^U)^{1-v_u}} \tag{10}$$

where $\phi_{t+1} \equiv \phi\left(\frac{I_{t+1}^U}{K_{t+1}^U}\right)$, $\phi'_{t+1} \equiv \phi'\left(\frac{I_{t+1}^U}{K_{t+1}^U}\right)$ and $\Lambda_{t,t+1} \equiv \beta\left(\frac{C_{t+1}^U}{C_t^U}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)$ is the stochastic discount factor for nominal payoffs. The Euler equation (6) is isomorphic to the representative agent economy. The real shadow value of capital, i.e. Tobin´s Q, is denoted by $Q_{k,t} \equiv \frac{\zeta_{k,t}}{\xi_t^U}$, where

$\zeta_{k,t}$ and ξ_t^U are the Lagrangian multiplier associated with (4) and (3), respectively. Its human capital counterpart is given by $Q_{h,t}^U \equiv \frac{\zeta_{h,t}^U}{\xi_t^U}$, where $\zeta_{h,t}^U$ is the Lagrangian multiplier for (5). The capital Euler equations are defined in, (7) and (8), respectively, stating that the shadow price must equal future expected discounted returns of each capital investment. On the other hand, (9) and (10) are the intratemporal equations of K_t^U and H_t^U. In particular, for the latter, the agent optimally decides education effort today by weighting the opportunity cost of studying (effective foregone wages) with its returns (increase in human capital).

Spender Households

The spender discounts future utility (1) by factor $\beta \in [0, 1]$. Their lifetime utility is defined by,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^S, N_t) \tag{11}$$

Given his restricted access to the credit market, the constrained agent must consume according to his budget constraint

$$P_t C_t^S = W_t N_t H_t^S - P_t T_t \tag{12}$$

where T_t denotes real taxes. The knowledge of the hand-to-mouth is described by H_t^S, which evolves according to

$$H_{t+1}^S = (1 - \delta_h) H_t^S + G_t^{\mu} \left(X_t^S\right)^{v_s} (H_t^S)^{1-v_s-\mu} \tag{13}$$

where $v_s \in [0, 1]$. Contrariwise to the unconstrained agent, the human capital of the spender evolves through a combination of effort X_t^S, and public investment in education G_t, where these are imperfect substitutes in producing it. An interpretation is that, in spite of individual effort being the main driver of human capital accumulation, the government can incentivize it by improving quality of education. To capture such feature, it is assumed $v_s > \mu$, where these parameters capture the pace at which time spent studying is accumulated into human capital and the quality of state schooling institutions, respectively. Both exhibit decreasing marginal returns.[9]

Following the lines of Costa and Maestri (2007), uncertainty arises in this economy since the hand-

9. See, for example, Mincer (1958).

to-mouth is not fully aware of its innate talent. The latter is measured v_s as it affects how quickly effort is accumulated into human capital. It may have high ability v_h with certain probability p or low ability v_l with probability $1-p$. Assuming the former equals that of the unconstrained agent, its expected ability is formally defined as: $v_s = pv_u + (1-p)v_l$ where $v_u > v_l$, such that $v_u > v_s$. Despite not optimally choosing the consumption path, spender households do maximize (11) subject to (12) and (13),

$$H_{t+1}^S : P_t Q_{h,t}^S = E_t \Big\{ \Lambda_{t,t+1} \left[W_{t+1} N_{t+1} + P_{t+1} Q_{h,t+1}^S \left((1-\delta_h) + (1-\mu-v_s) G_{t+1}^{\mu} (X_{t+1}^S)^{v_s} (H_{t+1}^S)^{-(\mu+v_s)} \right) \right] \Big\} \quad (14)$$

$$X_t^S : P_t Q_{h,t}^S = \frac{W_t H_t^S}{v_s G_t^{\mu} (X_t^S)^{v_s - 1} (H_t^S)^{1-\mu-v_s}} \quad (15)$$

The real shadow price of human capital of spender agent is $Q_{h,t}^S \equiv \frac{\zeta_{h,t}^S}{\xi_t^S}$, where $\zeta_{h,t}^S$ is the Lagrangian multiplier associated with (13) and ξ_t^S is the Lagrangian multiplier for (12). However, the spender does not have access to the credit market, hence (6) does not hold. Instead, he discounts future education returns according to β.

The optimality conditions demonstrate how the spender agent internalizes the level of public investment on education in its schooling effort decision. In particular from (15), it becomes clear how public spending generates two contradicting effects. It incentivizes spender households to raise time spent studying, by increasing its marginal productivity, captured by the denominator.[10] This is the direct (or partial equilibrium) effect (DE), as it directly affects demand for education X_t^S. On the other hand, it increases wages as a consequence of boosting the aggregate demand, captured by the numerator. Since the former raises opportunity cost of schooling, it deters agents from studying. This is the indirect (or general equilibrium) effect (IE), as it indirectly, through labour market, affects demand for education. Insofar as DE outweigh the IE, then schooling effort is increasing with public spending.

10. To see this: $\frac{d^2 H_{t+1}}{dX_t^U dG_t} = \frac{v\mu (H_t^U)^{1-v-\mu}}{G_t^{1-\mu} (X_t^U)^{1-v}} > 0$

2.2 Wage Schedule

Departing from a perfect labour market, it is assumed a continuum of of unions, indexed by $z \in [0, 1]$, each with bargaining power to set wages on behalf of the group of workers it represents. In addition, following Colciago (2011), nominal wage rigidities are introduced as each period only a fraction 1-θ_w of unions are able to reoptimize, independent of the time elapsed since last adjustment. As in GLV(2007), these set W_t^* to maximize a weighted average of agents´ net discounted utility obtained from working, during the k periods the union cannot reset wages (with probability θ_w^k). The weights are given by share of the group in population. Formally, it solves

$$\max_{W_t^*} E_t \sum_{k=0}^{\infty} (\theta_w \beta)^k \left\{ \lambda \left[\frac{1}{(C_{t+k}^S)^\sigma} W_t^* H_{t+k}^S N_{t+k}(z) \right] + (1-\lambda) \left[\frac{1}{(C_{t+k}^U)^\sigma} W_t^* H_{t+k}^U N_{t+k}(z) \right] - \frac{N_t^{1+\varphi}(z)}{1+\varphi} \right\}$$

subject to a labour demand schedule

$$N_{t+k}(z) = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\epsilon_w} N_{t+k}$$

as well as (3) and (12), where ϵ_w is the elasticity of substitution between labour types. Since consumption typically differs across households, the union weighs the marginal utility of consumption of each agent $[1/(C_{t+k}^j)^\sigma]$ for $j = U, S$. Given that firms allocate labour demand uniformly across different groups of workers, the z index can be dropped. Hence, the optimization problem yields the wage inflation curve,

$$\pi_t^w = \beta E_t \{\pi_{t+1}^w\} - \lambda_w \hat{\mu}_t^w \tag{16}$$

where $\lambda_w \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}$ and $\hat{\mu}_t^w = (\hat{w}_t - \hat{p}_t) + \hat{h}_t - (\hat{c}_t + \varphi \hat{n}_t)$.

2.3 Firms

The supply side of the economy consists of a final and intermediate goods firms. The former produces a single final good in a perfectly competitive market, using as inputs the goods produced by the latter under a monopostically competitive setting.

Final Goods Firm

The perfectly competitive firm transforms the goods produced by the intermediate sector $Y_t(j)$, for $j \in [0,1]$, into a single final one Y_t, according to a constant returns to scale (CRS) production function

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{17}$$

where ϵ_p denotes elasticity of substitution between goods. Taking the price of intermediate goods $P_t(j)$ as well as the final goods price P_t as given, the profit maximization yields the set of demand schedules

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_p} Y_t \tag{18}$$

where $P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_p} \right)^{\frac{1}{1-\epsilon_p}}$.

Intermediate Goods Firm

The intermediate sector is composed of a continuum of monopolistically competitive firms, each producing a differentiated good, indexed by $j \in [0,1]$, with CRS technology: $Y_t(j) = K_t(j)^{\alpha} Z_t(j)^{1-\alpha}$ where $Z_t(j) \equiv N_t(j) H_t(j)$ is the effective labour input hired by firms, and total factor productivity is normalized to unity.

The firms´ decision is twofold. It consists of choosing the cost-minimizing bundle of labour and capital and, in a second stage, setting a price as means to maximize profits. Departing from the former, taking wage and rental cost of capital as given, yields the optimality condition

$$\frac{K_t(j)}{Z_t(j)} = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{W_t}{R_t^k} \right) \tag{19}$$

From (19) it becomes clear how every firm will choose the same capital to effective labour ratio. Hence, real marginal cost is common across firms and given by,

$$MC_t = \frac{1}{\Phi} \left(\frac{R_t^k}{P_t} \right)^{\alpha} \left(\frac{W_t}{P_t} \right)^{1-\alpha} \tag{20}$$

where $\Phi \equiv \alpha^\alpha(1-\alpha)^{1-\alpha}$.

Following the market power owned by intermediate goods firms, these set price above marginal cost to achieve the desired markup, albeit at the expense of an inefficiently low level of output.[11] Each firm faces same demand (18) and takes aggregate price level P_t as well as consumption index $C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ as given. Hence, in equilibrium, every reoptimizing firm will choose the same price. Analogously to wages, price rigidities *à la* Calvo (1983) are introduced, as only a fraction 1-θ_p of randomly selected firms reoptimizes each period.

Solving the firms optimization problem obtains the price inflation curve,

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p \hat{\mu}_t^p \tag{21}$$

where $\lambda_p \equiv \frac{(1-\theta_p)(1-\theta_p\beta)}{\theta_p}$ and $\hat{\mu}_t^p = -\hat{mc}_t$.

2.4 Fiscal Policy

The government has education outlays G_t, financed by (lump-sum) taxation T_t and bonds B_t. The government budget constraint reads as

$$P_t T_t + R_t^{-1} B_{t+1} = B_t + P_t G_t \tag{22}$$

where taxes evolves according to

$$\hat{t}_t = \phi_b \hat{b}_t + \phi_g \hat{g}_t \tag{23}$$

and variables have been normalized by steady state GDP: $\hat{t}_t = \frac{T_t - T}{T}$, $\hat{b}_t = \frac{B_t/P_t - B/P}{Y}$ and $\hat{g}_t = \frac{G_t - G}{Y}$. Government expenditures follow an AR(1) process: $\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^{MP}$, with shock persistence ρ_g. The parameters ϕ_b and ϕ_g measure the elasticity of taxes to government debt and spending, respectively. Setting $\phi_g = 1$, renders the latter fully tax-financed.

11. Typically in the literature it is assumed an employment subsidy to ensure marginal cost pricing and, consequently, zero-profits in equilibrium. We shall not pursuit it to allow hours worked and wages to co-move positively.

The log-linearization of (22) combined with (23) obtains debt dynamics equation

$$\hat{b}_{t+1} = (1+\rho)\left[(1-\phi_b)\hat{b}_t + (1-\phi_g)\hat{g}_t\right] \tag{24}$$

where it is assumed $\phi_b > \frac{\rho}{1+\rho}$ to ensure stationary debt path.

2.5 Monetary Policy

Following Auclert et al. (2020), the central bank sets the nominal interest rate $\hat{r}_t$ according to,

$$\hat{r}_t = (1-\rho_r)\,\phi_\pi \pi_t + \rho_r \hat{r}_{t-1} + v_t \tag{25}$$

where $\hat{r}_{t-1}$ is the smoothing parameter, with weight ρ_r. In addition, v_t denotes the monetary policy shock, following an AR(1) process: $v_t = \rho_v v_{t-1} + \varepsilon_t^{MP}$.

2.6 Market Clearing

The goods market clears when $Y_t = C_t + I_t + G_t$, where $C_t \equiv \lambda C_t^S + (1-\lambda)C_t^U$ and $I_t \equiv (1-\lambda)I_t^U$, while the labour market is in equilibrium when $N_t = \int_0^1 N_t(j)dj$. Furthermore, market clearing for physical and human capital requires $K_t = \int_0^1 K_t(j)dj$ and $H_t = \int_0^1 H_t(j)dj$.

3 Results

The following section consists of log-linearizing the optimality conditions around the steady state, as well detailing the parametrization, from which the main results of the model shall be derived (see appendix A.1 for all derivations).[12]

12. The hat notation denotes the log deviation of the variable from steady state. Formally, $\hat{z}_t \equiv \ln Z_t - \ln Z \simeq \frac{Z_t - Z}{Z}$ where Z_t is a vector of variables.

3.1 Calibration

The baseline parametrization is close to GLV (2007) while the remaining parameters follow human capital literature. The length of each period is one quarter.

Departing from the parameters which shall remain unchanged, the discount factor β is equal to 0.99. The steady state average price markup μ^p is set to 0.2 and the elasticity of output with respect to capital, α, to 1/3. The depreciation rate of physical δ_k and human capital δ_h are equal to 0.025 and 0.01, respectively. In addition, the degree of price θ_p and nominal wage rigidity θ_w are both calibrated to 0.75, rendering an average price and wage duration of one year. It is assumed a log-utility, $\sigma = 1$, and an elasticity of investment with respect to Tobin´s Q, η, equal to 1. Government spending share of GDP γ_g is set to 0.2 and the fraction of hand-to-mouth households, λ, is set to 1/2, consistent with the range estimated in Mankiw and Campbell (1989). Turning to the policy parameters, ϕ_b is equal to 0.33 with a shock persistence ρ_g of 0.9. Finally, ϕ_π is set to 1.5, thus satisfying the Taylor principle ($\phi_\pi > 1$), and the weight of smoothing parameter ρ_r is calibrated to 0.89, following Auclert et al. (2020).

The remaining parameters are set to their baseline value but are subject to a sensitivity analysis. The elasticity of human capital with respect to public investment on education μ and effort of unconstrained agent v_u is equal to 0.1 and 0.75, respectively. The spender agent has an expected innate ability v_s equal to 0.55, yielding an aggregate average elasticity of 0.65.[13] These values lie well within the range calibrated in the literature (see Annabi et al., 2011; Dissou et al., 2016). The responsiveness of taxation to government spending ϕ_g is calibrated to 0.1. Finally, the inverse Frisch elasticity φ is equal to 1, thus departing from GLV (2007) parametrization, as it is more in line with the literature.

Determinacy and Taylor Principle As documented in Colciago (2011), the Taylor principle is both sufficient and necessary to guarantee equilibrium determinacy, under reasonable parametrization. The model is always calibrated to satisfy such condition.[14]

13. Under a probability p equal to 0.5, the low-type ability v_l is equal to 0.35, slightly above Blankenau and Simpson (2004) calibration.

14. For a full discussion see, for example, Bilbiie (2008), Colciago (2011), GLV (2003).

3.2 Fiscal Policy

The following section focuses on education inequality dynamics following an increase in public investment on two schooling cycles: i) Elementary/Secondary; ii) Higher education. Consistent with education laws, it is implicitly assumed that households must attend the former in the district they reside. In particular, public investment on mandatory education is channelled towards the districts chiefly populated by the lower-income group, agents S, where the share of the unconstrained agent is negligible and thereby ignored. On the other hand, state universities are freely accessible to every household.[15] Formally, education inequality is defined as: $\Gamma_t = \hat{h}^U_{t+1} - \hat{h}^S_{t+1}$.[16,17] Since wages and labour are not discriminated across types, henceforth it shall also be referred to as labour income inequality.

3.2.1 Elementary and Secondary Education

In the following subsection, government spending is channelled towards mandatory education of spender agent. Hence, public spending only enters in the human capital accumulation of the spender, in line with baseline model outlined in previous section. Notwithstanding, to pin down the overall impact on inequality Γ_t, it is necessary to understand the drivers behind education effort decision $\hat{x}_t$ of each agent.

General and Partial Equilibrium Effects. Public investment on education produces two contradicting effects on human capital accumulation of spender households. On the one hand, it creates incentives to become more knowledgeable by improving quality of education. This captures the DE, or partial equilibrium effect. On the other hand, it creates inflationary pressures by boosting aggregate demand and, in turn, wages. Since the latter raises opportunity cost of schooling, it deters agents from studying. This captures the IE, or general equilibrium effect. Proposition 1 states the conditions for which the former dominates, such that human capital of agent S rises.

Proposition 1. *Public spending on mandatory education increases spenders stock of human capital*

15. Alternatively, one may interpret it as a secondary, or elementary school, whose student body is representative of both groups.

16. Indeed human capital is, by definition, broader than education, for it encompasses on job training, health, etc. For the purposes of this paper, it solely refers to education.

17. Since $\hat{h}_t$ is a predetermined variable, it reacts to the shock with a one-period lag.

$\hat{h}^S_{t+1}$ *if, and only if,*[18]

$$\underbrace{\frac{\mu}{\gamma_g}}_{Partial\ Equilibrium} > \underbrace{v_s \frac{d\hat{\omega}_t}{d\hat{g}_t}}_{General\ Equilibrium} \tag{26}$$

where $\hat{\omega}_t \equiv (\hat{w}_t - \hat{p}_t)$ denotes the real wage. The partial equilibrium effect is rising with the elasticity of $\hat{h}^S_{t+1}$ with respect to public investment in education, μ, for it increases human capital response as well as the marginal productivity of time allocated to studying. The net effect of v_s, however, is less straightforward. On the one hand, it fastens the pace at which schooling effort is accumulated into human capital, but on the other it amplifies IE, as seen in (26). The overall impact will depend on the strength of DE relative to IE. Under reasonable parametrization, the former is generally stronger, such that $\hat{h}^S_{t+1}$ is increasing with v_s.[19,20] Therefore, as long as agent S is more uncertain of its innate ability ($v_s < v_u$), it demands less education than unconstrained households, *ceteris paribus*. This represents the uncertainty effect. Importantly, notice the DE is independent of how expenditures are financed and labour market assumptions. Instead, these are reflected on the general equilibrium effects which, under nominal wage rigidities, render the level of public investment in education $\hat{g}_t$ the main driver of $\hat{h}^S_{t+1}$. In addition, the IE are increasing with the government spending share of GDP γ_g, as it amplifies the aggregate demand response.

The unconstrained agent, on the other hand, is not subject to said direct effect, but the general equilibrium effect is magnified for it discounts education returns according to the real interest rate. When the latter is raised by the central bank, in response to fiscal policy generated inflationary pressures, it lowers the present value of future expected education returns. Hence, demand for schooling falls, *ceteris paribus*, a finding consistent with Dellas and Koubi (2003) and Kar (2013). A condition analogous to proposition 1 is thereby obtained.

Proposition 2. *Public spending on mandatory education increases savers stock of human capital*

18. For a proof of the following propositions and corollaries, see appendix A.2-A.4.
19. This finding is at odds with Blankenau and Simpson (2004), but is consistent with Dissou et al. (2016).
20. Under wage flexibility, the net effect is less straightforward as wage response is amplified. In fact, under particular parametrizations, it becomes non-linear over time, for $\hat{h}^S_{t+1}$ is initially decreasing with v_s but increasing afterwards. See appendix A.2 for a complete discussion.

$\hat{h}^U_{t+1}$ *if, and only if,*

$$\underbrace{0}_{Partial\ Equilibrium} > \underbrace{v_u \left(\frac{d\hat{\omega}_t}{d\hat{g}_t} + \frac{d\hat{rr}_t}{d\hat{g}_t} \right)}_{General\ Equilibrium} \tag{27}$$

where $\hat{rr}_t \equiv \hat{r}_t - E_t\{\pi_{t+1}\}$ is the real interest rate. Insofar as wages and the real rate react positively to a government spending shock, (27) does not hold, rendering public investment in education of spender households to deter the unconstrained agent from studying. Notice, however, this is not a general result, as the most plausible scenario would be $\hat{h}^U_{t+1}$ remaining constant, or even slightly increasing. Instead, proposition 2 stems from zero participation assumption of agent U in elementary and secondary schools invested by the state.

The effect of v_u on $\hat{h}^U_{t+1}$ is fairly straightforward since the DE are muted. The human capital of uncostrained households is falling with v_u as it renders its schooling decision more sensitive to a unit change in IE.[21] The condition for which state-financed schooling is labour income inequality reducing is outlined in corollary 1.

Corollary 1. *Public spending on mandatory education decreases labour income inequality* Γ_t *if, and only if,*

$$\frac{\mu}{\gamma_g} > \frac{1}{1-v_u} \left((v_s - v_u) \frac{d\hat{\omega}_t}{d\hat{g}_t} - v_u(1-v_s) \frac{d\hat{rr}_t}{d\hat{g}_t} \right) \tag{28}$$

The latter demonstrates the three effects shaping Γ_t. On the one hand, the DE boost education effort of agent S while the central bank response deters that of unconstrained households. Clearly, both diminish labour income inequality. On the other hand, there is the uncertainty effect which increases Γ_t. The spender agent is less certain of its innate ability, thus demands less education than agent U, *ceteris paribus*, as discussed in proposition 1. To the extent that (26) holds, but (27) does not, condition (28) is true as Γ_t falls unambiguously. Under reasonable parametrization, corollary 1 always holds.

Quantitative Results. Figure 1 exhibits the response of the main macroeconomic and education variables to a government spending shock on mandatory education of agent S. As the schooling decision of spender households mainly depends on level of public education expenditures, it opti-

21. However, this is not central to the analysis as it shall focus chiefly on v_s.

mally devotes more effort. On the other hand, both general equilibrium channels deter agent U from studying, while the direct effect is absent. Therefore, its time allocated to education is crowded-out. Clearly, fiscal policy decreases labour income inequality, while the aggregate stock of human capital in the long-run rises. Furthermore, the model complies with *Lucas´ less famous Critique*, as it is reflected in wages acyclical behaviour.[22]

The macroeconomic variables path is standard albeit muted by nominal wage rigidity. Stemming from the presence of rule-of-thumb consumers and price rigidity, consumption has an initial positive response.[23] The former has a high marginal propensity to consume, hence reacts strongly to a rise in labour earnings. The latter allows wages to increase, even if marginal product of labour falls, as long as the price markup $\hat{\mu}_t^p$ falls sufficiently. Investment response tracks closely Tobin´s Q. Since government intervention decreases the latter, private investment is crowded-out.

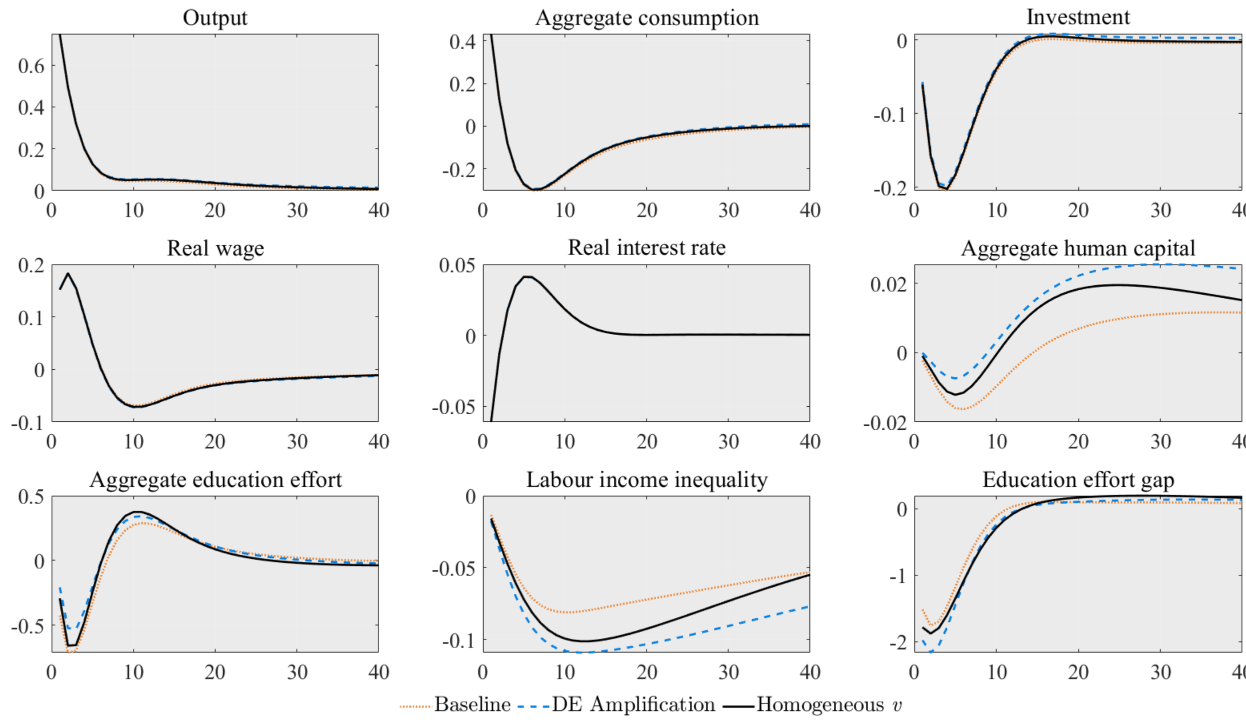
Given the statistical uncertainty of human capital elasticities estimates, the robustness of the results are studied under two alternative parametrizations. Without innate ability uncertainty of agent S ($p = 1$), implying $\upsilon_s = \upsilon_u = 0.75$, and amplification of DE by increasing quality of state schooling institutions μ. In particular, the latter is set to 0.18, following Blankenau et al. (2007). Since both render effort of the spender households more productive, the drop in schooling effort gap and, consequently, Γ_t is amplified, while simultaneously increasing aggregate human capital.

In what follows, an alternative method of financing $\hat{g}_t$ is analysed. In particular, it is momentarily assumed to be fully tax-financed ($\phi_g = 1$). As shown in Figure 4 in Appendix B, the impact is not so straightforward. Under baseline scenario, taxation is delayed into the future, while in the alternative taxes reach their peak today and then gradually fall. Hence, the IE are initially higher in the former calibration. Since these impact more $\hat{h}_{t+1}^U$ than $\hat{h}_{t+1}^S$, implied by ability uncertainty and credit market channel (see proposition 2), labour income inequality is lower as well. Only as taxation increases, eventually becoming higher than in a tax-financed scenario, do the IE become

22. Lucas (1977, p. 226) states that: "*observed real wages are not constant over the business cycle, but neither do they exhibit consistent pro- or countercyclical tendencies.*" For a full discussion see, e.g., Christiano and Eichenbaum (1992), Bilbiie and Straub (2004).

23. Evidence from SVAR suggests a strong and persistent increase in consumption following an expansionary fiscal policy shock (see, for instance, Blanchard and Perotti, 2002; Fatás and Mihov, 2001; GLV, 2007). However, DSGE models, in particular the real business cycle, have struggled to replicate such finding.

Figure 1: Government spending shock on mandatory education of spender agent



Note: The graph shows the impulse response function after a shock to government spending on elementary/secondary education of spender households. Three scenarios are distinguished: Baseline calibration, DE amplification of public expenditures ($\mu = 0.18$) and absence of uncertainty ($p = 1$).

lower (8 quarters in the graph). At that point, labour income inequality becomes higher in the baseline scenario. In sum, the initial increase in Γ_t, relatively to the debt-financing scheme, is only temporary. In the long-run, it is permanently lower under tax-financed public spending.

Figure 4 also displays the impact of such taxation path on aggregate stock of human capital. The difference is significant. It boosts incentives to attend school of both agents, by dampening IE, rendering the aggregate stock of human capital unambiguously higher than in a debt-financing scenario.

Furthermore, the model assigns considerable general equilibrium labour market effects in shaping schooling decision, as documented in Eckert and Kleineberg (2019) and Khanna (2015). To demonstrate this, labour supply is momentarily assumed to be elastic by setting φ equal to 0.5.

As the latter reduces the marginal disutility of working, unions need not to raise wages as much. Lower opportunity cost of education implies dampening of IE, culminating in an increase in Γ_t and stock of human capital, as shown in Figure 5 in Appendix B. An inelastic labour supply produces the opposite effect. Since empirical evidence documents a higher Frisch elasticity for poor income households,[24] implying a lower wage relatively to unconstrained households, it may imply the hitherto labour income inequality predicted by the model overestimated.

Policy Implications. The discussion in this section illustrated one important result for fiscal policy. In light of this model, the more spillover effects a unit increase in $\hat{g}_t$ has to the overall economy, the more likely it is that labour income inequality Γ_t falls. Importantly, this is independent of the type of education invested. Instead, this finding hinges on IE playing a larger role in schooling decision of unconstrained households, as discussed in proposition 2, although this is empirically debatable.[25] On the reverse side of the coin, a government attempting to maximize aggregate human capital should minimize IE, consistent with the literature.

3.2.2 Higher Education

The previous analysis concluded that it is possible to diminish inequalities through state-financed education. However, it hinged on government channelling those funds towards the spender agent. In what follows, it is considered public investment on universities, which do not discriminate between type. Human capital of the unconstrained agent now evolves according to,

$$H^U_{t+1} = (1-\delta_h)H^U_t + G^\mu_t \left(X^U_t\right)^{v_u} (H^U_t)^{1-v_u-\mu} \tag{29}$$

Solving the optimization problem subject to (3) and (29) obtains

$$H^U_{t+1} : P_t Q^S_{h,t} = E_t \left\{ \Lambda_{t,t+1} \left[W_{t+1}N_{t+1} + P_{t+1}Q^S_{h,t+1} \left((1-\delta_h) + (1-\mu-v_u) G^\mu_{t+1} (X^U_{t+1})^{v_u} (H^U_{t+1})^{-(\mu+v_u)} \right) \right] \right\} \tag{30}$$

$$X^U_t : P_t Q^S_{h,t} = \frac{W_t H^U_t}{v_u G^\mu_t (X^U_t)^{v_u-1} (H^U_t)^{1-\mu-v_u}} \tag{31}$$

24. See, for example, Juhn et al. (2002), McClleland and Mok (2012).

25. As documented in Beegle et al. (2006) and Kruger (2012), shocks to economic activity tend to affect more the time allocation set by the poor.

Under such setup, the saver internalizes the level of public investment on education into its optimal effort decision. As a result, proposition 2 is generalized.

Proposition 2´. *Public spending on higher education increases savers stock of human capital* $\hat{h}^U_{t+1}$ *if, and only if,*

$$\underbrace{\frac{\mu}{\gamma_g}}_{Partial\ Equilibrium} > \underbrace{v_u\left(\frac{d\hat{\omega}_t}{d\hat{g}_t}+\frac{d\hat{r}r_t}{d\hat{g}_t}\right)}_{General\ Equilibrium} \tag{32}$$

Comparing (32) with (27) illustrates the presence of the hitherto muted DE of $\hat{g}_t$ on human capital of agent U. Hence, labour income inequality Γ_t may no longer decrease, as discussed in corollary 1´.

Corollary 1´. *Public spending on higher education decreases labour income inequality* Γ_t *if, and only if,*

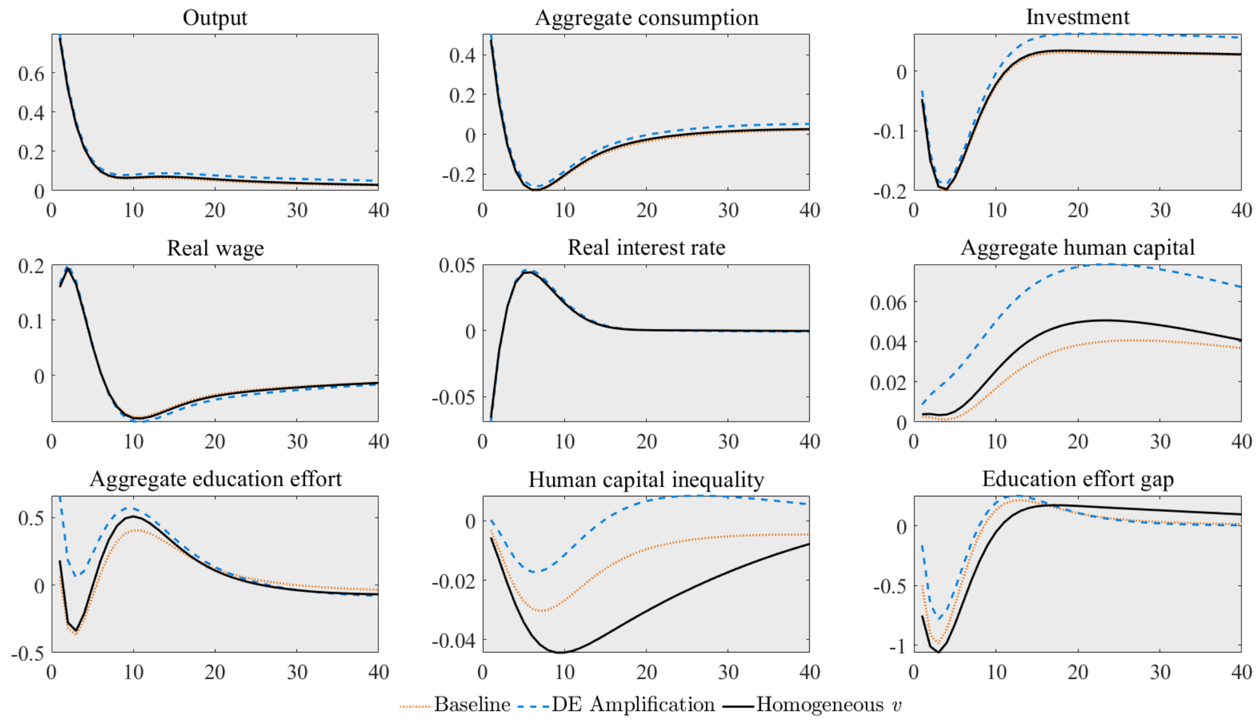
$$\frac{\mu}{\gamma_g} > \frac{d\hat{\omega}_t}{d\hat{g}_t} - \tilde{v}\frac{d\hat{r}r_t}{d\hat{g}_t} \tag{33}$$

where $\tilde{v} \equiv \frac{v_u(1-v_s)}{v_u-v_s} > 0$. In contrast with (28), only two contradicting effects shape Γ_t: uncertainty and credit market. The overall effect depends on the quality of state education. While it does not impact the latter, the former becomes increasingly stronger with μ, rendering the effort gap to rise as well. Hence, Γ_t may no longer fall with $\hat{g}_t$ if quality of tertiary education is sufficiently high. In fact, it exhibits a positive response for μ above 0.16, as it can be shown by numerical simulations, such that agent U reaps most of the benefits from public investment on universities.

Quantitative Results. Under the baseline calibration, labour income inequality is decreasing with state investment on higher education, as shown in Figure 2, while aggregate stock of human capital is increasing in all three scenarios. Hence, a preliminary answer can hereby be given to Stiglitz (1973). In light of the baseline model, the educational system decreases Γ_t regardless of the schooling rank invested in. If uncertainty is eliminated ($p = 1$), such that $v_s = v_u = 0.75$, the drop in labour income inequality is amplified, as it fastens the pace at which human capital of agent S is accumulated.

However, this result may no longer hold under amplification of DE for it strengthens the uncertainty effect, as hinted in the discussion above. Figure 2 illustrates the results when μ is set to 0.18. Under

Figure 2: Government spending shock on higher education



Note: The graph shows the impulse response function after a shock to government spending on higher education. Three scenarios are distinguished: Baseline calibration, DE amplification of public expenditures ($\mu = 0.18$) and absence of uncertainty ($p = 1$).

such calibration, the educational system perpetuates income inequality.

The issue hereby lies in pinning down the value of μ, for researchers are yet to reach a consensus. Previous studies indicate it lies between 0.1 (Glomm and Ravikumar, 1998) and 0.12 (Card and Krueger, 1992), whereas Blankenau et al. (2007) estimates a range between 0.18 and 0.22. Since the latter changes the sign response of labour income inequality, the predictions of the model are not robust for every empirically valid calibration of μ. Obtaining more accurate estimates of such elasticity is therefore a priority for future research.

Hence, a more concise answer can be given to Stiglitz (1973). It is indeed possible to diminish labour income inequality via the educational system, for a sufficiently low μ. If instead the quality of state universities is such that spender households no longer believe they can be admitted, then schooling system reproduces inequality.

Policy Implications. Insofar as the uncertainty effect decreases with one´s educational achievements, as documented in Höhne and Zander (2019b) and Marsh and O´Mara (2008), the model advocates an increase in public spending on elementary and secondary education of spender agent.

4 Discussion: Inequality vs Efficiency

The previous analysis focused on effects of state financed education, from an inequality perspective. The ensuing section enriches the discussion by studying it from an efficiency point of view, captured by the aggregate stock of human capital, as it measures workers´ productivity.

It follows that, under reasonable parametrization, such investment is always desirable. In addition, stock of human capital is increasing with tax responsiveness to fiscal spending ϕ_g, as shown in Figure 4 and 6 in Appendix B. Raising government debt is thereby a less preferred method of finance.

Section 3 highlighted the negative effect ability uncertainty of spender agent has on its demand for education and the implied fiscal policy. As shown in Figure 2 when uncertainty is eliminated, such policy is also desirable on efficiency grounds as it increases the aggregate stock of human capital.

These findings are of particular importance in countries with superior universities and high educational inequality during mandatory years, such as the United States, where ability uncertainty is likely more pronounced (see Figure 7 in Appendix B). In such case, subsidizing higher education without first addressing the gap in access to quality primary and secondary education, may simultaneously amplify the ongoing inequality and imply not exhausting the benefits from such investment, as measured by the stock of human capital. On the other hand, the same investment in Europe is more likely to reap higher returns and decrease labour income inequality.

5 Conclusion

This paper revisited Stiglitz (1973) under a TANK framework, distinguishing between public investment on mandatory and higher education. The former decreases labour income inequality if it is channelled towards the spender households, while the net effect of the latter is less clear. It will depend on quality of tertiary education. If it is too high, then schooling system perpetuates income inequality. To the extent that ability uncertainty decreases with one´s educational achievements, these findings call for higher public investment on mandatory schooling years of spender households.

Furthermore, the paper contributes to literature studying the link between public schooling expenditures and human capital. The model predicted a positive relationship between the two, which is increasing with tax responsiveness to government spending.

However, the model has one key limitation. The unconstrained agent is more sensitive to general equilibrium effects than the spender, *ceteris paribus*, as implied by ability uncertainty and credit market channel. This finding is seemingly at odds with empirical evidence as it indicates that the time allocation set by the poor is more sensitive to economic activity than the rich. An immediate implication is the hitherto predicted labour income inequality is underestimated. Such limitation can be corrected by modelling gap in education effort of agents through uninsurable risk to human capital accumulation, following Gottardi et al. (2015), instead of ability uncertainty, as it does not affect optimal decision of education effort.

Throughout it was also assumed the government financed its expenditures via lump-sum taxation. Introducing a labour income tax, for instance, would decrease present value of expected future education returns, thus lowering education demand.[26] In addition, explicitly incorporating demand for each schooling cycle, as in Violante et al. (2008), would deepen the analysis carried throughout and likely affect results. Extending the model along these lines is left for future research.

26. However, this argument may be flawed if risk to human capital accumulation is present in the model, as discussed in Gottardi et al. (2015).

Appendix

A Model Details and Derivations

A.1 Log-linearized Equilibrium

The following section details the derivation of the log-linearized equilibrium conditions of the model. The hat notation denotes log deviation from steady state. Formally, $\hat{z}_t = \ln Z_t - \ln Z \simeq \frac{Z_t - Z}{Z}$, where Z_t represents a vector of variables.

Steady State Analysis

In steady state it holds that $\beta = \frac{1}{RR}$, derived from (6), where RR stands for the real rate. From marginal cost of intermediate goods firms it follows that

$$\frac{WNH}{PC} = \frac{1-\alpha}{(1+\mu_p)\gamma_c} \tag{34}$$

$$\frac{R^k}{P} = \frac{\alpha}{1+\mu^p}\frac{Y}{K} \tag{35}$$

In addition, (7) implies $\frac{R^k}{P} = (\rho + \delta_k)$. Throughout, it is assumed $\phi(\delta_k) = \delta$ and $\phi'(\delta_k) = 1$, where $\delta_k = \frac{I}{K}$ in steady state, as implied by (4).

Households

The consumption of agent U is defined by its Euler equation,

$$\hat{c}_t^U = E_t[\hat{c}_{t+1}^U] - \frac{1}{\sigma}\left(\hat{r}_t - E_t\{\pi_{t+1}\}\right) \tag{36}$$

while the spender consumes according to its budget constraint,

$$\hat{c}_t^S = \frac{1-\alpha}{(1+\mu_p)\gamma_c}[(\hat{w}_t - \hat{p}_t) + \hat{n}_t + \hat{h}_t^S] - \frac{1}{\gamma_c}\hat{t}_t \tag{37}$$

The log-linearization of (7) and (9) obtains, respectively, the Tobin´s Q dynamics

$$\hat{q}_{k,t} = \beta E_t\{\hat{q}_{k,t+1}\} + [1 - \beta(1 - \delta_k)]E_t\{\hat{r}^k_{t+1} - \hat{p}_{t+1}\} - (\hat{r}_t - E_t\{\pi_{t+1}\}) \tag{38}$$

and its relationship with physical investment

$$\hat{i}_t - \hat{k}_t = \eta \hat{q}_{k,t} \tag{39}$$

where $\eta = -\frac{1}{\phi''(\delta_k)\delta_k}$ is the elasticity of investment to capital ratio with respect to Tobin´s Q. In addition, from physical capital accumulation (4) it follows that,

$$\hat{k}_{t+1} = \delta_k \hat{i}_t + (1 - \delta_k)\hat{k}_t \tag{40}$$

The human capital counterpart of (38) for the unconstrained agent is given by,

$$\begin{aligned} \hat{q}^U_{h,t} = {} & \beta(1 - \delta_h)E_t\{\hat{q}^U_{h,t+1}\} + [1 - \beta(1 - \delta_h)](\hat{w}_{t+1} - \hat{p}_{t+1}) + [1 - \beta(1 - \delta_h v_u)]E_t\{\hat{n}_{t+1}\} + \\ & + \beta(1 - v_u)\delta_h E_t\{\hat{x}^U_{t+1}\} - (\hat{r}_t - E_t\{\pi_{t+1}\}) \end{aligned} \tag{41}$$

while that of (39) reads as

$$\hat{q}^U_{h,t} = (\hat{w}_t - \hat{p}_t) + v_u \hat{h}^U_t + (1 - v_u)\hat{x}^U_t \tag{42}$$

where (41) and (42) stem from the log-linearization of (8) and (10), respectively. From the log-linearization of (14) and (15) the intertemporal,

$$\begin{aligned} \hat{q}^S_{h,t} = {} & \beta(1 - \delta_h)E_t\{\hat{q}^S_{h,t+1}\} + (1 - \beta(1 - \delta_h))E_t\{\hat{w}_{t+1} - \hat{p}_{t+1}\} + \\ & + [1 - \beta(1 - \delta_h(\mu + v_s))]E_t\{\hat{n}_{t+1}\} + \beta(1 - \mu - v)\delta_h E_t\{\hat{x}^S_{t+1}\} \end{aligned} \tag{43}$$

as well as the intratemporal optimality condition for education effort of agent S

$$\hat{q}^S_{h,t} = (\hat{w}_t - \hat{p}_t) + (\mu + v_s)\hat{h}^S_t - \frac{\mu}{\gamma_g}\hat{g}_t + (1 - v_s)\hat{x}^S_t \tag{44}$$

are derived. From (38), (39) and (41)-(44) it follows that physical and human investment are an intertemporal decision. It involves foregoing consumption today in exchange of future returns, discounted to present terms. In particular, attending school implies sacrificing wages in return of higher labour earnings in the future. To close the education block, the system needs the human capital accumulation of unconstrained,

$$\hat{h}^U_{t+1} = (1 - \delta_h v_u)\hat{h}^U_t + \delta_h v_u \hat{x}^U_t \tag{45}$$

and spender agent,

$$\hat{h}^S_{t+1} = \left[1 - \delta_h(v_s + \mu)\right]\hat{h}^S_t + \delta_h\left(\frac{\mu}{\gamma_g}\hat{g}_t + v_s\hat{x}^S_t\right) \tag{46}$$

In what follows, the equation describing wage dynamics is derived. In this economy there is a continuum of unions, indexed by $z \in [0, 1]$, each setting wages on behalf of the group of workers they represent. Formally, it solves

$$\max_{W^*_t} E_t \sum_{k=0}^{\infty}(\theta_w\beta)^k\left\{\lambda\left[\frac{1}{(C^S_{t+k})^\sigma}W^*_t N_{t+k}(z)H^S_{t+k}\right]+(1-\lambda)\left[\frac{1}{(C^U_{t+k})^\sigma}W^*_t N_{t+k}(z)H^S_{t+k}\right]-\frac{N^{1+\varphi}_{t+k}(z)}{1+\varphi}\right\}$$

subject to a labour demand schedule

$$N_{t+k}(z) = \left(\frac{W^*_t}{W_{t+k}}\right)^{-\theta_w} N_{t+k}$$

as well as (3) and (12). The FOC reads as,

$$E_t\sum_{k=0}^{\infty}(\theta_w\beta)^k\left\{\lambda\frac{H^S_{t+k}}{MRS^S_{t+k}} + (1-\lambda)\frac{H^U_{t+k}}{MRS^U_{t+k}}\right\} = \mathcal{M}^w E_t\sum_{k=0}^{\infty}(\theta_w\beta)^k N^{1+\varphi}_{t+k}(z) \tag{47}$$

where $MRS^j_{t+k} = (N_{t+k})^{\varphi}(C^j_{t+k})^{\sigma}$, for $j = U, S$. Under labour market flexibility ($\theta_w = 0$), unions reset wages each period. Hence, (47) boils down to a one period equation

$$\frac{W_t H_t}{P_t} = \mathcal{M}^w MRS_t \tag{48}$$

Clearly, the efficiency condition equating effective real wages to the marginal rate of substitution no longer holds. From (48) it follows that $\mathcal{M}^w$ is the markup desired by unions, as it is the markup consistent with efficient allocation. Hence, wages are set accordingly by reoptimizing unions. The log-linearization of (47) around zero steady state wage inflation obtains,

$$w_t^* = \mu^w + (1-\theta_w\beta)E_t\sum_{k=0}^{\infty}\xi^k\left(w_{t+k} - \mu_{t+k}\right) \tag{49}$$

where $\mu^w \equiv (w-p) + h - mrs$ and $\xi \equiv (\theta_w\beta)$. The next step is to combine (49) with aggregate wage dynamics. However, before doing so re-write it as,

$$\begin{aligned} w_t^* &= w_t + (1-\theta_w\beta)E_t\sum_{k=0}^{\infty}\xi^k\left(\hat{w}_{t+k} - \hat{\mu}^w_{t+k}\right) \\ &= w_t + (1-\theta_w\beta)E_t\sum_{k=0}^{\infty}\xi^k(w_{t+k} - w_t) - (1-\theta_w\beta)E_t\sum_{k=0}^{\infty}\xi^k\hat{\mu}^w_{t+k} \end{aligned} \tag{50}$$

The first summation in the RHS can be simplified to,

$$\begin{aligned} \sum_{k=0}^{\infty}\xi^k(w_{t+k} - w_t) &= \xi(w_{t+1} - w_t) + \xi^2(w_{t+2} - w_t) + ... \\ &= (1+\xi+\xi^2+...)\xi\pi^w_{t+1} + (1+\xi+\xi^2+...)\xi^2\pi^w_{t+2} + ... \\ &= \frac{1}{1-\xi}\left(\xi\pi^w_{t+1} + \xi^2\pi^w_{t+2} + ...\right) \\ &= \frac{1}{1-\xi}\left(\sum_{k=0}^{\infty}\xi^k\pi^w_{t+k} - \pi^w_t\right) \end{aligned} \tag{51}$$

since $\xi < 1$. Furthermore, let F be a forward operator such that $F^k z_t = z_{t+k}$ and,

$$\sum_{k=0}^{\infty} \xi^k z_{t+k} = \sum_{k=0}^{\infty} (\xi F)^k z_t = \frac{z_t}{1 - \xi F} \tag{52}$$

Combing (50) with (51) and (52) obtains,

$$\begin{aligned} w_t^* - w_t &= (1 - \theta_w \beta) \frac{1}{1 - \xi} \left(\sum_{k=0}^{\infty} \xi^k \pi_{t+k}^w - \pi_t^w \right) - (1 - \theta_w \beta) E_t \sum_{k=0}^{\infty} \xi^k \hat{\mu}_{t+k}^w \\ \iff (1 - \xi F)(w_t^* - w_t) &= (1 - \theta_w \beta) \frac{\xi}{1 - \xi} E_t \{ \pi_{t+1}^w \} - (1 - \theta_w \beta) \hat{\mu}_t^w \end{aligned} \tag{53}$$

Aggregate price dynamics evolve according to,

$$(\Pi_t^w)^{1-\epsilon_w} = \theta_w + (1 - \theta_w) \left(\frac{W_t^*}{W_{t-1}} \right)^{1-\epsilon_w} \tag{54}$$

Log-linearizing (54) yields

$$\begin{aligned} \pi_t^w &= (1 - \theta_w)(\hat{w}_t^* - \hat{w}_{t-1}) \\ \iff \hat{w}_t^* - \hat{w}_t &= \frac{\theta_w}{1 - \theta_w} \pi_t^w \end{aligned} \tag{55}$$

Combining (53) and (55) obtains the wage inflation curve,

$$\pi_t^w = \beta E_t \{ \pi_{t+1}^w \} - \lambda_w \hat{\mu}_t^w \tag{56}$$

where $\lambda_w \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}$ and $\hat{\mu}_t^w = (\hat{w}_t - \hat{p}_t) + \hat{h}_t - (\hat{c}_t + \varphi \hat{n}_t)$.

Final Goods Firms

Under a perfectly competitive market, the firm seeks to solve

$$\max_{Y_t, Y_t(j)} P_t Y_t - P_t(j) Y_t(j) \tag{57}$$

subject to technology constraint (17). Taking P_t and $P_t(j)$ as given, the optimality conditions read as,

$$Y_t : P_t = -\chi_t \tag{58}$$

$$Y_t(j) : Y_t(j) = \left(\frac{-P_t(j)}{\chi_t}\right)^{-\epsilon_p} Y_t \tag{59}$$

where χ_t is the Lagrangian multiplier for (17). Solving the system of equations obtains the set of demand schedules (18). Plugging the latter into the production function yields

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_p}\right)^{\frac{1}{1-\epsilon_p}} \tag{60}$$

as in the text.

Intermediate Goods Firms

The intermediate goods firms decision is twofold. The first stage consists of choosing the cost-minimizing capital K_t and effective labour Z_t. It seeks to solve,

$$\min_{Z_t(j),K_t(j)} TC_t = \frac{W_t}{P_t} Z_t(j) + \frac{R_t^k}{P_t} K_t(j) \tag{61}$$

subject to

$$Y_t(j) = K_t(j)^{\alpha} Z_t(j)^{1-\alpha} \tag{62}$$

The first-order conditions read as,

$$Z_t(j) : \frac{d\mathscr{L}}{dZ_t(j)} = 0 \Longleftrightarrow \frac{W_t}{P_t} = \zeta_t(1-\alpha)K_t(j)^{\alpha} Z_t(j)^{-\alpha} \tag{63}$$

$$K_t(j) : \frac{d\mathscr{L}}{dK_t(j)} = 0 \Longleftrightarrow \frac{R_t^k}{P_t} = \zeta_t \alpha K_t(j)^{\alpha-1} Z_t(j)^{1-\alpha} \tag{64}$$

where $\zeta_t \equiv MC_t$. Solving for the system yields,

$$\frac{K_t(j)}{Z_t(j)} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{W_t}{R_t^k}\right) \tag{65}$$

(65) is the optimal capital to effective labour ratio demanded by firms. Clearly, it is common across firms. To obtain real marginal cost as a function of input prices, substitute (65) into (63),

$$MC_t = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\left(\frac{R_t^k}{P_t}\right)^\alpha \left(\frac{W_t}{P_t}\right)^{1-\alpha} \tag{66}$$

The second stage is to choose the profit-maximizing price. Firms choose P_t^* to maximize the discounted stream of profits obtained while that price is still charged. Denoting θ_p^k as the probability of keeping P_t^* for k periods, the firm solves

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty}(\theta_p\beta)^k \Lambda_{t,t+k}\left[P_t^* - P_{t+k}MC_{t+k}\right]\left[\frac{P_t^*}{P_{t+k}}\right]^{-\epsilon_p} Y_{t+k} \tag{67}$$

subject to (6). The FOC reads as,

$$P_t^* = \mathcal{M}\frac{E_t\sum_{k=0}^{\infty}(\theta_p\beta)^k C_{t+k}^{1-\sigma}P_{t+k}^{\epsilon_p}MC_{t+k}}{E_t\sum_{k=0}^{\infty}(\theta_p\beta)^k C_{t+k}^{1-\sigma}P_{t+k}^{\epsilon_p-1}} \tag{68}$$

Under flexible prices ($\theta_p = 0$) firms reoptimize every period. Hence, (68) boils down to a one period equation

$$P_t^* = \mathcal{M}^p\frac{C_t^{1-\sigma}MC_tP_t^{\epsilon}}{C_t^{1-\sigma}P_t^{\epsilon-1}}$$

$$\Longleftrightarrow P_t^* = \mathcal{M}^p MC_t P_t \tag{69}$$

From (69) it follows that $\mathcal{M}^p$ is the desired markup, as it is the markup consistent with price flexibility, or efficient allocation. That is, reoptimizing firms set P_t^* to respect $\mathcal{M}^p$.

Log-linearizing (68) around zero steady state price inflation obtains,

$$p_t^* = \mu^p + (1-\theta\beta)E_t \sum_{k=0}^{\infty} \xi^k \left[mc_{t+kt} + p_{t+k}\right] \tag{70}$$

where $\mu^p \equiv -mc = \log \mathcal{M}^p$. Before proceeding to aggregate price dynamics, apply some algebraic manipulations to (70),

$$p_t^* = p_t + (1-\theta_p\beta)E_t \sum_{k=0}^{\infty} \xi^k \left[\hat{m}c_{t+k} + (p_{t+k} - p_t)\right] \tag{71}$$

Then simplify the second summation to obtain,

$$\begin{aligned}
\sum_{k=0}^{\infty} \xi^k (p_{t+k} - p_t) &= \xi(p_{t+1} - p_t) + \xi^2(p_{t+2} - p_t) + ... \\
&= (1 + \xi + \xi^2 + ...)\xi\pi_{t+1}^p + (1 + \xi + \xi^2 + ...)\xi^2\pi_{t+2}^p + ... \\
&= \frac{1}{1-\xi}\left(\xi\pi_{t+1}^p + \xi^2\pi_{t+2}^p + ...\right) \\
&= \frac{1}{1-\xi}\left(\sum_{k=0}^{\infty} \xi^k \pi_{t+k}^p - \pi_t^p\right)
\end{aligned} \tag{72}$$

Combining (71) with (52) and (72) yields,

$$\begin{aligned}
p_t^* &= p_t + (1-\theta_p\beta)E_t \sum_{k=0}^{\infty} \xi^k \hat{m}c_{t+k} + (1-\theta_p\beta)\frac{1}{1-\xi}\left(\sum_{k=0}^{\infty} \xi^k \pi_{t+k}^p - \pi_t^p\right) \\
&= p_t + \frac{1-\theta_p\beta}{1-\xi F}\hat{m}c_t + \frac{1-\theta_p\beta}{1-\xi F}\frac{\xi F}{1-\xi}\pi_t^p
\end{aligned} \tag{73}$$

Multiplying (73) on both sides by $(1-\xi F)$,

$$(1-\xi F)(p_t^* - p_t) = (1-\theta_p\beta)\hat{m}c_t + (1-\theta_p\beta)\frac{\xi}{(1-\xi)}E_t\{\pi_{t+1}^p\} \tag{74}$$

Finally, aggregate price dynamics evolve according to,

$$\left(\Pi_{t-1}^{p}\right)^{1-\epsilon_p} = \theta_p + (1-\theta_p)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon_p} \tag{75}$$

Re-writing (75) in log-linear terms obtains

$$\begin{aligned} \pi_t^p &= (1-\theta_p)(\hat{p}_t - \hat{p}_{t-1}) \\ \iff p_t^* - p_t &= \frac{\theta_p}{1-\theta_p}\pi_t^p \end{aligned} \tag{76}$$

Combining (74) and (76), yields the New Keynesian Philips curve,

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p \hat{\mu}_t^p \tag{77}$$

where $\lambda_p \equiv \frac{(1-\theta_p)(1-\theta_p\beta)}{\theta_p}$ and $\hat{\mu}_t^p = -\hat{mc}_t$. Notice how neither (77) nor (56) are affected by the presence of hand-to-mouth consumers.

Market clearing

The goods market clearing requires $Y = C + I + G$. In log-linear terms, it reads as

$$\hat{y}_t = \gamma_c \hat{c}_t + \gamma_i \hat{i}_t + \hat{g}_t \tag{78}$$

Furthermore, the aggregate production function can be written, up to first-order approximation, as

$$\hat{y}_t = (1-\alpha)\hat{z}_t + \alpha \hat{k}_t \tag{79}$$

where $\hat{z}_t \equiv \hat{n}_t + \hat{h}_t$.

A.2 Derivation of Proposition 1

From (46) it follows that human capital of agent S is increasing with government spending if, and only if,

$$\frac{d\hat{h}^S_{t+1}}{d\hat{g}_t} = \delta_h \left(\frac{\mu}{\gamma_g} + v_s \frac{d\hat{x}^S_t}{d\hat{g}_t} \right) > 0 \tag{80}$$

To draw insightful conclusions from (80), it is useful to breakdown $(d\hat{x}^S_t / d\hat{g}_t)$ one step further. Hence, combine (43) and (44),

$$\begin{aligned}(\hat{w}_t - \hat{p}_t) + (\mu + v_s)\hat{h}^S_t - \frac{\mu}{\gamma_g}\hat{g}_t + (1 - v_s)\hat{x}^S_t = E_t\{\hat{w}_{t+1} - \hat{p}_{t+1}\} + \\ + \beta(1 - \delta_h)E_t\Big\{(\mu + v_s)\hat{h}^S_{t+1} - \frac{\mu}{\gamma_g}\hat{g}_{t+1} + (1 - v_s)\hat{x}^S_{t+1}\Big\} + \\ + [1 - \beta(1 - \delta_h(\mu + v_s))]\, E_t\big\{\hat{n}_{t+1}\big\} + \\ + \beta\delta_h(1 - \mu - v_s)E_t\big\{\hat{x}^S_{t+1}\big\}\end{aligned} \tag{81}$$

Total differentiation of (81) with respect to $\hat{g}_t$ yields,

$$\frac{d\hat{x}^S_t}{d\hat{g}_t} = \frac{1}{1 - v_s} \left(\frac{\mu}{\gamma_g} - \frac{d\hat{\omega}_t}{d\hat{g}_t} \right) \tag{82}$$

where $\hat{\omega}_t \equiv (\hat{w}_t - \hat{p}_t)$. Combining (80) with (82) obtains

$$\frac{\mu}{\gamma_g} > v_s \frac{d\hat{\omega}_t}{d\hat{g}_t} \tag{83}$$

as in the text.

Detailing impact of elasticity parameter v_s

To clarify the effect of v_s on $\hat{h}^S_{t+1}$, plug (82) into (80) to obtain,

$$\frac{d\hat{h}^S_{t+1}}{d\hat{g}_t} = \frac{\delta_h}{1 - v_s} \left(\frac{\mu}{\gamma_g} - v_s \frac{d\hat{\omega}_t}{d\hat{g}_t} \right) \tag{84}$$

The υ_s impact is not so straightforward for it also amplifies the general equilibrium labour market effects. Ultimately, the overall impact depends on the magnitude of DE comparatively to IE. To see this, take derivative of (84) with respect to υ_s: $\frac{d^2\hat{h}^S_{t+1}}{d\hat{g}_t d\upsilon_s}$. It follows that stock of human capital of spender is increasing with υ_s if, and only if,

$$\frac{\mu}{\gamma_g} > \frac{d\hat{\omega}_t}{d\hat{g}_t} \tag{85}$$

Under baseline setup, (85) holds, such that υ_s incentivizes spender households to accumulate human capital and, consequently, reduces labour income inequality. However, under IE amplifying scenarios, such as more flexible wage setting environment, it may no longer be true.

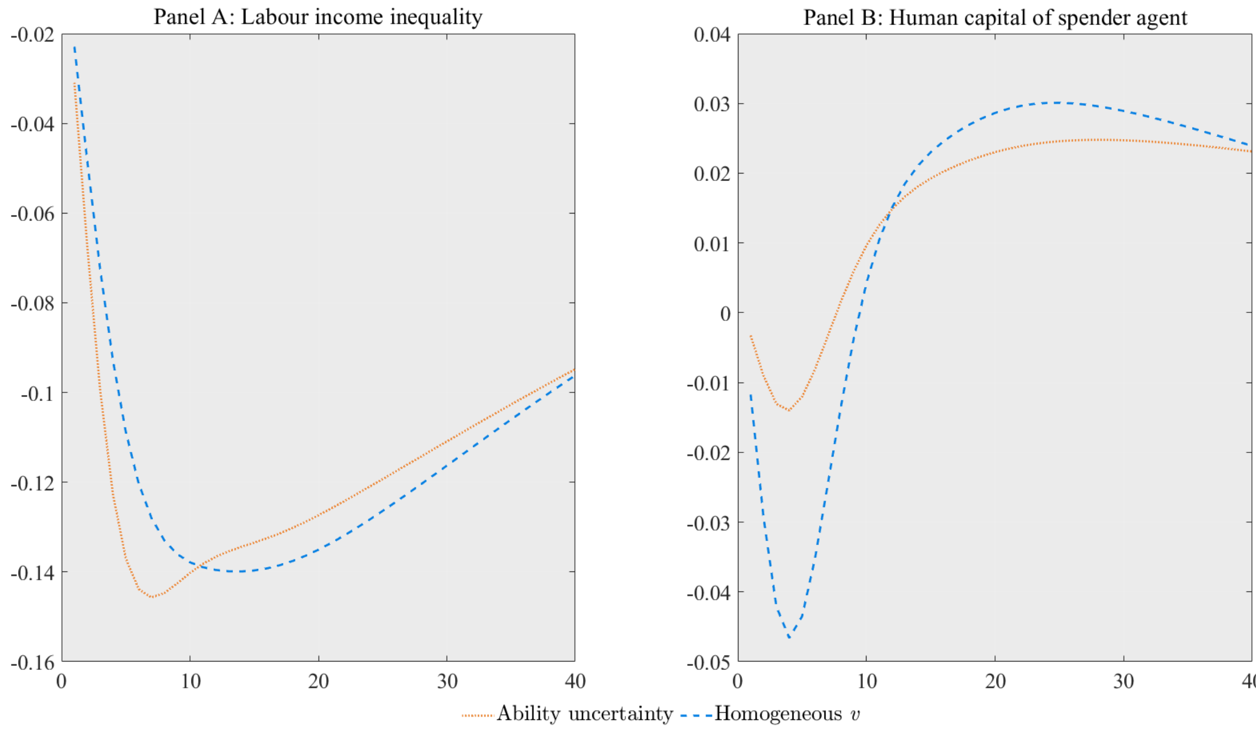
Figure 3 exhibits the impact of an increase in υ_s, such that $\upsilon_s = \upsilon_u = 0.75$, where $\theta_w = 0.6$ and $\varphi = 2$. In fact, the impact of υ_s on labour income inequality, becomes non-linear over time. Initially, the general equilibrium outweigh the partial equilibrium effects, such that $\hat{h}^S_{t+1}$ is decreasing with υ_s, which is reflected in a higher Γ_t. However, afterwards (3 years in the graph), it is the DE that dominate, such that (85) holds again and labour income inequality decreases with υ_s.

Hence, the initial response is consistent with Blankenau and Simpson (2004) findings, whereas in the long-run it corroborates with Dissou et al. (2016). In order to simplify the analysis, the impact of υ_s is studied under $\theta_w = 0.75$ and $\varphi = 1$.

A.3 Derivation of Proposition 2 and 2´

Since proposition 2´ is a generalization of 2, the former shall be derived and then shown to nest the latter. As it shall be clear below, the derivation is quite similar to proposition 1. Under public investment on higher education, human capital of unconstrained households evolves according to

Figure 3: Sensitivity to v_s



Note: The graph shows sensitivity of labour income inequality and human capital of agent S to v_s following a government spending shock on mandatory education. In both cases, it is imposed $\theta_w = 0.6$ and $\varphi = 2$.

(29), from which optimality conditions (30) and (31) are derived. The log-linearization yields

$$\hat{h}^U_{t+1} = (1 - \delta_h(v_u + \mu))\hat{h}^U_t + \delta_h \left[\frac{\mu}{\gamma_g}\hat{g}_t + v_u \hat{x}^U_t\right] \tag{86}$$

$$\begin{aligned} \hat{q}^U_{h,t} = {} & \beta(1-\delta_h)E_t\{\hat{q}^U_{h,t+1}\} + (1 - \beta(1-\delta_h))E_t\{\hat{w}_{t+1} - \hat{p}_{t+1}\} + \\ & + [1 - \beta[1 - \delta_h(\mu + v_s)]E_t\{\hat{n}_{t+1}\} + \beta\delta_h(1 - \mu - v)E_t\{\hat{x}^U_{t+1}\} \end{aligned} \tag{87}$$

$$\hat{q}^U_{h,t} = (\hat{w}_t - \hat{p}_t) + (\mu + v_s)\hat{h}^U_t - \frac{\mu}{\gamma_g}\hat{g}_t + (1 - v_s)\hat{x}^U_t \tag{88}$$

From (86) it follows that the human capital of the unconstrained agent is increasing with public investment in higher education if, and only if,

$$\frac{d\hat{h}^U_{t+1}}{d\hat{g}_t} = \delta_h\left(\frac{\mu}{\gamma_g} + v_u \frac{d\hat{x}^U_t}{d\hat{g}_t}\right) > 0 \tag{89}$$

Similarly to previous section, the next step consists of breaking down (89) one step further. Thus, combine (87) and (88) to obtain

$$
\begin{aligned}
(\hat{w}_t - \hat{p}_t) + (\mu + v_s)\hat{h}_t^U - \frac{\mu}{\gamma_g}\hat{g}_t + (1 - v_s)\hat{x}_t^U = {} & E_t\{\hat{w}_{t+1} - \hat{p}_{t+1}\} + \\
& + \beta(1 - \delta_h)E_t\Big\{(\mu + v_s)\hat{h}_{t+1}^U - \frac{\mu}{\gamma_g}\hat{g}_{t+1} + (1 - v_s)\hat{x}_{t+1}^U\Big\} + \\
& + \left[1 - \beta\left(1 - \delta_h(\mu + v_s)\right)\right] E_t\big\{\hat{n}_{t+1}\big\} + \\
& + \beta\delta_h(1 - \mu - v_s)E_t\big\{\hat{x}_{t+1}^U\big\} - (\hat{r}_t - E_t\{\pi_{t+1}\}) \qquad (90)
\end{aligned}
$$

Total differentiation of (90) with respect to $\hat{g}_t$ yields,

$$
\frac{d\hat{x}_t^U}{d\hat{g}_t} = \frac{1}{1 - v_u}\left[\frac{\mu}{\gamma_g} - \left(\frac{d\hat{\omega}_t}{d\hat{g}_t} + \frac{d\hat{rr}_t}{d\hat{g}_t}\right)\right] \qquad (91)
$$

Combining (91) and (89) implies that human capital of agent U is rising with public investment on higher education if, and only if,

$$
\frac{\mu}{\gamma_g} > v_u\left(\frac{d\hat{\omega}_t}{d\hat{g}_t} + \frac{d\hat{rr}_t}{d\hat{g}_t}\right) \qquad (92)
$$

as in the text. Under the baseline scenario, where $\mu = 0$ in (29), obtains proposition 2.

A.4 Derivation of Corollary 1 and 1´

The proof of each corollary is straightforward and follows from previous propositions. Departing from corollary 1, government spending on higher education decreases labour income inequality if, and only if,

$$\begin{aligned}
&\frac{d\hat{h}^S_{t+1}}{d\hat{g}_t} > \frac{d\hat{h}^U_{t+1}}{d\hat{g}_t} \\
&\iff \frac{1}{1-v_s}\left(\frac{\mu}{\gamma_g} - v_s\frac{d\hat{\omega}_t}{d\hat{g}_t}\right) > -\frac{v_u}{1-v_u}\left(\frac{d\hat{\omega}_t}{d\hat{g}_t} + \frac{d\hat{r}r_t}{d\hat{g}_t}\right) \\
&\iff \frac{1}{1-v_s}\frac{\mu}{g} > \frac{v_s - v_u}{(1-v_s)(1-v_u)}\frac{d\hat{\omega}_t}{d\hat{g}_t} - \frac{v_u}{1-v_u}\frac{d\hat{r}r_t}{d\hat{g}_t} \\
&\iff \frac{\mu}{\gamma_g} > \frac{1}{1-v_u}\left((v_s - v_u)\frac{d\hat{\omega}_t}{d\hat{g}_t} - v_u(1-v_s)\frac{d\hat{r}r_t}{d\hat{g}_t}\right) \qquad (93)
\end{aligned}$$

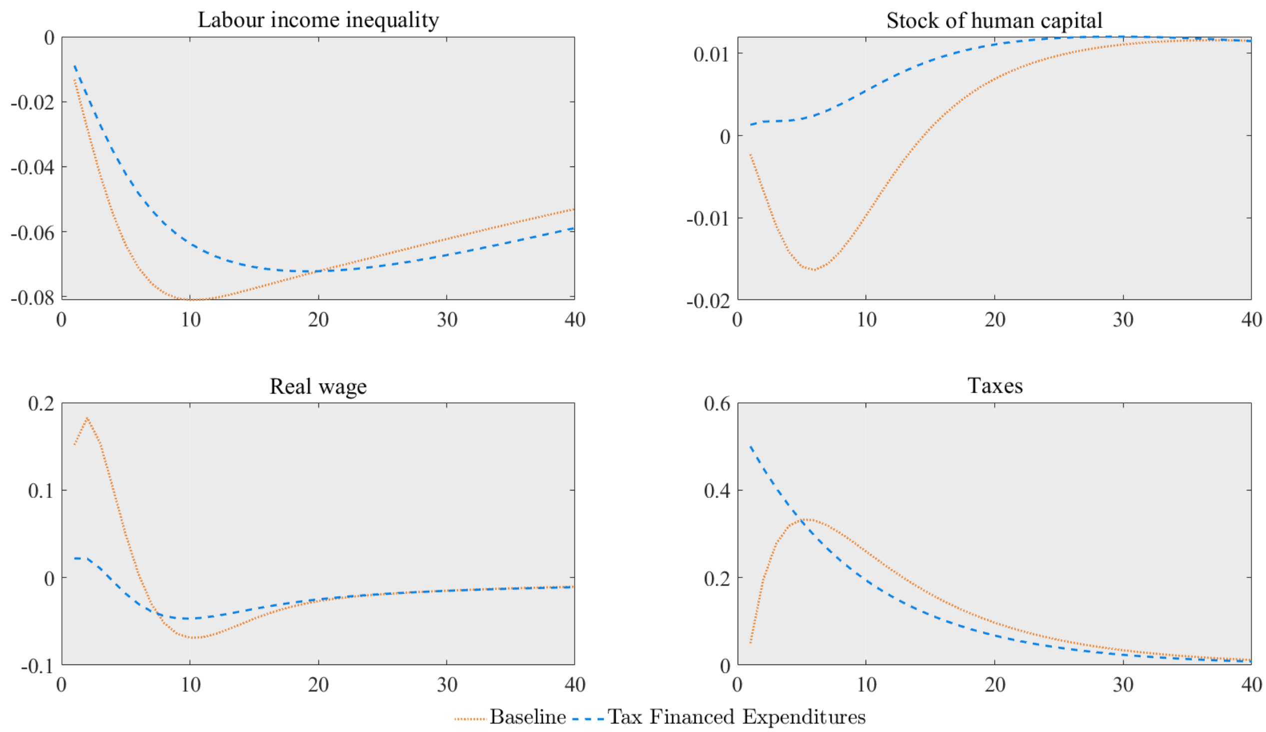
If fiscal authorities invest in universities, human capital accumulation of unconstrained households evolves, in log-linear terms, according to (86). Hence, (93) is generalized to obtain

$$\begin{aligned}
&\frac{d\hat{h}^S_{t+1}}{d\hat{g}_t} > \frac{d\hat{h}^U_{t+1}}{d\hat{g}_t} \\
&\iff \frac{1}{1-v_s}\left(\frac{\mu}{\gamma_g} - v_s\frac{d\hat{\omega}_t}{d\hat{g}_t}\right) > \frac{1}{1-v_u}\left[\frac{\mu}{\gamma_g} - v_u\left(\frac{d\hat{\omega}_t}{d\hat{g}_t} + \frac{d\hat{r}r_t}{d\hat{g}_t}\right)\right] \\
&\iff \frac{v_s - v_u}{(1-v_s)(1-v_u)}\frac{\mu}{\gamma_g} > \frac{v_s - v_u}{(1-v_s)(1-v_u)}\frac{d\hat{\omega}_t}{d\hat{g}_t} - \frac{v_u(1-v_s)}{(1-v_s)(1-v_u)}\frac{d\hat{r}r_t}{d\hat{g}_t} \\
&\iff \frac{\mu}{\gamma_g} > \frac{d\hat{\omega}_t}{d\hat{g}_t} - \tilde{v}\frac{d\hat{r}r_t}{d\hat{g}_t} \qquad (94)
\end{aligned}$$

where $\tilde{v} \equiv \frac{v_u(1-v_s)}{v_s - v_u}$, as in the text.

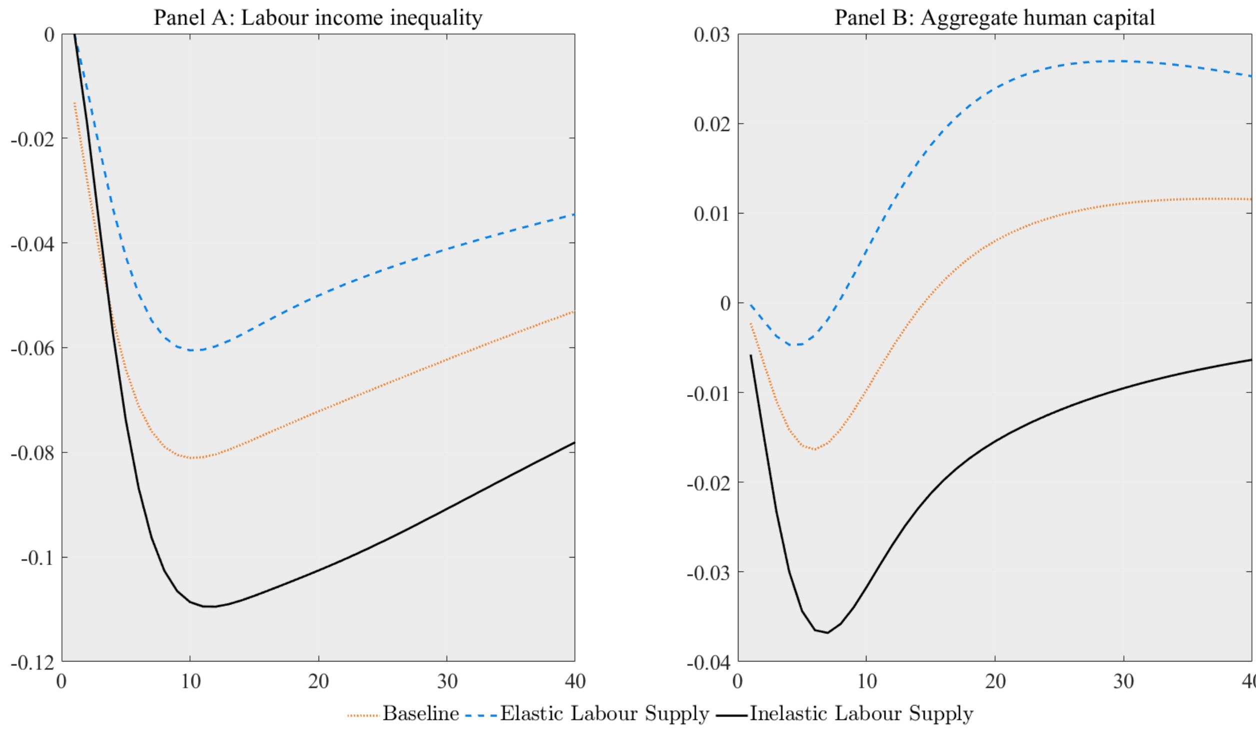
B Complementary figures

Figure 4: Sensitivity to ϕ_g



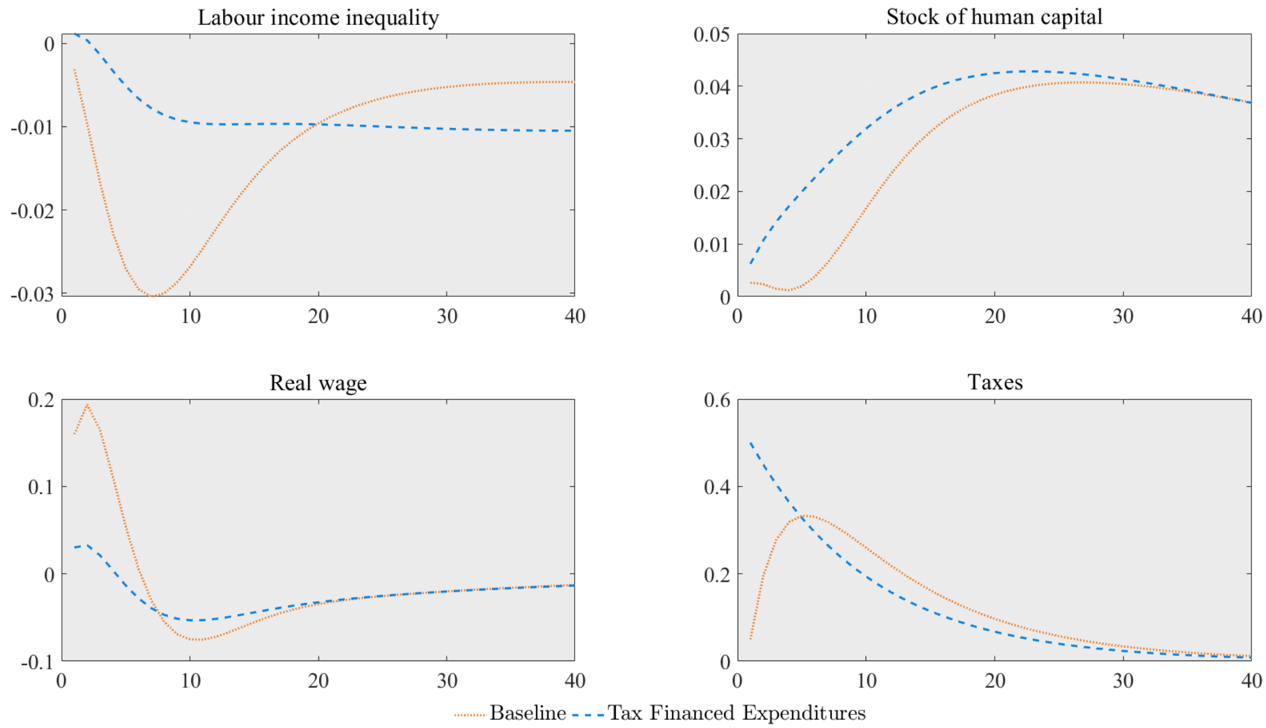
Note: The graph shows sensitivity of IRF to alternative methods of financing government spending on mandatory education. Two calibrations are distinguished: baseline setup and fully tax financed expenditures ($\phi_g = 1$).

Figure 5: Sensitivity to φ



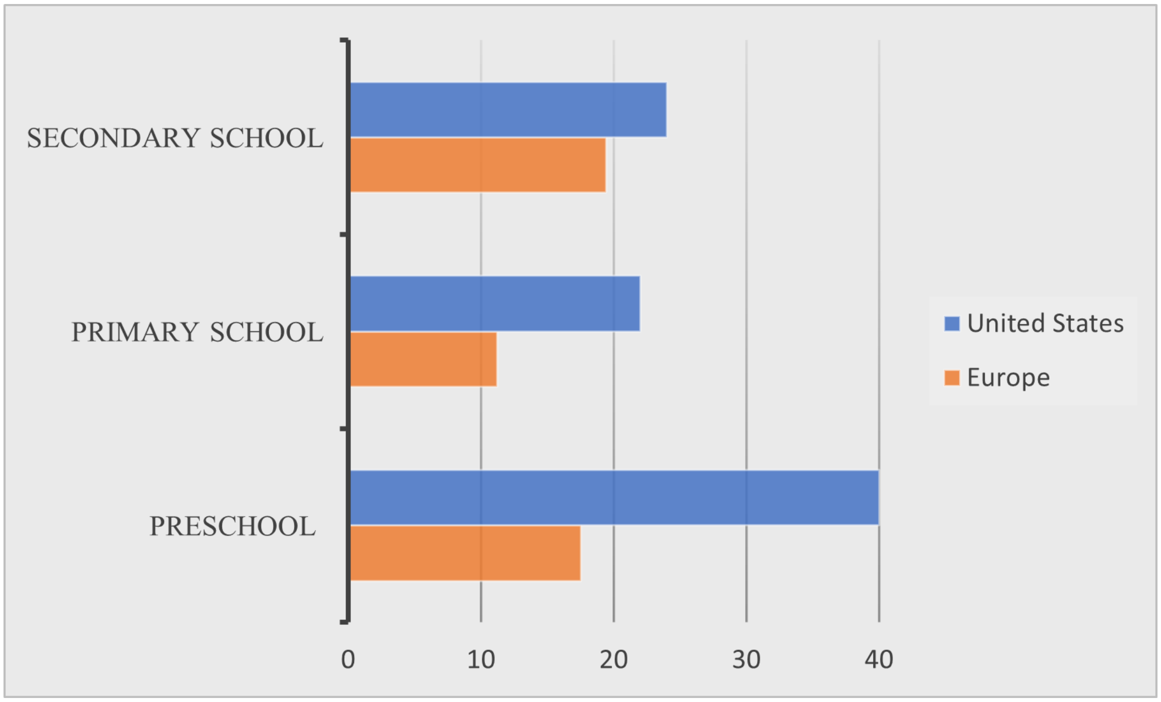
Note: The graph shows sensitivity of labour income inequality and aggregate stock of human capital to φ following government spending shock on mandatory education. Three calibrations are distinguished: baseline setup, elastic ($\varphi = 0.5$) and inelastic ($\varphi = 2$) labour supply.

Figure 6: Sensitivity to ϕ_g



Note: The graph shows sensitivity of IRF to alternative methods of financing government spending on higher education. Two calibrations are distinguished: baseline setup and fully tax financed expenditures ($\phi_g = 1$).

Figure 7: Education inequality throughout mandatory schooling years: U.S. vs Europe



Note: The graph shows the education inequality rank of Europe compared to United States, out of a total of 41 countries. Each country is classified in terms of pre, primary and secondary schooling.

Source: Gromada, A. et al., 'Measuring Inequality in Children's Education in Rich Countries', Innocenti Working Paper 2018-18, UNICEF Office of Research – Innocenti, Florence, 2018.