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Valuation of ATM European Swaptions with the Libor Market Model

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Abstract:

This working project developped a pricing tool for at-the-money European swaptions using the LMM. The tool is benchmarked by transforming quoted implied volatiliy of swaptions to market prices in euro, using Black's formula. When including a volatility structure calibrated on market data, the tool succeeds in pricing swaptions with a 2 till 5 years tenor. For shorter tenors, 1 year, or longer tenors, 7-10 years, the tool does not perform well, due to issues fitting today's unique volatility term structure. Improvements on the volatility calibration are suggested, as well as possible extensions of the tool towards non-at-the-money and exotic products.

Keywords:

Interest rate derivatives, Interest rate modelling, Libor market model, Swaptions

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1 Introduction

For my Direct Research Internship, I joined PwC's Governance, Risk and Compliance team as a Quant and risk consultant. As a Quant our job partly consists of providing support to the audit teams of financial institutions e.g., banks and insurance companies, or providing consultancy services to financial institutions directly. Often the support or services includes the pricing of financial instruments. It is within this pricing scope my work project is situated, the objective of this Direct Research internship is to create a pricing tool for ATM European Swaptions that can become part of our Quant team's toolbox and can be used as a basis for further development and/or usage in practice. This means that the tool must try to go beyond a mere academic tool and must be based on up-to-date models that are industry standard. But first, what is a swaption and why do financial institutions use it?

1.1 What are Swaptions and when are they used

1.1.1 Context

The financial sector is heavily regulated due to its essential service to society e.g., connecting borrowers and lenders, running payment systems etc.; its interconnectedness i.e., the failure of one component can spread throughout the sector; and its information asymmetry i.e., ordinary savers do not have the capacity to evaluate a bank's health. Therefore, public supervisors control the risk culture and governance within banks (European Central Bank 2018). Because the EU banking sector's assets amounted to 292% of EU's GDP in 2020 supervisors must try to anticipate on distress in the sector because systemic failure would have dramatic consequences (Eurostat 2020). Furthermore, banks are fragile by design, and reputational damage or market sentiment can easily push a bank towards the edge of a cliff, see recent events at Silicon Valley Bank and Credit Suisse (Choi 2023; Financial Times 2023). All this regulatory and reputational pressure make risk management core business of banking[1].

Within a financial institution the three largest sources of risk are credit risk, market risk and operational risk. (Bank of England 2020). Credit risk arises from a counterparty not being able to (fully) repay its debts, while market risk originates from banks being exposed to fluctuations in market prices. Finally, operational risk stems from (human) mistakes or errors in processes. An important example of market risk is interest rate risk. Changes in interest rates mainly influence the profitability of banks by affecting the Net Interest Income or NII. The Q4 2022 risk dashboard of the European Banking Authority (EBA) shows that recent interest rate increase has caused the Net Interest Income (NII) of European banks to rise by 14.1% on average between Q4 2021 and Q4 2022. The improvement of the NII is mainly coming from the increase in Net Interest Margin (NIM), 10.9%. This is the difference between the interest a bank earns on its assets and the interest it needs to pay to fund its assets. The remaining part, 3.2% is coming from a change in composition of its interest earning assets (IEA) (European Banking Authority 2023). This improvement in profitability shows that on average banks income is more sensitive to interest rates than its funding. This contrasts with the traditional view on banking in which a bank funds itself with short-term or sensitive deposits and supplies long-term, non-sensitive loans. This is called the maturity transformation and allows banks to earn a term premium. However, Drechsler, Savov and Schnabl (2021, 1094) argue that banks use their market power to do maturity transformation without or less interest rate risk because they do not fully implement, or delay interest rates rises on their deposits. Furthermore, Hoffmann et al. (2018, 5) show that the effect of interest rate rises on banks is very heterogenous across the European banking sector. Explanations are the different conventions within a country towards fixed or floating loans, mainly on mortgages, and the differences in interest rate

[1]"…, better risk management may be the only truly necessary element of success in banking,"
~ Alan Greenspan, former chair of the US Federal reserve.

hedging between banks. The recent events at SVB which came under pressure because of the negative impact the Fed's interest rate raises had on its assets and NIM, and the lack of hedging they hold to counter this effect, are an excellent example of the effect of interest rate risk and the failure of risk management (Thomadakis 2023).

Managing the exposure to interest rate risk and deciding how much risk a bank is willing to take is done in the asset and liability management commission or ALM commission. In this process of matching the cashflows that are streaming out of the bank with the cashflows coming in, financial instruments called swaps, play an important role (Hoffmann, et al. 2018, 5). If a bank receives a fixed interest rate on a pool of loans or bonds, while in the meantime it must pay a floating interest rate on a pool of deposits, and the bank wanted to match sensitivity of its assets closer to that of its funding, it could try to attract more term deposits and sell more floating loans. However, this can be a long and intensive process which requires marketing resources, potentially discounts on rates coming in and/or promising to pay higher deposit rates. Furthermore, the success of this campaign is not guaranteed. A much easier strategy is to enter a swap in which the bank receives floating and pays fixed, which is called a payer swap. Opposite, a swap in which you receive the fixed rate is called a receiver swap. By entering a payer swap, the bank is also able to match its assets and liabilities in a much easier way. Furthermore, at moment of inception the value of a swap equals zero, meaning that the Net Present Value (NPV) of the floating leg, paying a floating rate, equals the NPV of the fixed leg, paying a fixed rate called the swap rate (Lesniewski 2013a, 8; Lesniewski 2019a, 44). This means that entering a swap does not cost the bank money, besides limited transaction costs. Because of this ease-of-use, the over-the-counter interest rate swap market in USD amounted to more than 160 trillion dollars in June 2022 (Bank for International settlements 2022).

1.1.2 What is a swaption and when is it used

In the example above, the bank does not take a position in future interest rates. If the rates go up, the floating rate on its liabilities go up, but the floating rate the bank receives in its swap also increases. If the hedge is “perfect” the total effect of the rising interest rates will be zero. However, if the bank excepts interest rates to drop in the future, but it does not want or is not allowed e.g., by the ALM commission, to keep its position fully unhedged, it can buy an option on a payer swap or payer Swaption. By holding a payer swaption, an investor holds a call option on the forward swap rate, because it buys the option to enter a payer swap at a certain fixed swap rate in the future. If at the expiration date this fixed swap rate, strike price, is lower than the market swap rate at the time, the investor will exercise the swaption and pay this preset swap rate (Hull 2018, 684). So, for the bank that bought the payer swaption this means that if the rates go down, the bank will not exercise the swaption because the swap rate in the future will be lower than the swap rate, strike rate, in their swaption. A Strike price that lies higher than the market price means a call option is out of the money (OTM). If at this point the bank still wants to hedge its floating liabilities it can enter a payer swap at lower swap rate than before in the market, due to the drop in interest rates. On the other hand, if rates do go up, the bank has the option to enter a payer swap. This payer swap has a lower swap rate than the one currently in the market, because it was fixed when the floating rate was lower. A strike price which is lower than the market price means the call option is in the money, (ITM). When the strike swap rate is equal to the market swap rate, the bank will be indifferent in exercising the option or not, this is called an at the money option (ATM). Hence, in this case the payer swaption allows the bank to hedge itself against rising interest rates, while keeping the potential to enjoy the gain of a drop in interest rates. This opportunity to hedge risk while keeping the upside potential is popular in pension funds which invest in fixed income assets and wants to hedge itself against a possible decline in interest. However, they hedge this by buying receiver swaptions. Another popular use is the hedging of prepayment risk on mortgages. A financial institution holding a mortgage in which he receives a fixed rate is exposed by prepayment risk, because in most-countries a client is allowed to refinance his mortgages at any time (European Union 2022). This means that if interest rates drop, the client will refinance, and the bank will need to borrow at a lower rate. However, when rates go up the bank is stuck with its lower earning mortgage. By entering a receiver swaption the bank can replace its future cash flows in case of refinancing. Nevertheless, these often need to be American style swaptions or more exotic swaptions due to the non-fixed refinancing date and possibly changing cash-flows (Kosowski and Neftci 2015, 591). A study by ISDA in 2014 showed that the over-the-counter (OTC) swaption market that does not use a central clearing house, amounted to around $30 trillion dollar at year-end 2013 (ISDA - International Swaps and Derivatives Association 2014, 5) This popularity of swaptions stems from its ability to hedge while keeping the opportunity to have gains if the interest rates move in the anticipated direction.

In this part we have shown what a swaption is and that it can be used to hedge interest rates while keeping exposure to upward potential, which of course comes at a cost, the price of the swaption. Moreover, section 1.1 shows the relevance of interest rate risk in the current financial environment. It becomes clear how interest rates derivatives can play an important role for financial institutions while managing interest rate risk. Hence, a tool to price interest rate derivatives is valuable for PwC when consulting and/or auditing financial institutions.

1.2 How are swaptions priced?

1.2.1 Black's model

Black-Scholes-Merton model (BSM)

First a step back to stock options. The Black-Scholes-Merton (BSM) model is the absolute benchmark model to price equity options. In the BSM-world the price of stock follows a geometric Brownian motion, see equation 1a. From this equation it becomes clear that only two parameters are needed to model the stock price, being the volatility and the expected return of the stock, both are assumed to be constant. Furthermore, under the complete markets and no-arbitrage assumption, equity derivatives can be priced in the risk-neutral world or under the risk-neutral measure. This means that probability of decreasing or increasing of the underlying stock is not relevant for the price of the option. Hence, the expected return of a stock equals the risk-free rate under the risk-neutral measure, see equation 1b. This is because the complete markets and no-arbitrage assumptions allow for changes in a stock underlying an option to be completely hedged, resulting in a replication portfolio that has zero risk. Therefore, in the BSM world for a given strike price and maturity one can price a stock option with only the risk-free rate and the volatility of the stock. Unfortunately, it is not possible to apply the same model on bond options. The pull to par effect of a bond, and accompanying decrease in volatility, clashes with the constant volatility assumption. An even bigger problem is the constant interest rate assumption in the traditional risk-neutral world, which is incompatible with stochastic bond prices. Deterministic interest rates and bond prices would mean no uncertainty or volatility of interest rates, which would make interest rate derivatives redundant and pricing them unnecessary. Therefore, we need to move to a different risk-neutral world with stochastic interest rates (Rebonato 2002, 3; Hull 2018, 321).

Black's model

Black's model (1976) is an adaptation of the BSM model that allows for stochastic interest rates and solves the problems of the BSM model by changing to a different risk-neutral measure. Key in this is the "equivalent martingale measure result". This principle goes as following, if the price of a traded security is used as the unit of measurement, numeraire, then there is a market price of risk for which all security prices follow martingales, zero-drift stochastic process, (Hull 2018, 655). This means that the price of a security is no longer measured in euros or dollars, but relative to another traded security. The payoff at time T of the numeraire e.g., the price of a risk-free zero-coupon bond, is known, and the current market price of the numeraire is known. Therefore, the current price of the derivative is the current value of the numeraire multiplied with the value of the derivative at time T relative to the numeraire at time T. In other words, the current market info on the numeraire is used to discount the future payoff of the derivative. Because the derivative is priced relative to the forward price of the numeraire, this world is called “forward risk-neutral”. This is like the traditional risk-neutral world in which an option is priced in terms of the spot or current price of the underlying. In both cases, the probability of the underlying, forward rate or sport rate, to go up or down is not relevant for the option price, because it is captured in the measure, making the option price (forward) risk neutral. Furthermore, by picking the right numeraire, security prices follow a zero-drift process which resembles the process of equity prices in the Black-Scholes model, and similar formulas can be obtained. This brings us back to Black’s model. It assumes that the forward rate follows a driftless, due to picking the right numeraire, geometric Brownian motion similar to the BSM model, see equation 2. However, now under a forward risk-neutral measure. Furthermore, by modelling the forward rate instead of the spot rate the pull to par effect is surpassed. This is because forward rates are interest rates between two dates in the future, but under terms agreed upon today, see equation 3 (Rebonato 2002, 3 ; Hull 2018, 655; lesniewski 2019b,).

Valuation of Caps and floors

A broadly used implementation of a Black’s model is the valuation formula for caps and floors, which is still the market standard for quoting the prices of these products (Hull 2018, 673). A cap is an interest rate derivative that provides financial insurance against rising interest rates. Each reset date the forward rate is compared with a fixed cap rate and the holder of a cap has the right to receive the difference between the two on the next reset date. Hence, an investor is able to limit the floating rate e.g., linked to the Libor rate, and has to pay a certain cap rate by entering into a cap agreement. One single right within the cap is called a caplet and can also be seen as a European call option on a forward rate. Black’s model is able to price this Interest rate option by taking a risk-free zero-coupon bond as numeraire. The formula that follows, is closely related to the BSM formula for a European Call option on a stock, see equation 4. The value of the Cap can be obtained by adding up all caplets of which the cap contains (Lesniewsk 2013b, 7; Lesniewski 2019c, 22).

Valuation of European Swaption

A second popular expansion of Black's model, and more important for this paper, is the formula to price European swaptions. A European swaption is a European option on a swap. European means that the option can only be exercised at maturity. This is a necessary condition for Black's formula. This is not the case for more complexed pricing methodologies discussed later in the paper. However, this working project focusses on European swaptions only. As already mentioned, a payer swaption is the call on this forward swap rate with repeated payoffs. Forward swap rate is shown in equation 5. The repeated payoffs are due to the repeated payments of a swap contract, and at every payment date there is a potential payoff coming from the option. It seems that these repeated payoffs make it more complex to extend black's model, but by picking an annuity that pays €1 at each coupon date of the swap, as numeraire, the forward swap rates follow a driftless geometric Brownian motion, see equation 6. Hence, Black's formula to valuate a payer swaption under the swap measure, is closely related to the BSM formula of a European call option, see equation 7. Similarly, a receiver swaption can be seen as a European put option on the forward swap rate with repeated payoffs, because it gives an investor the right to receive a certain preset swap rate (Hull 2018, 673; Lesniewski 2013b, 9; lesniewski 2019c; 29).

Shortcomings of Black's model

Black's model is still broadly used in the market. Brokers give implied volatilities coming from Black's model and extensions when quoting caps, floors and European swaptions. An important principle of the model is that the variable underlying the option has a lognormal distribution at maturity. Although this assumption works for all extensions separately making them internally consistent, they are not consistent with each other. To price caps the model assumes Libor forward rates to be lognormally distributed. This while to price swaptions, Black's formula assumes swap rates to lognormally distributed. Both assumptions cannot occur at the same time. This possesses a problem for pricing non-ATM swaptions because the non-ATM swaption market is much less liquid as the non-ATM caps and floors market. The higher liquidity results in more reliable quotes. Therefore, it would be very useful to integrate this information and market expectations on non-ATM Caps and floors when pricing non-ATM swaptions. However, using Black's implied volatility on forward rates coming from these non-ATM caps/floors, is not possible due to the inconsistency in assumptions between the two formulas. In addition, the model does not describe the movement of interest rates through time, it is hard to value American style options. Furthermore, the volatility is assumed to be constant. This is not supported by interest markets. Here volatilities exhibit term structure, depended on the option's expiration date, and smiles or skews emerge in implied volatility curves (Hull 2018, 673; Rebonato 2002, 3; Brigo and Mercurio 2006, 195).To overcome these shortcomings, the Libor Market Model was developed based on the fundamentals of Black's models. It prices caplets and European swaptions consistently with Black's model, while consistently pricing different types of derivatives. Furthermore, it allows for the modelling of the entire term structure and is therefore able to price more exotic options. Finally, a user can fit his preferred volatility structure into the LMM. An overview of the LMM model and how to price swaptions with this model is given in the next part (Brigo & Mercurio 2006, 195; Hull 2018, 740).

1.2.2 Libor Market Model (LMM)

The forward dynamics

The model generally known as the Libor Market Model or the BGM model, Brace-Gatarek-Musiela model. LMM is the industry standard for interest rate modelling and the pricing of interest-rate derivatives (Lesniewski 2013c, 2). It builds further upon Black's model by assuming forward Libor rates are lognormal distributed (Brigo and Mercurio 2006, 195). This assumption has as a consequence that interest rates cannot become negative. In recent history this would have caused problems. A solution is to use shifted lognormal interest rates in which the lognormally distributed interest rates are shifted so negative interest rates become possible. Another possible solution is to assume interest rates are normally distributed, also called the Bachelier model (Hull 2016). However, since interest rates have become strongly positive again, this paper will focus on the lognormal LMM, which is consistent with Black's model. Under this lognormal assumption, the forward-measure dynamics of the LMM shows that forward rate $f_k(t)$ only follows a driftless geometric Brownian motion if a zero-coupon bond that pays 1€ at time T_{k+1} is used as numeraire, see equation 8. The other forward rates also follow a geometric Brownian motion, but one that has a drift. Furthermore, the volatility is no longer constant and the correlation between different forward rates becomes important when the process consists of a drift. For pricing swaption it is important to look at the dynamics of this model for forward rates that do not become driftless under the given measure. This will become clear later in this section (Lesniewski 2019d.; 13)

Introducing the Libor and Euribor

Until now while talking about the modelling of risk-free interest rates we mentioned a general forward rate. In this paragraph the Libor rate is introduced. The "London Interbank Offered Rate" or Libor is an unsecured short-term borrowing rate between British banks. It is published daily by the British banking association and is available in different currencies and ranges for a period between one day to one year (Hull 2018, 77). The market standard model for interest rates is named after Libor rates, because Libor rates are still the most used benchmark rate in financial contracts like swaps. PwC Belgium is mostly working with continental banks or the continental branch of international banks. Therefore, the benchmark rate it encounters is Euribor rather than Libor or the "European Interbank Offered Rate". This rate follows the exact same mechanism as Libor but for continental European banks. We can switch between the Libor and Euribor in our equations or models without any issue. Therefore, from now on we will still talk about the Libor Market model because it is the convention, but the forward rates that are modelled are the Euribor rates (Kiff 2012).

An important remark on the Ibor rates that impacts our pricing tool is the risk related to Ibor rates. Because Ibor represents the short-term borrowing rates of AA banks, and hence has little amount of credit risk embedded, Libor used to be a proxy for the risk-free rate while pricing derivatives. However, the financial crisis of 2008 has shown that in times of market distress the credit spread in the Ibor can surge, making it not an ideal proxy for the risk-free rate. Therefore, nowadays closer proxies for the risk-free rate are used. The market standard is Overnight Indexed Swap or OIS discounting. OIS rates are overnight unsecured borrowing rates between banks. The very short horizon makes it even more unlikely for banks to default. Hence, the credit risk is limited to the minimum (Hull 2018, 77). Furthermore, OIS rates are transactions based and not based on a survey like Ibor, making it less prone for manipulation. The OIS rate for the Euro is called the ESTR rate or Euro short-term rate. The use of OIS discounting impacts the dynamics of the LMM. The forward rate used for discounting is no longer equal to the forward rate that is modelled. To keep the complexity of the pricing tool under control we assume the OIS/Euribor-spread to be deterministic, see equation 9. This allows us to describe the forward-measure dynamics of the LMM in the following way, see equation 10 (Lesniewski 2019d, 13).

Swaptions as foreigners in the LMM

To price swaptions and caplets in a consistent way, it is needed to bring them under the same measure, use the same numeraire. The LMM uses a zero-coupon bond as numeraire and hence assumes the forward Euribor rates to be lognormally distributed. This means caplets play a home game in comparison with swaptions. To price swaptions we need to link swap rates with the modelled forward Euribor rates, see equation 11. This swap rate can afterwards be used to compute the payoff of a swaption, see equation 12. These equations also show why it is important to look at the dynamics of LMM when the forward rate is not driftless. In the payoff of a swaption the swap rate lies within the expectations, and this swap rate consists of multiple forward rates. Hence, it is impossible to make the process driftless because there are multiple forward rates within the expectations. Furthermore, due to the modeling of the multiple forward rates the correlation becomes important as well, see equation 9. As a result, multiple correlated Brownian motions need to be modelled, therefore swaption pricing using LMM can only be done by Monte Carlo simulations (Brigo and Mercurio 2006, 195).

We have now shown two distinctive ways to price swaptions. First, by using black's analytical formula and secondly, by modelling the entire term structure using the LMM. The later approach solves shortcomings of black's approach, but it uses Monte Carlo simulations. Hence, the computational effort is higher, and the calculations are more complex. The remaining part of this paper is structured in the following way. Section 2 shows the methodology used in the tool and calibration of the parameters. This part is based on the theoretical foundations from section 1.2. The tool is tested using quoted implied swaption volatilities derived from Bloomberg. Afterwards, the swaption prices are obtained using black's formula. These swaption prices will then be used as benchmark for the LMM calculations using Monte Carlo simulations. These results are discussed in section 3. Finally, in section 4 a conclusion will be derived from the results and some interesting directions for further development of the tool are discussed. The pricing tool itself consists of an excel input file containing the data from Bloomberg and a bit of data calibration, see next section, and a Juyper Notebook consisting of Python code for the main calculations.

2 Methodology and Calibration

2.1 Benchmarking

2.1.1 Implied volatilities from the market

Quotes for swaptions on Bloomberg are used to benchmark the pricing tool. These quotes represent hypothetical contracts consisting of an option maturity of which the expiry coincides with the start, first reset date of the underlying swap, a tenor or maturity of the underlying swap, and the underlying forward swap rate. On Bloomberg these hypothetical contracts are quoted in implied volatility of the underlying forward swap rate. Figure 1 shows a typical volatility surface, the implied volatilities per tenor and maturity, for ATM swaptions (Brigo and Mercurio 2006, 21). Per tenor, the implied volatility is showed to be humped, with higher volatility for the medium range swap rates. This humped volatility structure is also clearly displayed in figure 2, which shows the volatility term structure of an ATM European cap (Brigo and Mercurio 2006, 18). A possible explanation is the control of the short-term interest rates by central banks, which is linked to the swap rate, for short maturities. Medium range maturities are mainly impacted by trading activities and these traders tend to overreact on changes in the short-term rates by monetary authorities. Hence, the volatility at medium range is higher. Furthermore, interest rates tend to be mean reverting, so on the long run volatility declines. Mean reversion on the long run is also why the hump is less pronounced for swaps with a longer tenor (Hull 2018, 673; Rebonato, Mckay and White 2009, 12).

Table 1a shows the quoted implied volatility of ATM swaptions for a maturity going from 1m till 25 years and tenor going from 1 till 25 years. The underlying forward swap rate is the Euribor 6 months rate, Bloomberg code 45, and the valuation date is 31/03/2023. The tables show Euribor implied vols, and used Euribor discounting, even though we mentioned earlier that OIS discounting would be used in the pricer. However, Bloomberg can provide an OIS calibrated “vcube” and Euribor calibrated one. Both approaches are different quotation mechanisms for the same instrument and give the same “correct” market price for a swaption if plugged into the right formula, equation 7. For OIS this means the Annuity and forward should be based on the OIS-curve, and for Euribor they should be based on the Euribor curve. The OIS implied volatilities tend to be lower because they are constructed using a lower yield curve than the Euribor curve, and hence need to offset an increase in annuity. For this table the Euribor-discounted vols are used because they are more readily available according to Bloomberg (2021). However, this is converted into euro prices in the next section to form the benchmark for the pricing tool, which will make use of OIS discounting. Figure 3 shows that the current implied volatility does follow its usual humped shape if we leave out the very illiquid longer maturities and tenor with very high volatilities. Nevertheless, the level of implied volatilities is much higher as in figure 1, screenshot of ATM Europeans swaptions on 08/02/2001. This shows the high uncertainty concerning the underlying of the swaptions, which is the swap rate. Hence, it displays a high uncertainty concerning the evolution of interest rates.

2.1.2 Benchmark price: Black’s formula for swaptions

As mentioned in the previous paragraph, Bloomberg quotes the “Black” implied volatility, so lognormal distributed volatility. By plugging this implied volatility into Black’s formula for swaptions, see equation 7, and using the same parameters as for the hypothetical contract, we obtain a swaption price in euro instead of a quote in implied volatility. This translation of the swaption price into euro can later be used to benchmark the pricing tool, which is done using a specific metric, see section 2.1.3.

Most of the data needed for Black’s formula comes from table 1a and 1b, where table 1b shows the ATM strike swap rate for each swaption. For the annuity the Euribor 6m curve is obtained from Bloomberg. Because the swap is paying every 6m the rates for every 6m are needed, which are found using linear interpolation. Furthermore, we will be looking at 1x1- till 10x10-Swaptions, maturity of the option x Tenor of the swap. Therefore, we need a maximum Euribor

6m term of 20years. This choice is made, because swaptions with longer maturities are less liquid and, hence the implied volatilities are less reliable. The same reasoning holds for the swaps that are used for the construction of the forward Euribor curve. These forward rates are transformed into discount factors, see equation 3, to calculate the Annuity needed to discount value obtained by Black's formula, see equation 13. Finally, a notional of €1000 is chosen for the calculation of the swaption prices.

Table 2 clearly shows that the prices of a Swaption increases with the maturity of the option and the Tenor of the swap. This makes economic sense because the longer the option the larger the probability it will end up in the money, hence the higher the value. Same for the tenor, the higher the tenor, the bigger the opportunity once the option is in the money (Hull 2018, 234).

2.1.3 Benchmarking: performance metric

The performance of the pricing tool is measured by expressing the annual basis points difference between the prices obtained by the model with Black's benchmark prices, relative to the notional of the contract and adjusted for the maturity of the contract. This metric is obtained by dividing the difference between the two prices by the nominal multiplied with the maturity of the option. Afterwards, the outcome is scaled to display the difference in basis points, see equation 14. This is an internal metric used at PwC when auditing the value of financial instruments. For quoted instruments, such as European swaption the threshold is 10 bps.

2.2 LMM: Monte Carlo simulations

2.2.1 Methodology

To obtain the price of a swaption, the Euribor forward curve at moment when the option expires, is needed. This future Euribor forward curve can be modelled using Monte Carlo simulations. Each simulation makes a projection of the Euribor forward curve in the future using today's information. Afterwards, the swaption payoff that corresponds to this projection is calculated and discounted. This is done multiple times, a thousand in our case, and the average discounted payoff of all the projections is the value of the swaption. To start the simulation an equation describing the path of the forward rates is needed, this is where LMM comes in. Using the dynamics of forward rates described by LMM, see equation 10, the following path for forward Euribor rates can be constructed after some calculus, see equation 15. However, to simulate the path in a Monte Carlo simulation one must discretize this equation. Discretization can be done in multiple ways, but because the diffusion coefficient is deterministic, Euler's scheme coincides with Milstein's scheme. Applying either one gives the following result, see equation 16. As mentioned in section 1.2.2, it is not possible to make all forward rates driftless using the same numeraire, hence the modelled forward rates follow a geometric Brownian motion with a drift. In equation 16 the drift is represented by the summation in the second term. Finally, because the discretization results in a violation of the no-arbitrage condition, it is needed to take sufficiently small-time intervals while simulating the forward rates (Brigo and Mercurio 2006, 195). To overcome this problem, 100 time-steps per year are taken. Once, the path of the forward rate is known the modelling can be done by starting from the initial forward rates. The initial rates for e. g., a 2x5-swap, will be the Euribor 6m rates with maturity 2,5-7 years, because these correspond to the payments of the swap, which starts in two years and runs for 5 years. These initial rates are derived from Bloomberg, see section 2.1. In addition, to construct the path of the MC simulation, the volatility surface, correlation and forward ESTR rates need to be calibrated, which is done in the next part.

2.2.2 Calibration

ESTR/Euribor spread

Like the forward Euribor 6m curve, the forward ESTR curve is obtained from Bloomberg for the valuation day, 31/03/2023. After the same manipulations as for the Euribor, see section 2.1, we obtain figure 4 and table 3, which shows the corresponding rates. Both curves are upward sloping until the 2-year point, afterwards they decline until a maturity of 6 year, and stay flat until the 20-year point. An inverted to flat yield curve means, a lot of uncertainty about the macroeconomic outlook. This is also visualized by the high levels of implied volatility showed in section 2.1. A flat yield curve occurs in the transition of a normal, upward sloping yield curve to a downward sloping or inverted yield curve. Figure 5 shows yield curves corresponding to triple A European government bonds on 31/03/2022 and 31/03/2023. We can see that in this case the yield curve goes from a normal yield curve towards a more inverted curve due to an increase in short-term interest rates (European Central Bank 2023a). This is often seen as predictor of a recession (Bauer and Mertens 2018). However, currently there is no sign of recession in Europe, but the Fed has warned for the possibility of recession later this year. In the US the impact of the tighter monetary policy is already visible in the financial sector (Smith 2023). Furthermore, a government yield curve is not the same as the Euribor curve but both curves are linked with each other. Table 3 displays the ESTR/Euribor 6m spread. These spreads are the forward spreads that are currently implied by the market, they incorporate all market expectations at that point in time, and hence are the best proxy for the spreads in the future, when using deterministic spreads.

Correlation

The modelling of multiple forward rates also requires the calibration of the correlation between the forward rates, as can be seen in equation 16. A functional form for correlation is introduced, which reduces the dimensionality e.g., 40 forward rates, 20 years with a 6m interval, would result in the need to estimate 820 correlations. Furthermore, when using historical estimates without functional form problems with outliers and discontinuities often emerge (Lesniewski 2013c, 12; Brigo and Mercurio 2006, 195). The functional form for the correlation that is used in the pricing tool corresponds to the Doust (2010) correlation function, see equation 17, This function allows for a convex front and concave back of the correlation surface, which aligns to a higher decorrelation between two short-term rates than two long-term rates with the same maturity difference, which is supported by empirical evidence. An addition of the "long correlation"-factor to Doust's correlation, makes sure that the correlation does not go to zero when the maturity interval between two rates increases, see equation 18 (Rebonato, Mckay and White 2009, 21). This asymptotical minimum correlation, LongCorr, is the historical correlation between Euribor 6m forward rate at 0,5 year and at 20 years. This is calculated by taking a monthly snapshot of the Euribor 6m curve form April 2022 till March 2023 and taking the lognormal returns on the zero-coupon bonds which corresponds with the Euribor 6m forward rates for 0,5 year and 20 years. The prices of the zero-coupon bonds that pay 1€ at maturity are equal to the discount factor which can be found using equation 3. Only, 12 datapoints are used to estimate the correlation, which is rather low. However, making use of a functional form partially solves this issue. Furthermore, Rebonato, Mckay and White (2009, 17) argue that for European swaptions the impact of the correlation on the price is rather modest. To obtain the distance dependent decay coefficient β_k, the same is done for the other maturities. The coefficients are found by minimizing the total squared difference between the historical correlation and the correlation obtained from Doust functional form (Brigo and Mercurio, 2006, 195). Afterwards this correlation matrix is adjusted to include the LongCorr-factor. Table 4 shows a part of the 40x40-correlation surface, 20 years of rates with 6m intervals. Figure 6 visualises the convex decorrelation between the first Euribor rate with a 6m term and the other Euribor forward rates.

Volatility

One of the biggest perks of the LMM is that user can fit her/his preferred volatility structure into the model. Equation 19 shows the functional form of the volatility that is used in this pricing tool. This form allows for the humped volatility which his commonly witnessed in normal trading periods as described in section 2.1.1. Furthermore, it allows for gradually decaying volatility in periods with a lot of (expected) changes in interest rates by central banks. In these periods, traders anticipate on short-term rate changes by taking positions on them. Hence, high volatility on the short-term rates. Finally, by assuming the same volatility function for all forward rates, LMM becomes time homogenous. This means that the future volatility surface will look the same as today's surface, and the difference between the volatility of forward rates is only due to the difference in time to maturity (Rebonato, Mckay and White 2009, 12). Table 5a shows the parameters under difference scenarios found in papers from Brigo and Mercurio(2001; 2006); Rebonato, Mckay and White (2009); and an own calibration of the functional form, see next section. The first scenario from Brigo and Mercurio is calibrated in their book from 2001, the second in 2006. The scenarios from Rebonato, Mckay and White are displayed in their book from 2009 and display a typical volatility term structure in normal and exciting times. The effect of these different parameters on the volatility structure of forward rates is displayed in figure 7. A parallel upward shift is clearly seen between Brigo and Mercurio 2001 and 2006, and afterward another similar shift between Brigo & Mercurio 2006 and "normal times"- scenario from Rebonato, Mckay and White. Although the decay of volatility is a bit smaller. Furthermore, the "exciting times" - scenario displays the rapidly decaying curve as mentioned earlier. Finally, the calibrated curve also follows a humped pattern although the level of volatility lies much higher as in the other scenarios. This "Own Calibration" - scenario is calibrated in the following way. Firstly, Brigo and Mercurio (2006, 313) link the forward rate volatility with the swap rate volatility, see equation 20. This link is based on the "freezing the curve"-approximation from Rebonato shown in equation 21. Under this assumption the forward swap rate is a linear combination of forward rates. Furthermore, it assumes that the forward rates and the weights of each forward rate remain equal to their initial value. Although this is a strong assumption, previous research has shown that Rebonato's approximation leads to reasonable results (Brigo and Mercurio 2006, 195; Brace, Barton and Dun 1998). The initial forward volatilities are calculated by plugging initial values, (a = 0,0570; b = 0,0285; c =

0,20004; d = 0,1100), into the functional form. Afterwards a swaption implied volatility matrix can be filled using this parameter with equation 20. The obtained volatility matrix is compared with the one derived from the market, and by minimizing the total squared difference between the two, new calibrated parameters are found. To have parameters that make economic sense it is needed to set the following boundaries, see table 5b. As can be seen in both tables, the calibrated parameters move towards these boundaries to accommodate for the high implied volatility by the swaptions observed in the market. Table 6a shows the obtained swaption volatility matrix after using calibrated parameters, and in table 6b the differences with the market swaption volatilities in percentage-points are displayed. The differences are significant, which points towards issues in fitting the right volatility structure. A possible explanation could be that the current functional form does not have enough parameters or degrees of freedom to create a good fit for today's volatility term structure. By calibrating forward specific scaling factors for the volatility on caps/floors, it is possible to fit today's structure exactly, see equation 22. Afterwards, this information can be included into the functional form of volatilities and used to calibrate parameters a,b,c, and d, see equation 23 (Brigo and Mercurio, 2006). This extra calibration is complex and requires the bootstrapping of caplet volatilities out quoted cap volatilities. Hence it lies outside the scope of this work project.

3 Performance of the LMM pricing tool

3.1 LMM prices: Comparing the different volatility scenarios

The market prices, section 2.1- table 2, show increasing swaption values when the tenor of the underlying swap increases. This dynamic is also visible under each scenario, see table 7, which displays the MC prices under the different volatility scenarios. This is because an increase in the period in which potential gains can occur, results in an increase in the value of the contract. Furthermore, the value of an option also tends to increase with the maturity of the option, because it leaves more room for the option to end up "in-the-money". This result is witnessed in the market prices, the “normal times”-scenario of Rebonato, Mckay and White, and the “own calibration”-scenario. For the Brigo and Mercurio scenarios it is not the case. Here the value of the option seems to decrease with the maturity of the option. However, this is not due to the higher maturity, but could be due to the lower volatility of the underlying swap rate. An option is an insurance contract, and the value of an insurance increases when the uncertainty, volatility of the underlying increases. The volatility decreases with the term of the forward rate as can be seen on figure 7. If the maturity of the option increases, we move to the right of the volatility structure, and the forward rates which make up the swap rate, will have a lower volatility. Hence, the volatility of the swap rate decreases. It could be that for the Brigo and Mercurio scenarios the decrease in volatility, offsets the increase in maturity. For the other scenarios, the volatility of the swap rate, also decreases with the maturity, but here the decrease does not seem to outweigh the longer maturity (Hull 2018, 234).

3.2 LMM prices vs Black’s prices: the performance of the pricing tool

3.2.1 Performance of the pricing tool

Table 8 shows the performance metric for the pricing tool under each volatility scenario as described in section 2.1.3. Under the scenarios coming from the literature, the threshold of 10bps is breached for most swaptions. The scenarios from Brigo and Mercurio, and Rebonato, Mckay and White succeed in pricing the swaptions with a 1-year tenor, first column, quite well. However, if we move towards longer tenors the performance drops and differences go up. The “normal times”-scenarios of Rebonato, Mckay and White performs a bit better as the other scenarios from the literature, which could be explained by the humped shape of the volatility structure and the higher volatility level as the other non-calibrated scenarios. This corresponds to the shape of the swaption implied volatility surface observed in the market. However, the performance is still rather poor. This shows that given the unusually high levels of volatility on interest rates derivatives present in the market, figure 3, it is not possible to price swaptions using parameters form the literature which represent normal markets conditions. Even the "exiting times"-scenario from Rebonato, Mckay and White is not able to factor in the high implied volatility for long term forward rates and hence undervalues swaptions with longer tenors. The uniqueness of current interest rate conditions is further demonstrated by Figure 8, which shows that since 1980s, the Federal reserve has never increased its Federal funds target rate as fast as during this monetary tightening cycle (Richter 2023). Although, the ECB was lagging at first, it has also been hiking its rates at an unprecedented speed, see figure 9 (Trading Economics 2023). This brings us to the final scenario which does consider current market information on the volatility of forward rates. Therefore, we could already assume this model to outperform the other scenarios. This is indeed the case, the calibrated model succeeds in pricing much more swaption within the acceptable threshold. It is by far the best performing model. Especially, for the medium-term tenors the performance is very good. Nevertheless, it overprices short-term tenor contracts while heavily underpricing swaptions with long-term tenors and shorter maturities. These differences in performance of the pricing tool using the calibrated volatility are visualized in figure 10. This figure shows the performance metric for the tenor with the best performance, 3 years, and the worst performance, the 1- and 10-years tenor.

3.2.2 Drivers behind the mismatch between the pricing tool and the benchmark

Previous section has shown that the volatility structure seems to be the most important driver behind the mismatch between the market prices and the ones obtained with the pricing tool. The "own calibration" - scenario outperforms the non-calibrated scenarios, but even the calibrated volatility struggles in capturing volatility dynamics form the market. It is overstating the volatility on the short term while trying to reconcile with the higher volatility on the medium to long run. This problems of imperfections in the fitting are already highlighted in section 2.2 calibration – volatility. Figure 11 shows the change in value of a 5x5-swaption if we alter the implied volatility and keep everything else constant. This figure makes it clear that a mismatch in volatility heavily influences the final value of a swaption. Hence, we can assume that the biggest part of the price difference can be explained by imperfections in fitting the current volatility structure. The low amount of datapoints on which the functional form for the correlation is calibrated, could be another source for the suboptimal performance of the pricing tool. A final consideration in comparing the MC prices with benchmark prices is the standard error of MC simulations. The prices obtained from MC simulations are based on simulating multiple Euribor rate paths and the according payoffs. The simulated payoffs are independent identically distributed random variables (i.i.d), and by the central limit theorem we can assume that they are normally distributed. Hence, if we run enough simulations the average would be an unbiased prediction of the expected value, and the MC price converges with the "real" price of the LMM corresponding to the input parameters. For computational reasons the prices in table 8 are obtained using 1000 simulations. However, by increasing the number of simulations, a more correct MC price can be obtained. Table 9a shows "own calibration"- scenario prices after increasing the number of simulations to 10 000. These more "correct" prices also seem to be closer to the benchmark prices. Table 9b shows the performance metric for these prices, and the price difference seems to fall within the threshold more often. Because full convergence only happens when the number of trials moves towards infinity, it is more accurate to provide a confidence interval instead of looking at the obtained MC price. This interval shows a window in which the "real" price is situated with x% confidence, see equation 24. A 98% confidence level is used because this is often adopted in the literature (Brigo and Mercurio 2006, 195). Table 10 shows the confidence interval for MC prices from the "own calibration"-scenario, and increased numbers of simulations. If we compare these confidence intervals with the benchmark prices, we see that for most swaptions with tenors between 2-5 years, the Black's prices fall within the confidence interval, same for swaptions with long maturities. Hence, a part of the mismatch between our pricing tool and the benchmark prices in table 8, is due to noise from the MC simulations.

4 Conclusion

The objective of this work project was to create a pricing tool for ATM European swaptions using the LMM that can form the foundation for further development within the Quant team and/or can be used in practice. The results have shown that the pricing tool using the calibrated volatility structure, works well for swaptions with a tenor between 2-5 years. However, for long tenors and short maturities the pricing tool is performing less good. This is mainly due to problems in fitting today's volatility structure. The importance of the volatility term structure in pricing swaptions with LMM is also displayed by the week performance of the pricing tool for non-calibrated volatility scenarios. The uniqueness of current interest rates environment makes it impossible to price swaptions without using today's marked information on the volatility term structure of forward rates. Hence, we can conclude that the calibration of the volatility structure is key for a good pricing tool, and although the current tool already shows a lot of improvement in comparison with the non-calibrated scenarios, there is still need for refinement before the tool can be used by PwC while performing audit support or consultancy services. However, the tool forms a great base for further development of an inhouse tool for pricing interest rate derivatives. Suggestions for improvement and further development of the tool are discussed in section 5. Besides the pricing tool, this paper also forms a great starting point for new members or less experienced members of PwC's quant team to get more acquainted with interest rate derivatives and interest rate modelling. Therefore, although the tool is not yet able to price European ATM swaptions fully consistent with the market, I do believe this work project has succeeded in its goal.

5 Further development of the tool

First suggestion to improve the performance of the pricing tool is to increase the number of observations on which the correlation matrix is build. A second and more important suggestion is to develop better ways to calibrate the volatility structure on market information. Section 2.2.2 calibration – volatility already proposes a solution by calibrating forward rate specific scaling factors using caplet volatilities. This would allow to perfectly fit today's volatility term structure of forward rates. An easier solution could be to calibrate the volatility parameters using the current method but only on the tenor that is need. E.g., for a 5x5-swaption, the parameters are calibrated on the 5-years tenor column. The tool can also be extended to price more than only ATM European swaptions. By pricing swaptions using the LMM under the forward measure the tool allows for the integration of cap/floor information while pricing swaptions. This is useful for non-ATM options for which the market information on swaptions is scarce. The tool can be extended to include for volatility skews with the deterministic Constant Elasticity of Variance (CEV) extension from Andersen and Andreasen (2000). Adding stochastic SABR-volatility also allows to model a volatility skew and smile. Furthermore, SABR-LMM can even accommodate for the fact that implied volatilities in the future often move in an opposite direction as currently observed in the market. Hence, this SABR-LMM model has become the go-to model used by banks to quote implied volatility smiles and skews (Rebonato, Mckay and White, 2009). Given the forthcoming transition towards Secured Overnight Funding rates (SOFR), it can also be interesting to develop the tool more in that direction, by deriving a forward-looking term structure of SOFR rates from the future market (Federal Reserve Bank of New York 2023a). Finally, by changing the current lognormal LMM to a normal LMM, which can easily be done by using the Bachelier model instead of Black's model (Hull 2016), the tool could allow for negative interest rates. This could be interesting if in the (distant) future, interest rates will become negative again.

6 References

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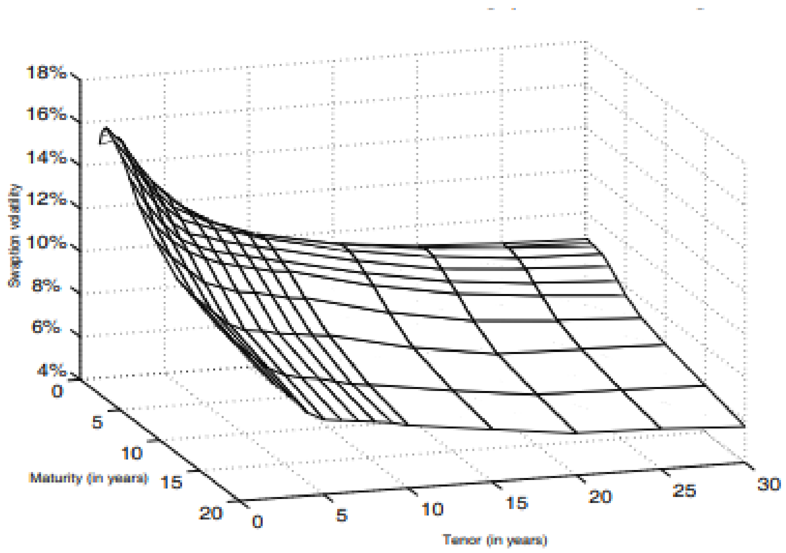
7 References

7.1 Figures

Figure 1 Typical volatility surface for ATM European swaption,

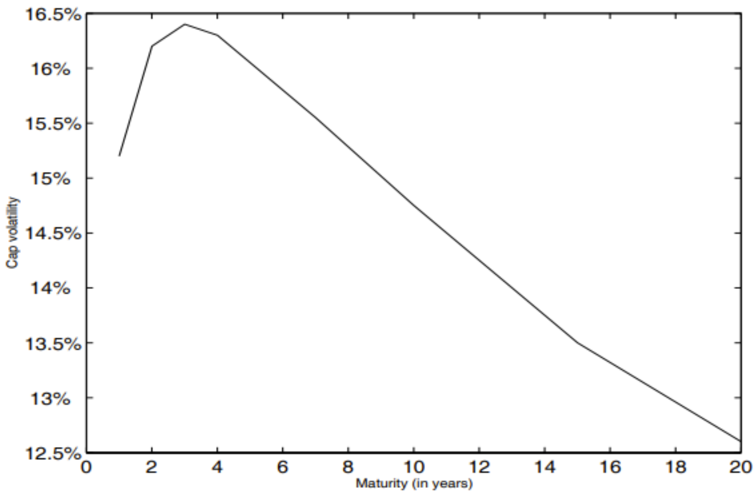
snapshot 08/02/2001 EOB



Source: Brigo and Mercurio (2006, 21)

Figure 2 Typical volatility term structure ATM European cap,

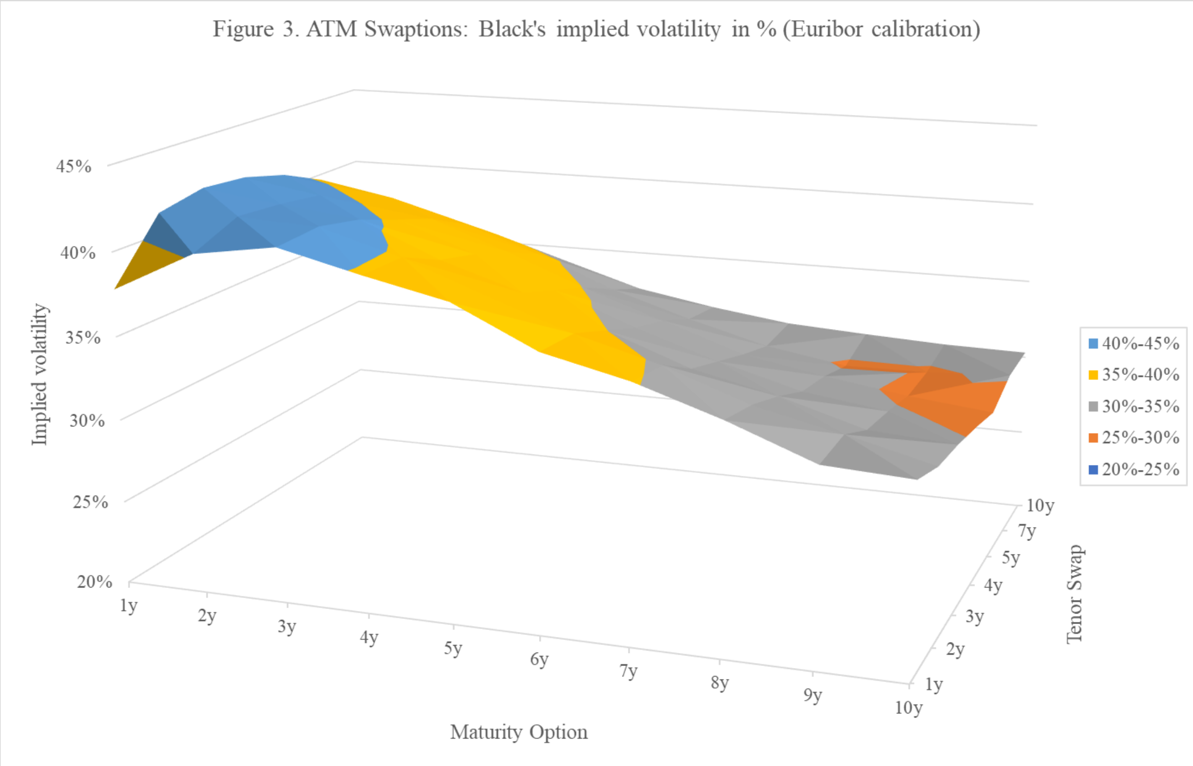
snapshot 13/02/2001 EOB



Source: Brigo and Mercurio (2006, 18)

Figure 3. Current volatility surface for ATM European swaption

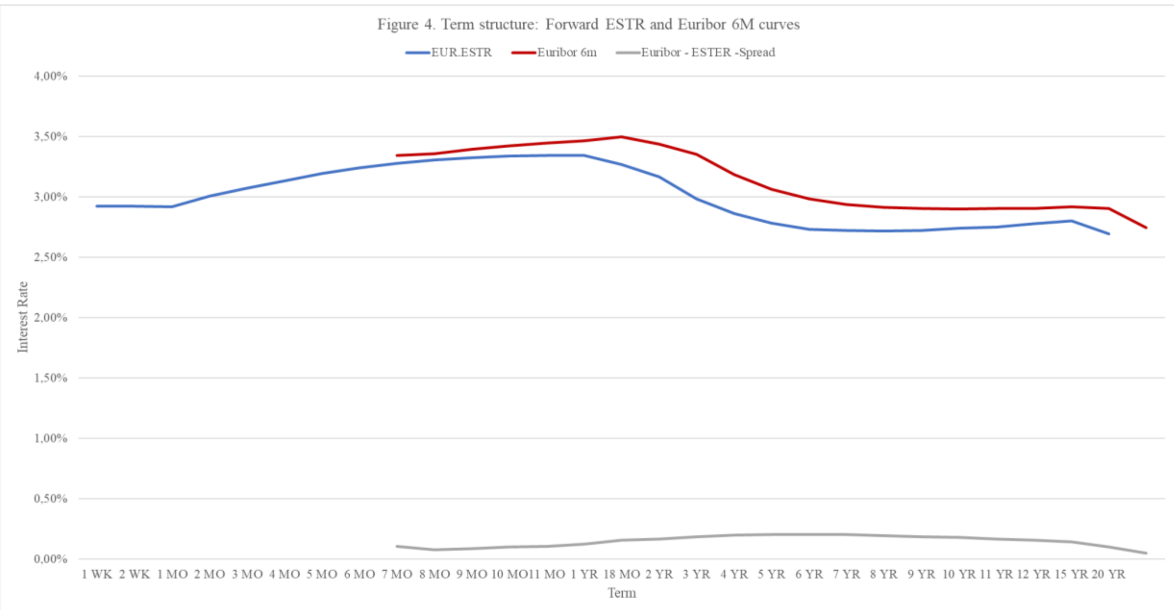
Snapshot 31/03/2023 EOB



Source: Bloomberg

Figure 4. ESTR and Euribor 6m months forward curve, and spread

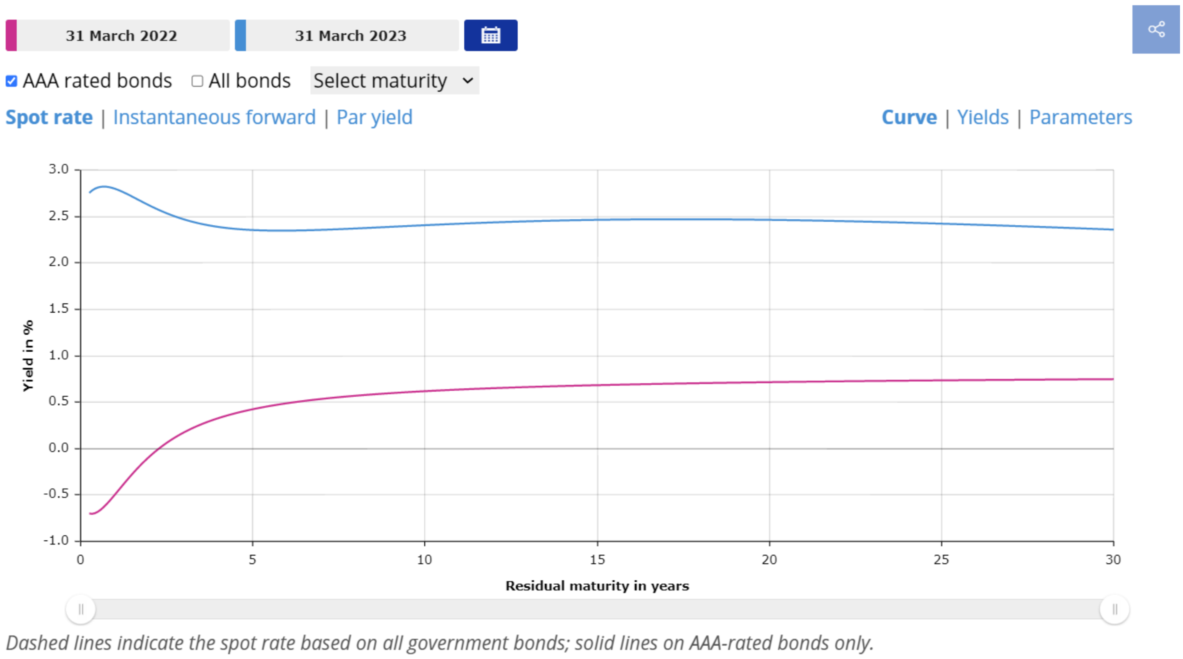
Initial forward rates at valuation date 31/03/2023 EOB



Source: Bloomberg

Figure 5. Triple A European government yield curve

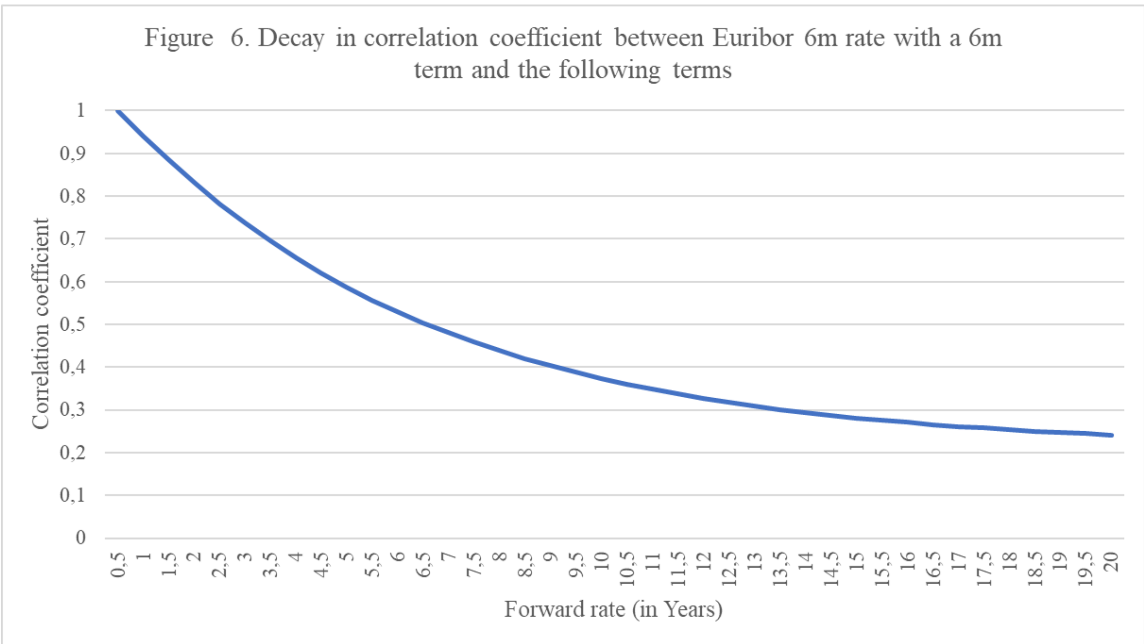
On the valuation date 31/03/2023 and1 year prior



Source: European Central Bank (2023a)

Figure 6. Decay in correlation coefficient between Euribor 6m rate with a 6m term and the following terms

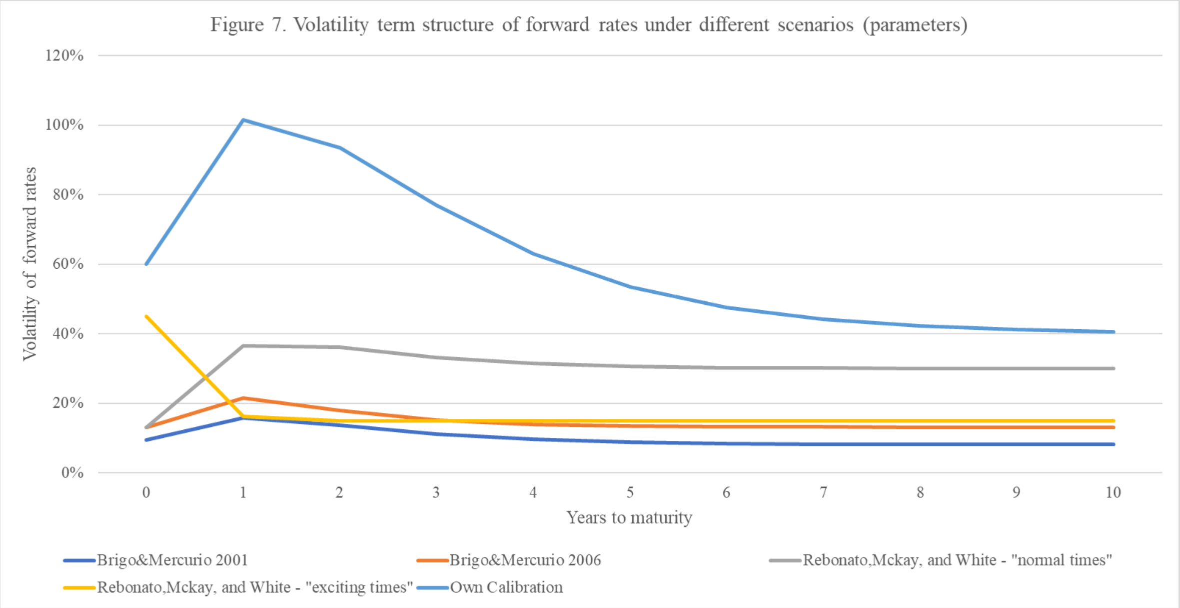
Correlation coefficients at valuation date 31/03/2023 EOB.



Source: own calculations

Figure 7. Different volatility scenarios: volatility term structure of the forward rates

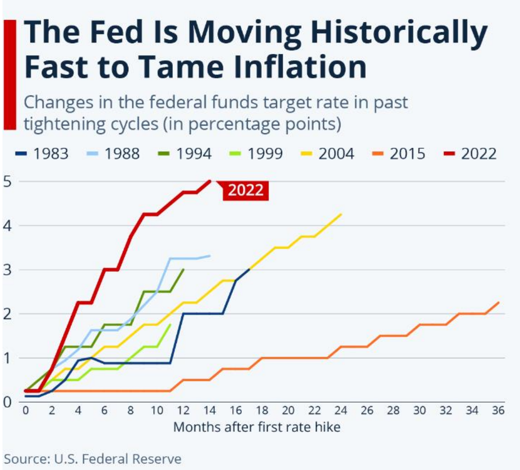
Volatility term structure following equation 18 and parameters from table 5



Source: own calculations

Figure 8. Target rates since the first month of a monetary tightening cycle.

The effective federal funds market is the unsecured borrowing market for financial institutions in the US, and the fed fund rate is the overnight rate at excess cash is lend out. The rate calculated as a volume-weighted overnight federal funds transactions. (Federal Reserve Bank of New York 2023b)



Source: Richter (2023)

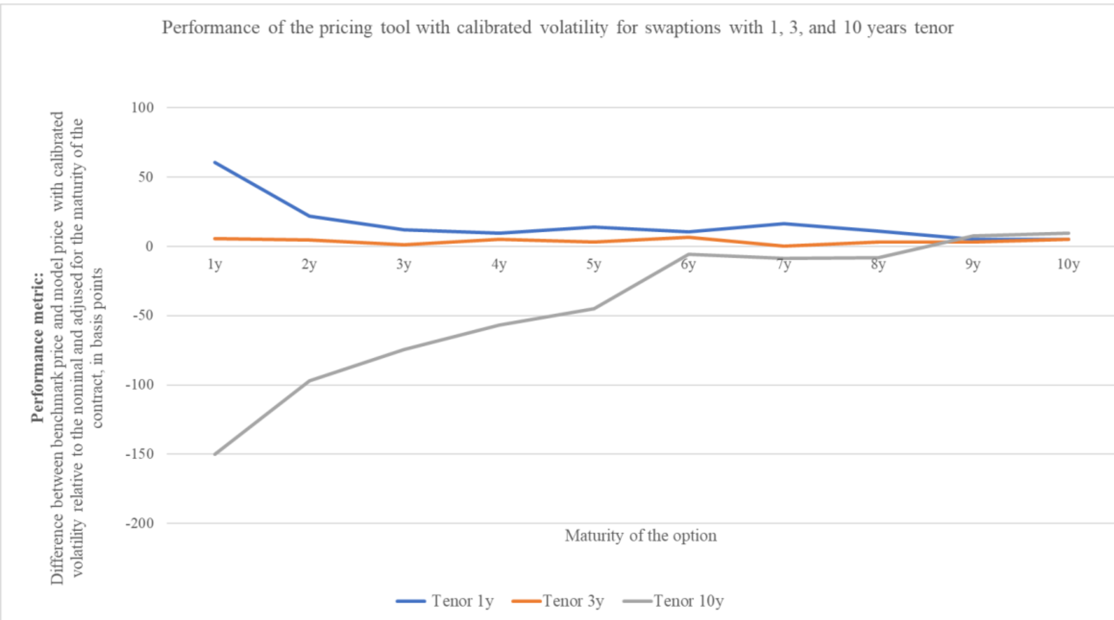
Figure 9. Evolution of the ECB's interest rate on its main refinancing operations (MRO)

The MRO provide the bulk of liquidity to the banking system, and the figures represent the variable rate tenders Minimum bid rate (European Central Bank 2023b).

Source: Trading Economics (2023)

Figure 10. Performance metric of the pricing tool using the calibrated volatility for tenors 1, 3, and 10 years.

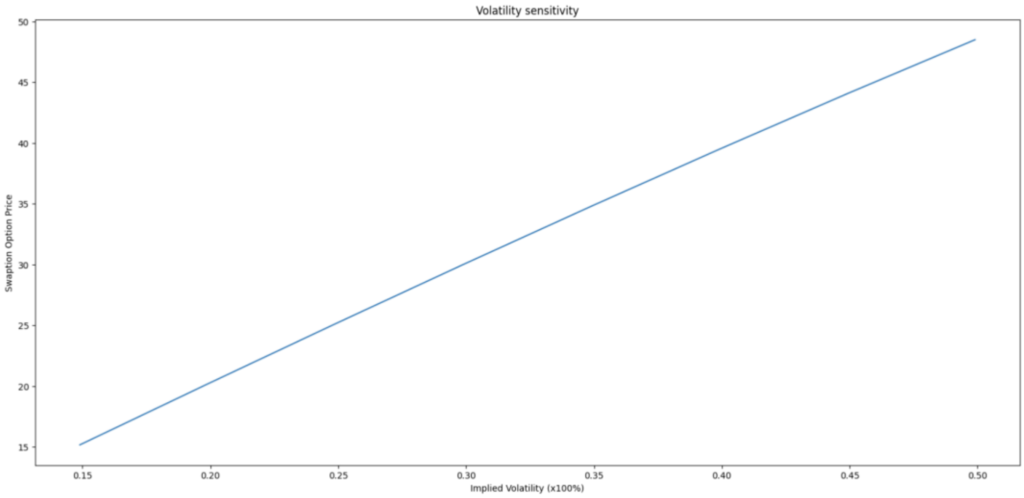
Valuation data 31/03/2023, data is coming from table 8 panel 5.



Source: own calculations

Figure 11. Price sensitivity of an 5x5 ATM European swaption to changes in the volatility when everything else is held constant.

Valuation date 31/03/2023



Source: own calculations

7.2 Tables

Table 1a, and 1b. ATM European swaptions: Black's implied volatility and strike rate

Table 1.a ATM European Swaptions: Black's implied volatility in % (Euribor calibration)

This table shows Black's implied volatility and strike rate of ATM European swaptions per option maturity and swap tenor

Valuation date 31/03/2023

σ		Tenor 1y	2y	3y	4y	5y	7y	10y	12y	15y	20y	25y	30y
Maturity	1m	34,41%	43,55%	44,77%	45,46%	44,72%	43,71%	40,08%	39,39%	38,75%	38,70%	39,74%	40,64%
	3m	35,02%	43,25%	44,18%	43,96%	43,10%	42,14%	39,53%	38,81%	38,19%	38,43%	39,73%	40,84%
	6m	35,49%	41,44%	42,18%	42,21%	41,41%	40,39%	37,67%	36,96%	36,43%	36,86%	38,43%	39,57%
	9m	36,29%	40,81%	41,61%	41,58%	40,93%	39,98%	37,31%	36,58%	36,10%	36,32%	37,89%	38,97%
	1y	37,80%	41,29%	41,86%	41,61%	40,86%	39,62%	37,27%	36,48%	35,99%	35,91%	37,45%	38,73%
	2y	40,26%	41,42%	41,18%	40,79%	39,75%	38,71%	36,82%	36,00%	35,60%	36,03%	37,57%	39,07%
	3y	41,01%	41,18%	40,59%	39,59%	38,65%	37,29%	35,58%	34,77%	34,77%	35,48%	37,05%	38,39%
	4y	39,80%	39,73%	38,48%	37,76%	36,87%	35,64%	34,04%	33,78%	33,76%	34,84%	36,41%	37,53%
	5y	38,71%	37,89%	36,90%	35,89%	34,90%	33,61%	32,53%	32,78%	32,68%	34,06%	35,62%	36,79%
	6y	36,34%	36,11%	35,05%	33,96%	33,18%	32,05%	31,65%	31,83%	32,01%	33,40%	34,89%	35,90%
	7y	35,17%	34,75%	33,19%	32,31%	31,44%	30,69%	30,96%	31,07%	31,58%	32,97%	34,38%	35,29%
	8y	33,52%	32,94%	32,03%	30,92%	30,51%	29,84%	30,66%	30,65%	31,45%	32,77%	34,16%	35,00%
	9y	31,59%	31,76%	30,53%	30,20%	29,73%	29,88%	30,41%	30,76%	31,36%	32,70%	34,06%	34,79%
	10y	31,32%	30,47%	30,19%	29,74%	29,18%	30,16%	30,31%	31,09%	31,47%	32,80%	34,06%	34,69%
	12y	31,07%	30,77%	30,01%	31,04%	31,35%	31,54%	32,43%	32,82%	33,11%	34,28%	35,25%	35,78%
	15y	38,87%	38,18%	37,02%	36,03%	34,90%	36,32%	36,65%	36,92%	36,40%	37,34%	37,48%	37,85%
	20y	56,88%	54,88%	52,59%	50,25%	47,96%	47,96%	46,03%	45,03%	42,69%	41,83%	42,17%	41,22%
	25y	101,34%	83,42%	74,88%	67,13%	61,69%	58,73%	54,18%	50,72%	45,77%	45,06%	43,91%	42,22%

Table 1.b ATM European Swaptions: Strike rates (Euribor calibration)

k		Tenor 1y	2y	3y	4y	5y	7y	10y	12y	15y	20y	25y	30y
Maturity	1m	3,62%	3,42%	3,24%	3,12%	3,04%	2,97%	2,96%	2,97%	2,96%	2,82%	2,64%	2,49%
	3m	3,64%	3,40%	3,22%	3,10%	3,03%	2,96%	2,95%	2,97%	2,95%	2,81%	2,63%	2,48%
	6m	3,56%	3,29%	3,13%	3,03%	2,97%	2,93%	2,93%	2,94%	2,92%	2,78%	2,61%	2,46%
	9m	3,43%	3,19%	3,05%	2,97%	2,92%	2,90%	2,91%	2,93%	2,90%	2,75%	2,58%	2,44%
	1y	3,25%	3,07%	2,96%	2,90%	2,87%	2,86%	2,89%	2,90%	2,87%	2,73%	2,56%	2,41%
	2y	2,89%	2,81%	2,78%	2,76%	2,78%	2,80%	2,86%	2,87%	2,81%	2,65%	2,49%	2,35%
	3y	2,73%	2,72%	2,72%	2,75%	2,77%	2,82%	2,86%	2,87%	2,77%	2,60%	2,43%	2,29%
	4y	2,72%	2,72%	2,75%	2,78%	2,80%	2,85%	2,88%	2,84%	2,75%	2,55%	2,38%	2,25%
	5y	2,71%	2,77%	2,80%	2,83%	2,86%	2,91%	2,90%	2,81%	2,72%	2,51%	2,34%	2,20%
	6y	2,83%	2,84%	2,87%	2,90%	2,91%	2,94%	2,87%	2,79%	2,67%	2,45%	2,28%	2,15%
	7y	2,85%	2,88%	2,92%	2,93%	2,96%	2,95%	2,82%	2,74%	2,60%	2,39%	2,22%	2,10%
	8y	2,92%	2,96%	2,96%	3,00%	2,98%	2,96%	2,77%	2,70%	2,53%	2,32%	2,16%	2,04%
	9y	3,00%	2,98%	3,02%	2,99%	2,98%	2,87%	2,71%	2,61%	2,45%	2,24%	2,09%	1,98%
	10y	2,97%	3,04%	2,99%	2,97%	2,96%	2,78%	2,64%	2,50%	2,36%	2,16%	2,02%	1,92%
	12y	2,90%	2,90%	2,90%	2,75%	2,66%	2,56%	2,37%	2,27%	2,14%	1,97%	1,86%	1,77%
	15y	2,29%	2,29%	2,28%	2,28%	2,28%	2,12%	2,00%	1,92%	1,83%	1,71%	1,64%	1,57%
	20y	1,70%	1,70%	1,69%	1,69%	1,69%	1,63%	1,57%	1,53%	1,49%	1,45%	1,39%	1,35%
	25y	1,45%	1,44%	1,44%	1,44%	1,44%	1,41%	1,38%	1,37%	1,36%	1,30%	1,26%	1,24%

Source: Bloomberg

Table 2. ATM European swaptions: Black's benchmark price

Table 2. ATM European Swaptions: Black's benchmark price

This table shows Black's of ATM European swaptions per option maturity and swap tenor

Valuationd date 31/03/2023

Notional € 1.000

price (€)		Tenor 1y	2y	3y	4y	5y	7y	10y
Maturity	1y	4,60	9,34	13,51	17,31	20,76	27,37	35,73
	2y	5,94	11,71	17,06	22,08	26,76	35,81	47,79
	3y	6,76	13,35	19,49	25,32	30,74	41,20	54,76
	4y	7,31	14,41	20,92	27,33	33,18	44,53	59,03
	5y	7,68	15,18	22,15	28,68	34,79	46,50	61,58
	6y	8,01	15,77	22,92	29,56	35,77	47,60	62,98
	7y	8,18	16,12	23,16	29,79	36,14	47,93	63,42
	8y	8,29	16,30	23,49	30,27	36,59	48,47	63,98
	9y	8,27	16,28	23,51	30,29	36,67	48,35	63,92
	10y	8,29	16,30	23,50	30,27	36,54	48,28	63,53

Source: own calculations

Table 3. ESTR and Euribor 6m initial forward rates, and forward spread

Table 3. ESTR rates and Euribor 6m rates				
This table shows the ESTR and Euribor 6m rates for a maturity going from 1 week till 20 years				
Valuation date				31/03/2023
Date	**Maturity**	**EUR.ESTR**	**Euribor 6m**	**Euribor - ESTER -Spread**
07/04/2023	1 WK	2,92%		
14/04/2023	2 WK	2,92%		
30/04/2023	1 MO	2,92%		
30/05/2023	2 MO	3,01%		
29/06/2023	3 MO	3,07%		
29/07/2023	4 MO	3,13%		
28/08/2023	5 MO	3,19%		
27/09/2023	6 MO	3,24%	3,34%	0,11%
27/10/2023	7 MO	3,28%	3,36%	0,08%
26/11/2023	8 MO	3,30%	3,39%	0,09%
26/12/2023	9 MO	3,32%	3,42%	0,10%
25/01/2024	10 MO	3,34%	3,44%	0,11%
24/02/2024	11 MO	3,34%	3,47%	0,13%
25/03/2024	1 YR	3,34%	3,50%	0,16%
23/06/1901	18 MO	3,27%	3,44%	0,17%
20/03/2025	2 YR	3,17%	3,35%	0,19%
15/03/2026	3 YR	2,98%	3,18%	0,20%
10/03/2027	4 YR	2,86%	3,06%	0,20%
04/03/2028	5 YR	2,78%	2,99%	0,20%
27/02/2029	6 YR	2,73%	2,93%	0,20%
22/02/2030	7 YR	2,72%	2,91%	0,19%
17/02/2031	8 YR	2,72%	2,90%	0,19%
12/02/2032	9 YR	2,72%	2,90%	0,18%
06/02/2033	10 YR	2,74%	2,90%	0,16%
01/02/2034	11 YR	2,75%	2,91%	0,16%
27/01/2035	12 YR	2,78%	2,92%	0,14%
11/01/2038	15 YR	2,80%	2,91%	0,10%
16/12/2042	20 YR	2,70%	2,74%	0,05%

Source: Bloomberg

Table 4: Forward Euribor 6m correlation matrix (partially)

Table 4. Euribor 6m - Correlation matrix													
This table shows a snipped of the correlation matrix of Euribor 6m forward rates													
Valuation date													31/03/2023
LongCorr	0,2110												
ρ	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y
1Y	1	0,99361	0,98404	0,97297	0,96091	0,94964	0,93916	0,92784	0,91651	0,90475	0,88179	0,851771	0,81073
2Y	0,99361	1	0,99723	0,99227	0,98592	0,97977	0,9741	0,96753	0,96079	0,95348	0,93892	0,920541	0,89742
3Y	0,98404	0,99723	1	0,99823	0,99476	0,99101	0,98746	0,9831	0,97847	0,97326	0,96267	0,949665	0,93489
4Y	0,97297	0,99227	0,99823	1	0,99868	0,99659	0,99446	0,99159	0,98842	0,98466	0,97679	0,96737	0,95808
5Y	0,96091	0,98592	0,99476	0,99868	1	0,99921	0,99813	0,99643	0,99437	0,99174	0,98595	0,97918	0,97385
6Y	0,94964	0,97977	0,99101	0,99659	0,99921	1	0,99981	0,9991	0,99799	0,99633	0,99234	0,987777	0,98569
7Y	0,93916	0,9741	0,98746	0,99446	0,99813	0,99981	1	0,99963	0,99882	0,99749	0,99402	0,989785	0,98777
8Y	0,92784	0,96753	0,9831	0,99159	0,99643	0,9991	0,99963	1	0,99965	0,99881	0,99618	0,992738	0,99159
9Y	0,91651	0,96079	0,97847	0,98842	0,99437	0,99799	0,99882	0,99965	1	0,99964	0,99782	0,995145	0,9948
10Y	0,90475	0,95348	0,97326	0,98466	0,99174	0,99633	0,99749	0,99881	0,99964	1	0,99901	0,99711	0,99759
12Y	0,88179	0,93892	0,96267	0,97679	0,98595	0,99234	0,99402	0,99618	0,99782	0,99901	1	0,999443	1
15Y	0,85177	0,92054	0,94967	0,96737	0,97918	0,98778	0,98979	0,99274	0,99515	0,99711	0,99944	1	1
20Y	0,81073	0,89742	0,93489	0,95808	0,97385	0,98569	0,98777	0,99159	0,9948	0,99759	1	1	1

Source: own calculations

Table 5: Different volatility scenarios - parameters

Table 5a. Volatility - different scenaios				
This table shows the different volatility scenarios are used in the pricing tool				
Valuation date				31/03/2023
Scenario	**a**	**b**	**c**	**d**
Brigo&Mercurio 2001	0,0134	0,1909	0,9746	0,0809
Brigo&Mercurio 2006	0,0000	0,2934	1,2508	0,1315
Rebonato,Mckay, and White - normal times	-0,1700	0,3700	1,1200	0,3000
Rebonato,Mckay, and White - exciting times	0,3000	1,5000	5,0000	0,1500
Own Calibration	0,2000	1,1000	0,7500	0,4000
Table 5b. Own Calibration - boundaries parameters				
Own Calibration	**a**	**b**	**c**	**d**
Lower boundary	-0,5000	0,0000	1,0000	0,0000
Upper boundary	0,3000	1,0000	2,0000	0,4000

Source: own calculations

Table 6: ATM European swaptions: calibrated swap rate volatility using own calibrated forward rate volatility

Table 6a. ATM European Swaptions: Calibrated implied volatility								
This table shows the implied volatility for ATM European Swaptions using the calibrated volatilty scenario. The panel below shows the difference of this obtained implied volatility with the quoted implied volatility shown in table 1a.								
Valuationd date							31/03/2023	
σ		Tenor						
		1y	2y	3y	4y	5y	7y	10y
Maturity	1y	5,14%	10,78%	15,30%	18,82%	21,69%	26,36%	32,17%
	2y	5,33%	10,92%	15,20%	18,53%	21,06%	25,45%	30,90%
	3y	5,27%	10,61%	14,67%	17,62%	20,08%	24,11%	29,60%
	4y	5,01%	10,09%	13,85%	16,69%	19,05%	22,94%	28,32%
	5y	4,81%	9,52%	13,09%	15,80%	18,00%	21,71%	27,20%
	6y	4,44%	8,97%	12,36%	14,93%	17,14%	20,84%	26,65%
	7y	4,27%	8,57%	11,78%	14,34%	16,36%	20,17%	26,32%
	8y	4,04%	8,10%	11,30%	13,62%	15,80%	19,54%	26,01%
	9y	3,83%	7,83%	10,78%	13,29%	15,37%	19,58%	25,81%
	10y	3,76%	7,47%	10,59%	13,01%	15,04%	19,63%	25,71%
Table 6b. ATM European Swaptions: Difference in Pct-points between Calibrated implied swaption volatility and the one quoted in the market								
%-points		Tenor						
		1y	2y	3y	4y	5y	7y	10y
Maturity	1y	- 0,33	- 0,31	- 0,27	- 0,23	- 0,19	- 0,13	- 0,05
	2y	- 0,35	- 0,30	- 0,26	- 0,22	- 0,19	- 0,13	- 0,06
	3y	- 0,36	- 0,31	- 0,26	- 0,22	- 0,19	- 0,13	- 0,06
	4y	- 0,35	- 0,30	- 0,25	- 0,21	- 0,18	- 0,13	- 0,06
	5y	- 0,34	- 0,28	- 0,24	- 0,20	- 0,17	- 0,12	- 0,05
	6y	- 0,32	- 0,27	- 0,23	- 0,19	- 0,16	- 0,11	- 0,05
	7y	- 0,31	- 0,26	- 0,21	- 0,18	- 0,15	- 0,11	- 0,05
	8y	- 0,29	- 0,25	- 0,21	- 0,17	- 0,15	- 0,10	- 0,05
	9y	- 0,28	- 0,24	- 0,20	- 0,17	- 0,14	- 0,10	- 0,05
	10y	- 0,28	- 0,23	- 0,20	- 0,17	- 0,14	- 0,11	- 0,05

Source: own Calculations

Table 7: ATM European swaptions: Mont Carlo prices under the different volatility scenarios

Table 7. ATM European Swaption: Monte Carlo prices - Comparison different scenarios

This table shows the outcome of the pricing tool for each volatility scenario

Valuationd date: 31/03/2023

Notional: € 1.000

Scenario 1 – Brigo & Mercurion scenario 1

price (€)		Tenor						
		1y	2y	3y	4y	5y	7y	10y
Maturity	1y	4,75	2,34	4,59	7,51	9,26	10,81	10,78
	2y	3,91	5,50	8,06	9,76	11,30	11,82	11,68
	3y	3,09	6,28	8,03	9,39	9,63	10,22	9,04
	4y	2,45	5,57	6,93	7,67	6,88	7,59	6,91
	5y	2,08	5,13	5,32	5,63	5,31	5,35	4,05
	6y	2,45	3,87	4,48	4,10	4,23	3,83	3,45
	7y	2,73	3,42	3,64	3,40	3,53	2,88	3,35
	8y	3,01	3,17	2,83	2,85	2,63	2,97	3,07
	9y	3,50	2,49	2,96	2,31	3,02	2,98	5,73
	10y	3,82	2,92	2,44	2,92	3,71	3,47	9,42

Scenario 2 – Brigo & Mercurion scenario 2

price (€)	Tenor						
	1y	2y	3y	4y	5y	7y	10y
1y	4,91	2,80	5,38	7,53	9,79	11,27	11,39
2y	4,45	5,81	8,50	10,19	11,31	12,10	12,86
3y	3,69	6,82	9,19	9,88	9,87	11,20	10,58
4y	3,33	6,22	7,87	8,69	9,02	9,16	8,91
5y	2,94	6,31	6,51	7,18	7,47	7,57	7,20
6y	3,06	4,50	5,93	6,22	5,93	6,82	5,81
7y	3,27	5,01	5,60	5,37	5,98	6,20	6,56
8y	4,03	4,30	4,71	5,19	4,71	5,76	6,39
9y	4,45	3,93	4,54	4,53	5,53	6,38	10,00
10y	4,79	4,29	4,65	6,06	6,32	7,08	12,98

Scenario 3 – Rebonato, Mckay and with - "normal times"

price (€)		Tenor						
		1y	2y	3y	4y	5y	7y	10y
Maturity	1y	6,23	4,80	7,15	9,80	12,08	14,12	14,44
	2y	6,49	8,75	11,00	13,80	16,06	16,80	19,10
	3y	6,11	10,50	13,05	15,00	16,23	17,90	19,25
	4y	6,54	10,20	13,69	14,60	16,09	19,24	20,31
	5y	6,57	10,18	13,46	14,95	15,80	17,96	19,35
	6y	6,59	10,08	13,59	13,48	16,55	19,58	21,54
	7y	7,77	11,05	13,97	14,96	18,38	19,20	24,17
	8y	9,24	10,78	12,92	16,69	15,96	22,86	29,15
	9y	7,99	12,99	13,19	15,61	18,78	24,46	33,86
	10y	8,37	11,72	15,13	16,53	21,03	25,98	39,81

Scenario 4 – Rebonato, Mckay and with - "exciting times"

price (€)	Tenor						
	1y	2y	3y	4y	5y	7y	10y
1y	10,85	14,17	16,83	20,04	23,09	23,55	25,03
2y	14,38	19,93	25,46	29,88	32,42	35,28	38,32
3y	15,53	22,81	28,81	37,67	37,26	40,71	45,25
4y	18,26	25,01	31,79	36,37	36,79	48,16	53,52
5y	14,20	24,80	32,97	42,60	45,89	48,99	60,49
6y	17,76	26,88	45,16	42,03	50,42	50,56	61,87
7y	18,49	40,43	44,39	46,11	36,73	54,93	80,92
8y	16,14	28,55	33,48	63,27	53,05	63,68	91,37
9y	19,51	38,85	38,47	48,95	57,94	59,53	100,91
10y	20,20	35,50	48,39	50,81	60,75	87,24	107,79

Scenario 5 – Own calibration

price (€)		Tenor						
		1y	2y	3y	4y	5y	7y	10y
Maturity	1y	10,67	9,88	14,05	16,09	17,25	20,29	20,73
	2y	10,31	15,65	17,93	23,38	24,28	27,53	28,35
	3y	10,32	18,26	19,77	23,61	28,06	28,99	32,43
	4y	11,01	16,60	22,99	25,94	28,70	32,65	36,35
	5y	14,63	22,87	23,80	28,39	26,89	30,57	39,08
	6y	14,29	16,31	26,83	26,48	28,11	35,91	59,54
	7y	19,73	24,15	23,42	28,24	31,42	39,81	57,20
	8y	17,06	22,56	26,08	31,41	37,08	49,90	57,54
	9y	12,82	22,96	26,34	37,98	36,92	50,32	70,72
	10y	13,19	21,15	28,56	37,47	39,78	43,93	72,90

Source: own calculations

Table 8: ATM European swaptions: Mont Carlo prices under the different volatility scenarios vs Black's prices

Table 8. ATM European Swaptions: Monte Carlo simulations price vs Black's price

This table shows for each scenario the difference in basis points and adjusted for the maturity of the option, between Black's benchmark prices (table 2) and the prices obtained from the pricing tool (table 7) relative to the notional of the contract.

Valuationd date: 31/03/2023
Notional: € 1.000

Scenario 1 – Brigo & Mercurion scenario 1

bps (Maturity \ Tenor)	1y	2y	3y	4y	5y	7y	10y
1y	1,50	-69,98	-89,21	-98,02	-115,01	-165,53	-249,54
2y	-10,13	-31,08	-45,00	-61,59	-77,31	-119,95	-180,55
3y	-12,23	-23,58	-38,22	-53,11	-70,36	-103,25	-152,40
4y	-12,15	-22,12	-34,99	-49,14	-65,76	-92,36	-130,30
5y	-11,20	-20,10	-33,65	-46,10	-58,97	-82,30	-115,07
6y	-9,27	-19,83	-30,74	-42,44	-52,57	-72,95	-99,21
7y	-7,79	-18,14	-27,88	-37,69	-46,58	-64,35	-85,81
8y	-6,60	-16,42	-25,82	-34,28	-42,45	-56,88	-76,14
9y	-5,30	-15,32	-22,83	-31,09	-37,38	-50,42	-64,65
10y	-4,47	-13,37	-21,06	-27,35	-32,82	-44,81	-54,11

Scenario 2 – Brigo & Mercurion scenario 2

bps (Maturity \ Tenor)	1y	2y	3y	4y	5y	7y	10y
1y	3,14	-65,42	-81,26	-97,85	-109,65	-160,96	-243,40
2y	-7,45	-29,50	-42,77	-59,44	-77,26	-118,55	-174,62
3y	-10,25	-21,76	-34,34	-51,47	-69,58	-100,00	-147,26
4y	-9,97	-20,49	-32,62	-46,60	-60,41	-88,43	-125,29
5y	-9,48	-17,75	-31,27	-42,99	-54,66	-77,87	-108,77
6y	-8,25	-18,78	-28,32	-38,90	-49,73	-67,97	-95,28
7y	-7,01	-15,88	-25,09	-34,88	-43,09	-59,61	-81,23
8y	-5,33	-15,00	-23,47	-31,35	-39,86	-53,39	-71,99
9y	-4,25	-13,73	-21,08	-28,62	-34,60	-46,64	-59,91
10y	-3,49	-12,01	-18,85	-24,20	-30,21	-41,20	-50,56

Scenario 3 – Rebonato, Mckay and with - "normal times"

bps (Maturity \ Tenor)	1y	2y	3y	4y	5y	7y	10y
1y	16,31	-45,42	-63,57	-75,07	-86,83	-132,48	-212,98
2y	2,77	-14,81	-30,27	-41,41	-53,51	-95,01	-143,45
3y	-2,17	-9,50	-21,48	-34,42	-48,36	-77,67	-118,36
4y	-1,95	-10,52	-18,08	-31,82	-42,71	-63,23	-96,80
5y	-2,23	-10,00	-17,38	-27,45	-37,99	-57,09	-84,47
6y	-2,36	-9,48	-15,55	-26,80	-32,03	-46,70	-69,06
7y	-0,58	-7,25	-13,12	-21,18	-25,37	-41,04	-56,08
8y	1,19	-6,91	-13,21	-16,97	-25,79	-32,01	-43,53
9y	-0,32	-3,66	-11,47	-16,31	-19,88	-26,55	-33,40
10y	0,08	-4,58	-8,37	-13,74	-15,51	-22,30	-23,72

Scenario 4 – Rebonato, Mckay and with - "exciting times"

bps (Maturity \ Tenor)	1y	2y	3y	4y	5y	7y	10y
1y	62,50	48,31	33,19	27,28	23,26	-38,13	-107,00
2y	42,19	41,08	42,02	39,01	28,30	-2,64	-47,34
3y	29,22	31,52	31,05	41,16	21,73	-1,62	-31,68
4y	27,36	26,50	27,17	22,60	9,03	9,06	-13,78
5y	13,03	19,23	21,65	27,84	22,19	4,98	-2,19
6y	16,24	18,51	37,07	20,78	24,41	4,93	-1,86
7y	14,72	34,72	30,33	23,33	0,84	10,01	25,00
8y	9,81	15,30	12,49	41,25	20,57	19,01	34,24
9y	12,49	25,07	16,61	20,73	23,63	12,41	41,10
10y	11,92	19,21	24,89	20,54	24,21	38,96	44,25

Scenario 5 – Own calibration

bps (Maturity \ Tenor)	1y	2y	3y	4y	5y	7y	10y
1y	60,73	5,42	5,45	-12,25	-35,07	-70,72	-149,99
2y	21,86	19,71	4,34	6,48	-12,39	-41,38	-97,20
3y	11,87	16,37	0,91	-5,72	-8,94	-40,71	-74,44
4y	9,25	5,47	5,17	-3,46	-11,20	-29,70	-56,70
5y	13,89	15,38	3,30	-0,58	-15,80	-31,87	-45,01
6y	10,46	0,90	6,52	-5,13	-12,76	-19,49	-5,73
7y	16,50	11,47	0,38	-2,20	-6,74	-11,59	-8,90
8y	10,96	7,82	3,24	1,43	0,60	1,78	-8,05
9y	5,05	7,43	3,14	8,55	0,28	2,19	7,55
10y	4,91	4,85	5,06	7,21	3,24	-4,35	9,37

(Green shading: Within the 10bps threshold)

Source: own calculations

Table 9: ATM European swaptions: Mont Carlo prices under the calibrated volatility scenario with an increased amount of simulations

Table 9a. ATM European Swaption: Monte Carlo price - own calibration + increased amount of simulations								
This table shows for each scenario the differene in basispoints and adjusted for the maturity of the option, between Black's benchmark prices (table 2) and the prices obtained from the pricing tool (table 6)								
Valuationd date							31/03/2023	
Notional							€ 1.000	
Scenario 1		**Own Calibration + 10 000 simulations**						
price (€)		Tenor						
		1y	2y	3y	4y	5y	7y	10y
Maturity	1y	9,69	11,39	13,70	15,76	17,87	19,55	21,05
	2y	11,35	16,26	19,43	21,63	23,92	26,72	29,49
	3y	12,22	18,00	21,20	24,60	25,66	29,01	32,98
	4y	12,25	18,95	23,88	25,21	26,56	31,95	37,60
	5y	12,87	19,78	23,51	25,57	29,65	34,21	40,86
	6y	12,28	19,34	23,74	28,70	32,21	36,49	44,51
	7y	14,56	21,26	25,61	31,13	33,19	39,55	53,12
	8y	12,94	21,33	25,62	31,96	34,10	43,32	58,68
	9y	15,17	23,56	31,00	33,40	38,70	48,56	67,96
	10y	15,71	23,36	31,06	37,29	43,28	53,33	77,58

Table 9b. ATM European Swaptions: Monte Carlo simulations price vs Black's price								
This table shows for each scenario the differene in basispoints and adjusted for the maturity of the option, between Black's benchmark prices (table 2) and the prices obtained from the pricing tool (table 9a) relative to the notional of the contract.								
bps		Tenor						
		1y	2y	3y	4y	5y	7y	10y
Maturity	1y	50,97	20,47	1,88	-15,49	-28,94	-78,13	-146,88
	2y	27,05	22,72	11,84	-2,25	-14,21	-45,43	-91,48
	3y	18,19	15,49	5,69	-2,42	-16,94	-40,63	-72,59
	4y	12,35	11,35	7,38	-5,28	-16,55	-31,45	-53,58
	5y	10,38	9,19	2,73	-6,21	-10,28	-24,58	-41,44
	6y	7,12	5,95	1,37	-1,43	-5,94	-18,52	-30,79
	7y	9,12	7,35	3,51	1,92	-4,22	-11,96	-14,72
	8y	5,81	6,28	2,67	2,11	-3,11	-6,44	-6,63
	9y	7,67	8,09	8,32	3,45	2,25	0,23	4,49
	10y	7,42	7,07	7,56	7,03	6,74	5,05	14,05

Source: own calculations

Table 10: ATM European swaptions: 98%-Confidence interval of the MC price under the calibrated volatility scenario with an increased amount of simulations

Table 10. ATM European Swaptions: 98% Confidence interval - Monte Carlo price (own calibration+increased amount of simulations)															
This table shows the 98%-confidence interval from the prices obtained from the pricing tool using the own calibrated volatility structured and the increased amount of Monte Carlo simulations (table 9a)															
Valuationd date														31/03/2023	
Notional														€ 1.000	
price (€)		Tenor													
		1y		2y		3y		4y		5y		7y		10y	
		lb	ub	lb	ub	lb	ub	lb	ub	lb	ub	lb	ub	lb	ub
Maturity	1y	8,36	11,03	9,78	13,00	12,01	15,39	14,00	17,52	16,08	19,65	17,74	21,36	19,27	22,82
	2y	9,30	13,39	13,48	19,03	16,85	22,00	19,05	24,21	21,18	26,65	23,98	29,46	26,67	32,32
	3y	9,40	15,04	15,04	20,95	17,98	24,42	21,30	27,89	22,52	28,79	25,71	32,31	29,40	36,56
	4y	9,55	14,96	14,60	23,31	20,06	27,69	21,17	29,26	22,84	30,28	28,15	35,75	33,57	41,63
	5y	9,61	16,14	16,09	23,47	18,62	28,41	21,66	29,49	24,71	34,60	29,77	38,65	36,40	45,33
	6y	9,20	15,37	15,01	23,66	19,08	28,41	23,83	33,57	27,01	37,41	30,96	42,02	39,30	49,71
	7y	10,50	18,62	16,84	25,69	20,77	30,46	25,62	36,64	27,42	38,96	33,96	45,14	46,96	59,28
	8y	9,41	16,47	16,11	26,54	20,19	31,05	24,59	39,32	28,34	39,87	36,35	50,30	51,37	65,98
	9y	8,70	21,65	16,98	30,13	18,99	43,00	26,94	39,85	31,90	45,50	41,14	55,98	59,92	76,01
	10y	10,52	20,90	16,83	29,90	21,73	40,39	29,93	44,65	34,07	52,49	44,57	62,09	68,34	86,82

Black's price falls within the 98%-confidence interval

Source: own calculations

8 List of equations

Equation 1: (a) Stochastic differential equation describing the movement of the price of a stock. (b) Stochastic differential equation describing the movement of the price of a stock under the risk-neutral measure.

$$dS = S\left(\mu dt + \sigma\sqrt{dt}\epsilon\right) \quad \text{(a)}$$

$$dS = S\left(\mu dt + \sigma\sqrt{dt}\epsilon\right) \quad \text{(b)}$$

With dS = an infinitesimally small change in stock price S, μ= the expected return of the stock per time unit, r_f = the risk-free rate per time unit, dt = an infinitesimally small-time interval, σ = the volatility of the stock, and ϵ = a standard normal distributed variable (Hull 2018, 234).

Equation 2: Stochastic differential equation describing the movement of the forward rate under a forward risk-neutral measure.

$$df(t, T_k, T_{k+1}) = f_k(t)\sigma_k\sqrt{dt}\epsilon$$

With $df_k(t)$ = $df(t, T_k, T_{k+1})$ = an infinitesimally small change in forward rate f_k at time t, that goes from T_k till T_{k+1}, dt = an infinitesimally small-time interval, σ_k = the constant volatility of $f_k(t)$, and ϵ = a standard normal distributed variable (Hull 2018, 655).

Equation 3: Forward rate starts at T_k and matures at T_{k+1}

$$f(t, T_k, T_{k+1}) = \frac{1}{\tau_k}\left(\frac{1}{P(t, P_k, P_{k+1})} - 1\right)$$

With $f_k(t)$ = $f(t, T_k, T_{k+1})$ = a forward rate f_k at time t, that goes from T_k till T_{k+1}, $P_t(T_k, T_{k+1})$=P(t, T_k, T_{k+1}) = the price at time t forward contract on a risk-free zero-coupon bond or a forward discount factor starting at time T_k and paying 1€ at T_{k+1}, and τ_k = the appropriate year fraction (Lesniewski 2013a, 14).

Equation 4: Black's extension, the valuation of a caplet

$$caplet = L\tau_k P_0(T_k, T_{k+1})E_{k+1}[\max(f(0, T_k, T_{k+1}) - cap\ rate_k, 0)]$$

$$= L\tau_k P_0(T_k, T_{k+1})\left[E_{k+1}\left(f(0, T_k, T_{k+1})\right)N(d_1) - cape\ rate_k N(d_2)\right]$$

$$= L\tau_k P_0(T_k, T_{k+1}) f(0, T_k, T_{k+1}) N(d_1) - cap\ rate_k N(d_2)]$$ [2]

$$d_1 = \frac{\ln\left[f(0, T_k, T_{k+1}) / cap\ rate_k \right] + \sigma_k^2(T_k)/2}{\sigma_k \sqrt{T_k}}$$

$$d_2 = d_1 - \sigma_k \sqrt{T_k}$$

With L = principal value of the contract, τ_k = the appropriate year fraction, $P_t(T_k, T_{k+1})$ = the price at time t forward contract on a risk-free zero-coupon bond starting at time T_k and paying 1€ at T_{k+1}, E_{k+1} = the expectations in a world that is forward risk-neutral with respect to $P_t(T_k, T_{k+1})$ and $f(0, T_k, T_{k+1})$ = lognormal distributed forward rate at time 0 between T_k and T_{k+1} with constant volatility σ_k (Hull 2018, 655).

Equation 5: Forward swap rate

$$\text{Fixed: } s(t, T_m, T_n) * \sum_{i=m}^{n} \alpha_i P(t, T_{i+1}) = s_{m,n}(t) * A(t, T_m, T_n)$$

$$Floating: \sum_{j=a}^{b} \tau_j f(t, T_j, T_{j+1}) P(t, T_{j+1})$$

$$\Leftrightarrow s_{m,n}(t) = \frac{\sum_{j=a}^{b} \tau_j f(t, T_j, T_{j+1}) P(t, T_{j+1})}{A(t, T_m, T_n)}$$

With $s_{m,n}(t) = s(t, T_m, T_n)$ = forward swap rate observed at time t that settles at T_m and matures at T_{bn}. $P(t, T_i)$ = price of a zero-coupon bond at time t that pays €1 at time T_i , α_i = is the appropriate year fraction of the fixed leg, τ_j = is the appropriate year fraction of the floating leg, $f_k(t) = f(t, T_k, T_{k+1})$ = a forward rate f_k at time t, that goes from T_k till T_{k+1}, and $A(t, T_m, T_n)$ = forward annuity observed at time t that pays €1 that starts T_m and matures at T_n (Lesniewski 2013a, 16)

Equation 6: Stochastic differential equation describing the movement of the forward swap rate under a forward swap measure.

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[2] $E_{k+1}(f(t, T_k, T_{k+1})) = f(0, T_k, T_{k+1})$, todays forward rate between T_k and T_{k+1} is equal to the expected future rate between T_k and T_{k+1} in a forward risk-neutral world with respect to $P_t(T_k, T_{k+1})$.

$$ds(t, T_m, T_n) = s_{m,n}(t)\sigma_{m,n}\sqrt{dt}\epsilon$$

With $s_{m,n}(t) = s(t, T_m, T_n)$ = an infinitesimally small change in forward swap rate observed at time t that settles at T_m and matures at T_{bn}, dt = an infinitesimally small time interval, $\sigma_{m,n}$ = the constant volatility of the forward swap rate $s_{m,n}(t)$, and ϵ = a standard normal distributed variable (Brigo and Mercurio 2006, 195).

Equation 7: Black's extension, the valuation of a payer swaption

$$Swaption = A(0, T_m, T_n)L\left[E_A\left(s_{m,n}(t)\right)N(d_1) - s_K N(d_2)\right]$$

$$= AL[s_{m,n}(0)N(d_1) - s_k N(d_2)] \text{ }(^3)$$

$$d_1 = \frac{\ln\left[{}^{s_{m,n}(0))}\!/_{s_K}\right] + \sigma_{m,n}^2\,{}^{T_m}\!/_2}{\sigma_{m,n}\sqrt{T_m}}$$

$$d_2 = d_1 - \sigma_{m,n}\sqrt{T_m}$$

With L = principal value, m = amount of payments within a year, n = maturity of the swap in years, $A(t, T_m, T_n)$ = forward annuity observed at time t that pays €1 that starts T_m and matures at T_n, E_A = the expectations in a world that is forward risk-neutral with respect to $A(t, T_m, T_n)$, s_k is the fixed rate of a swap in the market at time T_m and $s_{m,n}(t) = s(t, T_m, T_n) =$ lognormally distributed forward swap rate observed at time t that settles at T_m and matures at T_{bn} with a constant volatility $\sigma_{m,n}$, (Hull 2018, 655).

Equation 8: LMM forward-measure dynamics under forward measure Q^i

$$df_k(t) = \sigma_k(t) * f_k(t)\, X \begin{cases} -\sum_{j=i+k}^{k} \frac{\rho_{kj}\,\tau_j\,\sigma_j(t) f_j(t)}{1 + \tau_j\, f_j(t)} dt + dZ_k(t), & if\ i > k, \\ dZ_k(t), if\ i = k, & \\ +\sum_{j=i+k}^{k} \frac{\rho_{kj}\,\tau_j\,\sigma_j(t) f_j(t)}{1 + \tau_j\, f_j(t)} dt + dZ_k(t), & if\ i < k. \end{cases}$$

With Z_k = Wiener process under measure Q^i, ρ_{kj} = correlation coefficient between forward rate

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[3] $E_A(s_N(t)) = s_N(0)$, todays forward swap rate is equal to the expected future swap rate in a forward risk-neutral world with respect to A(t).

k and j, τ_j = the appropriate year fraction, $f_k(t)$ = lognormal distributed forward rate at time t between t_k and t_{k+1} with volatility $\sigma_k(t)$ (Brigo and Mercurio 2006, 195).

Equation 9: Forward Euribor rate

$$\mathrm{L}(\mathrm{t}, T_m, T_n) = f(t, T_m, T_n) + B(t, T_m, T_n)$$

$$\Leftrightarrow \mathrm{L}(\mathrm{t}, T_m, T_n) = \frac{1}{\tau_n} \frac{P(t,t,T_m) - P(t,t,T_n) + \tau_n P(t,t,T_n) * B(t,T_m,T_n)}{P(t,t,n)}$$

With $L_n(t) = \mathrm{L}(\mathrm{t}, T_m, T_n)$ = forward Euribor rate observed at time t that starts at T_m and matures at T_n. $P(t, T_j, T_i)$ = forward price of a zero-coupon bond or discount factor at time t that starts at T_j, and pays €1 at time T_i , τ_j = is the appropriate year fraction of the floating leg, and $B(t, T_m, T_n)$ = Spread between forward ESTR rate observed at time t that starts at T_m and matures at T_n and the corresponding forward Euribor rate (Lesniewski 2013b, 8).

Equation 10: LMM forward-measure dynamics under forward measure Q^i, with OIS discounting.

$$dL_k(t) = \sigma_k(t) * L_k(t)\ X \begin{cases} -\sum_{j=i+k}^{k} \frac{\rho_{kj}\ \tau_j\ \sigma_j(t) L_j(t)}{1 + \tau_j f_j(t)} dt + dZ_k(t), & if\ i > k, \\ dZ_k(t), if\ i = k, & \\ +\sum_{j=i+k}^{k} \frac{\rho_{kj}\ \tau_j\ \sigma_j(t) L_j(t)}{1 + \tau_j f_j(t)} dt + dZ_k(t), & if\ i < k. \end{cases}$$

With Z_k = Wiener process under measure Q^i, ρ_{kj} = correlation coefficient between forward rate k and j, τ_j = the distance between the forward rates, $L_k(t)$ = lognormal distributed forward Euribor rate at time t between t_k and t_{k+1} with volatility $\sigma_k(t)$, and $f_k(t)$ = ESTR rate at time t with maturity t_k. (Lesniewski 2019d, 16).

Equation 11: Forward swap rate 2.0

$$s_{m,n}(t) = \frac{\sum_{j=m}^{n} \tau_j L(t, T_j, T_{j+1}) P(t, T_{j+1})\ )}{A(t, T_m, T_n)}$$

Equation 12: Payoff payer swaption

$$Payoff\ swaption = P(0,T_m)E_m\left[\max(\, s_{m,n}(t) - s_K) * \sum_{j=m+1}^{n} \tau_j P\big(t, T_m, T_{j+1}\big)\right]$$

With $s_{m,n}(t) = \mathrm{s}(\mathrm{t}, T_m, T_n)$ = forward swap rate observed at time t that settles at T_m and matures at T_n. $P(t, T_i)$ = price of a zero-coupon bond or discount factor at time t that pays €1 at time T_i , τ_j = is the appropriate year fraction of the floating leg, $L(t, T_k, T_{k+1})$ = a forward Euribor rate L_k at time t, that goes from T_k till T_{k+1}, s_k is the fixed rate of a swap in the market at time T_m and $A(t, T_m, T_n)$ = forward annuity observed at time t that pays €1 that starts T_m and matures at T_n (Brigo and Mercurio 2006, 195).

Equation 13: Value of an Annuity

$$A(t, T_m, T_n) = \frac{1}{m}\sum_{i=1}^{mn} P(t, P_i)$$

With $A(t, T_m, T_n)$ = value of an annuity at time t, that goes from T_m till T_n and pays €1/m at each T_i, P(t, T_i) = discount factor at time t corresponding to rate with maturity T_i (Hull 2018, 655).

Equation 14: Comparison metric: yearly NPV difference as % of notional

$$Metric_{m,n} = \frac{MC\ price_{m,n} - Black\ price_{m,n}}{Notional * Maturity\ option}\ 10000$$

With $Metric_{m,n}$ = metric for a mxn-swaption (m =maturity of the option and n = tenor of the swap). (Internal source)

Equation 15: Forward dynamics of LMM after taking the natural log and applying Itô's Lemma

$$dln\big(L_k(t)\big) = \sigma_K(t)\sum_{j=i+k}^{k}\frac{\rho_{kj}\,\tau_j\,\sigma_j(t)L_j(t)}{1+\ \tau_j\,f_j(t)}dt - \frac{\sigma_k^2(t)}{2}dt + \sigma_K(t)\,dZ_k(t)\,,\ \ for\ k = m+1, \dots, n$$

With Z_k = Wiener process under measure Q^i, ρ_{kj} = correlation coefficient between forward rate k and j, τ_j = the appropriate year fraction, $L_k(t)$ = lognormal distributed forward Euribor rate at time t between t_k and t_{k+1} with volatility $\sigma_k(t)$, and $f_k(t)$ = OIS rate at time with maturity t_k. (Lesniewski 2019d, 16; Brigo and Mercurio 2006, 195).

Equation 16: Discretization of the forward dynamics of the LMM

$$ln\left(L_k^{\Delta t}(t+\Delta t)\right)$$

$$= ln\left(L_k^{\Delta t}(t)\right) + \ \sigma_K(t) \sum_{j=i+1}^{k} \frac{\rho_{kj}\ \tau_j\ \sigma_j(t) L_j^{\Delta t}(t)}{1+\ \tau_j\ f_j^{\Delta t}(t)} \Delta t$$

$$- \frac{\sigma_k^2(t)}{2} \Delta t + \sigma_K(t)\ (Z_k(\ t+\Delta t) - \ Z_k(t))$$

With Z_k = Wiener process under measure Q^i, ρ_{kj} = correlation coefficient between forward rate k and j, τ_j = the appropriate year fraction, $L_k(t)$ = lognormal distributed forward Euribor rate at time t between t_k and t_{k+1} with volatility $\sigma_k(t)$, and $f_k(t)$ = OIS rate at time with maturity t_k. (Lesniewski 2019d, 16; Brigo and Mercurio 2006, 195)

Equation 17: Doust's Functional form for correlation between forward Euribor rates

$$\rho_{ij} = e^{-\sum_{k=j}^{i-1} \beta_k \Delta T}$$

With $\rho_{i,j}$ = the correlation between of a forward Euribor rate that T_i and a forward Euribor rate that matures at T_j, and β_k = positive constant which represents the decay rate of correlation between different forward rates (Rebonato, Mckay and White 2009, 20).

Equation 18: Functional form for the correlation between forward Euribor rates

$$\rho_{ij} = Longcorr + (1 - Longcorr) e^{-\sum_{k=j}^{i-1} \beta_k \Delta T}$$

With $\rho_{i,j}$ = the correlation between of a forward Euribor rate that T_i and a forward Euribor rate that matures at T_j, Longcorr = The asymptotic minimum correlation, which equals the historical average correlation between the two furthest forward Euribor rates, and β_k = positive constant which represents the decay rate of correlation (Rebonato, Mckay and White 2009, 21).

Equation 19: Functional form for the volatility of forward Euribor rates

$$\sigma_k(t) = \left(a + b(T_k - t)\right)e^{c(T_k - t)} + d$$

With $\sigma_k(t)$ the volatility of a forward Euribor rate that starts at t and matures at T_k (Rebonato, Mckay and White 2009; 12).

Equation 20: Rebonato's forward swap rate approximation

$$\sigma_{m,n}^2 = \sum_{i,j=m+1}^{n} \frac{w_i(0)\, w_j(0)\, L_i(0)\, L_j(0)\, \rho_{i,j}}{s_{m,n}(0)^2} * \int_0^{T_m} \sigma_i(t)\sigma_j(t)dt$$

Equation 21: Forward swap rate 3.0 – "Freezing the curve" approximation

$$s_{m,n}(t) = \sum_{i=m+1}^{n} w_i(t)L_i\,(t)$$

$$\text{With } w_i(t) = \frac{\tau_i\, P(t,T_i)}{\sum_{k=m+1}^{n} \tau_l P(t,T_k)}$$

$$\text{“Freezing the curve”} \Leftrightarrow s_{m,n}(t) \approx \textstyle\sum_{i=m+1}^{n} w_i(0) f_i\,(t)$$

With $s_{m,n}(t) = s(t, T_m, T_n)$ = forward swap rate observed at time t that settles at T_m and matures at T_n. $P(t, T_i)$ = price of a zero-coupon bond at time t that pays €1 at time T_i , τ_j = is the appropriate year fraction, $L_k(t) = L(t, T_i, T_{i+1})$ = a forward Euribor rate L_i at time t, that goes from T_i till T_{i+1}, and $\sigma_i(t)$ = the volatility of $L_i(t)$ at time t, and $\sigma_{m,n}(t)$ the volatility of the swap rate at time t for a swap that starts at time T_m and expires at T_n (Brigo and Mercurio 2006, 195).

Equation 22: Forward rate specific scaling factor for caplet volatilities

$$k_{T_i}^2 = \frac{\hat{\sigma}_{T_i}^2\, T_i}{\int_0^{T_i} \left(\left(a + b(\tau)\right)e^{c(\tau)} + d\right)^2 d\tau} \quad (8)$$

With $\hat{\sigma}_k(t)$ the market implied volatility of a forward Euribor rate that matures at T_i, and K_{T_i} the forward specific scaling factor for forward Euribor rate that matures at T_i (Rebonato, Mckay and White 2009, 15; Brigo and Mercurio 2006, 195-313).

Equation 23: Functional form for the volatility of forward Euribor rates 2.0

$$\sigma_i(t) = K_{T_i}[\left(a + b(T - t)\right)e^{c(T_i - t)} + d]$$

With $\sigma_i(t)$ the volatility of a forward Euribor rate that starts t at T_i, K_{T_i} the forward specific scaling factor for forward Euribor rate that matures at T_i (Rebonato, Mckay and White 2009, 15 ; Brigo and Mercurio 2006, 195-313).

Equation 24: Confidence interval of Monte Carlo prices

$$\left[\frac{\sum_{j=1}^{N} Price_j}{N} - 2{,}33 * \frac{Std(\widehat{Price}, N)}{\sqrt{N}}, \frac{\sum_{j=1}^{N} Price_j}{N} + 2{,}33 * \frac{Std(\widehat{Price}, N)}{\sqrt{N}}\right]$$

With $Price_j$ = the MC price of simulation j, 2,33 = Z-factor of a standard normal distribution which is not surpassed with 98% confidence if a random variable out of the standard normal distribution is drawn, $Std(\widehat{Price}, N)$ = sample standard deviation of the MC price used as predictor for the unknown “real” or population standard deviation, and N = the amount of simulations in the MC (Brigo and Mercurio 2006, 195).