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Bachelor in Mathematics

COMPARATIVE EFFICIENCY ANALYSIS OF BANK BRANCHES - CAPE VERDEAN BANK

MEASURING BANK BRANCHES EFFICIENCY THROUGH DEA

MASTER IN MATHEMATICS AND APPLICATIONS

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Comparative efficiency analysis of bank branches - Cape Verdean bank

To my mom

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“Stand firm for what you believe in, until and unless logic and experience proves you wrong; Remember, when the emperor looks naked, the emperor Is naked; The truth and a lie are not "sort of the same thing"; And there is no aspect, no facet, no moment of life that can’t be improved with pizza.” (Daria)

Abstract

Data Envelopment Analysis (DEA) is a mathematical programming-based approach that evaluates the relative efficiency of a set of DMUs (Decision-Making Units) where the conversion of multiple inputs into multiples outputs complicates the comparison of the DMUs performances. A DMU is considered efficient if it belongs to the efficiency frontier created by the performance evaluation of all the DMUs in the set we are evaluating.

Portela (2003) developed a model to evaluate bank branches' efficiency that considered the influence and/or consequences of the new alternative distribution channels for banking transactions. This model divides Bank Branche's efficiency into three viewpoints: transactional efficiency, operational efficiency, and profit efficiency. Based on this work we will adapt this perspective to a Bank in Cape Verde. Considering Cape Verde's reality, the Bank showed a lot of interest in the transactional efficiency because they haven't been able to explore it as well as they wished. We will evaluate the Bank Branches of a Cape Verdean Bank considering two perspectives: transactional efficiency and operational efficiency. To do so we will use DEA models.

The goal on the transactional perspective is to assess the branch's capacity to redirect branch transactions to alternative channels, i.e., assess the performance of bank branches in encouraging the use of alternative means by its customers. To do this analysis we are going to use the BCC model and a Geometric Distance Function.

The operational measure of efficiency translates all types of operations that go in a bank branch considering the Bank goals. In this perspective, we had to deal with negative data. To do this analysis we are going to use the Range Directional Model (RDM), the Modified Slacks Based Model (MSBM), and a modified version of the Modified Slacks Based Model(MSBM).

Results show that most bank branches are efficient in both perspectives.

Keywords: Data envelopment analysis, DEA, Banking, Efficiency, Negative data

Resumo

Data Envelopment Analysis (DEA) é uma abordagem baseada em programação matemática que avalia a eficiência relativa de um conjunto de DMUs (Decision Making Units) onde a conversão de múltiplos inputs em múltiplas outputs complica a comparação dos desempenhos das DMUs. Uma DMU é considerada eficiente se pertencer à fronteira de eficiência estabelecida com base no desempenho de todas as DMUs do conjunto sobre avaliação.

Portela (2003) desenvolveu um modelo de avaliação da eficiência de agências bancárias que considera a influência e/ou consequências da utlização dos canais de distribuição alternativos das transações bancárias. Este modelo divide a eficiência das agências bancárias em três ópticas: eficiência transacional, eficiência operacional e eficiência de lucro. Com base neste trabalho vamos adaptar esta perspectiva a um Banco em Cabo Verde. Considerando a realidade de Cabo Verde, o Banco expressou bastante interesse no estudo da eficiência transacional porque é uma perspectiva que ainda não foram capazes de explorar de forma satisfatória. Avaliaremos as Agências Bancárias do Banco Cabo-verdiano considerando duas perspectivas: eficiência transacional e eficiência operacional. Para tal, usaremos modelos DEA.

O objetivo na perspectiva transacional é avaliar a capacidade da agência em redirecionar as transações da agência para os canais alternativos, ou seja, avaliar o desempenho das agências bancárias no incentivo ao uso de meios alternativos por seus clientes. Para fazer esta análise, vamos usar o modelo BCC e uma Função de Distância Geométrica.

A medida operacional de eficiência traduz todos os tipos de operações que são realizadas numa agência bancária considerando os objectivos do Banco. Nesta perspectiva, tivemos que trabalhar com dados negativos. Para fazer esta análise, vamos usar the Range directional model (RDM), the Modified Slacks based Model (MSBM), e uma versão modificada do Modified Slacks Based Model (MSBM).

Palavras-chave: Data envelopment analysis, DEA, Banco, Eficiência, Dados negativos

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Acronyms

ATM	Automated Teller Machine
CRS	Constant Return to scale
DEA	Data Envelopment Analysis
DFA	Distribution-free approach
DMU	Decision Making Unit
DRS	Decreasing Return to scale
ETM	Everywhere Teller Machine
FDH	Free disposal hull
GDF	Geometric Distance Function
IRS	Increasing Return to scale
MSBM	Modified Slacks Based Measure
ObsD	Observations made by the director of the Bank
POS/TPA	Automatic Payment Terminal
RDM	Range Directional Model
SBM	Slacks Based Measure
SFA	Stochastic frontier approach
TFA	Thick frontier approach
VRS	Variable Return to Scale

Chapter 1

Introduction

Efficiency and productivity are performance measures. According to Fried et al. (1993) the interest in measuring efficiency and productivity has two reasons. First of all, they allow us to measure performance. Second, only by measuring efficiency and productivity and, separating their effects from the effects of the production environment, can we explore their sources. Portela (2003) defends the importance of studying efficiency, through it's most simple definition as "doing this right".

Bogetoft and Hougaard (2001) said that inefficiency can be measured because inefficiency is a waste. Farrell (1957) identified two components in productive efficiency: technical (or physical), and allocative (or price). Koopmans (1951) provided a definition for technical efficiency: a producer is technically efficient if an increase in any output (and/or reduction in any input) requires a reduction in at least another output or an increase at least one input. Farrell (1957) introduced a measure of technical efficiency as the maximum equiproportionate reduction in all inputs that allows equivalent production of observed output. Frontier efficiency analysis is a performance measurement approach applied to bank branches. Frontier methods can be parametric and non-parametric. This paper focuses on a non-parametric approach to efficiency measurement: Data Envelopment Analysis (DEA).

DEA is a non-parametric technique for measuring and evaluating the relative efficiency of a set of Decision Making Unit (DMU) with alternative inputs and outputs. The Data Envelopment Analysis (DEA) first model was proposed by Charnes et al. (1978) and further studied in literature by others like Banker et al. (1984) and so much more. The rapid growth and widespread (and almost immediate) acceptance of the DEA method serves as testimony to its strengths and applicability (Cooper et al., 2011) . DEA has been recognized by researchers in several fields as an excellent methodology for modeling operational processes, (Cvetkoska, 2011). Some of DEA advantages are minimization of priori assumptions and applicability to multiple input and output cases.

In this thesis, we are going to analyze the efficiency of bank branches of a Cape Verdean Bank. To do so we will adapt the model for bank branch efficiency assessment present in Portela (2003) and Portela and Thanassoulis (2007), where she divided bank branch efficiency analysis into: 1) transactional efficiency; 2) operational efficiency; and 3) profit efficiency. To choose the direction of this study we had a few meetings with a director of the Bank we are working with. He expressed a lot of interest in studying bank efficiency from the transactional perspective since it was a study that they had always wanted to conduct themselves. Cape Verde is a 10 island (9 populated) archipelago, therefore the bank and the clients have a lot to benefit from the use of the new distribution channels, especially when it comes to the online platform. Cape Verde has a population of 555 988 and Santiago has nearly half of this population with the capital Praia having nearly 60% of Santiago's population. In most of the other islands, we have very scattered population centers, and in a lot of cases, there isn't a bank Branch nearby. The director explained to me that a situation that happens with frequency is: "person X lives in location A, but it has to go to location B every time it wants access to his money. That means that, in reality, person X's money value is the money withdrawal minus transportation costs" . This is as an example he used to express some of the importance of the new distribution channels in this specific context for Cape Verde clients. Another important point is that Cape Verde has a lot of emigrants, they estimate that there is probably more cape-verdeans living outside of Cape Verde than in, and the culture of cape-verdeans emigrants is to send money to their families in Cape Verde.

The goals of this study will be the analyses of the efficiency of the bank branches of the Cape Verdean Bank considering the transactional efficiency and the operational efficiency. The transactional efficiency aims to assess the branch's capacity to redirect branch transactions to alternative channel. The operational measure of efficiency translates all types of operations that go in a bank branch.

The data we will be using is mostly from the year 2019. The Bank in the study preferred these data as they felt they were a more accurate representation of the bank activities (pré covid19).

Besides the Introduction, this dissertation is divided in 5 other chapters:

- **Background** - Literature review on the context of efficiency and bank branches efficiency measurement. The objectives of this study are also presented in this chapter.

- **DEA Models** - In this chapter we will first introduce Data Envelopment Analysis, explore it's development and the traditional methods (CRR Model). After that we will present other approaches and new DEA models that we will use in this study. The BCC model used to assess transactional efficiency. The RDM, MSBM and modified MSBM model used to asses operational efficiency.

- **Measuring the performance of Bank Branches in a Cape Verdean Bank** - Here we will present the case study and the measurement model we will use to asses the efficiency of the bank branches.
- **Empirical Analysis** - In this chapter we will present and discuss the practical results we obtained.
- **Conclusions** - Final considerations on the study, difficulties found when developing the study, suggestions and possibilities for future studies.

Chapter 2

Background

2.1 Efficiency Measurement

2.1.1 Productivity, Efficiency and Effectiveness

Productivity of a production unit is a "measure of the relationship between the outputs produced and the inputs used by a production unit" (Portela, 2003). Lovell (1993) presented this measure as the ratio of the outputs and the inputs of the production unit (productivity ratio). He also claimed that when the production unit uses several inputs to produce several outputs, to guarantee that productivity remains the ratio of two scalars, the outputs in the numerator must be aggregated in some economically sensible fashion, as must the inputs in the denominator. He declared that the difference in productivity depends on production technology, differences in the efficiency of the production process, and differences in the environment in which production occurs. His interest was to isolate the efficiency component and measure its contribution to productivity.

Efficiency of a production unit will be defined in this dissertation as the comparison between observed and optimal value of the output and input of the production unit. "The comparison can take form of the ratio of observed to maximum potential output obtainable from given input, or the ratio of minimum potential to observed input required to produce given output, or some combination of the two" (Lovell, 1993). The former ratio focuses on outputs (it is output oriented) and the latter focuses on inputs (it is input oriented). Depending on the optimal values taken as reference we obtain a different efficiency measure. If the optimal is defined in terms of production possibilities, efficiency is *technical*. On another hand, if the optimal is defined is terms of the behavioral goal of the production unit, efficiency is *economic*, i.e, when the optimum relates to any economic (cost, revenue or profit) function.

Effectiveness can be defined as the extent to which an organisation meets its objectives or goals (Klassen et al., 1998 ; Agrell and Bogetoft, 2001). A measure of effectiveness is the ratio between produced outputs and desired outputs. To understand the difference between effectiveness and efficiency, we can consider two different questions "*What to do*" and "*How to do it*". Effectiveness is the answer to the former question as *to do the right thing* while efficiency is the answer to the latter question as *to do things right*. Or, in the words of Golany et al. (1993), efficiency is related to *performing current activities as well as possible* an effectiveness is related to *choosing the proper activities*. Therefore efficiency is the best way to implement strategies, while effectiveness represents the strategy formulation (strategic goals and choices).

2.1.2 Definitions and Measure of Productive Efficiency

According to Fried et al. (1993) the interest in measuring efficiency and productivity has two reasons. First of all, they allow us to evaluate/measure production units performance and therefore are success indicators. Second, we can only explore hypotheses concerning the sources of efficiency or productivity and therefore improve the performance of the production unit by measuring efficiency and productivity and separating their effects from the effects of the production environment.

Another perspective, that compliments the first one, was presented by Portela (2003) where she defended that the traditional definition of efficiency as 'doing things right' validates the need to measure efficiency. Organizations are constituted by human beings and not by machines so, is only reasonable to accept that production units don´t always do things right, i.e., more often than desired they do things wrongly. She believes that even if two units were set under the same conditions and had access to the same resources, because of the human component, these units wouldn't operate in the same manner or present the same outcome. Therefore, in the limit, efficiency is always related to people.

We will consider in this paper the perspective presented by Bogetoft and Hougaard (2001) where: inefficiency can be measured because inefficiency is a waste. In this context, inefficient units employ too many inputs to produce a certain level of outputs or produce too few outputs from a given set of inputs.

Farrell (1957) identified two components in productive efficiency: technical (or physical), and allocative (or price). The former refers to the "ability to avoid waste by producing as much output as input usage allows, or by using as little input as output production allows". Depending on the objective, the analysis of technical efficiency can be output-oriented when the goal is output augmentation or input-oriented when the goal is input conserving. The latter component refers to the "ability to combine inputs and outputs in optimal proportions in light of prevailing prices". In this paper we will focus on the concept of technical efficiency.

Koopmans (1951) provided a definition for technical efficiency: a producer is technically efficient if an increase in any output (and/or reduction in any input) requires a reduction in at least another output or an increase in at least one input. Thus, a production unit is considered technically inefficient if it could produce more with the same resources, or if could use less resources to produce the same outcome.

In the context of technical production,the construction of a production line (efficient frontier) aims to define a limit where the more efficient production unit will be located on this boundary, and the less efficient will be situated in the below border area, known as a set of possible production.

Farrell (1957) introduced a measure of technical efficiency as the maximum equiproportionate reduction in all inputs that allows equivalent production of observed output. One may also convert the Farrel measure to equiproportionate output expansion with given inputs. 'Farrell measure of efficiency' was defined by him as the amount of "waste" that can be eliminated without worsening any input or output.

Even more importantly, Farrell (1957) used the performances of other DMUs to evaluate the behavior of each DMU comparatively to the outputs and the inputs used by each one of the other DMUs. Which made it possible to measure their relative efficiencies. A DMU is relatively efficient "on the basis of available evidence if and, only if, the performances of other units does not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs" (Cooper et al., 2007). Since, in most management or social science applications the theoretically possible levels of efficiency will not be known, Cooper states that efficiency can be measured only with the information that is empirically available.

An important note, pointed out by Lovell (1993) is that after Farrell's efficiency is achieved we may still have some slacks. In classic radial models, the slacks relate to the additional increases in output or reduction in input that could be gained beyond that implied by the radial projection (i.e. equal increase in all outputs, or reduction in all inputs). As a result, the Farrell measure may not measure Koopmans efficiency. The Farrell measure identifies all production units that are Koopmans efficient as being technical efficiency, however, not all the production units who are found technically efficient by the measure are Koopmans efficient. (Consequently, Farrell technical efficiency is necessary but not sufficient for Koopmans technical efficiency.).

2.2 Bank Branches Efficiency Measurement

Bank Branch performance can help improve understanding of the basis of efficiency at bank level and help resolve some measurement problems in the standard bank-level analysis (Berger et al., 1997). Bank´s branch network normally represents the largest source of operational expenses for a bank and, from the administration point of view, cost management is more efficiently controlled at branch level (Joseph & Haiyan, 2013). In this context, we defend the importance of efficiency analyses at branch level.

However, according to Joseph and Haiyan (2013), bank branch performance measurement is not an easy task. Branches are diverse in size and offer different services to different customers while operating in distinct economic regions. Therefore, an appropriate bank branch performance evaluation should be able to: (1) capture the essential aspects of the bank's internal operating processes; (2) enlighten these processes in terms of what is achieved and how it is achieved; (3) provide management improvement guidelines by identifying the best practices and the worst practices; and (4) allow significant research of various hypotheses concerning the sources of inefficiency.

2.2.1 Performance measurement approaches applied to bank branches

The business environment's evolution and complexity have increased the challenge that is the determination of the best methodology to evaluate bank branch performance. Joseph and Haiyan (2013) identified at least four different approaches that are commonly used: ratio analysis, regression analysis, frontier efficiency analysis, and other performance evaluation methods that include multivariate statistical analysis (Zopounidis et al., 1952; Canbas et al., 2005), analytic hierarchy process (Frei and Harker, 1999; Seçme et al., 2009), gray relation analysis (Ho & Wu, 2006), balanced scorecard (Kim and Davidson, 2004; Wu et al., 2009).

2.2.1.1 Ratio analysis

Historically, ratio analysis has been the standard technique used by regulators, industry analysts, and management to examine performance at all levels. Ratios measure the relationship between two variables that are chosen to better understand the different aspects of the branch´s diversified operations, such as profitability, capital adequacy, asset quality, risk management, etc. Depending on the objective of the analysis any number of ratios can be designed. Some of the traditional financial ratios for estimating bank branch performance are: return on total assets; return on investment; loans per employee; deposits per employee; cost to income (Noulas et al., 2008).

Although the traditional ratio measures are attractive to analysts due to their simplicity and ease of understanding, there have been many methodological problems and limitations that must be considered (Noulas et al., 2008; Fernandez-Castro and Smith, 1994).

The main weakness of ratio analysis is that each one of the ratios only represents an aspect of the production unit activity, which means the method does not reflect the bank branch's multidimensional nature and, therefore, fails to provide the necessary performance information. Another observation is that the number of ratios that can be developed from financial statement data seems to be unlimited, leading to results that can be contradictory and confusing, hence, ineffective for the evaluation of a branch's overal performance. Furthermore, although ratios do provide certain useful information on the performance of a unit on specific aspects they are not adequate to set improvement targets for inefficient units (Joseph & Haiyan, 2013).

Thanassoulis et al. (1996) compared DEA and ratio analysis as alternative tools for assessing the performance of District Health Authorities in England. With this comparison, he aimed to study how well the two methods agreed on the performance of a unit relative to that of other units and the estimates of performance improvement targets. The authors found that the two methods could disagree substantially on the relative performance of individual units since DEA took simultaneous account of all resources and outputs in assessing performance while ratio analysis related only one resource to one output at a time.

2.2.1.2 Regression analysis

Regression analysis is another common methodology used in previous studies of the bank sector based on input-output analyses, such as Olsen and Lord (1979), Murphy and Orgler (1982), Berger et al. (1993), Boufounou (1995), Avkiran (1997), and Hensel (2003).

The main advantage of regression analysis is that it allows for statistical inference and measurement errors. Another advantage of this method, compared to the ratio analysis, is that it allows a simultaneous evaluation of the influence of multiple independent variables on the dependent variable. Regression Analysis provides the average performance information of all branches included in the sample and can also be used to estimate the expected performance of new branches (Joseph & Haiyan, 2013).

Joseph and Haiyan (2013) stated that even though regression analysis is effective in many circumstances (used to measure just about every aspect or to compare similar branches), it has a number of inherent limitations making it unsuitable for fully reflecting the increasingly complex nature of branch banking. Namely:

1. Regression analysis is a parametric method that requires a general production model to be specified where you need to make large assumptions about the underlying function.

2. Regression analysis is a central tendency method where predicted values result from a regression model provide the average or expected level of outcome given certain inputs, instead of the maximum achievable outcome (Ray, 1991; Soteriou et al., 1998).

3. It is only suitable to model single input-multiple outputs or multiple inputs-single output systems.

2.2.1.3 Frontier efficiency methodologies

The limitations associated with ratio and regression analysis encouraged the development of more advanced tools to measure units performance. Research in this domain has increasingly focused on benchmarking technique-based models, i.e, models that evaluate decision making unit performance by comparing it to the best of their peers operating under the same conditions. An important class of benchmarking methods is the frontier efficiency methodology. Given a data set, the best units are identified and they form the empirically efficient frontier. The main advantage of frontier efficiency over other methods is that it offers overall objective numerical efficiency scores with economic optimization mechanisms in complex operational environments and summarizes the performance in a single statistic (Berger & Humphrey, 1997).

Frontier efficiency analysis results can help identify the operational areas that need improvement, set future development strategies, identify attractive targets for mergers and acquisitions, etc. It can also determine achievable targets for inefficient units and the effects of the environmental variable, which ultimately provides additional insights and improve the understanding of their production systems (Joseph & Haiyan, 2013).

Frontier methods can be parametric and non-parametric. Both parametric and non-parametric frontier methods were inspired by the work of Farrell (1957). Non-parametric frontier methods gained notoriety after Charnes et al. (1978), that gave rise to Data Envelopment Analysis (DEA). The most common non-parametric techniques are: (i) Data envelopment analysis (DEA), and (ii) Free disposal hull (FDH). Berger and Humphrey (1997) classified parametric techniques into three categories: (i) Stochastic frontier approach (SFA), (ii) Thick frontier approach (TFA), and (iii) Distribution-free approach (DFA).

The main differences between these approaches are in the assumptions imposed on the specifications of the efficient frontier, the existence of random error, and the distribution of the inefficiencies and random error (Berger and Humphrey, 1997; Thanassoulis et al., 1996). Parametric frontier analyses require an a priori specification of the form of the production function and normally considers two error components: an error term that captures inefficiency and a random error. On the other hand, non-parametric methods require few assumptions when specifying the best-practice frontier but don't usually account for random errors (Joseph & Haiyan, 2013). Another advantage of non-parametric methods is that they give an individual efficiency measure for each observation (Førsund et al., 1980).

Since the ideal level of (in)efficiency is unknown we cannot state without doubts whether parametric techniques outperform non-parametric techniques or vice-versa (Portela, 2003). Berger et al. (1993) compared average efficiency for the two methods in US banking, with parametric methods giving higher estimates of efficiency and lower dispersion on estimate efficiency. Ferrier and Lovell (1990) also applied SFA and DEA to 575 banks in the U.S and concluded that "the two approaches are in substantial agreement on several important issues". Giokas (1991) compared DEA and the log-linear estimation function using bank branches in Greece, the models gave similar results in efficiency values, but there were differences concerning the classification of units. Bauer et al. (1993) investigated the consistency between DEA and the parametric methods: SFA, TFA, and DFA. They concluded that although the parametric methods were generally consistent with one another in terms of distributions of efficiency, rank order, and identification of the best-practice and worst-practice, the parametric and non-parametric methods were not mutually consistent. Weill (2004) observed the lack of consistency between parametric frontier approaches, SFA and DFA, and the non-parametric approach, DEA, when these approaches were applied on five European banking sectors. Gong and Sickles (1992) in a study made outside of banking conclude that stochastic frontier methods outperformed DEA when the chosen functional form was close to the underlying technology, however, when the functional form was not close to the underlying technology DEA outperformed stochastic frontier. Since in real-world applications it's almost impossible to know if the functional form resembles the underlying technology, DEA gains an advantage over stochastic methods. Banker (1993) and Banker and Natarajan (2008) have shown that DEA estimators had good statistical properties and could be used to provide consistent estimators of the effects of environmental variables on firm performance. The studies above suggest that more important than choosing the technique to be employed is the choice of the model specifications within that technique.

2.3 Model for Bank Branch Efficiency Assessment

Portela (2003) defended the development of a Bank Branch Efficiency assessment that would also take into consideration the changing role of bank branches from transactional based to sales based. This study was made to a Portuguese Bank Branches, and it took into consideration the opinions and concerns of the managers of the bank under analysis.

The analysis concluded that, in general, the main objectives of the bank agencies are to:

- Allow their customers to access banking services anywhere in the country (or in abroad) which is done by providing several different distribution channels of their services.
- Encourage the use of alternative distributions channel so that branch personnel can use their time on value-added activities.
- Increase sales and broaden customer base of the branch, while serving the clients that visit the branch with high level of service quality.
- Manage the product mix in a way that generates high profitability, without reducing quality of the service.
- Have maximum possible profitability given bank branches resources and environment.

The first two objectives are related to the accessibility of banking services, the third one relates to operational skills and sales volume, and the two last ones are a mix of management and profit of the products. Therefore, the efficiency of a bank branch can be defined as the extent to which the objectives are achieved, given their external environment. These goals are naturally connected, and through Figure 2.1 we can observe these interconnections, that work as the basis for the development of some measures of efficiency in the context of bank branches.

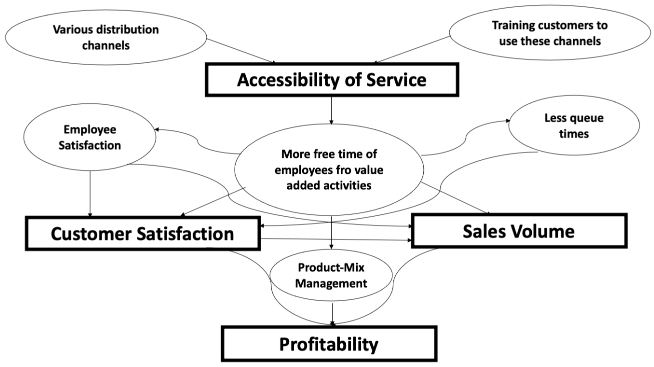


Figure 2.1: Model of Bank Branches Objectives

One of the main objectives of a bank is to provide access to its services. Efficient accessibility comes not only from the existence of different means of distribution but also from the training and motivation of customers to use these means. This is a responsibility of the bank branches that are the main interface between the bank and the customers.

When good accessibility to services is achieved, bank branches staff are allowed to devote less time to transaction activities and therefore have more time to carry out other activities, such as selling new products, attracting new customers, managing the product mix, etc. This will directly impact employee satisfaction (who will have more time to devote to more stimulating activities) and will reduce queues. These factors, happier employees, smaller queues, less waiting time, higher quality of service due to greater accessibility of services are likely to improve customer satisfaction. Higher customer satisfaction will have an impact on customer retention, sales volume, and market share. The sales volume will probably also be influenced by the fact that employees will have more time for selling new products to existing customers or using more proactive strategies to attract new customers.

Both management of the product mix and augmentation in sales volume will contribute to an increase in profits. Better profits may also be the consequence of superior service quality levels, as high service quality levels result in reduced price elasticity, i.e., loyal and satisfied customers tend to be willing to pay for the benefits they receive and eventually be more tolerant of price increases.

Considering the three main objectives of bank branches (accessibility, sales volume, and profit) and the model interconnecting these objectives (represented in Figure 2.1), Portela (2003) developed three concepts of efficiency. These concepts reflect the extent to which these objectives are being fulfilled. These three performance measures are *transactional*, *operational*, and *profit*. They used the DEA method to assess the performance of

Bank Branches, taking into account that the intention was comparative analysis between them, i.e., the assessment of relative efficiency. This method was privileged for two reasons: first, because it allows the evaluation of multiple inputs against multiple outputs and, second, because it avoids the necessity to make assumptions about the link between the inputs and outputs, which would be necessary if we approached it through Parametric models. The main limitation of the DEA is that it does not capture noise derived from the randomness of the data however, this limitation is not so relevant when we want to analyze general trends, rather than concrete numbers for evaluating the performance of agencies (Portela, 2003; Portela and Thanassoulis, 2007).

Based on this analysis, we concluded that it's possible to replicate this study to a specific Cape Verdean Bank assuming the same goals. In this thesis, we will assess the performance of the Bank Branches of this Cape Verdean Bank in the following perspectives: transactional and operational. **Transactional Efficiency** intends to measure the extent to which, given accessibility of services, customers use new distribution channels for transactional activities. **Operational Efficiency** intends to measure the extent to which a branch, given its resources and environment, increases its sales and is customer base while serving its existing customers with adequate service quality.

We chose to use the DEA method to evaluate the performance of the Bank Branches under study. DEA is a non-parametric approach which is recognized as being a robust efficiency analysis tool since it uses actual data from the units under evaluation to construct the efficiency frontier without setting up a specific functional form. We also so felt that in the context of Cape Verde banking, the individual efficiency measure for each observation provided by DEA was of great interest.

Even though DEA has two potential problems: (1) is non-stochastic and cannot allow for random error, hence any deviation from the efficient frontier is associated with inefficiency; (2) since the efficient frontier must cover all the units in the sample, some a-typical units may be self-identified as efficient simply because there are no similar units with which to compare them. Since the goal aren't definitive numerical measures but general indications of performance these disadvantages are not as relevant.

DEA accounts for the main problem of the stochastic frontier methods, namely, the arbitrary imposition of a specific functional form, i.e., when using DEA there isn't the need to make prior assumptions as to the functional form linking inputs and outputs. Which reduces the possibility of a bias measure of efficiency due to specification error. Additional advantages of DEA are that: it permits efficiency to change over time and it is also capable of dealing with the analysis of systems with multiple inputs and outputs. Some studies have combined other operational research techniques with the DEA model to make the efficiency estimation more accurate and to extend the model's application range.

Chapter 3

Data Envelopment Analysis (DEA) Models

3.1 History

As explored in the previous chapter, efficiency measurement has always been a subject of a lot of interest since organizations have been continuously concerned with productivity improvement. Farrell (1957) best explored the explanations for this interest in his seminal paper on the measurement of productive efficiency: “The problem of measuring the productive efficiency of an industry is important to both the economic theorist and the economic policy maker. If the theoretical arguments as to the relative efficiency of different economic systems are to be subjected to empirical testing, it is essential to be able to make some actual measurements of efficiency. Equally, if economic planning is to concern itself with particular industries, it is important to know how far a given industry can be expected to increase its output by simply increasing its efficiency, without absorbing further resources”. This work, as stated by Cooper et al. (2011) “represents the inception of Data Envelopment Analysis (DEA)”. Farrell (1957) targeted the development of better methods and models for evaluating productivity. He claimed that while attempts to solve the problem usually produced careful measurements, they were also very restrictive because they failed to combine the measurements of multiple inputs into any satisfactory overall measure of efficiency. These, that he nominated as inadequate approaches, included forming an average productivity for a single input (ignoring all other inputs), and building an index of efficiency in which a weighted average of inputs is compared with output. Responding to these inadequacies of separate indices of labor productivity, capital productivity, etc., Farrell proposed an activity analysis approach that could more adequately deal with the problem. His approach was intended to be applicable to any productive organization: “... from a workshop to a whole economy”. In the process, Farrel extended the concept of ’productivity’ to the more general concept of 'efficiency'. Although he was able to formulate a multiple output case, he confined his numerical examples and discussion to single output situations.

About twenty years after Farrell's work, and on the basis of those ideas, Charnes et al. (1978) , as a response to the demand for satisfactory procedures to evaluate the relative efficiencies of multi–input multi–output production units, developed a mathematical programming technique that has been named Data Envelopment Analysis (DEA). According to Cook and Seiford (2009), the original idea behind DEA was to provide a powerful methodology whereby, within a set of comparable decision making units (DMUs), those exhibiting best practice could be identified, and would form an efficient frontier. Furthermore, the methodology enables one to measure the level of efficiency of non-frontier units, and to identify benchmarks against which such inefficient units can be compared.

Since the initial study by Charnes et al. (1978), i.e, the introduction of DEA in 1978, there has been a remarkable growth both in theoretical developments and applications of the concepts to practical situations. One may argue that the fast growth and widespread (and almost immediate) acceptance of the methodology of DEA serves as a witness to its strengths and applicability (Cooper et al., 2011).

3.2 Definition

In their originating study, Charnes et al. (1978) described DEA as a 'mathematical programming model applied to observational data that provides a new way of obtaining empirical estimates of relations - such as the production functions and/or efficient production possibility surfaces – that are cornerstones of modern economics'.

DEA is a non-parametric approach that studies the efficiency of peer organizational units called Decision Making Units (DMUs) where the conversion of multiple inputs into multiples outputs complicates the comparison of the DMUs performances. Given a set of DMUs, the DEA model evaluates if, compared to the rest of the DMUs, the DMU_o under study can be considered efficient or not. Within the given set, the model identifies the DMUs whose characteristics are similar to DMU_o and evaluates its relative efficiency. If DMU_o isn´t efficient, the reference DMU(s) is(are) identified, and based on them goals are set for DMU_o to become efficient. A reference DMU is an efficient DMU that serves as an example to the DMU that is considered inefficient, they are called peers or benchmarks. Since the measurement by DEA is within reference to some set of units can be said to be a method for measuring comparative or relative efficiency. The efficiency score is usually expressed as either a number between 0-1 or 0-100%. A DMU with a rating of 100% (or 1) is considered to be relatively efficient, and a DMU with a rating of less than 100% (< 1) is deemed inefficient relative to other units Cooper et al. (2007).

As presented in Sherman and Ladino (1995) DEA determines the following:

- The best-practise - most productive group of DMUs;
- The less-productive DMUs compared to the best–practice units;

- The amount of excess resources used by each of the less–productive DMUs;
- The amount of excess capacity or ability to increase service outputs in less–productive DMUs without utilizing added resources;
- The set of best–practice DMUs most similar to the less-productive DMUs;

3.2.1 Efficiency in DEA

On the subsection 2.1.2, we explored the definitions and measures of efficiency provided by Farrell (1957) and Koopmans (1951).

Farrell (1957) introduced a measure of technical efficiency as the maximum equiproportionate reduction in all inputs that allows equivalent production of observed output, or, the equiproportionate output expansion with given inputs. The efficiency score provided by the DEA method is usually related to Farrell's measure of efficiency. This efficiency score, from 0 to 1, translates whether or not the DMU under study belongs to the efficiency frontier (=1) and, if the DMU is not on the frontier ($\neq 1$), the radial projection (i.e. equal increase in all outputs, or reduction in all in- puts) in which the inputs or outputs should move in the direction of the efficiency frontier. From this point on, we will refer to this efficiency score as *Radial Efficiency*. On another hand Koopmans (1951) definition of efficiency says that a DMU is efficient if an increase in any output (and/or reduction in any input) requires a reduction in at least another output or an increase in at least one input. Even though the DEA score is radial the model identifies additional increases in output or reduction in input that can be made (slacks). The efficiency score that considers the influence of these values we will refer to as *Non-radial efficiency*. A DMU is Koopmans efficient only when the non-radial efficiency score is 1. So, even after Farrell's efficiency is achieved, the unit may not be Koopmans efficient. Therefore the radial efficiency score of 1 of the model, is a necessary but not sufficient condition for efficiency. Considering these concepts one may also differentiate between *Radial Goals/Objectives (RTargets)* and *Non-radial Goals/Objectives (NRTArgets)*. The first one reflects only the values to be improved considering proportion according to radial efficiency, while the second, reflects the value of the slacks identified by the model.

These concepts will be applied in the practical application. Even though most DEA models only provide the score of radial efficiency, a few techniques can be applied with the results of the DEA models to obtain the non-radial efficiency scores.

3.2.2 Data

In DEA, the organization under study is called a DMU (Decision Making Unit). The categorization of the unit of assessment as "decision making" implies that it has control over the process it employs to convert its resources into outcomes. Therefore, generically a DMU is regarded as the entity responsible for converting inputs into outputs and whose performances are to be evaluated. The definition of DMU allows flexibility in its use over a wide range of possible applications (Cooper et al., 2007).

In management application DMUs can be organizational units such as local authority departments (Thanassoulis et al., 1987), schools (Kirjavainen & Loikkanent, 1998), hospitals (O'Neill et al., 2008), shops (Clarke & Gourdin, 1991), bank branches, on another hand, in engineering, DMUs may take such forms as airplanes or their components such as jet engines, as long as there is a relatively homogeneous set of units.

Thanassoulis (2001) explains that in DEA resources are typically referred to as "inputs" and the outcomes as "outputs". The identification of the inputs and the outputs in an assessment of DMUs is as difficult as it is crucial. The inputs should capture all resources which impact the outputs. The outputs should reflect all useful outcomes on which we wish to assess the DMUs. Furthermore, any environmental factors which impact the transformation of resources into outcomes should also be reflected in the inputs or the outputs depending on the direction of that impact. Cooper et al. (2007) reinforces that with DEA we study relative efficiency, since a group of DMUs is used to evaluate each other with each DMU having a certain degree of managerial freedom in decision making.

According to Cooper et al. (2007) if we have a set of n DMUs: DMU_1, DMU_2,..., and DMU_n. When selecting some common input and output items for each of these j = 1,...,n, we must consider the following:

1. Numerical data are available for each input and output, and the data is assumed to be positive for all DMUs.

2. The inputs, outputs and choice of DMUs that will enter the relative efficiency evaluations of the DMUs should reflect what an analyst's or a manager's is interested in evaluating.

3. In principle, we want do reduce inputs and increase outputs so, efficiency scores should reflect these principles.

4. The measurement units of the different inputs and outputs don't have to be the same. Some may involve number of persons, or areas or money expended, etc.

3.3 Traditional DEA models

3.3.1 Introduction

Data envelopment analysis is an approach that evaluates performance by comparing the efficiency of organizational units such as business firms, local authority departments, schools, hospitals, bank branches, and similar instances where there is a relatively homogeneous set of units (Boussofiane et al., 1991).

For the simple case where a process or a unit has a single input and a single output efficiency is defined simply as the ratio between the output and the input:

$$Efficiency = \frac{\text{output}}{\text{input}}$$

However, more often than not, DMUs have multiple inputs and outputs, whose relationship is usually not proportionate. We can incorporate this complexity in an efficiency measure by defining the efficiency as:

$$Efficiency = \frac{\text{weighted sum of outputs}}{\text{weighted sum of inputs}}$$

This definition requires a set of weights to be defined, however, this task may be difficult, especially if the goal is a common set of weights to be applied across the set of organizational units. A solution to this problem is arguing that individual units may have their particular value systems and therefore may legitimately define their own peculiar set of weights (Boussofiane et al., 1991).

3.3.2 The CCR Model

Charnes et al. (1978) proposed the *CCR Model* that is one of the most simple DEA models. For each DMU, Charnes et al. (1978) formed the virtual input and output by (yet unknown) weights (v_i) and (u_r):

$$\text{Virtual input} = v_1 * x_{1o} + \ldots v_m * x_{mo}$$
$$\text{Virtual output} = u_1 * y_{1o} + \ldots u_s * y_{so}$$

Then the goal is to determine the weight, using linear programming to maximize the ratio between virtual input and virtual output:

$$\frac{\text{Virtual Output}}{\text{Virtual Input}}$$

The weights are decision variables, therefore, the optimal weights can (and probably will) vary depending on the DMU. DEA allows each DMUs to define their own set of weights. Thus, the weights in DEA come from the given data instead of being fixed in advance.

Consider a set of n DMUs, with each DMU_j (j=1, ..., n) using m inputs x_{ij} (i = 1,...,m) and generating s outputs y_{rj} (r = 1,...,s) . We measure the efficiency of each DMU once and therefore n optimizations are required, one for each DMU_j to be evaluated. We designate as DMU_o the DMU_j in evaluation. Charnes et al. (1978) proposed deriving appropriate multipliers for a given DMU by solving a particular non-linear programming problem. For each DMU_o under consideration, we solve the following fractional programming model to obtain values for the input weights (v_i) (i = 1,...,m) and the output weights (u_r) (r = 1,...,s) as variables.

$$
\begin{aligned}
&(FP_o) \qquad && \max_{v,u} \theta = \frac{\sum_r u_r * y_{ro}}{\sum_i v_i * x_{io}} && (3.1)\\
&\text{subject to} && \frac{\sum_r u_r * y_{rj}}{\sum_i v_i * x_{ij}} \leq 1,\ \forall j && (a)\\
& && u_r, v_i \geq \epsilon \quad \forall (r,i) && (b)
\end{aligned}
$$

.

The constraints assure the ratio between the “virtual output” and the “virtual input” should not exceed 1 for every DMU. The objective function is to obtain weights (v_i) and (u_r) that maximize the ratio for DMU_o. Therefore the solution for this model is the maximized efficiency of DMU_o subject to efficiencies of all unit in set having an upper bound 1. Considering the weights (v_i) and (u_r) as variables and therefore as unknown is the key feature of the model. They will be chosen so as to maximize the efficiency of DMU_o, that will either equal 1 in wich case it is efficient relative to the other units or will be inferior to 1 in which case it is considered inefficient.

We study the efficiency of each unit at a time, and hence we'll make n optimizations. The weights values will generally differ from one DMU to another, this flexibility is both a weakness and a strength of this method. It is a weakness because since the weights chosen present the best scenario, there may be concern the efficiency of the DMU has more to do with the choice of weights than any inherent efficiency. However, this flexibility is also a strength since, if a DMU is considered inefficient even in the best scenario then this is a strong statement and in particular the argument that the weights are inappropriate is not sustainable.

In reality, the model proposed by Charnes et al. (1978), simply restricts the variables to be non-negative ($\epsilon = 0$). However, in the follow-up paper, Charnes et al. (1981) introduced the condition of a strictly positive lower limit ($\epsilon > 0$). But, for convenience, we shall refer to (3.1) as the original CCR model.

The model (3.1) is a fractional linear program, applying the Charnes and Cooper (1962) theory of fractional programming we can convert it into a linear program, hence, allowing us to apply the methods of linear programming. The above fractional program 3.1 is converted into the following linear program (LP_o)

$$
\begin{aligned}
&(LP_o) \qquad && \max_{\nu,\mu} \theta = \sum_r \mu_r * y_{ro} && (3.2)\\
&\text{subject to} && \sum_i \nu_i * x_{io} = 1 && (a)\\
& && \sum_r \mu_r * y_{rj} - \sum_i \nu_i * x_{ij} \leq 0\ \forall j && (b)\\
& && \mu_r, \nu_i \geq \epsilon\ \forall (r,i) && (c)
\end{aligned}
$$

•

Theorem *The fractional program* (FP_o) *is equivalent to* (LP_o)

Proof. Under the nonzero assumption of $\boldsymbol{\nu}$ and $\mathbf{X} > 0$, the denominator of the constraints of (FP_o) is positive for every j , and hence we obtain 3.2.b) by multiplying both sides of 3.1.a) by the denominator. Next, we note that a fractional number is invariant under multiplication of both numerator and denominator by the same nonzero number. After making this multiplication, we set the denominator of the objective function of (FP_o) equal to 1, move it to a constraint, as done in 3.2.a), and maximize the numerator, resulting in (LP_o). Let an optimal solution of (LP_o) be ($\nu = \nu^*$, $\mu = \mu_*$) and the optimal objective value θ^*. The solution ($\nu = \nu^*$, $\mu = \mu^*$) is also optimal for (FP_o), since the above transformation is reversible under the assumptions above. (FP_o) and (LP_o) therefore have the same optimal objective value θ^* (Cooper et al., 2007) •

Theorem (Units Invariance Theorem) *The optimal values of max* $\theta = \theta^*$ *in 3.1 and 3.2 are independent of the units in which the inputs and outputs are measured provided these units are the same for every DMU*

Proof. Let θ^*, u_r^*, v_i^* be optimal for (3.1)-(3.2). Now replace the original y_{rj} and x_{ij} by $\rho_r y_{rj}$ and $\delta_i x_{ij}$ for some choices of δ_i, $\rho_r > 0$. But then choosing $u_r' = u_r^*/\rho_r$ and $v_i' = v_i^*/\delta_i$ we have a solution to the transformed problem with $\theta' = \theta^*$. An optimal value for the transformed problem must therefore have $\theta'^* \geq \theta^*$. Now suppose we could have $\theta'^* > \theta^*$. Then, however, $u_r' = u_r^* \rho_r$ and $v_i' = v_i^* \delta_i$ satisfy the original constraints so the assumption $\theta'^* > \theta^*$ contradicts the optimally assumed for θ^* under these constraints. The only remaining possibility is $\theta' = \theta^*$. This proves the invariance claimed for 3.2. The Theorem above demonstrated the equivalence of (LP_o) to (FP_o) and thus the same result must hold and the theorem is therefore proved• (Cooper et al., 2007) •

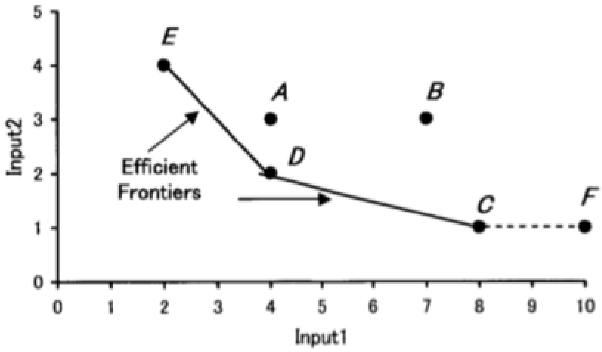
3.3.2.1 CCR Efficiency

Considering the model 3.2, a DMU_o is **efficient** if $\theta^* = 1$ and there exists at least one optimal (v^*, u^*), with $v^* > 0$ and $u^* > 0$. A DMU is CCR-inefficient if either (i) $\theta^* < 1$ or (i) $\theta^* = 1$ and at least one element of (v^*, u^*) is zero for every optimal solution of 3.2.

In the Figure 3.1, we can see a small example of an application of the CCR method. In this example, we are analysing 6 DMUs with two inputs and one output.

The Efficient Frontier was built based on the DMUs that were considered CCR efficient by the model, namely, E, D and C.

Analysing unit A, we can see in the Table b) Figure 3.1 that the optimal solution is $(\nu_1^* = 0.1429$, $\nu_2^* = 0.1429$, $\mu_1^* = 0.8571$, $\theta^* = 0.8571)$ and the CCR-efficiency score of A is 0.8571. This is the case where a DMU is CCR-inefficient because $\theta^* < 1$. For A to become efficient we must reduce the inputs to 85.71 %. That would located A in the efficient frontier between units E and D , that are her reference set of A.



(a) Example Efficiency Frontier

(b) Results of Example

DMU	x1	x2	y	Effiency Score	Reference	v_1	v_2	μ_1
A	4	*3*	1	0.8571	*D E*	0.1429	0.1429	0.8571
B	7	*3*	1	0.6316	*C D*	0.0526	0.2105	0.6316
C	8	*1*	1	1	*C*	0.0833	0.3333	1
D	4	*2*	1	1	*D*	0.1667	0.1667	1
E	2	*4*	1	1	*E*	0.2143	0.1429	1
F	10	*1*	1	1	*C*	0	1	1

Figure 3.1: Example (Cooper et al., 2007)

Lets analyse unit F. As we can see by the Table b) Figure 3.1 F has an efficiency score of $\theta = 1$ but, v_1 is zero, therefore F is not efficient because at least one element of (v^*, u^*) is zero for every optimal solution of the model. If we study Figure a) Figure 3.1 we can see that for F to be a part of the efficient frontier we had to reduce only input v_1. If we analyse the table again we can see that unit F reference is unit C, therefore, by direct comparison we can identify the fact that F uses an excessive amount of input v_1 . The input v_1 of F is 10, however, unit C uses only 8 units of input v_1 but produces the same results. That is, we can reduce input v_1 in unit F and continue with the same output.

However, excess in an input (or shortage in output) it's not always easy to see through the optimal solution of the CCR model. Therefore, in the next subsection I will approach the CCR model from the dual side of the linear program and which will enable us to determine the excess and shortfalls.

3.3.3 Dual Problem and the CRR model

The dual problem of (LP_o) (when $\epsilon = 0$) is expressed with a real variable θ and a non-negative vector $\lambda = (\lambda_1, ..., \lambda_n)^T$ of variables as follows:

$$\begin{aligned} (DLP_o) \qquad & \theta^* = \min_{\theta,\lambda} \theta \\ \text{subject to} \quad & \theta x_{io} - \sum_{j=1}^{n} x_{ij}\lambda_j \geq 0 \ \forall i \\ & \sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{ro} \ \forall r \\ & \lambda_j \geq 0 \ \forall j \end{aligned} \tag{3.3}$$

.

According to Charnes et al. (1978) let's consider the optimal solution in the equivalent equation form of 3.3 with slack variables be represented by s^+ and s^-. Where s^+ represents a vector of non-negative slack associated with the output inequalities and s^- represents a vector of non-negative slack associated with the input inequalities. If s^+ has any positive components these represent output *shortfalls*, on another hand, if s^- has any positive components these represent input *excess*.

Correspondences between the primal (LP_o) and the dual (DLP_o) constraints and variables are displayed in Table 3.1.

Table 3.1: Primal and Dual Correspondence - ref: Cooper et al. (2007)

Constraint (LP_o)	Dual variable (DLP_o)	Constraint (DLP_o)	Primal variable (LP_o)
$\sum_i v_i * x_{io} = 1$	θ	$\theta x_{io} - \sum_{j=1}^{n} x_{ij}\lambda_j \geq 0 \ \forall i$	$v_i \geq 0$
$\sum_r u_r * y_{rj} \leq \sum_i v_i * x_{ij} \quad (\forall j)$	$\lambda_j \geq 0 \ \forall j$ >0	$\sum_{j=1}^{n} x_{rj}\lambda_j \geq y_{ro} \ \forall r$	$u_r \geq 0$

Charnes et al. (1978) stated that no DMU can be rated as **efficient** unless the following conditions are both satisfied

1. $\theta^* = 1$, and

2. The slack variables are all zero.

3.3.4 Model Orientation

An inefficient DMU can be made more efficient by projection onto the frontier. Model orientation determines the direction of the projection for the inefficient DMUs. The DEA models can be input or output oriented.

Input-oriented model contracts the inputs as far as possible while controlling the outputs. In an input-oriented model, an inefficient unit is made efficient through proportional reduction of its inputs while its outputs proportions are held constant.

The output-oriented model expands the outputs as far as possible while controlling the inputs. In an output-oriented model, an inefficient unit is made efficient through proportional increase (augmentation) of its outputs while the inputs' proportions remain unchanged

Up to this point we have been dealing with input-oriented models, the models (3.1), (3.2) and (3.3) are input-oriented.

We can formulate the linear **output-oriented** model correspondent to (3.2) by inverting the efficiency ratio to the ratio of virtual input to output. This would reorient the objective from maximization to minimization. Considering once again the transformation from fractional to linear we obtain,

$$\begin{aligned} (LPO_o) \qquad & \min_{v,u} \beta = \sum_i v_i * x_{io} \\ \text{subject to} \quad & \sum_i u_r * y_{ro} = 1 \\ & \sum_r v_i * x_{ij} - \sum_i u_r * y_{rj} \geq 0 \ \forall j \\ & u_r, v_i \geq \epsilon \ \forall (r,i) \end{aligned} \tag{3.4}$$

•

On another hand, we can also formulate the output-oriented correspondent to (3.3) as:

$$\begin{aligned} (DLPO_o) \quad & \beta^* = \max_{\beta,\Lambda} \beta \\ \text{subject to} \quad & x_{io} \geq \sum_{j=1}^{n} x_{ij}\Lambda_j \ \ \forall i \\ & \sum_{j=1}^{n} y_{rj}\Lambda_j - y_{ro}\beta \geq 0 \ \forall r \\ & \Lambda_j \geq 0 \ \forall j \end{aligned} \tag{3.5}$$

•

The optimal solution of the output-oriented model 3.5 relates to that of the input-oriented model 3.3 as following:

$$\beta^* = 1/\theta^*, \quad \Lambda^* = \lambda^*/\theta^* \tag{3.6}$$

3.4 BCC Model

The CCR models are Constant Return to Scale (CRS) because they assume that an increase in inputs results in the same proportional increase in outputs, i.e, the returns are proportionate to increase in efforts. However, studies began to incorporate the concept of scale in their analysis, i.e., they started considering that increasing the volume of production inputs in DMUs may lead to gain, loss, or constancy of productivity. Introducing the possibility of Variable Return to Scale (VRS), i.e., the returns are not proportionate to increase in efforts (Benicio & de Mello, 2015). The variable return to scale may be: (i) Increasing Return to Scale (IRS) if the variation of inputs is smaller than the output variation, i.e, the average productivity increases with increasing production scale; (ii) Decreasing Return to Scale DRS DRS if the variation of inputs is larger than the output variation, i.e, the average productivity decreases with increasing production scale. The Figure 3.2 shows the scale returns from possible change in the amount of inputs used, where the different behaviors of the efficient frontier curve production can be verified.

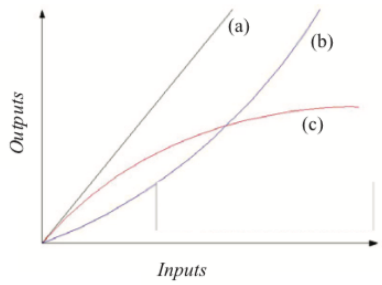


Figure 3.2: (a) Constant Return to Scale (CRS); (b) Increasing Return to Scale (IRS) ; (c) Decreasing Return to Scale DRS - (Benicio & de Mello, 2015)

Banker et al. (1984) extended the earlier work of Charnes et al. (1978), by introducing into the CCR model a modification that would allow the analyses of the variable returns to scale (VRS) in DEA. Banker et al. (1984) wanted to acknowledge that in different situations, the conditions that influence the productivity of a production unit are also diverse. Hence, the BCC model aims to interpret that DMUs could have different productivity at different scales and still be efficient.

The BCC ratio model differs from the CRR model 3.2, by an additional variable, the free variable u_o, as an indicator of returns to scale:

$$
\begin{aligned}
(LP_o) \qquad & \max_{v,u} \theta = \sum_r u_r * y_{ro} - u_o \\
\text{subject to} \quad & \sum_i v_i * x_{io} = 1 \\
& \sum_r u_r * y_{rj} - \sum_i v_i * x_{ij} - u_o \leq 0 \; \forall j \\
& u_r, v_i \geq \epsilon \; \forall (r,i)
\end{aligned} \tag{3.7}
$$

•

Increasing returns to scale are indicated for $u_o < 0$, constant returns for $u_o = 0$, and decreasing returns to scale for $u_o > 0$. CRS efficiency is the same regardless of the model orientation, however VRS efficiency is not. Scale efficiency is always dependent on the model orientation.

The Dual BCC model differs from the Dual CRR model (3.3), by setting $S = \sum_j \lambda_j = 1$. Therefore the Dual BCC output-oriented model is defined as:

$$
\begin{aligned}
\beta^* = \; & \max_{\beta,\lambda} \beta \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = x_{io} \;\; \forall i \\
& \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{ro}\beta \;\forall r \\
& \sum_j \lambda_j = 1 \\
& \lambda_j, s_i^-, s_r^+ \geq 0
\end{aligned}
\tag{3.8}
$$

•

The solution of an envelopment model 3.8 gives us the values of input and/or outputs that a DMU should attain to be efficient. Such values are usually called targets and, in this output oriented model, they are defined as:

$$
\begin{aligned}
y_{ro}^* &= \sum_{j=1}^{n} \lambda_j y_{rj} = \beta_o^* y_{ro} + s_r^+ \;\forall r \\
x_{io}^* &= \sum_{j=1}^{n} \lambda_j x_{ij} = \theta_o^* x_{io} - s_i^- \;\forall i
\end{aligned}
$$

•

Theses targets take into consideration the slacks variables s_r^+ and s_i^- and therefore can be considered non-radial targets. However, the model doesn't guarantee that the value of the slacks variables s_r^+ and s_i^- are maximum, hence, we can't say if the targets are in fact Pareto-efficient. To guarantee that the targets are Pareto-efficient one must solve a second stage model where the slacks are maximized.

$$
\begin{aligned}
\max \quad & \sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^- \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = x_{io}^* \ \ \forall i \\
& \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{ro}^* \ \forall r \\
& \sum_{j} \lambda_j = 1 \\
& \lambda_j, s_r^+, s_i^- \geq 0
\end{aligned} \tag{3.9}
$$

If the optimal solution of this second stage model is zero (all slacks variables are zero) then the optimal target level obtained in the first stage is Pareto efficient. Otherwise, the second stage model provides Pareto efficient targets and peers.

3.5 Geometric Distance Function (GDF)

Portela (2003) developed a new measure of efficiency based on geometric mean measure of distance between the point whose efficiency is being measured and the maximum efficiency point.

$$
\begin{aligned}
& \text{Geometric Distance Function (GDF)} = \frac{(\prod_i \theta_i)^{1/m}}{(\prod_r \beta_r)^{1/s}} \\
& \text{where:} \quad \theta_i = \frac{x_i^*}{x_i} \qquad \beta_r = \frac{y_r^*}{y_r}
\end{aligned} \tag{3.10}
$$

Where x_i^* and y_r^* represent the target input an output levels respectively, and x_i and y_r represent the observed input and output levels respectively. Portela (2003) named this measure geometric distance function (Geometric Distance Function (GDF)), since it represents the ratio of the geometric mean distance between observed and target input levels and the corresponding distance between observed and target output levels. The geometric function distance can be used to measure the distance between any two points. If the relationship is between a technically efficient point and the current point, the resulting GDF is a general measure of *technical efficiency*.

3.6 RDM Model

Data Envelopment Analysis (DEA) models as presented by Charnes et al. (1978) is an efficiency assessment tool that assumes non-negativity of all inputs and outputs. However, in many real life contexts, not all inputs and outputs are positive for all operating units, therefore, the assumption is not always satisfied. Traditionally negative data was handled in efficiency applications through some data transformation (Portela et al., 2004). In this context, where no method could be used with negative data directly without needing to transform them and that maintained an efficiency score that could be used to compare units, Portela (2003) proposed a DEA model.

A DEA model that provides efficiency scores, similar in interpretation to efficiency scores given by CRR models, where negative data can be used without the need to transform them. This model is the modification of the generic directional distance model of Chambers et al. (1996, 1998).

The Range Directional Model (RDM) was defined as shown in 3.11

$$
\begin{aligned}
& \max \beta_o \\
\text{subject to} \quad & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \beta_o R_{ro} \ \forall r \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} - \beta_o R_{io} \ \ \forall i \\
& \sum_{j} \lambda_j = 1 \\
& \beta_o \geq 0, \ \lambda_j \geq 0 \ \ \forall j
\end{aligned}
\tag{3.11}
$$

•

where R_{ro} and R_{io} represent the **range of possible improvement of unit *o***, Portela denominated this measure **SP Range**

$$
\begin{aligned}
R_{io} &= x_{io} - \min_j \{x_{ij} : j = 1, \ldots, n\}, \quad i = 1, \ldots, m, \\
R_{ro} &= \max_j \{y_{rj} : j = 1, \ldots, n\} - y_{ro}, \quad r = 1, \ldots, s,
\end{aligned}
\tag{3.12}
$$

•

The RDM **efficiency measure** is given by $1 - \beta$. Efficiency of 1 will only be achieved when the observed are also the target values, i.e, when DMU_o (the DMU under evaluation) lies on the efficient frontier. Simliarly, to the traditional DEA models proposed by Charnes et al. (1978).

This efficiency measure can not incorporate all sources of inefficiency, because value of slacks variables are not reflected in the value of β_o. To overcame this obstacle, Portela (2003) and Portela and Thanassoulis (2007) defined the following ratio norm:

$$
(\|A\| / \|B\|) = (\|I - T\| / \|I - O\|) \tag{3.13}
$$

where I represents *ideal point*, T represents *target level* and O represents *observed level*.

When using ratio (3.13), the goal is to account for all inefficiencies, including those given by slacks values. To achieve this the target levels A and B must be Pareto-efficient. This eliminates the risk of biased ranking based on an efficiency measure that does not account for all sources of inefficiency, especially if the slacks are high.

The model 3.11 is non-oriented. To define the **output-oriented RDM** model we set $R_{io} = 0$.

$$
\begin{aligned}
& \max \beta_o \\
\text{subject to} \quad & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \beta_o R_{ro} \ \forall r \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} \ \ \forall i \\
& \sum_{j} \lambda_j = 1 \\
& \lambda_j \geq 0 \ \ \forall j
\end{aligned}
\tag{3.14}
$$

3.7 MSBM Model

Tone (2001) proposed a Slacks Based Measure (SBM) for a system with m positive inputs and s positive outputs as follows:

$$
\begin{aligned}
& \min \ p = \frac{1 - \sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 - \sum_{r=1}^{s} \frac{s_r^+}{y_{ro}}} \\
\text{subject to} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \ \forall i \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \ \ \forall r \\
& \sum_{j} \lambda_j = 1 \\
& \lambda_j, s_r^+, s_i^- \geq 0
\end{aligned}
\tag{3.15}
$$

Based on the Tone (2001) SBM model (3.15), Sharp et al. (2007) proposed an Modified Slacks Based Measure (MSBM) model where he extended the SBM model to solve the problem of dealing with negative data. Sharp et al. (2007) based their modifications to

SBM on the SP Range (3.12) introduced by Portela (2003): (i) $R_{io} = x_{io} - \min_j\{x_{ij} : j = 1,\ldots,n\}$ (i=1,..., m); (ii) $R_{ro} = \max_j\{y_{rj} j = 1,\ldots,n\} - y_{ro}$ (r=1,..., s).

The modified slack based model developed by Sharp et al. (2007) can be formulated as follows:

$$
\begin{aligned}
\min\ p = \frac{1 - \sum_{i=1}^{m} \frac{w_i s_i^-}{R_{io}}}{1 - \sum_{r=1}^{s} \frac{v_r s_r^+}{R_{ro}}} \\
\text{subject to} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}\ \forall i \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}\ \ \forall r \\
& \sum_{j} \lambda_j = 1,\ \sum_{i=1}^{m} w_i = 1,\ \sum_{r=1}^{s} v_r = 1 \\
& \lambda_j, w_i, v_r, s_r^+, s_i^- \geq 0
\end{aligned}
\tag{3.16}
$$

.

where:

s_i^-: the value of the input slack

s_r^+ the value of the output slack

w_i and v_r are specific weights to reflect the strength of improving inputs or outputs. When R_{io} and R_{ro} are equal to zero, the corresponding term is dropped, respectively, from the nominator and denominator.

The advantage of the slacks-based measure model is that the efficiency score is not radial, however, as earlier exposed, the basic SBM model cannot handle negative data, so Sharp et al. (2007) modified the basic SBM model so that it would give an efficiency score between 0 and 1 with negative data. Therefore unlike the efficiency score given by the RDM model, MSBM model efficiencies account for individual input and output slacks values.

The **Output-oriented MSBM** is defined in 3.17

$$
\begin{aligned}
\max\ h = 1 + \sum_{r=1}^{s} \frac{v_r s_r^+}{R_r o} \\
\text{subject to} \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \ \ \forall i \\
\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \ \ \forall r \\
\sum_{j} \lambda_j = 1, \ \sum_{r=1}^{s} v_r = 1 \\
\lambda_j, v_r, s_r^+, s_i^- \geq 0
\end{aligned}
\tag{3.17}
$$

.

3.8 Modified MSBM Model

However there are some disadvantages in the MSBM model proposed by Sharp et al. (2007). Namely when $x_{io} = \min x_{ij}$ or $\max y_{rj} = y_{ro}$, R_{io} or R_{ro} are zero, and it is necessary to eliminate the corresponding terms from the respective numerator or denominator. To avoid this problem, Wei et al. (2019) proposed a New MSBM, which uses the range R_{io}^- and R_{ro}^+ to replace R_{io} and R_{ro} (SP Range)[1]. The modified MSBM (Wei et al., 2019) is:

$$
\begin{aligned}
\min\ p = \frac{1 - \sum_{i=1}^{m} \frac{w_i s_i^-}{R_{io}^-}}{1 - \sum_{r=1}^{s} \frac{v_r s_r^+}{R_{ro}^+}} \\
\text{subject to} \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \ \forall i \\
\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \ \ \forall r \\
\sum_{j} \lambda_j = 1, \ \sum_{i=1}^{m} w_i = 1, \ \sum_{r=1}^{s} v_r = 1 \\
\lambda_j, w_i, v_r, s_r^+, s_i^- \geq 0
\end{aligned}
\tag{3.18}
$$

.

with

$$
\begin{aligned}
R_{io}^- &= \max_j \{X_{ij} : j = 1, \ldots, n\} - \min_j \{X_{ij} : j = 1, \ldots, n\}, \quad i = 1, \ldots, m, \\
R_{ro}^+ &= \max_j \{Y_{rj} : j = 1, \ldots, n\} - \min_j \{Y_{rj} : j = 1, \ldots, n\}, \quad r = 1, \ldots, s,
\end{aligned}
\tag{3.19}
$$

[1](i) $R_{io} = x_{io} - \min_j \{x_{ij} : j = 1, \ldots, n\}$ (i=1,..., m); (ii) $R_{ro} = \max_j \{y_{rj} j = 1, \ldots, n\} - y_{ro}$ (r=1,..., s)

•

This new MSBM model measures efficiency in the presence of negative data, but avoids the inconvenience to eliminate the corresponding items.

The **Output-oriented MSBM** is defined in 3.20

$$
\begin{aligned}
\max \; h = 1 + \sum_{r=1}^{s} \frac{v_r s_r^+}{R_{ro}^+} \\
\text{subject to} \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \quad \forall i \\
\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad \forall r \\
\sum_{j} \lambda_j = 1, \; \sum_{r=1}^{s} v_r = 1 \\
\lambda_j, v_r, s_r^+, s_i^- \geq 0
\end{aligned}
\tag{3.20}
$$

•

Where the efficiency rating for DMU_o is $1/z^*$ where z^* is the optimal value of z in the model (3.20).

3.9 Applied Models

In this paper, we will evaluate the **Transactional Efficiency** of a Branch using the DEA model introduced by Banker et al. (1984), the BCC model. As explored in the previous section, the BCC model aims to take into account the fact that in different situations the conditions that influence the productivity of production are also diverse, i.e., it's a variable return to scale (VRS) model. We will use VRS because it's assumption is less restrictive than CRS and there is no priori reason to assume that for efficient branches outputs and inputs are proportional. We will designate the efficiency level given by this model by BCC. The orientation we followed from transactional perspective was towards output augmentation, because the branches' goals is to increase transactions in alternative distribution channels, considering their available resources. In an output-oriented model an inefficient unit becomes efficient by proportionally increasing its outputs, while the inputs remain unchanged. However, BCC efficiency scores only translates the radial efficiency. To identify the non-radial efficiency scores we will use a output-oriented Geometric Distance Function (GDF) measure. The output-oriented GDF measure, presented in the section 3.5, is an efficiency measure that considers the slacks at output level to present new efficiency scores. It uses the efficiency score and the output slacks identified by the BCC model to calculate a new efficiency score. Let's emphasize that since the model used is output-oriented, and the outputs used are the transactions in the alternative distribution channels, the efficiency scores represents the ability of the Bank Branch in redirecting the transactions to these alternative distribution channels. And, since the efficiency is relative, we are measuring bank Branch capacity to do so, when compared to the other branches.

To evaluate the **operational efficiency**, we will use outputs that may have negative values, therefore, we will use the RDM model presented by Portela (2003). The orientation we are following is towards output augmentation, since branch's goals are to: 1) increase the customer base and sales; 2) increase sales of the products the branches have to offer, and 3) serve it's clients. Even though transactions are outputs we assumed the bank is not interest in increasing the number of transaction in the bank branches and, therefore, this is a non-discretionary[2] output. Because our goal is output augmentation, we will use the output oriented model (3.14). However, the RDM efficiency score doesn't account for the influence of the value of slacks variables, therefore we will also use the MSBM model presented by Sharp et al. (2007), more specifically the output oriented model (3.17). We also felt it would be interesting to apply the new MSBM model and compare the results, so we will also apply the modified MSBM output oriented model (3.20).

With these methods we will be able to identify, in the context of transactional and operational efficiency:

- The group of branches with the highest productivity - *Efficient branches*.
- The group of branches with lower productivity compared to more productive branches - *Inefficient branches*.
- The set of branches with greater productivity whose characteristics are identical to the less productive branches which will allow us to understand which factors to improve that will lead to an increase in relative efficiency - *Reference Branches*.
- Margin of capacity/ability to increase output (results) in the less productive branches without increasing resources - Deviation to the goals/objectives of the branches.

[2]Portela (2003) explained that an non-discretionary output is an output that must be considered in the model but whose increase shouldn't be considered for the model optimal solution. Adopting the Banker and Morey (1986) approach, Portela set that for output r, that is non-discretionary, we set in RDM models R_{ro} to zero

Chapter 4

Measuring the performance of Bank Branches in a Cape Verdean Bank

4.1 Case Study

In this thesis we want to evaluate the performance of the bank branches of a certain Cape Verdean Bank, that has a total of 33 bank branches. The branches are scattered through all the islands of Cape Verde. We are going to assess the relative efficiency of the bank branches in the following perspectives: transactional and operational.

The data provided by the bank is from the year 2019. The Bank in the study preferred these data as they felt they were a more accurate representation of the bank activities (pré covid19).

In the following sections, we are going to explore in detail each efficiency measure and how we adapted it to our particular case study. We will present the set of inputs and outputs that should ideally be used, and those that we will use given the data availability of the company.

The input and output selection is a crucial part of a good DEA analysis. One of the advantages of the DEA model is that it avoids the problem of agreeing on a common set of weights for the inputs and outputs. However, the models can not avoid the problems related to data selection (input, output). The inputs must consider all the resources used by the DMUs. The output must represent all the amounts of products or services produced by the DMU. Since these products (or services) may be produced at different levels of quality the outputs may include a range of performance and activity measures. The model must also consider all the environmental factors that may affect the conversion of inputs into outputs (Boussofiane et al., 1991).

4.2 Transactional Efficiency

This measure of efficiency reflects the objective of bank branches to shift their focus from transactional activities to value-added activities. In this context, transactions are all types of activities performed by branch personnel that take place inside the bank branch itself, but that could have been carried out through other means of distribution (e.g. withdrawals, balance inquiries, etc.). Therefore the goal is to assess the branch's capacity to redirect these transactions to alternative channels, i.e., assess the performance of bank branches in encouraging the use of alternative means by its customers.

4.2.1 Data - Input/Output

Portela (2003) defines as ideal inputs (resources that allow the branch to encourage the use of alternatives channels) and outputs (results obtained by the agency using available resources) of the transactional efficiency analysis the ones presented in Table 4.1.

Table 4.1: Ideal Inputs and Outputs for Transactional Efficiency

Inputs	Outputs
Number of ETMs	Number of Transactions in ETMs
Average Age Clients	Number of Transaction by Phone
Socio-economic background	Number of Transaction in Internet
Total Number of Transactions	

The ideal inputs refer to the relevant resources in the agency for its transactional activities and aspects of the environment that affect customers' ability to use alternative means of distribution. The agency's resources are the number of Everywhere Teller Machine (ETM)s, which includes Automated Teller Machine (ATM)s and Automatic Payment Terminal (POS/TPA). Other environmental factors are the location of the bank branch and the socio-economic context of the customers, which can influence costumers willingness to use alternative distribution channels. For example, in certain areas, customers may not have the knowledge, equipment and/or willingness to use these channels. The customer's age can also influence the willingness to use distribution channels alternatives: younger customers are more likely to try new options, use the Internet and ETMs, in opposition to older and traditional customers that usually offer more resistance. The input "total number of transactions" includes transactions carried out at the bank branch by branch employees and also transactions carried out through alternative distribution channels. This input is considered because we want to assess to what extent the branch transfers transactions to the alternative distribution channels.

The measurement of transactional efficiency is naturally oriented towards output maximization. It's possible to notice that there is no interest in modifying the inputs since our objective is to increase the number of transactions in alternative distribution channels. Therefore, the ideal outputs pointed out for this evaluation are precisely the transactions carried out in these distribution channels alternatives.

Table 4.2: Actual Inputs e Outputs for Transactional Efficiency

Inputs	Outputs
NºATMs per Branch	Nº Transactions in ATM e POS/TPA
Average Age Clients	Nº Transactions trough Bank online plataform - phone & net
Proportion of population >10 years who used the Internet	
Business Volume (Municipality)	
Population (Municipality)	
Total nº Transactions	

The actual input and outputs used in transactional efficiency assessment are represented in Table 4.2. We were able to gather most of the data considered ideal. The only input data that we couldn't collect was the input "Rent", which is a variable commonly used to translate both the location and the size of the branches. In other words, this input translates the socio-economic background of the branches clients and the location of the branches to allow a fair comparison. That said, we chose to carry out two analyses, considering two different inputs. So that, by comparing the results, we could identify which of the inputs best expresses the reality in which we intend to assess the branches. As a substitute for the "Rent" input, we first chose to use the "Proportion of the population over 10 years old who used the Internet in the last 3 months" and the "Business Volume by Municipality", INE-CaboVerde (2015). In the second analysis we kept the "Proportion of population over 10 years old who used the Internet in the last 3 months", but we replaced the input "Business Volume by Municipality" with the input "Population of the Municipality", also provided by the Bank in study. We chose this input because, considering Cape Verde reality, usually the municipalities with higher population are the cities, where people tend to me more open towards the new distribution channels. The empirical analysis of transactional efficiency will therefore be divided into two cases:

- **Case 1** - Business Volume by Municipality
- **Case 2** - Population of the Municipality

By comparing the results, we want to identify which of the inputs best expresses the reality in which we intend to evaluate the branches.

4.3 Operational Efficiency

The operational measure of efficiency translates all types of operations that go in a bank branch.

Portela (2003) defended that since this efficiency measure receives influence mainly from value-added operations (more product sales), it seems to be linked to transactional efficiency. Because the more transactions the branch redirects to alternative channels, the more time it will have to carry out operations with greater added value, meaning that branches with greater transactional efficiency will tend to have greater operational efficiency.

4.3.1 Data - Input/Output

Portela (2003) defines as ideal inputs (resources used by the branch to carry out operations and the context in which it operates) and outputs (results obtained by the agency using available resources) of the Operational efficiency analysis the one represented in Table 4.3

Table 4.3: Ideal Inputs e Outputs for Operational Efficiency

Inputs	Outputs
Number of Staff	Total Number of Transactions
Competition	New clients
Age of branch	Demand Deposit
Market Potential	Term Deposit
Socio-economic background	Standard Credit
	Overdue Loans
	Banking Product

This measure has more similarities with the usual production efficiency in banking literature. However, it has some differences, namely, it considers the market efficiency measure as defined in Athanassopoulos and Thanassoulis (1995) and Anthanassopoulos (1998). Anthanassopoulos (1998) defines market efficiency as "the extent to which individual bank branches, given their capacity and resources available, utilize their market potential by maximizing sales". This applies in this context because one of the objectives of the branch is to increase its sales and customer base, something that depends on its market conditions (competition in the marketplace, location of the branch, socio-economic conditions of people in this location). The market efficiency measure is naturally output-oriented.

The measure of operation efficiency presented by Portela (2003) fits the definition above but it also incorporates other operational aspects. For instance, they still wanna offer quality service to the clients that go there to perform general transactions, which is why transactions appear as an output of operational efficiency. But the bank doesn't want to increase this input so, it represents a non-discretionary. Competition, market potential (reflects "size"), socio-economic background (reflects "quality") of the branch clients are environmental factors. The branch age is an important input if we want to study branches growth.

There was no consensus in the Portela (2003) study whether more competition or less competition was better. Therefore, this input won't be included.

The operational input number of staff is a particularly important input in the operational activities of bank branches.

The output of the operational efficiency assessment reflect the number of new clients and the number of sales. Besides selling, staff also serves clients that go to the branch for transactional purposes. Let's emphasize that, this output is non-discretionary, because the bank is not interested in increasing it. Another note is that, in this efficiency assessment, the total number of transactions refers to all the transactions made in the branch, without discriminating between branch clients or not.

Portela (2003) also considered as an ideal input the service quality index, however, decides against it, because she believes that is not a practical input to measure and use.

Given data availability, the actual inputs and outputs used in the operational efficiency assessment are represented in Table 4.4

Table 4.4: Actual Inputs and Outputs for Operational Efficiency

Inputs	**Outputs**
Number of Staff	Total Number of Transaction
Municipality Population	New clients
Business Volume	Demand Deposit
	Term Deposit
	Standard Credit
	Overdue Loans
	Δ Banking Product

Concerning inputs we will use "Business Volume" as a translation of the "socio-economic background", and "Municipality Population" as the representation of "Market Potential" as suggested by the director of the Bank.

In the transactional efficiency analyses we used both these inputs separately as a representation of the socio-economic background, dividing the analyses in two different cases. However in the operational efficiency we will use both inputs simultaneously. This is because we believe that this scenario will more accurately represent the reality in which we are studying the bank branches. Even so we´ll also explore a little of the cases in which when we use only the "Municipality Population" input or only use the "Business Volume" input and present the found differences, and possible conclusions.

Reinforcing that the ideal outputs are the adaptation of the ideal outputs presented in Portela to the reality of the bank we are studying. The outputs variables in Table 4.3 represent the number of the operations. The bank was able to provides this data, except for the output "Banking Product". For this output we will use information that relates to the amount of money kept in the various accounts: we will use the changes in the values of Banking product from 2018 to 2019.[1]

[1] Δ - Variation between two consecutive years

Chapter 5

Empirical Analysis

5.1 Transactional Efficiency

Through this section we will be exploring the results found on the analysis of transactional efficiency of the 33 branches of the Cape Verdean Bank under study. The analysis will be divided into two cases: Business Volume and Population by Municipality.

We were able to receive feedback from the Bank when it comes to the results of the transactional efficiency, therefore, through this analyse I will include some comments made by the director of the Bank in his feedbak. Every time I do so I will refer to this comments as Observations made by the director of the Bank (ObsD). I decided to include the comments in this way to present the director's point of view after seeing the results of this analyses.

All the efficiency results shown in this subsection were produced using the software GAMS, a modeling system for mathematical optimization that uses the simplex method.

5.1.1 CASE 1: Business Volume

In this case, the input considered to translate the environment reality of the branches was the "Business Volume".

5.1.1.1 Transactional Efficiency Score

The results of this subsection were obtained through the BCC output-oriented model 3.8, to these results we also applied the GDF function 3.10. Table 5.1 shows the transactional efficiency scores by branch, of branches that were considered inefficient. The remaining branches have an efficiency score of 1 and slack values are zero, therefore they are considered efficient. We can see from Table 5.1 that the values of BCC and GDF are the same, this allows to conclude the percentage of inefficiency in the form of slacks values is non-existent for all branches shown in Table 5.1.

Consider Branch F, which has the lowest degree of efficiency, with an efficiency score equivalent to 69%. It can be interpreted that with this current level of inputs, Branch F is producing only 69% of it's maximum possible outputs. In this sense, the outputs of Branch F should increase by about 144.9% (1/0.69) for the branch to reach the efficient frontier. An analogous interpretation can be done to the other inefficient branches.

ObsD: The Bank Branch F is in rural location, where we assumed people have the tendency to use the Branch counter more that the alternative distributions channels, (due to a certain resistance to the new technologies). Which corroborates with the observed efficiency score.

Table 5.1: Transactional Efficiency Score per Branch for inefficient branches - Business Volume

Branch	**BCC**	**GDF**
F	0.69	0.69
E	0.85	0.85
P	0.92	0.92
S	0.94	0.94
V	0.98	0.98

ObsD: Branch F and E are branches from rural locations, and branch P although is located in the capital city it is historically used by clients that come from rural locations.

5.1.1.2 Target level

Another point of interest for the Bank are the goals (target levels) to be achieved by inefficient branches. Table 5.2 shows how much each observed output must improve for the inefficient branch in question to be considered efficient. Let's take for example Branch E, it's target levels indicate that considering it's inputs (average age of client, number of ATMs per branch, the proportion of the population over 10 years old using the Internet, business volume, and the total number of transactions at the branch), the Branch E could present a higher number of transactions in ATM and POS/TPA (about 1500 transactions more) and higher number of transactions performed through the Bank online platform (about 1600). That is, the Branch is not making the most of the available inputs to generate the maximum possible outputs. A similar evaluation can be done for the remaining inefficient branches.

5.1.1.3 Peers or Benchmarks

The goals explored in the section above were based on the efficient branches with whom we compared each inefficient branch. These efficient branches are called Peers or Benchmarks. Table 5.3 shows the peer branches for each inefficient branch, i.e., branches whose

Table 5.2: Observed and Target levels Transactions for inefficient branches - Business Volume

Branch	**Output**					
	ATMs			**Online Platform**		
	Observed	Target	Target deviation	Observed	Target	Target deviation
F	11 913	17 259	5 346	11 252	16 301	5 049
E	8 506	9 970	1 464	9 402	11 020	1 618
P	45 297	49 215	3 828	47 304	51 396	4 092
S	38 200	40 498	2 298	31 309	33 192	1 883
V	37 560	38 101	541	42 798	43 414	616

transactional practices could be emulated. For each branch, we also identified the proportions that each peer branch contributed to the point used to evaluate the branch under study, i.e., the contribution of each reference branch to the optimal value of BCC model and for the optimal target levels. The higher the proportion, the greater is the similarity between the branch on evaluation and its peer. The colors represented in Table 5.3 differentiate this proportion as follows: (i) proportion smaller that 0.2 has no color; (ii) proportion between 0.2 and 0.4 is red; and (iii) proportion higher than 0.4 is yellow.

Table 5.3: Peer or Benchmarks of each inefficient Branch and the proportion of their relation - Business Volume

Peer\Branch	F	E	P	S	V
A	-	-	-	0.426	-
C	-	-	-	0.344	-
H	-	0.097	-	-	-
I	0.206	0.307	-	-	-
J	0.204	-	-	-	-
K	0.507	0.301	-	-	-
M	-	0.295	-	-	-
N	-	-	-	0.278	-
O	-	-	0.467	-	0.315
Q	-	-	0.425	-	0.293
Z	-	-	0.104	-	0.212
AF	0.083	-	-	0.052	-
AH	-	-	0.04	-	0.181

Through Table 5.3 we can see that most of the branches have 3 significant peer branches that contribute for their target values. Branch P however, has two significant peers, Branch O and Branch Q. More than half of the Peers branches are the benchmarks for two bank branches.

To better understand what the proportion in the Table 5.3 represent, lets consider the example of branch F. Considering the proportions contributed by the branches K, I, J and AF for the point used to assess Branch F, trough model 3.8, we can express the output targets of Branch F as:

$(Output\ Branch\ F) * (1/0.69) = 0.507 * (Output\ Branch\ K) + 0.206 * (Output\ Branch\ I) +$

$0.204 * (Output\ Branch\ J) + 0.083 * (Output\ Branch\ AF)$

We can also say that due to the magnitude of the coefficients it appears that Branch F has more similarity with Branch K. ObsD: This is an interesting conclusion since the branches can be considered similar when it come to their clients.

ObsD: In general the director could see the similarities between the Branch and its Peers.

Table 5.4: Peers or benchamrks of Branch F - Business Volume

	Branch	F	K	I	J	AF
Inputs	Number of ETMs	4	2	1	3	1
	Average age	53	52	50	42	37
	Prop. Internet	33.6	31.2	30.4	42.8	33.6
	Business Volume	2 139	646	326	7 688	2 139
	Total nºtransactions	66 125	62 601	39 383	119 524	23 167
Output	ATMs	11 913	15 314	5 248	38 219	7 563
	Online Platform	11 252	13 748	11 378	31 602	6 598

Table 5.4 shows all input and output data related to Branch F and its peers. We can see that the values of the inputs of Branch K are smaller than the values of inputs of Branch F, with the Business Volume is significantly smaller (3 times smaller). However, all the output values are greater. On another hand Branch I which has almost half of the transactions numbers of Branch F, still manages to redirect more transactions to the online platform than Branch F.

ObsD: The Branch K can be seen in certain ways as similar to Branch F, even though is located in another island. However Branch K has a large number of emigrant clients which may have contributed to stimulate the higher use of the alternative distribution channels.

This last comment of the director made us think that in the future it could be pertinent to include as an input, “the number of emigrant clients per branch” and analyse it's effects.

5.1.2 CASE 2: Population by Municipality

In this case, we will use the input “Population by Municipality” to replace the input “Business Volume” used in the first case. Since the BCC Model compares a DMU to other DMUs similar in scale, we considered it would be pertinent to present two options to replace the input "Rent". This so that we can see which input best reflects the reality in which we conduct this study i.e, the location and socio-economic context of bank branches.

5.1.2.1 Transactional Efficiency Score

In the Table 5.5 we can see presented the BCC model efficiency scores and the GDF efficiency scores of the branches that were considered inefficient in the transactional point of view.

Table 5.5: Transactional Efficiency Score per inefficient Branch - Population by Municipality

Branch	BCC	GDF
F	0.47	0.47
M	0.54	0.54
E	0.59	0.59
T	0.78	0.61
G	0.80	0.76
D	0.85	0.68
P	0.92	0.92
V	0.99	0.99
S	1.00	1.00

In this case, the model identifies four additional inefficient branches when compared to the solution in Case 1. All branches that were considered inefficient in Case 1 continue to be considered inefficient in this context. Figure 5.1 represents the efficiency scores of all the branch considered inefficient, considering case 1 and case 2.

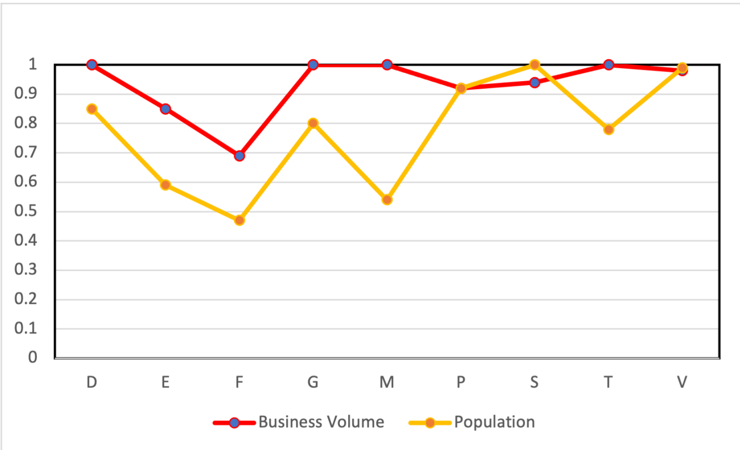


Figure 5.1: Transactional efficiency values considering the cases: 1) Business Volume, 2) Population

Let's take a closer look at Branch F, which continues to be the Branch with the highest level of inefficiency, but that now has an efficiency score of only 47%, when, it had an efficiency score of 69% on Case 1. In other words, Branch F is considered almost 22% less efficient when considering the input "Population of the Municipality" instead of the input "Business Volume". Branch E, which in Case 1 presents a score of 85% efficiency, now has an efficiency score of only 59%, losing 26% of its efficiency, one of the most significant changes. But the biggest change comes when we analyze Branch M, which in Case 1 is considered efficient but in Case 2 presents an efficiency score of just 54%. On the other hand, the Branch P efficiency score does not change, the efficiency of Branch V slightly improves (just 1%), and Branch S improves almost 3%, with an efficiency score that almost rounds off 100% (it rounds to 1 when we round the values to 3 decimals).

In this analysis, in contrast to Case 1 (where the GDF efficiency scores remained unchanged), 3 branches were identified with slacks at the output level, and therefore present smaller GDF efficiency. This is the case for Branch D, G, and T. Branch D has a BCC efficiency score of 85%, when the slacks are not taken into account but, on the other hand, it has the GDF efficiency of 68%, which indicates that when slacks are accounted for, the Branch's efficiency reduces by about 17%. This means that slacks represent a major source of inefficiency for Branch. I.e, the branch has at least an output that can be improved besides the equiproportional radial improvement. Similarly, when we analyze Branch T, we can see that it has a BCC efficiency score of 78% but a GDF efficiency score of 61%, which indicates that when slacks are accounted for, the efficiency of the branch also reduces about 17%, which indicates that, for this branch, the slacks also represent a major source of inefficiency. As for Branch G, the level of efficiency reduces only 4% when we consider the slacks, i.e., most of the Branch inefficiency can be explained by radial efficiency and not slacks.

ObsD: Branch D and Branch T both belong to the same island.

This lead us to believe that when we change the input to population these branches are now compared with another that is in a county that has different business volume but similar population. In this context, if the results are influenced by the population, these branches should be doing better.

5.1.2.2 Target level

Another point of interest, as we know, are the target levels to be achieved by inefficient branches in order to be considered efficient.

The observed and targed values of the branches at the output level are presented in Table 5.6. In contrast to Case 1, there are slack values different from zero in some branches. Thus, in Table 5.6, radial targets (RTargets), that don't include the value of slacks are distinguished from non-radial targets (NRTargets) that include the value of slacks. The difference between these two targets reflects the value of the slacks identified by the model.

The non-radial targets for Branch D indicate the existence of slacks at the output level "Number of Transactions in the ATMs" (column ATM of Table 5.6. That is, for Branch D to reach the efficient frontier, "Number of Transactions in ETMs" must reach RTarget (13 869) but considering the available inputs of the Branch, without the risk of worsening another input or output, it would still be possible for the Branch to increase the "Number of Transactions in ETMs" to the NRTarget value (21 973). An identical analysis can be made to the Branch T.

On the other hand, Branch G has slack values in the output "Number of Transactions at the online platform". That is, for the branch to reach the efficient frontier, the "Number of Transactions in the online platform" (column "Online Platform" at table 5.6) should reach the RTarget (15 304) but, considering the available inputs of the branch, without the risk of worsening other output or input, it would still be possible for the branch to increase the “Number of Transactions in the online platform” to reach the value NRTarget (16 806).

We can also see that when it comes to Branch D, RTarget represents only 63% of the maximum possible output for the branch , and that, when it comes to Branch T, RTarget represents only 60% of the maximum possible output for the branch, these values are consistent with the significant influence of slacks on the branch’s GDF inefficiency value (68% and 60% respectively). On the other hand, when it comes to Branch G, RTarget represents 91% of the maximum possible output for the Branch, which is also consistent with the small influence of slacks on the GDF inefficiency value of the Branch.

Table 5.6: Observed and Target Output levels of Inefficient Branches - Population by Municipality

Branch	Output									
	ATMs					Online Platform				
	Observed	RTarget	RTargert Deviation	NRTarget	NRTarget Deviation	Observed	RTarget	RTargert Deviation	NRTarget	NRTarget Deviation
D	11 821	13 869	2 048	21 973	8 104	17 018	19 966	2 948	19 966	0
E	8 506	14 335	0	14 335	0	9 402	15 845	6 443	15 845	0
F	11 913	25 483	13 570	25 483	0	11 252	24 069	12 817	24 069	0
G	18 957	23 838	4 881	23 838	0	12 170	15 304	3 134	16 806	1 502
M	6 972	12 899	5 927	12 899	0	7 547	13 962	6 415	13 962	0
P	45 297	49 215	3918	49 215	0	47 304	51 396	4 092	51 396	0
S	38 200	38 251	51	38 251	0	31 309	31 351	42	31 351	0
T	2 165	2 762	597	4 592	1 830	6 016	7 676	1 660	7 676	0
V	37 560	38 101	541	38 101	0	42 798	43 414	616	43 414	0

5.1.2.3 Peer or Benchmarks

Table 5.7 shows the peer branches for each inefficient branch considering the input "Population of the Municipality". These are peers branches whose transactional practices can be used as reference. For each branch, we identified the proportions contributed by the peer branches to the point used to assess the branch under study (optimal solution) and, as we know, the greater the magnitude of this proportion, the greater the similarity between the branch in study and the peer branch.

The colors represented in Table 5.7continue to differentiate this proportion, the same way presented in Case 1 [1]. Although the branches usually have more peers branches than in Case 1, most branches have a peer branch whose contributed proportion is higher than 0.4 and, even higher than 0.5 in more than half of the cases. This means that, although there are more peer branches, it is easier to identify the peer branch more similar to the branch under study.

[1]Proportion smaller that 0.2 has no color, proportion between 0.2 and 0.4 is red, if the proportion is higher than 0.4 the is yellow.

Table 5.7: Peer or Benchmarks of each Inefficient Branch and the respective proportion - Population by Municipality

Peers \Branch	F	M	E	T	G	D	P	V	S
A	0.031	0.072	0.556	0.007	-	0.04	-	-	0.05
B	-	0.011	0.238	-	-	-	-	-	-
C	-		-	-	0.153	-	-	-	0.345
H	-	0.088	0.12	-	0.494	0.620	-	-	-
I	-	0.125	0.059	0.507	-	0.173	-	-	-
J	-	-	-	-	0.075	0.167	-	-	0.13
K	-	-	-	-	0.101	-	-	-	-
N	0.295	-	-		-	-	-	-	0.018
O	0.081	-	-	-	-	-	0.47	0.293	-
Q	-	-	-	-	-	-	0.426	0.212	-
Z	-	-	-	-	-	-	0.10	0.181	0.07
AC	-	-	-	0.022	-	-	-	-	-
AD	0.593	0.704	-	0.286	0.176	-	-	-	-
AE	-	-	0.027	0.177	-	-	-	-	-
AH	-	-	-	-	-	-	0.04	0.314	0.387

5.2 Operational Efficiency

In this section we are going to present and discuss the results found upon the evaluating of the operational efficiency of 33 bank branches of a Cape Verdean Bank.

Operational efficiency of a branch will be evaluated using the RDM model output-oriented model 3.14, the MSBM output-oriented model presented 3.17 and the modified MSBM output-oriented model 3.20 and comparing these results.

All the efficiency results shown in this subsection were produced using the software GAMS, a modeling system for mathematical optimization that uses the simplex method.

5.2.1 Efficiency Scores

Figure 5.2 presents the operational efficiency scores for the branches that were considered inefficient by each model (RDM, MSBM and modified MSBM). The remaining branches are not shown because they have an efficiency score of 1, and the slacks values are all zero, and therefore are considered efficient.

As we can see, the MSBM model usually presents the lowest efficiency scores because, unlike the RDM model, it considers the influence of the slacks values. The modified MSBM efficiency scores tend to be lower than the RDM scores but higher than the MSBM (except for two cases). We believe this is related to the improvement range that uses the minimum and maximum input. The modified MSBM aimed to solve the limitation that arises in the MSBM model because of the cases where the SP range[2] is equal to zero. However, the presented solution does not compensate for the advantages of using the SP range, which seems a more precise improvement measurement. Besides, it makes more sense to compare the RDM model and the MSBM model because they both use the SP

[2](i) $R_{io} = x_{io} - \min_j\{x_{ij} : j = 1, \ldots, n\}$ (i=1,..., m) , (ii) $R_{ro} = \max_j\{y_{rj} j = 1, \ldots, n\} - y_{ro}$ (r=1,..., s)

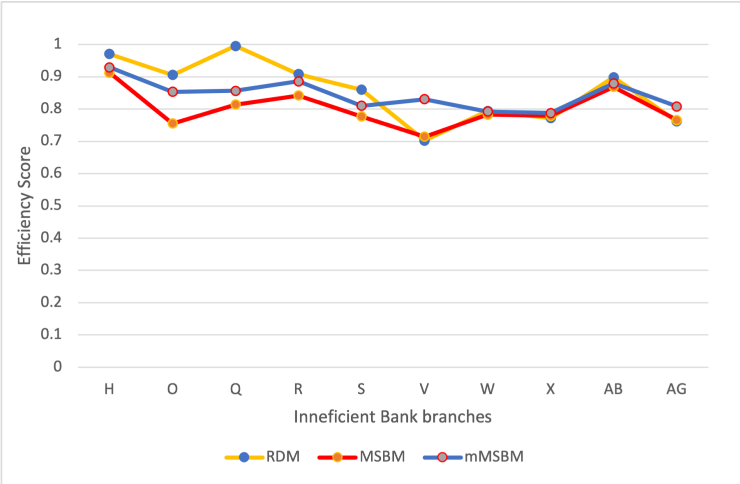


Figure 5.2: Operational Efficiency scores using models RDM, MSBM and modified MSBM

range. Therefore from this point forward, we will discuss the results from these two models.

Figure 5.3 and Table 5.8 show the results of the RDM and the MSBM models. The major difference between this scores happens in the the inefficient Branches O, Q, R and S. Which means these are the bank branches where the slacks have a bigger impact. The Branch V has the lowest efficiency score, with an efficiency score of about 70 % in both models.

Table 5.8: Operation efficiency Score of Inefficient branches given by models RDM and MSBM

Branch	**RDM**	**MSBM**
H	0.970	0.913
O	0.906	0.755
Q	0.995	0.814
R	0.908	0.842
S	0.860	0.777
V	0.702	0.715
W	0.794	0.783
X	0.772	0.779
AB	0.899	0.868
AG	0.761	0.766

Another interesting characteristic of the MSBM model is the weight component present in the models that allows the study of the efficiency score in different scenarios. The Figure 5.4 shows the efficiency scores considering 7 different scenarios, where scenario 0, is the one we have been seeing and that represents equal weights for all the outputs.

These are possible scenarios that I considered, each scenario represents a reality where an output, or a set of outputs is considered more important than the others. The more relevant outputs per scenario (3 first scenarios) are: (i) Scenario 1 - New clients; (ii)

Figure 5.3: Comparison of Operational Efficiency Score of Inefficient branches given by model RDM and MSBM

Scenario 2 - Deposits ;(iii) Scenario 3 - Credit and Loans. In Scenario 4 we only consider the outputs Deposits, Credits and Loans. In Scenario 5, New clients and Banking product are considered but are still less relevant than the rest. In Scenario 6, Banking product is less relevant than all the other outputs.

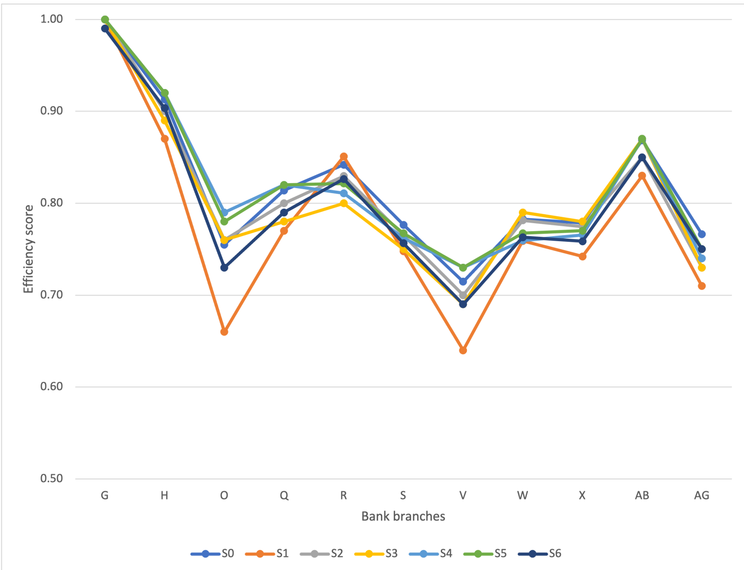


Figure 5.4: Operational Efficiency of Inefficient branches using MSBM and considering 7 different scenarios

The scenario that differs the most from the others, is scenario 1. In this scenario, we considered that the most important output was the output "New Clients". We can see that most of the branch's efficiency scores got lower which means that this is an output they are especially lacking, i.e, these branches are failing in bringing new clients to the branch. On the other hand, Branch R is the only branch whose efficiency got higher in scenario 1, meaning that this branch is doing a better job bringing new clients compared to the other outputs.

5.2.2 Target Settings

Another point of interest are target levels to be achieved by inefficient branches for them to be considered efficient. Table 5.9 presents the targeted values of the branches considered inefficient given by the models RDM and MSBM. The RDM target values are radial (slacks values are not accounted for), while the MSBM targets values are non-radial (slacks values are accounted for).

Table 5.9: Observed and Target levels of RDM and MSBM model of inefficient branches

Branch	Output								
	New clients			Demand Deposit			Term Deposit		
	Obs	RDM	MSBM	Obs	RDM	MSBM	Obs	RDM	MSBM
H	111	128	253	9231	10050	9901	1394	1548	1789
O	284	321	660	9890	12425	19497	895	1432	1728
Q	313	315	479	8876	9002	16515	1021	1046	2693
R	423	446	460	8720	11292	14391	736	1272	2219
S	198	265	413	9246	13107	14602	494	1348	3342
V	402	483	661	8182	16696	17580	593	2378	1413
W	126	239	322	1550	8809	9775	92	1430	2251
X	25	173	322	396	8698	9775	15	1515	2251
AB	74	135	244	845	4484	6802	59	720	1524
AG	389	457	567	2921	10996	11597	183	1710	662

Branch	Output								
	Standard Credit			Overdue Loans			Δ Banking Prodcut (contos cv)		
	Obs	RDM	MSBM	Obs	RDM	MSBM	Obs	RDM	MSBM
H	645	708	764	107	116	184	2490	3620	2621
O	1707	1808	1808	306	316	355	18240	20336	23288
Q	632	642	1102	81	82	193	16628	16736	16628
R	772	956	1165	141	165	254	7245	10292	10237
S	449	775	1020	59	108	240	1765	7195	6811
V	1112	1607	1700	236	287	349	10590	19488	21381
W	180	714	680	44	119	161	-29717	-15248	5561.
X	108	716	680	28	115	161	-14749	-2144	5561
AB	261	516	470	43	80	110	-944	3250	3877
AG	580	1103	1247	109	180	282	4040	12729	14725

Trough Table 5.9 we can see the difference between the RDM and MSBM targets for the outputs “New Clients”, “Demand Deposit"” and “Term Deposit” for Branch O. Which allows to conclude that the 20% difference Branch O has between it’s RDM efficiency score and it’s MSBM efficiency score comes from the slack values related to these outputs. In the same line of logic, the MSBM efficiency score of Branch Q is smaller that his RDM efficiency score because of the influence of slack values related to the outputs “Demand Deposit” and “Term Deposit”.

When we analyse the Branch AB we can see that it's MSBM efficiency score is just slightly bigger than it's RDM efficiency (0.5 %), but the differences in the targeted levels are quite considerable. The MSBM target level for the input “Term Deposit” is a lot bigger the RDM target level, however, the MSBM target level of the input “Standard Credit” is smaller than the RDM target level, which indicates that the models prioritized different outputs when studying Branch AB. A similar interpretation can be made to Branch H.

Another observation is that the difference in the target levels given by the models are higher when the output “Δ Banking Product” has negative values. This happens because the MSBM always pushes for a positive target level, however, even though the RDM model defines an improvement that improvement doesn't necessarily have to result in positive values of output.

Overall, the MSBM targets tend to have much higher values than the RDM targets, and the outputs "Demand Deposits"and "Term Deposits"are the outputs were there can be identified the major differences. With the targets given by RDM and MSBM are in some cases almost the double form the other, even though the efficiency score is not that different.

5.2.3 Peers or Benchmarks

The goals explored in the section above were based on the efficient branches with whom we compared each inefficient branch. These efficient branches are called Peers or Benchmarks.

We'll only present the peers identified by the MSBM model. To explain why, let's first recall that when we identify branch's peers we also identify the proportions that each peer Branch contributed to the point used to evaluate the Branch under study. The higher the proportion, the greater is the similarity between the Branch on evaluation and its peers. That said, comparing the peers identified by RDM and MSBM we could see that the RDM model identified more peers than the MSBM model, however, the proportion that these extra peers contribute to evaluate the branch is usually very very low (<0.001) which mean these branch have very little similarity with branch under study . Therefore for analysis purposes, we decided that it would be more relevant to evaluate the peers with higher similarities to the branch under evaluation which are given by the MSBM.

Table 5.10 shows the peer branches for each inefficient branch, i.e., branches whose operational practices could be assimilated. The colors represented in Table 5.10 differentiate the proportion as follows: proportion smaller that 0.2 has no color, proportion between 0.2 and 0.5 is red, if the proportion is higher than 0.5 the is yellow. We chose 0.5 because in the operational efficiency perspective there are a significant number of peer branches that represent a proportion of more than half, and we decided it would be interesting to have these stand out.

We can see that most branches have a peer whose proportion is bigger than 0.5 meaning these are very similar branches, and they should be studied with attention to improve

Table 5.10: Peer or Benchmarks of each Branch of inefficient branches using RDM model for Operational Efficiency

Peer\Branch	H	O	Q	R	S	V	W	X	AB	AG
A		0.25				0.167				
B			0.37	0.07						
D	0.31			0.42	0.25					
E	0.39				0.01					
F			0.01	0.13	0.73		0.6	0.6	0.4	
L	0.29									
P		0.75	0.31	0.38		0.83				0.83
Y					0.004					
AD			0.319				0.4	0.4	0.6	0.167

the inefficient branches practices. As for the Branches H, O and R, they have a set of peer branches whose contribution are significant.

5.2.4 Results with alternative inputs

As discussed in section 4.3, we felt that it would be relevant to conduct a few extra analyses in the Operational efficiency evaluation. In this section we'll discuss the major differences we found when we considered two different cases:

- CASE 1 - using only the input "Population"
- CASE 2 - using only the input "Business Volume"

All branches considered inefficient in the previous analysis are also inefficient in both of these cases. However, in these cases, we found two more inefficient branches. One branch appears in both cases (Branch G) but the other two are different per case. We decided that these branches are within the threshold of efficiency, meaning that they are very susceptible to change depending on the input assumptions.In the following analysis we are going to explore: 1) the branches that aren't considered inefficient in the first analysis and 2) the difference that occurs in the branches that are considered inefficient in all the cases.

5.2.4.1 CASE 1 - Population by Municipality

Efficiency Scores

Table 5.11 displays the efficiency scores that, in this case, are different from the ones found in the first operational analysis. Branch G and Branch AF are the branches that are considered inefficient in this case but are efficient when we consider both inputs. However we can see that their efficiency score are still very close to 1. The Branch H is displayed because in the RDM model it's efficiency dropped a little bit.

Table 5.11: Operational Efficiency Score of each Branch with different efficiency using the Population by Municipality

Branch	RDM	MSBM
G	0.983	0.948
H	0.964	0.913
AF	0.976	0.952

Target Settings

Table 5.12 shows the target levels found in this case for the branches that showed different results. Branch H shows differences in all the RDM targets but they don't seem to be very significant.

Table 5.12: Observed and Target levels of each Branch with different efficiency using Population

Branch	Output								
	New clients			Demand Deposit			Term Deposit		
	Obs	RDM	MSBM	Obs	RDM	MSBM	Obs	RDM	MSBM
G	235	243	235	8528	9013	10169	960	1057	1772
H	111	131	253	9231	10225	9901	1394	1581	1789
AF	203	214	251	4045	4836	6499	412	561	1300

Branch	Output								
	Standard Credit			Overdue Loans			Δ Banking prodcut (contos cv)		
	Obs	RDM	MSBM	Obs	RDM	MSBM	Obs	RDM	MSBM
G	685	721	738	117	122	161	1803.647	2467.753	1988.673
H	645	722	764	107	118	184	2490.073	3861.373	2621.249
AF	405	462	481	111	118	112	4303.149	5177.299	4303.149

Peers or Benchmarks

In Table 5.13 we display the peers in this case. The colors in the table still represent the same proportions as defined above [3]. We can see that Branch G and Branch D both have branches whose proportion is bigger than 0.5. The only new branch that appeared as a reference is Branch J.

5.2.4.2 CASE 2 - Business Volume

Efficiency Scores

Table 5.14 displays the efficiency results found in this case that are different from the original operational efficiency analyses. In this case the only branches that are considered inefficient here that weren't in the firs analysis are Branches G and L. However, Branch H, Q, R and S all show different efficiency scores. Branch H suffered the greatest change,

[3]proportion of 0.2 has no color, proportion between 0.2 and 0.5 is red, if the proportion is higher than 0.5 is yellow

Table 5.13: Peer or Benchmarks of each Branch with different efficiency using Population on RDM model

Peer\Branch	G	H	AF
D	0.51	0.31	
E	0.13	0.39	
F			0.33
J	0.13		
L	0.23	0.29	
P			0.06
AD			0.61

losing more than 12% of its efficiency score when we consider only the "Business Volume"input, another branch who also suffers a lot of change is the Branch R. Branch Q only loses efficiency score in the MSBM model.

Table 5.14: Operational Efficiency Score of each Branch with different efficiency using Business Volume

Branch	**RDM**	**MSBM**
G	0.91	0.862
H	0.84	0.793
L	0.93	0.876
Q	0.96	0.742
R	0.77	0.755
S	0.79	0.747

Target Settings

Table 5.15 represents the target levels found in this case. We can see that when we considered only “Business Volume”, the target levels for the Branch H are bigger in all the outputs, and the same goes for the Branch R and Branch S. As for Branch Q, what seems to present the most changes are the target levels for outputs “New clients” and “Overdue Loans”, in the MSBM targets, i.e, the targets with the influence of slacks.

Peers or Benchmarks

Table 5.16 represents the peers identified in this case. The colors maintain the same meaning[4]. A very interesting observation is that in the previous study cases, considering both inputs and considering only input “Population” the Branch L is identified as a peer of Branch H, however is this case it is considered inefficient. Therefore the peers for Branch H change. Branch G on another hand preserves the peer Branch D with the same proportion even though the other peers change (Branch J e L) or change proportion (Branch E). Branches Q, R and S all suffered change in the peers considered, but Branch F

[4]proportion of 0.2 has no color, proportion between 0.2 and 0.5 is red, if the proportion is higher than 0.5 the is yellow

Table 5.15: Observed and Target levels of each Branch with different efficiency using Business Volume

Branch	Output								
	New clients			Demand Deposit			Term Deposit		
	Obs	RDM	MSBM	Obs	RDM	MSBM	Obs	RDM	MSBM
G	235	273	319	8528	10953	12623	960	1443	2653
H	111	199	418	9231	13526	14081	1394	2204	3101
L	159	197	283	5531	7828	9722	549	993	1751
Q	313	328	600	8876	10010	17551	1021	1247	2733
R	423	480	586	8720	15117	15302	736	2070	2084
S	198	298	488	9246	15030	15182	494	1774	3308

Branch	Output								
	Standard Credit			Overdue Loans			Δ Banking prodcut (contos cv)		
	Obs	RDM	MSBM	Obs	RDM	MSBM	Obs	RDM	MSBM
G	685	864	888	117	142	211	1803.647	5124.220	4121.288
H	645	977	1000	107	154	248	2490.073	8413.947	7890.386
L	589	750	729	130	150	186	-500.021	2512.863	5601.895
Q	632	719	1391	81	94	294	16628.910	17598.218	16752.055
R	772	1228	1372	141	202	309	7245.406	14822.688	14822.688
S	449	937	1118	59	132	265	1765.945	9900.277	9687.224

still remain the closest to Branch S, which can mean that Branch S and Branch F are from municipality's whose business volume and population are very close in proportion..

Table 5.16: Peer or Benchmarks of each Branch with different efficiency using Business Volume on RDM model

Peer\Branch	G	H	L	Q	R	S
A					0.03	
B				0.17		
D	0.51		0.11		0.004	
E	0.26	0.41	0.63			
F	0.23	0.59		0.34	0.40	0.87
M			0.13			
P				0.50	0.56	0.11
AD			0.13			0.02

5.3 Considerations

In the transactional efficiency analysis, it wasn't possible to obtain one of the necessary inputs to evaluate the branches. This input was the "Rent" input, which is commonly used to translate the location and socio-economic context of the Branches under study. To replace this data, we chose to use the "Proportion of population over 10 years old who used the Internet in the last 3 months" and the "Volume of Business per Municipality". Analyzing the results we could verify the influence that this data has on the results. Therefore, we decided that it would be prudent to also make a second analysis, considering the data "Population by Municipality", replacing the input "Business Volume Municipality". With these two analyses, the intention was to understand which of the inputs best reflects the reality in which we are studying the bank branches. In the first Case 1 we were able to identify 5 inefficient branches, and in Case 2 those same 5 branches were identified plus another 4 inefficient branches, with some additional information, we can see that the two analyzes complemented each other. But it seems that in the case of transactional efficiency the input "Population" seems to find fragility's in the bank branches efficiency. In Cape Verde, county with similarly populations tend to be very similar, and this variable seems to account for a source of inefficiency that the Business Volume isn't. Population seems to be considering more possibilities for improvement. On the transactional analysis, in a total of 33 bank branches, 24 branches were always considered efficient independently of the inputs, 5 branches were always considered inefficient and for 4 branches the efficiency depends of the chosen input.

To study the operational efficiency we had to deal with negative data. Therefore we used the RDM model and the MSBM model that can handle negative data without needing to transform them. Both these models identified the same inefficient branches for all cases. The efficiency score of the MSBM model, as expected, is lower than the RDM model in most cases, because of the influence of slacks. We felt that using both models was an interesting way to identify the influence of slacks, and which are the outputs where they were more present. The major difference between the model, came in the analyses of the targets. Overall the MSBM are much bigger than the RDM and, in some cases the MSBM targets double the RDM model, especially in the cases where we have negative outputs. It seems that RDM targets are in general more achievable and, therefore, it may be pertinent to focus on the RDM targets and study the MSBM targets to understand what may be influencing the considerable difference in the output target. On another hand, MSBM model offers the advantage of using different scenarios depending on the different importance one may attribute to the different outputs, which may be of interest in certain for certain situations. In the operational efficiency context we choose the inputs "Business Volume" and "Population" simultaneously. For a complementary analysis we also decide to study the two cases where we use only one of these inputs. We found in both cases 2 additional inefficient bank branches, that should also be looked in. These analyses are complementary because they all bring to light new information. Using both inputs as a starting point proved to be a good decision because the model identified the bank branches that are most inefficient, i.e, inefficient independently of the case and therefore should be analysed more closely. On the operational context, in a total of 33 bank branches, 20 branches are always considered efficient independently of the input, 10 branches are always considered inefficient and for 3 branches the efficiency depends on the chosen input.

When we defined operational measure of efficiency we speculated that this measure of efficiency would be linked to transactional efficiency, i.e., the banks who are more effective considering the transactional measure would also be more effective in the operational measure. Because the more transactions the branch redirects to alternative channels, the more time it will have to carry out operations with greater added value, meaning that branches with greater transactional efficiency will tend to have greater operational efficiency. However, that doesn't seem to be the case in this bank. From the inefficient branches, we identified in transactional efficacy analysis only Branches S and V are inefficient regardless of the mcase. Branch G is inefficient in the transactional analysis when we considered input population and it is also inefficient in the operational efficiency analysis when we consider input population (and business volume), however is not an inefficient branch when we consider both inputs. Another remark is that even though Branches E, F, and P were considered inefficient in the transactional analysis not only they are efficient in operational analysis but they are also peers or benchmarks for the Bank Branches who are considered inefficient. Therefore in the case of the bank branches of this Cape Verde bank the operational efficiency and the transnational efficiency don't seem to be linked.

Let's remember that because the weights in DEA are the variables, they represent the best possible scenario for each unit. If a unit is considered inefficient even in the best scenario, then this is a strong statement. Therefore we decided that it would still be pertinent to present and consider the inefficient units whose efficiency score was approximately 1, but not 1. Because there are sources of inefficiencies. That even though , are not alarming, can still be improved. Overall, considering both perspectives (transactional and operational) in the 33 bank branch of the Cape Verde Bank, 2 branches are always inefficient, 16 branches efficiency depend on the perspective or the input considered and 15 branches are always efficient.

Chapter 6

Conclusion

This dissertation aimed to analyse the comparative efficiency of the bank branches of a Cape Verdean Bank using Data Envelopment Analysis (DEA). Since resources can't be changed, the objective is to analyse the ability of the bank branches in producing results given their resources. The efficiency that we studied is technical efficiency and in this context it represents the ability to avoid waste by producing as much outcome as resource allows. For this purpose we based the efficiency measurements on the model for bank branch efficiency presented by Portela (2003), and the goal we set for this dissertation was to study the efficiency of the bank branches considering two different optics: 1) transactional efficiency and 2) operational efficiency. Transactional efficiency aims to measure the extent to which, given accessibility of services, customers use new distribution channels for transactional activities. Operational efficiency aims to measure the extent to which a branch, given its resources and environment, increases its sales and its costumer base, while serving its existing customers with adequate service quality.

To analyse transitional efficiency we used the output oriented model BCC model that considers variables returns to scale. On the transactional analysis, in a total of 33 bank branches 24 branches were always considered efficient independently of the inputs, 5 branches are always considered inefficient and 4 branches efficiency depends of the chosen input. The director of the Bank under study was able to gave feedback on the transitional analysis. The director seem pleased with the results, since they gave new information and reinforce the empirical notion he had of the effectiveness of bank agencies.

To analyse operational efficiency we used the output oriented models RDM, MSBM and modified MSBM, we used these models because they can handle negative data. On the operational context, in a total of 33 bank branches, 20 branches are always considered efficient independently of the input, 10 branches are always considered inefficient and 3 branches efficiency depends of the chosen input.

Overall, considering both perspectives (transactional and operational) in the 33 bank branch of the Cape Verd Bank, 2 branches are always inefficient, 16 branches efficiency depend on the perspective or the input considered and 15 branches are always efficient.

There was no evidence to relate the transactional efficiency to the operational efficiency in the case of this Cape Verdean Bank.

The choice of input to translate the socioeconomic context raised many questions. Not being able to say with certainty which input best reflects the reality of Cape Verde, we had to resort to different analyzes to fill in possible shortcomings. Handling negative data is a limitation of the traditional DEA model, but various author have been trying to deal with this problem and we resorted to the 3 different models presented by them, each on with their own limitations and advantages.

Given Cape Verde reality after this analyses it seems it would also be relevant to study the efficiency of the bank branches in the optic of transactional efficiency considering an additional input: "Emigrant Clients". An additional interesting study would be the analysis of the of bank branches efficiency from the perspective of profit efficiency and the evolution of the branches efficiency through time. On another hand, an interesting project would be studying comparative efficiency between all the Cape Verdean bank branches (national level).

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Appendix

A

Transactional Efficiency

Branch	BCC	GDF	Branch	BCC	GDF
A	1	1	R	1	1
B	1	1	S	0.94	0.94
C	1	1	T	1	1
D	1	1	U	1	1
E	0.85	0.85	V	0.98	0.98
F	0.69	0.69	W	1	1
G	1	1	X	1	1
H	1	1	Y	1	1
I	1	1	Z	1	1
J	1	1	AB	1	1
K	1	1	AC	1	1
L	1	1	AD	1	1
M	1	1	AE	1	1
N	1	1	AF	1	1
O	1	1	AG	1	1
P	0.92	0.92	AH	1	1
Q	1	1			

Table A.1: Transactional Efficiency Score per Branch - Business Volume

Branch	BCC	GDF	Branch	BCC	GDF
A	1	1	R	1	1
B	1	1	S	0.999	0.999
C	1	1	T	0.78	0.61
D	0.85	0.68	U	1	1
E	0.59	0.59	V	0.99	0.99
F	0.47	0.47	W	1	1
G	0.80	0.76	X	1	1
H	1	1	Y	1	1
I	1	1	Z	1	1
J	1	1	AB	1	1
K	1	1	AC	1	1
L	1	1	AD	1	1
M	0.54	0.54	AE	1	1
N	1	1	AF	1	1
O	1	1	AG	1	1
P	0.92	0.92	AH	1	1
Q	1	1			

Table A.2: Transactional Efficiency Score per Branch - Population

Appendix

B

Operational Efficiency

Branch	RDM	Norm	MSBM	mMSBM	Branch	RDM	Norm	MSBM	mMSBM
A	1	1	1	1	R	0.908	0.908	0.842	0.885
B	1	1	1	1	S	0.860	0.860	0.777	0.810
C	1	1	1	1	T	1	1	1	1
D	1	1	1	1	U	1	1	1	1
E	1	1	1	1	V	0.702	0.702	0.715	0.830
F	1	1	1	1	W	0.794	0.794	0.783	0.792
G	1	1	1	1	X	0.772	0.772	0.779	0.788
H	0.970	0.970	0.913	0.929	Y	1	1	1	1
I	1	1	1	1	Z	1	1	1	1
J	1	1	1	1	AB	0.899	0.899	0.868	0.879
K	1	1	1	1	AC	1	1	1	1
L	1	1	1	1	AD	1	1	1	1
M	1	1	1	1	AE	1	1	1	1
N	1	1	1	1	AF	1	1	1	1
O	0.906	0.906	0.755	0.854	AG	0.761	0.761	0.766	0.808
P	1	1	1	1	AH	1	1	1	1
Q	0.995	0.995	0.814	0.857					

Table B.1: Operational Efficiency Score per Branch - Population & Business Volume

Branch	RDM	Norm	MSBM	mMSBM	Branch	RDM	Norm	MSBM	mMSBM
A	1	1	1	1	R	0.908	0.908	0.842	0.885
B	1	1	1	1	S	0.860	0.860	0.777	0.810
C	1	1	1	1	T	1	1	1	1
D	1	1	1	1	U	1	1	1	1
E	1	1	1	1	V	0.702	0.702	0.715	0.830
F	1	1	1	1	W	0.794	0.794	0.783	0.792
G	0.983	0.983	0.948	0.959	X	0.772	0.772	0.779	0.788
H	0.964	0.964	0.913	0.929	Y	1	1	1	1
I	1	1	1	1	Z	1	1	1	1
J	1	1	1	1	AB	0.899	0.899	0.868	0.879
K	1	1	1	1	AC	1	1	1	1
L	1	1	1	1	AD	1	1	1	1
M	1	1	1	1	AE	1	1	1	1
N	1	1	1	1	AF	0.976	1	0.952	0.958
O	0.906	0.906	0.755	0.854	AG	0.761	0.761	0.766	0.808
P	1	1	1	1	AH	1	1	1	1
Q	0.995	0.995	0.814	0.857					

Table B.2: Operational Efficiency Score per Branch - Population

Branch	RDM	Norm	MSBM	mMSBM	Branch	RDM	Norm	MSBM	mMSBM
A	1	1	1	1	R	0.772	0.772	0.755	0.835
B	1	1	1	1	S	0.790	0.790	0.747	0.786
C	1	1	1	1	T	1	1	1	1
D	1	1	1	1	U	1	1	1	1
E	1	1	1	1	V	0.702	0.702	0.715	0.830
F	1	1	1	1	W	0.794	0.794	0.783	0.792
G	0.914	0.914	0.8622	0.893	X	0.772	0.772	0.779	0.788
H	0.844	0.844	0.793	0.831	Y	1	1	1	1
I	1	1	1	1	Z	1	1	1	1
J	1	1	1	1	AB	0.899	0.899	0.868	0.879
K	1	1	1	1	AC	1	1	1	1
L	0.926	0.926	0.876	0.901	AD	1	1	1	1
M	1	1	1	1	AE	1	1	1	1
N	1	1	1	1	AF	1	1	1	1
O	0.906	0.906	0.755	0.854	AG	0.761	0.761	0.766	0.808
P	1	1	1	1	AH	1	1	1	1
Q	0.959	0.959	0.742	0.799					

Table B.3: Operational Efficiency Score per Branch - Business Volume