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Compensating Variation and the Young Portuguese Emigrants:

A Study on how to keep our own.

Manuel Maria Zuzarte Reis Piedade

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Professor Pedro Pita Barros

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ABSTRACT

Emigration has been a very present word in Portugal. Due to the effects of the Economic Crisis and the Memorandum of Understanding policies, we have witnessed a significant yearly migration outflow of people searching for better conditions. This study aims to measure the factors affecting this flow as well as how much the probability of emigrating has evolved during the years bridging 2006 to 2012. I shall consider the decision of emigrating as Discrete Choice Random Utility maximization use a conditional Logit framework to model the probability choice for 31 OECD countries of destination. Moreover I will ascertain the compensating variation required such that the probability of choice in 2012 is adjusted back to 2007 values, keeping all other variables constant. I replicate this exercise using the unemployment rate instead of income. The most likely country of destination is Luxembourg throughout the years analyzed and the values obtained for the CV is of circa 1.700€ in terms of Income per capita and -11% in terms of the unemployment rate adjustment.

Keywords: emigration, compensating variation, probability of choice, Logit

1. INTRODUCTION

The Portuguese People have, historically, a propensity to emigrate. As a small yet quite open economy, emigration dates as far back as the XV century, with the pioneers in the Discoveries searching for fame and fortune around the World. Recently this has been viewed as an adverse effect of the European Economic Crisis and a consequence of the Memorandum of Understanding signed in 2011. With the deterioration of key aspects in the Portuguese Economy – when comparing to other countries–, it is not surprising that we might have witnessed a change in the Probabilities of Choice when deciding to migrate or not as well as a change in the Probabilities of Choice for the countries considered. This study's core consists on the modeling of these Probabilities of Choice, via a Conditional Logit models such as the one employed by Greenwood et. all (2001). Given the nature of the Logit model we can apply the concept of Compensating Variation to the Probabilities of Choice (de Palma, Killani, 2009) and calculate what amount should be given to an individual such that his probabilities of choice are adjusted to the ex-ante values. In this case it would be interesting to bring back the probabilities' values to pre-troika years.

I condition the probability of choice on the values of the explanatory variables for each year and calculate the subsequent probability. This means that, given the parameters estimated, I will be able to calculate for Norway–, conditional on the values that the explanatory variables present for this country in particular on a given year–, what was the probability of choosing this country as the Destination for an individual who is deciding on the place to go to. Given that this calculation is done on a yearly basis, the probabilities will evolve as the variables evolve. This is not surprising and we can expect that if the probabilities will behave in such way that they increase when Portugal

is relatively worse comparing to the Destination. For example, if Portugal's Income per capita decreases and Germany's is kept constant from one year to the other, all else being equal we can expect that the probability of choosing Germany will increase. Regarding how the individual characteristics of the decision maker are approached, I will follow the work of Greenwood et.all (2001) and impose two additional restrictions on the model. Firstly I will focus on the effect of choice-specific attributes on the migration decision and observe how the characteristics of a country affect the individual's destination choice. This model is not able to identify the individual characteristics, as opposed to a Multinomial Logit for example which incorporates individual characteristics on the decision making process, yielding different parameters for different choices. There would be a great computation cost in terms of the number parameters when incorporating individual characteristics and I do not have individual level data, as such I will not employ this approach. Secondly, given that the conditional Logit cannot identify directly the effects of individual characteristics, I will incorporate relative measures between Portugal and the destination. This seems to meet the notion of comparing countries when deciding to migrate, and the characteristics of specific country should be thought f as of a proxy for an individual characteristic in the sense that individuals will view differently the characteristics of a country. For example, a high income individual will not consider the income per capita like a low income individual.

This paper is organized as follows: in Section 2. I will review the literature; in Section 3. I will describe the methodology employed; in Section 4. I will present and discuss the results; Section 5. I will conclude. The Annex will consist of complementary tables and graphs, as well as relevant data

2. LITERATURE REVIEW

The study of migration flows has been quite vast and has carried out many paths in the literature. As one may imagine, the implications of migration in a country are immense and visible in many aspects of the society. They can range from economic issues such as the equilibrium of the labor market to the impact in the productivity, the effect on wages and long run sectoral bias, to social and cultural issues such as the integration of migrant populations, the effect on crime, on social services costs, etc. In fact, Hooge et. Al (2008) center the economic literature to explain migration in three theories: *economic and labor-based*, where the factors boosting migration are connected to Standards of Living indicators, such as Income per capita, Unemployment rate, GDP growth among others, and the decision is based on the relative measure between Origin and Destination countries; *cultural and hegemonic theories*, where the factors are related to language and culture similarities, and a flow periphery-to-core is assumed; *social theories*, where the factors are mainly the so called *network effect***,** or the effect of friends and family already established in the Destination or possessing some sort of information, influencing the decision of the individual as to where to go and if he should or not. This particular study will undergo the first road and hence I will base my rational for migrating on *economic and labor-based factors.* It is common ground in migration models to refer to the Roy model (1951) to mathematically explain the incentives to migrate, which was further adapted to the labor market and migration modeling by Borjas (1987) where he builds the notion of comparing Origin and Destination when deciding to migrate in terms of potential earnings. This is interesting because it should be close to what an individual thinks when taking the decision of migrating. He would wonder if the advantages he could achieve in the Destination are

attractive enough compared to the advantages of staying, given that there are associated costs of moving.[1] To this idea that the individual will compare aspects between the Destination and the Origin and base his decision accordingly, many authors have extended the notion of indicators. They need not to be only wages, income or wealth, but also other components of an economy can be taken into consideration. Otrachsenko and Popova (2013) investigate the effect of Life Satisfaction, using the Eurobarometer Survey 27, for migration flows in Central and Eastern European countries and state that people have greater intention to move when dissatisfied with life and that socioeconomic and macroeconomic variables affect indirectly this decision through their influence in life satisfaction. Stark (2005) studies the relation between income inequality and the incentive to migrate, reaching the conclusion that a higher total relative deprivation[2] leads to higher incentives to migrate –given a constant income level and that deprivation is positively correlated with the Gini coefficient– and hence this measure of inequality is positively correlated with migration holding income constant. Finnie (2004) posed the question "who moves?" to analyze the inter-provincial migration in Canada by using panel data for tax fillers from the LAD dataset, that collects detailed information on income, taxes, demographic characteristics such as the province of residence. The author uses a Logit model to compute the probability of moving from one province to another (by setting the dependent variable as whether or

[1] According to this stream, the equation that models migration is: $d_i = W_{di} - W_{oi} - z_i - c$, where the decision to migrate of individual "i" (d_i) is conditioned by the difference between the wage he receives in the destination country (W_{di}) and the origin country (W_{oi}), sustracting the cost of migrating (c) and the personal loses of the migrant (z_i).

[2] Relative Deprivation is defined by the author as the proportion of individuals in a group that have an income higher than the individual in question times their mean excess income. So the individual looks at the members of the group that have more income than he does. The formula is as follows.

$$RD(w) \equiv [1 - F(w)]E(x - w \mid x > w) = \int_{w}^{\infty} [1 - F(x)]dx,$$

not the individual's province of residence changed from one year to another) as a function of "environmental" factors such as the current province of residence, the provincial unemployment rate and the area size, personal characteristics such as language, age, marital situation and presence of children, key labor market indicators such as earnings, receipt of unemployment insurance and social assistance. Their analysis is performed for a span of circa 13 years and allows such detail mainly due to the nature of the dataset.[3] Similarly, Greenwood, Li and Davies (2001) also use a Logit model applied to the inter-state migration in the United States of America for the years of 1986 to 1997. The usage of panel data is particularly interesting as it captures time effects that static analyses cannot do. The authors investigate the responses to relative economic performance measurements, modeled by ratios between destination and origin locations for income per capita and unemployment rate, and to the cost of moving, modeled by the distance between locations. They also employ a fixed effects framework to capture unobserved factors such as amenities and finally undertake a study on the trade-off between variables that their model controls for. For instance, they compute the unemployment rate – per capita income trade-off which is, as the authors calculate it, the change in the destination Income pc required to offset an increase in the unemployment rate. It can be seen as a Compensating Variation for unemployment increases, since the econometrical model is solidly founded in Discrete Choice Theory and derive from an individual's utility maximization (McFadden 1973). This particular paper will be the main reference for this WorkProject and I will derive to a similar model, however directed at the emigration from Portugal to other countries.

[3] The Longitudinal Administrative Database, or LAD as referred above, is 10% representative sample of Canadian tax fillers and identified spouses, and as such followed as individuals over time and matched into family units on an annual basis, thereby providing individual and family-level information. The LAD's coverage of the adult population is very good since, unlike some other countries the rate of tax filing in Canada is very high. The estimated coverage of the targeted adult population is 91% to 95%.

3. METHODOLOGY

Database description

The data used in this Work Project is comprised of Inflows of Portuguese population, income per capita, unemployment rate, GDP growth factor[4] and distance for 31 countries in the OECD. This data selection is compiled for 6 years, from 2006 to 2012, and follows the literature set up such as Greenwood et. al (2001) and Finnie (2004) regarding the nature of the variables selected. These authors control for economic, social and demographic factors with the selected variables as well as unobserved factors with the model specification. This matter will be deferred to the section bellow.

The Inflows of Portuguese population data was extracted from the OECD International Migration Database. This dataset contains figures for both stocks and flows of total immigrant population, immigrant labor force and data on acquisition of nationality. The OECD compiled this dataset using individual contributions of national correspondents appointed by the OECD Secretariat with the approval of the authorities of Member countries. Because of the great variety of sources used, different populations may be measured. In addition, the criteria for registering population and the conditions for granting residence permits, for example, vary across countries, which mean that measurements may differ greatly even if a theoretically unique source is being used. Also due to the fact that not many data sources are specifically designed to record migration movements, presentation of the series in a relatively standard format does not imply that the data have been fully standardized and are comparable at an international level. However it appears to be a reliable source with a solid reputation regarding the

[4] (1+Gdp growth rate) – measures how much the GDP grew as a multiplicative factor.

quality of the data it produces. I have chosen to extract data concerning foreign population inflow as a whole, opposed to simply foreign workers, because I believe it adds more information to the analysis and captures better immigrant's decision towards moving. For instance, foreign workers data, which comprises solely the foreign labor force that enters a certain country, hence not taking into account migrants that are not part of the work force like spouses or retirees, restricts the population more when compared to the total foreigners entering the same country. As such, we can lose some relevant information if we do not use a broader span of population, and for an early study I believe it is more interesting looking at the general population. In further work, it would surely be attention-grabbing to categorize the inflows of population and compare the differences in the conclusions.

The data related to the economic and social indicators, these being the Income per capita,[5] GDP growth[6] factor and the unemployment rate[7] are extracted from the World Bank data library. This data set consists of WB national accounts data as well as OECD national accounts data files for the series Income per capita and GDP growth and of Key Indicators of the Labor Market database from the International Labor Organization for the unemployment rate series. The way this data is incorporated to the study followed by Greenwood et. al (2001) and previous literature rational on comparing destination and origin countries. Naturally there is space to incorporate more variables that could be pertinent for explaining the decision to migrate, for example total taxation, borrowing

[5] Income per capita is gross domestic product divided by midyear population. GDP is the sum of gross value added by all resident producers in the economy plus any product taxes.

[6] Annual percentage growth rate of GDP at market prices based on constant local currency. Aggregates are based on constant 2005 U.S. dollars. GDP is the sum of gross value added by all resident producers in the economy plus any product taxes and minus any subsidies not included in the value of the products. It is calculated without making deductions for depreciation of fabricated assets or for depletion and degradation of natural resources.

[7] Unemployment refers to the share of the labor force that is without work but available for and seeking employment. Definitions of labor force and unemployment differ by country.

costs, costs of utilities, etc., which can be interesting for further research. Nonetheless, the variables selected aim to control for the income factor, the employment factor and the economic prosperity factor, which I believe the factors that matter the most for the individual given the stream of events of recent years. The income per capita should serve as a proxy for how much will an individual earn on a yearly basis, the unemployment rate should serve as a proxy for the possibilities of finding a job (the lower the value the more likely and easier it will be to get employed) and the GDP growth factor should serve as a proxy for the strength of the destination's economy, in a way that the lower it is, the less dynamic the economy should be.

I have built ratios for the variables, using the framework

$$\frac{Destination\ Variable}{Portugal\ Variable}$$

If we are looking at a ratio that is larger than 1, then we can conclude that $Destination\ Variable > Portugal\ Variable$, and this allows an easy and understandable comparison of the indicator in question. Taking the example of Russia and Switzerland, in 2011, they yield Income per capita ratios of circa 0.59 and 3.69 and as so we can easily read that Russia has a lower Income per capita and Switzerland a higher one and hence the probability of migrating to Russia should surely be lower than for Switzerland, all else being equal. For the GDP growth factor and unemployment we can do the same reading of the ratios, bearing in mind that the unemployment factor in the Destination should have a negative propensity in the probability to migrate to that country.

Finally, the variable Distance was extracted from an Online Distance Calculator.[8] This source uses the country's mid-point as reference and calculates the distance between those midpoints. This variable should capture the cost and effort associated with moving, as well as the psychological effect of being away from the Origin country. In previous work (Greenwood et. al 2001) distance is introduced in a quadratic form, in order to capture decreasing marginal costs for very long distances. However I have reached the conclusion that this step was deemed statistically insignificant. The distance should have, *a priori*, a negative effect on the probability of emigrating. All else being equal, an individual should have a larger propensity to choose a country closer to his current location. We can also look at this variable as a proxy for the risk profile of the individual. Although this approach is not analyzed in this study, for further reference, it could be interesting to try to infer if distance can capture the "adventurous" spirit of certain individuals. For example, if we sectorize the ages of the migrating population we may find that younger people are more prone to accept longer distances than older people. This idea is left for further research.

Estimation Strategy

The assumption that I betake to model the behavior of the individual is that the choice of emigrating is founded on a random utility model. The literature on this type of model is quite vast and dates back to the 20's and to the study of food preferences. This type of model is nested in both economic and psychological premises and as such is suited for cases where an individual has to choose between mutually exclusive alternatives and hence has to be able to rank the options he has consistently and unambiguously to choose the best one. This is referred to as a deterministic choice process (de Palma,

[8] MapCrow.info

Anderson, Thisse, 1992).[9] In the particular case of the migrating decision, an individual choses one country given certain characteristics and this choice invalidates any other country being chosen at the same time (hence being mutually exclusive). The total number of Portuguese people entering a specific country is therefore the number of individuals that chose that country through this channel. However, due to the unpredictability of the human being, we can observe inconsistencies in the behavior of an individual. There are cases where, in repeated experiences, he will not chose the same option under the same circumstances, by so rendering the ranking system above ineffective and spur issues of intransitivity. This utterly reflects the fact that the deterministic choice model is not suitable for human behavior and so we must not consider Man as a perfect choosing instrument but as a stochastic one (Georgescu-Roegen, 1958). We must thereafter accept that an individual is sensitive to the circumstances he faces, and his behavior is influenced, which leads us to the need of identifying the reasons that ultimately are determinant of the choice outcome. We can counter this problem by turning to probabilistic choice models, where we aim to reach the probability of choosing *a* rather than *b* from the set *A* that contains both *a* and *b* (de Palma, Anderson, Thisse, 1992; Tversky, 1972a).

The model employed in this study is a Conditional Logit Model, and follows Greenwood et. All (2001) in its application to migration flows. I assume, as stated above, that the individual follows a Random Utility Model, and faces *J* choice options, from where he will choose option *j* that will yield the highest utility.[10]

I will assume that each individual models his behavior with the utility function:

[9] "Discrete Choice Theory for Product Differentiation", MIT Press.

[10] $U_j > U_k \; for \; all \; k \neq j \; and \; \in J$

$$U_{ij} = \beta' X_{ij} + \varepsilon_{ij}$$

where X_{ij} is a vector of specific choice-characteristics. The parameter β is constant across choices for the conditional Logit. The individual will therefore choose the option *j* that yields the highest utility, as said before. Hence, if this is the choice to be made, the statistical model for the probability of choosing *j* can be represented as

$$P(y_i = j) = P(U_{ij} > U_{ik}) \; \forall \; k \neq j$$

If the J disturbances are i.i.d with the Extreme Value Distribution, or Weibull Distribution, then we can model the probability of an individual *i* choosing *j* as[11]

$$P(y_i = j) = \frac{e^{\beta' X_{ij}}}{\sum_k e^{\beta' X_{ik}}}$$

This framework will allow for the computation of the probability of choosing as the destination country *j*. The X_{ij}vector contains choice-specific characteristics that are related to the Destination Country, and are composed by economic factors, such as the Income per capita and GDP growth factor, social factors, such as the unemployment rate and geographical factors, such as the distance between countries. This framework looks at all "positive outcomes", equal to the number of individuals choosing country *j* as the destination country, and computes, given the explanatory variables, the probability of choice of that country being the destination out of all the possible countries and the option of remaining in Portugal.

It is widely accepted in the literature that more than merely observable variables condition the decision of the country of choice. Other unobservable effects such as the

[11] Greenwood et. all (2011); de Palma, Killani (2009)

climate, the culture, the proximity of the language to the native one, political history, are also factors that play a role in the individual's decision process. As such, I employ a fixed effects panel regression methodology to try and capture these aspects. This methodology is useful in such cases since it takes into account time independent effects that might be correlated to the regressors for each entity. In this case, for example, there might a correlation in the culture and the unemployment rate if the country in question is "hard working". Normally we associate to the Northern Countries a culture of hard-work and this may be translated into lower unemployment and greater wages and GDP growth. The inverse might be visible in Southern Countries. Moreover, I will estimate a linear panel data regression and a per year regression in order to compare the results of each type of estimation. Finally, this model requires that the Independence of Irrelevant Alternatives proposition holds. This assumption builds up on the independent and homoscedastic disturbances already mentioned before for the random utility model and imply that relative probabilities of choice must be independent of other alternatives. Greenwood et al. test this property using a Hausman-type[12] specification test (Hausman and McFadden 1984) and a Lagrange-type test (McFadden 1987) and find that it holds for their cross-state migratory flows study. The nature of such tests require eliminating a subset of the choices from the set and re-estimating the model and checking if the parameters are systematically similar to the full model. The setback is that for a model with many destinations, as is the case of this one with 31 countries, the computational

[12] The test statistic for this test is $\chi^2 = (b_s - b_f)'(V_s - V_f)^{-1}(b_s - b_f)$ and it has the χ^2 distribution with *k* degrees of freedom, where *k* is the rank of $(V_s - V_f)$ and asymptotic covariances matrices.

effort is considerable[13] and due to time restrictions I forego this test and assume that given the similarity of the work with what these authors do, the result should be akin.

4. RESULTS

Description

In this section the estimated results are presented. Given the aforementioned regressions performed, I have 3 types to report: a linear panel regression, a fixed effects panel regression and a linear regression for each year under analysis. I will follow this order when analyzing the results for consistency. The regressions are the result of estimating the model:

$$\ln\left(\frac{\boldsymbol{Inflow_i}}{\boldsymbol{Active\ population}}\right) = \beta_1 Incpcratio_i + \beta_2 Unempratio_i + \beta_3 Gdpgrowthratio_i + \beta_4 Distance_i + \beta_5 + \varepsilon_i$$

The dependent variable of this model is the natural logarithm of the **Share of Migrants** to country *i*, and this formulation allows to approximate to the probability of choosing country *i* as the Destination. Following Ivaldi and Verboven (2005) and their study in differentiated product markets, the market share of a product at an aggregate level coincides with the choice probability of the same product, and so we can think of the choice of a destination country as the differentiated product, and the market share as the total share of individuals choosing that country as the destination in each year, by so using the above dependent variable as a proxy to estimate the coefficients relevant to the choice probability. In the annex I present the shares for every country in every year.

The table 1 reports for each one of the approaches the coefficients as well as the t-statistics in italic beneath the coefficients. The F-statistic is also reported, as well as the

[13] If we take out a single country there are 31 different combinations to test. If we take two there are C_{31}^{2}.

R-Squared. Note that the fixed effects approach reports three types of R-Squared, which are explained further below. The correlation matrix can be found in the Annex.

	Linear	Fixed Effects	By year: 2006	By year: 2007	By year: 2008	By year: 2009	By year: 2010	By year: 2011	By year: 2012
Incomepcratio	1.256	0.742	1.228	1.624	1.267	1.355	1.29	1.078	1.069
	-4.68	*2.78*	*3.72*	*5.07*	*3.55*	*3.87*	*3.22*	*3.64*	*4.03*
Unemploymentratio	0.637	-0.454	1.298	2.826	1.431	0.386	0.0194	-0.092	0.586
	0.6	*-1.97*	*1.2*	*1.81*	*0.95*	*0.26*	*0.01*	*-0.05*	*0.31*
Distance	-0.00013	-	-0.00008	-0.00008	-0.0001	-0.0001	-0.0002	-0.0002	-0.0002
	-2.61	-	*-1.16*	*-1.19*	*-2.14*	*-1.79*	*-2.05*	*-2.76*	*-2.92*
GDPgrowthratio	-1.183	-1.72	-19.18	8.605	6.856	-1.128	-17.437	-9.526	2.738
	-0.66	*-2.28*	*-1.23*	*0.74*	*0.41*	*-0.11*	*-1.15*	*-0.86*	*0.38*
constant	-6.456	-4.87	11.285	-19.117	-15.083	-6.384	10.29	3.164	-10.377
	-2.39	*-6.94*	*0.7*	*-1.55*		*-0.68*	*0.66*	*0.26*	*-1.23*
Number of obs	211	211	30	30	30	30	30	31	30
Prob > F	0.0006	0.0138	0.0016	0.0004	0.0018	0.001	0.0015	0.0008	0.0009
R-Squared	0.407	-	0.454	0.4635	0.437	0.419	0.398	0.425	0.437
R-Squared within	-	0.108	-	-	-	-	-	-	-
R-Squared between	-	0.288	-	-	-	-	-	-	-
R-Squared overall	-	0.271	-	-	-	-	-	-	-

Table 1.

Linear panel approach

The linear panel approach allows us to infer that, for the years bridging 2006 to 2012, the Income per capita ratio and the distance were the only explicative variables with statistical significance to explain the probability of migrating to country *i*. The marginal effects of these variables appear to take the expected direction, with Income per capita showing a positive sign and Distance a negative one. This means that if the Income per capita ratio increases one unit–,– which given the formulation of the ratio (refer to the section of Database Description) can only happen if either the Destination's value for Income per capita increases or if Portugal's decreases–,– then the probability of choosing country *i* as the Destination country increases. This makes sense in an empirical framework, since an individual would more likely move to a country where, all else being equal, he would have a higher income comparing to the where he currently is, and, if facing several options of Destination country, the highest probability should be with the one offering the highest income ratio among all. Regarding Distance,

it is simple to understand that the negative impact of the coefficient means that the longer the distance between the Destination and Portugal, the lower will be the probability of choosing that country as the Destination. This is quite straightforward since greater distances translate into greater costs not in terms of money and time but also psychologically speaking. This may capture the effect that being away from Portugal and home, family and friends has on the decision of an individual and supports the thought that all, else being equal, an individual will more likely choose the Destination closest to Portugal.

Fixed effects panel approach

This method of estimation permits to control for time invariant aspects that might be correlated with the regressors of the model. This is particularly useful for panel data related to various countries, as it is this case. This model specification involves taking the time average of the explanatory variables and given the assumption that there are time invariant factors accounted for, as would be the case of the Distance variable or even other unobserved factors, that do not vary in time (culture, climate, political context, etc.), taking the difference between the initial model and these time averages for all variables and by so *time-demeaning* the model. This procedure hence permits the estimation of the model via Pooled OLS and is named the *within estimator* or *fixed effects estimator*. The *between estimator* is the result of an OLS regression on the cross-sectional equation for the time average explanatory variables.

Note that we employ this procedure if we believe there could be some correlation between the time invariant variable and the regressors. Also this model controls for time invariant factors that are unobservable. As such, it can be pertinent in this case to

employ this analysis and compare the results with the previous regression. We can observe that the results change in comparison. Income per capita and GDP growth rate ratio are both significant at 5% level and the unemployment rate ratio is significant at 10% significance level, and misses the 5% level by only 0.9%. Vis-à-vis with the previous regression, we have now all coefficients showing statistical significance. Regarding the marginal effects: Income per capita shows the expected direction, with a positive signal, meaning that if the ratio between Destination and Portugal increases –,– which happens either if the Destination's Income per capita increases or if Portugal's decrease–, then the probability of choosing that Destination also increases, since the Destination country offers a higher Income comparing to Portugal; the Unemployment rate ratio also shows the expected direction, with a negative signal, meaning that if the ratio between Destination and Portugal increases–, which happens either if the Destination's Unemployment rate increases or if Portugal's decrease–, then the probability of choosing that Destination will decrease, since the Destination country will show an unemployment rate evolution less attractive comparing to Portugal than it did before.[14] The GDP growth factor ratio presents a coefficient that is less straightforward to interpret, with a negative signal, and the implication of this is that if the ratio between Destination's growth factor and Portugal's growth factor increases, which happens if the numerator increases or if the denominator decreases, then the probability of choosing that country decreases, and this is somewhat counter intuitive since if the growth factor in Portugal increases this should translate into a higher GDP and this should work as a pull factor instead of a push factor. Nonetheless, regarding

[14] Note that this does not mean that the Destination country will have a higher unemployment rate than Portugal (the ratio may still be lower than one). It means that, given the probability of choosing a country at one specific point in time, if this country's unemployment rate increases or if Portugal's decrease, this will affect the Unemployment rate ratio in a way that translates into a decrease in the probability of choice.

this we can argue that, given that in the time span analyzed, Portugal's growth factor has been mostly below 1, meaning that GDP decreased, the effect of the growth factor in the decision to emigrate may work as a deterrent for traveling or exert costs of any kind to change country because of the effect of a shrinking economy and how that fact affects the individual. This could be a stretched conclusion, and widening the year span may shed more light on this particular matter.

By year approach

In this approach I ran a by year OLS regression so that I could compare the coefficients and their significance evolution throughout the years. Given the number of regressions, I will present for the years of 2007 and 2012 and please refer to the annex for the remaining years. For the sake of simplicity I will analyze one coefficient for every year before moving to the next.

Starting with Income per capita, this variable seems to be consistently significant throughout the years and always showing, in terms of marginal effects, the expected direction with a positive signal. As mentioned before, the positive signal means that if the Income per capita ratio increases, which can only happen either if the numerator increases or if the denominator decreases, then the probability of choosing that country also increases. Also this variable consistently shows coefficient above 1.1 so the effect on Income per capita is quite significant, since for country *i*, an increase in the ratio of 1 unit will at least yield 1.1% more probability of choosing that country as the Destination, all else being equal.

Distance appears as insignificant for the years of 2006 and 2007 and significant at a 5% level for the subsequent years, except 2009 which is significant at a 10% level. The marginal effect of Distance behaves as expected, influencing negatively the probability with an increase of one unit, which, as already aforementioned, is understandable given increased costs related to longer travels. The insignificance of early years is interesting, and one reason may be due to the increase in migratory flows from Portugal correlated with the Financial Crisis of 2008.

Finally, both unemployment ratio and GDP growth factor ratio are statistically insignificant throughout the years considered. This is somewhat puzzling since these variables, especially the unemployment rate, should be of interest to the individual when taking the decision regarding the destination country, and perhaps given that only 31 countries are analyzed[15] and so there might be a problem regarding the small number of observations. However, this problem is related to the availability of data and as such in further research one can try to enlarge this sample and perform a more detailed analysis. To conclude this section, I have also performed additional regressions not reported here that included dummies variables for the years of "post-crisis". I created separately a dummy for all years after 2009, 2010 and 2011 not only to check if there was a difference of the years after the crisis in the decision to migrate but also if there was any delay for this effect to take action (hence the 2010 and 2011 dummies). However these estimations did not add any explicative power and the dummies where consistently insignificant.

[15] Only 31 countries were available on the OECD migration flows dataset described previously

Probabilities report

After estimating the above models, I have computed the probabilities of choice for each country and for each year, using the linear approach, the fixed effects approach and the by year approach. The result is quite large and not practical to report, as such please refer to the annex for all Countries probabilities. I will here report solely, table 2 and table 3, the top 5 countries for years 2007 and 2012 in terms of probabilities of choice calculated through each approach. The remaining years will also be re-laid to the annex.

country	year	Inflow	inc pc ratio	unemp ratio	distance	gdp growth ratio	1st approach
Luxembourg	2007	4,39	4,86	0,51	1592,48	1,04	4,82%
Norway	2007	0,16	3,80	0,31	2780,25	1,00	1,83%
Iceland	2007	0,24	2,98	0,29	2905,82	1,04	0,83%
Switzerland	2007	15,47	2,71	0,45	1535,39	1,01	0,80%
Denmark	2007	0,16	2,59	0,47	2259,07	0,99	0,66%
							2nd approach
Luxembourg	2007	4,39	4,86	0,51	1592,48	1,04	3,08%
Norway	2007	0,16	3,80	0,31	2780,25	1,00	1,64%
Iceland	2007	0,24	2,98	0,29	2905,82	1,04	0,85%
Switzerland	2007	15,47	2,71	0,45	1535,39	1,01	0,67%
Denmark	2007	0,16	2,59	0,47	2259,07	0,99	0,63%
							3rd approach
Luxembourg	2007	4,39	4,86	0,51	1592,48	1,04	75,26%
Norway	2007	0,16	3,80	0,31	2780,25	1,00	4,97%
Germany	2007	5,38	1,84	1,08	1952,78	1,01	1,61%
Finland	2007	0,04	2,12	0,85	3506,29	1,03	1,53%
Sweden	2007	0,15	2,30	0,76	2942,43	1,01	1,06%

Table 2.

country	year	Inflow	inc pc ratio	unemp ratio	distance	gdp growth ratio	1st approach
Luxembourg	2012	3,51	5,15	0,33	1592,48	1,03	6,77%
Norway	2012	0,58	4,94	0,21	2780,25	1,06	4,62%
Switzerland	2012	14,39	3,91	0,27	1535,39	1,04	2,32%
Denmark	2012	0,41	2,79	0,48	2259,07	1,03	0,73%
Netherlands	2012	1,73	2,28	0,25	1785,97	0,97	0,61%
							2nd approach
Luxembourg	2012	3,51	5,15	0,33	1592,48	1,03	4,21%
Norway	2012	0,58	4,94	0,21	2780,25	1,06	3,61%
Switzerland	2012	14,39	3,91	0,27	1535,39	1,04	1,69%
Australia	2012	0,28	3,34	0,33	16203,29	1,07	0,69%
Denmark	2012	0,41	2,79	0,48	2259,07	1,03	0,41%
							3rd approach
Luxembourg	2012	3,51	5,15	0,33	1592,48	1,03	38,31%
Norway	2012	0,58	4,94	0,21	2780,25	1,06	25,71%
Switzerland	2012	14,39	3,91	0,27	1535,39	1,04	10,32%
Denmark	2012	0,41	2,79	0,48	2259,07	1,03	3,02%
Sweden	2012	0,31	2,73	0,51	2942,43	1,04	1,29%

Table 3.

We can see that the panel regressions ran, such as in the 1st and 2nd approach, provide rankings much more consistent throughout the years, and rank consistently Luxembourg

and Norway as the most likely choices for an individual. This is not very farfetched as this country not only provides attractive conditions in terms of the variables controlled by the model (high Income, low unemployment and a reasonable GDP growth factor) and particularly Luxembourg has a very deep rooted community of Portuguese immigrants which can be an observed pull factor towards this country. As to what concerns Norway regarding unobserved pull factors I cannot pinpoint any particular one, but perhaps the culture around efficiency and hard work and also the quality of the country's Institutions may be an unobserved pull factor. Also, if we assume that many of the immigrants from Portugal are a young person, which is an empirically plausible assumption, then Norway's free Universities may be a great comparative advantage to attract individuals. Many young people in Portugal, given the lack of job opportunities and the higher costs of education, may opt to go abroad to study in a Country where the Schools are free of charge and that will add value to their future prospects. Regarding the 3rd approach, this is more volatile in its ranking and the reason for this is that shocks to the variables are less smoothed out as when considering the entirety of the years analyzed. As such, if there is an alteration of the Income per capita or the Unemployment rate of a country in a particular year, this approach should be more sensitive to this change than the others and reflect this sensitivity on its ranking. Also, note that the probabilities for top countries are much higher in the 3rd approach than the 1st or 2nd and this is due to the fact that the calculation of these probabilities uses as denominator the sum of all countries (refer to the previous section) and in the 1st and 2nd approaches this sum encompasses all years whilst the 3rd encompasses solely the year in question. In the case of 2007 the denominator for the 3rd approach is the sum of all

countries' values just for 2007 as opposed to the 1st and 2nd that sum every year's values.

Compensating Variation

One of the purposes of this study is to calculate how we could compensate monetarily the migrating individual such that he would be indifferent between moving and staying. To understand how the Compensating Variation fits in this framework, assume that there is a price of spending one year of your life in a country, and that this price reflects the opportunity cost that you incur by submitting yourself to the characteristics of that country, these being economic ones, such as the Income per capita, social ones, such as the unemployment rate, or any other characteristic that is inherent with that country. These characteristics are the ones used to model the probability of choice and we have already seen that they change over time and by so changing the probabilities of choice. For example, the probability of choosing Luxembourg in 2007 was, using the 2nd approach, 3.1 % and in 2012 4.2% and the Income per capita ratios were, for the respective years, 4.9 and 5.1. The exercise I try to do is to adjust the probability of the top 5 countries in 2012 back to the 2007 probability via calibrating the Income per capita value. I leave all other variables unchanged in order to reduce the disturbances. The choice of the year 2007 is a somewhat subjective but since this was the year in which Portugal performed better in terms of GDP growth it should work as a proxy for a "good year". We can use any other year to do this calibration. After obtaining the new value for the Income per capita ratio, I calculate the value that Portugal's Income per capita should have in order to produce that ratio, leaving the other country's constant. The method used to perform these steps was via Excel's Solver and a Generalized

Reduced Gradient Non Linear optimization method where all Kash-Kuhn-Tucker conditions for optimization[16] are satisfied for the reported result. The results thatin tables 4 and 5 are the amounts that would have to be given to an individual so that the probabilities for the top 3 countries in 2012 are the same as in 2007, given that all other variables are constant at 2012 values.

Table 4.

country	year	Compensation
Luxembourg	2012	1.649,89 €
Norway	2012	5.497,07 €
Switzerland	2012	9.110,89 €

Using the 1st approach

Table 5.

country	year	Compensation
Luxembourg	2012	1.790,03 €
Norway	2012	5.532,30 €
Switzerland	2012	9.355,33 €

Using the 2nd approach

The values in the tables above are the required increases in the annual Income per capita indicator for Portugal such that the probabilities of choice for each of the countries are kept at 2007 values, given that all other variables are constant at 2012 values. There are some interesting remarks regarding these values. Countries that had a larger increase in the probability of choice from 2007 to 2012 show larger values for the respective compensation, as are the cases of Norway and Switzerland, which show an increase of their probabilities of 150% and 190%. For countries where the probability increase is lower, the Compensation seems to be smaller. Luxembourg's probability increases only 40%. Hence what seems to matter is the magnitude of the transition in the probability and not the magnitude of the probability per se. What may shed light on these statements is the fact that the model is inherently connected to a Random Utility framework and that the subsequent behavior of the individual is the result of maximization of utility and of responding in such way that is the best for him. This is why there is a connection between the Compensating Variation and the adjustment of

[16] See the annex for the conditions

the probabilities via Income as done here. The probabilities of choice have an intrinsic utility value. This model specification allows us to compute the Compensating Variation using the choice probabilities and correcting for the transition in these choice probabilities via the correct channel (de Palma, Killani, 2009). As a final note on this analysis, I have used, for tables 6 and 7, the same process above to calculate the adjustment via the Unemployment rate. Even though it is not a Compensation of any kind I found that it was interesting to report.

Table 6.

country	year	Compensation
Luxembourg	2012	-11%
Norway	2012	-14%
Switzerland	2012	-14%

Using the 1st approach

Table 7.

country	year	Compensation
Luxembourg	2012	-11%
Norway	2012	-14%
Switzerland	2012	-14%

Using the 2nd approach

These tables show us what adjustment to Portugal's unemployment rate[17] would be required so that the probabilities of choice would remain in 2007 values, all else being equal. Hence, the unemployment rate in Portugal should be 11% lower than it was to keep the probability of choosing Luxembourg at 2007 values, all else being equal. Again there seems to be the case that higher transitions in the probabilities of choice imply higher adjustments.

[17] For reference, the Unemployment Rate in Portugal was 15,6%.

5. CONCLUSION

This study focused on the migratory flows from Portugal to 31 Destination countries during the years 2006 to 2012. To ascertain what causes were relevant to explain these movements, I have used a Conditional Logit Model to calculate the Choice Probabilities for each country in each year and to evaluate the significance of Income per capita, Unemployment Rate, GDP growth factor and Distance in the individual's decision process. The first three factors are brought to the data in a ratio format since the most likely and accurate way that an individual will absorb the information is by comparing factors in Portugal and some Destination, and taking his decision based on compared values instead of simple absolute values. The estimation strategy I used was segmented in a linear, a fixed effects and a by year approach, in which Income per capita is consistently significant cross approaches. Moreover I have calculated the Compensation required to keep the probability of choice at 2007 values for the top 5 countries in 2012, as a mean of reaching the value that would have to be given to an individual to present choice probabilities equal to 2007, all else being equal. The fact that the model is rooted in Discrete Choice Theory allows for this type of compensation to be computed, since it derives from a Random Utility function maximization framework. Also, I have employed the same exercise using the Unemployment Rate and calculated the value by which this indicator would have to be adjusted in order to yield 2007 choice probabilities, all else being equal.

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Annex

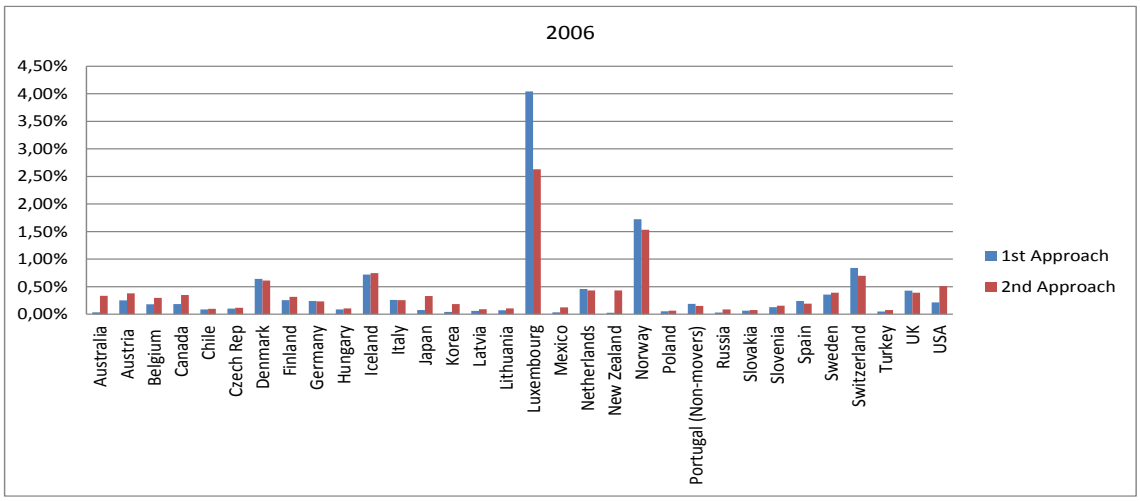
Figure 1. Descriptive Statistics of the Shares of Migrants of each country.

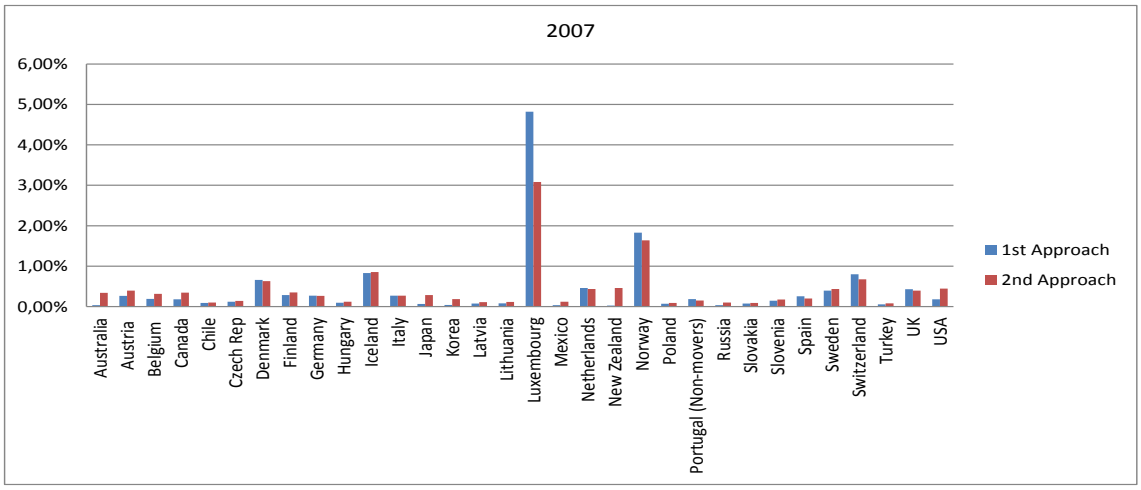
	2006	2007	2008	2009	2010	2011	2012	Total
Australia	.00291669	.00209551	.00267164	.0033728	.00442374	.00350536	.00450231	.00335543
Austria	.00449724	.00423264	.00516712	.00647361	.00900407	.0092827	.00835913	.00671664
Belgium	.03307751	.03182115	.04697385	.05175259	.05318275	.05095748	.03613144	.04341382
Canada	.00659921	.00553713	.0095856	.01106135	.01184231	.00819539	.00843981	.00875154
Chile	.00034218	.00019429	.00048442	.00047147	.001057	.0009088	.00140717	.00069505
Czech Rep	.	.	.	.	.	.00061668	.	.00061668
Denmark	.00211826	.00215102	.00199639	.00275627	.00328844	.00295359	.00656789	.00311884
Finland	.00073325	.0005551	.00082204	.00072533	.00080254	.0006816	.00080687	.00073239
Germany	.08011927	.07459165	.08676951	.12292599	.12748592	.13464786	.1461072	.1103782
Hungary	.00048883	.00013878	.00101287	.00088853	.00070467	.00110354	.00109734	.00077637
Iceland	.00581708	.0033306	.00421297	.0010336	.00043063	.00058423	.00058094	.00228429
Italy	.0057682	.00548162	.00364047	.00382614	.00450204	.00373255	.00371158	.00438037
Japan	.00213456	.00136	.00176152	.00210347	.00252505	.00152548	.00151691	.00184671
Korea	.00076583	.00040245	.00036698	.00041707	.00058722	.00047063	.00046798	.00049688
Latvia	.00021183	.00023592	.00013211	.00018133	.00021531	.00017851	.00017751	.00019036
Lithuania	.00006518	.00009714	.00027891	.0001088	.0000783	.00006491	.00006455	.00010826
Luxembourg	.06185332	.06085291	.06651205	.06970461	.07526229	.08076924	.05656127	.06735938
Mexico	.00014665	.0001249	.0003523	.00068907	.00066552	.00074651	.00074232	.00049532
Netherlands	.02263284	.02474362	.0350102	.04306671	.03832603	.03401493	.02786914	.03223764
New Zealand	.000277	.00027755	.00024955	.0005984	.00025446	.00027588	.00027433	.00031531
Norway	.00158055	.00216489	.0039781	.00466027	.00555903	.00743265	.00939191	.00496677
Poland	.00158055	.00215102	.00174684	.00295574	.00197698	.00230445	.00213012	.00212081
Russia	.00030959	.00034694	.00045506	.00050773	.00068509	.00089257	.00088755	.00058351
Slovakia	.00034218	.00034694	.0003523	.00068907	.00025446	.0006816	.00067777	.00047776
Slovenia	.00211826	.00018041	.01115629	.000816	.00001957	.00149302	.00080687	.00237006
Spain	.3366085	.37716317	.24744947	.17660071	.15028968	.12048037	.10006744	.21552277
Sweden	.00272116	.00208163	.00293587	.00378987	.00364078	.00306719	.00495415	.00331295
Switzerland	.20363039	.21468519	.26088104	.24788295	.251057	.24923727	.23218359	.23707963
Turkey	.00205309	.00142938	.00117435	.00101547	.00064594	.00053554	.00053253	.00105519
UK	.1955321	.16708532	.19053769	.2217709	.23645474	.26533592	.32989499	.22951595
USA	.02295873	.01414119	.01133244	.01715415	.01477842	.0133236	.01308736	.0152537
Total	.03333333	.03333333	.03333333	.03333333	.03333333	.03225807	.03333333	.03317536

Figure 2. Correlation Matrix

	Incomepcratio	Unemploymentratio	Distance	GDPgrowthratio
Incomepcratio	1	-	-	-
Unemploymentratio	-0,477	1	-	-
Distance	0,024	-0,354	1	-
GDPgrowthratio	0,056	-0,202	0,027	1

Figure 3. All Choice probabilities for approach 1 and 2. Approach 3 is left out (can be reported on demand from the author) due to scale of the graph.

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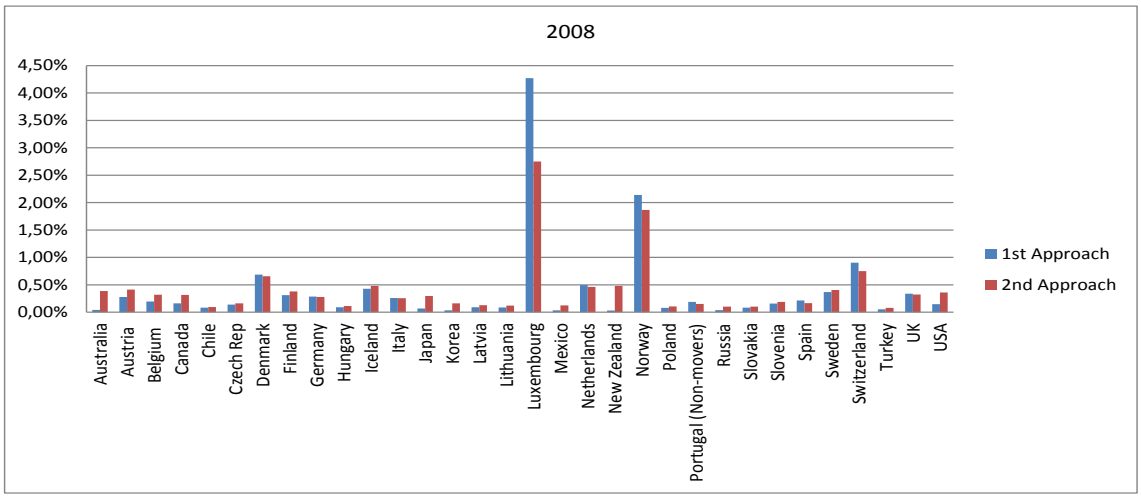
1st Approach
2nd Approach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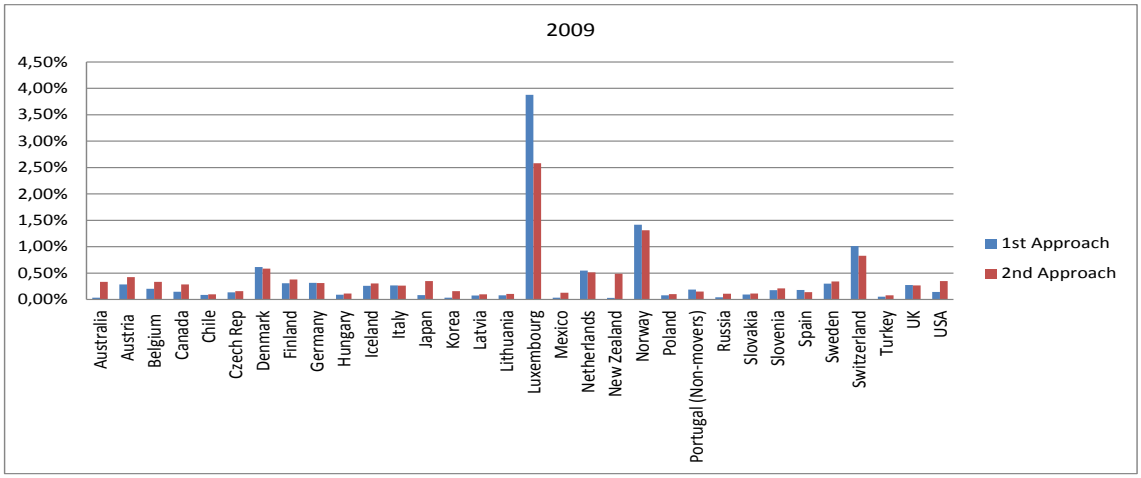
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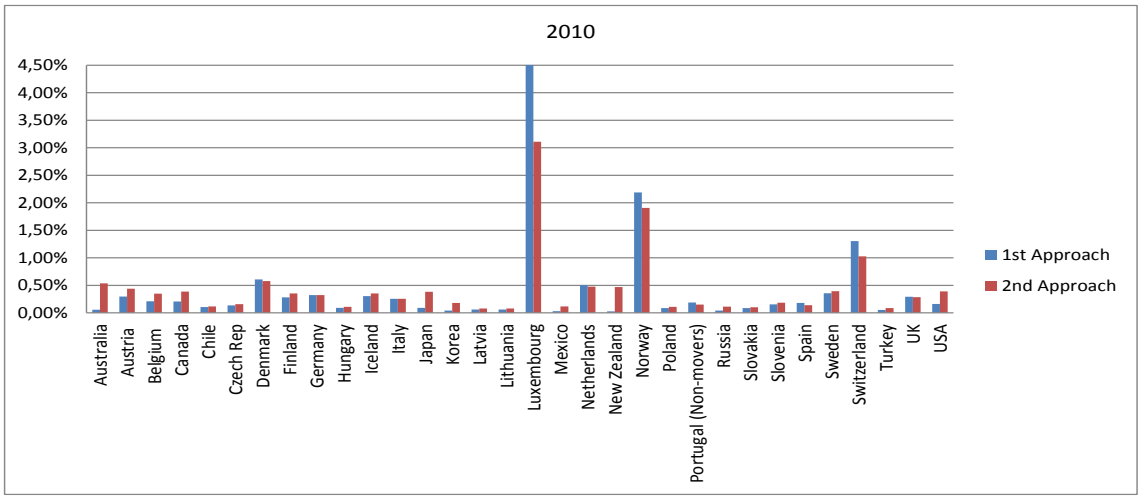
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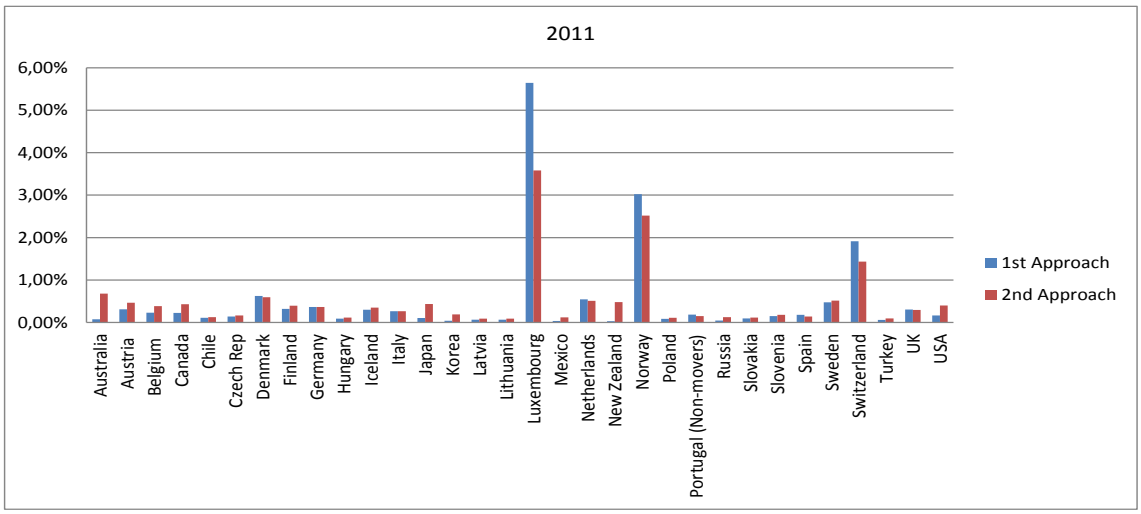
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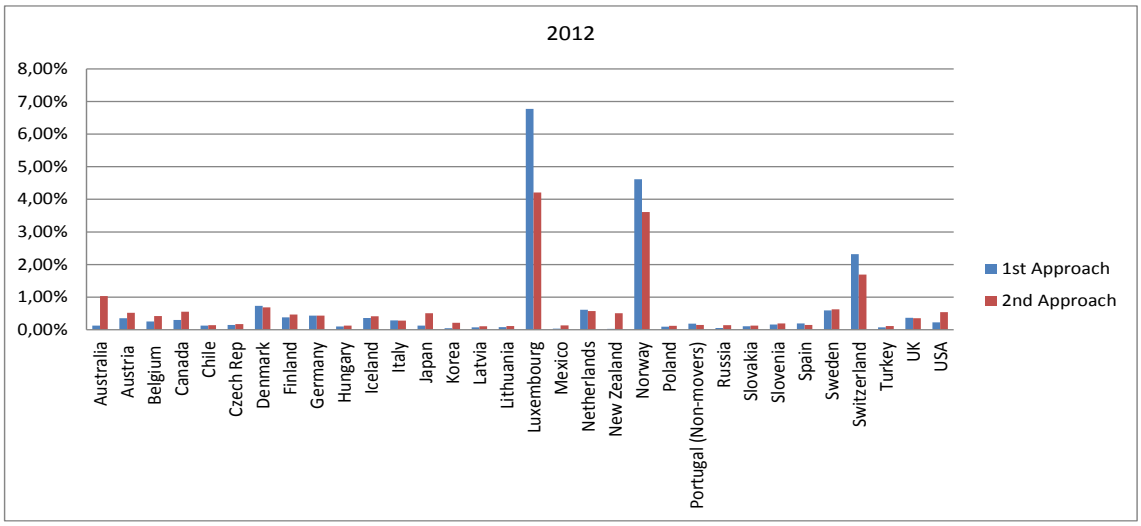
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Portugal (Non-movers)
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USA
1st Approach
2nd Approach

Figure 4. Top 5 countries in terms of Choice Probabilities for the excluded years.

country	year	Inflow	inc pc ratio	unemp ratio	distance	gdp growth ratio	1st approach
Luxembourg	2006	3,796	4,693827433	0,610389601	1.592,48	1,03437907	4,0420%
Norway	2006	0,097	3,804460953	0,441558465	2.780,25	1,008383465	1,7228%
Switzerland	2006	12,497	2,823136871	0,519480532	1.535,39	1,022703813	0,8401%
Iceland	2006	0,357	2,858254021	0,389610399	2.905,82	1,032141271	0,7201%
Denmark	2006	0,13	2,631335083	0,506493531	2.259,07	1,019186293	0,6401%
							2nd approach
Luxembourg	2006	3,796	4,693827433	0,610389601	1.592,48	1,03437907	2,6301%
Norway	2006	0,097	3,804460953	0,441558465	2.780,25	1,008383465	1,5341%
Iceland	2006	0,357	2,858254021	0,389610399	2.905,82	1,032141271	0,7468%
Switzerland	2006	12,497	2,823136871	0,519480532	1.535,39	1,022703813	0,6972%
Denmark	2006	0,13	2,631335083	0,506493531	2.259,07	1,019186293	0,6119%
							3rd approach
Luxembourg	2006	3,796	4,693827433	0,610389601	1.592,48	1,03437907	39,4567%
Norway	2006	0,097	3,804460953	0,441558465	2.780,25	1,008383465	15,8550%
Switzerland	2006	12,497	2,823136871	0,519480532	1.535,39	1,022703813	4,4287%
Germany	2006	4,917	1,837452192	1,337662396	1.952,78	1,022195605	3,7243%
Denmark	2006	0,13	2,631335083	0,506493531	2.259,07	1,019186293	3,4660%

country	year	Inflow	inc pc ratio	unemp ratio	distance	gdp growth ratio	1st approach
Luxembourg	2008	4,531	4,695110221	0,671052627	1.592,48	0,992738632	4,2687%
Norway	2008	0,271	3,989402722	0,342105255	2.780,25	1,000760796	2,1381%
Switzerland	2008	17,772	2,873152351	0,447368439	1.535,39	1,021729431	0,9051%
Denmark	2008	0,136	2,623415836	0,447368439	2.259,07	0,992245842	0,6844%
Netherlands	2008	2,385	2,219175281	0,513157914	1.785,97	1,003506861	0,4942%
							2nd approach
Luxembourg	2008	4,531	4,695110221	0,671052627	1.592,48	0,992738632	2,7513%
Norway	2008	0,271	3,989402722	0,342105255	2.780,25	1,000760796	1,8654%
Switzerland	2008	17,772	2,873152351	0,447368439	1.535,39	1,021729431	0,7489%
Denmark	2008	0,136	2,623415836	0,447368439	2.259,07	0,992245842	0,6545%
Iceland	2008	0,287	2,22243396	0,394736847	2.905,82	1,011966675	0,4811%
							3rd approach
Luxembourg	2008	4,531	4,695110221	0,671052627	1.592,48	0,992738632	52,4066%
Norway	2008	0,271	3,989402722	0,342105255	2.780,25	1,000760796	11,9689%
Switzerland	2008	17,772	2,873152351	0,447368439	1.535,39	1,021729431	4,6506%
Spain	2008	16,857	1,453192365	1,486842149	346,17	1,009002754	3,6861%
Denmark	2008	0,136	2,623415836	0,447368439	2.259,07	0,992245842	2,5009%

country	year	Inflow	inc pc ratio	unemp ratio	distance	gdp growth ratio	1st approach
Luxembourg	2009	3,844	4,481620576	0,536842095	1.592,48	0,972728417	3,8783%
Norway	2009	0,257	3,541600857	0,33684211	2.780,25	1,013116187	1,4146%
Switzerland	2009	13,67	2,969791176	0,431578937	1.535,39	1,010006286	1,0132%
Denmark	2009	0,152	2,538091361	0,631578947	2.259,07	0,971594279	0,6132%
Netherlands	2009	2,375	2,174590306	0,410526326	1.785,97	0,949571648	0,5476%
							2nd approach
Luxembourg	2009	3,844	4,481620576	0,536842095	1.592,48	0,972728417	2,5827%
Norway	2009	0,257	3,541600857	0,33684211	2.780,25	1,013116187	1,3128%
Switzerland	2009	13,67	2,969791176	0,431578937	1.535,39	1,010006286	0,8269%
Denmark	2009	0,152	2,538091361	0,631578947	2.259,07	0,971594279	0,5855%
Netherlands	2009	2,375	2,174590306	0,410526326	1.785,97	0,949571648	0,5132%
							3rd approach
Luxembourg	2009	3,844	4,481620576	0,536842095	1.592,48	0,972728417	55,0013%
Norway	2009	0,257	3,541600857	0,33684211	2.780,25	1,013116187	11,8137%
Switzerland	2009	13,67	2,969791176	0,431578937	1.535,39	1,010006286	6,5744%
Denmark	2009	0,152	2,538091361	0,631578947	2.259,07	0,971594279	3,7898%
Netherlands	2009	2,375	2,174590306	0,410526326	1.785,97	0,949571648	2,3069%

country	year	Inflow	inc pc ratio	unemp ratio	distance	gdp growth ratio	1st approach
Luxembourg	2010	3,845	4,742012247	0,407407409	1.592,48	1,01142889	4,7732%
Norway	2010	0,284	3,978918552	0,333333319	2.780,25	0,985694046	2,1867%
Switzerland	2010	12,826	3,249896632	0,416666659	1.535,39	1,009974966	1,3027%
Denmark	2010	0,168	2,607109039	0,694444432	2.259,07	0,994608888	0,6085%
Netherlands	2010	1,958	2,16013258	0,361111114	1.785,97	1,000782987	0,5013%
							2nd approach
Luxembourg	2010	3,845	4,742012247	0,407407409	1.592,48	1,01142889	3,1091%
Norway	2010	0,284	3,978918552	0,333333319	2.780,25	0,985694046	1,9071%
Switzerland	2010	12,826	3,249896632	0,416666659	1.535,39	1,009974966	1,0250%
Denmark	2010	0,168	2,607109039	0,694444432	2.259,07	0,994608888	0,5757%
Australia	2010	0,226	2,393423314	0,481481455	16.203,29	1,001482947	0,5347%
							3rd approach
Luxembourg	2010	3,845	4,742012247	0,407407409	1.592,48	1,01142889	47,2875%
Norway	2010	0,284	3,978918552	0,333333319	2.780,25	0,985694046	23,0753%
Switzerland	2010	12,826	3,249896632	0,416666659	1.535,39	1,009974966	7,1416%
Denmark	2010	0,168	2,607109039	0,694444432	2.259,07	0,994608888	3,6701%
Iceland	2010	0,022	1,824557412	0,703703682	2.905,82	0,940794588	3,0990%

country	year	Inflow	inc pc ratio	unemp ratio	distance	gdp growth ratio	1st approach
Luxembourg	2011	4,977	4,966743417	0,385826785	1.592,48	1,031954289	5,6415%
Norway	2011	0,458	4,401313039	0,25984252	2.780,25	1,026245015	3,0223%
Switzerland	2011	15,358	3,68743035	0,314960635	1.535,39	1,030792699	1,9145%
Denmark	2011	0,182	2,658271785	0,598425198	2.259,07	1,023511691	0,6266%
Netherlands	2011	2,096	2,213969485	0,307086626	1.785,97	0,99503433	0,5428%
							2nd approach
Luxembourg	2011	4,977	4,966743417	0,385826785	1.592,48	1,031954289	3,5811%
Norway	2011	0,458	4,401313039	0,25984252	2.780,25	1,026245015	2,5163%
Switzerland	2011	15,358	3,68743035	0,314960635	1.535,39	1,030792699	1,4332%
Australia	2011	0,216	2,755174132	0,401574802	16.203,29	1,037303303	0,6820%
Denmark	2011	0,182	2,658271785	0,598425198	2.259,07	1,023511691	0,5944%
							3rd approach
Luxembourg	2011	4,977	4,966743417	0,385826785	1.592,48	1,031954289	43,0529%
Norway	2011	0,458	4,401313039	0,25984252	2.780,25	1,026245015	20,4079%
Switzerland	2011	15,358	3,68743035	0,314960635	1.535,39	1,030792699	11,1384%
Denmark	2011	0,182	2,658271785	0,598425198	2.259,07	1,023511691	3,3882%
Netherlands	2011	2,096	2,213969485	0,307086626	1.785,97	0,99503433	3,0656%

Karush-Kuhn-Tucker Conditions for Local Optimality

Consider the following nonlinear optimization problem:

$$\begin{aligned} &\text{Maximize} \quad f(x) \\ &\text{subject to} \\ &g_i(x) \leq 0, h_j(x) = 0 \end{aligned}$$

where *x* is the optimization variable, f is the *objective* or *cost* function, $g_i \ (i = 1, \ldots, m)$ are the inequality constraint functions, and $h_j \ (j = 1, \ldots, l)$ are the equality constraint functions. The numbers of inequality and equality constraints are denoted *m* and *l*, respectively.

Necessary conditions for optimality:

Suppose that the objective function $f : \mathbb{R}^n \to \mathbb{R}$ and the constraint functions $g_i : \mathbb{R}^n \to \mathbb{R}$ and $h_j : \mathbb{R}^n \to \mathbb{R}$ are continuously differentiable at a point $\boldsymbol{x^*}$. If $\boldsymbol{x^*}$ is a local minimum that satisfies some regularity conditions then there exist constants $\mu_i \ (i = 1, \ldots, m)$ and $\lambda_j \ (j = 1, \ldots, l)$, called KKT multipliers, such that

Stationarity

For maximizing f(x):

$$\nabla f(x^*) = \sum_{i=1}^{m} \mu_i \nabla g_i(x^*) + \sum_{j=1}^{l} \lambda_j \nabla h_j(x^*),$$

Primal feasibility

$$g_i(x^*) \leq 0, \text{ for all } i = 1, \ldots, m$$
$$h_j(x^*) = 0, \text{ for all } j = 1, \ldots, l$$

Dual feasibility

$$\mu_i \geq 0, \text{ for all } i = 1, \ldots, m$$

Complementary slackness

$$\mu_i g_i(x^*) = 0, \text{for all } i = 1, \ldots, m.$$

In the particular case $m = 0$, i.e., when there are no inequality constraints, the KKT conditions turn into the Lagrange conditions, and the KKT multipliers are called Lagrange multipliers.