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Econometric approach for forecasting stock indices price

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Abstract

This work proposes to build a profitable dynamic trading strategy. In order to do that it is necessary to forecast the future stock indices prices. First we exploit the forecast power of stock indices assuming that they follow a Geometric Brownian motion. Next, we present an alternative forecasting model that involves cross sectional regression between indices. The latter proves to be more profitable on average than the former.

Keywords: **cross sectional regressions, stock indices, stochastic processes, trading strategies**

1 Introduction

Einstein used to say that *God does not play dices with the Universe.* In the early 1900s the idea of *quantum physics* was born, revolutionizing the deterministic concept of nature. As a byproduct of the quantum physics success, there was the initial impression that all the natural phenomena were governed by probabilistic laws. Einstein argued that nature should evolve according to deterministic mathematical laws. The complexity of nature and of all these deterministic rules lead us to use probabilistic explanations of whatever is observed - not because the underlying phenomena are intrinsically probabilistic, but simply to hide the ignorance of the true mechanisms.

This paper is based on the idea that any financial market follows a logical structure where the underlying laws of demand and supply determine its outcome. On the top of that basic mechanism, there are random fluctuations. In other words, God rolls dices on the top of a deterministic system, and those who know how to assess this specific randomness will have more foreshadowing power. The objective of this work is to show that, by considering this additional randomness, it is possible to improve drastically the forecasting power of future stock indices and build a subsequent more efficient strategy.

As referred in Kim and Han (2016), there are three alternative approaches to best forecast stock prices: technical analysis, fundamental analysis and time series. As defined by Murphy (1999), technical analysis forecasts future price trends mainly studying charts. One of the most famous strategies is based on the Relative Strength Index[1] (an index that can take value $0 \leq RSI \leq 100$). Menkhoff and Taylor (2007) studied the literature and the reasons of using technical analysis in foreign exchange market. Using moving averages

[1] Go long on the asset that are oversold (i.e. RSI<30) and short on the asset that are overbought (i.e. RSI>70)

should be profitable as well. However, strong empirical evidence says that technical analysis can not be used to predict excess return. Daniotti (2012) studied a method referred to the cross of two moving averages. Applying this method to three equity indices[2] it has resulted to be not profitable after transaction costs. The overall result is quite disappointing since it did not outperform a simple buy and hold strategy.

The fundamental analysis, instead, tries to forecast returns looking at the *intrinsic value* of a company as depending on macroeconomic (e.g. overall economy and industry conditions) and microeconomic factors (e.g. financial conditions and company management). Predicting excess returns using past information contradicts the *Efficient Market Hypothesis* as defined by Fama (1969). There are however many models based on this fundamental analysis assuming that markets are inefficient to some extent. One of the first models used to do that (see Ang (2014)) is the Capital Asset Pricing Model (CAPM) [3]. Many authors tested the relationship

$$R_{i,t+1} = \alpha + \beta R_{m,t+1} + \epsilon_{t+1}. \tag{1}$$

Under the CAPM null hypothesis the expected result should be $\alpha = 0$ and $\beta = 1$. Unfortunately, regressing a portfolio of US stocks and the S&P 500, the α resulted too large and the R^2 of the regression too small. In order to better explain the excess stock returns Fama and French (1993) added two risky factors related to size and value of the portfolios (namely the so-called SMB and HML factors). This is the model that seems to better explain portfolio excess returns in terms of fundamental analysis.

Models that try to predict excess returns use time series data. A time series is a par-

[2]Dow Jones Euro Stoxx for Europe, the S&P 500 for the USA and the Topix for Japan

[3]it has been developed by Treynor (1961), Sharpe (1964), Lintner (1965), and Mossin (1966), based on Harry Markowitz (1952)

ticular realization of a stochastic process, which is a set of random variables indexed by time (Stirzaker (2005)). A stochastic process is said to be discrete if it is a measurable set of random variables, and continuous if it is a non countable set of random variables. A time series $\{x_t\}$ is thus a sample path of the underlying stochastic process, and can be either discrete or continuous, depending on whether the series is recorded on discrete or continuous time as defined in (Brockwell and Davis (2016)).
The literature on using continuous time models to predict prices and backtesting investment strategies is summarized by Sonono and Mashele (2015). This paper contributes by proposing an alternative continuous-time model that uses multivariate rolling regressions and fitted values to predict future prices. For each implemented strategy the results are back-tested.

This paper is structured as follows. Next section presents the most important factor models, connected with the fundamental analysis. The third section explains the data set and presents the computation for every implemented strategy. The fourth section brings the result of a simple buy and hold strategy, used as a benchmark. In the fifth paragraph, the attention is switched to the underlying stochastic process, the Geometric Brownian motion. Additionally, the backtesting of a potential strategy involving GBM is computed. The sixth section provides an alternative model to forecast future indices price. The associated backtesting strategy results provide evidence that this new model beats the Geometric Brownian motion hypothesis and the buy and hold one. Last section concludes.

2 Multi-factor models

One of the first models that exploited the trade-off between risk and return, thus explaining portfolio returns, was the CAPM. Since in this model all the efficient portfolios are well diversified, the systematic risk is the only one to be priced. The model itself states that the market portfolio, with respect to which the systematic risk is measured, is efficient. It thus follows that the expected excess return on a particular asset is a linear function of its systematic risk, as measured by $\frac{cov(\sigma_m \sigma_i)}{\sigma_m^2}$ (see Ericsson and Karlsson (2004)).
The CAPM is consistent with a single factor market model where the portfolio returns are explained by the market portfolio. As such, the market return variable captures all the systematic risk (see also Campbell et al. (1997)). This model has been used as a workhorse of finance, particularly in capital budgeting decisions. As it is said in the first section this model does not allow to best forecast future portfolio returns. If an investor is willing to use a multi-factor model he is implying that factors, individually, explain the systematic risk (Berk and DeMarzo (2007)). The general equation of a multi-factor model is the following:

$$R_p = \alpha + \sum_{n=1}^{N} \beta_i^{F_n} R_{F_n} + \epsilon$$

where the βs are the coefficients for each factor. By taking expectations it follows that summing the excess return of every factor, multiplied by the Beta [4], it is possible to obtain the asset risk premium (Berk and DeMarzo (2007)).

2.1 Three-Factor Model by Fama and French

Multi-factor models pick variables that may best predict average returns and capture risk premiums. Fama and French (1993) elaborated one of the most used models in empirical

[4] Sensitivity of the asset with that explanatory factor

research and industry application (see (Bodie et al., 2014)):

$$R_p = \alpha + \beta_m R_M + \beta_2 SMB + \beta_3 HML + \epsilon, \tag{2}$$

where SMB (Small minus Big) identifies the market capitalization and HML (High minus Low) describes the book-to-market ratio. SMB measures the historic returns of small companies in excess of those of large companies. HML measures the return of value companies in excess of those of growth companies. Small and value firms empirically outperform big and growth firms, respectively. As a result, building a SMB portfolio means to take a long position in the small and a short position in the stocks of big firms. On the other hand, building a HML portfolio is equivalent to buy value stocks and sell growth stocks. It has been observed that small companies can change quicker business condition in case of economy crisis. In spite of this, value companies seem to be the first ones to face financial crisis when the economy slowdown.

This model is also used as a benchmark in asset management companies. Accordingly the manager beats the benchmark if its $\alpha > 0$ in equation (2). However, this coefficient incorporates selection and timing ability. The former is referred to the skill of picking stocks that outperform the market, the latter is referred to the ability of leveraging and overweighting the portfolio when the market return is high and reverting this position when it is low. There are methods to show how to calculate separately these two components of the alpha, but this is not the purpose of this work.

One of greatest problems with multi-factor models is that some variables may predict well average returns only in some period of time, as referred in Bodie et al. (2014). The ones, willing to search for explanatory factors on security returns, might find that returns are explained by risky factors purely based on the particular period of time analyzed (Black (1993)).

2.2 Four-Factor Model by Carharth

The Cahart (1997) four-factor model is an extension of the Fama–French three-factor. It can be constructed adding a momentum factor to equation (2). Momentum is an "anomaly" empirically identified in the market and refers to the fact that a stock that showed positive average returns in the prior twelve months will continue to do well. A simple strategy based on momentum is one that goes long on stocks that are *winners* and short on *losers* (stocks that performed poorly in the previous months). The four factor model is reflected in the following equation:

$$R_p = \alpha + \beta_m R_m + \beta_2 SMB + \beta_3 HML + \beta_4 MOM + \epsilon \quad (3)$$

In his work Cahart (1997) run the regressions (1), (2) and (3) on several US index[5]. As expected, adding factors will reduce the observed average error. The average error for the CAPM is $E(\epsilon) = 0.35\%$, for the Fama and French three-factor model is $E(\epsilon) = 0.31\%$, and for the Carharth four-factor model is $E(\epsilon) = 0.14\%$. The four-factor model eliminates almost all the pricing errors and is an improvement as compared to the others.

2.3 Five-Factors model by Fama and French

Fama and French (2014) improved their model trying to better explain expected asset returns:

$$R_p = R_f + \beta_m R_m + \beta_2 SMB + \beta_3 HML + \beta_4 RMW + \beta_5 CMA + \epsilon,$$

[5]New York stock exchange (NYSE), American stock Exchange (Amex) and Nasdaq stocks

where Robust Minus Weak identifies the profitability factor and Conservative Minus Aggressive explains the *Conservative/ Aggressive* investment policy of firms. The former can be computed as the difference between returns of companies with robust and weak profitability. The latter, instead, is related to the returns of conservative minus aggressive investment companies. Fama and French (2014) discovered that the five-factor model explains in a proper way the asset return with respect to the three-factor model Fama and French (1993). However, the former breaks down in capturing the HML factor in the sample (July 1963– December 2013). Run an hypothetical four-factor model without the HML, would performs the same as the five-factor [6].

3 Data

All the data are taken from the Bloomberg platform. For the purpose of this work, several stock indices are analyzed:

Name	Aex	Atx	Cac	Ccmp	Dax	Ftse100	Ftse Mib	S&P/TSX	Ibex	Nikkei	S&P500
Country	The Netherlands	Austria	France	Usa	Germany	UK	Italy	Canada	Spain	Japan	Usa

Table 1: Stock Indices

Open and close prices were taken for all the indices in the sample from 01/01/2000 to 31/12/2016.

In order to test different strategies it is necessary to include transaction costs for each of them, computed as the 2% of every return, either positive or negative (in the case of zero return, unlikely possible, a constant negative return of -0.05% is considered). The percentage is high because it includes brokers' commissions, spreads ($bid - ask$) and the

[6] The high minus low factor is necessary when investors interested in portfolio tilts toward size, value, profitability, and investment excess return.

cost of the short selling positions. Regarding the taxation framework it has been assumed the prospective of an Italian Investor (nation with one of the higher taxation), meaning 26% of the profit, for the case of the future derivatives, due to the state. On the other hand, having a negative position allow the investor to reduce future tax payments. The percentage of losses that can be discounted from future profit positions varies across investors. For simplicity it is settled at the 26% level. As a result the figure $r_{taxcost}$ includes all the costs and it is the real profit that an investor earns. For every strategy the out-of-sample returns are computed[7]. In order to do that the strategies require to estimate some parameters, explained below. These coefficients are estimated as those that maximize the total returns obtained following the strategy in the previous year. The in-sample results are not presented since that kind of strategy is not achievable in reality, because it uses forward looking information. Furthermore, for every strategy the standard deviation (σ) of the after tax return and the Info Sharpe[8] are computed. The info Sharpe is referred to the annual return after taxes. Additionally, skewness and kurtosis of after-tax returns are analyzed[9]. Skewness is used to describe asymmetry deviations from the normal distribution in a set of statistical data. A positive skewness means that the shape of the distribution has a long tail on the right (positive side) so there is an higher probability of having *positive days*. On the contrary, a negative skewness means that there is more probability of having *losses days*. The kurtosis identifies the *tailedness* of the probability distribution. Usually, this number is compared to the kurtosis of the normal distribution that is 3. A distribution that has kurtosis greater than 3 is said to be *leptokurtic*, on the other hand if it is smaller than 3 is said *platykurtic*. In finance, investors want to have positive skewness, because it allows to have less negative days. Regarding the kurtosis,

[7]Investors are allowed to use only past informations.

[8]Basically it is $IS = \frac{annualreturn}{annual\sigma}$ and it identifies how was the return of the strategy comparing to the volatility.

[9]Analyzing the statistics for an in sample strategy is almost useless.

more risk adverse investors search for leptokurtic distribution of return. On the contrary, risk lover investors want platykurtic distribution that can guarantee more "abnormal" returns.

4 Buy And Hold

The Buy and hold strategy is used as a benchmark to beat. This clearly means to buy at the beginning of the first year (01/01/2005) of the sample and selling at the last day of the sample (31/12/2016). The results presented below included the taxes. It is not necessary to include the transaction costs because investor does only two operations, buying at the beginning and selling at the end of the sample. The Dax index was the best one with a total return of 123.97%, annual average return of 6.89% and standard deviation of 22.16%. The resulting info Sharpe would be 0.31. The FTSE 100 index had a total return of 35.79%, annual average return of 2.98% and standard deviation of 18.93%. The resulting info Sharpe would be 0.16. The Japanese index had a total return of 49.12%, annual average return of 4.09% and standard deviation of 24.48%. The resulting info Sharpe would be 0.17. The ATX index had a total return of 5.03%, annual average return of 0.42% and standard deviation of 18.06%. The resulting info Sharpe would be 0.02. As expected, this strategy is not exceptional but, at least, perform on average better than the one based on the GBM (showed below).

5 Geometric Brownian motion

There are many stochastic process that can describe the path of an asset. However, the most used one is the Geometric Brownian motion (GBM), a continuous-time stochastic process. To identify a specific GBM it is necessary that the logarithm random part of the equation[10] (4) follows a Wiener process (Brownian motion) with drift (Ross (2014)):

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{4}$$

Where $0 \leq t \leq T$, μ is a drift and dW_t is the factor that describes the Wiener process. It is possible to demonstrate that equation 4 can be written as:

$$S_{t+\Delta t} = S_t \cdot \exp\{(\mu - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t} \cdot \epsilon_{t+1}\}$$

where $\Delta t = t_{i+1} - t_i$ and ϵ_{t+1} is a standard normal random variable that it is needed to generate different scenarios. In fact, to run this kind of processes there are needed Monte Carlo simulations. The most difficult part in this process is the estimations of the parameters. In order to implement the strategy it has been assumed that the drift μ and σ are respectively the average return and the volatility (standard deviation) of the θ previous periods of a given stock index. To generate the different scenarios for a given ϵ it has been used the Matlab command *randn*, providing a random number from the standard normal distribution. To estimate θ it is maximized the total return that an investor would have get following this strategy for the previous year. In this way it is not used forward looking information. Regarding Δt it has been used the value $\frac{1}{260}$ because the interval is daily. Matlab is used for all the computation. For every iteration the software produce 10000 S_t starting from S_{t-1}. This is done either for the open either for the close price of the index taken in consideration. Having the open and the close price of t+1 day it is possible to compute the hypothetical future return $\phi = \frac{S_{close}}{S_{open}} - 1$. Basically, if the investor is at time t and he sees that $\phi > 0$, he decides to go long, otherwise he goes short. As it has been said, all the information used are the ones available at time t.

[10] The random varying quantity.

5.1 Geometric Brownian motion strategy results

In order to implement an out of sample[11] strategy it has been imposed a maximization problem ($\max y = f(\theta)$) with the constraint of $\theta \in \mathbb{N}, \theta \neq 0$. The function to maximize is the total return of the previous year that an investor would have get following the strategy[12]. Regarding ϵ, it is chosen to have 10000 figures (everyone of them simulate a different scenario and it is set with the Matlab command *randn*).

The four table below (2, 3, 4, 5) show the results for the Japanese, UK, Austrian and German stock indices respectively.

NKY	No costs	Transaction	Tax	θ	σ	IS	Skew	Kurt
2005	6.29%	3.42%	2.64%	32	8.31%	0.318	0.1285	4.0338
2006	33.98%	28.81%	20.90%	3	11.87%	1.761	-0.1130	3.2995
2007	-9.87%	-12.78%	-9.46%	3	10.08%	-0.938	-0.7322	6.4947
2008	16.67%	7.51%	7.03%	7	28.27%	0.249	-0.5358	9.1913
2009	-22.43%	-26.18%	-19.81%	2	14.82%	-1.337	0.0950	4.0230
2010	-12.79%	-15.61%	-11.65%	17	9.96%	-1.170	0.0418	3.8288
2011	-20.63%	-23.07%	-17.44%	1	11.45%	-1.523	-3.2650	32.5977
2012	9.43%	6.55%	4.93%	27	8.10%	0.608	0.1817	3.8017
2013	-27.48%	-30.88%	-23.59%	8	15.36%	-1.536	-1.0906	8.9179
2014	4.03%	0.74%	0.73%	2	10.25%	0.071	-0.1342	5.7202
2015	-19.93%	-22.59%	-17.07%	3	11.06%	-1.543	-1.2006	9.0843
2016	14.51%	9.40%	7.38%	41	16.18%	0.4558	-0.9150	11.6897

Table 2: GBM NKY out of sample

[11] To implement the out-of-the sample strategy it is not possible to use forward looking information but only past informations.

[12] For example if the current year is 2016, in order to obtain the θ to use in the strategy it has been maximize the cumulative annual return of the previous year (2015).

UK	No costs	Transaction	Tax	θ	σ	IS	Skew	kurt
2005	12.65%	10.16%	7.50%	20	6.48%	1.1581	0.2222	3.5848
2006	-20.51%	-22.93%	-17.40%	2	9.43%	-1.8453	-0.2617	4.3857
2007	7.75%	3.27%	2.71%	1	12.94%	0.2944	-0.0127	4.5276
2008	-29.50%	-35.26%	-26.49%	6	27.93%	-0.9485	-0.0034	6.5029
2009	-20.81%	-25.23%	-18.92%	1	17.46%	-1.0834	-0.1115	4.3321
2010	8.71%	4.23%	3.42%	241	13.02%	0.2630	-0.5646	5.3129
2011	35.20%	28.50%	20.90%	54	15.64%	1.3363	0.5582	3.9447
2012	-4.46%	-7.69%	-5.56%	7	10.51%	-0.5292	-0.1131	3.4724
2013	-5.52%	-8.34%	-6.10%	1	9.08%	-0.6720	0.4661	4.4842
2014	13.43%	10.42%	7.74%	31	8.41%	0.9209	-0.0875	4.9720
2015	25.37%	20.33%	15.00%	15	12.77%	1.1754	-0.2368	4.9762
2016	18.64%	13.91%	10.43%	4	12.59%	0.8285	-0.0250	4.1792

Table 3: GBM UK out of sample

ATX	No costs	Transaction	Tax	θ	σ	IS	Skew	Kurt
2005	-17.06%	-19.86%	-14.95%	3	10.31%	-1.4502	-0.1764	4.7030
2006	-11.23%	-15.43%	-11.28%	35	15.49%	-0.7282	-1.3239	8.6898
2007	14.58%	8.97%	6.98%	5	15.04%	0.4645	-0.0598	4.7587
2008	37.99%	23.52%	19.51%	6	35.53%	0.5493	0.4348	5.3655
2009	41.10%	28.61%	21.98%	11	26.70%	0.8234	0.1625	3.1922
2010	66.46%	57.03%	40.42%	5	17.79%	2.2717	0.7358	7.0576
2011	26.01%	17.47%	13.61%	6	21.93%	0.6205	-0.1076	4.4705
2012	54.45%	46.40%	33.19%	33	16.02%	2.0723	0.0619	3.6484
2013	30.95%	25.81%	18.81%	124	11.91%	1.5795	0.2189	4.4592
2014	55.08%	48.74%	34.51%	5	12.28%	2.8106	0.0709	3.7528
2015	10.67%	5.31%	4.31%	16	14.94%	0.2886	0.2094	3.6367
2016	30.60%	23.93%	17.73%	3	15.99%	1.1087	0.1235	4.9458

Table 4: GBM ATX out of sample

DAX	No costs	Transaction	Tax	θ	σ	IS	Skew	Kurt
2005	6.40%	3.71%	2.83%	5	7.57%	0.3743	0.0822	3.1062
2006	-12.35%	-15.15%	-11.28%	2	10.09%	-1.1184	-0.2546	4.3949
2007	0.58%	-2.61%	-1.77%	4	9.74%	-0.1820	0.0573	3.4653
2008	19.29%	11.36%	9.43%	28	24.63%	0.3830	0.7602	10.0379
2009	25.45%	17.54%	13.43%	26	19.03%	0.7054	0.0400	3.2653
2010	42.51%	37.06%	26.60%	21	12.00%	2.2160	-0.2354	4.0373
2011	-10.26%	-15.09%	-10.86%	1	18.48%	-0.5877	-0.3170	6.1887
2012	20.64%	16.00%	11.89%	40	11.94%	0.9955	-0.3411	4.1952
2013	28.48%	24.79%	17.97%	4	8.92%	2.0154	0.3832	4.3875
2014	-2.43%	-5.89%	-4.19%	4	10.92%	-0.3838	-0.0888	3.5550
2015	-19.92%	-23.66%	-17.82%	33	14.08%	-1.2657	-0.0083	3.5126
2016	27.41%	22.45%	16.48%	198	12.32%	1.3381	0.3284	4.8198

Table 5: GBM DAX out of sample

As expected the strategy on average does not perform so well. The ATX on average shows a positive average return of 14.40% with an average standard deviation of 17.83%, resulting an info sharpe of 0.86. Furthermore, having invested 1 Euro from the 1^{st} of January 2005 to the end of December 2016, you would have ended up with 4.9071. The DAX on average shows a positive average return of 4.39% with an average standard deviation of 13.32%, resulting an info sharpe of 0.33. Additionally, having invested 1 Euro from the 1^{st} of January 2005 to the end of December 2016, you would have ended up 1.5197. The NKY on average shows a negative average return of -4.62% with an average standard deviation of 12.98%, resulting an info sharpe of -0.36. Having invested 1 Euro from the 1^{st} of January 2005 to the end of December 2016, you would have ended up with 0.5059. The UK on average shows a negative average return of -0.56% with an average standard deviation of 12.77%, resulting an info sharpe of -0.04. As a result, having invested 1 Euro from the 1^{st} of January 2005 to the end of December 2016, you would have ended up with 0.8247. It is clearly observable that the GBM is not a good proxy for simulating the path of a stock index since it is not able to beat a simple buy and hold strategy.

6 Alternative Econometric Models

6.1 Methodology

Below it is explained a new model that tries to forecast indices prices. In order to forecast future prices several steps are required.

Firstly, multiple rolling linear regressions have been run for open and close prices:

$$y_t = \alpha + \beta_1 x_1 + \beta_2 x_2 + \epsilon$$

Where y_t is a vector of prices of the index to be predicted that goes from t to $t-\delta$. x_1 and x_2 are the independent variables, meaning the two indices used to forecast the dependent variable. In order to choose which indices should forecast the future price of the others it has been computed a correlation table between all indices. The independent variable chosen are the ones with the highest correlation with the dependent one.

Once it has been estimated the coefficients of the regressions, it is possible to forecast the future prices (open and close):

$$y_{t+1} = \hat{\alpha} + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2$$

Until now there is not something innovative. What is really innovative is the following.

After having estimated the futures open and closing prince, it has been computed the hypothetical arithmetic return [13]:

$$\phi = \frac{y_{t+1}^{close}}{y_{t+1}^{open}} - 1$$

[13] In the long run it is better to use the arithmetic return instead of the logarithmic. This because the log returns are an approximation given by the fact that $\ln(1+r) \approx r \iff r$ is small.

From this point it is possible to implement two different strategies:

1. A simple strategy that do not involve using boundaries:
   - if $\phi < 0$ the strategy involves investing in a short position in the future;
   - if $\phi > 0$ the strategy involves investing in a long position in the future.
2. An aggressive strategy that uses some boundaries explained below:
   - If $\phi < \gamma_1$ the strategy involves investing in two short positions in the future;
   - if $\gamma_1 \leq \phi \leq \gamma_2$ the strategy involves investing in one short position in the future;
   - if $\gamma_2 \leq \phi \leq \gamma_3$ the strategy involves to not invest;
   - if $\gamma_3 \leq \phi \leq \gamma_4$ the strategy involves investing in one long position in the future;
   - If $\phi > \gamma_4$ the strategy involves investing in two long positions in the future;

The most difficult part, now, is to set the time of the rolling regression and the boundaries (γ_i with $i = 1, 2, 3, 4$). In order to estimate these coefficients, it has been imposed an optimization problem given some constraint. The objective function to maximize is the cumulative return that an investor would have get for the previous year following the strategies. It is possible to define the two maximization problems as the following:

1.
$$\max y = f(t)$$

2.
$$\max y = f(t, \gamma_1, \gamma_2, \gamma_3, \gamma_4)$$

The constraints are the following:

- the variable t (time) is an integer constraint.
- $\gamma_1 \leq \gamma_2 \leq \gamma_3 \leq \gamma_4$
- $-2 \leq \gamma_i \leq 2, i = \{1, 2, 3, 4\}$

6.2 Alternative model Results

We present below the results the simple and the aggressive strategies.

As for the GBM, the strategies have been implemented for four different stock indices.

6.2.1 Simple Strategy

Table 6 summarize the dependent and independent variable of the model.

Y	X_1	X_2
Atx	FtseMib	Ibex
Dax	S&P 500	Ccmp
Ftse100	S&P 500	Smi
Nikkei	Aex	Smi

Table 6: Resume indices

The Y is the index in which the investor should make operations. The independent variable, instead, are the indices that one needs to forecast prices. In the tables 7, 8, 9 and 10, there are the statistical of the out of sample strategies of the first model.

NKY	No costs	Transaction	Tax	σ	IS	Skew	Kurt	Rolling period
2005	1.39%	-1.32%	-0.86%	13.26%	-0.0650	0.2865	4.4380	25
2006	-15.98%	-19.27%	-14.43%	8.14%	-1.7736	-0.0709	3.9627	9
2007	21.01%	17.14%	12.62%	12.38%	1.0188	-0.0370	2.9951	102
2008	148.14%	128.88%	87.13%	9.25%	9.4241	0.8634	7.4969	77
2009	138.67%	127.25%	84.27%	14.17%	5.9474	0.3627	3.6429	394
2010	37.95%	33.47%	24.03%	9.98%	2.4075	-0.1103	3.8101	397
2011	31.81%	27.81%	20.17%	10.20%	1.9779	-0.1457	3.7066	58
2012	-13.46%	-15.75%	-11.18%	11.73%	-1.0064	2.6238	26.3132	53
2013	-35.36%	-38.40%	-29.82%	8.07%	-3.6973	-0.1800	4.1947	24
2014	20.87%	17.09%	12.59%	15.32%	0.8216	-1.0202	9.2569	171
2015	-2.20%	-5.46%	-3.86%	9.74%	-0.3964	-0.0132	3.9428	75
2016	26.13%	20.52%	15.35%	16.15%	0.9564	0.2886	5.9384	230

Table 7: S1 NKY out of sample

UK	No costs	Transaction	Tax	σ	IS	Skew	Kurt	Rolling period
2005	22.14%	19.45%	14.14%	8.10%	1.7466	0.0974	3.4731	69
2006	-5.59%	-8.46%	-6.19%	9.71%	-0.6368	-0.1094	4.2839	322
2007	30.10%	24.70%	18.09%	8.23%	2.1990	0.1337	3.8816	267
2008	114.70%	97.38%	67.62%	11.47%	5.8937	0.0699	5.4828	59
2009	36.11%	28.58%	21.09%	27.96%	0.7543	0.7717	5.2505	98
2010	57.08%	50.61%	35.80%	13.45%	2.6618	-0.6163	4.9958	257
2011	64.40%	56.22%	39.71%	14.94%	2.6578	0.0747	4.1329	354
2012	12.91%	9.12%	6.88%	15.26%	0.4504	0.2437	3.5444	282
2013	-2.00%	-4.93%	-3.53%	10.33%	-0.3414	-0.0560	3.4705	148
2014	-1.16%	-3.78%	-2.69%	6.85%	-0.3929	-0.0512	3.4705	331
2015	44.06%	38.28%	27.47%	7.55%	3.6388	0.2831	4.3516	106
2016	23.62%	18.69%	13.84%	12.75%	-0.0552	4.1190	1.0988	234

Table 8: S1 UK out of sample

ATX	No costs	Transaction	Tax	σ	IS	Skew	Kurt	Rolling period
2005	23.74%	19.57%	14.35%	10.26%	1.3988	0.3808	4.4468	21
2006	11.80%	6.52%	5.23%	15.45%	0.3384	-0.9555	8.9649	1
2007	45.59%	38.47%	27.74%	14.92%	1.8588	0.0722	8.969	71
2008	37.44%	22.93%	19.19%	35.63%	0.5386	-0.4292	4.7151	104
2009	35.77%	23.79%	18.59%	26.74%	0.6950	0.0910	3.2258	18
2010	55.82%	46.94%	33.72%	17.88%	1.8853	-0.9035	8.4845	148
2011	13.65%	5.96%	5.26%	21.91%	0.2402	-0.0450	4.4508	28
2012	27.85%	21.12%	15.77%	16.26%	0.9699	-0.2958	3.7610	7
2013	44.30%	38.62%	27.65%	11.86%	2.3319	0.2150	4.4456	1
2014	46.85%	40.85%	29.20%	12.32%	2.3690	-0.0066	3.7926	5
2015	12.02%	6.59%	5.25%	14.97%	0.3505	-0.0154	3.6791	49
2016	21.83%	15.62%	11.85%	16.03%	0.7390	-0.4842	5.2833	19

Table 9: S1 ATX out of sample

DAX	No costs	Transaction	Tax	σ	IS	Skew	Kurt	Rolling period
2005	-10.15%	-12.42%	-9.26%	7.58%	−1.2212	-0.0308	3.1312	106
2006	-14.89%	-17.61%	-13.20%	10.12%	-1.3044	-0.6069	4.2872	26
2007	9.19%	5.74%	4.39%	9.71%	0.4516	0.0545	3.4548	227
2008	0.38%	-7.06%	-4.22%	24.79%	-0.1701	-0.7464	10.5419	81
2009	109.67%	96.51%	65.91%	18.77%	3.5119	-0.3894	3.6187	342
2010	-36.36%	-38.83%	-30.30%	12.06%	-2.5119	-0.5609	3.5547	51
2011	1.66%	-3.75%	-2.21%	18.32%	-0.1207	-0.0190	6.2697	1
2012	-0.23%	-4.12%	-2.81%	12.18%	-0.2310	-0.1569	4.0901	5
2013	3.35%	0.37%	0.42%	9.04%	0.0463	-0.0877	4.5620	22
2014	10.82%	6.91%	5.29%	10.88%	0.4859	-0.0993	3.5954	4
2015	46.31%	39.52%	28.39%	13.93%	2.0378	0.1722	3.3805	58
2016	21.66%	16.94%	12.58%	12.52%	1.0050	0.3773	4.6659	55

Table 10: S1 DAX out of sample

The ATX on average shows a positive average return of 17.82% with an average standard deviation of 18.10%, resulting an info sharpe of 0.98. Furthermore, having invested 1 Euro from the 1^{st} of January 2005 to the end of December 2016, you would have ended up with 6.8737. The DAX on average shows a positive average return of 4.58% with an average standard deviation of 13.33%, resulting an info sharpe of 0.34. Furthermore, having invested 1 Euro from the 1^{st} of January 2005 to the end of December 2016, you would have ended up with 1.3227. The NKY on average shows a positive average return of 16.33% with an average standard deviation of 11.53%, resulting an info sharpe of 1.42. Furthermore, having invested 1 Euro from the 1^{st} of January 2005 to the end of December 2016, you would have ended up with 3.8217. The UK on average shows a positive average return of 19.35% with an average standard deviation of 12.22%, resulting an info sharpe of 1.58. Furthermore, having invested 1 Euro from the 1^{st} of January 2005 to the end of December 2016, you would have ended up with 7.0894. This strategy clearly outperform the GBM results. It is interesting to notice that the info Sharpe are really high. The are all above 1, only the DAX has a IS of 0.34 (not so efficient).

6.2.2 Aggressive Strategy

In the following tables 11, 12, 13 and 14 are shown the results of the second strategy for the different indices.

UK	No costs	T.cos	TT.cos	σ	IS	Skew	Kurt	R. period	γ_1	γ_2	γ_3	γ_4
2005	118.25%	108.96%	73.01%	12.32%	5.9283	0.3306	3.5176	342	-0.320%	-0.297%	-0.271%	-0.240%
2006	30.92%	23.09%	17.35%	18.87%	0.9196	-0.4372	4.6337	371	-0.373%	-0.371%	-0.864%	-0.606%
2007	39.63%	29.45%	22.32%	24.27%	0.9197	-0.3568	5.1272	302	-24.751%	-5.612%	-6.143%	-0.622%
2008	305.22%	242.75%	162.36%	55.13%	2.9451	0.4168	6.3302	98	-8.036E-03%	-8.093E-03%	-7.666E-03%	8.549E-03%
2009	73.31%	54.90%	41.16%	34.42%	1.1958	0.0594	4.4341	261	-2.747E-03%	-2.748E-03%	-2.746E-03%	1.928E-03%
2010	161.20%	140.37%	93.62%	25.48%	3.6736	-0.4503	5.7595	284	-1.294 E-03%	1.496 E-06%	5.432E-05%	1.401E-02%
2011	158.42%	133.42%	90.48%	31.15%	2.9047	0.0688	4.1932	354	-1.183E-15%	3.656E-11%	-2.647E-09%	7.708E-10%
2012	35.06%	26.14%	19.67%	20.92%	0.9402	-0.0737	3.5029	147	-3.470 E-10%	-2.151E-11%	1.575E-11%	-1.528E-11%
2013	-5.41%	-11.00%	-7.71%	18.27%	-0.4221	-0.7589	4.5973	148	9.457E-10%	-6.026E-10%	-3.265E-07%	-4.208-8%
2014	-3.57%	-8.63%	-5.99%	16.90%	-0.3542	-0.5357	5.0141	331	9.144E-06%	-3.277E-06%	-8.143E-06%	-5.033E-06%
2015	101.48%	85.68%	59.88%	25.31%	2.3655	0.1307	4.8281	106	0	0	0	0
2016	46.46%	35.05%	26.30%	25.14%	1.0461	-0.3166	4.3332	234	-1.809E-02%	5.393E-05%	-3.053E-06%	7.209E-04%

Table 11: S2 UK out of sample

UK	No costs	T.cos	TT.cos	σ	IS	Skew	Kurt	R. period	γ_1	γ_2	γ_3	γ_4
2005	1.67%	-3.60%	-2.23%	16.16%	-0.1380	0.0788	4.1673	25	-0.01761%	-0.00181%	0.00009%	0.01987%
2006	-28.20%	-33.30%	-25.22%	16.16%	-1.1199	-0.0069	3.4763	9	-0.25770%	-5.6246E-03%	9.0070E-03%	3.4171E-02%
2007	12.41%	5.34%	4.65%	20.03%	0.2322	0.55927	6.1420	55	0	0	0	0
2008	1433.04%	1211.04%	606.13%	54.26%	11.1701	1.3335	8.2106	203	-3.2499E-02	-3.2414E-02	-2.8354E-02	-1.9340E-02
2009	454.46%	402.72%	235.31%	28.45%	8.2697	0.3736	3.6334	396	0	0	0	0
2010	81.42%	69.95%	49.08%	19.67%	2.4950	-0.1044	3.9184	397	-3.0286E-07	-9.7535E-08	-7.7336E-08	2.8588E-08
2011	69.90%	59.80%	42.66%	22.50%	1.8962	2.9588	30.1696	58	0	0	0	0
2012	-25.42%	-29.27%	-22.25%	16.09%	-1.3825	-0.3624	3.9189	53	-2.3270E-04	-1.0597E-04	-7.2119E-03	-4.3934E-03
2013	-60.05%	-63.75%	-51.96%	30.63%	-1.6961	-1.2281	8.7871	24	0	0	0	0
2014	-8.16%	-13.61%	-9.63%	19.97%	-0.4822	-1.2281	6.0790	128	-41.47%	-23.37%	-17.35%	-1.38%
2015	-6.52%	-12.66%	-8.72%	22.23%	-0.3923	-0.8701	9.3128	75	0	0	0	0
2016	46.41%	33.78%	26.41%	32.20%	0.8202	-1.2022	12.1614	251	-0.2651%	-0.2587%	-0.2408%	-0.1929%

Table 12: S2 NKY out of sample

2005	55.78%	48.18%	34.29%	14.71%	2.3309	-0.2239	3.3459	199	-16.3459%	-3.6993%	-3.5054%	-1.8618%
2006	11.98%	5.45%	4.69%	19.24%	0.2439	-0.5829	5.0758	333	-1.4586%	-1.3891%	-1.1336%	-0.0875%
2007	63.33%	53.34%	38.10%	19.11%	1.9930	0.1949	3.4378	76	-0.1751%	-0.1707%	-0.1547%	-0.1485%
2008	19.90%	4.44%	7.87%	49.34%	0.1595	0.0492	10.4652	103	-3.1183E-05	-3.1182E-05	-3.1189E-05	3.1194E-05
2009	48.41%	30.92%	25.14%	37.37%	0.6728	0.0492	3.5832	18	-3.3330E-02%	-2.5139E-02%	0.1025%	0.1414%
2010	6.99%	-1.10%	0.23%	24.36%	0.0093	-0.2326	3.8332	187	-1.5720%	-1.5717%	-1.5715%	-1.5657%
2011	6.77%	-4.31%	-0.87%	36.68%	-0.0237	-0.2147	6.2862	28	0	0	0	0
2012	32.53%	22.41%	17.36%	24.29%	0.7147	-0.1787	4.1204	148	–4.0127E-07%	4.0150E-07%	4.3526E-07%	6.1159E-07%
2013	7.65%	3.07%	2.62%	14.12%	0.1857	-0.1489	5.4261	42	-0.8301%	-2.1104E-02%	-3.3776E-03%	0.1661%
2014	-35.20%	-39.71%	-30.65%	21.74%	-1.4097	0.0775	3.5936	9	0	0	0	0
2015	22.79%	11.63%	10.00%	28.14%	0.3555	-0.0418	3.4896	49	0	0	0	0
2016	14.67%	8.79%	7.05%	18.25%	0.3865	0.1664	6.65239	294	-0.5524%	-0.2268%	-1.1511E-02%	-1.1208%

Table 13: S2 DAX out of sample

UK	No costs	T.cos	TT.cos	σ	IS	Skew	Kurt	R. period	γ_1	γ_2	γ_3	γ_4
UK	No costs	T.cos	TT.cos	σ	IS	Skew	Kurt	R. period	γ_1	γ_2	γ_3	γ_4
2005	158.07%	142.04%	93.60%	18.93%	4.9437	-0.1937	5.3461	98	-113.7989%	-61.6739%	-12.4074%	-1.0590%
2006	96.12%	78.25%	55.98%	30.94%	1.8360	-0.8376	9.4889	38	-0.5975%	-0.5615%	-0.5255%	-0.5194%
2007	130.03%	108.14%	74.74%	29.77%	2.5104	0.0220	4.7546	69	-0.1487%	-0.1487%	-0.1487%	-0.1487%
2008	57.30%	25.64%	30.06%	71.25%	0.4219	-0.4387	5.7653	103	-2.6770E-03	-1.7822E-03	-1.2258E-03	2.7044E-03
2009	30.68%	8.94%	11.93%	52.93%	0.2254	0.0896	3.3384	18	0.1560%	0.1609%	0.2265%	0.2557%
2010	372.53%	324.27%	197.72%	33.91%	5.8312	-1.1100	10.6233	334	-1.1460%	-0.8518%	-0.6913%	-0.6210%
2011	26.53%	2.85%	5.61%	43.81%	0.1282	-0.0450	4.4508	28	0	0	0	0
2012	26.53%	13.51%	16.39%	32.62%	0.3659	-0.4640	3.7768	148	5.1472E-08%	-8.5844E-10%	-6.1736E-08%	2.4762E-08%
2013	101.40%	85.86%	11.94%	23.75%	2.5173	-0.0095	4.5951	392	0	0	0	0
2014	49.34%	37.36%	27.87%	24.87%	1.1207	-0.1084	3.8164	9	0	0	0	0
2015	20.46%	9.05%	8.31%	29.94%	0.2777	-0.0154	3.6791	49	0	0	0	0
2016	39.82%	27.05%	21.32%	30.10%	0.7083	-0.5325	6.0876	264	-1.1570%	-1.1417%	-1.0637%	-0.3988%

Table 14: S2 ATX out of sample

The overall average return are much higher comparing to the buy and hold, the GBM and the simple strategy. The ATX on average shows a positive average return of 46.29% with an average standard deviation of 35.24%, resulting an info sharpe of 1.31. Furthermore, having invested 1 Euro from the 1^{st} of January 2005 to the end of December 2016, you would have ended up with 51.8738. The DAX on average shows a positive average return of 9.65% with an average standard deviation of 25.61%, resulting an info sharpe of 0.38. Furthermore, having invested 1 Euro from the 1^{st} of January 2005 to the end of December 2016, you would have ended up 1.5611. The NKY on average shows a positive average return of 70.35% with an average standard deviation of 24.86%, resulting an info sharpe of 2.83. Furthermore, having invested 1 Euro from the 1^{st} of January 2005 to the end of December 2016, you would have ended up with 14.0061. The UK on average shows a positive average return of 49.37% with an average standard deviation of 25.68%, resulting an info sharpe of 1.92. Furthermore, having invested 1 Euro from the 1^{st} of January 2005 to the end of December 2016, you would have ended up with 70.1168. This strategy clearly outperforms the GBM results and the buy and hold one. It is interesting to notice that the info Sharpe are really high. The are all above 1, only the DAX has a IS of 0.38 (not so efficient, as the previous strategy).

There are two main reasons why this strategy is so profitable on average. The former is due to the protective boundary (the one that suggests to stay cash) that allows to avoid false signals. The latter is mainly due to the aggressive position of having two contracts allowing to double the profit but, eventually, the losses. The great result is in the fact that all the strategies statistic are free of taxes. Thus, the figures shown identify real profit.

7 Conclusion

In this work it has been analyzed how it is possible to best forecast indices pricing.

The Geometric Brownian motion is supposed to be a good proxy for simulating the path of stock indices but of course has a lot of bias inside, mostly given by the parameters estimated in the model. This results to have a poor average performance for the four indices taken in analysis. The Austrian index shows on average an IS of 0.86, the England index figures on average a IS -0.04, the Japanese index a negative average IS of -0.36 and the German index a positive average IS of 0.33.

The simple alternative strategy performs really well on average, much better than the GBM and the buy and hold. The Austrian index shows on average an IS of 0.98, the England index figures on average a IS 1.58, the Japanese index an average IS of 1.42 and the German index a positive average IS of 0.34.

The aggressive alternative strategy performs really well on average, much better than the GBM, the simple model and the buy and hold. The Austrian index shows on average an IS of 1.31, the England index figures on average a IS 1.92, the Japanese index an average IS of 2.83 and the German index a positive average IS of 0.38. This can be achieved thanks to the protective boundary that allows to stay cash and avoid false signal and the aggressive boundaries.

Further improvements can be done studying different indices and constructing equally weighted or risk parity portfolios. Diversifying on different indices can allow to avoid single shock on specific countries. Furthermore, including more independent variables may improve the forecast power, allowing to increase more the profit.

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