

**TELEPHONE DEMAND: A
THEORETICAL APPROCH AND ITS
APPLICATION TO THE PORTUGUESE
CASE**

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A Theoretical Approach
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Abstract: We develop a theoretical model which explains the joint decision for joining a telephone network and using it. We arrive at testable propositions and use data for the Portuguese telephone services to test them. Results suggest that positive network externalities are very important. A 10% increase in the network would increase the demand for access by 7-8% in the long run and an increase in traffic by 6-7% in the long run (assuming constant population). The income elasticity for traffic is greater than one in the long run. Both these results imply a strong pressure in the telephone network and a need for planning so that the quality of the services does not deteriorate.

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Introduction:

There are many services for which the consumer must pay an access fee prior to use. Examples of these are the telephone, telex, telefax services or social clubs.

For the services presented above, the well-being of a consumer joining the network (club) depends on the number of people in the network. There is a network externality which can be either positive or negative depending on how the dimension of the network affects the quality of the service (possibility of the existence of congestion).

There is a vast literature in optimal pricing of access and usage of services with network externalities (see Willig (1979) and Curien (1987)). The main conclusion of this literature are: 1) The access fee should be set below the marginal customer access cost and the usage fee should be set below the marginal usage cost if there are network externalities. The existence of positive network externality implies that to obtain the optimal number of subscribers the access fee should be set below the marginal customer access cost.

The purpose of this paper is to develop a theoretical model which explains the demand for these services assuming there is a two-part tariff, and empirically test it with Portuguese telephone services data, to see if there are positive network externalities.

In this paper we present a simple theoretical model which

allows us to arrive at a sufficient number of testable propositions with a small set of assumptions; for instance, we do not assume differentiability of the utility function or use the money metric, and we show the existence and uniqueness of a lower bound income above which all consumers will want to access the service.

As in the Portuguese case, there is a waiting list demand for telephone services, i.e., waiting list for the telephone to be installed, we suggest two alternative ways of empirically evaluating the price, income and network elasticities of the usage demand. Literature, so far, when testing the theory did not consider the effect of a waiting list in the actual demand, so this paper is the first to consider the difference between usage per subscriber and average usage per capita.

In the Portuguese case the waiting list is not a rationing device but the result of a temporary shortage of supply, that the firms promised to solve in the near future. A person is counted in the waiting list if he tells the company he wants a telephone installed. He will have his telephone installed when there is a line available for the place he asked it for, so there are some persons who stay in the list for a short period while others will stay in the list for a long period. There is no payment till the telephone is installed.

For most people obtaining a telephone is a kind of a lottery.

Our estimations show that empirical results depend on the estimation procedure used, but both alternatives point to a

positive network externality.

There is a long series of studies dealing with telephone demand. Some of the better ones are Taylor (1983) and Curien and Gensollen (1987). Studies dealing with Portuguese data are Melo (1986) and Luz (1990).

Without going into the details of these studies, their main assumptions can be easily stated. All these studies consider that the demand depends on the real price of telephones (access and usage), income, population. Some of these studies point out the existence of externalities in the usage of telephones.

In this paper we consider and find evidence of the possibility of partial adjustment in the access demand.

The theoretical results are tested using data for the Portuguese telephone services. We explore some alternative ways of estimation when there is a waiting list.

Finally, we compare the results of both estimations. The results suggest a positive network externality in the demand for telephone services.

In the Portuguese case the satisfaction of the waiting list would (in 1988) have increased the demand for access by 7 - 8 % and the usage demand by 6 - 7%.

The existence of network externality points to the need of more research to be done in the study of the optimal pricing of telephone services in Portugal.

The Model:

The following model addresses the example of telephone services, but is similar to the cases of the other services presented in the introduction.

It is assumed that each consumer's well being depends upon the usage of telephone (e.g. number of phone calls), the consumption of other goods (numeraire) and the number of people connected to the network at the time the decision is made (N).

Each consumer has a positive exogenous income and preferences which can be represented by a utility function

$$U(dN, du, x)$$

where $d=1$ if the consumer has access to the telephone network and $d=0$ otherwise; u is a measure of telephone usage and x is the quantity of other goods.

We assume the following conditions to be verified:

A.1) $U(N, u, x)$ is continuous in u and x

$U(0, 0, x)$ is continuous in x

A.2) there is no satiation in x and u , meaning that if $x_1 > x$ then $U(dN, du, x_1) > U(dN, du, x)$ for all u , d and N , and if $u_1 > u$ then $U(N, u_1, x) > U(N, u, x)$ for all x and N .

A.3) there is perfect divisibility of u and x .

A.4) $U(N, u, 0) < U(0, 0, x)$ for all $x > 0$ and $u \geq 0$ and N ; people cannot survive by telephone use, without consuming other goods.

A.5) $\lim U(N,0,x) > \lim U(0,0,x)$ as x goes to infinity; meaning that after the basic needs are satisfied, people prefer having a telephone to not having it. At the same time this assumption considers the option demand for telephone.

A.6) if $x_1 > x_2$ then $[U(dN,du,x_2+h) - U(dN,du,x_2)] > [U(0,0,x_1+h) - U(0,0,x_1)]$ for all $h > 0$ and for all d and u ; the same amount of increase in the consumption of the other goods increases the utility more, the smaller the consumption of the other goods is which the consumer started with. This assumption is similar to saying that "marginal utilities" are decreasing.

Each consumer solves the problem

$$\text{Max}_{d,u,x} U(dN,du,x)$$

s.t.

$$d(P_a + P_u u) + x \leq y$$

$$x, u \geq 0$$

$$d=1 \text{ or } 0$$

where P_a is the access charge, P_u is the usage charge and y is the exogenous income.

As d takes two distinct values, the easiest way to discuss the solution of the above problem is to solve it for $d=0$ and then for $d=1$

Case of $d=0$

$$\text{Max}_x U(0,0,x) \quad (0)$$

s.t.

$$x \leq y$$

$$x \geq 0$$

by conditions A.1), A.2) and A.3) the above problem has at least one solution for $y \geq 0$.

The set of optimal solutions to (0) is denoted $\{x(y)\}$ and $V(y) = U(0,0,x(y))$

$V(y)$ satisfies the following properties:

P01 - $V(y)$ is continuous for $y \geq 0$

P02 - $V(y)$ is increasing in y

Case of $d=1$

$$\text{Max}_{u,x} U(N,u,x) \quad (1)$$

s.t.

$$Pu + x \leq y - Pa$$

$$u, x \geq 0$$

by conditions A.1), A.2) and A.3) the above problem has at least one solution for $y - Pa \geq 0$.

The set of optimal solutions to (1) is denoted $\{u(Pu, y - Pa, N), x(Pu, y - Pa, N)\}$ and $V(Pu, y - Pa, N) = U(N, u(Pu, y - Pa), x(Pu, y - Pa))$

$V(P_u, y - P_a, N)$ satisfies the following properties:

P11 - $V(P_u, y - P_a)$ is continuous in P_u , y and P_a for $y - P_a \geq 0$ and $P_u > 0$

P12 - $V(P_u, y - P_a)$ is increasing in y and non increasing in P_u and P_a .

P13 - $V(P_u, y - P_a)$ is quasi-convex in P_u .

Proposition 1:

There is a y^* such that $V(y^*) = V(P_u, y^* - P_a, N)$ for all $P_u, P_a > 0$, i.e., there a level of income for which the consumer is indifferent between having or not telephone.

Proof:

Suppose the proposition is false. There is a P_u and a P_a , both positive, such that $V(y) \neq V(P_u, y - P_a, N)$ for all y .

As both functions are continuous in y either $V(P_u, y - P_a, N) > V(y)$ for all y or $V(P_u, y - P_a, N) < V(y)$ for all y .

The first case is false because if $y - P_a = 0$, and $V(P_u, y - P_a, N) = U(N, 0, 0) < V(y)$ by conditions A.1) and A.4).

The second case is false because $V(P_u, y - P_a, N) \geq U(N, 0, y - P_a)$ by definition $> V(y)$ as y goes to infinity (by condition A.5) and A.1).

Therefore, there is a y^* such that $V(P_u, y^* - P_a, N) = V(y^*)$. ■

Proposition 2:

The y^* in Proposition 1 is unique.

Proof:

Suppose not. There are y_1 and y_2 such that $V(P_u, y_1 - P_a, N) = V(y_1)$ and $V(P_u, y_2 - P_a, N) = V(y_2)$ and $y_1 \neq y_2$.

Let us assume $y_1 > y_2$, this implies that $y_1 = y_2 + h$ ($h > 0$)

$V(P_u, y_2 + h - P_a, N) - V(P_u, y_2 - P_a, N) \geq U(N, u(P_u, y_2 - P_a), x(P_u, y_2 - P_a) + h) - V(P_u, y_2 - P_a) > V(y_1) - V(y_2)$ by condition A.6) as $x(P_u, y_2 - P_a) < y_2$.

Therefore, there is a contradiction which proves the uniqueness of y . ■

Proposition 3:

If $y_1 > y^*$ (of Proposition 1) $V(P_u, y_1 - P_a, N) > V(y_1)$, i. e., consumers with higher income than y^* will have telephone.

Proof:

$y_1 = y^* + h$ ($h > 0$)

$V(P_u, y_1 - P_a, N) \geq U(N, u(P_u, y^* - P_a, N), x(P_u, y^* - P_a, N) + h) = U(N, u(P_u, y^* - P_a, N), x(P_u, y^* - P_a, N) + h) - U(N, u(P_u, y^* - P_a, N), x(P_u, y^* - P_a, N)) + V(y^*)$ by definition of $y^* > V(y_1) - V(y^*) + V(y^*) = V(y_1)$ by A.6). ■

We assume that all the consumers are equal, face the same prices and have different incomes.

If the income is distributed with density $f(y)$, then the demand for access (A) (number of telephones a certain population wants to have installed) is

$$A = \text{POP} \int_{y^*}^{+\infty} f(y) dy$$
 where POP is the total population; the demand for access is the number of consumers with income higher than y^* in a certain population.

(Note: if $A > \text{the installed network } (N)$, the network will increase)

$\text{Apc} = A/\text{POP}$ - access demand per capita or proportion of population with telephone.

and the demand for usage (U) is given by

$$U = \text{POP} \int_{y^*}^{+\infty} u(P_u, y - P_a, N) f(y) dy$$

$\text{Upc} = U/\text{POP}$ - usage demand per capita or average number of calls or periods of usage.

Proposition 4:

y^* increases with P_u and P_a . If $U(N, u, x)$ is increasing in N , y^* decreases with N .

Proof:

This results from the fact that $V(P_u, y - P_a, N)$ decreases with P_u and P_a and increases with N , and $V(y)$ does not depend on P_u and P_a .

Corollary 1:

A_{pc} is decreasing in P_u and P_a , and increasing with N .

Proof:

Using Proposition 4 and definition. ■

Corollary 2:

U_{pc} is decreasing in P_a and increasing with N if $u(P_u, y - P_a, N)$ is increasing in y and N .

Proof:

An increase in P_a (decrease in N) decreases $V(P_u, y - P_a, N)$ and therefore increases y^* (by Proposition 4).

U is then the integral of the same (smaller) positive function in a smaller interval and therefore decreases. ■

Corollary 3:

If $u(P_u, y - P_a, N)$ is increasing in y , then U_{pc} is non-increasing in P_u .

Proof:

Be $Plu > Pu$ and $y1 = Plu u(Pu, y - Pa, N) + x(Pu, y - Pa, N) > y$.

$$u(Plu, y - Pa, N) - u(Pu, y - Pa, N) = [u(Plu, y - Pa, N) - u(Plu, y1 - Pa, N)] (1) + [u(Plu, y1 - Pa, N) - u(Pu, y - Pa, N)] (2).$$

(1) is negative by assumption of this proposition.

(2) is non-positive by the theory of Revealed Preference, therefore

$u(Plu, y - Pa, N) - u(Pu, y - Pa, N)$ is negative.

As y^* is decreasing with Pu , $U1pc$ is the integral of a smaller positive function in a smaller interval, therefore $U1pc < Upc$. ■

From the propositions above we conclude that the per capita demand for access and for usage is decreasing in Pa and Pu and increasing with N .

If there is a general increase in y , meaning a shift to the right of the distribution of y , then the per capita demand for access and for usage increase. If we characterize the distribution of y by its mean (my), a shift to the right implies a higher mean.

In conclusion, from the above model we have:

$$Apc = Apc (\overset{-}{Pa}, \overset{-}{Pu}, \overset{+}{N}, \overset{+}{my})$$

and

$$Upc = Upc (\overset{-}{Pa}, \overset{-}{Pu}, \overset{+}{N}, \overset{+}{my})$$

where the signs of influence of the explanatory variables appear above them.

Data and Estimation Procedures:

Our estimations are based on the available annual information for the telephone services in Portugal (mainland) for the period 1963-1988.

In Portugal the usage of the telephone by a subscriber (at home) has 3 costs:

- 1) the cost to have the telephone installed (P_i)
- 2) the monthly rent of the telephone (P_r)
- 3) the price of each period for which the telephone is used (P_{ul})

If we compare these three costs with the costs that appear in the above model, we see that P_{ul} is the empirical counterpart of P_u and P_a is positively related with P_i and P_r .

We do not know the income distribution in Portugal. The data we have available is the real income per capita (the mean of the distribution). We use this information in the estimation to see the influence of income in the demand for access and the demand for use.

The total population in Portugal in each year is known and is POP in the above model.

The demand for telephones has not been satisfied in Portugal; there are a number of consumers that are on a waiting list.

This waiting list accounted in one year for more than 16% of consumers that wanted to have a telephone installed (people with telephone plus people on the waiting list).

We assumed the sum of telephones installed plus the number of people on the waiting list as the total demand for access; as explained in the introduction we do not believe the waiting list to be a rationing device in this particular case..

The number of periods the telephones were used in Portugal in a particular year is available. This number does not correspond to U above because of the existence of the waiting list, therefore there is people who would have telephone and used it but had not his telephone installed.

Let us call VU the verified traffic for which we have data available.

If we assume that people in the waiting list are randomly selected from all the people who wish to have a telephone, we can use the number of periods used (VU) multiplied by the number of telephones plus waiting list divided by the number telephones as a measure of U .

As people have telephones to contact or be contacted by other people. Therefore, the dimension of the telephone network is going to influence the utility that results from having a telephone.

We can assume there is a positive network externality in the case of the telephone services.

The demand for access is not attained instantly. Therefore, there may be some temporary disequilibrium. We can capture this temporary disequilibrium through a partial adjustment process (see Cabral & Leite 1989).

To keep the demand equations simple we assumed that they are log-linear as the goal of this paper is to show the existence of network externalities.

The data consists of the following variables:

Endogenous variables are:

A - telephones installed plus waiting list. Data available for 1968 -1988; there are no data collected on the waiting list for the period prior to 1968. From our examination of the data available we presume that there was a waiting list before 1968.

VU - number of periods used. Data available for the period 1963 - 1988. In 1985 there was a break in the series as the result of the splitting of the two companies that offer telephone services in Portugal. We use the series 1963 - 1985.

U - number of periods used multiplied by A divided by the number of telephones installed.

Note that all data is for private plus business demand and

therefore is different from the ideal for testing the model above.

The exogenous variables are:

Pu - real price per period of use of the telephone.

Pr - real monthly rental price of the telephone

Pi - real installation price of the telephone. The Consumer Index price was used as deflator.

Y- real income per capita. Data from the Portuguese National Statistics Office (INE).

POP - population resident in Portugal (mainland). Data from the INE.

N - number of telephones installed (number of subscribers).

So in what follows:

W - waiting list

$$Apc = \frac{N + W}{POP}$$

$$Upc = VU * \frac{N + W}{N} \cdot \frac{1}{POP}$$

The functional forms of the equations to be estimated are (log-linear forms are used for simplicity reasons¹)

¹ The main results do not change if we assume linear relations.

$$\ln A_{pc} = b_0 + b_1 \ln P_i + b_2 \ln P_r + b_3 \ln P_u + b_4 \ln A_{pc}(-1) + b_5 \ln N(-1) + b_6 \ln Y + u$$

for the access per capita equation with partial adjustment ($b_1, b_2, b_3 < 0$ and $b_5, b_6 > 0$ by the above model),

$$\ln U_{pc} = c_0 + c_1 \ln P_i + c_2 \ln P_r + c_3 \ln P_u + c_4 \ln N(-1) + c_5 \ln U_{pc}(-1) + c_6 \ln Y + u$$

for the usage per capita equation ($c_1, c_2, c_3 < 0$ and $c_4, c_6 > 0$ by the above model).

These equations were estimated using ordinary least squares and we used Maximum Likelihood for the detection of first order autocorrelated errors. No coefficient of autocorrelation was significantly different from zero at 10% level.

Empirical Results:

Following, we present the elasticities of the access demand and usage demand. The absolute value of the t - ratios for the coefficients appear between brackets.

1) Access Demand Per Capita

	Short Run	Long Run	No Partial Adj.
Pi	-.0409 (2.62)	-.0683	-.0065 (.49)
Pr	-.0875 (3.43)	-.1461	-.1431 (6.45)
Pu	-.0272 (.70)	-.0454	-.0415 (.89)
Y	.2357 (3.09)	.3936	.3907 (5.59)
N(-1)	.4642 (4.80)	.7751	.7387 (25.40)
Apc(-1)	.4011 (2.97)		
Adj.R2	.998		.998

The results above show that with exclusion of the one time installation price, the long run estimates and the estimates without partial adjustment are very similar.

There is a positive network externality, whose elasticity in the long run is around .7 - .8. As there is a waiting list of 9.6% (1988 data), the satisfaction of this waiting list would increase the access demand per capita by 7 - 8% in the long run. The total access demand would increase 7 - 8 % plus the increase (decrease) in population.

Another point worthy of attention is the fact that the usage fee does not significantly influence the access demand.

2) Usage Demand per Capita

As there is a waiting list the verified traffic is below the theoretical level; there are some people who would like to use their own telephone and cannot because it has not been installed.

To have a proxy of the theoretical traffic we assume that the people with telephones are a random selection from the people who want to have telephones; therefore, as explained above, the theoretical usage is the subscribers usage multiplied by the the number of subscribers plus the waiting list divided by the number of subscribers.

$$U = VU * A / N$$

where VU is the verified traffic, U is the theoretical traffic and N is the number of subscribers.

$$Upc = U / POP \quad U \text{ per capita}$$

and we estimate

- 1) U per capita directly
- and

2) VU/N the verified traffic per subscriber; as $Upc = VU/N * A/POP = VU/N * Apc$ and therefore the Upc elasticities are the elasticities of U/N plus the Apc elasticities found above; this means that we take the verified traffic per subscriber as a measure of the average usage for the non-truncated sample and multiply by A to obtain U .

2.1) Traffic per capita estimated directly

	Short Run	Long Run	No Partial Adj.
Pi	-.0504 (1.73)	-.0786	-.0006 (.01)
Pr	-.0850 (1.62)	-.1326	-.2832 (5.13)
Pu	-.1023 (1.54)	-.1596	-.0321 (.35)
Y	.7918 (4.47)	1.2352	1.5418 (11.61)
N(-1)	.4097 (3.83)	.6392	.6923 (6.44)
Upc(-1)	.3589 (3.24)		
Adj. R2	.997		.996

We see that there are positive network externalities. The elasticity of the usage per capita in terms of the network in the

previous year is .4 in the short run and around .6 - .7 in the long run. The satisfaction of the waiting list would increase the traffic per capita in 6 - 7%, as the waiting list in 1988 was 9.6% of the network.

The results above show that there is a large difference in the value of the monthly rental price and usage price elasticities estimates. With such a small sample this can easily happen.

2.2) Estimation of the elasticities of the traffic per capita using the verified traffic per subscriber equation ($VU_{ps} = VU/N$)

We assume that in the traffic per subscriber there is not a partial adjustment process, because once the telephone is installed the individual can use it as many times as he/she wants.

The equation to be estimated is

$$\ln VU_{ps} = c_0 + c_1 \ln P_i + c_2 \ln P_r + c_3 \ln P_u + c_4 \ln N(-1) + c_5 \ln Y + u$$

where VU_{ps} is the verified traffic per subscriber.

The results for the traffic per subscriber equation are the following:

P1	-.0122
	(.39)
Pr	-.1535
	(3.22)
Pu	-.0368
	(.47)
Y	1.0934
	(9.99)
N(-1)	-.0429
	(.46)
Ad. R2	.984

From the estimation above we see that the existing network in the previous period does not positively influence the traffic demand by subscriber.

This does not imply the inexistence of network externalities because the network externalities have to be seen in the traffic per capita and not in the traffic per subscriber, as explained above.

To have the elasticities of the traffic per capita, we have to add the above elasticities to the ones for demand for access.

We obtain the following results

	Short Run	Long Run
Pi	-.0531 (1.52)	-.0805
Pr	-.2410 (4.46)	-.2996
Pu	-.0640 (.73)	-.0822
Y	1.3291 (9.96)	1.4870
N(-1)	.4213 (3.13)	.7322

And again we see the existence of a positive network externality.

From the results above we can conclude that the elasticity of the traffic demand in relation to the network is around .6 to .7 in the long run, which means that in the Portuguese case, the fact of the telephone company satisfying the waiting list (1988 data), would, by itself, increase traffic by 6 to 7%.

The elasticity traffic demand / monthly rental price is much higher (in absolute values) in this estimation than it was in the estimation using total traffic per capita with partial adjustment. This can result from the fact of having a small sample and the standard errors of the estimates being very large.

The income elasticity is greater than one. This means that in terms of usage, the telephone is still a luxury good for the Portuguese population.

Conclusions:

We use Table 1 from Taylor (1983) to compare our results with those obtained previously.

Point and Interval Estimates of Price and Income Elasticities of Demand for Selected Telephone Services

Type of Demand	Elasticity		Toll Price	Inc.
	Service Con. Charge	Monthly Serv. Charge		
Access	-0.03 (± 0.01)	-0.10 (± 0.09)		0.50 (± 0.10)
(ours)	-0.07	-0.15		.39
Local Calls		-0.20 (± 0.05)		1.00 (± 0.40)
Toll Calls (conversation- minutes)				
Intrastate			-0.65 (± 0.15)	1.25 (± 0.25)
Interstate			-0.75 (± 0.20)	1.50 (± 0.40)
International Calls			-0.90 (± 0.30)	1.70 (± 0.40)
Duration of Toll Calls			-0.015 (± 0.05)	0.25 (± 0.10)
(ours - usage)		-.24		1.23

Source: These estimates refer to long-run, steady state elasticities. The numbers in parentheses are subjective standard errors.

From the above table we see that our results appear in the range of value plus or minus a standard error with exception for the coefficient of the installation price in the access demand equation and the one of the rental price in the usage equation. This can happen because the countries Taylor takes into consideration have a much higher average income per capita than Portugal.

In conclusion, this paper shows that network externalities are very important in the Portuguese telephone services. Our results suggest that a 10% increase in the network (approximately the satisfaction of the waiting list) would increase the number of subscribers in the long run by 7 - 8 % and the traffic by 6 - 7% (assuming a constant population).

This increase has to be accounted for, so the quality of the service does not deteriorate.

The income elasticity (greater than 1) of the traffic shows that with the growth process expected in Portugal there will be a pressure on the installed network of telephone services and it is therefore important to have a short and long run planning of this sector.

The demand of telephone services does not seem to be significantly (10% level) influenced by the usage price. This can be the result of the aggregation process and micro data should be necessary to test it.

The one-time installation fee negatively influences the access demand while the monthly rental fee negatively influences

both the access demand and the usage demand.

The existence of network externalities showed in this paper is an incentive to allocate more time to the research of the optimal pricing policy for those services and to the problem of privatisation of the firms providing those services.

References

Cabral, Luis and Leite, Antonio (1989) "Network Consumption Externalities: The Case of Telex Service ", WP 135, Faculdade de Economia, Universidade Nova de Lisboa.

Curien, Nicolas (1987), " L' Acces et l' Usage Telephonique: Modelization Conjointe et Tarification Optimale ", Revue Economique, 2, 415 - 458.

Curien, Nicolas and Gensollen, Michel (1987), " Les Theories de la Demande de Raccordement Telephonie ", Revue Economique, 2, 203 - 255.

Littlechild, S. C. (1975), " Two Part Tariffs and Consumption Externalities ", Bell Journal of Economics, 6, 661 - 670.

Melo, Paulo (1986), " Procura de Telefones em Portugal ", WP 53, Faculdade de Economia, Universidade Nova de Lisboa.

Luz, Silvia (1990), " Estudo Microeconómico da Procura de Telefone: Análise do Impacto de uma Isenção Tarifária ", WP 150, Faculdade de Economia, Universidade Nova de Lisboa.

Taylor, Lester D. (1980), " Telecommunications Demand: A Survey and Critique, Cambridge, Massachusetts: Ballinger Publishing Company.

Taylor, Lester D. (1983), " Problems and Issues in Modeling Telecommunications Demand ", in L. Courveille & al. (ed), " Economic Analysis of Telecommunications ", North Holland, Amsterdam.

Varian, Hal R. (1984), " Microeconomic Analysis ", New York, W.W. Norton Company.

Varian, Hal R. (1987), " Intermediate Microeconomics ", New York, W.W. Norton Company.

Willig, Robert D. (1979), " The Theory of Network Access Pricing ", in Harry M. Trebing (ed.), "Issues in Public Utility Regulation ", Michigan State University, East Lansing.