DEPARTMENT OF
MATHEMATICS

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MASTER IN MATHEMATICS AND APPLICATIONS
SPECIALIZATION IN COMPUTATIONAL FINANCIAL MATHEMATICS

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*F*or my beloved brother, Manuel

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"Out of clutter, find simplicity. From discord, find harmony. In the middle of difficulty lies opportunity."

— Albert Einstein

Abstract

This dissertation investigates how the Shapley Value can contribute to a more rigorous analysis of portfolio performance, examining how companies with different levels of ESG (Environmental, Social, and Governance) risk influence portfolio outcomes. Traditional approaches, such as the Capital Asset Pricing Model, rely heavily on beta as a measure of systematic risk. However, beta has proven insufficient to capture the precise contribution of individual assets in a diversified portfolio. To address this limitation, cooperative game theory methods, in particular the Shapley Value, are applied to provide a more comprehensive decomposition of asset contributions.

The study proceeds in two main steps. First, the Shapley Value is used to replicate and extend an established framework, applying it to major European stock indices in order to demonstrate how assets contribute to overall portfolio risk along the efficient frontier.

Second, the focus shifts to Socially Responsible Investments. Using a data sample of 30 U.S. firms from 10 industrial sectors, grouped by different sustainability profiles according to Sustainalytics (2024) ESG risk ratings, three portfolios are constructed: sustainable, intermediate, and less sustainable. Performance is evaluated for both the Minimum Variance Portfolio and the Maximum Sharpe Ratio Portfolio, using two characteristic functions: the Sharpe ratio and Value at Risk.

The results reveal that the impact of ESG profiles on portfolio performance depends on market conditions. In more volatile markets, portfolios composed of firms with medium levels of sustainability show stronger risk-adjusted performance, while in calmer periods portfolios of more sustainable firms tend to outperform. The results show that the effect of ESG profiles on portfolio performance changes with market conditions, and that the Shapley Value offers a more precise and informative attribution than beta.

Resumo

Esta dissertação investiga de que forma o Shapley Value pode contribuir para uma análise mais rigorosa do desempenho de carteiras financeiras, procurando perceber de que forma empresas com diferentes níveis de risco ESG (Ambiental, Social e de Governação) influenciam o desempenho das carteiras. As abordagens tradicionais, como o Capital Asset Pricing Model, assentam fortemente no beta como medida de risco sistemático. No entanto, o beta tem-se mostrado insuficiente para captar a contribuição exata de cada ativo numa carteira diversificada. Para colmatar esta limitação, aplica-se a teoria dos jogos cooperativos, em particular o Shapley Value, que possibilita uma decomposição mais rigorosa das contribuições dos ativos.

O estudo desenvolve-se em duas secções principais. Numa primeira fase, o Shapley Value é utilizado para replicar um enquadramento já existente, aplicando-o a três grandes índices do mercado europeu, de modo a demonstrar como os ativos contribuem para o risco global de uma carteira ao longo da fronteira eficiente.

Numa segunda fase, esta dissertação foca-se nos Investimentos Socialmente Responsáveis. Com base numa amostra de 30 empresas norte-americanas de 10 setores industriais diferentes, as empresas são classificadas de acordo com o nível de risco ESG atribuído pela Sustainalytics (2024). A partir desta classificação, são formadas três carteiras: uma composta pelas empresas mais sustentáveis, outra pelas empresas de nível intermédio e uma terceira pelas empresas menos sustentáveis. Para avaliar o desempenho, utilizam-se duas funções características: o rácio de Sharpe e o Value at Risk, calculados para a Carteira de Variância Mínima e para a Carteira de Máximo Rácio de Sharpe.

Os resultados mostram que o impacto dos perfis ESG no desempenho das carteiras depende das condições do mercado. Em períodos de maior volatilidade, as carteiras compostas por empresas com níveis intermédios de sustentabilidade apresentam melhor rendibilidade ajustada ao risco, enquanto em contextos mais estáveis são as carteiras com empresas mais sustentáveis que tendem a obter melhores resultados. No geral, conclui-se que o impacto dos diferentes níveis de risco ESG varia consoante as condições do mercado e que o Shapley Value fornece uma atribuição mais precisa e informativa do que o beta.

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Acronyms

CAPM	Capital Asset Pricing Model
ESG	Environmental, Social and Governance
MSRP	Maximum Sharpe Ratio Portfolio
MVP	Minimum Variance Portfolio
SML	Security Market Line
SRI	Socially Responsible Investment
VaR	Value at Risk

1

Introduction

1.1 Motivation

In recent decades, the increasing awareness of financial risk, market volatility, and sustainability has stimulated significant developments in portfolio theory and investment practice. Traditional models such as the mean–variance optimization of Markowitz (1952) and the Capital Asset Pricing Model (Lintner 1965; Sharpe 1964) established the foundations of modern portfolio management, providing a framework to balance return and risk. However, these models rely heavily on beta as a measure of systematic risk, which has shown important limitations in capturing the precise contribution of individual assets within a diversified portfolio.

To overcome the limitations of traditional risk measures, recent approaches apply cooperative game theory. In particular, the Shapley value (Shapley 1953) offers a more precise decomposition of each asset's contribution to portfolio performance, capturing all possible asset combinations and providing a more complete perspective.

Meanwhile, Socially Responsible Investments (SRI) are becoming increasingly relevant, with Environmental, Social, and Governance (ESG) criteria integrated into portfolio decisions. Although many studies link ESG practices to lower risk and greater resilience, others point to diversification constraints, inconsistent ratings and the possibility that firms may exaggerate or misrepresent their sustainability practices to appear more responsible. This mixed evidence highlights the need for more rigorous methods to assess the role of ESG in portfolio performance.

Against this backdrop, this dissertation aims to combine the Shapley value with ESG-based portfolio construction in order to assess how different ESG profiles influence portfolio performance, namely whether portfolios composed of more sustainable firms deliver superior results compared to those with weaker ESG practices. By analyzing U.S. companies across multiple sectors and ESG profiles, this study examines whether using the Shapley value provides a clearer perspective than traditional risk measures. It also addresses how ESG levels affect portfolio results, especially under different market conditions.

1.2 Objectives and Contributions

This dissertation aims to contribute to the existing literature by assessing whether performance evaluation based on the Shapley Value provides a more comprehensive understanding of an asset's role in a portfolio than the traditional beta. In particular, it applies the Shapley Value to assess the contribution of socially responsible investments to portfolio performance by comparing two distinct market regimes: one characterized by high volatility (2022) and another by lower volatility (2023). To evaluate how sustainability profiles affect both performance and exposure to extreme losses, the analysis is based on two characteristic functions: the Sharpe ratio (Huang and Litzenberger 1988) and the Value at Risk (Jorion 2007). The results are examined under two types of portfolio weighting, the Minimum Variance Portfolio and the Maximum Sharpe ratio Portfolio (Huang and Litzenberger 1988).

Before examining the impact of socially responsible investments on portfolio performance, this dissertation first focuses on portfolio risk through an empirical analysis of each asset's contribution to the overall portfolio risk. In Chapter 4, we replicate a framework based on the Shapley Value (Shalit 2020, 2021) to decompose portfolio risk, applying it to major European indices. The results show that this methodology provides consistent and meaningful risk attributions, offering valuable insights into how assets contribute to portfolio risk along the efficient frontier.

In Chapter 5, this dissertation examines the impact of ESG ratings on portfolio performance using the Shapley Value. The analysis considers two characteristic functions, namely the Sharpe ratio and the Value at Risk, calculated on both the Minimum Variance Portfolio and the Maximum Sharpe Ratio Portfolio. To capture potential differences in the contributions, the results are evaluated separately for periods of high and low market volatility.

The empirical analysis covers 30 U.S. companies, distributed across 10 industrial sectors to ensure diversification and limit dependency patterns. The observation period spans from January 3rd, 2022 to December 29th, 2023. During this time, the S&P 500 index recovered, with 2022 showing higher volatility and 2023 comparatively lower volatility. We based our study on the ESG risk ratings sourced from Sustainalytics (2024), referring to the 12-month period between March 2023 and March 2024. For consistency, these ratings are assumed constant across both years. The ESG risk ratings from Sustainalytics (2024) measure a company's exposure to industry-specific ESG risks and its ability to manage them. A higher rating signals greater unmanaged risk and weaker sustainability practices, while a lower rating reflects stronger management of ESG issues and better alignment with responsible investing principles. From each of the 10 industrial sectors, three firms were selected to represent different levels of sustainability: one more sustainable, one less sustainable, and one with an intermediate level. This gives a total of 30 firms (10 sectors, with 3 firms per sector). For each year under analysis, three separate portfolios are then constructed: a portfolio of the more sustainable firms (10 firms, one per sector), a portfolio of the less sustainable firms (10 firms, one per sector), and a portfolio of firms with intermediate sustainability (10 firms, one per sector).

The results indicate that the impacts of ESG ratings on portfolio performance changes with market volatility. When markets are highly volatile, portfolios with intermediate sustainable firms achieve stronger risk-adjusted returns, showing an unexpected resilience. On the other hand, in calmer periods, portfolios with more sustainable firms tend to perform better. This suggests that socially responsible investments influence portfolio outcomes differently depending on market conditions.

1.3 Thesis Structure

This dissertation is organized as follows. Chapter 1 discusses the motivation behind this study, sets out the research objectives, and highlights the main results. Chapter 2 reviews the relevant literature, which serves as the basis for the subsequent analysis. Chapter 3 introduces the key concepts used throughout the study and provides a real-world example of the application of the Shapley Value. Chapter 4 illustrates the application of the Shapley Value to financial markets by replicating the example presented in Shalit (2020, 2021) with European indices. Finally, Chapter 5 develops the theme of Socially Responsible Investments (SRI) and presents the main empirical study of this dissertation, which examines the impact of ESG risk ratings on portfolio performance.

2

Literature Review

The mean-variance optimization framework, developed by Markowitz (1952), became the foundation of modern financial mathematics. Assuming that investors are averse to risk and that their decisions are driven solely by the expected return and variance of their investments, this theory states that investors choose efficient mean-variance portfolios. In other words, they select portfolios that minimize variance (risk) for a given expected return and maximize expected return for a given level of variance.

Later, Sharpe (1964) and Lintner (1965) introduced the Capital Asset Pricing Model (CAPM) to examine the relationship between expected return and systematic risk, which refers to the type of risk that affects the whole market and cannot be eliminated through diversification. In this model, an asset's exposure to systematic market risk is measured by its beta, and the risk premium for each asset is derived accordingly. Additionally, CAPM introduced the perspective of an investor whose investment choices are based on excess return (earned by investing in a risky asset rather than a risk-free asset) instead of absolute return. This means that investors seek portfolios that maximize the expected excess return per unit of risk. This framework fundamentally shaped modern portfolio theory, emphasizing the trade-off between expected excess risk and return, captured by Sharpe ratio. Building upon this, asset pricing models introduced systematic risk as a key factor in determining expected returns, with beta emerging as a widely used measure of market exposure. Since the market portfolio has, by definition, a beta equal to 1, assets with $\beta_i > 1$ are interpreted as riskier than the market, while assets with $0 < \beta_i < 1$ are considered less risky. The further an asset's beta deviates from 1, the greater the associated risk exposure. A negative beta indicates that the asset tends to move in the opposite direction of the market.

However, despite its prominence, beta presents significant limitations, particularly in accurately assessing an asset's true contribution to portfolio risk. These limitations have led to the exploration of alternative methodologies that aim to provide a more robust assessment of an asset's impact on overall portfolio risk. Shalit (2020, 2021) proposed the Shapley Value (Shapley 1953) as an alternative to beta in portfolio optimization. Interpreting portfolios as cooperative games among assets, the Shapley Value measures each asset's exact contribution to overall risk by averaging its marginal impact across all possible coalitions. This approach provides a more precise decomposition of portfolio risk and clarifies how adding or removing assets affects the total risk of the portfolio.

Socially Responsible Investments (SRI) play a crucial role in fostering a sustainable and inclusive economy. SRI integrate social, environmental, and ethical criteria into the investment selection and analysis process. By combining financial performance with the ethical and social values of market participants, SRI serve as a central driver of sustainable economic growth. Financial market companies have progressively adopted Environmental, Social, and Governance (ESG) principles. ESG practices have shown potential to mitigate corporate risk, particularly in the financial sector (Renneboog et al. 2008).

Lee and Koh (2024) argued that firms with strong ESG strategies exhibit lower beta values, suggesting reduced systematic risk. ESG initiatives help reduce the financial risks that come from taking on large amounts of debt, which has encouraged more financial firms to adopt them and has provided investors with a more stable investment option. In line with these risk-reducing effects, Morelli (2024) proposed a risk management approach that integrates ethical concerns into portfolio selection by minimizing Conditional Value at Risk (CVaR) while constraining environmental scores. The results showed that environmentally irresponsible firms contribute more to extreme market risk, while sustainable portfolios proved both ethically sound and financially efficient.

Albuquerque et al. (2020) demonstrated that the resilience of firms with strong ESG practices is also evident during market crises. Particularly, companies with strong environmental and social (ES) ratings performed better during the COVID-19 crash, benefiting from customer loyalty and ESG-driven institutional investment. These factors contributed to higher returns and lower volatility, reinforcing ESG as a key component of financial stability in times of stress. From a different angle, Engle et al. (2020) addressed climate risk by developing hedge portfolios responsive to climate-related news. Using a climate news index based on media coverage, they showed that ESG-aligned portfolios outperform conventional industry risk-mitigation strategies. This highlights ESG investing as a practical tool for managing risks related to climate change and sustainability. Moreover, Baselli (2024) showed that companies with stronger ESG practices tend to outperform over the long term and are more resilient during crises, particularly in stable sectors like healthcare and utilities. These portfolios not only achieve better performance but also offer superior downside protection.

While the popularity of SRI continues to rise, its effectiveness is still questioned. Recent studies offer mixed evidence on the performance of ESG investments. According to Sourd (2024), ESG constraints may limit diversification and lower Sharpe ratios. Although ESG portfolios sometimes reduce volatility during crises, these effects are often attributed to sector or factor exposures rather than ESG characteristics themselves. Nevertheless, the authors suggest that investing in companies that are improving their ESG scores can be a promising strategy. Morgan Stanley Investment Management, Emerging Markets Equity Team (2023) suggested that companies with mid-range ESG performance (neither strongest nor weakest) may offer attractive opportunities, especially where ESG integration is still developing. These firms may benefit investors as they improve their practices and move up the sustainability trajectory. In addition, Vu et al. (2025) found no consistent evidence that high ESG-rated firms outperform lower-rated ones across developed markets. Their results suggest that ESG ratings are already reflected in prices, which limits the potential for consistent outperformance. However, they also showed that unexpected increases in global attention to sustainability can temporarily boost the returns of firms with strong governance and environmental profiles, highlighting the importance of context.

The sensitivity of ESG assets to market conditions is also visible during crises. Döttling and Kim (2024) showed that retail investors (individual investors) significantly reduced their holdings in sustainable funds during the COVID-19 crisis, while institutional investors (such as pension funds or asset managers) maintained their positions. This suggests that retail demand for ESG is more fragile and closely tied to income shocks. In a similar line, Perote et al. (2023) reported that sustainable investments and green bonds responded positively under moderate pandemic stress, but their performance weakened when the shock intensified. This points to a nonlinear relationship, where ESG assets appear resilient in mild downturns but vulnerable in severe crises.

Gao et al. (2025) emphasized that not all ESG practices are equally effective. Their study founds that only robust and strategic ESG integration, where firms include sustainability in their core management processes, leads to greater opportunities for gains and lower downside risk. By contrast, superficial or symbolic ESG commitments do not provide the same financial benefits. This highlights that the quality of ESG implementation is crucial for delivering long-term value.

Other studies, however, suggest that investors may not always be motivated by financial outcomes. Ciciretti et al. (2023) reported that firms with higher ESG scores have lower expected returns, as investors are willing to sacrifice performance in favor of sustainability. Building on this distinction between financial and ethical motives, Kräussl et al. (2024) pointed out that ESG investing can be driven by either financial goals or social responsibility, with concerns such as misleading sustainability claims, often referred to as greenwashing. Extending this discussion, Lopez-de-Silanes et al. (2024) highlighted the implications for investor behavior. They argued that institutional investors often include highly rated ESG firms in their portfolios, but the size of these holdings is typically small, which suggests that the allocations are more symbolic than substantial, possibly driven by reputation concerns. The study also warned that the pressure to appear sustainable can increase the risk of greenwashing, where firms exaggerate or misrepresent their ESG achievements, making it harder for investors to judge the true value of these companies.

Beyond motivations, a further challenge lies in the measurement of ESG itself, as inconsistent ratings create uncertainty for both firms and investors. Avramov et al. (2022) showed that ESG uncertainty increases perceived risk and weakens investor demand, particularly for firms with very high or very low ESG scores. When ESG ratings are unreliable, the typical outperformance of firms with stronger ESG profiles disappears and it is precisely these firms that are then perceived as riskier.

Berg et al. (2022) argued that most of the disagreement between rating agencies comes from the way ESG attributes are measured, while differences in scope and weighting matter less. This lack of alignment makes it harder for investors to assess company performance, discourages firms from improving their practices and reduces confidence in ESG-based analysis. In a related study, Bissoondoyal-Bheenick et al. (2024) compared three major providers and found clear contradictions in their ESG scores, with correlations varying strongly across agencies. They also showed that European markets tend to display lower divergence than other regions. The authors argue that using multiple rating sources and applying aggregation methods can help reduce this variation and provide more reliable information for financial decisions.

A more granular view of ESG attribution is offered by Wijnen et al. (2023), who applied the Shapley Value to decompose portfolio returns into the contributions of financial strategy and non-financial choices. These non-financial elements include excluding controversial sectors such as energy or weapons, giving greater weight to companies with stronger ESG characteristics, and adjusting the portfolio to meet carbon reduction goals. This framework also captures the interaction between ESG constraints and active strategies like stock selection, offering clearer insights into how sustainability policies influence excess return. Their results showed that while the financial strategy contributed positively to performance, exclusions and carbon targets reduced returns, whereas favoring companies with stronger ESG profiles had only a marginal positive effect.

3

Methodology

3.1 Introduction

This chapter outlines the methodological foundations that guide the empirical analysis of this dissertation. The aim is to present the main theoretical models and risk–return measures that will be employed to evaluate portfolio performance and to assess the role of ESG risk ratings. The chapter begins with the CAPM, which establishes the relationship between expected return and systematic risk, and then introduces the concepts of efficient portfolios and the MVP. Performance metrics such as the Sharpe Ratio and Value at Risk (VaR) are subsequently presented, offering complementary perspectives on portfolio efficiency and downside risk. The chapter concludes with the framework of cooperative game theory and the Shapley Value, which provide a more comprehensive approach to decomposing the contribution of each asset within a portfolio. To illustrate its practical relevance, this chapter also presents a numerical application of the Shapley Value.

3.2 Capital Asset Pricing Model

The CAPM is a financial model that establishes a relationship between the expected return of an investment and its associated risk. In this model, an investor selects a portfolio at time $t-1$ that produces a return at time t. The model assumes that investors are averse to risk and, when choosing between portfolios, they are concerned only with the mean and variance of the investment return over a given period. As a result, investors select portfolios that minimize the variance of the portfolio return for a given expected return or portfolios that maximize the expected return for a given variance.

The CAPM is based on the relationship between the market beta of the asset ($\beta_{i,m}$), which quantifies the asset's sensitivity to fluctuations in the overall market return; the risk-free rate (R_f), which represents the expected return of an investment with no risk; and the market risk premium ($R_m - R_f$), which is the expected return of the market portfolio minus the risk-free rate, reflecting the additional return that investors expect to receive for investing in a portfolio with associated risk. Widely applied in the financial sector for pricing risky assets and estimating expected returns, this model allows investors to measure the systematic risk associated with investments. According to CAPM, the expected return of an investment in asset i, denoted as $E(R_i)$, can be calculated using the following formula:

$$E(R_i) = R_f + \beta_{i,m}(R_m - R_f) \tag{3.1}$$

where $\beta_{i,m}$ measures the asset's sensitivity to market movements, capturing the systematic risk that cannot be eliminated through diversification. It is defined as the covariance between the returns of asset i and the market portfolio, divided by the variance of the market returns:

$$\beta_{i,m} = \frac{Cov(R_i, R_m)}{Var(R_m)} \tag{3.2}$$

Figure 3.1 illustrates the graphical representation of this model, with portfolio risk (represented by $\beta_{i,m}$) plotted on the x-axis and expected return on the y-axis.

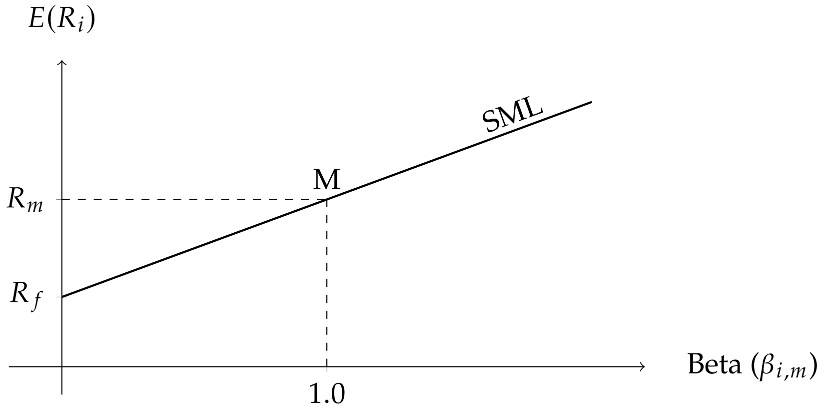


Figure 3.1: Market Portfolio and Security Market Line

The SML line, known as the Security Market Line, represents the relationship between an asset's expected return and its market risk ($\beta_{i,m}$) in the context of the CAPM. The SML plots different levels of systematic risk (on the x-axis) against expected return (on the y-axis), illustrating the required return that compensates investors for taking on the risk associated with the market, also referred to as the market risk premium. According to this model, any asset positioned above the SML is considered undervalued, as it provides an excessive return for the given level of risk. On the other hand, assets below the SML are overvalued, offering insufficient returns for the amount of risk involved. The SML is given by Equation (3.1), previously presented, which shows the relationship between expected return and systematic risk. The market portfolio, represented by point M in Figure 3.1, with $\beta_{i,m} = 1$, is a theoretical portfolio that includes all assets in the market, weighted according to their proportion in the total market. This means that the expected return of the market portfolio is the same as the expected return of the market as a whole.

Furthermore, since this is a fully diversified portfolio, unsystematic risk is eliminated, leaving only systematic risk (which can be measured by beta) as the primary focus of the model.

Consider a financial market with n assets, where the returns are denoted by $r_1, r_2, \ldots, r_n$. A portfolio can be represented as the vector $p = (x_1, x_2, \ldots, x_n)$, with the constraint $x_1 + x_2 + \cdots + x_n = 1$, where x_i ($i = 1, \ldots, n$) represents the proportion of wealth invested in asset i. The return of the portfolio, in this case, is a random variable:

$$r_p = x_1 r_1 + x_2 r_2 + \ldots + x_n r_n \tag{3.3}$$

Let $E[r_i]$ represent the expected return of asset i, σ_i^2 its variance, and σ_{ij} the covariance between assets i and j, where $i = 1, \ldots, n$ and $j = 1, \ldots, n$. The expected return of portfolio p, denoted as $E[r_p]$, is the weighted sum of the expected returns of the individual assets:

$$E[r_p] = x_1 E[r_1] + x_2 E[r_2] + \ldots + x_n E[r_n] = x_1\mu_1 + x_2\mu_2 + \cdots + x_n\mu_n = x'\mu \tag{3.4}$$

The variance of portfolio p, denoted as σ_p^2, is given by:

$$\sigma_p^2 = \sum_{i=1}^{n} x_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} x_i x_j \sigma_{ij} = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = x'Vx \tag{3.5}$$

where V is the covariance matrix.

Using Equation (3.5), we can determine the contribution of each asset to the total portfolio risk by rearranging the terms as follows:

$$\begin{aligned} \sigma_p^2 &= \sum_{i=1}^{n} x_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} x_i x_j \sigma_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \\ &= (x_1^2\sigma_1^2 + \cdots + x_1 x_i \sigma_{1i} + \cdots + x_1 x_n \sigma_{1n}) \\ &+ \cdots + (\boldsymbol{x_i x_1 \sigma_{i1}} + \cdots + \boldsymbol{x_i^2 \sigma_i^2} + \cdots + \boldsymbol{x_i x_n \sigma_{in}}) + \cdots + \\ &(x_n x_1 \sigma_{n1} + \cdots + x_n x_i \sigma_{ni} + \cdots + x_n^2\sigma_n^2) \\ &= x_1\sigma_{1p} + \cdots + \boldsymbol{x_i \sigma_{ip}} + \cdots + x_n \sigma_{np} \end{aligned} \tag{3.6}$$

To obtain the proportion of risk attributed to asset i, we divide its absolute contribution to the total risk ($x_i\sigma_{ip}$) by the total portfolio risk (variance).

$$\varphi_i = x_i \frac{\sigma_{ip}}{\sigma_p^2} = x_i \beta_i. \tag{3.7}$$

The parameter β_i represents the relationship between the return of asset i and the return of the portfolio p, indicating how sensitive asset i is to the portfolio's performance. The parameter φ_i expresses how much asset i contributes to the total portfolio risk in percentage terms. Although widely used, this parameters have proven to be incomplete and insufficient in capturing the true impact of an asset within a portfolio. By analyzing Equation (3.6), which decomposes portfolio variance, it becomes evident that the portion attributed to each asset does not fully account for all of its interactions within the portfolio. For example, the bold expressions related to asset i show that the term $(x_i \sigma_{ip})$ does not capture all the interactions of asset i in terms of risk. Instead, there are also interactions between the portfolio p and asset i, which appear in the terms measuring the contributions of the other assets to the portfolio $(\sigma_{jp}, j \neq i)$.

3.3 Efficient Portfolios

The ability to explicitly recognize and quantify risk through variance plays a fundamental role in portfolio construction. In this context, a portfolio is considered efficient when it optimally balances risk and return. Specifically, an efficient portfolio minimizes variance for a given expected return while also maximizing expected return for a fixed level of risk. Under the assumption that return and variance are the only determinants of a portfolio's performance, efficiency implies that no alternative portfolio offers a higher expected return for the same level of risk. Likewise, if expected return is held constant, an efficient portfolio is the one with the lowest possible variance.

The efficient frontier, as its name suggests, is a curve that contains all efficient portfolios. Portfolios lying on this frontier represent asset combinations for which it is impossible to obtain a higher return without taking on additional risk, or to reduce risk without giving up return. The efficient frontier is a subset of the portfolios that solve the optimization problem (3.8).

$$\min_{x_1, x_2, \ldots, x_n} \quad \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} x_i x_j \sigma_{ij} \tag{3.8}$$

subject to:

$$\sum_{i=1}^{n} x_i = 1,$$

$$\sum_{i=1}^{n} x_i E(r_i) = \mu_p.$$

The portfolios that solve problem (3.8) form a hyperbola in the risk-return space, as illustrated in Figure 3.2.

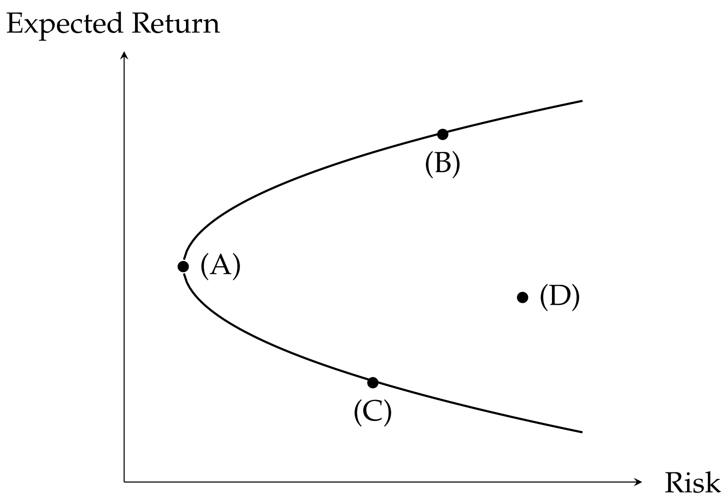


Figure 3.2: Illustration of the Solution to Problem (3.8)

Notice that portfolios C and D are not efficient. At their given risk levels, it is possible to find portfolios with higher expected returns. Likewise, if we fix their expected returns, there are always alternatives with lower risk. In contrast, portfolios A and B lie on the efficient frontier. For their respective levels of risk, they provide the highest possible expected return. Portfolio A, which is the vertex of the hyperbola, is not only efficient but also corresponds to the portfolio with the minimum achievable risk, which is the MVP.

Thus, the efficient frontier is formed by all portfolios located on the upper part of the hyperbola. Starting from vertex A, every portfolio combines the lowest possible risk for a given target return with the highest achievable return for a given risk level.

3.4 Minimum Variance Portfolio

In portfolio theory, the MVP plays a crucial role in risk management, as it represents the portfolio that achieves the lowest possible risk for a given set of assets. Constructing such portfolio is particularly relevant when investors seek to minimize uncertainty while targeting a specific level of return. Therefore, determining the portfolio with the lowest variance for a given expected return is a key optimization problem in investment analysis. For a given return μ_p, the optimal portfolio x that minimizes risk, with risk defined as variance, can be determined by solving the optimization problem (3.9).

$$\min_{x_1, x_2, \ldots, x_n} \quad x^T V x \tag{3.9}$$

subject to

$$x^T \mu = \mu_p,$$
$$x^T \mathbf{1} = 1.$$

where x is the vector of portfolio weights, V is the variance–covariance matrix of asset returns, $\mathbf{1}$ is a vector of ones, $\mu$is the vector of expected asset returns, and μ_p is the target expected return of the portfolio.

This problem can be solved with Lagrange multipliers. The variance of the portfolio is originally written as $x^T V x$, as proved in Equation (3.5). However, in quadratic programming it is common to use the equivalent form $\frac{1}{2}x^T V x$, since the off-diagonal covariance terms appear twice in $x^T V x$. This formulation has the advantage that its derivative with respect to x is simply Vx. With this in mind, by applying the Lagrangian function (3.10), we obtain the following first-order conditions:

$$\min_{x,\lambda,\tau} \quad L = \frac{1}{2}x^T V x + \lambda(\mu_p - x^T\mu) + \tau(1 - x^T\mathbf{1}) \tag{3.10}$$

$$\frac{\partial L}{\partial x} = Vx - \lambda\mu - \tau\mathbf{1} = 0 \tag{3.11a}$$

$$\frac{\partial L}{\partial \lambda} = \mu_p - x^T\mu = 0 \tag{3.11b}$$

$$\frac{\partial L}{\partial \tau} = 1 - x^T\mathbf{1} = 0 \tag{3.11c}$$

Solving equation (3.11a) for x, the result is

$$x = \lambda(V^{-1}\mu) + \tau(V^{-1}\mathbf{1}). \tag{3.12}$$

Multiplying both sides of (3.12) by μ^T gives

$$x\mu^T = \lambda(\mu^T V^{-1}\mu) + \tau(\mu^T V^{-1}\mathbf{1}) \Leftrightarrow$$

$$\mu_p = \lambda(\mu^T V^{-1}\mu) + \tau(\mu^T V^{-1}\mathbf{1}). \tag{3.13}$$

Similarly, multiplying (3.12) by $\mathbf{1}^T$ yields

$$x\mathbf{1}^T = \lambda(\mathbf{1}^T V^{-1}\mu) + \tau(\mathbf{1}^T V^{-1}\mathbf{1}) \Leftrightarrow$$

$$1 = \lambda(\mathbf{1}^T V^{-1}\mu) + \tau(\mathbf{1}^T V^{-1}\mathbf{1}). \tag{3.14}$$

From (3.13) and (3.14), it follows that λ and τ can be written as

$$\lambda = \frac{\mu_p - \tau(\mu^T V^{-1}\mathbf{1})}{\mu^T V^{-1}\mu}, \tag{3.15}$$

$$\tau = \frac{1 - \lambda(\mathbf{1}^T V^{-1}\mu)}{\mathbf{1}^T V^{-1}\mathbf{1}}. \tag{3.16}$$

Solving this system leads to explicit expressions for the multipliers:

$$\lambda = \frac{\mu_p(\mathbf{1}^T V^{-1}\mathbf{1}) - (\mu^T V^{-1}\mathbf{1})}{(\mathbf{1}^T V^{-1}\mathbf{1})(\mu^T V^{-1}\mu) - (\mu^T V^{-1}\mathbf{1})(\mathbf{1}^T V^{-1}\mu)} \Leftrightarrow \lambda = \frac{C\mu_p - A}{D}, \tag{3.17}$$

$$\tau = \frac{(\mu^T V^{-1}\mu) - (\mathbf{1}^T V^{-1}\mu)\mu_p}{(\mu^T V^{-1}\mu)(\mathbf{1}^T V^{-1}\mathbf{1}) - (\mu^T V^{-1}\mathbf{1})(\mathbf{1}^T V^{-1}\mu)} \Leftrightarrow \tau = \frac{B - A\mu_p}{D}. \tag{3.18}$$

where

$$A = \mathbf{1}^T V^{-1}\mu, \quad B = \mu^T V^{-1}\mu, \quad C = \mathbf{1}^T V^{-1}\mathbf{1}, \quad D = BC - A^2.$$

Finally, substituting λ and τ into (3.12) gives the expression for the optimal weights of the minimum variance portfolio with target return μ_p:

$$x^* = \lambda(V^{-1}\mu) + \tau(V^{-1}\mathbf{1}) \Leftrightarrow x^* = \frac{C\mu_p - A}{D}(V^{-1}\mu) + \frac{B - A\mu_p}{D}(V^{-1}\mathbf{1}). \tag{3.19}$$

Consequently, the portfolio x^* will have an expected return $E[r_p]$ and variance $Var[r_p]$, given by Equations (3.20) and (3.21).

$$E[r_p] = \mu_p = x^*\mu \tag{3.20}$$

$$Var[r_p] = \sigma_p^2 = x^{*'} V x^* = \frac{C}{D}\left(\mu_p - \frac{A}{C}\right)^2 + \frac{1}{C}. \tag{3.21}$$

The expected return and variance of the MVP, located at the vertex of the hyperbola, are directly determined by the following expressions:

$$E[r_{MVP}] = \frac{A}{C} \tag{3.22}$$

$$Var[r_{MVP}] = \frac{1}{C} \tag{3.23}$$

Therefore, the efficient frontier consists of all portfolios whose expected return satisfies the condition $E[r_p] > \frac{A}{C}$.

3.5 Sharpe Ratio

The Sharpe ratio is one of the most widely used tools for evaluating the performance of investment portfolios. This ratio measures the risk premium of a portfolio per unit of risk. In other words, it quantifies the additional return obtained by investing in a risky asset instead of a risk-free asset, considering the unit of risk assumed. Thus, a higher Sharpe ratio indicates better portfolio performance in terms of the risk-return balance. This index establishes the relationship between the portfolio return (R_p), its volatility or standard deviation (σ_p), and the risk-free rate (R_f) as expressed in Equation (3.24).

$$SR = \frac{R_p - R_f}{\sigma_p} \tag{3.24}$$

The portfolio with the maximum Sharpe ratio, commonly referred to as the tangency portfolio, will be denoted in this thesis as the Maximum Sharpe Ratio Portfolio (MSRP). It can be determined using Equation (3.25), where V^{-1} denotes the inverse of the variance-covariance matrix of asset returns, μ represents the expected returns of the individual assets, r_f denotes the risk-free rate, $A = (\mathbf{1}^T V^{-1}\mu)$, and $C = (\mathbf{1}^T V^{-1}\mathbf{1})$.

$$x^T = \frac{V^{-1}(\mu - r_f\mathbf{1})}{A - Cr_f}. \tag{3.25}$$

3.6 Value at Risk

Value at Risk is a widely used statistical measure that summarizes the potential loss in value of a portfolio over a given time horizon, for a specific confidence level. In essence, it estimates the maximum expected loss that is unlikely to be exceeded within a defined probability. Losses beyond the VaR threshold occur with a small, predefined probability and are considered extreme events.

Let R be the portfolio return over the chosen time horizon, and let R^* be a threshold return that marks unusually adverse outcomes. We define a small probability α as the chance that the return is at or below this threshold:

$$\alpha = P(R \leq R^*) \tag{3.26}$$

This means that R^* is a low return that is only reached in a small fraction of cases. For example, with a 95% confidence level, we usually set $\alpha = 5\%$, meaning that only about 5% of the time the return would be equal to or worse than R^*. In practice, VaR is the loss level linked to this threshold: most of the time losses should be smaller than the VaR, but in a small number of cases they can be larger.

In this study, VaR will be calculated using the historical simulation method, which relies on actual past return data to estimate the empirical distribution of potential losses. The process consists of ordering the historical returns and then identifying the return value at the position corresponding to the chosen confidence level. For a 95% confidence level, this means selecting the return at the 5th percentile of the ordered distribution. If the required position lies between two observations, the average of the two is taken to obtain a more precise estimate.

3.7 Game Theory

Game theory is a field that studies decision-making in various scenarios involving multiple players, where the outcome for each player depends not only on their own choices but also on the choices of others. Cooperative games are a branch of game theory in which players work together to achieve a common goal rather than competing against each other. In this type of game, participants form coalitions and seek to maximize collective benefits, with the intent of sharing the gains obtained through cooperation.

Additionally, players make their decisions simultaneously and independently. While they may communicate and reach informal agreements before the game begins, it is assumed that they lack the means to establish binding commitments. The central question in cooperative game theory revolves around determining the fair allocation of rewards to each player, considering the final outcome of the cooperation. In a financial context, this theory can be applied to assess how different assets contribute to the overall performance of a portfolio.

3.7.1 Shapley Value

Cooperative game theory introduces the fundamental concept of the Shapley Value, which seeks to answer the question: how should a given amount of profits (or costs) be allocated among a specific set of participants? This concept determines the fair contribution of each player to the total value generated by a coalition and assigns each participant a proportional share of the total benefit, considering all possible orders in which players can be added to the coalition.

In the context of a financial portfolio, the Shapley Value provides an effective method for decomposing overall portfolio performance, attributing to each asset a proportion of the final result, based on the interactions it has with the other assets. Compared to other metrics, such as β_i presented in the CAPM, this method stands out as a more robust and fair indicator, as it considers all possible interactions among assets and distributes the total value proportionally to the impact of each asset. Thus, the Shapley Value not only helps to understand how each asset contributes to the portfolio's performance but also provides a key tool for efficient portfolio management, ensuring that each asset is rewarded according to its true value in the portfolio composition.

The computation of the Shapley Value is based on the average marginal contributions of each player to the total benefit, meaning the difference each asset makes when joining the coalition. To determine the marginal contributions and Shapley Value of each asset within a portfolio, it is necessary to define the characteristic function for each participant. The characteristic function is a central concept in cooperative game theory, used to describe the total value that any group of players, known as a coalition, can achieve through cooperation. We denote by $v(S)$ the function that quantifies the total benefit that group S can generate collectively.

A cooperative game is defined by a set of players N and a characteristic function $v : 2^N \to \mathbb{R}$, where $v(S)$ is interpreted as the value of the coalition $S \subseteq N$. The Shapley Value specifies a fair allocation of the total value of the grand coalition, $v(N)$, among all participating players. This allocation is determined by averaging each player's marginal contribution across all possible permutations of coalition formation. The Shapley Value is formally defined by the Equation (3.27):

$$Sh_k(N, v) = \sum_{S \subseteq N} \frac{(n-s)!(n-1)!}{n!} \left[v(S) - v(S \setminus \{k\}) \right] \quad \forall k \in N \tag{3.27}$$

Since the value of the grand coalition is decomposed into the Shapley Values of each player, the sum of these values must satisfy Equation (3.28).

$$v(N) = \sum_{k=0}^{n} Sh_k(N, v). \tag{3.28}$$

Consider ϕ, a function that assigns to each possible characteristic function v of a game with n players an n-tuple $\phi(v) = (\phi_1(v), \phi_2(v), \ldots, \phi_n(v))$ of real numbers. Here, $\phi_i(v)$ represents the reward to which player i is entitled. Using this function, we can enumerate the four axioms of the Shapley Value:

1. **Efficiency:** $\sum_{i \in N} \phi_i(v) = v(N)$, meaning the total value created by the coalition is fully distributed among all players.
2. **Symmetry:** If i and j are such that $v(S \cup \{i\}) = v(S \cup \{j\})$, then $\phi_i(v) = \phi_j(v)$. In other words, if the characteristic function assigns the same value to i and j, then their rewards must be equal. This guarantees that players who contribute equally receive the same reward.
3. **Null Player:** If i is such that $v(S) = v(S \cup \{i\})$, then $\phi_i(v) = 0$, meaning that any player who adds no value to any coalition should receive nothing.
4. **Additivity:** If u and v are characteristic functions, then $\phi(u + v) = \phi(u) + \phi(v)$, meaning that the value a player receives in two combined games equals the sum of their rewards in each game separately.

3.8 A Numerical Example of Shapley Value Application

For a clearer understanding, consider an example of the Shapley Value in a context outside financial markets. Suppose store X conducts all its sales in person, and at the time of each purchase customers are invited to complete a short satisfaction survey. In this survey, the customer rates the quality of the service received, while the employees record how many colleagues were present in the store during the interaction, since more than one staff member may attend the same customer. The highest satisfaction score is observed when all three employees are working together in the store, reaching a total of 300 satisfaction points.

The store owner intends to reward the employees with a monetary bonus based on the high satisfaction score achieved. This bonus should be distributed fairly, reflecting the contribution of each employee to the total of 300 points. To properly formulate the problem, suppose that Ana (A), Bernardo (B), and Carlos (C) are the only employees of the store. Furthermore, assume that the entire bonus is to be allocated exclusively among the three of them.

According to the survey, when working alone in the store, Ana obtained 0 satisfaction points, Bernardo obtained 30, and Carlos obtained 60. When Ana and Bernardo worked together, they reached 150 points; Ana and Carlos together achieved 120 points; and the combination of Bernardo and Carlos resulted in 160 points. In this three-player game, the characteristic values are therefore defined for each individual and for every possible coalition, as summarized in Table 3.1.

Table 3.1: Characteristic Functions of Each Player and Coalition

Characteristic Functions
$v(A) = 0$
$v(B) = 30$
$v(C) = 60$
$v(AB) = 150$
$v(AC) = 120$
$v(CB) = 160$
$v(ABC) = 300$

To determine the fair share of the prize for each player, it is necessary to compute their marginal contributions. As an example, Ana's marginal contributions are presented in Table 3.2.

Table 3.2: Marginal Contribution of Player Ana

Entry Order	**Ana's Marginal Contribution**
ABC	$v(\{A\}) - v(\emptyset) = 0 - 0 = 0$
ACB	$v(\{A\}) - v(\emptyset) = 0 - 0 = 0$
BAC	$v(\{A,B\}) - v(\{B\}) = 150 - 30 = 120$
BCA	$v(\{A,B,C\}) - v(\{B,C\}) = 300 - 180 = 120$
CAB	$v(\{A,C\}) - v(\{C\}) = 120 - 60 = 60$
CBA	$v(\{A,B,C\}) - v(\{C,B\}) = 300 - 180 = 120$

First, it is necessary to consider the possible orders in which the workers join the coalition. In this case, there are 2^3 possible combinations of contributions. For example, in the first row of Table 3.2, Ana is the first to arrive at the store, so all sales she generates are evaluated according to her own characteristic function. On the other hand, in the last row, Ana is the last to arrive; by that point, Bernardo and Carlos have already achieved a satisfaction score without her, and Ana's marginal contribution is therefore given by the characteristic value of the three together minus the value obtained by Bernardo and Carlos alone.

Following the same procedure for all three players and averaging their marginal contributions, the corresponding Shapley Values are obtained. Table 3.3 summarizes the results, showing that, out of the 300 satisfaction points, 30 are attributed to Ana, 115 to Bernardo, and 115 to Carlos. In terms of the monetary prize, regardless of its total value, Ana should receive 23.3%, while the remaining 76.7% should be equally divided between Bernardo and Carlos.

Table 3.3: Marginal Contributions and Shapley Value

Entry Order	**MC of Player A**	**MC of Player B**	**MC of Player C**
ABC	0	150	150
ACB	0	180	120
BAC	120	30	150
BCA	120	30	150
CAB	60	180	60
CBA	120	120	60
Sh_k	70	115	115
Sh_k (%)	**23.3**	**38.3**	**38.3**

4

The Shapley Value as a Risk Measure

4.1 Introduction

Shalit (2020, 2021) proposed the use of the Shapley Value as an alternative to the traditional beta for evaluating systematic risk in optimal portfolios. By interpreting portfolio construction as a cooperative game played by assets, he argued that the Shapley Value provides a more precise decomposition of each asset's contribution to overall portfolio risk. In his 2020 paper, based on U.S. stocks and indices between 2016 and 2019, Shalit claimed that the Shapley Value captures the a more accurate marginal impact of assets across all possible coalitions, thereby overcoming several of the limitations associated with market beta which only looks at how an asset moves with the market. In his 2021 paper, Shalit used historical data from major U.S. asset classes to explore how individual assets contribute to portfolio risk. Instead of comparing each asset's Shapley Value to the market beta, he focused on the asset's own beta which measures how much the asset contributes to the total risk of the portfolio. The results showed that the Shapley Value may tell a very different story. Some assets that appear risky when judged by their individual beta actually help reduce overall portfolio risk, while others that seem safe may increase it. This suggests that relying solely on an asset's beta can be misleading when assessing its true role in a diversified portfolio. Shalit illustrated these two approaches using both the minimum variance portfolio and mean–variance efficient portfolios, showing that the Shapley Value provides a clear breakdown of each asset's role in portfolio risk and how these roles shift along the efficient frontier. This suggests that relying solely on an asset's beta can be misleading when assessing its true impact in a diversified portfolio.

In this chapter, Shalit (2020, 2021) framework is replicated by applying his methodology in a European context. While his empirical work focused on U.S. indices, we apply the Shapley Value to three major European indices, using their historical returns to construct optimal portfolios and decompose risk contributions. As in the 2020 paper, we begin with the construction of the MVP and proceed to evaluate efficient portfolios along the frontier, computing the Shapley Values of the constituent indices in each case. As in the 2021 paper, we compare the results using each asset's own beta. Since the analysis is entirely focused on risk decomposition, the beta of the individual asset offers a more meaningful point of comparison than the market beta.

This exercise not only confirms the robustness of the Shapley methodology across markets but also provides an important comparative perspective, laying the groundwork for the subsequent analysis of ESG-based portfolios in Chapter 5. In addition, replicating Shalit's framework with European indices provided a deeper understanding of the mechanics of the Shapley Value and its practical application in portfolio analysis, thereby consolidating the theoretical foundations presented in the earlier chapters. All calculations related to this chapter were carried out using the R programming language and can be viewed at the following GitHub link: The Shapley Value as a Risk Measure — GitHub.

4.2 Data

For the European application, the investment portfolio was built using three major benchmark indices: the STOXX Europe 50 (STOXX50), which tracks the performance of 50 leading companies across European markets; the DAX 40 (GDAXI), which reflects the performance of the 40 largest and most liquid blue-chip stocks listed on the Frankfurt Stock Exchange; and the FTSE 100 (FTSE), which represents the 100 most significant companies traded on the London Stock Exchange. These indices were chosen because they cover key parts of the European market and include well-known and highly traded companies. This makes them useful to compare how different assets contribute to portfolio risk in a European context.

The historical data for each index were obtained from Yahoo Finance (2024), using the adjusted closing prices (which account for dividend distributions). The daily return of each index i was then calculated according to Equation (4.1), where P_t denotes the adjusted closing price of index i on day t and P_{t-1} is the adjusted closing price of the same index on the previous trading day $(t-1)$. To ensure consistency, any day with missing data for at least one index was excluded from the analysis.

The subsequent calculations rely on the mean and standard deviation of returns for each index, reported in Table 4.1, along with the variance-covariance matrix presented in Table 4.2.

$$R_i = \ln\left(\frac{P_t}{P_{t-1}}\right) \times 100 \tag{4.1}$$

Table 4.1: Mean and Standard Deviation of Indices Returns

	STOXX	GDAXI	FTSE
Mean	0.026	0.023	0.022
Standard Deviation	1.156	1.088	0.828

Table 4.2: Variance-Covariance Matrix of Indices Returns

	STOXX	GDAXI	FTSE
STOXX	1.34	1.20	0.761
GDAXI	1.20	1.18	0.704
FTSE	0.761	0.704	0.685

The descriptive statistics above provide some insights into the behavior of the three indices. The average returns are relatively low and very similar across the STOXX, GDAXI and FTSE, indicating that none of the indices delivered substantially higher mean returns over the sample period. In terms of volatility, measured by the standard deviation, the STOXX and GDAXI exhibit higher fluctuations, whereas the FTSE appears less volatile. This suggests that the FTSE index was relatively more stable compared to the other two. The covariance values are all positive and relatively high, particularly between the STOXX and GDAXI (1.20), showing that these two indices tend to move together. The positive and relatively high covariances suggest that the indices move in the same direction.

4.3 Shapley Values for MVP

In the financial context, and particularly in this study, the characteristic function of each asset corresponds to its own standard deviation. For each set of assets and all possible combinations, the characteristic function corresponds to the standard deviation of the MVP, formed by the respective combination of assets. Consider a portfolio consisting of three assets, A, B and C, whose standard deviations of returns are σ_A, σ_B, and σ_C, respectively, then we have $v(A) = \sigma_A$, $v(B) = \sigma_B$, and $v(C) = \sigma_C$. Since this section relies on the MVP, either with all three assets or with pairs, the characteristic functions $v(ABC)$, $v(AB)$, $v(AC)$, and $v(BC)$ represent the minimum achievable standard deviation for each portfolio. In other words, they are obtained from the vertex of the efficient frontier formed by each asset set in the return–standard deviation space. These values are then used to compute the marginal contribution of each asset, considering all possible entry orders into the portfolio. The marginal contribution of each asset is obtained by the difference in the characteristic function as it enters the coalition. For example, in the coalition ABC, the marginal contribution of asset A is simply its own characteristic function $v(A)$ since it is the first asset to enter the coalition. As B enters next, its contribution is given by the difference $v(AB) - v(A)$. Finally, as AB is already included in the coalition, the contribution of C is determined by $v(ABC) - v(AB)$. The Shapley Value of each asset is obtained as the average of all its marginal contributions.

In our specific case, the characteristic function of each index corresponds to its standard deviation, that is, $v(S) = 1.156$, $v(G) = 1.088$, and $v(F) = 0.828$. In the context of the MVP, the characteristic functions of the indices are obtained by identifying the vertex of the efficient frontier associated with each combination of indices. Thus, to compute $v(SG)$, it is sufficient to determine the allocation of capital between STOXX and GDAXI so that the resulting portfolio yields the lowest possible variance. By taking the square root of Equation (3.23), the standard deviation of this minimum-risk portfolio is obtained, giving $v(SG) = 1.0856$. The same reasoning applies to the portfolio comprising the three indices, where the standard deviation corresponds to the vertex of the efficient frontier formed by all indices, resulting in $v(SGF) = 0.807$. This means that the MVP comprising the grand coalition has a standard deviation of 0.870.

To compute the marginal contributions of each index, an algorithm was implemented that subtracts the characteristic functions as assets enter the portfolio in different orders. For instance, in the grand coalition SGF, the marginal contribution of STOXX is $v(S) = 1.156$, since it enters first. When GDAXI is added, its contribution is $v(SG)-v(S) = 1.085-1.156 = -0.070$ and when FTSE joins, its contribution is $v(SGF) - v(SG) = 0.807 - 1.085 = -0.278$. The Shapley Value of each index is then obtained by averaging its marginal contributions across all permutations. The results are reported in Table 4.3.

Table 4.3: Shapley Values for MVP

Entry Order	Marginal Contribution		
	S	**G**	**F**
SGF	1.156	-0.070	-0.278
SFG	1.156	-0.013	-0.335
FSG	-0.007	-0.013	0.828
FGS	-0.020	-0.0005	0.828
GSF	-0.002	1.088	-0.278
GFS	-0.020	1.088	-0.260
Shapley Value	**0.377**	**0.346**	**0.084**

Table 4.3 reports the Shapley Values for the MVP with the three indices, which has a total standard deviation of 0.870. Of this risk, FTSE contributes 0.084, GDAXI 0.346, and STOXX 0.377. STOXX is therefore the main contributor to portfolio risk, followed by GDAXI, while FTSE plays a comparatively smaller role.

4.4 Shapley Values along the Efficient Frontier

Instead of focusing solely on the portfolio with the lowest possible risk, it may be useful to set a desired expected return and determine the portfolio that, for that level of return, offers the minimum risk (or standard deviation). Moreover, by fixing a desired return level and identifying the portfolio (i.e., the proportion of wealth to be invested in each index) that minimizes risk, we can construct the efficient frontier for this set of indices.

To determine the portfolio x^* that minimizes risk for a given expected return μ_p, Equation (3.19) is applied to each target return level. This calculation uses the variance-covariance matrix in Table 4.2, from which the optimal portfolio weights are derived. The risk of each portfolio is then obtained by taking the square root of Equation (3.21). Table 4.4 reports the proportion of wealth (in percentage) allocated to each index in the minimum-risk portfolio corresponding to the different return targets. These results form the basis for the efficient frontier, which consists of six portfolios achieving the lowest possible risk at expected returns of 0.020, 0.021, 0.022, 0.023, 0.024 and 0.025. The frontier itself is illustrated in 4.1.

Table 4.4: Optimal Weights of Minimum-Risk Portfolios by Target Return

Expected Return	Standard Deviation	Weights (%)		
		S	**G**	**F**
0.020	0.8076	-0.5168	0.4411	1.0757
0.021	0.8165	-0.1898	0.1725	1.0173
0.022	0.8395	0.1372	-0.0960	0.9589
0.023	0.8754	0.4642	-0.3646	0.9004
0.024	0.9229	0.7912	-0.6332	0.8420
0.025	0.9801	1.1182	-0.9018	0.7836

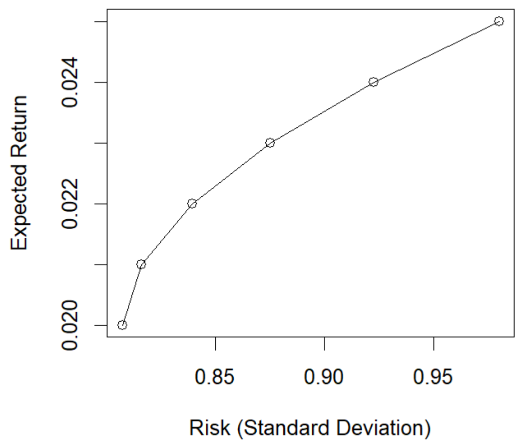


Figure 4.1: Efficient Frontier of Portfolios Formed by the Three Indices

From Table 4.4, some conclusions can already be drawn. As expected, as the target return increases, the portfolio's risk also rises. Furthermore, while risk increases, the absolute proportion of wealth allocated to the STOXX and GDAXI indices grows, while allocation to the FTSE index decrease. Since STOXX and GAXI are the indices with the highest expected return and variance, it is natural for their weights in the optimal portfolio to increase as the target return and risk level rise. Moreover, given that STOXX exhibits the highest risk and expected return among all indices, it is expected to hold the largest proportion of wealth in portfolios with the highest levels of return and risk. Note that negative weights for STOXX, GDAXI and FTSE suggest the strategy of short selling these indices. Short selling involves borrowing and selling an asset with the expectation that its price will decrease. This approach can help minimize overall portfolio risk, but it also carries the risk of losses if the shorted assets increase in value instead.

The Shapley Values of the three indices for each of the six portfolios are calculated under the assumption that the characteristic functions $v(SD)$, $v(SF)$, $v(DF)$, and $v(SDF)$ follow a different approach from that described in Subsection 4.3. In this case, we cannot use the formula for the abscissa of the efficient frontier vertex, as it does not correspond to the portfolio being used. Instead, for each pair or triplet of indices, the portfolio x^* that minimizes risk for a given fixed return is determined using the Equation (3.19). Once x^* is obtained, its variance is calculated using Equation (3.5), where V denotes the variance-covariance matrix of the indices in the chosen portfolio configuration. As shown in Section 3.4, replacing x^* in this variance equation leads to Equation (3.21), which explicitly gives the variance of a portfolio with expected return μ_p.

For each set of indices and each fixed expected return, the variance of the minimum-risk portfolio is calculated, and its square root is taken to obtain the characteristic functions for each set. Consequently, six distinct values of $v(SD)$, $v(SF)$, $v(DF)$, and $v(SDF)$ are derived, one for each fixed expected return. With all characteristic functions determined, the marginal contributions and Shapley Values of the indices are then computed using the same approach described in Subsection 4.3. Tables 4.5 and 4.6 present the marginal contributions of each index for the fixed expected returns of 0.020 and 0.023, respectively. Table 4.7 displays the Shapley Values of each index for all six expected returns.

Table 4.5: Shapley Values for the Expected Return of 0.020

Entry Order	Marginal Contributions		
	S	**G**	**F**
SGF	1.1558	-0.0377	-0.3105
SFG	1.1558	-0.0284	-0.3198
FSG	0.0083	-0.0284	0.8277
FGS	-0.3257	0.3055	0.8277
GSF	0.0303	1.0877	-0.3105
GFS	-0.3257	1.0877	0.0455
Shapley Value	0.2831	0.3977	0.1267

Table 4.6: Shapley Values for the Expected Return of 0.023

Entry Order	Marginal Contributions		
	S	**G**	**F**
SGF	1.1558	-0.0671	-0.2133
SFG	1.1558	-0.0180	-0.2623
FSG	0.0658	-0.0180	0.8277
FGS	-0.2543	0.3020	0.8277
GSF	0.0010	1.0877	-0.2133
GFS	-0.2543	1.0877	0.0420
Shapley Value	0.3116	0.3957	0.1681

Table 4.7: Shapley Value for Each Portfolio

Expected Return	Standard Deviation	Shapley Value		
		S	**G**	**F**
0.020	0.8076	0.2831	0.3977	0.1267
0.021	0.8165	0.3686	0.3577	0.0901
0.022	0.8395	0.3785	0.3568	0.1041
0.023	0.8754	0.3116	0.3957	0.1681
0.024	0.9229	0.2107	0.4536	0.2586
0.025	0.9801	0.1004	0.5187	0.3610

4.5 Discussion

For clearer interpretation and comparison, the Shapley Value is expressed relative to the total risk of the portfolio. In other words, it shows, in percentage terms, the share of the portfolio's risk (or standard deviation) attributable to each index. This is obtained by dividing the Shapley Values in Table 4.7 by the total standard deviation of the corresponding portfolio.

As a comparative measure, φ_i was employed, representing the individual beta of each index expressed in percentage terms. According to CAPM, φ_i shows how much of the total portfolio risk is explained by that specific index. Since the analysis is entirely focused on risk decomposition, the beta of the individual index in percentage offers a more meaningful point of comparison that the maket beta. The φ_i of each index was calculated using Equation (3.7). Table 4.8 presents a side-by-side comparison of these two parameters.

Table 4.8: Relative Shapley Value Vs. φ_i

Expected Return	Std Deviation	Relative Shapley Value			φ_i		
		S	**G**	**F**	**S**	**G**	**F**
0.020	0.8076	0.3506	0.4925	0.1569	-0.5229	0.4437	1.0791
0.021	0.8165	0.4515	0.4381	0.1104	-0.2074	0.1789	1.0285
0.022	0.8395	0.4508	0.4251	0.1241	0.1551	-0.0990	0.9439
0.023	0.8754	0.3560	0.4520	0.1920	0.5243	-0.3625	0.8382
0.024	0.9229	0.2283	0.4915	0.2802	0.8678	-0.5927	0.7248
0.025	**0.9801**	**0.1024**	**0.5292**	**0.3683**	**1.1674**	**-0.7815**	**0.6141**

As a first observation, it is important to note that as the fixed expected return increases, the risk also rises, which is naturally expected. By analyzing the relative Shapley Values, several conclusions can be drawn regarding the contributions of each index to the total portfolio risk. For instance, in the portfolio with the highest expected return and risk, 53% of the total risk is attributed to the GDAXI index, 37% to the FTSE index, and only 10% to the STOXX index. Regarding STOXX, the relative Shapley Values exhibit a downward trend as portfolio risk increases. This pattern suggests that STOXX plays a role in mitigating portfolio risk. On the other hand, as portfolio risk grows, the GDAXI and FTSE Shapley values tend to rise (with a stronger emphasis on FTSE), indicating that these two indices are increasingly responsible for driving portfolio risk higher.

Notably, the values and conclusions drawn from the betas of the individual indices in percentage differ significantly from those obtained through the Shapley Value. In fact, when using φ_i, the conclusions are precisely the opposite of those derived from Shapley analysis. Most strikingly, STOXX is the only index whose φ_i value increases as portfolio risk rises, whereas GDAXI and FTSE exhibit a decreasing trend as risk increases. Note that every index had a negative beta at some point. This means that, at certain moments, these indices moved inversely to the portfolio. In other words, when the returns of these indices increased, the portfolio's return tended to decrease, and vice versa.

These results reinforce the notion that the Shapley Value provides a more robust and comprehensive framework for allocating portfolio risk among assets. By averaging each asset's marginal contributions across all possible portfolio combinations, it delivers a consistent decomposition that reflects each asset's true role. Moreover, in line with Shalit (2021), the findings confirm that Shapley-based conclusions frequently diverge from those obtained through beta-based measures. Similarly to his work on U.S. indices, this application on European markets shows that the conclusions drawn from each asset's beta can, in some cases, be completely opposite to those based on the Shapley Value. Assets that appear to increase portfolio risk when judged by their individual beta may actually reduce it when evaluated through the Shapley Value, and vice versa. This reinforces the idea, already evident in the 2021 study, that beta alone may not reflect an asset's complete impact on portfolio risk.

4.6 Conclusion

In summary, the analysis confirms the expected trade-off between return and risk: as target returns increase, portfolio risk also rises. The Shapley decomposition highlights important differences in how each index contributes to this risk. STOXX tends to mitigate portfolio risk, particularly as risk levels increase, whereas GDAXI and FTSE account for a growing share of the total risk. Overall, the results emphasize that the Shapley Value offers a more consistent and informative framework to decompose portfolio risk, capturing the more complete role of each index within the efficient frontier.

5

Socially Responsible Investments

5.1 Introduction

Socially responsible investments have become increasingly important in the financial world due to growing awareness of environmental, social, and governance issues. These investments aim to balance financial returns with a positive impact on public well-being and environmental sustainability.

The ESG classification assesses companies based on three fundamental pillars: the environmental one examines how a company reduces the environmental impact of its operations; the social pillar considers aspects such as respect for human rights, inclusion, and labor practices; the governance one evaluates transparency, ethics, and corporate management. This classification is publicly available and allows companies to be ranked according to their level of ESG commitment (high, medium, or low).

This chapter aims to evaluate whether socially responsible investments are associated with superior risk-adjusted performance and reduced downside risk. The analysis focuses on two characteristic functions, the Sharpe ratio and the Value at Risk, and is applied to both the Minimum Variance Portfolio and the Maximum Sharpe Ratio Portfolio. Results are examined separately for periods of higher and lower market volatility to identify potential differences in ESG contributions. All calculations related to this chapter were carried out using the R programming language and can be viewed at the following GitHub link: Socially Responsible Investments — GitHub.

It is important to note that, differently from some other studies in the literature, we rely on ESG risk ratings provided by Sustainalytics (2024), where a High rating indicates companies with greater ESG risk exposure, meaning they are less committed to sustainability practices and may be more vulnerable to environmental and social controversies. Conversely, a Low ESG risk rating reflects companies with stronger sustainability practices, signifying a lower risk of ESG-related incidents and a greater alignment with responsible investing principles. The ESG Risk Ratings from Sustainalytics (2024) are calculated by assessing a company's exposure to industry-specific ESG risks and how well the company is managing those risks. The methodology separates risks that companies can control from those they cannot, and focuses on how well companies manage the controllable risks. The final score reflects the portion of unmanaged ESG risk, combining both exposure and management performance.

5.2 Methodological Approach

This study specifically analyzes how different ESG categories influence portfolio financial outcomes. The Shapley Value is computed using marginal contributions (following the reasoning outlined in Subsection 3.7.1), measuring how each ESG group affects key metrics such as the Sharpe ratio and the Value at Risk. Applying the Shapley Value in this context provides deeper insights into how companies with High, Medium and Low ESG ratings contribute to overall portfolio performance.

We consider four cooperative games, all defined on the same set of three players. Each player is a collection of 10 assets with similar ESG ratings. Specifically, let $N = \{H, M, L\}$, where H denotes a collection of 10 assets with High ESG rating, M a collection of 10 assets with Medium ESG score and L a collection of 10 assets with Low ESG score. The coalition $\{H, M\}$ is the collection of 20 assets comprising companies with High and Medium ESG ratings. Similarly, $\{H, L\}$ consists of 20 assets including companies with High and Low ESG ratings, while $\{H, M\}$ includes 20 assets with Medium and Low ESG-rated companies. The grand coalition $\{H, M, L\}$ encompasses 30 assets, incorporating all companies across the High, Medium, and Low ESG categories.

The first part of the study evaluates the impact of ESG levels on portfolio financial performance using the Sharpe ratio as the primary metric. The value of the characteristic function for each coalition is defined as the Sharpe ratio of a reference portfolio that includes all the assets in the coalition. To assess whether the results vary depending on the reference portfolio used to construct the characteristic function, the analysis considers two distinct portfolio configurations: the Minimum Variance Portfolio (MVP) and the Maximum Sharpe Ratio Portfolio (MSRP) (Huang and Litzenberger 1988). The rationale behind these constructions is straightforward. For example, in the case of MVP, the characteristic function for a coalition consisting of High and Low ESG-rated companies corresponds to the Sharpe ratio of the MVP composed of all companies in these two categories. The same reasoning applies to MSRP: for the High-Low coalition, the characteristic function is the Sharpe ratio of the MSRP formed by all High and Low-rated companies.

The second part of the study shifts the focus to risk assessment, specifically evaluating the maximum potential loss of portfolios. Instead of analyzing the Shapley Value based on the Sharpe ratio, this section assesses the Shapley Value using the Value at Risk of the portfolios. The portfolio construction process and the determination of characteristic functions follow the same logical framework as previously described. For example, when considering the grand coalition (comprising High, Medium, and Low ESG-rated companies), the characteristic function is defined as the VaR of the MVP constructed with all companies in the sample. Similarly, when the MSRP is used as the reference portfolio, the characteristic function for the total coalition corresponds to the VaR of the MSRP composed of all selected companies.

By analyzing both financial performance and risk exposure through the Shapley Value framework, this study provides a comprehensive evaluation of the contributions of ESG-classified companies to investment portfolios.

Based on this structure, we define four different cooperative games. In all four games, the set of players is $N = \{H, M, L\}$ and the set of all possible coalitions is by $2^N = \{\{\emptyset\}, \{H\}, \{M\}, \{L\}, \{HM\}, \{HL\}, \{ML\}, \{HML\}\}$.

- **Game 1:** $v(S)$ = Sharpe ratio (MVP(S)), that is, the value of the coalition S is the Sharpe ratio of the Minimum Variance Portfolio, constructed using all the assets in S.
- **Game 2:** $v(S)$ = Sharpe ratio (MSRP(S)), that is, the value of the coalition S is the Sharpe ratio of the Maximum Sharpe Ratio Portfolio, constructed using all the assets in S.
- **Game 3:** $v(S)$ = Value at Risk (MVP(S)), that is, the value of the coalition S is the Value at Risk of the Minimum Variance Portfolio constructed using all the assets in S.
- **Game 4:** $v(S)$ = Value at Risk (MSRP(S)), that is, the value of the coalition S is the Value at Risk of the Maximum Sharpe Ratio Portfolio, constructed using all the assets in S.

The Sharpe Ratio is calculated using Equation (3.24) , assuming a risk-free rate (R_f) of 0. This choice is made for simplicity, as the risk-free rate has been often very close to zero in practice and has little impact on the results. The VaR is calculated as described in Section 3.6, using a 95% confidence level.

5.3 Data

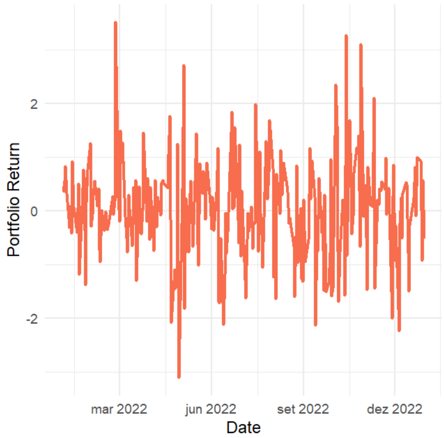
The empirical analysis includes 30 companies from the United States, distributed across 10 different industrial sectors. This selection ensures a diverse representation of industries and minimizes dependency patterns. The analysis period spans from January 3rd, 2022 to December 29th, 2023. For the purposes of this analysis, we used the adjusted closing prices of each stock, ensuring that dividends and stock splits were taken into account.

Examining the performance of the S&P 500 index, these two years are particularly noteworthy not only due to the market's recovery from a downward trend but particularly because of a decline in market volatility from one year to the next. Figure 5.1, sourced from Yahoo Finance (2024), illustrates the contrast between these two years. The blue line represents the overall performance of the index, while the black line outlines its historical volatility. The red line, indicating the minimum recorded historical volatility in 2022, highlights the decline in volatility the following year.

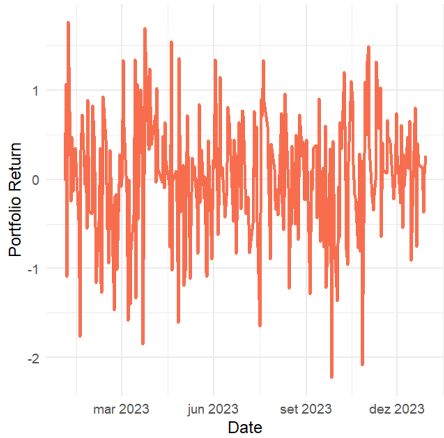
Figure 5.1: Performance of the S&P 500 Index

The sectors selected include oil and gas producers, energy services, banking, healthcare, telecommunications services, pharmaceuticals, services, software and services, construction materials, and transportation. Within these 10 sectors, three companies were chosen - one from each level of ESG risk (High, Medium, and Low) - based on the ratings available at Sustainalytics (2024). These ratings correspond to the 12-month period between March 2023 and March 2024. For consistency, it is assumed that the ratings remained as such during 2022 and 2023. Table A.1 in Appendix A presents the 30 selected companies, classified by sector and ESG risk rating.

Figures 5.2 and 5.3 illustrate the return dynamics of the MVP and the MSRP over the years 2022 and 2023, constructed using all sampled companies classified as High, Medium, and Low ESG risk. These time-series plots provide insights into the fluctuations and volatility patterns across different portfolio types and market conditions.

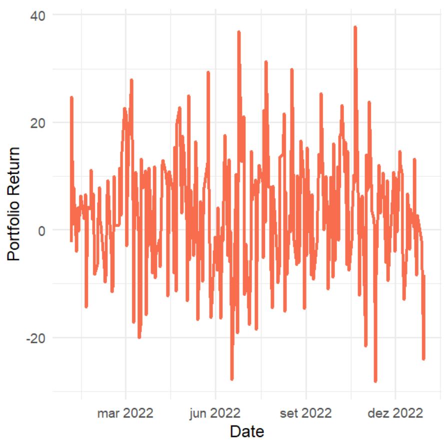


(a) 2022

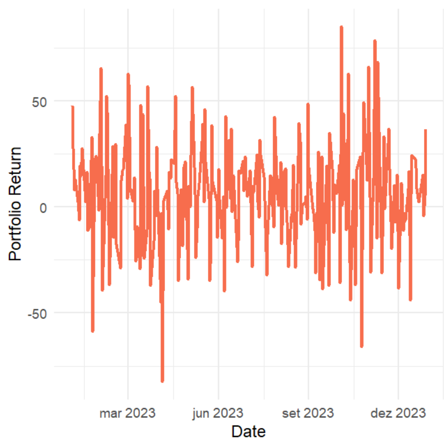


(b) 2023

Figure 5.2: Daily Returns of the MVP



(a) 2022



(b) 2023

Figure 5.3: Daily Returns of the MSRP

The MVP showed higher volatility in 2022, in line with the more unstable markets of that year. In 2023, the variations were smaller, reflecting a more stable environment. The volatility in the MSRP was much higher than in the MVP in both years. Furthermore, while the MVP became more stable in 2023, the MSRP experienced even stronger swings in that year compared to 2022, as shown in Figure 5.3.

The descriptive statistics in Table 5.1 reveal important differences in MVP behavior across years with distinct market conditions.

Table 5.1: Return, Standard Deviation, Coefficient of Variation, Skewness and Kurtosis for MVP

	2022					2023				
	Mean	**Std Dev**	**CV**	**Skew**	**Kurt**	**Mean**	**Std Dev**	**CV**	**Skew**	**Kurt**
H	0.008	1.190	157.1	0.403	1.228	0.007	0.998	140.7	0.117	1.521
M	**0.024**	1.105	46.42	0.239	0.434	**-0.014**	0.822	-57.44	-0.188	0.072
L	**-0.029**	1.180	-40.30	-0.044	1.300	**0.034**	0.874	26.02	-0.106	-0.157
HM	0.032	1.027	31.66	0.301	0.814	-0.011	0.780	-69.99	-0.168	0.206
HL	-0.004	1.024	-281.0	0.032	1.314	0.020	0.810	41.25	-0.116	0.184
ML	0.015	1.045	69.61	0.290	0.942	0.006	0.759	128.0	-0.281	0.020
HML	0.022	0.973	44.87	0.197	1.173	0.006	0.730	131.2	-0.298	0.089

In 2022, a volatile year with falling markets, portfolios of moderately sustainable firms (M) achieved the highest average return (0.024), while the most sustainable firms (L) performed the worst (-0.029). In 2023, a period of recovery and lower volatility, the pattern reversed: the most sustainable portfolios led (0.034), while the moderate group posted negative returns (-0.014). Although L firms achieved the highest returns, the portfolio that included all firms (HML) recorded very low returns, suggesting that the addition of HM firms reduced the overall returns. Volatility fell across all portfolios, consistent with calmer conditions. Skewness shifted from positive to negative, pointing to greater downside risk. Kurtosis also declined in most cases, suggesting fewer extreme return events.

The descriptive statistics presented in Table 5.2 highlight significant differences in the behavior of the MSRP across years with varying market conditions.

Table 5.2: Return, Standard Deviation, Coefficient of Variation, Skewness, and Kurtosis for MSRP

	2022					2023				
	Mean	**Std Dev**	**CV**	**Skew**	**Kurt**	**Mean**	**Std Dev**	**CV**	**Skew**	**Kurt**
H	**7.362**	**37.10**	5.039	-0.191	1.096	3.038	20.66	6.799	-0.067	0.561
M	**1.130**	7.611	6.738	0.018	0.375	**-1.245**	7.666	-6.159	-0.379	2.153
L	**-1.576**	8.657	-5.491	0.135	0.561	**0.968**	4.692	4.845	-0.087	0.421
HM	1.495	6.973	4.665	-0.174	0.469	-2.167	10.88	-5.019	-0.347	1.403
HL	**-16.08**	68.03	-4.232	0.027	0.383	1.845	7.853	4.257	-0.017	0.541
ML	3.560	16.09	4.520	0.098	0.533	5.230	22.54	4.309	-0.079	0.797
HML	3.073	11.58	3.769	0.078	0.114	6.470	24.89	**3.847**	0.033	0.764

In the MSRP, the data show clear differences in performance and risk across sustainability profiles. In 2022, the least sustainable assets had the highest mean return (7.362) but also the largest volatility (37.10). On the other hand, the most sustainable group recorded negative returns (-1.576). The HL portfolio performed particularly poorly, with a strongly negative return (-16.08). Since the H portfolio recorded the highest return in 2022 while the HL portfolio recorded the lowest, this suggests that adding L firms to the portfolio reduced the returns. In 2023, the most sustainable assets turned positive (0.968), while the moderate group shifted to negative (-1.245). Skewness and kurtosis stayed close to zero in both years, indicating more balanced return distributions with fewer extreme events.

5.4 Results

Table 5.3 presents the market beta of each ESG level relative to the S&P 500, which serves as the market benchmark, calculated through Equation (3.2). Unlike Chapter 4, this chapter uses each asset's market beta, as in Shalit (2020). In this setting, market beta provides a common external benchmark that is more suitable than an asset specific beta, since the analysis centers on performance measures rather than risk alone. Given the focus on risk adjusted outcomes such as the Sharpe ratio, alongside downside exposure captured by Value at Risk, the market beta is the most appropriate choice for comparison.

Table 5.3: Market Beta for each Portfolio

		High	**Medium**	**Low**
MVP	**2022**	0.487	0.499	**0.503**
	2023	0.712	0.596	**0.726**
MSRP	**2022**	-7.709	-1.662	**1.866**
	2023	**7.058**	-2.409	2.268

The MVP show moderate and positive betas, yet these values provide little meaningful information. The values are too close to each other and not far from 1, making it hard to draw meaningful conclusions based only on this metric. In general, the results suggest stability, but the differences between the risk levels of ESG are minimal.

When looking at the MSRP, the patterns become more distinct. Here, the beta values show much greater variation. For the High and Medium ESG risk portfolios, some betas turned negative, which points to movements opposite to the market. The High ESG risk portfolio stands out. Its deeply negative beta in 2022 shows that it moved against the market and carried higher risk than the market as a whole. Next year, its beta reversed from strongly negative to strongly positive, showing a shift from moving against the market to moving in the same direction with very high sensitivity. The most sustainable companies behaved differently, since their betas remained positive in both years and showed a consistent connection to the market.

Table 5.4 presents the values of the characteristic function $v(S)$ for each ESG group and coalition in different years, portfolio weights (MVP and MSRP), and evaluation metrics (Sharpe ratio and VaR).

Table 5.4: Characteristic Functions for each Portfolio

			Characteristic Functions[1]						
			v({H})	**v({M})**	**v({L})**	**v({HM})**	**v({HL})**	**v({ML})**	**v({HML})**
Sharpe ratio	**MVP**	**2022**	0.006	0.021	-0.025	**0.032**	-0.004	0.014	0.022
		2023	0.007	-0.017	0.038	-0.014	0.024	0.008	0.008
	MSRP	**2022**	0.198	0.148	-0.182	0.214	-0.236	0.221	**0.265**
		2023	0.147	-0.162	0.206	-0.199	0.235	0.232	**0.260**
Value at Risk	**MVP**	**2022**	1.853	1.836	2.046	1.655	1.715	1.764	1.618
		2023	1.721	1.551	1.457	1.467	1.329	1.440	1.278
	MSRP	**2022**	57.51	11.90	17.18	**9.613**	**131.3**	22.99	16.33
		2023	33.57	12.70	6.856	19.59	11.97	34.94	34.94

[1] $\mathbf{v}(\emptyset) = 0$

For the Sharpe ratio, higher values indicate better performance. In 2022, the coalition $\{HM\}$ delivered the highest Sharpe ratio for MVP. However, the grand coalition $\{HML\}$ as the highest Sharpe ratio for MSRP both years, suggesting that diversification across ESG risk levels may improve overall portfolio efficiency. Some combinations showed negative Sharpe ratios, indicating that these combinations may be less effective in enhancing portfolio performance. With respect to VaR, higher values indicate a greater exposure to downside risk. In general, MVP displays lower levels of risk, whereas MSRP reach much higher values. In 2022, the MSRP recorded its highest risk in the $\{HL\}$ coalition (v = 131.3), while the $\{HM\}$ coalition showed the lowest risk in both years (v = 9.613).

These values are essential for computing Shapley Values, which enables a fair assessment of each ESG risk level's contribution to the portfolio's risk-adjusted returns or risk.

Table 5.5 provides a consolidated view of the Shapley Values for each level of ESG risk, organized according to two distinct characteristic functions: the Sharpe ratio and VaR. Within each characteristic function, the results are divided into two portfolio weightings: the MVP and the MSRP. The table also reports $v(\{HML\})$, which corresponds to the value of the characteristic function for the full coalition of High, Medium, and Low firms. Because the sum of the individual Shapley Values must equal $v(\{HML\})$, including this measure helps us to understand more clearly how each group contributes to the portfolio's performance. The tables detailing the calculations of the marginal contributions for each ESG risk level can be found in Appendix A.

Table 5.5: Summary of Shapley Values

			ESG Risk Rating			
			High	**Medium**	**Low**	**v({HML})**
Sharpe ratio	**MVP**	**2022**	0.010	**0.026**	**-0.014**	0.022
		2023	0.001	**-0.020**	**0.027**	0.008
	MSRP	**2022**	0.083	**0.286**	**-0.104**	0.265
		2023	0.057	**-0.099**	**0.302**	0.260
Value at Risk	**MVP**	**2022**	**0.484**	0.499	**0.635**	1.618
		2023	**0.484**	0.455	**0.339**	1.278
	MSRP	**2022**	**35.59**	**-41.37**	22.11	**16.33**
		2023	13.19	14.24	7.509	**34.94**

For the Sharpe ratio, the results show a clear shift between the two years. In MVP (2022), firms in the Medium ESG risk group contributed the most to increase the Sharpe ratio (0.026), while the most sustainable firms had a negative impact (-0.014). In 2023 this pattern reversed: the more sustainable firms became the strongest contributors (0.027), and the Medium group showed the weakest performance (-0.020). A similar trend appears in the MSRP. In 2022, Medium ESG risk group (0.286) had the highest positive effect, while the most sustainable group reduced the risk-adjusted returns (-0.104). However, in 2023, the most sustainable firms led (0.302), and the Medium group turned negative again (-0.099).

For VaR, the shifts are also notable. In MVP (2022), the most sustainable firms (0.635) added the most to portfolio risk, followed by the Medium group (0.499) and the least sustainable (0.484). By 2023, the risk of the most sustainable group dropped to 0.339, while the least sustainable remained stable (0.484). The MSRP show an even stronger change. In 2022, moderately sustainable firms (-41.37) reduced risk the most, while least sustainable (35.59) amplified it. In 2023, all groups contributed positively, reflecting increased overall exposure. This is confirmed by the VaR of the total portfolio $v(\{HML\})$, which increased significantly from 16.33 in 2022 to 34.94 in 2023.

5.5 Discussion

For a clearer discussion and interpretation of the findings, Table 5.6 highlights the ESG levels with the best and worst Shapley Value results for each characteristic function and portfolio weight.

Table 5.6: Best and Worst Contributors for Each Characteristic Function and Portfolio Weight

		Characteristic Function			
		Sharpe ratio		Value at Risk	
		Best Contributor	**Worst Contributor**	**Best Loss Reducer**	**Worst Loss Reducer**
MVP	**2022**	M	L	H	L
	2023	L	M	L	H
MSRP	**2022**	M	L	M	H
	2023	L	M	L	M

L = Low ESG Risk Rating; M = Medium ESG Risk Rating; H = High ESG Risk Rating

Regarding the Shapley Values, the analysis reveals a clear shift in ESG risk level contributions to portfolio performance between periods of high and low market volatility. In 2022, a year of increased market volatility, Medium ESG risk assets most effectively enhanced the risk-adjusted returns (Sharpe ratio) of both MVP and MSRP. This finding aligns with Morgan Stanley Investment Management, Emerging Markets Equity Team (2023), who highlight the investment potential of mid-range ESG performers, particularly in contexts where ESG integration is still maturing. The results are also consistent with Sourd (2024), who argue that firms improving their ESG scores can be particularly appealing, since markets often take time to adjust to the advantages of such improvements. On the other hand, under the lower volatility conditions of 2023, the most sustainable assets delivered superior performance in terms of contributions to increase the Sharpe ratio.

However, it is also worth noting that the less sustainable ones (H) were among the worst performers in terms of downside risk reduction (VaR), particularly in 2022 for MSRP and in 2023 for MVP. Furthermore, the most sustainable assets (L) emerged as the best loss reducers in the more stable market environment of 2023. This finding is consistent with Morelli (2024), who report that less sustainable companies are generally more volatile and less effective in providing downside protection and that firms with stronger environmental performance are generally more stable and less exposed to market shocks. This pattern is less consistent with Baselli (2024), who argue that sustainable firms often show their strength in times of crisis. It also contrasts with Albuquerque et al. (2020), whose results suggest that firms with strong ESG practices remained resilient even during severe downturns, whereas in our analysis such resilience was not evident in 2022. Our findings point in another direction. They are closer to Perote et al. (2023), who argue that sustainable investments hold up under moderate shocks but lose strength under more extreme shocks. In our study, 2022 was a year of high volatility and can be viewed as one of those severe shocks. In this period, the portfolios of more sustainable firms did not show resilience. This suggests that the link between ESG and risk management is dependent on market conditions.

The results also vary according to the characteristic function and portfolio weighting. In the least volatile year, the most sustainable firms made the strongest contributions, both in increasing the Sharpe ratio and reducing the VaR for both the MVP and MSRP. However, in the most volatile year, firms with medium sustainability levels had the greatest impact on increasing the Sharpe ratio and effectively reducing VaR, particularly in the MSRP. For the MVP in that same year, the contributions differed by characteristic function: while intermediate sustainability firms contributed the most to increasing the Sharpe ratio, the least sustainable firms were most effective in reducing VaR. In general, these variations emphasize the importance of preferences and how they can change depending on market conditions.

These results further support the usefulness of the Shapley Value as an attribution tool in ESG investing, as proposed by Wijnen et al. (2023). Unlike traditional beta-based models, the Shapley Value captures the effects of ESG profiles on portfolio outcomes in a more precise way, across different metrics, as highlighted by Shalit (2020). As we have seen, for the MVP, beta proved to be of very limited use. In the case of the MSRP, beta indicated that the most sustainable companies behaved more in line with the market and exhibited lower volatility for both years, a conclusion that differs slightly from the patterns revealed by the Shapley Value.

5.6 Conclusion

This chapter applied the Shapley Value methodology to evaluate how assets with different ESG risk profiles contribute to portfolio performance across different market environments. The results show that both sustainability and market conditions do matter. Firms with stronger sustainability records generally outperformed those with weaker practices. The more sustainable companies added the most value in calmer markets, improving risk-adjusted returns and limiting downside risk, while firms with moderate sustainability showed greater resilience in volatile periods. This suggests that moderately sustainable businesses may provide stronger protection when markets are unstable, whereas the most sustainable ones tend to be rewarded in steadier environments. On the other hand, less sustainable firms contributed less to downside protection, particularly in the MSRP in 2022 and the MVP in 2023. This weaker role likely reflects their higher perceived risk, linked to factors such as weaker governance practices, exposure to environmental liabilities or reduced investor confidence.

The outcomes differ based on the characteristic function and portfolio weighting. In the least volatile year, the most sustainable firms had the greatest impact on improving the Sharpe ratio and lowering VaR for both the MVP and MSRP. In contrast, during the most volatile year, firms with moderate sustainability levels were most effective in increasing the Sharpe ratio and reducing VaR, especially in the MSRP. For the MVP in the same year, contributions varied by characteristic function: firms with intermediate sustainability had the largest effect on the Sharpe ratio, while the least sustainable firms were most successful in reducing VaR. These differences highlight how preferences can shift with market conditions.

A look at the beta results provides a useful point of comparison. For the MVP, beta values were consistently below 1 and showed little meaningful variation. In the MSRP, beta suggested that the most sustainable companies behaved more in line with the market and displayed lower volatility. However, these results differ from the patterns revealed by the Shapley Value, which is consistent with the literature suggesting that Shapley Value provides a more precise perspective than traditional beta.

From a policy perspective, this findings suggest that the weaker financial contribution of sustainable firms during times of crisis may reduce their attractiveness to investors, particularly when regulatory support or structural incentives are lacking. Therefore, regulation is justified to promote the development and credibility of sustainable investment markets. Without these incentives, ESG assets are traded primarily by investors with a clear sense of social responsibility.

Finally, this chapter highlights the value of the Shapley Value approach in capturing the distinct contributions of ESG risk profiles to portfolio outcomes. By capturing non-linear and context-dependent contributions across both return and risk dimensions, this approach provides more information than traditional beta-based models and supports more informed decision-making in ESG investments.

6

Concluding Remarks

This thesis has examined the contribution of financial assets to portfolio risk and performance by applying the Shapley Value methodology in two distinct contexts: traditional financial indices and firms with different ESG risk profiles. Across both applications, the findings highlight the added value of using the Shapley Value compared to more traditional approaches such as beta-based measures.

The analysis of European stock indices shows that the Shapley Value provides a more consistent and reliable decomposition of portfolio risk. Unlike the results based on individual betas, which may suggest conflicting or even opposite conclusions, the Shapley framework captures each index's complete marginal contribution by considering all possible portfolio combinations. This leads to insights that are not only more robust but also more aligned with the real behavior of assets within a portfolio. In particular, the study demonstrates that assets which appear risky when judged in isolation through their betas may, in fact, mitigate portfolio risk when assessed through the Shapley Value.

The second application, focusing socially responsible investments, further underlines the advantages of this approach. The results show that more sustainable companies contribute most positively to portfolio performance in stable market conditions, enhancing risk-adjusted returns and reducing downside risk. Companies with medium sustainability tend to be more resilient in volatile periods, supporting both stability and performance. In contrast, less sustainable companies add less value, especially in terms of reducing portfolio risk, which may reflect structural weaknesses or lower investor confidence. These findings suggest that sustainability can play a significant role in portfolio construction, though its benefits vary across market environments.

From a risk manager's perspective, the Value at Risk results also suggest that the conclusions depend strongly on market conditions and on the type of portfolio used. In the MVP, the most sustainable firms were less effective at reducing the Value at Risk during the more volatile period, while in the calmer year the least sustainable firms showed the weakest performance. On the other hand, in the MSRP, the least sustainable firms performed worst in the volatile year, whereas in the lower-volatility year the mid-sustainability group was the least effective at reducing the Value at Risk.

Taken together, the results of this thesis reinforce the importance of moving beyond traditional beta-based measures when evaluating portfolio risk and performance. The Shapley Value framework offers a richer and more comprehensive perspective, capturing nonlinear and context-dependent effects that are otherwise overlooked. From an investment perspective, this provides stronger guidance for building portfolios that balance risk and return in different market conditions. From a policy perspective, the findings also highlight the need for regulatory support to strengthen sustainable investment markets, particularly given the weaker financial contribution of more sustainable companies during times of crisis.

Overall, this thesis aims to contribute to the growing field of portfolio analysis and sustainable finance by showing that the Shapley Value can provide deeper insights into both traditional financial indices and sustainability-focused investments. It demonstrates that understanding marginal contributions in a more precise way is essential for investors and policymakers seeking to manage risk, enhance returns, and support the development of more sustainable financial markets.

For future work, it would be interesting to compare short-term and long-term investment strategies in order to better understand how ESG factors influence performance over different time horizons. It would also be useful to rely on several ESG rating providers instead of a single source by combining different ESG risk ratings through a weighting scheme, which could lead to a more robust and realistic composite ESG score, for example by using MSCI ESG Ratings, FTSE Russell ESG Ratings, and Bloomberg ESG Disclosure Scores. In addition, extending the sample to include more firms from a wider range of sectors would improve the representativeness of the analysis and strengthen the reliability of the results.

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A

Appendix

Table A.1: Selected Companies by Sector and ESG Risk Level

Sector	**ESG Risk Rating**	**ESG Risk Level**	**Company Name**
Oil and Gas Producers	38.4	High	Chevron Corp.
	22.0	Medium	Viper Energy Inc.
	14.1	Low	Texas Pacific Land Corp.
Energy Services	32.0	High	Forum Energy Technologies Inc.
	27.8	Medium	Aspen Aerogels Inc.
	19.1	Low	Baker Hughes Co.
Banking	35.0	High	1st Source Corp.
	29.1	Medium	Arrow Financial Corp.
	19.8	Low	BGC Group Inc.
Healthcare	30.2	High	908 Devices Inc.
	22.2	Medium	Abbott Laboratories
	18.5	Low	Addus HomeCare Corp.
Telecommunication Services	32.5	High	Anterix Inc.
	22.1	Medium	AT&T Inc.
	19.3	Low	Verizon Communications Inc.
Pharmaceuticals	35.9	High	2seventy bio Inc.
	22.5	Medium	10x Genomics Inc.
	18.4	Low	Adaptive Biotechnologies Corp.
Basic Services	32.8	High	ALLETE Inc.
	27.0	Medium	Ameren Corp.
	19.1	Low	American Water Works Co. Inc.
Building Materials	36.0	High	Eagle Materials Inc.
	23.9	Medium	Martin Marietta Materials Inc.
	17.6	Low	CRH plc
Software and Services	31.6	High	Grid Dynamics Holdings Inc.
	20.5	Medium	A10 Networks Inc.
	14.1	Low	Adobe Inc.
Transportation	34.1	High	Allegiant Travel Co.
	23.6	Medium	Air Transport Services Group Inc.
	14.7	Low	ArcBest Corp.

Table A.2: Shapley Values for Sharpe Ratio, MVP (2022)

Entry Orders	High	Medium	Low
HML	0.0064	0.0252	-0.0093
HLM	0.0064	0.0258	-0.0099
MHL	0.0100	0.0215	-0.0093
MLH	0.0079	0.0215	-0.0072
LHM	0.0213	0.0258	-0.0248
LMH	0.0079	0.0392	-0.0248
Shapley Values	**0.0100**	**0.0265**	**-0.0142**

Table A.3: Shapley Values for Sharpe Ratio, MVP (2023)

Entry Orders	High	Medium	Low
HML	0.0071	-0.0214	0.0219
HLM	0.0071	-0.0166	0.0171
MHL	0.0031	-0.0174	0.0219
MLH	-0.0002	-0.0174	0.0252
LHM	-0.0142	-0.0166	0.0384
LMH	-0.0002	-0.0306	0.0384
Shapley Values	**0.0005**	**-0.0200**	**0.0272**

Table A.4: Shapley Values for Sharpe Ratio, MSRP (2022)

Entry Orders	High	Medium	Low
HML	0.1984	0.0159	0.0510
HLM	0.1984	0.5016	-0.4348
MHL	0.0659	0.1484	0.0510
MLH	0.0441	0.1484	0.0728
LHM	-0.0542	0.5016	-0.1821
LMH	0.0441	0.4034	-0.1821
Shapley Values	**0.0828**	**0.2866**	**-0.1040**

Table A.5: Shapley Values for Sharpe Ratio, MSRP (2023)

Entry Orders	High	Medium	Low
HML	0.1471	-0.3463	0.4592
HLM	0.1471	0.0251	0.0878
MHL	-0.0369	-0.1624	0.4592
MLH	0.0279	-0.1624	0.3944
LHM	0.0285	-0.0251	0.2064
LMH	0.0279	-0.0257	0.2064
Shapley Values	**0.0569**	**-0.0992**	**0.3022**

Table A.6: Shapley Values for VaR, MVP (2022)

Entry Orders	High	Medium	Low
HML	1.8534	-0.1986	-0.0365
HLM	1.8534	-0.0970	-0.1380
MHL	-0.1812	1.8360	-0.0365
MLH	-0.1455	1.8360	-0.0722
LHM	-0.3308	-0.0970	2.0461
LMH	-0.1455	-0.2823	2.0461
Shapley Values	**0.4840**	**0.4995**	**0.6348**

Table A.7: Shapley Values for VaR, MVP (2023)

Entry Orders	High	Medium	Low
HML	1.7213	-0.2543	-0.1891
HLM	1.7213	-0.0509	-0.3926
MHL	-0.0839	1.5509	-0.1891
MLH	-0.1617	1.5509	-0.1114
LHM	-0.1287	-0.0509	1.4575
LMH	-0.1617	-0.0179	1.4575
Shapley Values	**0.4844**	**0.4547**	**0.3388**

Table A.8: Shapley Values for VaR, MSRP (2022)

Entry Orders	High	Medium	Low
HML	57.507	-47.894	6.7195
HLM	57.507	-114.95	73.778
MHL	-2.2829	11.896	6.7195
MLH	-6.6533	11.896	11.090
LHM	114.11	-114.95	17.179
LMH	-6.6533	5.8068	17.179
Shapley Values	**35.588**	**-41.366**	**22.111**

Table A.9: Shapley Values for VaR, MSRP (2023)

Entry Orders	High	Medium	Low
HML	33.572	-13.977	15.349
HLM	33.572	22.973	-21.601
MHL	6.8977	12.697	15.349
MLH	0.0047	12.697	22.242
LHM	5.1142	22.973	6.8566
LMH	0.0047	28.082	6.8566
Shapley Values	**13.194**	**14.241**	**7.5086**