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Dynamic Firm Valuation: A Real Options Approach to Strategic Reinvestment Using Schwartz & Moon

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Abstract:

This thesis evaluates the adequacy of different valuation methods in high-uncertainty environments, focusing on the dynamic internet sector. It presents an innovative valuation approach through the development of an Extended Schwartz & Moon model that integrates real options to account for conditional accelerated reinvestment periods. This adaptation better captures strategic flexibility often overlooked by traditional methods. A case study of a leading internet company, then illustrates the substantial discrepancies in valuation outcomes, highlighting the inadequacies of conventional approaches. The findings underscore the necessity for more sophisticated valuation methodologies to accurately reflect the volatile dynamics of contemporary business environments.

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Keywords:

Corporate Finance, Real Options, Valuation Methods, Investment Decisions, Directed Research, Empirical Analysis

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1 Introduction

1.1 Background

Recent years have witnessed the emergence, and disruption brought about by a new breed of firms, specifically high-growth tech companies whose business models are predominantly internet-based (Manyika, 2016). Examples of such market leaders include relatively recent entrants like Facebook and Uber. Notably, these companies have rapidly ascended to become some of the highest-valued entities globally. A distinctive feature of these firms, however, is their initial public offering (IPO) valuations. At the time of their IPOs, many of these had not yet achieved profitability. For instance, Uber went public in 2019 with losses amounting to millions, yet it was valued at $82.4 billion on its IPO date (Merced and Conger, 2019), which can be attributed to two inherent characteristics: high growth and significant market uncertainty. These attributes present unique challenges in accurately valuing such companies, which often defy conventional financial metrics and predictive models, emphasizing the need for robust valuation frameworks that can navigate the complexities of the technology-driven market.

1.2 Problem Introduction and Purpose

This phenomenon has sparked an ongoing debate over the justification for the high valuations assigned to these innovative yet unprofitable enterprises. As the sector evolves, so to does the importance of developing advanced valuation methods that can keep pace with the rapid innovation and shifting business models characteristic of tech companies. Thus, the sector not only warrants but necessitates a deeper examination to ensure that financial valuations reflect its true potential and risk, and so, enabling stakeholders to achieve their strategic objectives through more precise valuation of the companies involved. Luehrman, (1998) describes DCF analysis as a powerful tool and “the heart of most corporate capital-budgeting systems”, widely used across sectors. However, while DCF may serve mature companies with predictable cash flows, it struggles in rapidly changing industries. Traditional valuation methods, as Penman, (2013) notes, often fall short by assuming a perpetual growth rate, unrealistic in dynamic sectors. Furthermore, such methods fail to capture the impact of managerial flexibility and unpredictability, overlooking the crucial role of strategic planning in the valuation of internet-based companies (S. C. Myers, (1984)). Recognising these flaws, Real-Option theory has sought to bridge the gap between financial analysis and strategic planning. A significant contribution in this area is the Schwartz and Moon, (2000) model, which employs Monte Carlo simulations to provide Real Options Valuation (ROV) methods tailored to these challenges. This method is particularly adept at addressing the unpredictability of a firm´s development of current assets. Despite these advancements, there remains a notable absence of models that effectively measure how managerial flexibility, especially through reinvestments, impacts the intrinsic valuation of high-growth technological companies. This oversight is significant given the increasing prominence and importance of these companies in financial discussions worldwide.

1.3 Research Objectives

This thesis aims to explore the impacts of a lack of accountability for managerial flexibility, traditionally overlooked in company valuation. Initially, acknowledging the strengths and weaknesses of the Discounted Cash Flows (DCF) method, a hybrid model will employ the DCF for forecast valuation and Schwartz & Moon for Terminal calculation. Additionally, the study will extend the Schwartz & Moon ROV approach to include the optimal decision-making process regarding accelerated investment periods. Subsequently, a case study where the three methods are applied, aims to get inferences from the differences in valuation results. Ultimately, this thesis seeks to answer the research question:

Are traditional valuation methods deficient in their scope due to their failure to adequately *incorporate a more refined terminal value and managerial flexibility into their analyses? If so, can it be quantified by expanding novel Real-Option pricing models?*

1.4 Thesis Structure

My research is divided into four distinct parts, each serving a specific purpose. Section 2 presents a literature review that explores traditional valuation methods, their advantages and drawbacks, and the application of Real Options theory as a novel approach to company valuation. This chapter also delves into the importance of managerial flexibility and introduces the Schwartz and Moon, (2000) model. In Section 3, I adapt existing models to develop an Hybrid and Expanded version of the Schwartz & Moon model, with the latter incorporating accelerated investment periods as a Real-Option for companies. This adaptation is then applied, in Section 4, in a real-world scenario through a case study on Uber, assessing the model's validity. The thesis concludes with Section 5, where I discuss the results obtained and provide conclusions, respectively, offering final remarks and suggestions for further research on the topic.

2 Literature Review

2.1 The Discounted Cash Flow Model for Firm Valuation

The DCF model is celebrated for its principled approach, converting expected economic benefits into a present value metric that serves as a solid basis for investment decisions, proving to be particularly effective in sectors where future cash flows can be anticipated with greater certainty and stability. However, its application to companies experiencing irregular growth presents notable challenges. The model's reliance on a perpetual growth assumption for the Terminal Value is a critical weakness, particularly in rapidly evolving sectors such as technology. Penman, (2013) highlights that this assumption can lead to significant valuation discrepancies, as the model is highly sensitive to long-term growth projections. Moreover, the ease with which these assumptions can be adjusted, often by mere basis points, introduces a potential for manipulation in the valuation analysis (Steiger, 2010). Further compounding these limitations, Matthews, (2018) criticizes the perpetual growth assumption for overlooking the impact of creative destruction and disruptive innovation, as well as companies' eventual decline and potential failure, and ignoring these realities is bound to result in materially overstated valuations.

2.2 Real Options Literature

S. C. Myers, (1984) critically observed that "DCF is not as helpful in valuing companies with significant growth opportunities...In other words, it is not the whole answer when options account for a large fraction of a business' value." This premise sets the stage for exploring Real Options, a concept that extends the framework of traditional valuation methods to include strategic managerial decisions and future opportunities. Real Options are effectively defined by Iyer et al., (2000) as analogous to financial options. This analogy builds on foundational ideas by S. Myers, (1977) and Dixit and Pindyck, (1994), who first explored the rights but not obligations related to a company's assets. However, Real Options are also a way of thinking which helps managers equate their strategic options, specifically the future opportunities that are created by today's investments, and thus capture the value of managerial flexibility, first coined by Luehrman, (1998). In practical terms, Copeland and Antikarov, (2001) further elaborated on the flexibility provided by Real Options by categorizing them into five types: the options to defer, abandon, contract, expand, and extend. These types illustrate how Real Options are applied in real-life scenarios, showcasing their utility in providing flexibility in strategic business decisions.

Turning the focus to the intersection of valuation and strategic planning within the literature, S. C. Myers, (1984) offers a thorough exploration of the distinctions and overlaps between financial and strategic analyses. He posits that traditional financial theories, particularly Discounted Cash Flow (DCF) analysis, have been underwhelming in strategic planning. Myers argues that for DCF to be effective in strategic contexts, it must evolve and integrate strategic option values linked to new business opportunities. Concluding his discussion, the author advocates for future research in order to bridge the gap between financial theory with strategic planning. He specifically suggests expanding financial theory to incorporate option-pricing theory to model time-series interactions between investment decisions and firm valuation over time.

Further literature by Triantis and Borison, (2001) explains how in order to achieve the premise of S. C. Myers, (1984) Real Options introduce a paradigm shift in strategic decision-making. They argue that viewing projects through a "real options lens" significantly enhances managerial flexibility, enabling companies to better navigate uncertainties by recognizing and capitalizing the added value embedded in options within investments. This approach is not merely analytical but is fundamental in shaping strategic thought processes. Managers are encouraged to perceive uncertainty as a spectrum of opportunities rather than risks, fostering a proactive stance in corporate strategy formulation. They also elucidate how real options are applied in diverse scenarios, from simple project evaluations to complex, multistage investment decisions. The use of advanced option pricing models, such as binomial trees, Black-Scholes and Monte Carlo simulations, allows firms to explicitly value the strategic choices available to them, integrating investment decisions with operational and financial strategies.

Regarding the instrumentalization of strategic real options, it is imperative to observe the nuances of Dixit and Pindyck, (1994) and their broadening of corporate investments as decisions involving upfront costs with potential future benefits. To identify it the authors, formulate a triad of characteristics investments must have: *irreversibility, uncertainty, and timing flexibility*. These factors necessitate a departure from orthodox investment theories, which fail to account for the strategic timing and the option-like nature of many investment decisions. This critique underpins the theoretical motivation for adopting Real Options, which provide a framework to value flexibility and strategic decision-making in face of uncertain future outcomes. The

Real Options approach, as developed further by Dixit and Pindyck, offers not only a more robust analytical tool compared to traditional methods but also enhances the theoretical basis for integrating financial theory with strategic planning, as also suggested by S. C. Myers, (1984) and further explored in the works of Copeland and Antikarov, (2001).

The exploration of Real Options underscores their critical role in enhancing firm valuation by incorporating strategic flexibility and managerial decision-making under uncertainty (Leslie and Michaels, 1997). This approach is essential, especially in industries where investment conditions are volatile and the potential for technological advancements is high. Transitioning to the Schwartz & Moon model, this thesis will delve into how such methods integrate Real Options to offer more nuanced valuation techniques that are vital for high-growth firms in the modern economic landscape.

2.3 Schwartz & Moon Model (2000)

The authors develop a simulation-based pricing model for high-growth companies through real options valuation and capital budgeting techniques, during a time when internet stocks were being subjected to sky-rocketing valuations, despite many generating losses and having little intrinsic value to show. This topic "elicited" strong feelings in the finance community, creating the preposition and research question behind the Schwartz and Moon, (2000) model. With it, the authors aimed to create a valuation method that would incorporate the growth potential inherent to the technology industry, which traditional valuation methods have failed to capture. A clear advantage over the DCF methodology for company valuation is its autonomy of subjective growth forecasts for the highly important terminal valuation, often overlooked in traditional valuation methods. Secondly, by using a Monte-Carlo simulation approach it also provides a more thorough analysis of the potential outcomes that a simple sensitivity analysis fails to (Schosser and Ströbele, (2019)). Ultimately, the model can rationalize the apparent irrationality behind the high valuations of Internet Companies at the time, by demonstrating that under high enough growth rates and volatilities of certain parameters, these valuations would be able to be numerically justifiable. Further research on this model, conducted by Klobucnik and Sievers, (2013), infers that the model can indicate severe market fundamental misevaluations, demonstrating the investment value of a trading strategy based on Schwartz and Moon, and confirming its applicability.

2.3.1 Walkthrough

In the paper from 2000, the authors produce the model in both a discrete and continuous time approximation, provide guidelines on how to estimate the parameters (Figure A.1) and follow it with a case study evaluation and sensitivity analysis. Starting in the formulation of the model, it has two stochastic sources of uncertainty, the revenues (R_t) and growth rate in revenues (μ_t). Besides this, the model presents four deterministic, path-dependent variables, the net after-tax rate of cash flow to the company (Y_t), the cost at time t ($Costs_t$), the loss carry-forward (L_t) and available cash (X_t). The model begins by deriving the dynamics of a firm´s Revenue and Revenue growth components, using empiric probability measures. These will later suffer a risk adjustment process to ensure that the model's forecasts and valuations are consistent with a risk-neutral valuation framework. The drift-adjusted processes are therefore formulated by the following stochastic differential equation:

$$\frac{dR_t}{R} = (\mu_t - \lambda_1 \sigma_t)dt + \sigma_t dz_1^* \tag{2.1}$$

where R_t denotes the instantaneous rate of revenues at time t, μ_t is the expected rate of growth in revenues, but is subject to random shocks dz_1 (increment of the Wiener Process), with σ_t measuring the intensity of these shocks. It reflects the unpredictable nature of Internet company revenues, which, while expected to grow, experience considerable volatility due to various factors such as market competition, technological changes, and consumer preferences. Young internet firms often experience exceptionally high growth rates at first, which are expected to stochastically settle into a more sustainable and industry-appropriate pace of expansion. Hence, it's posited that the growth trend, or drift μ_t, adheres to a mean-reverting process, characterized by an industry long-term average rate ($\overline{\mu}$) (Schwartz and Moon, (2000)):

$$\mu_t = [\kappa(\overline{\mu} - \mu_t) - \lambda_2 \eta_t]\, dt + \eta_t dz_2^* \tag{2.2}$$

Where $\eta(t)$ is the volatility of expected growth in revenues and κ the mean-reversion coefficient of the stochastic process, affecting how fast the firm converges to the long-term growth level. The Wiener process component dz_2 introduces randomness, acknowledging that the path towards the industry-level mean can be unpredictable. The risk adjustment process in the Schwartz & Moon model is based on Merton, (1973) ICAPM market price of risk principles. Brennan and Schwartz, (1985) enhanced this method by linking the market price of risk to the correlation between an asset's returns and market volatility. Schwartz and Moon, (2001) further refine this by focusing the adjustment solely on revenue fluctuations, arguing that this uncertainty should carry a risk premium (λ_R). They simplify the calculation of the market price of risk for the revenue stream using the stock's beta, which streamlines the process and improves the model's practical relevance. Then the authors model the volatility $d\sigma_t$ of the revenue growth rate as a mean-reverting process, where κ_1 represents the rate of reversion towards the long-term average volatility $\overline{\sigma}$:

$$d\sigma_t = \kappa_1(\overline{\sigma} - \sigma_t)dt \tag{2.3}$$

The formulation on the volatility of the expected rate of growth in revenues η_t also follows a mean-reverting process towards zero over time, with κ_2 being the rate of reversion:

$$d\eta_t = \kappa_2 \eta_t dt \tag{2.4}$$

In both equations, the Ornstein-Uhlenbeck process comes into play, where it tends towards a long-term mean over time, making it particularly useful for modelling variables that fluctuate around a stable long-term average. Furthermore, the model specifies that the Brownian motions, from equations (2.1) and (2.2) may be correlated:

$$dz_1 dz_2 = \rho dt \tag{2.5}$$

In essence, the model simulates revenues and expected growth of revenues up to a pre-determined time T. With these values, it allows for the calculation of the deterministic components of the model, which in turn cascade with each other to finally determine the value of the firm. This will have one component, expected net cash flow at time t, X_t, which once discounted under the equivalent martingale measure E_Q, indicates the model's usage of risk-neutral probabilities to discount the expected value of the firm:

$$V_0 = E_Q(X_t e^{-rT}) \tag{2.6}$$

2.4 Gap Identification

Influenced by Berk et al., (1999) model, developed to describe investment decision-making by companies and its valuation effects on a firm's cash flows resulting from the conditional growth options captured and optimal decision-making, while recognizing the limitations of the advanced processes from their work, my research proposes to bridge this gap with a hybrid model that integrates the valuation advantages of both DCF and the Schwartz & Moon approach, to offer a balance of complexity and usability, emphasizing the strategic elements of firm valuation. Triantis and Borison, (2001) emphasize that the widespread acceptance of real options hinges on a critical question: "Does real options help managers make better decisions—decisions that end up creating more wealth for the firm´s shareholders?" They note a prevailing sentiment across industries suggesting that the answer is affirmative. This research aims to build on this preliminary consensus by providing empirical evidence to substantiate the claim that real options, indeed, enhance managerial decision-making and shareholder wealth.

3 Methodology

3.1 Hybrid Model Development

In the realm of business valuation, particularly for internet firms characterized by high and volatile growth rates, traditional models often struggle to capture the full spectrum of value dynamics. As thoroughly discussed in the literature review the Discounted Cash Flow (DCF) model, despite its extensive use, has certain limitations that may not fully accommodate the dynamics of fast-evolving sectors such as technology.

The Schwartz & Moon model incorporates real options valuation, which is advantageous for valuing firms in the technology sector that face high uncertainty and have significant growth potential. This model acknowledges the real options regarding the development of current assets, allowing for a more flexible approach to firm valuation under uncertainty.

However, the Schwartz & Moon model can be complex and require extensive data, which may not be available for newer firms or those without substantial historical performance data and, as noted by Schwartz and Moon, (2001), this model can introduce a speculative element to valuation, heavily influenced by market volatility and investor sentiment, potentially leading to unstable valuations.

Lastly, an important aspect, and main motto for my extension of this model, comes from its critics regarding how it excludes the valuation of strategic options. Internet companies will present uncertainty regarding their growth and expansion options, which this model will fail to value. As Malmcrona and Andersson, (2018) expose, this fact can explain the shortcomings regarding empirical tests of the model that yields low prices compared to the market valuation.

Rationale for a Hybrid Approach: Combining the DCF and Expanded Schwartz & Moon models addresses these challenges by blending the structured, cash-flow-based approach of the DCF with the flexibility of the real options approach from the Schwartz & Moon model, together with mathematical adjustments to it, to try and capture strategic real options by modelling accelerated firm reinvestment periods. This hybrid model is particularly suited for internet firms, providing a balanced, realistic valuation method that captures both predictable cash flows and the potential impacts of strategic decisions and market changes.

I aim not to discard the DCF model entirely by relying solely on the Schwartz & Moon model for valuation. Instead, my goal is to develop a hybrid model that leverages the strengths of both approaches. This will enhance the calculation of the terminal value in a DCF by improving the perpetuity growth assumption, aligning with the primary objective of my research.

$$Forecasted\ DCF = \sum_{t=0}^{n} \frac{FCF_t}{(1+WACC)^t} \tag{3.1}$$

$$Terminal\ Value\ from\ Schwartz\ \&\ Moon = E_Q(X_{t_5} e^{-rt}) \tag{3.2}$$

$$Company\ Value = 3.1 + 3.2 \tag{3.3}$$

Here, equation term (3.1) represents the DCF component, and it is the sum of the forecasted discounted free cash flows up to the pre-determined t before the terminal value calculation. Consequently, the value of (3.2) is calculated through the Schwartz & Moon model adjusted to take in parameters from the forecasted t financials and to include a conditional providing a real option for an accelerated reinvestment period, thus laying out the terminal value. When summed both components return the present value of the company.

3.2 Accelerated Reinvestment Period Real-Option

Introducing one of the more innovative approaches to ROV firm valuation is essential before presenting my methodology. Berk et al., (1999) denote that "the random evolution of the firm's collection of projects determines how its risk and expected returns change over time". In essence, the model ties firm-specific variables to changes in a firm's risk profile. As firms seize investment opportunities, their systematic risk evolves, impacting their expected returns. By integrating the dynamics of firm-specific investment opportunities, this model provides a sophisticated tool to analyse how corporate policies influence firm valuation. Looking at their empirical valuation function the authors value the firm as the future cash flows of all projects still alive plus the value of future investment opportunities, and so accounting for current assets uncertainty and strategic real options:

$$P(t) = \sum_{j=0}^{t} V_j(t)\chi_j(t) + V^*(t) \tag{3.4}$$

Building on this foundation, my adaptation of the Schwartz & Moon model is inspired by Triantis and Borison, (2001) insights into the future of real options implementation. They suggest that "over long periods, companies that make better decisions tend to prosper relative to those with deficient decision processes ... if firms can create options and then monetize their value by divesting assets or businesses, the gains from creating options at a discount to their value will be more quickly recognized". This prompted the idea that testing the model on a single company would provide clear results. Therefore, the methodology will start with a conventional valuation method, integrate it with the hybrid Schwartz & Moon model, and then apply my adjusted model that accounts for optimal reinvestment real options. This approach is expected to demonstrate value creation when compared with the first two models, given the presence of strategic options.

3.2.1 Expanding the Model

To incorporate strategic options into the Schwartz & Moon model, it is necessary to integrate state-dependent growth rates and investment periods. This enhancement helps in assessing the optimal timing and scale of investment decisions within uncertain financial environments by directly linking them to forecasted financial metrics, thereby measuring their impact on the overall valuation of an individual firm.

To operationalize this dynamic environment, the mathematical model will employ two distinct operational states. The "free" state, where the firm operates under normal conditions and adheres to the baseline S&M model kinetics without additional investments. Conversely, the "drived" state activates when the firm decides to initiate an accelerated reinvestment phase, which involves injecting additional capital for further investments. This transition will directly adjust the expected growth rate of revenues, and the cost equation. Indirectly, the changes in the drift rate will affect the revenue, influencing the rest of the deterministic processes that follow (Algorithm 1). In the "free" state, the drift rate (μ_t) will evolve normally according to the SDE:

$$\mu_t = [\kappa(\overline{\mu} - \mu_t) - \lambda_2\eta_t]\, dt + \eta_t dz_2^* \tag{3.5}$$

But, when in a “drived” state, μ_t is adjusted to account for the impact that the additional investments have on the growth rate of revenues, measured by θ, which will be the proportion of investments over the revenues at t, meaning it will also evolve over time in absolute terms. This adjustment means that the drift will then be represented by:

$$\mu_t = [\kappa_2\theta + \kappa(\overline{\mu} - \mu_t) - \lambda_2\eta_t]\, dt + \eta_t dz_2^* \tag{3.6}$$

Here κ_2 is a new mean reversion coefficient which will explain how efficient will the additional investments translate to higher rates of growth in revenues. Inherent to both possible operational states is the transition rule to switch from one to the other: $Transition\ Rule = \mu_t > \mu^*$.

The firm will now shift from its initial state to “drived” whenever μ_t surpasses the critical threshold of μ^*, which marks the point at which the expected benefits from reinvestment outweigh the operational risks and costs. After reaching this point, the firm enters the “drived” state where it remains for a period of G. This duration is crucial as it represents the time-frame during which enhanced financial metrics-originating from new projects, increased R&D, and other initiatives-are expected to influence the firm’s operations.

This will also directly affect the cost equation where the additional investments will have to be accounted for, by an increase in variable costs incurred at t. A discussion point is if a firm in a “drived” state will continue with the same variable costs and have additional reinvestment costs, or as I interpret it, following Triantis and Borison, (2001), “firms are able to create options and then monetize their value by divesting assets or businesses the gains from creating options at a discount to their value will be more quickly recognized”, thus welcoming the possibility of divesting from less productive costs to incur the additional investment. Thus, in a “free” state the costs equation remains the same, but in a “drived” one is now represented by:

$$Costs_t = (\theta + \alpha + \beta)R_t + F \tag{3.7}$$

The deterministic components will be equated in the same manner across different states; however, they will produce distinct dynamic results due to the cascading effect of the sequential equations leading up to the value function:

$$V_0 = E_Q(X_t e^{-rT}) \tag{3.8}$$

To conclude, this transition rule essentially transforms the decision to accelerate reinvestments in a real-option. This threshold acts similarly to the strike price in a financial option, delineating the point at which it becomes advantageous for the firm to switch states. Moreover, the conditional execution makes it that the decision is not mandatory, however when incurred it´s irreversible until $t + G$, embedding the cost of *irreversibility* in the decision-making process. Furthermore, *uncertainty* management is present in the model by allowing the firm to defer or accelerate investment based on observable and forecastable financial metrics, optimizing the timing of investments in response to internal performance indicators. Furthermore, *Timing Flexibility*, ensures that the firm is not obligated to make the investment at any specific time but can instead wait until the trigger is reached, thus timing until favourable conditions are sustained enough to justify the increased investment. Once this period expires, much like an option's expiration, the benefits of accelerated investment are assessed, and the flexibility to revert to the "free state" becomes available. Therefore, this model encapsulates the three fundamental characteristics of Real Options, as laid out by Dixit and Pindyck, (1994), completing a robust framework for strategic investment decision-making.

3.3 Methodology Scenarios

The most fitting scenario involves applying the three models to a current high-growth technology company, mirroring the original Schwartz and Moon, (2000) research conditions on Amazon. This approach allows for direct comparisons between the outcomes of three distinct valuation models: the conventional Discounted Cash Flow (DCF), the hybrid Schwartz & Moon model, and the strategic real option-adjusted expanded model. The choice of a modern Internet company for the case study is strategic, as it enables an assessment of each model's adaptability and performance in today's volatile economic landscape. This three-way comparison not only tests each model's capacity to capture complex value drivers, typical of tech companies, but also highlights the potential enhancements of the hybrid and adjusted expanded models over traditional DCF valuation methods. Such an approach aligns with previous academic works that have employed the Schwartz & Moon model, reinforcing its validity (Schwartz and Moon, (2001); Malmcrona and Andersson, (2018); Schosser and Ströbele, (2019)). Furthermore, using an ongoing business for the study ensures that the findings are immediately relevant and can be validated or expanded upon in future research, underscoring its practical relevance and robust results for real-world financial analysis.

4 Valuation Case Study

4.1 Uber Company Presentation - Summary

In 2009, Uber began with the straightforward concept of tapping a button to get a ride, and quickly evolved into a major player in the ride-hailing market. Now, it operates across 70 countries, with a user base of 137 million people who use Uber at least once a month.

Uber's market cap has seen dramatic growth over the years, standing at $\$144.04$ billion as of April 2024. This reflects a substantial increase of almost 128% over the previous year (Uber, (2024)). Moreover, as per Figure A.4, 2023 marked the first full year that Uber achieved a net income profit, totalling $\$1.8$ billion. This milestone represents the beginning of the firm's consolidation process, whose financial success underscores Uber's evolution from a novel start-up to a major corporation in the global transportation and delivery market.

As of today, Uber is the undisputed leader in the $\$154$ billion ride-hailing market (Figure A.2). Uber, (2024) annual report outlines its strategy to cement its market leadership and capitalise on these growth projections, which is segmented into five action areas: enhancing margins through cost structure discipline, leveraging leading technology to maintain a cutting-edge supply engine, expanding consumer reach by entering underdeveloped markets, such as airport trips, and new demographics. Additionally, maintaining a balanced product portfolio is crucial for sustained growth in revenue and profits. Finally, the company is investing in the Autonomous Vehicle (AV) sector, aiming to shape the autonomous future of mobility.

4.2 Uber Hybrid Valuation

4.2.1 Discounted Cash Flows

Regarding what will be the baseline scenario of Uber´s valuation some points are important to define. First, a 5-year forecasting period is used, given that longer periods would mean weaker assumptions. As for the forecasts, a detailed methodology is described in section 4.2.2. The WACC used in discounts is then thoroughly calculated (Figure A.10) to be 9.57%. All of this together originates a projected period valuation of $\$30.6$ billion (Eq: 3.1). Lastly, an annual perpetual growth rate of 4% is used for the terminal valuation, in line with the global economy.

4.2.2 Schwartz & Moon component

The outcomes of the valuation approach are fundamentally influenced by the assumptions regarding future revenues, growth rates, and costs. The following section details the methodology employed to estimate the parameters required to conduct the Monte Carlo simulation from the Schwartz and Moon, (2000) model for Uber. A futuristic approach is needed given the particularity of the hybrid model, which simulates paths from the start of the terminal value calculation, unlike the original model. Given that most forecasts are typically provided on an annual basis, the simulations are conducted annually (Schwartz and Moon, (2001)).

Revenue Dynamics: The initial revenue for the simulation is set at $\$77,256$ billion, based on projected 2028 figures (Figure A.8). Despite a historical average growth rate of 54%, a more conservative forecast of 15.7% is adopted for the next five years based on analyst expectations, tapering to 10.98% as Uber approaches maturity, aligning the long-term rate with the 5-year average for mature "Online Services" companies. Initial revenue volatility, following methodologies from Schwartz and Moon, (2000) and Schosser and Ströbele, (2019), is derived from the standard deviation of forecasted revenue changes up to FY28, with Uber's stock price historical volatility providing a 23% initial rate (Figure A.15). This volatility is expected to decrease to 8.4% over time, reflecting the typical reduction observed in more mature counterparts.

Costs Dynamics: The Cost of Goods Sold as a percentage of revenues (α) is projected at 47% for FY28, derived from dividing forecasted COGS by revenues (Figure A.8). This forecast is based on analyst estimates, adjusting downward from 2023 levels to reflect a trend of decreasing COGS relative to revenue. The Fixed Component of other expenses (F) for FY28 is estimated at $\$14,203$ billion, calculated to be a consistent 18% of revenues. The Variable component of other expenses (β), representing additional variable costs, is set at 11% of revenues for the forecast period.

Speed of Adjustments, Correlations and Half-Time Deviations: As per Schwartz and Moon, (2000), all speed of adjustment coefficients are set equally and estimated based on the half-life of deviations from the long-term growth rate. Doffou, (2015) notes that the half-life of growth rates in highly competitive industries is typically shorter, as sustained above-average growth becomes difficult due to competitive pressures. This concept is particularly relevant to Uber, operating in the competitive ride-hailing and food delivery sectors. While intense competition may limit long-term growth, the rapid expansion of these markets could still allow for significant short-term gains. Based on Schosser and Ströbele, (2019) discussion regarding their case of Facebook, and the empirical study conducted on the calculation of optimal mean reversion coefficients, Uber follows their rationale and so a coefficient of 0,36 per annum is set. This therefore means a half-life of 1.93 years. Given that Uber´s scenario takes place in 2028, the assumption of a short time taken to reach industry-level growth rates seems reasonable.

Initial Cash Balance Available & Loss-Carry Forward: The FY23 value for cash and cash equivalents is sourced from Uber's 2023 annual report. Given the fluctuations observed in historical data and a consistent decrease in its percentage of revenues, it is projected that these trends will continue. Thus, the initial cash balance for the 2028 forecast is set at $\$5,786$ billion, based on the 8-year historical average, capturing both the fluctuation and the anticipated continued decline in revenue percentage (Figure A.14). The same dynamics and rationale come into play for the Initial Loss-Carry Forward used in the simulation, with its calculation for 2028 assuming it will consistently equal the historical average of $\$6,164$ billion.

Environmental and Risk Parameters: The corporate tax rate used in the simulations was taken directly from the company's annual report with a value of 25%. Furthermore, the risk-free interest rate is given by the 5-year US treasury bill equalling 3.75%. The market price of risk of the revenue process, is a parameter not directly observable and is implied from the beta of the stock, as detailed in Schwartz and Moon, (2001). Figure A.16 illustrates the regression used to estimate the beta using weekly stock returns for Uber and the MSCI world index, both because Uber operates globally and because it attracts investors from a diverse array of countries, from January 2020 to March 2024. Then the adjusted beta from this period is 1.06. Using the formulation provided in their revised model from 2001 it is possible to calculate the risk premium for the revenue factor of Uber, by multiplying its adjusted beta by the market risk premium ($\lambda_t = \beta_R(r_m - r)$).

Simulation Parameters: The simulations use a yearly time increment over a 20-year horizon, adjusted from the 25-year span typical in Schwartz & Moon model applications to account for the hybrid model's start at year 5. Consistent with previous studies, 10,000 simulations are run. These simulations are conducted using Wolfram's Mathematica due to its robust capabilities in solving stochastic differential equations and academic acceptance.

4.3 Strategic Real-Option to Accelerate Investments Setup

To extend the model, the new set of parameters, introduced in section 3.2, need to be accurately estimated, given that valuation outcomes are going to be fundamentally affected by it. In determining the reinvestment rate (θ), a value of 0.3 was selected, implying that 30% of the firm's revenues are reinvested back into Uber during periods of accelerated investment. This value was derived from an analysis of the company's historical reinvestment rates, which oscillated between 9% and 65%. Additionally, an optimization was performed to ascertain which rate, under *ceteris paribus* conditions, would maximize the company's value (Figure A.18). Upon careful examination, this value appears aggressive enough to fund substantial growth initiatives, such as technology upgrades or market expansion, without jeopardizing financial stability.

Then the state-transition threshold μ^* is set at 0.125, thus a growth rate of revenues of 12.5% serves as the trigger point where Uber decides that its operating conditions are favorable enough to justify increased investments. This value is strategically chosen to lie between Uber's initial growth rate (15.7%) and the industry mean (10.98%), balancing sensitivity to avoid frequent triggering and over-investment, and conservativeness to not miss growth opportunities. Optimizing μ^* in the context of real options models would typically involve using empirical data to find a value that maximizes strategic benefits. However, given the limited span of data and the technical complexity of the dynamic model, a programmatic optimization of μ^* was not feasible. Instead, the threshold was set by manually testing different values within the feasible range to, *ceteris paribus*, observe their impacts on the model's outcomes. This approach, while less precise than a fully programmatic optimization, allowed for a practical assessment of μ^* under the given constraints, leaving room for further research to model this programmatic optimization.

Regarding the duration of the Accelerated Investments (G), a 2-year period allows enough time for substantial projects to be implemented and start showing results, yet it's short enough to avoid long-term commitment to potentially unprofitable strategies. For Uber, this might cover completing some of the strategic action areas laid out in section 4.1, like entering underdeveloped markets and investing in autonomous mobility.

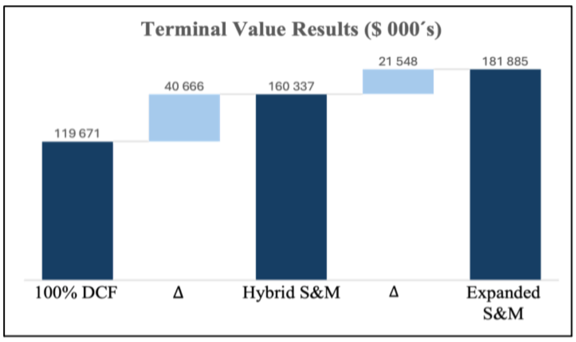
Lastly, a new mean reverting process k_2 of 0.36 is chosen to give theta, in the "drived-state" μ_t

the same weight, i.e., effect as the other variables. To test this value a regression analysis between reinvestment and revenue growth rate was conducted, presenting a coefficient of 0.47, but given its low R^2 (Figure A.6) it becomes no more than a good exercise to validate the value used.

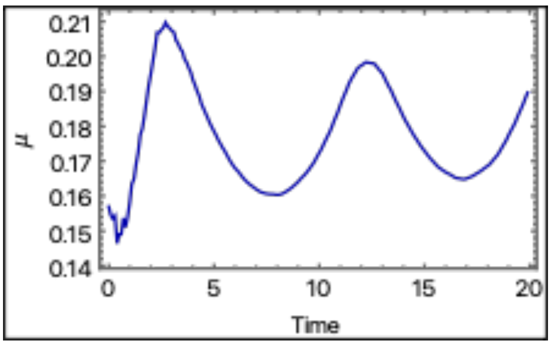
4.4 Valuation Results

Implementing the hybrid Schwartz & Moon yields a mean simulation Enterprise Value (EV) of $\$190$ billion. Furthermore, the Expanded model, which includes the reinvestment strategic option, yields an even higher mean simulation valuation with EV amounting to $\$212$ billion. Considering the DCF valuation of $\$150$ billion (Figure A.12), the potential development of current assets captures an additional $\$40$ billion, while managerial flexibility from accelerated investments adds $\$20, 5$ billion more.

Translating this into stock price comparisons, these differences are significant. The main benchmark is Uber's closing price on March 20th, $\$78.67$, the date the financial information was retrieved. The DCF model presents a price target of $\$70.38$, indicating Uber is overvalued. In contrast, the Hybrid and Expanded models offer more optimistic scenarios with price targets of $\$90.01$ and $\$100.42$, respectively (Figure A.17). Thus, Real-Option pricing identifies hidden value not captured by traditional methods, arising from both the present value of current assets and strategic growth options.



(a) Terminal Value Results



(b) Growth Rate of Revenues

Figure 4.1: (a) Shows the valuation results and differences between the 3 models; (b) Presents the dynamic evolution of the growth rate parameter, when subject to Reinvestment Real-Option.

Figure 4.1b displaying the dynamic evolution of the growth rate of revenues, (μ_t) under the adjusted Schwartz & Moon model is then the most revealing result. It reflects the implementation of strategic reinvestment options. Initially, it follows the baseline model kinetics in the "free" state, where no additional investments influence the firm's operations, representing the normal operational growth rate. As the simulation progresses, a significant upward shift in μ_t occurs when the firm transitions to the "drived" state-triggered when μ_t exceeds the critical threshold μ^*. From here onwards, the state-shifting dynamics are clearly represented, where after a period G, it reverts back to the free state. This repeats itself, adhering to the transition rules, until the end of the observation period at $T = 20$, showcasing the firm's ability to dynamically manage the investment timing real option in response to changing conditions.

5 Discussion and Conclusion

Empirical Implications: The empirical findings from this study explicitly addresses the research question excerpt: "If so, can it be quantified by expanding novel Real-Option pricing models?". The case study reveals a significant discrepancy of $\$40$ billion in favour of the Hybrid Schwartz & Moon model compared to traditional DCF valuations. This disparity underscores the substantial present value of current assets that traditional methods fail to capture, particularly due to their oversimplified perpetual growth assumptions.

Further, the Expanded Schwartz & Moon model, enhanced with strategic reinvestment options, offers additional insights to the forementioned excerpt. This model significantly shifts the paradigm of Schwartz & Moon diffusion process simulations by successfully incorporating conditional accelerated reinvestment periods. Analysis reveals a $\$21$ billion valuation increase for the expanded model over the hybrid, with a 28% higher target share price compared to the benchmark, demonstrating the hidden value potential that strategic real options integration unlocks.

These findings clearly quantify the added value provided by novel Real-Option pricing models, validating the model's effectiveness in capturing complex dynamic investment opportunities overlooked by traditional methods. This enables firms like Uber to make more informed strategic decisions regarding future investments, while providing finance professionals a framework to rationalize them in their recommendations.

Theoretical Findings: Building on the results beforehand, also results in several theoretical inferences that conclusively answer the first part of the research question of this thesis. Triantis and Borison, (2001) insights are validated, by demonstrating that firms with adept management of strategic options exhibit superior long-term performance. The dynamics of μ_t in the Expanded Schwartz & Moon model, which integrates strategic reinvestment decisions, explicitly showcase this effect. As μ_t evolves, the dynamic effects of "free" and "drived" states, reveals not just an increase in revenue growth rates but also uncovers hidden value that traditional valuation models overlook. This detailed observation of μ_t behaviour provides a quantifiable measure of how strategic flexibility can uncover opportunities and enhance firm valuation. These insights suggest that deeper understanding and optimization of such strategic decision-making processes essentially fulfills the premise laid by S. C. Myers, (1984), and it proves to bridge the gap between finance theory and strategic planning by using "option pricing theory to model the time-series interactions between investments".

Additionally, the adjustment of Schwartz & Moon addresses its main criticism of focusing solely on the development of current assets, which often leads to undervaluation. By expanding the model to include the reinvestment real option, it enhances the theoretical underpinnings while closely aligning it with real-world business conditions.

The last theoretical finding of this study tackles the critique by Matthews, (2018) that traditional models often overlook bankruptcy risks. By integrating the Schwartz & Moon model, that explicitly accounts for bankruptcy, it improves the model's realism in assessing distress, while maintaining the strongpoints of DCF.

Limitations: Limitations regarding the hybrid model mainly fall under the existing weaknesses of Schwartz & Moon, and it´s reliance on calculated input parameters, with the added fact that in an hybrid model the parameters taken will be assumptions of the short-term future development of a firm. As for the Expanded model, its main limitation is the subjectivity of μ^* and k_2, given the lack of data available from Uber and comparables to optimize the threshold condition, and thoroughly understand the correlation between reinvestments and revenue growth.

To address these issues while harnessing the potential of the model, a general algorithm has been developed that operates robustly with any external stimuli. This algorithm enhances the model's adaptability and is designed to work effectively across different conditions, providing a solid foundation for future research to further refine and optimize parameter selection. Future studies should aim to validate this model across a broader pool of firms, akin to Klobucnik and Sievers, (2013) approach. The use of cross-sectional data could also further enhance the accuracy of parameter estimations.

Conclusion: On a final note, this research conclusively addresses whether traditional valuation methods are adequate for assessing firms with dynamic and rapidly evolving business models, particularly in the technology sector. By quantifying these aspects through advanced Real-Option pricing adjustments, the model provides a more precise and reflective measure of firm value. It effectively changes the paradigm of option pricing in firm valuation, rationalizing the valuations behind the apparent irrationality inherent to internet companies, proving especially crucial in a sector demanding a more sophisticated appraisal approach, ensuring that firms are valued not just on static metrics but on their real strategic potential.

References

Berk, J. B., Green, R. C., & Naik, V. ((1999)). Optimal investment, growth options, and security returns. *The Journal of Finance*, *54*(5), 1553–1607.

Boston Consulting Group. (2023). *MA Is Looking Up After Bottoming Out* [Accessed 27th of April, from:]. https://www.bcg.com/publications/2023/m-and-a-outlook-looking-up-after-bottoming-out

Branch, M. A. ((2003)). *Real options in practice*. John Wiley Sons.

Brennan, M. J., & Schwartz, E. S. ((1985)). Evaluating natural resource investments. *The Journal of Business*, *58*(2), 135–157.

Buckley, A., Tse, K., Rijken, H., & Eijgenhuijsen, H. (2002). Stock market valuation with real options: *European Management Journal*, *20*, 512–526.

Copeland, T., & Antikarov, V. ((2001)). Real options: A practitioner's guide. *Texere*.

Dixit, A. K., & Pindyck, R. S. ((1994)). *Investment under uncertainty*. Princeton University Press.

Doffou, A. ((2015)). An improved valuation model for technology companies. *International Journal of Financial Studies*, *3*, 162–176.

Gamba, A. (2003). Real options valuation: A monte carlo approach. *SSRN Electronic Journal*.

Iyer, B., Kulatilaka, N., & Storck, J. ((2000)). Managing information technology investment using a real-options approach. *The Journal of Strategic Information Systems*, *9*, 39–62.

Klobucnik, J., & Sievers, S. ((2013)). Valuing high technology growth firms. *Journal of Business Economics*, *83*, 947–984.

Kulatilaka, N., & Marks, S. (1988). The strategic value of flexibility: Reducing the ability to compromise. *American Economic Review*, *78*, 574–80.

Leslie, K., & Michaels, M. (1997). The real power of real options. *The Mckinsey Quarterly*, *3*.

Longstaff, F., & Schwartz, E. (2001). Valuing american options by simulation: A simple least-squares approach. *Review of Financial Studies*, *14*, 113–47.

LSEG Data & Analytics. (2024). *LSEG* [Accessed 18th of March 2024, from: [online]. Available at: Subscription Service].

Luehrman, T. A. ((1998)). Strategy as a portfolio of real options. *Harvard Business Review, September-October*, (98506).

Malmcrona, A., & Andersson, M. ((2018)). Real option valuation of high growth tech firms. *Copenhagen Business School*.

Manyika, J. (2016). Digital globalization: The new era of global flows. *McKinsey Global Institute*.

Matthews, G. ((2018)). Creative destruction and the perpetual growth assumption. *37*, 150–157.

Merced, M., & Conger, K. (2019). *Uber I.P.O. Values Ride-Hailing Giant at* $82.4 Billion$ [Accessed 27th of April, from:]. https://www.nytimes.com/2019/05/09/technology/uber-ipo-stock-price.html

Merton, R. C. ((1973)). An intertemporal capital asset pricing model. *Econometrica*, *41*(5), 867–887.

Myers, S. ((1977)). The determinants of corporate borrowings. *Journal of Financial Economic*, *5*, 147–175.

Myers, S. C. ((1984)). The capital structure puzzle. *The Journal of Finance*, *39*(3), 574–592.

Pástor, Ľ., & Veronesi, P. (2006). Was there a nasdaq bubble in the late 1990s? *Journal of Financial Economics*, *81*, 61–100.

Penman, S. H. ((2013)). *Financial statement analysis and security valuation* (Vol. 5th edition). McGraw Hill:

Schosser, J., & Ströbele, H. ((2019)). What is the value of facebook? evidence from the schwartz/moon model. *The Journal of Risk Finance*, *20*, 267–290.

Schwartz, E. S., & Moon, M. ((2000)). Rational pricing of internet companies. *Financial Analysts Journal*, *56*(3), 62–75.

Schwartz, E. S., & Moon, M. ((2001)). Rational pricing of internet companies revisited. *The Financial Review*, *36*, 7–26.

Statista. ((2023)). *Makret Update: market data analysis* [Accessed 8th of March 2024, from:]. https://www.statista.com/outlook/mmo/shared-mobility/ride-hailing/worldwide#methodology

Steiger, F. (2010). The validity of company valuation using discounted cash flow methods. *arXiv.org, Quantitative Finance Papers*.

Triantis, A., & Borison, A. ((2001)). Real options: State of the practice. *Journal of Applied Corporate Finance*, *14*(2), 8–24.

Uber. ((2024)). *Investor Update February 2024* [Accessed 5th of March 2024, from:]. https://investor.uber.com/news-events/events-and-presentations/event-details/2024/Uber-2024-Investor-Update-2024-du4f3lzxqk/default.aspx

van Aarle, R. (2013). A real-options approach to company valuation. *University of Twente*.

Zeng, S., & Zhang, S. (2011). Real options literature review. *iBusiness*, *3*, 43–48.

Appendices

A Figures

Exhibit 1. Key Parameters of the Model

Parameter	Notation	Proposed Estimation Procedure
Initial revenue	R_0	Observable from current income statement
Initial loss carry-forward	L_0	Observable from current balance sheet
Initial cash balance available	X_0	Observable from current balance sheet
Initial expected rate of growth in revenues	μ_0	From past income statements and projections of future growth
Initial volatility of revenues	σ_0	Standard deviation of percentage change in revenues over the recent past
Initial volatility of expected rates of growth in revenues	η_0	Inferred from market volatility of stock price
Correlation between percentage change in revenue and change in expected rate of growth	ρ	Estimated from past company or cross-sectional data
Long-term rate of growth in revenues	$\bar{\mu}$	Rate of growth in revenues for a stable company in the same industry as the company being valued
Long-term volatility of the rate of growth in revenues	$\bar{\sigma}$	Volatility of percentage changes in revenues for a stable company in the same industry as the company being valued
Company's corporate tax rate	τ_c	Observable from tax code
Risk-free interest rate	r	One year U.S. T-bill rate
Speed of adjustment for the rate of growth process	κ	Estimated from assumptions about the half-life of the process to $\bar{\mu}$
Speed of adjustment for the volatility of revenue process	κ_1	Estimated from assumptions about the half-life of the process to $\bar{\sigma}$
Speed of adjustment for the volatility of the rate of growth process	κ_2	Estimated from assumptions about the half-life of the process to zero
COGS as a percentage of revenues	α	Analysts' future projections
Fixed component of other expenses	F	Analysts' future projections
Variable component of other expenses	β	Analysts' future projections
Market price of risk for the revenue factor	λ_1	Obtained from the product of the correlation between percentage changes in revenues and return on aggregate wealth multiplied by the standard deviation of aggregate wealth
Market price of risk for the expected rate of growth in revenues factor	λ_2	Obtained from the product of the correlation between changes in growth rates in revenues and return on aggregate wealth multiplied by the standard deviation of aggregate wealth
Horizon for the estimation	T	An arbitrary long-term horizon at which the company is deemed to become a "normal" company
Time increment for the discrete version of the model	Δt	Chosen according to data availability, which is usually quarterly

Figure A.1: Schwartz & Moon Proposed Estimation of Parameters

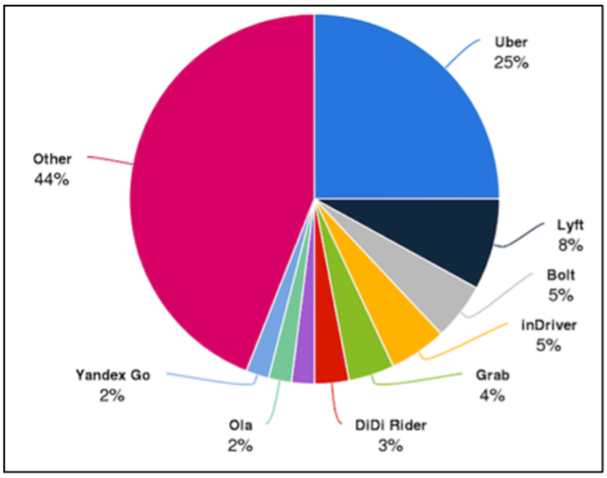


Figure A.2: Ride-Hailing Industry Market Share

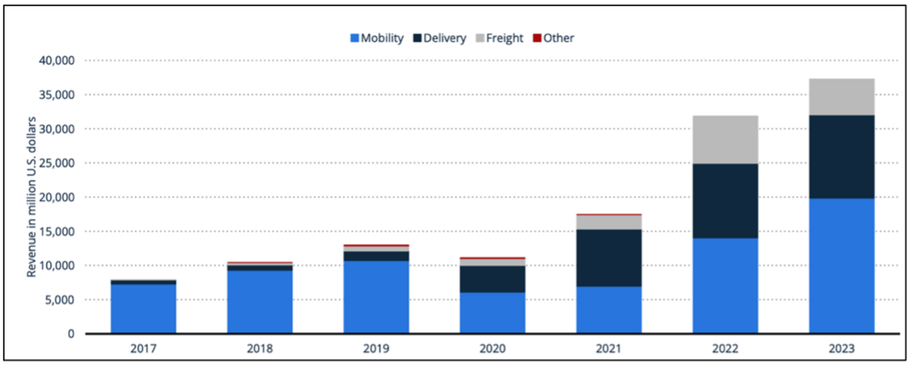


Figure A.3: Uber Segmentated Revenues per Business Area

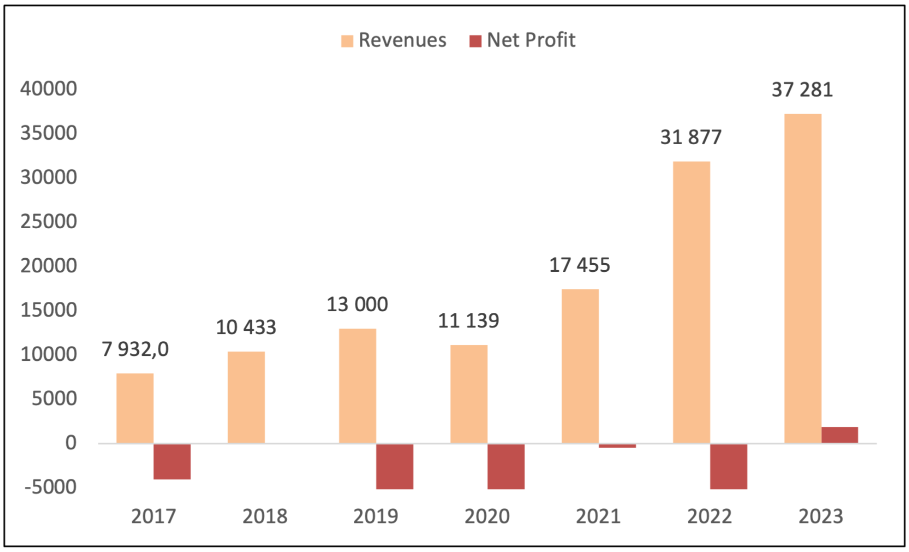


Figure A.4: Historical Revenues and Net Profit

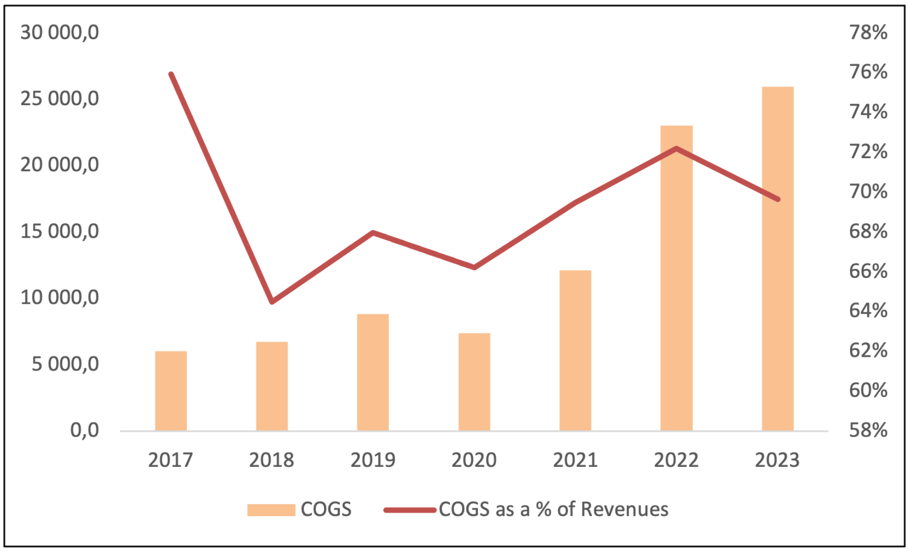


Figure A.5: Historical COGS and COGS as a % of Revenues

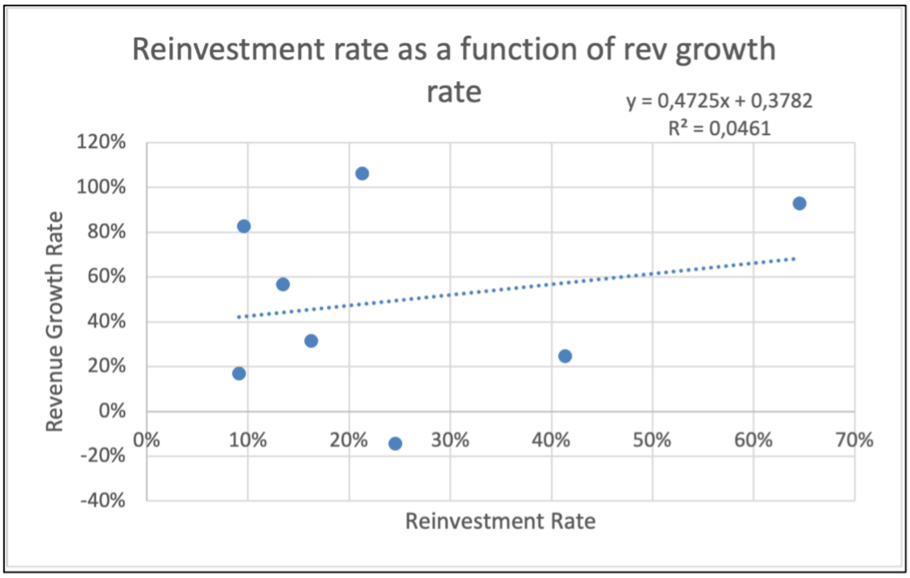


Figure A.6: Reinvestment Rate as a function of Revenue Growth Rate

Parameters (FY28)		Proposed Estimation
Revenues		77256,4
Loss Carry-Forward		6164
Cash and Cash Equivalents		5786,0
Initial Expected Growth rate of Revenues		30,1%
Initial Volatility of Revenues		16,7%
Initial volatility of expected rates of growth in Revenues		0,23
Correlation between percentage change in revenue and change in expected rate of growth		0
Long-term rate of growth in revenues		10,98%
Long-term volatility of the rate of growth in revenues		8,4%
Company´s corporate tax rate		25%
Rf interest rate (5year US T-bill)		3,75%
Speed of adjustement for the rate of growth process		0,36
Speed of adjustement for the volatility of revenues process		0,36
Speed of adjustment for the volatility of the rate of growth process		0,36
COGS as a percentage of revenues		47%
Fixed component of other expenses		14203,4
Variable component of other expenses		11%
Beta		1,01
Return on market portfolio (Msci world)		11,7%
Market price of risk for the revenue factor		0,080
Market price of risk for the expected rate of growth in revenues factor		0
Horizon of the estimation		20 years
Time increment for the discrete version of the model		1 year
Reinvestment Rate		0,3
Reinvestment Mean-Reverting Coefficient		0,36
Trigger Growth Rate of Revenues		12,50%

Figure A.7: Summary of Full List of Parameters Estimated

(USD in millions)	FY23	FY24	FY25	FY26	FY27	FY28
				Projections		
Revenue	**37 281**	**43 321**	**50 559**	**58 421**	**67 769**	**77 256**
% Growth	*54%*	*16%*	*17%*	*16%*	*16%*	*14%*
Cost of goods sold	(25 969)	(25 841)	(29 536)	(33 462)	(33 829)	(36 642)
% of Revenue	*70%*	*60%*	*58%*	*57%*	*50%*	*47%*
Selling, G&A expenses	(6 854)	(7 964)	(9 295)	(10 741)	(12 459)	(14 203)
% of Revenue	*18%*	*18%*	*18%*	*18%*	*18%*	*18%*
Research & Development	(3 164)	(3 677)	(4 291)	(4 958)	(5 751)	(6 557)
% of Revenue	*8%*	*8%*	*8%*	*8%*	*8%*	*8%*
Financial income/expense	(603)	(701)	(818)	(945)	(1 096)	(1 250)
% of Revenue	*1,6%*	*1,6%*	*1,6%*	*1,6%*	*1,6%*	*1,6%*
Other incomes/expenses	(221)	257	300	346	402	458
% of Revenue	*0,6%*	*0,6%*	*0,6%*	*0,6%*	*0,6%*	*0,6%*
Tax expense	(213)	(724)	(729)	(1 108)	(1 755)	(2 225)
Tax rate	*9%*	*13%*	*11%*	*13%*	*12%*	*12%*
Non-Recurring income/exp	1630					
Net profit	**1 887**	**4 671**	**6 190**	**7 554**	**13 280**	**16 838**
% Margin	*5%*	*11%*	*12%*	*13%*	*20%*	*22%*

Figure A.8: Revenues & Expenses Forecasts

(USD in millions except days)

	Historical Avg	FY23	FY24	FY25	FY26	FY27	FY28
			Projections				
Days receivables	39		*39*	*39*	*39*	*39*	*39*
Trade receivables		4 121	4 629	5 402	6 242	7 241	8 255
Days payables	17		*17*	*17*	*17*	*17*	*17*
Trade payables		(3 496)	(1 204)	(1 376)	(1 559)	(1 576)	(1 707)
Days inventory	0		-	-	-	-	-
Inventory		-	-	-	-	-	-
Net working capital		625	3 425	4 027	4 684	5 665	6 548
Change in NWC			**2 800**	**601**	**657**	**982**	**883**

Capital Expenditure Plan

(USD in millions)

	FY24	FY25	FY26	FY27	FY28
			Projections		
Capex	2 594	3 027	3 498	4 057	4 625
% of Revenue	*6.0%*	*6.0%*	*6.0%*	*6.0%*	*6.0%*

	8y Avg	2016	2017	2018	2019	2020	2021	2022	2023
				Historical					
Capex	*689*	*1 618*	*487*	*189*	*537*	*613*	*298*	*252*	*223*
% of Revenue	*6,0%*	*42,1%*	*6,1%*	*1,8%*	*4,1%*	*5,5%*	*1,7%*	*0,8%*	*0,6%*

Depreciation & Amortization Schedule

(USD in millions)

	FY24	FY25	FY26	FY27	FY28
			Projections		
Depreciation & Amortization	1 501	1 752	2 024	2 348	2 677
% of Revenue	*3,5%*	*3,5%*	*3,5%*	*3,5%*	*3,5%*

	8y Avg	2016	2017	2018	2019	2020	2021	2022	2023
				Historical					
Depreciation & Amortization	*461*	*320*	*511*	*426*	*472*	*575*	*902*	*947*	*823*
% of Revenue	*3,5%*	*4,9%*	*4,8%*	*3,3%*	*2,2%*	*3,8%*	*4,4%*	*2,9%*	*2,3%*

Figure A.9: Net Working Capital, CAPEX & Depreciation Forecasts

Category	**Low**	**High**
Long-term bond rate	3,90%	4,40%
Equity market risk premium	4,60%	5,60%
Adjusted beta	0,92	1,19
Additional risk adjustments	0,00%	0,50%
Cost of equity	8,13%	11,56%
Tax rate	14,13%	16,61%
Debt/Equity ratio	7,92%	7,92%
Cost of debt	5,64%	8,66%
After-tax WACC	7,89%	11,25%
Selected WACC	**9,57%**	

Figure A.10: Summary of WACC Calculation

Detailed calculations

Tax rate

	Company History									
	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Historical tax rate	0,00%	-0,81%	0,00%	0,00%	21,57%	0,00%	-2,76%	-48,00%	-1,92%	9,18%
Low tax rate	**14,13%**									
High tax rate	**16,61%**	17%								
Selected tax rate	15,37%	15,37%								

Cost of debt

Method 1 (effective interest rate)

(USD in millions)	2017	2018	2019	2020	2021	2022	2023
	Historical						
Interest expense	479	648	559	458	483	565	633
Total debt	3 476	7 442	6 120	8 232	9 537	9 691	9 962
Effective interest rate	**0,00%**	**18,64%**	**7,51%**	**7,48%**	**5,87%**	**5,92%**	**6,53%**
Cost of debt 1	**8,66%**						
(average effective interest rates)							

Method 2 (using synthetic ratings)

TTM EBIT	2 321		
(/) TTM Interest expense	633		
TTM Interest coverage ratio	**3,67**		
Estimated default spread	1,74%	*Source*	http://www.stern.nyu.edu/~adamodar/pc/ratings.xls
Estimated bond rating	BB+		
Cost of debt 2	**5,64%**		

Figure A.11: WACC Intermediate Calculations

(USD in millions)		Projections FY24	FY25	FY26	FY27	FY28	Terminal
Profit before tax		5 395	6 919	8 662	15 035	19 063	14 253
(-) Net interest income/expense		701	818	945	1 096	1 250	1 097
(+) Depreciation & Armortization		1 501	1 752	2 024	2 348	2 677	2 350
EBITDA		**6 195**	**7 853**	**9 741**	**16 287**	**20 490**	**15 506**
(-) Tax		724	729	1 108	1 755	2 225	1 696
(-): Capex		2 594	3 027	3 498	4 057	4 625	4 060
(-): Change in NWC		(2 800)	(601)	(657)	(982)	(883)	(841)
Free Cash Flow (FCF)		**5 678**	**4 698**	**5 793**	**11 456**	**14 523**	**10 591**
Terminal value							**197 811**
WACC / Discount rate	**9,57%**						
Long-term growth rate	**4,00%**						
Timing of FCF (mid year)		1,00	2,00	3,00	4,00	5,00	5,50
Present value of FCF		**5 182**	**3 913**	**4 404**	**7 949**	**9 197**	**119 671**

Enterprise value	150 316	
Projection period	30 645	*20,4%*
Terminal value	119 671	*79,6%*
(-) Current net debt	4 555	
Equity value	145 761	
(/) Outstanding shares	2 071	
Fair Price (USD)	**70,38**	

Figure A.12: DCF Valuation Results

Sensitivity Analysis

			FY24	FY25	FY26	FY27	FY28	Terminal
Low	WACC	11,25%						
	Long-term growth rate	3,0%						
	Terminal value							132 295
	Present value of FCF		5 104	3 796	4 208	7 480	8 524	73 620
	Enterprise value	102 731						
	Equity value	98 176						
	Fair Price (USD)	47,40						
High	WACC	7,89%						
	Long-term growth rate	5,0%						
	Terminal value							384 688
	Present value of FCF		5 262	4 036	4 612	8 455	9 934	253 336
	Enterprise value	285 636						
	Equity value	281 081						
	Fair Price (USD)	135,71						

Figure A.13: DCF Sensitivity Analysis

(USD in millions)

Projections	FY24	FY25	FY26	FY27	FY28
Cash & Equivalents	5 786	5 786	5 786	5 786	5 786
% of Revenue	*13,4%*	*11,4%*	*9,9%*	*8,5%*	*7,5%*

Historical	8y Avg	2017	2018	2019	2020	2021	2022	2023
Cash & Equivalents	*5 786*	*4 393*	*6 406*	*10 873*	*5 647*	*4 295*	*4 208*	*4 680*
% of Revenue	*25,3%*	*55,4%*	*61,4%*	*83,6%*	*50,7%*	*24,6%*	*13,2%*	*12,6%*

Figure A.14: Cash & Cash Equivalents Forecast

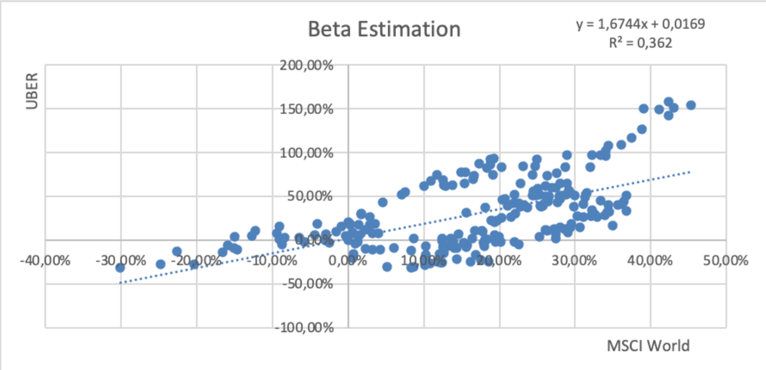
(USD in millions)

	FY23	FY24	FY25	FY26	FY27	FY28
		Projections				
Revenue	**37 281**	**43 321**	**50 559**	**58 421**	**67 769**	**77 256**
% Growth	*54%*	*16%*	*17%*	*16%*	*16%*	*14%*
% Change in growth		-37,9%	0,5%	-1,2%	0,5%	-2,0%

(USD in millions)

Historical	FY19	FY20	FY21	FY22	FY23
Revenue	**13 000**	**11 139**	**17 455**	**31 877**	**37 281**
% Growth	*25%*	*-14%*	*57%*	*83%*	*17%*
% Change in growth	-6,9%	-38,9%	71,0%	25,9%	-65,7%

Figure A.15: Calculation of the Initial Volatility of Revenues Parameter



Unlevering Beta	
Levered Beta	1,674
Debt	9 962
Equity	11 249
Tax Rate	25%
Unlevered Beta	1,006

Figure A.16: Calculation of Levered & Unlevered Beta

Enterprise/Equity Value Comparison		
Discounted Cash Flow Valuation Method		
Enterprise value	150 316	
Projection period (as a %)	30 645	*20,4%*
Terminal value (as a %)	119 671	*79,6%*
(-) Current net debt	4 555	
Equity value	145 761	
(/) Outstanding shares	2 071	
Fair Price (USD)	**70,38**	
(Percentage difference from 20th of March)	-11%	
Schwartz & Moon Hybrid Valuation Method		
Enterprise value	190 982	
Projection period (as a %)	30 645	*16,0%*
Terminal value (as a %)	160 337	*84,0%*
	40 666	
(-) Current net debt	4 555	
Equity value	186 427	
(/) Outstanding shares	2 071	
Fair Price (USD)	**90,01**	
(Percentage difference from 20th of March)	14%	
Expanded Schwartz & Moon Strategic ROV Valuation Method		
Enterprise value	212 530	
Projection period (as a %)	30 645	*14,4%*
Terminal value (as a %)	181 885	*85,6%*
	21 548	
(-) Current net debt	4 555	
Equity value	207 975	
(/) Outstanding shares	2 071	
Fair Price (USD)	**100,42**	
(Percentage difference from 20th of March)	28%	

Figure A.17: Enterprise and Share Value Results from the 3 Models

```
In[•]:= Quiet[SDEmodel[0.1, λ1Vec⟦idx⟧, λ2Vec⟦idx⟧, μtildeVec⟦idx⟧, ϰVec⟦idx⟧, k1Vec⟦idx⟧, k2Vec⟦idx⟧, σ0Vec⟦idx⟧, η0Vec⟦idx⟧, σbarVec⟦idx⟧, x0Vec⟦idx⟧,
          y0Vec⟦idx⟧, δVec⟦idx⟧, ρVec⟦idx⟧, Y0Vec⟦idx⟧, X0Vec⟦idx⟧, αVec⟦idx⟧, βVec⟦idx⟧, FVec⟦idx⟧, τ0Vec⟦idx⟧, rV0Vec⟦idx⟧]]

Out[•]= {170 114., 15 193.3, 0.105212}

In[•]:= Quiet[SDEmodel[0.3, λ1Vec⟦idx⟧, λ2Vec⟦idx⟧, μtildeVec⟦idx⟧, ϰVec⟦idx⟧, k1Vec⟦idx⟧, k2Vec⟦idx⟧, σ0Vec⟦idx⟧, η0Vec⟦idx⟧, σbarVec⟦idx⟧, x0Vec⟦idx⟧,
          y0Vec⟦idx⟧, δVec⟦idx⟧, ρVec⟦idx⟧, Y0Vec⟦idx⟧, X0Vec⟦idx⟧, αVec⟦idx⟧, βVec⟦idx⟧, FVec⟦idx⟧, τ0Vec⟦idx⟧, rV0Vec⟦idx⟧]]

Out[•]= {170 254., 15 508.6, 0.101899}

In[•]:= Quiet[SDEmodel[0.5, λ1Vec⟦idx⟧, λ2Vec⟦idx⟧, μtildeVec⟦idx⟧, ϰVec⟦idx⟧, k1Vec⟦idx⟧, k2Vec⟦idx⟧, σ0Vec⟦idx⟧, η0Vec⟦idx⟧, σbarVec⟦idx⟧, x0Vec⟦idx⟧,
          y0Vec⟦idx⟧, δVec⟦idx⟧, ρVec⟦idx⟧, Y0Vec⟦idx⟧, X0Vec⟦idx⟧, αVec⟦idx⟧, βVec⟦idx⟧, FVec⟦idx⟧, τ0Vec⟦idx⟧, rV0Vec⟦idx⟧]]

Out[•]= {169 665., 15 199.9, 0.0999283}

In[•]:= Quiet[SDEmodel[1.0, λ1Vec⟦idx⟧, λ2Vec⟦idx⟧, μtildeVec⟦idx⟧, ϰVec⟦idx⟧, k1Vec⟦idx⟧, k2Vec⟦idx⟧, σ0Vec⟦idx⟧, η0Vec⟦idx⟧, σbarVec⟦idx⟧, x0Vec⟦idx⟧,
          y0Vec⟦idx⟧, δVec⟦idx⟧, ρVec⟦idx⟧, Y0Vec⟦idx⟧, X0Vec⟦idx⟧, αVec⟦idx⟧, βVec⟦idx⟧, FVec⟦idx⟧, τ0Vec⟦idx⟧, rV0Vec⟦idx⟧]]

Out[•]= {166 861., 16 187.1, 0.100086}
```

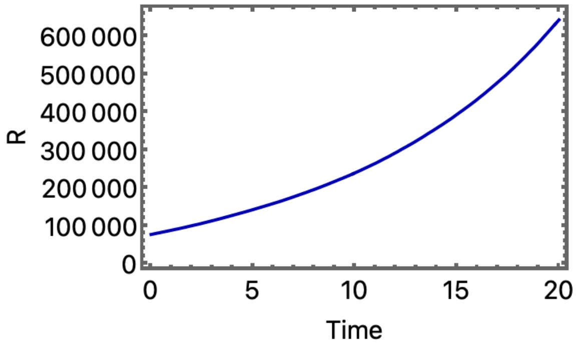
Figure A.18: Programmatic Optimization of θ - output in the form of (V_0, std. deviation, $\overline{\mu_t}$)

```
Two Different Regimes for μ:
        Free - No Investiment
        Drived - With Investiment

In[•]:= ModelYFree[initTime_, stepTime_, endTime_, θ_, λ1_, λ2_, μtilde_, ϰ_, k1_, k2_, σ0_, η0_, σbar_, x0_, y0_, δV0_, ρ_, Y0_, X0_, α_, β_, F_, τ0_, rV0_] := (
          σ[t_] := σ0 Exp[-k1 t] + σbar - σbar Exp[-k1 t];
          η[t_] := η0 Exp[-k2 t];
          ϰ2[t_] := ϰ;
          proc = ItoProcess[
            {dy[t] == (ϰ μtilde - ϰ y[t] - λ2 η[t]) dt + η[t] × dw2[t]}
            , {y[t]}, {y, y0}, {t, 0}, {w2 ≈ WienerProcess[0, ρ]}];

          simData = RandomFunction[proc, {initTime, endTime, stepTime}];
          simData
        )

        ModelYDrived[initTime_, stepTime_, endTime_, θ_, λ1_, λ2_, μtilde_, ϰ_, k1_, k2_, σ0_, η0_, σbar_, x0_, y0_, δV0_, ρ_, Y0_, X0_, α_, β_, F_, τ0_, rV0_] := (
          σ[t_] := σ0 Exp[-k1 t] + σbar - σbar Exp[-k1 t];
          η[t_] := η0 Exp[-k2 t];
          θFunc[t_] := θ;
          ϰ2[t_] := ϰ ;
          proc = ItoProcess[
            {dy[t] == (ϰ2[t] × θFunc[t] + ϰ μtilde - ϰ y[t] - λ2 η[t]) dt + η[t] × dw2[t]}
            , {y[t]}, {y, y0}, {t, 0}, {w2 ≈ WienerProcess[0, ρ]}];

          simData = RandomFunction[proc, {initTime, endTime, stepTime}];
          simData
        )
```

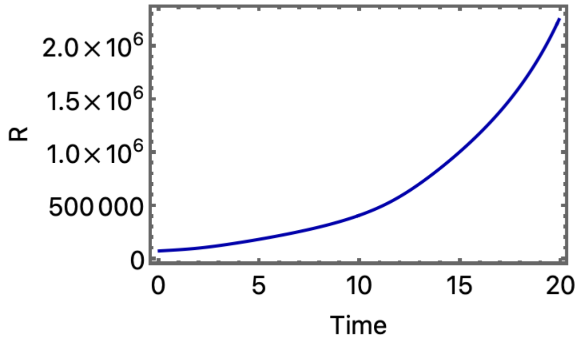
Figure A.19: Expanded Model - Code explaining the two regimes

```
In[•]:= computeV0[vecR_, vecμ_, t0_] := (
          trigger = computeTriggers[vecμ, μStar];
          trigger2 = trigger + Gstep;
          conditions = Table[{1, trigger⟦i⟧ ≤ x2 < trigger2⟦i⟧}, {i, 1, Length[trigger]}];

          funcTrigger[x_] := Piecewise[conditions /. {x2 → x}, 0]; (*1 inside the investiment and 0 outside*)

          fMeanR = Interpolation[vecR]; (*R(t)*)
          fMeanμ = Interpolation[vecμ];

          Cost[t_] := (αVec⟦idx⟧ + βVec⟦idx⟧ + θ * funcTrigger[t] ) fMeanR[t] + FVec⟦idx⟧;
          Y[t_] := (fMeanR[t] - Cost[t]) (1 - τ0Vec⟦idx⟧);
          X[t_] := NIntegrate[Y[t1], {t1, 0, t}] + X0Vec⟦idx⟧ ;
          V0[t_] := X[t] Exp[-rV0Vec⟦idx⟧ t];
          V0[t0]
        );
```

Figure A.20: Valuation through cascading equations

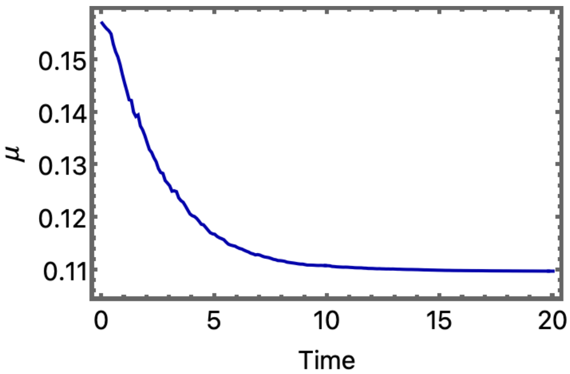


(a) Hybrid Model Mean Revenue Evolution

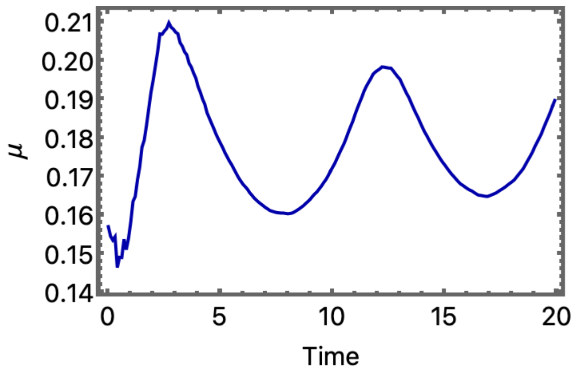


(b) Expanded Model Mean Revenue Evolution

Figure A.21: (a) Presents Moderate growth up to \$600M; (b) Presents the dynamic evolution of Revenues, which due to successful timing of Reinvestments grows up to \$2000M.

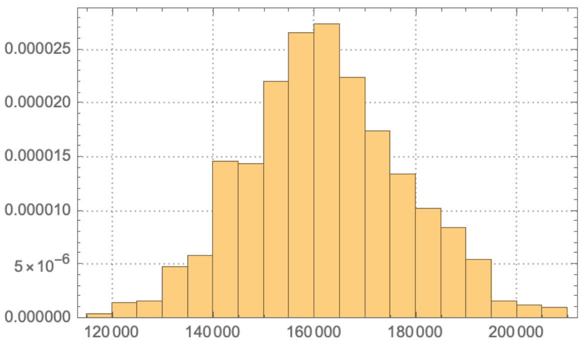


(a) Hybrid Model μ_t Evolution

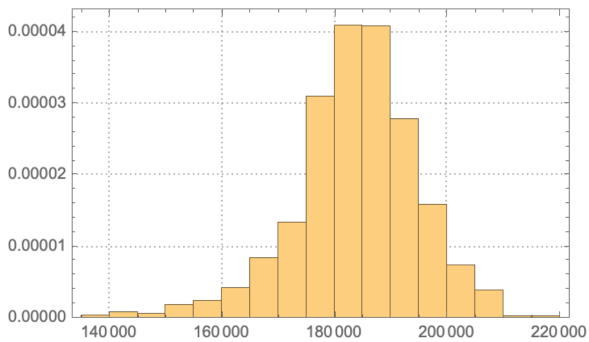


(b) Expanded Model μ_t Evolution

Figure A.22: (a) Presents μ_t strictly following the mean-reverting process to the industry average; (b) Presents the dynamic evolution of this same μ_t parameter, when subject to Reinvestment Real-Option.

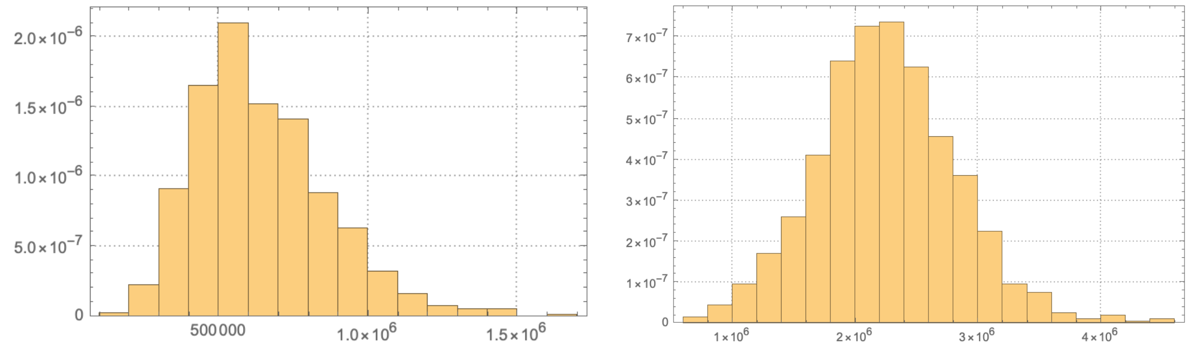


(a) Hybrid Model Valuation Simulation



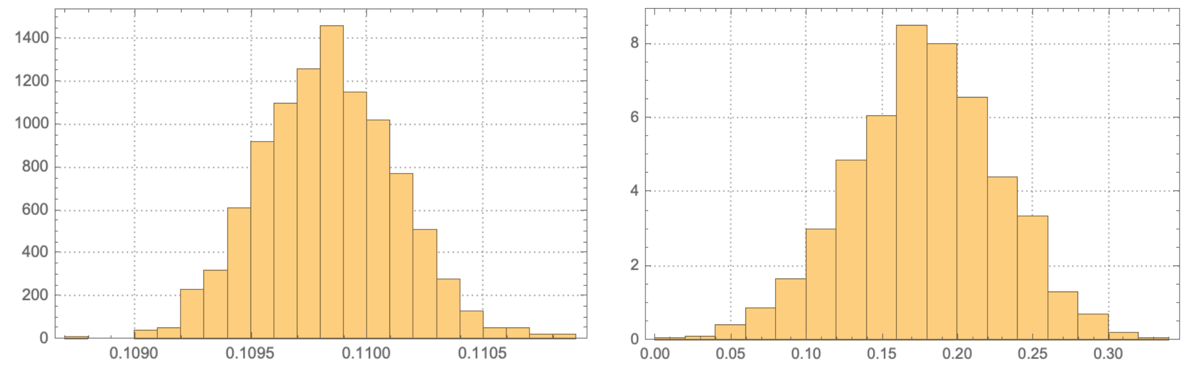
(b) Expanded Model Valuation Simulations

Figure A.23: (a) Presents a mean of \$160B; (b) With Dynamic Reinvestments has a higher mean valuation of \$181B.



(a) Hybrid Model Revenue Simulations **(b)** Expanded Model Revenue Simulations

Figure A.24: (a) Presents a lower tail averaging \$600B; (b) With the Real-Option present it is clear the effect on Revenues averaging \$2000B.



(a) Hybrid Model μ_t Simulations **(b)** Expanded Model μ_t Simulations

Figure A.25: (a) Shows the predominant effect of the Mean-Reverting Coefficient, with the long-term μ averaging at an industry-level of 10.98%; (b) The direct effect of Reinvestments on μ will make it grow to an average of 16%.

B Mathematical Derivations

B.1 Schwartz & Moon Derivation

The model begins by deriving the dynamics of a firm´s Revenue and Revenue growth components, using empiric probability measures characterized by the following stochastic differential equation:

$$dR_t = \mu_t dt + \sigma_t dz_t \tag{B.1}$$

where R_t denotes the instantaneous rate of revenues at time t, μ_t is the expected rate of growth in revenues, subject to random shocks dz_t (increment of the Wiener Process), with σ_t measuring the intensity of these shocks. It reflects the unpredictable nature of Internet company revenues, which, while expected to grow, experience considerable volatility due to various factors such as market competition, technological changes, and consumer preferences. Young internet firms often experience exceptionally high growth rates at first, which are expected to stochastically settle into a more sustainable and industry-appropriate pace of expansion, motivated by the author's justification that in the long run, a firm cannot grow faster than it. Hence, it's posited that the growth trend, or drift μ_t, adheres to a mean-reverting process, characterized by an industry-specific long-term average rate ($\overline{\mu}$)(Schwartz & Moon, 2000):

$$d\mu_t = \kappa(\overline{\mu} - \mu_t)dt + \eta_t dz_2 \tag{B.2}$$

where η_t is the volatility of expected growth in revenues and κ the mean-reversion coefficient of the stochastic process, affecting how fast the firm converges to the long-term growth level. The Wiener process component dz_2, introduces randomness, acknowledging that the path towards the industry-level mean can be unpredictable. Schwartz and Moon define the half-life of any deviations from the long-run growth rate as $ln(B.2)/\kappa$.

The first deterministic equation is defined, where, under quite simplifying assumptions the authors formulate the Costs at time t has:

$$\text{Cost}_t = \text{COGS}_t + \text{Other Expenses}_t = \alpha R_t + (F + \beta R_t) = (\alpha + \beta) R_t + F \tag{B.3}$$

The cost structure is composed of two components, the Costs of Goods Sold ($COGS_t$), which are proportional to the revenues α, and other expenses, composed of a Revenue proportional portion β, and a fixed portion F. In further revisions, Schwartz & Moon have developed this cost structure with a more complicated stochastic formulation aiming to better capture future competitive pressures and technological developments.

Having formulated both Revenues and Costs, Schwartz & Moon, move on to equating the net after-tax cash flow to the company given by:

$$Y_t = (R_t - \text{Cost}_t)(1 - \tau) \tag{B.4}$$

Here, τ represents the corporate tax rate, which is only to be paid in case the firm in question does not have a loss carry-forward at time t. Further innovations to the model have since been developed to account for both Depreciation and CAPEX in the cash flow calculation. The loss carry-forward dynamics are then given by:

$$dL_t = \begin{cases} -Y_t dt & \text{if } L_t > 0, \\ \max(-Y_t dt, 0) & \text{if } L_t = 0. \end{cases} \tag{B.5}$$

The last deterministic formulation is related to the amount of cash available at time t which is given by:

$$dX_t = Y_t dt \tag{B.6}$$

Here, the model introduces the concept of firm bankruptcy, which happens the first time X_t reaches zero. Furthermore, to sidestep the necessity of establishing a dividend strategy within the model, Schwartz & Moon operate under the premise that the cash flow produced by the company's operations is retained within the company. This retained cash earns interest at the risk-free rate and is anticipated to be distributable to the shareholders at a designated long-term point, T, by which the company is expected to have transitioned back to its "normal" state. Another concern of the model that needs to be addressed is the fact that equation (3.2) resorts to risk-neutral probabilities whereas the stochastic formulations in (1) and (2) refer to the empiric probability measure. This means that a risk adjustment process is needed to ensure that the model's forecasts and valuations are consistent with a risk-neutral world, allowing for a more straightforward valuation of future cash flows or derivative assets tied to the company's performance. It involves recalibrating the stochastic drivers of the model using the market price of risk derived from ICAPM principles (Merton, 1973). Building on Merton's 1973 model, which assumes investors have logarithmic utility functions, the market price of risk for an asset λ_i is determined by multiplying the correlation between the asset's return and the market portfolio's return ρ_{iM} by the volatility of the market portfolio's returns η_M as outlined by Brennan and Schwartz (1982). Schwartz and Moon (2001) identify two forms of uncertainty in their model: fluctuations in revenue and variability in the expected growth rate. They propose that only the uncertainty related to revenue is linked with a risk premium. This approach simplifies the risk adjustment process, enabling the market price of risk for the revenue stream λ_R to be calculated using the stock's beta, as discussed in their work:

The drift-adjusted processes are therefore formulated by:

$$\frac{dR_t}{R_t} = (\mu_t - \lambda_1 \sigma_t)dt + \sigma_t dz_1^*, \tag{B.7}$$

$$d\mu_t = [\kappa(\overline{\mu} - \mu_t) - \lambda_2 \eta_t]dt + \eta_t dz_2^*, \tag{B.8}$$

where $dz_1^* dz_2^* = \rho dt$. Finally, the firm's value, at any given time, is determined based on the previously mentioned state variables:

$$V \equiv V(R, \mu, L, X, t) \tag{B.9}$$

By applying Itô's Lemma, one can derive the company´s value dynamics and its volatility (Schwartz and Moon, 2000):

$$dV = V_R dR + V_\mu d\mu + V_L dL + V_X dX + V_t dt + \frac{1}{2}V_{RR}(dR)^2 + \frac{1}{2}V_{\mu\mu}(d\mu)^2 + V_{R\mu} dR d\mu. \tag{B.10}$$

Since these partial differential equations cannot be solved directly, numerical methods are employed, notably Monte Carlo Simulation. This requires discretizing the partially risk-adjusted processes [Equations (B.7)-(B.8)] along with differential equations (B.3) and (B.4):

$$R_{t+\Delta t} = R_t e^{\left(\mu_t - \lambda\sigma_t - \frac{\sigma_t^2}{2}\right)\Delta t + \sigma_t \sqrt{\Delta t}\epsilon_1} \tag{17}$$

$$\mu_{t+\Delta t} = e^{-\kappa\Delta t}\mu_t + (1 - e^{-\kappa\Delta t})\left(\overline{\mu} - \frac{\lambda_2 \eta_t}{\kappa}\right) + \sqrt{\frac{1 - e^{-2\kappa\Delta t}}{2\kappa}}\eta_t \sqrt{\Delta t}\epsilon_2 \tag{18}$$

where

$$\sigma_t = \sigma_0 e^{-\kappa_1 t} + \sigma_1(1 - e^{-\kappa_1 t}) \tag{19}$$

$$\eta_t = \eta_0 e^{-\kappa_2 t} \tag{20}$$

B.2 Berk, Green and Naik, (1999) Valuation Detailed Results

Looking at their empirical valuation function, it comprises two steps, with the first being the valuation of the cash flows from a given project:

$$V_j(t) = E_t \left[\sum_{s=t+1}^{\infty} \frac{z(s)}{z(t)} C_j(s) \chi_j(s) \right] \tag{B.11}$$

where $\chi_j(t) = 0$ for all $s > t$ and $V_j(t)$ must be zero because the project has expired or the options to invest in such projects in the future cannot be postponed. The value of each option is the maximum of the NPV of the project or zero.

The second component values the options to invest in such projects in the future, which given the impossibility to postpone, are valued has the maximum of the NPV of the project or zero:

$$V^*(t) = E_t \left[\sum_{s=t+1}^{\infty} \frac{z(s)}{z(t)} \max\left[V(s) - I_0, 0\right] \right] \tag{B.12}$$

where I_0 is the initial investment cost.

Thus, the authors value the firm as the future cash flows of all projects still alive plus the value of future investment opportunities, and so having a current assets uncertainty and strategic real options:

$$P(t) = \sum_{j=0}^{t} V_j(t)\chi_j(t) + V^*(t) \tag{B.13}$$

C Dynamic Model Logic Algorithm

Algorithm 1 State-Transitioning Dynamic Model

Require: Initial time $init\_time$, end time end_time, step time $step\_time$, parameters including μ, θ, κ, λ_1, λ_2, σ_0, η_0, σ_{bar}, ρ, μ^*, investment parameters G (duration of Drived model), etc.

Ensure: Simulation data of $y(t)$, $R(t)$, and $V(t)$ over time.

Initialize $y(t)$, $R(t)$, and $V(t)$ based on initial conditions.
Define time-dependent parameters $\sigma(t)$ and $\eta(t)$:
$\sigma(t) \leftarrow \sigma_0 \exp(-k_1 t) + \sigma_{bar}(1 - \exp(-k_1 t))$
$\eta(t) \leftarrow \eta_0 \exp(-k_2 t)$
Set $current\_time \leftarrow init\_time$
Set $model\_state \leftarrow$ "Free"
Set $time\_drived\_ends \leftarrow 0$
while $current\_time \leq end\_time$ **do**
    if $model\_state$ = "Free" and $y(t) \geq \mu^*$ **then**
        $model\_state \leftarrow$ "Drived"
        $time\_drived\_ends \leftarrow current\_time + G$
    end if
    if $current\_time \geq time\_drived\_ends$ **then**
        $model\_state \leftarrow$ "Free"
    end if
    if $model\_state$ = "Free" **then**
        Operate under Free Model:
        $dy(t) \leftarrow (\kappa\mu - \kappa y(t) - \lambda_2 \eta(t))dt + \eta(t) dw_2(t)$
        Update $y(t)$ for the next time step using stochastic calculus
    else
        Operate under Drived Model:
        $dy(t) \leftarrow (\kappa\theta + \kappa\mu - \kappa y(t) - \lambda_2 \eta(t))dt + \eta(t) dw_2(t)$
        Update $y(t)$ for the next time step as above
    end if
    Compute revenue $R(t)$ based on $y(t)$:
    $R(t) \leftarrow \text{RevenueFunction}(y(t), \text{other parameters})$
    Update the value $V(t)$ based on $R(t)$:
    $dV(t) \leftarrow \text{DiscountFunction}(V(t-1), R(t), current\_time, \text{parameters})$
    $V(t) \leftarrow V(t-1) + dV(t)$
    Record $(current\_time, y(t), R(t), V(t))$ in the simulation output
    $current\_time \leftarrow current\_time + step\_time$
end while
return the simulation output containing time series data of $y(t)$, $R(t)$, and $V(t)$

D Uber Company Presentation

In 2009, Uber began with the straightforward concept of tapping a button to get a ride, and quickly evolved into a major player in the ride-hailing market. Now, it operates across 70 countries, with a user base of 137 million people who use Uber at least once a month. Uber's business model exemplifies an Online-to-Offline (O2O) platform, streamlining the transition from digital service requests to physical world interactions via a sophisticated app. This app allows users to hail rides and track their progress with real-time updates and GPS technology, while handling payments electronically to eliminate cash transactions. Expanding its O2O model, Uber also introduced Uber Eats, leveraging its existing network to connect users with local restaurants for online ordering and delivery. Uber's market cap has seen dramatic growth over the years, standing at $\$144.04$ billion as of April 2024. This reflects a substantial increase of almost 128% over the previous year (Uber, (2024)). This financial success underscores Uber's evolution from a novel start-up to a major corporation in the global transportation and delivery market.

D.1 Financial Overview

Ever since its IPO, Uber has experienced sustainable growth, demonstrating a compounded annual growth rate (CAGR) of 30% in revenue since 2019. Moreover, as per FigureA.4, 2023 marked the first full year that Uber achieved a net income profit, totalling $\$1.8$ billion. This milestone represents the beginning of the firm's consolidation process. Cost of Revenue (COGS) represents most Uber's expenses, accounting for 70% of its revenues in 2023. As per FigureA.5, this percentage has fluctuated between 76% and 64%. From a segmentation perspective, mobility remains Uber's largest revenue source and has shown sustainable growth over the past five years, excluding 2020, accounting for $\$19.3B$ in revenues in 2023, with delivery experiencing significant growth since the pandemic (FigureA.3). In analysing Uber's financial performance by region, revenues from the U.S. and Canada dominate. Remarkable is the surge in the EMEA region's revenues in 2023, reflective of strategic growth, with the APAC region also displaying a marked increase in its contribution.

D.2 The Horizon of Ride-Hailing: Assessing Uber's Growth Opportunities

As of today, Uber is the undisputed leader in the $154 billion ride-hailing market (FigureA.2. This industry has been growing steadily, driven by user preferences for convenience and affordability. Also, according to Statista, (2023) forecast, this industry is expected to grow to $202 billion by 2027, spurred by significant global expansion, particularly in the Asia-Pacific (APAC). Uber, (2024) annual report outlines its strategy to cement its market leadership and capitalise on these growth projections, which is segmented into five action areas: enhancing margins through cost structure discipline, leveraging leading technology to maintain a cutting-edge supply engine, expanding consumer reach by entering underdeveloped markets, such as airport trips, and new demographics. Additionally, maintaining a balanced product portfolio is crucial for sustained growth in revenue and profits. Finally, the company is investing in the Autonomous Vehicle (AV) sector, aiming to shape the autonomous future of mobility.