A Work Project, presented as part of the requirements for the Award of a Master Degree
in Economics from the NOVA – School of Business and Economics

Differences in Natural Interest Rates in the Eurozone

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A project carried out on the Master in Economics Program under the supervision of:

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January 3rd, 2018

Abstract

Differences in the natural interest rates across the countries of the Eurozone implies that the monetary policy may not be optimal for each member state individually. This research uses the model developed by Laubach and Williams (2003, 2016) to estimate the natural interest rate for each country of the Eurozone-12. With these estimates I compute the natural interest rate for the Eurozone as a weighted average of the natural rates of each country. The results show that the differences in the natural rate across the Eurozone are higher in periods of asymmetric shocks. Most of the countries also present a large difference between their rates and the Eurozone natural rate. All these estimates are associated to a high degree of uncertainty.

Keywords: Natural interest rate; Kalman Filter; Optimal Monetary Policy; Eurozone

1. Introduction

During the Financial Crisis of 2008 and the Sovereign Debt Crisis of 2009, the European economy faced the most serious recession since the Great Depression of 1929. The European Central Bank (ECB) used all its tools of monetary policy to keep the euro as stable as possible. This involved a decrease of the overnight deposit facility rates to values never reached before (Cœuré, 2016). The lower rates created favorable conditions to the economic recovery of the European countries. Nevertheless, some states took much more time to recover than others. For example, in Portugal the economic recovery only occurred at the end of 2014 and the beginning of 2015. Greece had the first quarters of economic recovery during 2017. Many reasons may be behind this sluggish recovery. Structural differences between the Eurozone Members may imply that the monetary policy adopted was not the optimal policy for each country individually. Nechio (2011) pointed that between 2008 and 2011 the interest rate established by the ECB was not able to address the problems of the periphery countries. This implies that one policy may not be optimal for all member states. A method to evaluate the stance of monetary policy in a country is to use the real interest rate gap as discussed by Amato (2005). The real interest rate gap is the difference between the real interest rate that is observed in the economy and the natural interest rate. The concept of the natural interest rate was initially introduced by Wicksell (1898) that defined it as a “rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them”. This concept was further developed by Laubach and Williams (2003) that defined it as “the real short-term interest rate consistent with output converging to potential, where potential is the level of output consistent with stable inflation”. A positive real interest rate gap implies that the real interest rate is above the natural interest rate, which points towards a contractionary monetary policy stance.

By its own definition the natural interest rate is a good proxy for the target real interest rate that central banks should aim when there are no economic shocks. Following a Taylor rule specification (Taylor, 1993), the natural interest rate will be the target rate when the output level is equal to potential and inflation is equal to the desired level. The ECB when determining the monetary policy will consider the economic indicators of the Eurozone as a whole. So, we can argue that the target rate set by the ECB is exactly the natural interest rate of the Eurozone. However, this rate may not be similar to the natural interest rate of all member states. This implies that the monetary policy rule established by the ECB may not be optimal for each country individually. A way to understand this is comparing the natural interest rate of each member state with the natural rate of the Eurozone. The goal of this research is to calculate the natural interest rate for each state and for the Eurozone. To do so I used the methodology adopted by Laubach and Williams (Laubach and Williams, 2003) that was further developed by the same authors in 2016 (Holston, Laubach, and Williams 2016). This model uses a New Keynesian IS and Phillips curve to jointly estimate the output gap, the trend growth rate and the natural interest rate by Kalman filter. For each country the model was applied directly. For the natural interest rate of the Eurozone, I used the estimates of each country and computed a weighted average of these rates. This way the estimate is performed with a larger data pool which creates a more consistent estimation for the natural interest rate.

The results show that there are differences between the natural interest rate across the members of the Eurozone. There is also a common negative trend of the natural interest rates for all the countries analyzed, a result that is consistent with the Secular Stagnation Hypothesis developed by Hansen (1939) and, more recently by Eggertsson, Mehrotra and Summers (2016). The estimate for the natural interest rate of the Eurozone is also consistent with previous literature. The reaming sections of this paper are organized as follows: Section 2 discusses the previous literature related to the estimation of the natural interest rate, Section 3 presents the methodology used in more detail, Section 4 discusses the data used for the estimation, Section 5 presents the results and Section 6 concludes.

2. Literature Review

The renewed interest in the concept of natural interest rate introduced by Wicksell (1898) came from the discussion presented by Woodfort (2000) that presented the interest rate gap as a method to evaluate the monetary policy stance, relating the Wicksellian concept with a dynamic stochastic general equilibrium model (DSGE). Previous to this, the analysis of the monetary policy stance would be made based on data of the monetary aggregates or by the exchange rate. Both these methods presented flaws. As pointed by Neiss et al. (2000), central banks establish the level of the short run interest rate as the main tool of monetary policy and not the level of the monetary aggregates. So an instrument directly related to the interest rate will be best suited to make this analysis. King (1997) argued that exchange rate indicators (like the monetary conditions index) are also not optimal, since many external factors may cause changes in the exchange rate that are not related to monetary policy. This way, an optimal method to evaluate the policy stance should only depend on the level of the interest rate. The first attempt to calculate the real interest rate gap was made by Neiss et al. (2000) that followed directly the neo-Wicksellian model presented by Woodford (2000) for the United Kingdom. The authors conclude that the real interest rate gap is a good indicator to explain the behavior of inflation and thus can be used as an indicator for the monetary policy stance. The DSGE methodology was also adopted by Giammarioli and Valla (2003), where the authors computed the natural interest rate for the Eurozone. The authors built upon the work of Neiss et al. with the introduction of three different specifications of the monetary policy rule. This way the authors took into consideration the possible policy implications that central banks can make using the level of the natural interest rate and how these changes affect the model. The authors conclude that with a Taylor rule that takes into account the natural interest rate, Central Bank’s actions improve the overall stability of the economy. Cúrdia et al. (2015) use a New Keynesian DSGE model to compute the natural interest rate to understand if a Taylor rule that responds to deviations of the real rate from its natural level can fit better the U.S. data than a traditional output gap response Taylor rule. The authors found that the latest has more explaining power, implying that in the U.S. policy makers seem to be considering the concept of the natural interest rate. Despite all the developments, some arguments were made against the DSGE approach of estimating the natural interest rate. In Levin, Wieland and Williams (1999) the authors pointed that the DSGE approach was highly reliable on the structural assumptions of the model itself, and so the estimates would not be robust to any model uncertainty. To overcome this, an alternative method was presented by Laubach and Williams (2003) that applies the Kalman filter to jointly estimate the output gap, the trend growth rate of potential output and the natural interest rate for the U.S. For this the authors estimated a New Keynesian IS and Phillips curve via maximum likelihood and use the estimates into the state space model to filter the unobservable components. For these authors the concept of natural interest rate is “the real short-term interest rate consistent with output converging to potential, where potential is the level of output consistent with stable inflation”. This definition differs from the one pointed by Woolford (2000), given that it defines the natural interest rate as the intercept term of a Taylor rule, instead of having the natural interest rate as a specific guidance level for the real rate. The authors concluded that the natural interest rate in the U.S. had some variation and that is mostly determined by the variation of the trend growth rate. The same authors replicated this paper in 2015, (Laubach and Williams, 2015) for the data in the U.S. to look at the evolution of the natural interest rate during the financial crisis of 2008, to test if the abnormal decrease of the interest rates during this period was reflected in the natural interest rate. The authors found a significant decrease in 2008 and persistent lower levels of the natural interest rate, including values below zero, between 2012 and 2013, thus concluding that lower interest rates had become a new reality. In 2016 the same authors expanded their analysis outside the U.S. (Holston, Laubach and Williams, 2016), by calculating the natural interest rate for the U.S, U.K, Eurozone and Canada to understand if the behavior found in 2008/15 was also present around the world. The conclusion was that the natural interest rate decrease was present in all four estimates, implying that it was a global phenomenon. An important remark made by the authors was that the estimates for the Eurozone, Canada and the U.K were much more imprecise than the ones calculated for the U.S. This fact has to do with the overall significance of the parameters estimated of the IS and Phillips curve, suggesting that a different specification of the model can yield better estimates for these countries.

The methodology developed by Laubach and Williams was widely used in the literature for various countries. Garnier and Wilhelmsen (2005) and Constâncio (2016) apply this model to the Eurozone. Both point to the high level of uncertainty of the natural interest rate estimates, implying that any policy implications made from the results must be made with caution. Mendonça (2017) calculates the natural interest rate specifically for the Netherlands and Italy. By comparing both estimates the author concludes that there are significant differences, implying that a single monetary policy for both countries would be suboptimal. An alternative method to perform this estimation of the natural interest rate is the use of a Time-Varying parameter VAR. Lubik and Matthes (2015) present this type of models, arguing that they are particular useful to the analysis of macroeconomic series that usually present some sort of non-linear structure, yielding better results that regime-switching VAR's. The same authors use this approach to the estimation of the natural interest rate in the U.S. and compare the results with the ones of Laubach and Williams (Lubik and Matthes, 2015). The estimates of the TVP-VAR have less variation, but are consistent with the findings of Laubach and

Williams. The main difference is that the estimate of Lubik and Matthes doesn't reach negative values. A common trend between all these models is the sharp decrease in the natural interest rate during the period of the financial crisis, reaching in some cases negative values.

The most recent development on the estimation of the natural interest rate was made by Lewis and Vazquez-Grande (2017). The authors build upon the model of Laubach and Williams and use a Bayesian approach to better account for all the parameter uncertainty associated with the three-step maximum likelihood estimation and construct different models to test the different possible specifications of the level of the natural interest rate. Despite finding results that are not statistically different from Holston, Laubach and Williams (2016), the authors found a significant recovery in the levels of the American natural interest rate after the 2008 financial crisis, obtaining estimates for the end of 2016, 2% above the findings of Laubach and Williams (2016).

This paper builds upon the work of Holston, Laubach and Williams (2016) and Mendonça (2017), using the methodology developed by Laubach and Williams (2003) to calculate the natural interest rates for each member of the Eurozone-12 and a subsequent computation of the natural interest rate for the Eurozone.

3. Methodology

3.1 Model

The model adopted in this paper was developed by Laubach and Williams (2003) (LW). This model defines the natural interest rate as "the real short-term interest rate consistent with output converging to potential, where potential is the level of output consistent with stable inflation". The model uses a specification for the natural interest rate that is derived from the neoclassical growth model. In a steady-state the result of the maximization of the intertemporal household utility of a representative consumer with CES, implies that:

$$r^* = \frac{1}{\sigma} g_c + \theta \tag{1}$$

where σ represents the intertemporal elasticity of substitution, g_c represents the trend growth rate and θ is the rate of time preference. Being this a steady-state result, LW use variation of this relationship to express the natural interest rate. The model uses a reduced form New Keynesian IS curve and Phillips curve to express the relationship between the output gap, the real interest rate gap and the level of inflation:

$$\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \frac{a_r}{2}\sum_{j=1}^{2}(r_{t-j} - {r^*}_{t-j}) + \epsilon_{\tilde{y},t} \quad (2)$$

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi)\pi_{t-2,4} + b_y \tilde{y}_{t-1} + \epsilon_{\pi,t} \quad (3)$$

where $\tilde{y}_t = 100 * (y_t - {y^*}_t)$ represents the output gap, y_t and ${y^*}_t$ represent respectively the logarithms of real GDP and the potential real output, r_t denotes the real short-term interest rate, π_t is the inflation level and $\pi_{t-2,4}$ is the average of its second to forth lags[1]. The natural interest rate is represented by ${r^*}_t$. Equation (2) relates the output gap with its own lags and the real rate gap. The coefficients $a_{y,1}$ and $a_{y,2}$ will measure the persistency of the output gap in an economy[2]. To ensure the stability of the system, the sum of these two coefficients must not surpass one. The coefficient associated to the real rate gap (a_r) is expected to be negative: If the real rate is above the natural interest rate, the level of output is likely to be below potential. Equation (3) relates inflation with its own lags and the output gap. The restriction over the coefficient of the inflation lags follows LW. The coefficient b_y is the slope paramenter of the Phillips Curve, and following economic theory this coefficient should be positive: If the level of output is above potential it creates inflationary pressures in an economy. In the estimation LW impose restrictions on the coefficients a_r[3] (negative) and b_y[4](positive). The authors argue that these specifications don't affect the results but are useful to facilitate the Maximum likelihood optimization. Equations (2) and (3) will be the measurement equations for the State Space model. The unobservable components in this

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[1] $\pi_{t-2,4} = \frac{\sum_2^4 \pi_{t-i}}{3}$

[2] The model assumes that the output gap follows an AR(2) process. This fact is consistent with previous literature.

[3] $a_r \leq -0.0025$

[4] $b_y \geq 0.025$

model are the level of potential output and the natural interest rate. Using the result presented in Equation (1), the authors present the transition equation for the natural interest rate as[5]:

$$r^*{}_t = g_t + z_t \tag{4}$$

where g_t represents the growth rate of potential output and z_t represents a residual variable that captures all the variation of the natural interest rate that is not explained by the trend growth rate. For the log of potential output, the transition equation is described as a random walk with drift g_t:

$$y^*{}_t = y^*{}_{t-1} + g_{t-1} + \epsilon_{y^*,t} \tag{5}$$

This specification assumes that potential output follows an integrated process of order two. This result is consistent with Stock and Watson (1998). The trend growth rate is specified as a random walk:

$$g_t = g_{t-1} + \epsilon_{g,t} \tag{6}$$

A similar process is also used as a transition equation for the variable z_t:

$$z_t = z_{t-1} + \epsilon_{z,t} \tag{7}$$

It´s assumed that $\epsilon_{y^*,t}$, $\epsilon_{g,t}$ and $\epsilon_{z,t}$ are normally distributed disturbances with standard deviations σ_{y^*}, σ_g and σ_z , serially and contemporaneously uncorrelated. Equations (5)-(7) will be the transition equations for the unobservable components in the State Space model[6].

3.2 Empirical Methodology

Using the estimates of equations (2) and (3) via Maximum Likelihood estimation, the Kalman Filter is used to estimate potential output, trend growth rate and the variable z_t to calculate the natural interest rate following equation (4). The Kalman Filter is a viable method in this model since the unobservable components are linear in the transition equations. A recognized setback from directly estimating this model is the “pile-up problem”, discussed by Stock (1994). It’s expected that the unobservable components have large variation on its levels, but

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[5] In Laubach and Williams (2003) the authors use as measurement equation $r^*{}_t = cg_t + z_t$ to capture the effect of the intertemporal elasticity of substitution. The estimates for c were extremely close to 1, so the authors drop this variable in Holston, Laubach, and Williams (2016).

[6] See Appendix 1 for a complete representation of the State Space model.

not on the rates at which they grow. This lack of variation on the growth rates will create a bias towards zero of the disturbances of σ_g and σ_z in their maximum likelihood estimation. This reasoning is also presented in Roberts (2001) where the author argues a similar problem for the U.S economy data. To overcome this, LW use the median unbiased estimator developed by Stock and Watson (1998). This estimator involves computing the exponential Wald statistic for a structural break with unknown break of the estimated potential output and of the intercept of the IS curve to obtain estimates of $\lambda_g = \frac{\sigma_g}{\sigma_{y^*}}$ and $\lambda_z = \frac{a_r \sigma_z}{\sigma_{\tilde{y}}}$. λ_g and λ_z will represent a measure of the variation of the trend growth rate and the variable z. These ratios are then imposed in the estimation to obtain more consistent results. Nevertheless, the computation of these ratios involves a three step procedure. In the first step LW estimate the level of potential output using equations (2) and (3), ignoring the real rate gap component of the IS curve and assuming that the variable g in equation (5) to be constant. This is the same approach developed by Kuttner (1994). With the Kalman filter estimate of the level of potential output, the exponential Wald statistic is calculated on its first difference, and the result is used to obtain λ_g. In the second step, the authors estimate jointly the level of potential output and the trend growth rate, imposing λ_g and including the real rate gap on equation (2), but treat the variable z as a constant on equation (4). An exponential Wald statistic is computed for a structural break on the intercept of the IS curve to obtain the estimate of λ_z. In the final step, using the results for λ_g and λ_z, the entire model is estimated by Maximum likelihood and the unobservable components are estimated using the Kalman Filter. This filtering methodology involves the existence of a prior conditional expectation and covariance matrix for the unobservable components, so that the recursive algorithm has a starting point. For the lags of the level of potential output, LW apply the HP filter to the log of real GDP four quarters prior to the estimation sample with a smoothing parameter of λ=36000 and use the values of the trend component to the three quarters prior to the sample start. For the prior of the trend growth, the authors use the first difference of the HP trend and variable

z is set to start at zero. The initial covariance matrix of the states is computed from the gradients of the likelihood function. A procedure developed by Hamilton (1986) is used by the authors to calculate the standard errors for the unobservable components from the Kalman filter. This method uses a Monte Carlo simulation that draws multiple parameter vectors from a normal distribution with covariance matrix of the parameter vector of the final estimation. This creates a measure that accounts for both the filter and the parameter uncertainty. In this paper I follow all the specifications pointed in this section for the estimates of the natural interest rates for members of the Eurozone-12 and use these results to obtain an estimate of the Eurozone natural interest rate.

4. Data

The goal of this project is to compute the natural interest rate for the members of the Eurozone-12 and obtain an estimate for the natural interest rate for the Eurozone. The Eurozone-12 is composed by the European countries that use the euro as their national currency, excluding the countries from 2004 enlargement onwards. This choice of countries is related to the amount of data available, since most of the new member states have a lower span of available indicators. Moreover, the Eurozone-12 countries accounted for 97,8% of the GDP of the Eurozone in 2016, according to the Eurostat, thus an analysis of these countries will not result on a loss of generality of the results. These countries are: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain. The LW model uses data of the real GDP, short-term interest rate and inflation rate to compute the estimates of the natural interest rate. The measure for the real GDP adopted was the *Gross Domestic Product-expenditure approach* measured in U.S dollars, with reference year in 2010 and seasonally adjusted. This indicator was collected from the “Quarterly National accounts” data base of the OECD. The inflation was computed from *Core CPI* (measure of CPI that excludes the price of food and energy). This measure was collected from the “Prices: Consumer prices database” from OECD. The Short-term interest rates were collected from the “Finance” data base of the OECD. For the measure of the real rate, that is used in equation (2) LW suggest a difference between the short-run interest rates and inflation expectations. The calculation of inflation expectations adopted in Holston, Laubach and Williams (2016) uses an average of the inflation rate of the current period with 3 lags.[7] The same approach is adopted in this paper. The short-term interest rates are annualized manually and the series were seasonally adjusted before the start of the estimation. The amount of quarters of data available varies from country to country, implying that the estimates for each country will be made for different time periods. The starting periods for each country are the following: for Austria, the starting point is 1991:Q3; Belgium, 1978:Q3; Finland, 1989:Q1; France, 1972:Q1; Germany, 1964:Q1; Greece, 1996:Q2; Ireland, 1986:Q1; Italy, 1980:Q4; Luxembourg, 1996:Q1; Netherlands, 1984:Q1; Portugal, 1987:Q1 and Spain 1979:Q1. The last quarter for all estimates is 2017:Q2.

5. Results

5.1 Parameter Estimates

The parameter estimates for each country are presented in Table 1 in Appendix 2. The estimates for λ_g range from 0.2829 for Greece and 0.0102 for Ireland. These results imply a significant variation in the trend growth rate, particularly for the cases of Greece and Austria, that presents an estimate of 0.1096 for this coefficient. An important point to notice is that both these countries are the ones with the lowest quarters of data available, together with Luxembourg. In the case of Greece, the result is not unexpected since the country faced long periods of recession during the financial crisis of 2008 and the consequent sovereign debt crisis, which had impact on the growth rate of the level of potential output. This fact can also be seen in the estimate for Portugal that has the third highest estimate for λ_g (0.0935). For the remaining countries we have coefficients that are consistent with previous research. The estimates in Mendonça (2017) for Italy and the Netherlands are both 0.038, while here the estimates are 0.0496 and 0.0453 respectively. Holston, Laubach, and Williams (2016) present

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[7] $inflation\ expectations_t = \frac{\sum_{i=0}^{3} \pi_{t-i}}{4}$

an estimate of λ_g for the Eurozone as a hole of 0.031 that is in line with most of the estimates presented. Regarding λ_z, we observe again the highest estimate in Austria, with 0.1452 but the lowest is now observed in Germany with 0.0125. Again the results are consistent with the findings in Mendonça (2017) (in Italy and the Netherlands, the author obtained estimates of 0.041 and 0.122, respectively that are similar to the estimates of 0.0362 and 0.1205 found in this paper) and in Holston, Laubach, and Williams (2016) (the authors present a λ_z of 0.040 for the Eurozone). The sum of the coefficients of the output gap lags in equation (2) ($\sum a_y$) is a measure for the persistency of the output gap in each country. As it is usual in the literature, this sum is close to one for most of the countries in our sample, excluding Greece that presents an estimate of 0.6582. This implies that the output gap in Greece is much less persistent than in the remaining countries analyzed. As pointed before during the period of our sample Greece faced a period of adjustment that can explain the lower persistency of the output gap. The coefficients a_r and b_y are the coefficients that identify both the output gap and the real rate gap in model, implying that the statistically significance of these coefficients is crucial for the estimates of the natural interest rate. By looking at the estimates and the corresponding t-statistics it can be observed that the only countries in our sample that present statistically significant coefficients for both a_r and b_y are France and Italy. Spain only presents the coefficient associated with the real rate gap significant. The remaining countries in the sample analyzed have poorly identified output gaps and real rate gaps. This result is also found in Mendonça (2017) and Holston, Laubach, and Williams (2016) where the authors found insignificant coefficients for the Netherlands and the Eurozone, respectively. Consequently, the standard errors of the Kalman Filter estimates of the natural interest rate are significantly high in our sample. In the table the information presented in the standard errors, sample average and standard errors, final observations, are the results of the Monte Carlo simulation discussed on section 3. It can be observed that the countries that present a lower standard error are the ones with a correct specification of the real rate gap and the output gap: France, Italy and Spain present an average standard error throughout the sample of 2.0368, 2.2261 and 2.0159, respectively. On the other hand, countries like Ireland, Portugal and Austria present an extreme degree of uncertainty on the estimates. All the levels of the standard errors increase when looking at the level of the final observation. This measure of uncertainty is related to the one-sided estimates of the filter (it uses the information until period t, to obtain the estimates at period t, while a two-sided estimate uses all the information of the sample to obtain the same estimate. This is also the difference between a Filtered estimate and a Smoothed estimate). These estimates are more consistent with the real-time approximations, implying that the use of this model to estimate the current level of the unobservable components leads to an even higher degree of uncertainty. Nevertheless, this uncertainty is directly related to the uncertainty of the z variable. The standard errors associated with the estimation of the trend growth rate are much smaller than the standard errors of the estimates of r^*, even in the countries with an imprecise specification of a_r and b_y. Thus, the estimates on the trend growth rate are much more reliable than the estimates of the natural interest rate, meaning that the cause of a significant part of uncertainty is the random walk z that captures all the remaining sources that affect the natural interest rate. A final note must be made regarding Austria, Portugal and Ireland, since these countries present the highest degree of uncertainty on the estimates. This fact may be related with the incorrect specification of the model[8].

5.2 Natural Interest Rate Estimates

In the figures 1-3 in Appendix 3, I present the one-sided estimates of the natural interest rates for all the countries in our sample. Just like the behavior described in the literature, the natural interest rates of the Eurozone countries have a significant decrease in 2008. In some countries the decrease lead to negative levels of the natural interest (in the cases of Portugal, the

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[8] For each country of the Eurozone-12, the correlations between the standard errors for r^* and the degree of openness and between the same standard errors and the size of that country were calculated. The results show a positive s.e-openness correlation and a negative s.e-size correlation. This points that the model could be incorrectly specified for small open economies. These results are presented in the external appendix 1, since this is not the main goal of this research. For a measure of size I used the average GDP level of each country and for the openness level I used the average share of imports and exports over GDP.

Netherlands, Italy, Greece, Finland and Belgium).The pattern described by Holston, Laubach, and Williams (2016) is also observed in our sample: In many countries we see a small increase in the estimates immediately after 2008, only to fall back again in 2011. Such behavior is substantially different for the U.S, where the decrease of 2008 saw no significant rebound in the following periods. This scenario is consistent with the theory of Secular Stagnation, initially presented by Alvin Hansen and, more recently, discussed by Laurence Summers (Summers, 2014). Another significant result that can be observed is the overall difference between the natural interest rates across countries. Despite following the same downward trend, there are differences that go against the idea of an optimal currency area (OCA). In an OCA it's expected that shocks affect all its members in a similar way, or at least countries have alternative tools to cope with asymmetric shocks (fiscal transferences for instance). These results show that there are significant base differences in terms of output gap, trend growth and, consequently, natural interest rates between the Eurozone members. This implies that the ECB upon deciding on the monetary policy may affect the different members in different ways. Nechio (2011) analyzed the policy choices of the ECB between 2001 and 2011 and compared it to an optimal policy, resulting from a Taylor rule. The author found that the ECB generally followed an optimal policy for the Eurozone, but the policy was not optimal for every country. The author constructed a Taylor rule for the core (Austria, Belgium, France, Finland, Germany, Italy and the Netherlands) and for the periphery countries (Portugal, Spain, Greece and Ireland) and found that the policy of the ECB was far from optimal for the periphery members. This result was explained by the large unemployment gaps that these countries still presented after the financial crisis, while the core countries were already recovering. The estimates of the natural interest rate presented here are consistent with these findings.

The literature on OCA's show that there are stabilization costs associated with monetary policy under structural differences. In Corsetti (2008) the author goes through the theory of OCA using a micro-foundations model to evaluate the costs in terms of welfare from having a single monetary policy. Corsetti found that the costs from incomplete stabilization are directly related to the individual structural differences. Thus, it can be argued that the Eurozone countries suffered from high incomplete stabilization costs.

A possible way to evaluate these differences between the Eurozone members is constructing a measure of the standard deviation per year of all the natural interest rate of each country. This will create a measure of the volatility of the natural interest rates in the Eurozone. The measure is presented on Figure 4 in Appendix 3. The volatility of the natural interest rate remained constant for most of the sample period. Nevertheless, there can be observed two significant increases in 2005 and in 2011. These can be related with specific asymmetric shocks that affected the Eurozone as a hole during these periods. In 2004 the European Union had the largest enlargement of its history, with the entry of 10 new countries. This increased the number of participants on the Single European Market and that lead to an increase in demand faced by the remaining European countries. It also lead to a redirection of the European funds (see Pires (2015) for the evolution of the structural funds in Portugal). and of Foreign Direct Investment (FDI) to the new members. As pointed by Medve-Bálint (2014), the European Union was the main driver of the FDI to the Eastern Central Europe, that create new economic dynamics for these new countries. As it’s expected by economic theory on economic integration, the enlargement had larger impact on the neighbor countries (core European countries), than periphery countries. Braakmann and Vogel, (2011) measure the impact of the enlargement on the companies of the Eastern border of Germany and found significant increases in the level on employment when compared to other German companies. This shows that the impacts of the enlargement were asymmetric across regions. All these facts could explain potential differences in structural terms in the countries of the Eurozone-12. In 2011 the European countries were in the middle the financial crisis and the Sovereign debt crisis. It´s well known that the crisis had different impacts on different member-states. The increase in the risk premium associated to some countries (particularly in the periphery of Europe), created heterogenous financing conditions across the Eurozone that could explain specific structural differences across member states. These differences can explain the increase in the volatility of the natural interest rate during this period. Another fact that can also be observed is the decrease in volatility in 2008, at the beginning of the financial crisis.

Given the results presented, it's important to understand if there are differences in the volatility between the periphery and core countries[9] and between large and small countries[10]. The measures of the volatility for each group of countries is presented in Figure 5 (Appendix 3). A clear result that can be observed is the difference between the levels of volatility. For the periphery and small countries group the average volatility is always higher than their respective counter parts (For the periphery the average is 2.25 while the core countries present an average of 1.16; For the small countries the average volatility is 2.44 while for large countries have an average of 0.48). These differences between the different areas in the Eurozone create significant limitations to the application of an optimal monetary policy by the ECB. It can also be observed that the increase in volatility in Eurozone-12 in 2005 is mainly explained by the volatility in the core European countries, while the increase in 2011 is much more associated with the volatility of periphery countries.

5.3 The Natural Interest Rate in the Eurozone

The results presented above show that there is some volatility in the natural interest rates across the Eurozone. A relevant analysis that can be made is comparing the natural interest rate of each country with the natural interest rate in the Eurozone. If the ECB makes its decision based on a Taylor rule, a proxy for its intercept would be the natural interest rate of the Eurozone. To find an estimate for this indicator I perform a weighted average of the natural interest rates of all the countries in our sample, where the weights at each period are the shares of that country nominal GDP in the Eurozone's GDP[11]. It's expected that the real rate that is consistent with the level of potential output in the Eurozone is a weighted average

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[9] The Periphery countries criteria follows Nechio (2011), that include Portugal, Spain, Greece and Ireland. The remaining countries are part of the core countries.

[10] The large countries considered are France, Germany, Italy and Spain.

[11] $r^*_{EZ_t} = \sum w_{it} \times r^*{}_{it}$, where i represents all the countries in our sample.

based on GDP of all the rates that are consistent with potential output for all its members. The results are presented in figure[12] 6 (Appendix 3). As expected it has a large fall in 2008 that is consistent will the previous literature. This estimate is also similar to the findings of Holston, Laubach, and Williams (2016) where the authors estimate the natural interest rate for the Eurozone. The small increase in 2009/2010 and the subsequent decrease in 2011 are also observable. The main difference is that the estimate of these authors reaches negative values in 2012. A relevant finding that can be seen in the estimate is the increase of this indicator in last periods of the sample. The natural interest rate in the Eurozone reached levels of 0.7% in the second quarter of 2017. A similar result is also present in the U.S.. John Williams (Williams, 2017) argues that the level of the natural interest rate in the U.S. is currently around 0.5%, and points that this is a new reality that bankers and other economic agents should take this into account moving forward. The author believes that future interest rates will be close to 2.5%, in the U.S. that is a significantly different scenario from the one observed in the world 20 years ago. A similar argument can be made for the Eurozone, looking at the estimates for the natural interest rate and at the ECB's inflation target.

With the estimates of the Eurozone natural interest rate it's possible to perform a comparison with the natural interest rate of all the countries in our sample to understand the fit and the impact (positive or negative) of the monetary policy. To do this I present in figure 7 (Appendix 3), the difference between the natural interest rate of each specific country and the natural rate of the Eurozone. If this difference is positive it means that the target rate of the ECB is lower than the natural rate of the country, implying that the policy is favoring this country. If it's negative, the conclusion is exactly the opposite. By looking at the cases of Portugal and Greece, we can see that this difference becomes negative particularly during the periods of the financial crisis. In Finland, we observe a similar pattern but only after 2011. An almost symmetric result is present for countries like Germany, France and Spain where during

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[12] The analysis of the Eurozone natural interest rate stars at 1996 since this is the date of the start for the sample in Greece. An alternative computation was made without Greece but the data for the nominal GDP for each country only goes back to 1995 in Eurostat. Since the difference is not significant, the full sample rate was analyzed.

the crisis this gap became more and more positive. Concluding from these results that the monetary policy of the ECB during the period of the crisis was contractionary for Portugal and Greece can be misleading since we are not comparing the actual real interest rate established by the ECB. Nevertheless, and assuming that the ECB considers the information of natural interest rate into a Taylor rule, it can be said that during this period the policy specification was more harmful to countries like Portugal and Greece than towards countries like Spain, France and Germany. Regarding the remaining countries, the difference points to some inconsistent results. In Ireland and Luxembourg, the difference follows a similar pattern to Portugal and Greece, in the sense that they fall during the period of 2008, but the difference remains always positive throughout the sample period. Italy and the Netherlands present a result that is persistently negative. Belgium and Austria present the most random estimates for this indicator.

6. Conclusion

This research aimed at studying the differences in structural terms of the members of Eurozone 12 and how these differences affect monetary policy of the ECB. The Wickesillian concept of natural interest rate was adopted and estimated for all the countries in the sample using the model developed by Laubach and Williams (2003). The estimates show that there are significant differences between the natural interest rates across states, arguing against the existence of an Optimal Currency Area in the Eurozone. A downward trend is also observed for all countries particularly after 2008. Nevertheless, most of the estimates present a high degree of uncertainty associated to the poor specification of the coefficients of the IS and Phillips curve. This uncertainty is a limitation to this research since it takes power to any conclusion drawn from the results. A measure of volatility of the natural interest rates was also built, computing the standard deviation of the estimates across countries. It pointed that the volatility in the natural interest rates is directly associated to asymmetric shocks that affect the Eurozone, given that it increased during the period of the 2004 enlargement and the financial crisis, that affected the member-states in different ways. Computing this measure for specific groups of countries pointed that the periphery and small European countries have a higher level of volatility. Also, I computed the natural interest rate of the Eurozone by performing a weighted average of the estimates for the countries in the sample, and the result was consistent with previous estimates in the literature (Holston, Laubach, and Williams 2016). At the end, the difference between the natural interest rate of each country and the eurozone rate was computed to evaluate the fit of the ECB monetary policy. It pointed that some countries like Portugal and Greece were more harmed by the structure of the monetary policy than countries like Germany, France and Spain. It became clear that there are significant differences in structural indicators across the Eurozone. Further research in this topic can be made to understand the source of these differences, splitting between the trend growth rate and the z variable. An important step in this field would be to find an improved specification of the LW model for Europe, considering possible small open economy dynamics. The authors in Holston, Laubach and Williams (2016) pointed that more accurate estimates could be obtained if the specification of the model was better suited for the European case, and that fact is a major limitation for this research. Some work was already done to improve the specification of the LW model (Lewis and Vazquez-Grande, 2017), but not for the particular case of the Eurozone.

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Appendix 1 – State Space Representation

As mentioned in section 3, the model for the estimation of the natural interest rates uses the Kalman Filter to compute the unobservable components. The State space representation of the model using equations (2)-(7) can be represented in the following specification:

$$\gamma_t = Ax_t + H\varphi_t + \varepsilon_t$$

$$\varphi_t = F\varphi_{t-1} + \epsilon_t$$

where γ_t represents a vector of the observed variables, x_t represents a vector of the lagged observed variables. φ_t is a vector of the unobservable components that will be estimated with the Kalman Filter. As pointed before ε_t and ϵ_t are mutually uncorrelated disturbances with mean zero and covariance matrix R and Q respectively. For the final stage of the estimation, the matrices can be represented as following:

$$\gamma_t = [y_t \quad \pi_t]^T \qquad x_t = [y_{t-1} \quad y_{t-2} \quad r_{t-1} \quad r_{t-2} \quad \pi_{t-1} \quad \pi_{t-2,4}]^T$$

$$\varphi_t = [y_t^* \quad y_{t-1}^* \quad y_{t-2}^* \quad g_{t-1} \quad g_{t-2} \quad z_{t-1} \quad z_{t-2}]^T \quad A = \begin{bmatrix} a_{y,1} & a_{y,2} & \frac{a_r}{2} & \frac{a_r}{2} & 0 & 0 \\ b_y & 0 & 0 & 0 & b_\pi & 1-b_\pi \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & -a_{y,1} & -a_{y,2} & -\frac{a_r}{2} & -\frac{a_r}{2} & -\frac{a_r}{2} & -\frac{a_r}{2} \\ 0 & -b_y & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} (1+\lambda_g^2)\sigma_{y^*}^2 & 0 & 0 & (\lambda_g \sigma_{y^*})^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (\lambda_g \sigma_{y^*})^2 & 0 & 0 & (\lambda_g \sigma_{y^*})^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (\frac{\lambda_z \sigma_{\tilde{y}}}{a_r})^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Appendix 2: Table 1 – Parameters Estimates

Parameters	Austria	Belgium	Finland	France	Germany	Greece
λ_g	0,1096	0,0765	0,0227	0,0405	0,0689	0,2829
λ_z	0,1452	0,0258	0,0780	0,0422	0,0125	0,0867
a1+a2	0,8934	0,9026	0,9677	0,9537	0,9362	0,6582
a_r	-0,0080	-0,0135	-0,0660	-0,0376	-0,0805	-0,0622
(t-stat)	0,2146	1,0819	1,5550	2,9074	1,7270	0,6677
by	0,1193	0,1880	0,0250	0,1429	0,0996	0,6376
(t-stat)	0,6643	1,2168	0,8580	2,2967	1,2699	0,8234
σ_~y	0,2153	0,5168	0,4163	0,2247	0,8000	0,6073
σ_π	1,6457	1,1761	1,8407	1,5487	1,6257	2,5725
σ_y*	0,3711	0,3063	0,9060	0,3352	0,4749	0,8374
σ_g	0,0407	0,0234	0,0205	0,0136	0,0327	0,2369
σ_z	3,9009	0,9898	0,4921	0,2517	0,1242	0,8460
σ_r*	3,9011	0,9901	0,4925	0,2521	0,1284	0,8785
S.e (Sample Average)						
r*	11,4014	8,4324	5,9804	2,0368	3,1440	7,5578
g	0,4344	0,3407	0,5435	0,2260	0,4544	1,3861
y*	1,6322	1,2090	4,5802	1,3878	1,9748	1,1041
S.e (final observation)						
r*	16,1692	12,4353	8,6365	2,9241	4,6008	11,5152
g	0,5933	0,5018	0,6620	0,3116	0,6508	1,8296
y*	2,3586	1,9986	7,1639	2,0087	2,8861	1,4620

Parameters	Ireland	Italy	Luxembourg	Netherlands	Portugal	Spain
λ_g	0,0102	0,0496	0,0557	0,0453	0,0935	0,0265
λ_z	0,0867	0,0362	0,0555	0,1205	0,0519	0,0462
a1+a2	0,8861	0,9123	0,8253	0,9814	0,9341	0,9912
a_r	-0,2068	-0,0531	-0,5652	-0,0253	-0,0924	-0,0137
(t-stat)	0,9622	2,0395	1,5115	1,1224	1,1757	2,6812
by	0,0250	0,1449	0,0250	0,0250	0,0933	0,0418
(t-stat)	0,4367	2,0504	0,9879	0,4998	0,7497	1,2748
σ_~y	1,1959	0,2416	0,6639	0,1664	0,0000	0,0556
σ_π	2,7733	1,5028	1,1898	1,8034	1,4490	2,5722
σ_y*	1,8789	0,5176	1,2303	0,5986	0,8747	0,6147
σ_g	0,0192	0,0257	0,0686	0,0271	0,0818	0,0163
σ_z	0,5014	0,1646	0,0652	0,7911	0,0000	0,1882
σ_r*	0,5018	0,1666	0,0947	0,7916	0,0818	0,1889
S.e (Sample Average)						
r*	13,3274	2,2261	2,7618	9,3644	34254,9075	2,0159
g	0,9041	0,3631	0,8921	0,4960	1,0469	0,3589
y*	5,1906	1,4034	3,4879	3,5022	3,7088	3,3093
S.e (final observation)						
r*	18,5502	3,1109	3,9590	14,1609	48368,6300	2,9395
g	1,0174	0,4723	1,1013	0,7043	2,1650	0,4730
y*	7,0654	1,4364	5,6547	6,2012	6,9716	4,5080

Appendix 3 – Figures

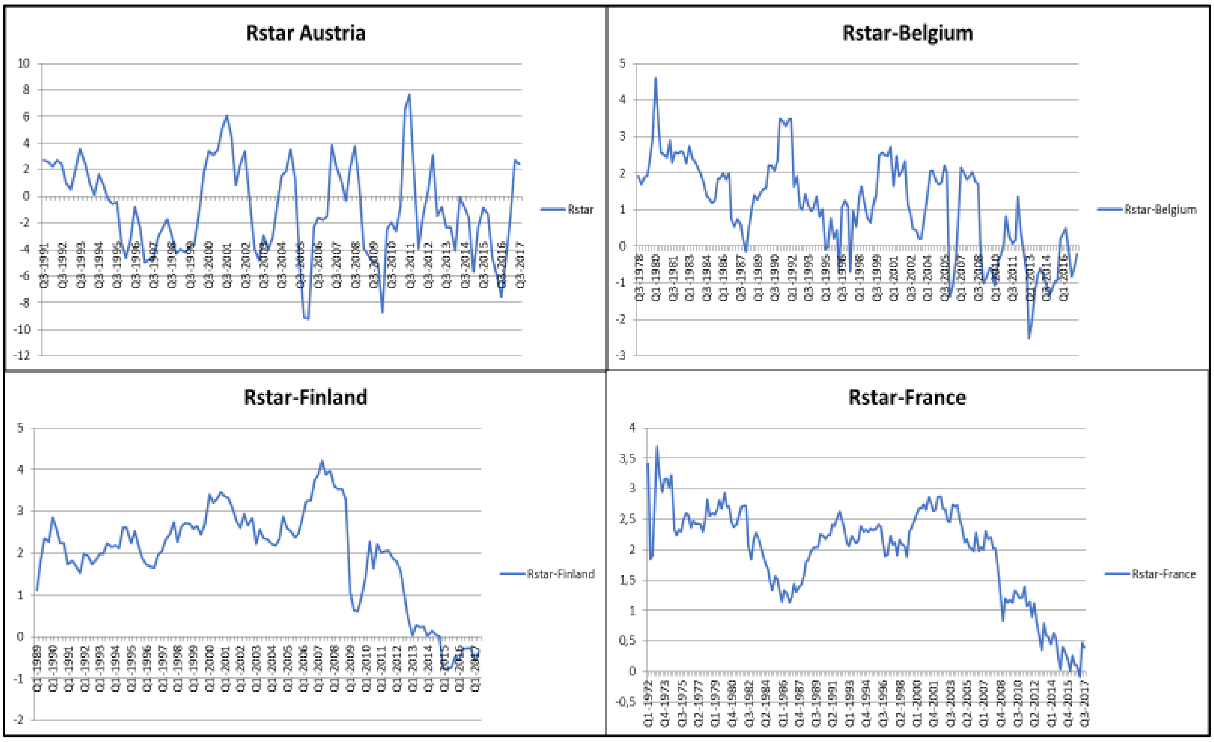

Fig.1:Natural Interest Rate Estimates – Austria, Belgium, Finland and France

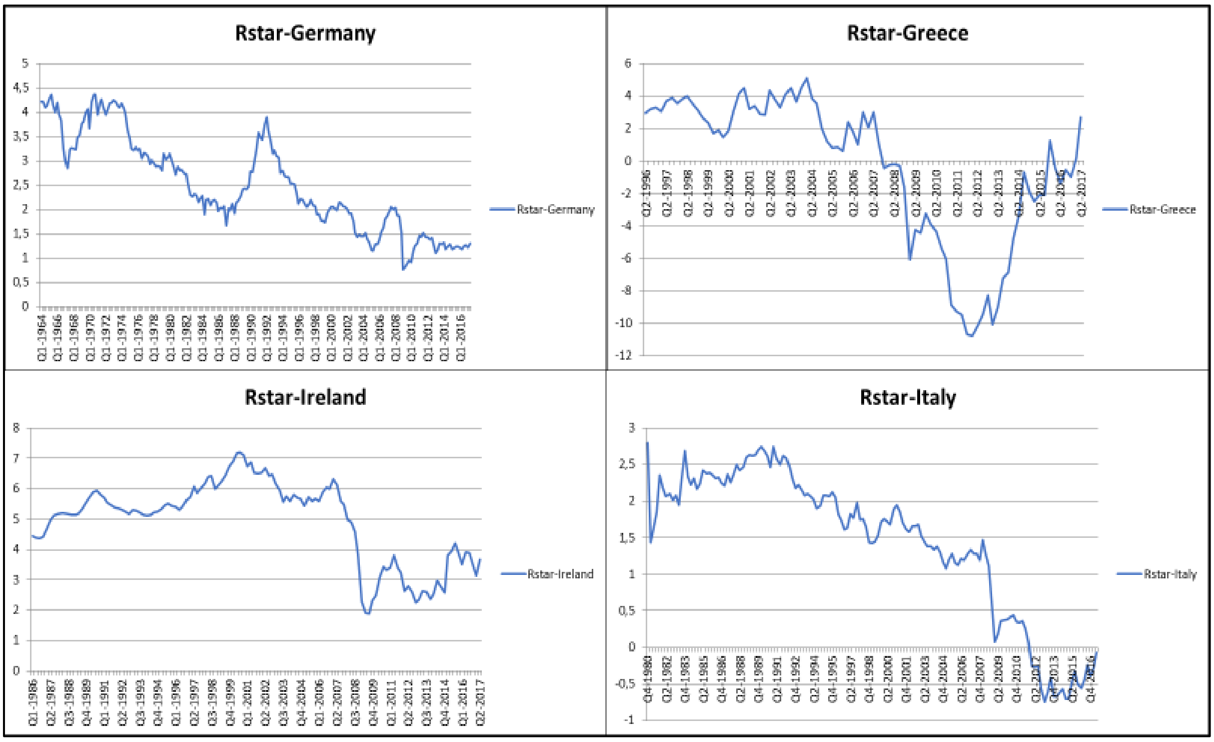

Fig. 2: Natural Interest Rate Estimates – Germany, Greece, Ireland and Italy

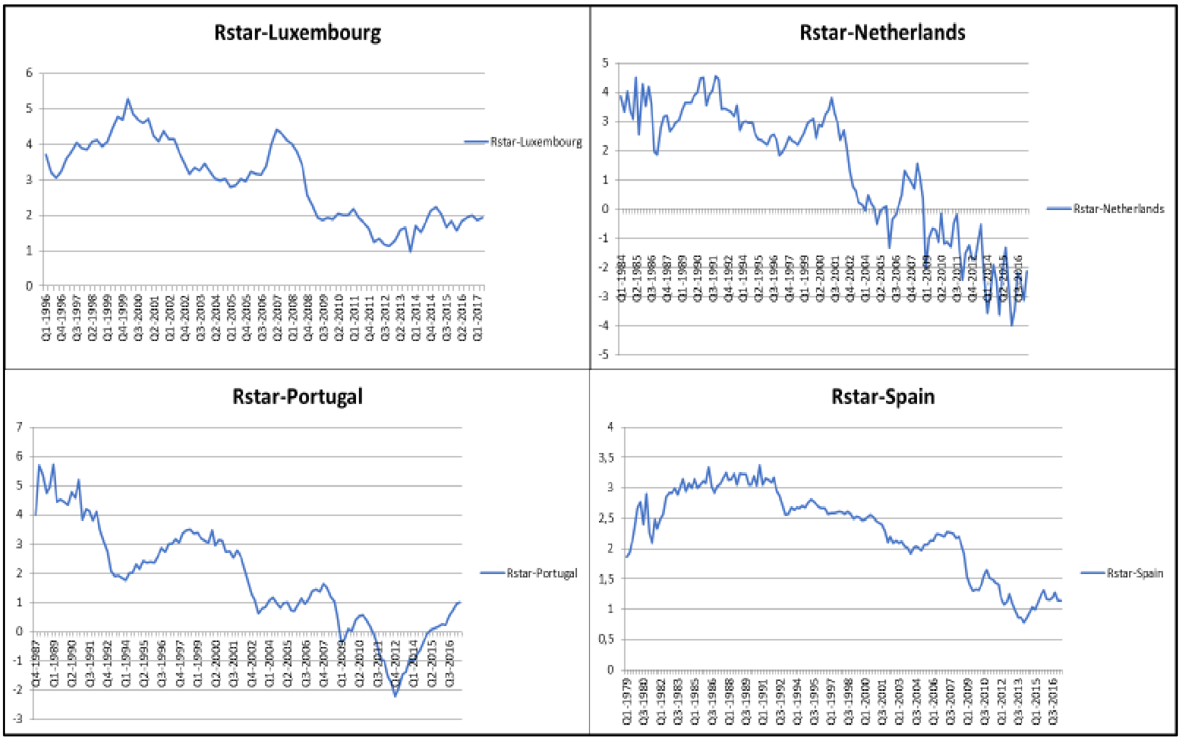

Fig. 3: Natural Interest Rate Estimates – Luxembourg, Netherlands, Portugal and Spain

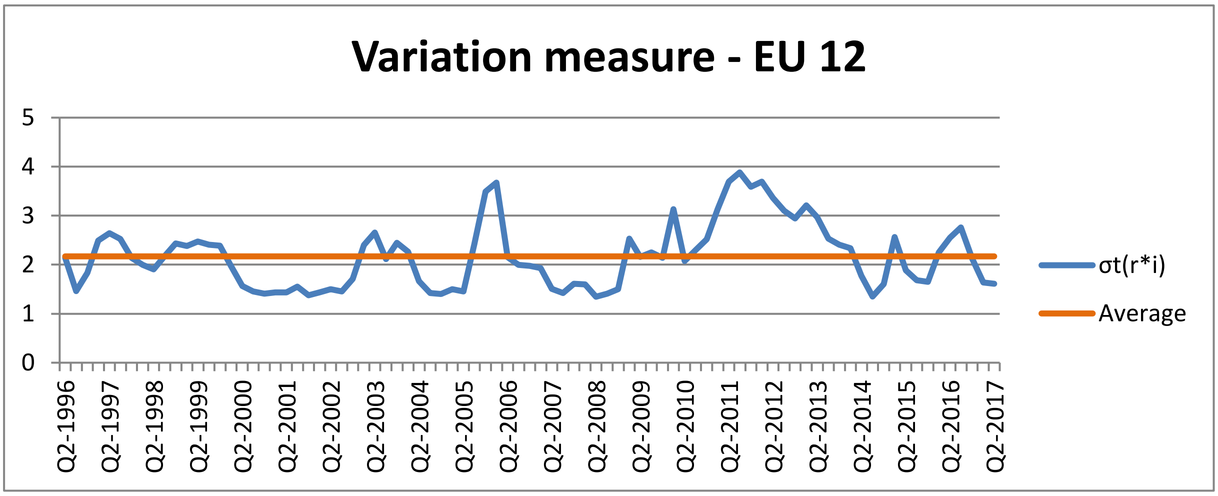

Fig. 4: Standard Deviation of the Natural Interest Rate Estimates

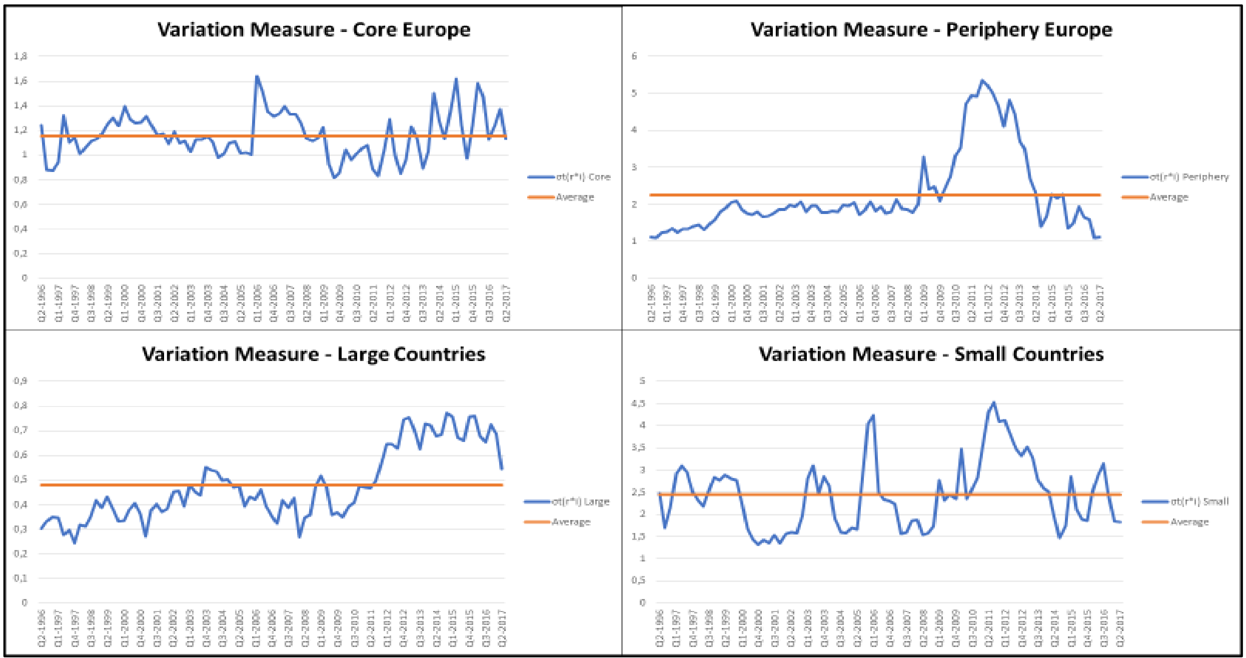

Fig. 5: Volatility measure per group of countries

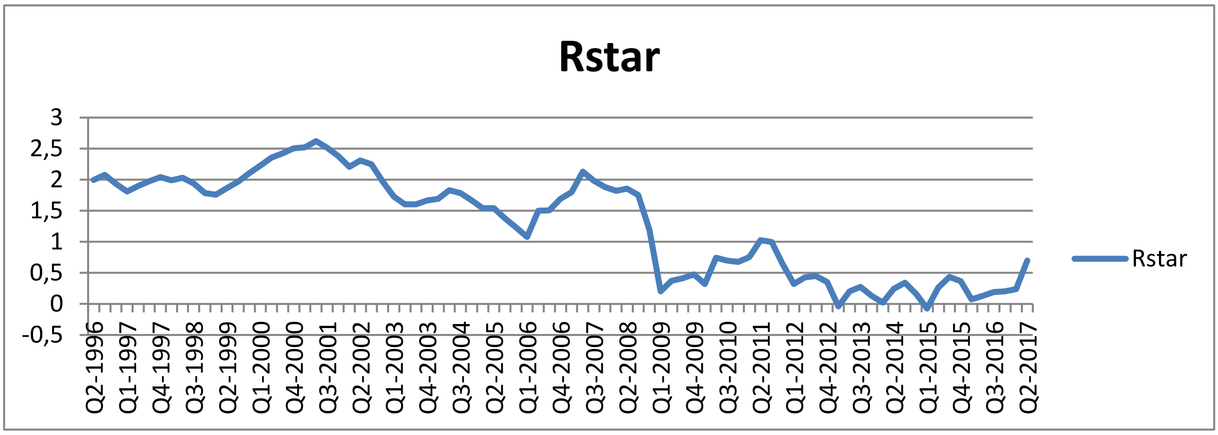

Fig. 6: Natural Interest Rate Estimates - Eurozone

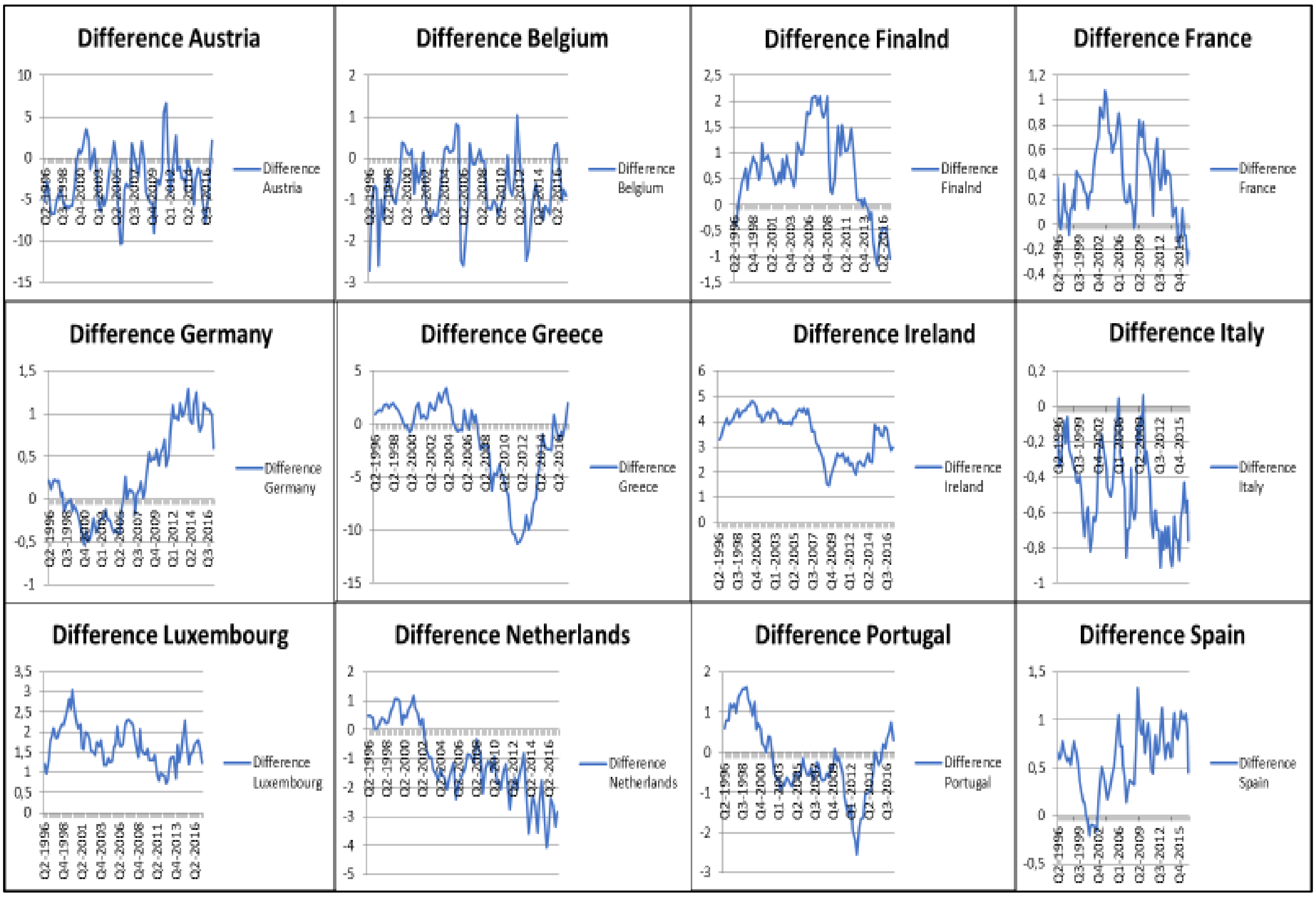

Fig. 7: Differences of natural interest rates per country