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Theory of regulation of trade repositories, with an illustration from Brazil

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This thesis investigates the impact of regulatory changes in Brazil's Trade Repository (TR) market, particularly focusing on the implications of enforced interoperability and transparency on market prices and competition dynamics. Utilizing a modified version of Laffont et al.'s (1998) model, the study reveals that regulatory interventions has the potential to significantly reduce both intra-network and inter-network prices, enhancing competition and efficiency within the TR industry. These findings contribute to a deeper understanding of the regulatory dynamics in network industries and offer insights for designing policies that foster competition and reduce costs in similar market settings.

Keywords: Network competition; Competition Economics; Network externalities; Trade repositories; Intra/inter-network access pricing.

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1. Introduction

Trade repositories (TRs) play a vital role as data warehouses, managing electronic information of over-the-counter (OTC) derivative and security transactions to reduce information asymmetry and enhance data integrity. Despite empirical evidence highlighting the significance of TRs in assessing financial stability and mitigating risks (Pezzini, 2015; Bank of England, 2013; Bank of England, 2016; Bank of England, 2017), financial market infrastructures (FMIs), including TRs, have been undervalued by economists (Bindseil and Pantelopoulos, 2023).

The present study focuses on a distinctive use of TRs in Brazil, where the Central Bank of Brazil (hereafter BCB) regulation aimed to reduce credit costs by designating revenues from credit card sales, known as "payment arrangement receivables" (hereafter RECAP), as financial assets. Moreover, the BCB provision created a two-sided market. On the one side, payment processors,[1] responsible for providing point-of-sale (POS) terminals or payment solutions for e-commerce platforms to merchants, were required to report retail sales to a TR, fostering transparency. On the other side, financers (e.g., banks, credit rights investment funds) compete by offering credit backed by these assets, which ultimately has the potential to decrease credit costs. The regulatory-led initiative thus prompted TRs to develop an ecosystem for interoperability between their networks,[2] compelling competing firms to cooperate, as discussed in previous literature (Rochet and Tirole, 2002; Laffont et al., 1998).

The main purpose of this research is to study how a regulatory-led process to ensure interoperability and data sharing among competitors affected the dynamics of TR industry in terms of impact on equilibrium prices. By examining the regulatory changes in Brazil and their

[1] Also referred in literature as payment facilitators/aggregators, acquires or sub-acquirers.
[2] Brazilian regulation extends beyond the interconnection between TRs for bit and byte exchange. In addition to data exchange, interoperability encompasses the portability of financial assets and derivatives, ensuring the uniqueness of these assets.

effects on TR networks, this research seeks to provide insights regarding the implications of market agreements and regulatory constraints in shaping price competition and cooperation within FMI industry.

This study makes relevant contributions to Brazilian competition policy by analyzing the TR market, where these networks have additional roles when compared to other peer economies. The findings suggest that regulatory constraints result in lower prices charged by TRs intra-network and inter-network, when compared with prices freely and individually chosen by the firms. We posit that understanding the dynamics in TR industry and its consequences for the competition requires examining of firm-level decisions and authority responses. The BCB board decided to observe the implications of regulation changes in RECAP market,[3] which lessons will be further applied in the regulation reform of a larger asset market - the virtual duplicata or "entry-book trade bills."[4]

To achieve this, I built the present study on the model developed by Laffont et. al. (1998) to investigate the network competition in the telecommunication industry, which requires cooperation among rival firms to set up standards and prices. I observe considerable similarities between both TR and telecommunication industries under the interoperability perspective. As Laffont et. al. (1998) suggest, their "model could be used as a backbone for a number of extensions such as non-balanced call patterns or vertical network differentiation." I want to extend their findings to the RECAP market, which encompasses both features non-balanced access pattern and vertical network differentiation, among other distinctions detailed in section 6.1 of the present work. A theoretical model is developed for two TRs, incumbent and entrant. Following the price equilibrium and impacts computed under free prices chosen individually, I extend the model for impacts caused by market agreement and/or regulatory constraints.

[3] BCB Board Vote available at: https://normativos.bcb.gov.br/Votos/BCB/2023135/Voto_do_BC_135_2023.pdf.
[4] Estimated to be around €3 trillion per year, with only 20% currently used in asset-backed credit.

Additionally, we examined quantitative and qualitative data to measure credit cost in revenues of credit card sales anticipation, and to analyze the price behavior of the TRs.

The main results suggest that regulation on prices or market share is effective in lowering prices for both incumbents and entrants, as prices that are freely and individually defined can lead to market distortions such as subsidization. The findings support Laffont et al. (1998), suggesting that unregulated access prices can impede competition, and corroborates Armstrong's (1998) suggestion that the regulator must play a role beyond an arbitrator. This highlights the potential for regulatory policies to enhance competition in broader asset markets.

This study departs from the literature on network competition, interconnection between networks, switching costs and network effects. Moreover, how competing firms access monopolistic or bottleneck facilities, and what determines the price of interoperability access.

Little literature exists in regulation and competition for FMI and, more specifically, dedicated to TRs despite their important role for mitigating systemic risk in capital market and decreasing asymmetric information (FSB, 2017). For this reason, this study draws on existing research on other industries with some similarities to perform a literature review.[5]

To accomplish the intended objectives, this study is organized as follows. Section 2 outlines the background of the literature on the main concepts to be developed in the research. Section 3 summarizes the main aspects of TRs and introduces the key players. Section 4 describes the main characteristics of the financial asset RECAP.[6] In Section 5 we present the methodology, the core assumptions of the benchmark model, and describe the data used. Section 6 presents the theoretical model setup and extends it for different regulatory constraints. Section 7

[5] Telecommunication, payment systems, banking, open banking network, card industry and digital markets.
[6] Brazilian legislation formally calls it as "*Unidade de Recebíveis*" (UR).

concludes, presents policy implications, and the limitations. Section 8 presents the references. Section 9 presents an appendix with complementary proofs and pieces of evidence.

2. Literature review

2.1. Industry dynamics

Classical theory of firms suggests that an increase in the number of firms within an industry is socially beneficial, improving allocative efficiency, consumer welfare, and overall welfare. Numerous studies have examined the implications of market structure on a firm's conduct and performance. Bain (1951), for instance, noted a positive correlation between higher profit rates and increased industry concentration. Additionally, the works of Beck et al. (2004), Lartey et al. (2023), Fraas and Greer (1977), and Cetorelli and Gambera (2023) reveal adverse effects of concentration, such as collusion, monopolistic pricing, and entry foreclosure.

In most countries, the TR industry exhibits high concentration, typically characterized by monopolies or duopolies. An exemption to this trend is found in Brazil, where eight firms operate in the sector. Alongside the PFMI provided by the BIS/CPMI,[7] the BCB imposes stringent regulations, outlining guidelines and standards for authorization. These regulations subject both entrants and incumbents to regular monitoring and on-site inspections, ensuring compliance with stablished standards. Notably, the BCB regulation mandates that TR platforms must be available at least 99.8% of the scheduled operational time. Consequently, it is reasonable to assume that the services offered by TRs are, at least *ex ante*, homogeneous.

2.2. Interconnected networks

The interaction between competition and regulation in network industries has been longstanding focus of investigation.[8] Efficient regulation aims to overcome issues such as

[7] These principles provide general guidance for legal basis, governance, risk management, operational risk, access, and participation etc.
[8] Telecommunication, electricity, cable TV, natural gas etc.

asymmetric information, regulatory capture, collusion, and market foreclosure. Nonetheless, Bianchi et. al (2023) argue that the pricing of interoperable systems is an under-explored subject. They conclude that lack of interoperability may affect competition both in telecom and in payment markets by discouraging entry and lessening competition.

In the context of "two ways" access, or network interconnection, firms require access to each other's network, typically subject to access charge payments. A competitive bottleneck arises when multiple networks compete in the same industry, each holding a monopoly over certain assets (essential facilities) to which access must be granted to rival networks.

In mature markets with multiple operators, the access price may be individually chosen by each operator. Alternatively, firms may engage in free negotiations to determine a reciprocal access price, with the regulator occasionally serving as a mediator when agreements prove elusive. A third approach involves the regulator actively setting the access charge, although this is significantly challenging due to the risk of establishing a price that may not be viable for certain operators (Bijl and Peitz, 2002).

Laffont et. al. (1998) developed a model for determining the competitive market equilibrium in telecommunications[9]. They concluded that a competitive equilibrium may fail to exist under large access prices or substantial network substitutability. Freely negotiated access charges may hinder competition through collusion and create barriers to entry, raising doubts about whether competition alone would discipline access and retail prices.

Armstrong (1998) suggests that rival firms can agree on a reciprocal access charge in a symmetric model without regulation, except if there is an incumbency advantage. The author

[9] The authors published two companion papers, the first assuming non-discriminatory pricing, and the second considering price discrimination.

proposes further research to explore how to control access charges between networks with distinct cost characteristics, where the reciprocity assumption may not be satisfactory.

2.3. Network externalities and switching costs

Network externality occurs when the utility a consumer derives from a product increases with the number of users of that product. This characteristic has been substantiated by several studies across various industries (Gruber and Verboven, 2001; He et al., 2023, Brynjolfsson and Keremer, 1996; Katz and Shapiro, 1994; Katz and Shapiro, 2001). The impact of network externality tends to intensify with the presence of compatibility, interoperability, and standardization.

Katz and Shapiro (2001) proposed that if consumers anticipate a firm attaining a dominant position, their willingness to pay a higher price for the product or service increases, thereby reinforcing the firm's inclination toward dominance. Gent et al. (2023) conclude that a monopolistic market structure is more likely to emerge when the network effect is robust.

Switching costs are typically observed in industries with differentiated products. However, as suggested by Padilla (1995), there is conceivable scenario where firms compete for demand for an *ex ante* homogeneous good. In this scenario, although the good is initially homogeneous, once customers decide to change to another firm, switching costs come into play, leading to the *ex-post* differentiation of services and an increase in market power due to the lock-in effect experienced by customers.

In line with Klemperer (1987), two primary types of switching costs can be identified. The first is transaction costs, arising from the losses on the initial investment (e.g., technological design, specific development of software or APIs) faced by consumers when changing to a different supplier. The second is learning costs, which entail the customer familiarizing themselves with a new process system (e.g., designing new standards for communication and data sharing). In the TR industry, switching costs can arise from factors such as data portability, system integration and customization, training and convenience, contractual agreements, and the evaluation and selection of a supplier.

Fostering industry competition has the potential to increase social welfare, however undesired effects can be faced by authorities, such as market foreclosure, locked-in customers (Farrell and Klemperer, 2007), requiring efficient regulation for achieving market equilibrium.

From the literature review, it is possible to extract some core concepts for the sequence of this study, including the roles of regulation, competition, incumbency, and *ex-post* product differentiation in determining market equilibrium. Additionally, the competitive bottleneck resulting from interconnected networks with considerable market power over valuable assets.

3. Trade repositories: Overview and economic market

Motivated by the competition policies and regulatory changes instituted by the BCB, the total number of TRs in Brazil experienced a notable increase, rising from two, in 2018, to eight, in 2023 (see Table 1 in Appendix 9.1).[10] Globally, the Bank for International Settlements (BIS) lists a total 39 TRs or TR-like entities.[11] It is expected that this expansion may be followed by a contraction in the market, with mergers and acquisitions taking place in the near future, as already observed in 2023.[12]

Regulatory adjustments concerning technological provisions in the financial system contribute to this phenomenon in Brazil. Resolution CMN no. 4,658/2018 allowed financial institutions to

[10] Available at: https://stats.bis.org/statx/toc/CPMI.html
[11] Entities not authorized as a TR but used by the market to report OTC derivatives and other securities trade data.
[12] The entity Nuclea acquired the CRT4 in 2023.

adopt cloud computing, [13] which decreases significantly fixed costs of data processing.[14] Although not directly applied to FMIs, this regulatory innovation served as a benchmark for the latter. Thereby, after years without contenders in the FMI industry, the BCB started to receive numerous authorization requests, with new entrants seeking to capitalize on the significant reduction in entry costs compared to traditional on-premises data centers.

In contrast to other countries, Brazilian legislation broadened the scope of mandatory transactions to be reported to TRs, encompassing a wide array of transaction types and nearly all asset classes, including OTC and exchange-traded derivatives, loans, foreign exchange, equity, private securities, and fixed income transactions. The Brazilian financial market encompasses approximately 200 different financial instruments, with around 50 classified as financial assets.

Competition among TRs is contingent on the authorization granted by the BCB, as each TR is authorized to receive trade reports for specific financial assets. Consequently, the product market is delineated by the BCB's authorization. Pricing dynamics are freely determined by market agents. In geographical terms, once authorized, TRs can operate throughout Brazilian territory without constraint.

Table 2 in Appendix 9.1 consolidates information about TRs and their authorized financial assets. Some financial assets are reported exclusively to one TR, which essentially operates as a local monopoly for those specific asset classes. In contrast, others financial assets are registered at up to five TRs, implying greater competition. For the purposes of this study,

[13] On demand service provided by companies (e.g., AWZ, Oracle, Azure) for data processing, data storage and database.
[14] Available at: https://www.bcb.gov.br/estabilidadefinanceira/exibenormativo?tipo=Resolu%C3%A7%C3%A3o&numero=4658. It was replaced by Resolution CMN no. 4,893/2021, available at: https://www.bcb.gov.br/estabilidadefinanceira/exibenormativo?tipo=Resolu%C3%A7%C3%A3o%20CMN&numero=4893

attention is focused on three TRs: CERC S.A. (CERC), TAG Tecnologia para o Sistema Financeiro S.A. (TAG), and CIP S.A. (Nuclea).[15] Additionally, the asset RECAP (highlighted in Table 2 in Appendix 9.1) is the subject of the present analysis.

4. Financial asset "payment arrangement receivable" (RECAP)

4.1. RECAP characteristics

Payments made with credit and debit card in the retail sector are widely popular in Brazil,[16] constituting nearly 30% of individual income commitments in 2022, the largest share among the credit operations (BCB, 2023). As a retail payment instrument, credit/debit cards are second only to the Brazilian instant payment service (Pix).

Each time a consumer uses their credit or debit card, it generates a credit right for the retailer. Typically, sales made by card are credited to the retail outlet's account within 32 days if paid in the credit function. In the context of the increased interest rate spread in Brazil, credit sources backed by assets can play a crucial role in reducing the cost of credit. Sales paid with a credit card can be advanced to sellers with interest, using the credit card invoice as collateral. In 2022 alone, there were over 30 trillion card transactions, totaling approximately €550 billion (R$3 trillion),[17] illustrating the substantial market potential for guaranteed financing backed with RECAP.

Prior to the establishment of RECAP reporting, merchants faced a local monopoly when negotiating their credit rights. Both payment processors and banks were vertically integrated with exclusive access to the available credit rights of a retailer. Prices were assumed to be above the competitive level, leading to monopoly profits in the credit market. Other potential

[15] The fourth TR authorized in the RECAP market, B3 S.A. - Brasil, Bolsa, Balcão (B3), has no operations performed. It was the last firm to join the ecosystem when the customers had already chosen their TRs.
[16] See BIS Statistics at: https://stats.bis.org/statx/srs/table/T4?c=BR. Access at: 19/11/2023
[17] See BCB's payment methods statistics at: https://www.bcb.gov.br/estatisticas/spbadendos. Access at: 19/11/2023

financiers were interested in obtaining information about the credit rights, but asymmetric information hindered this form of competition. A simple representation of this scenario is presented in Figure 3 in Appendix 9.1.

Figure 4, in Appendix 9.1, compares the annualized average interest rate in credit for firms and the interest rate for credit backed by RECAP, where we can observe a larger decrease in the interest rate for RECAP discount against to other credit options. Figure 5, in Appendix 9.1, depicts the increasing ratio of RECAP discount against total credit for firms.

4.2. Market functioning

The increment of firms in TR industry in Brazil, although encouraged, has led to fragmented transaction reporting standards. To address this, the BCB determined the implementation of interoperability mechanisms through compatible procedures and technology.[18] This ensures that participants can access data from any network, promoting portability and asset uniqueness. Each TR functions as an individual network and an essential facility.

Following Niels and Ralston's (2021) definition, the TR for RECAP operates as a two-sided market connecting both payment processors and financers, for intra-network, and inter-network asset consultation. The exclusive obligation of reporting RECAP lies with payment processors, creating an inelastic demand for TRs in this market side. However, this service cannot be charged to avoid a financial burden on payment processors, which, therefore, lack the incentive to change to a different TR based on prices or in network externalities.

On the other hand, the financers' demand for TR services is more elastic, and they are expected to leverage network externalities. Financers are charged for accessing market assets and for

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[18] Resolution BCB no. 264/2022. Available at: https://www.bcb.gov.br/estabilidadefinanceira/exibenormativo?tipo=Resolu%C3%A7%C3%A3o%20BCB&numero=264

reporting a credit contract backed with RECAP. These costs may be passed on to retailers through higher interest rates, reducing consumer surplus. There is also a risk of financers restricting credit offers to the network they participate in, posing undesirable consequences.

5. Data & Methodology

5.1. Methodology

This study is based on the development of a theoretical approach. First a benchmark model is designed to represent the current functioning of the RECAP market established in Brazil. After computing equilibrium prices in this setup, changes potentially set in place by the regulation are introduced and their impact in equilibrium prices observed.

5.1.1. Assumptions

The benchmark model assumes facilities-based competition involving two asymmetric TRs,[19] which networks have two different types of customers, the payment processors, and the financers. TR *i* is an incumbent network with a sizable market share of financers and payment processors, and TR *j* an entrant network with a smaller share of financers and payment processors.

The TRs are considered as single product firms, offering access to RECAP assets for financers, for simplicity. The payment processors' report of RECAP is considered as exogenously determined. On the other hand, financers exhibit relatively elastic demand for access to reported RECAP and relatively low elasticity of substitution between TRs. This is grounded in the fact that access costs for most financers are not significant relative to total revenues, and the low elasticity of substitution is justified by the *ex-ante* homogeneity and *ex-post* heterogeneity of services offered by TRs, resulting from switching costs.

[19] I follow the literature in network competition, which generally considers duopolies, but most the outcomes can be extended to markets with more than two firms.

Regulatory constraints dictate that payment processors can only connect and report to one TR. We assume that financers connect to a single TR for convenience due to discriminatory pricing based on demand, where increasing discounts are applied to higher transaction volumes.

The pricing model is linear. Each TR sequentially determines inter-network price and non-cooperatively, and potentially asymmetrically, chooses intra-network price. This approach is supported by prior research (Laffont et al., 1998; Armstrong, 2001). Asymmetric network interconnection is modeled following Carter and Wright (2003), while network competition with discriminatory prices, linear pricing, and non-cooperative access charge determination is modeled based on Laffont et. al. (1998).

5.2. Data

The data source encompasses the annualized average interest rate in Brazil in credit operations among firms, the average interest rate for RECAP discount, the total volume of credit operations for firms, and the volume of credit backed by RECAP. This information is disclosed by the BCB through its Time Series Management System,[20] covering the period from 2013 to 2023.

Furthermore, I used data obtained from the TRs, which includes the historic fee schedule. The data spans from 2021 (inauguration of the interoperability ecosystem) to 2023. The ecosystem is still under enhancements to achieve full functionality, enabling the observation of the real-time impact of regulatory measures on market structure. Figure 6 in Appendix 9.1 shows the price variation for consultation (fee per each access in Brazilian Real - BRL) of RECAPs in the interoperability ecosystem.

6. Theoretical model

[20] Available at:
https://www3.bcb.gov.br/sgspub/localizarseries/localizarSeries.do?method=prepararTelaLocalizarSeries

6.1. Model setup

In this Subsection, I develop a model for two competing TRs representing the dynamics in the industry. In Figure 1 below, period t = 1 represents the initial moment when the services provided by TRs are homogeneous, and the demand presented by financers is a function of the different prices. In period t = 2, switching costs borne by financers hinder them from moving away to other TR following price changes. Consequently, in t = 3, the period of interest in this study, the demand presented by financers is a function uniquely dependent on the price charged by the TR to which they are currently connected.

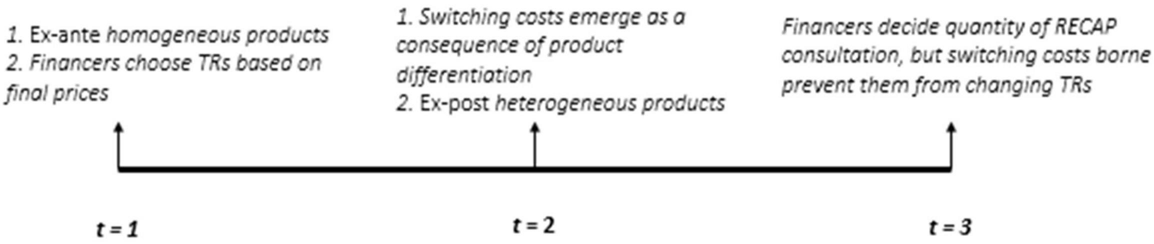

Figure 1 - Timeline

In TR market, the entrant network is a net receiver of access demand, contrary to the assumption in Laffont et. al. (1998), as we assume it to have a higher share of RECAP than of financers. Additionally, financers' demand for intra-network and inter-network access initiated by TRs is assumed to be uniformly distributed over the mass of RECAPs, proportional to the market share of TRs in terms of RECAPs reported by payment processors.[21] Incumbent and new entrant TRs present a clear imbalance. Using a telecommunication example, the incumbent TR resembles a telecom operator that has most clients in the phone-marketing segment, thus originating most of calls. On the other hand, entrant TRs may be compared to the telecom operator that has as customers mostly the households that receive the phone-marketing segment calls, thus profiting from terminating the incumbents' calls. Therefore, we cannot assume a balanced access pattern

[21] Although an inherent economic imbalance before a mature phase.

as employed by Laffont et. al. (1998), which implies balanced inflow and outflows of a network for equal marginal prices, even when market shares are not.

Figure 2 below illustrates a simplified representation of the interoperability between two networks, where the red dotted line represents the inter-network access through TRs.

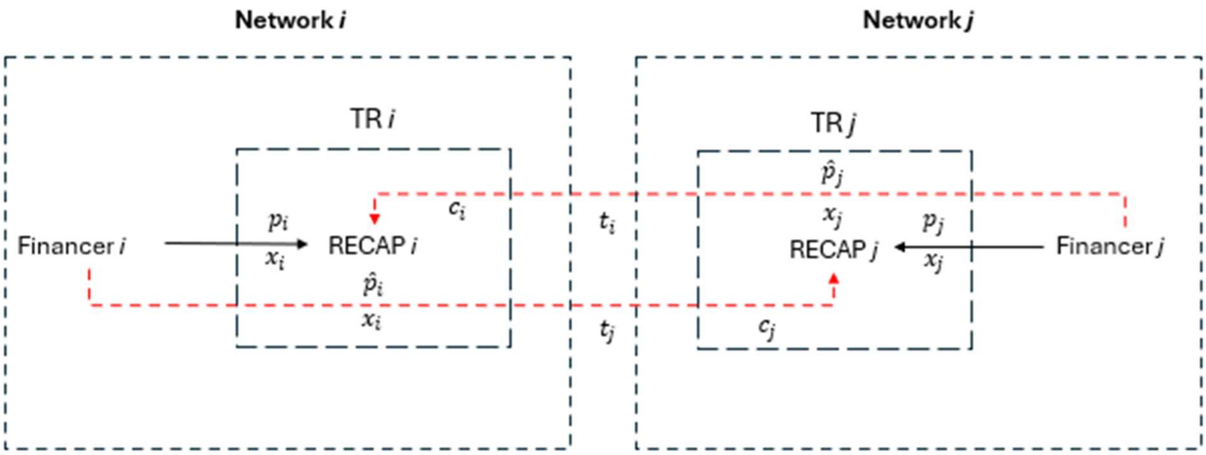

Figure 2 – Benchmark model

In the model, TR i, and respectively TR j, charges p_i to Financer i for intra-network access to RECAP i, $\hat{p}_i$ for inter-network access to RECAP j, and t_i represents the access fee imposed by TR i on TR j, for inter-network access, where $i \neq j$. TR i has cost x_i to provide intra-network to Financer i, and to initiate its inter-network access to RECAP j. Variable c_i represents the cost of TR i to terminate inter-network consultation demands. No fixed costs are considered in the model.

In the initial setup, α_i denotes the market share of financers connected to TR i, in terms of demand for RECAP consultation. Similarly, β_i depicts the market share of payment processors linked to TRs i, in terms of RECAP volume.[22] Market shares (α_i and β_i) are exogenous, thus

[22] Note that it is more relevant to analyze from RECAP mass perspective, given that payment processors have no incentive to move between TRs and that I assume no new payment processors in the market.

independent of price p_i. Moreover $\alpha_i + \alpha_j = 1$, $\beta_i + \beta_j = 1$, $\alpha_i > \alpha_j$, and $\beta_i > \beta_j$, indicating that the incumbent (TR *i*) has a larger market share than the entrant (TR *j*). I also assume $\beta_i,\ \beta_j \neq 0$ and $\beta_i,\ \beta_j \neq 1$.[23] For simplicity, I assume that no new financer or payment processor enters the market.[24]

TR *i*, and respectively TR *j*, receives from Financer *i* a total demand equal to $D_i(p_i, \hat{p}_i)$ for intra and inter-network, for given prices p_i, and $\hat{p}_i$, in that order, where a share of $\beta_i D_i(p_i, \hat{p}_i)$ is terminated intra-network, and $\beta_j D_i(p_i, \hat{p}_i)$ inter-network. Thus:

$$\boldsymbol{\beta_i D_i(p_i, \hat{p}_i) + \beta_j D_i(p_i, \hat{p}_i) = (\beta_i + \beta_j) D_i(p_i, \hat{p}_i)}, \tag{1}$$

where $\boldsymbol{\beta_i + \beta_j = 1}$.

Revenues for TR *i* can be obtained from intra-network consultation:

$$\boldsymbol{\beta_i D_i(p_i, \hat{p}_i)[p_i - x_i]}, \tag{2}$$

inter-network consultation outbound:

$$\boldsymbol{\beta_j D_i(p_i, \hat{p}_i)[\hat{p}_i - x_i - t_j]}, \tag{3}$$

and inter-network consultation incoming:

$$\boldsymbol{\beta_i D_j\left(p_j, \hat{p}_j\right)[t_i - c_i]}. \tag{4}$$

Total profit for TR *i* (respectively for TR *j*) is:

$$\boldsymbol{\pi_i(p_i, \hat{p}_i, t_i, t_j) = D_i(p_i, \hat{p}_i)\left[\beta_i(p_i - x_i) + (1 - \beta_i)\left(\hat{p}_i - x_i - t_j\right)\right] + \beta_i D_j\left(p_j, \hat{p}_j\right)[t_i - c_i]} \tag{5}$$

[23] Subsection 9.2 in Appendix presents additional features for this market.

[24] This assumption is realistic for payment processor and for banks, but not for credit rights investment funds. Nonetheless, banks account for most of credit provided in this market.

6.2. Price analysis under free prices

The objective function, for TR *i*, is given by the profit described in equation (5), respectively for TR *j*. TRs are free to choose their intra/inter-network prices.

Proposition 1. *A decrease in* $\boldsymbol{\beta_i}$ *leads to a decrease in* p_i *and* $\hat{p}_j$*, and to an increase in* $\hat{p}_i$ *and* p_j*, with* $\frac{dp_i}{d\beta_i} > 0$, $\frac{d\hat{p}_i}{d\beta_i} < 0$, $\frac{dp_j}{d\beta_i} < 0$, and $\frac{d\hat{p}_j}{d\beta_i} > 0$.

Proof. See Subsection 9.3 in Appendix.

The equilibrium prices and total differentiation analysis demonstrate a clear relationship between market share changes and pricing behavior in interoperable networks. As TR *i* loses market share of RECAP, it is forced to lower intra-network prices and increase inter-network prices, while TR *j* gains pricing power within its network and reduces inter-network prices.

Equilibrium prices under free prices

For profit function (5) faced by TR *i*, and respectively by TR *j*, the financers present the following linear demands:

$$\boldsymbol{D_i(p_i,\ \hat{p}_i) = a_0 - \frac{a_1 p_i}{2} - \frac{a_1 \hat{p}_i}{2}} \tag{6}$$

$$\boldsymbol{D_j(p_j, \hat{p}_j) = b_0 - \frac{b_1 p_j}{2} - \frac{b_1 \hat{p}_j}{2}} \tag{7}$$

Which yield the following equilibrium prices:

$$\boldsymbol{p_i^* = \frac{2a_0\beta_i - 2a_0 - a_1\beta_i t_j + a_1 t_j + a_1 x_i}{a_1(2\beta_i - 1)}} \tag{8}$$

$$\boldsymbol{\hat{p}_i^* = \frac{2a_0\beta_i + a_1\beta_i t_j - a_1 t_j - a_1 x_i}{a_1(2\beta_i - 1)}} \tag{9}$$

$$\boldsymbol{p_j^* = \frac{2b_0\beta_i - b_1\beta_i t_i - b_1 x_j}{b_1(2\beta_i - 1)}} \tag{10}$$

$$\widehat{p}_j^* = \frac{2b_0\beta_i - 2b_0 + b_1\beta_i t_i + b_1 x_j}{b_1(2\beta_i - 1)} \tag{11}$$

Proof. See Subsection 9.4 and Figure 7 in Appendix 9.13.

The price behavior of TRs demonstrates that, starting from a symmetric equilibrium, a slight change in market share of RECAP leads to significant and opposite reaction from the firms.

6.3. Price responses to regulatory constraints and market agreements

Departing from the conditions described in previous Subsection, we explore the consequences of market agreements over prices charged by TRs, as well as price regulation by the competition authority. In the next subsections, we consider strict price equality for intra and inter-network access, i.e., $\boldsymbol{p_i} = \widehat{\boldsymbol{p}}_i$ and $\boldsymbol{p_j} = \widehat{\boldsymbol{p}}_j$,[25] where the new objective function for TR *i*, and respectively for TR *j*, becomes:

$$\pi_i(p_i, t_i, t_j) = D_i(p_i)[(p_i - x_i - (1 - \beta_i)t_j)] + \beta_i D_j(p_j)[t_i - c_i] \tag{12}$$

For profit function (12) faced by TR *i*, and respectively by TR *j*, the financers present the following linear demands for $\boldsymbol{D_i(p_i)}$, and $\boldsymbol{D_j(p_j)}$:

$$D_i(p_i) = a_0 - a_1 p_i \tag{13}$$

$$D_j(p_j) = b_0 - b_1 p_j \tag{14}$$

6.3.1. Price equality for intra-network and access fee

We suppose mandatory price equality for intra-network and access fee imposed by TRs to each other, i.e., $\boldsymbol{p_i} = \boldsymbol{t_i}$, and $\boldsymbol{p_j} = \boldsymbol{t_j}$.[26]

[25] This is assumed for simplicity and because it was observed for most TRs, which charge the same price for a specific service provided either intra-network or inter-network.
[26] TRs are allowed to set in place market agreements regarding standards and prices.

Proposition 2. *An increase in* $\boldsymbol{\beta_i}$ *implies lower prices of TR i, but the behavior of rival TR j is different depending on the magnitude of market share loss or gain, as* $\frac{d\boldsymbol{p_i}}{d\boldsymbol{\beta_i}} < \mathbf{0}$, *and* $\frac{d\boldsymbol{p_j}}{d\boldsymbol{\beta_i}} \gtrless \mathbf{0}$.

Proof. See Subsection 9.5 in Appendix.

A higher $\boldsymbol{\beta_i}$ implies higher revenues from intra-network and inter-network incoming access for TR *i*, allowing for price decreases due to increased demand. However, the impact on TR *j* is more complex. For moderate $\boldsymbol{\beta_i}$ levels (i.e., $\boldsymbol{\beta_i} = \mathbf{0.51}$ and $\boldsymbol{\beta_i} = \mathbf{0.75}$), TR *j* responds with higher prices, possibly as a measure to maintain profitability despite reduced market share. For high $\boldsymbol{\beta_i}$ levels (i.e., $\boldsymbol{\beta_i} = \mathbf{0.99}$), TR *j* lowers prices, which may indicate aggressive competitive behavior to retain any remaining market share or to counteract the dominant position of TR *i*.

Equilibrium prices under price equality for intra-network and access fee

Assuming $\boldsymbol{p_i} = \boldsymbol{t_i}$ and $\boldsymbol{p_j} = \boldsymbol{t_j}$, equilibrium prices are:

$$\boldsymbol{p_i^1} = \frac{2a_0b_1+(1-\beta_i)^2a_0a_1-\beta_i(1-\beta_i)a_0b_1+\beta_ib_0b_1+(1-\beta_i)a_1b_0+2a_1b_1x_i+(1-\beta_i)a_1b_1x_j-\beta_ib_1^2x_j}{4a_1b_1-2\beta_i(1-\beta_i)a_1b_1+(1-\beta_i)^2a_1^2+\beta_i^2b_1^2} \quad (15)$$

$$\boldsymbol{p_j^1} = \frac{2a_1b_0+2a_1b_1x_j+(1-\beta_i)a_0a_1+\beta_ia_0b_1+\beta_ia_1b_1x_i-(1-\beta_i)a_1^2x_i+\beta_i^2b_0b_1-(1-\beta_i)\beta_ia_1b_0}{4a_1b_1-2\beta_i(1-\beta_i)a_1b_1+(1-\beta_i)^2a_1^2+\beta_i^2b_1^2} \quad (16)$$

Proof. See Subsection 9.6 and Figure 8 in Appendix 9.13.

The results observed in the price equilibrium analysis demonstrate that the reaction of TR *j* for higher $\boldsymbol{\beta_i}$ levels is not definite, with a slight increase in price. This is not confirmed in the analyses with the total differentiation of $\boldsymbol{p_j}$ with respect to $\boldsymbol{\beta_i}$, which remains inconclusive.

6.3.2. Reciprocal access fee (*t*)

A common access fee for consultation is determined by regulator $\boldsymbol{t} = \boldsymbol{t_i} = \boldsymbol{t_j}$, where $\boldsymbol{t} \geq \boldsymbol{c_i}$ and $\boldsymbol{t} \geq \boldsymbol{c_j}$ are necessary conditions to guarantee that TRs do not incur in losses providing inter-network incoming access. TRs can still choose their intra-network prices $\boldsymbol{p_i}$ and $\boldsymbol{p_j}$, which can be equal or not to the access fee defined by the regulator.

Proposition 3. *Increases of reciprocal access fee* $\boldsymbol{t}$ *implies that both TRs would increase intra-network price charged, with* $\frac{d p_i}{dt} > \mathbf{0}$, *and* $\frac{d p_j}{dt} > \mathbf{0}$.[27]

As $\boldsymbol{t}$ decreases, the burden borne by each TR in the outbound inter-network access diminishes, allowing for price reductions. However, it is challenging to the regulator to ensure that the conditions $\boldsymbol{t} \geq \boldsymbol{c_i}$ and $\boldsymbol{t} \geq \boldsymbol{c_j}$ are met.

Proof. See Subsection 9.7 in Appendix.

Equilibrium prices under reciprocal access fee (t)

Assuming $\boldsymbol{t} = \boldsymbol{t_i} = \boldsymbol{t_j}$, equilibrium condition results in:

$$\boldsymbol{p_i^2} = \frac{a_0 + a_1 x_i + a_1 t(1-\beta_i)}{2a_1} \tag{17}$$

$$\boldsymbol{p_j^2} = \frac{b_0 + b_1 x_j + b_1 t \beta_i}{2b_1} \tag{18}$$

Proof. See Subsection 9.8 and Figure 9, Figure 10, and Figure 11in Appendix 9.13.

Both TRs are sensitive to changes in reciprocal access fee, although this sensitivity decreases as $\boldsymbol{\beta_i}$ increases. Specifically, TR *i* becomes non-sensitive to changes in $\boldsymbol{t}$ at higher levels of market share for RECAP.

6.3.3. Asymmetrically regulated access fees $\boldsymbol{t_i^r}$ and $\boldsymbol{t_j}$

Alternatively, to the imposed reciprocity in interconnection price suggested in previous subsection, the competition authority can regulate access prices asymmetrically. This could be

[27] Assuming $\boldsymbol{t}$ = [0.40, 0.45, 0.50].

used to impose a decrease in access fee charged by the incumbent TR i, $\boldsymbol{t_i^r}$, as the entrant TR j can choose its prices $\boldsymbol{p_j}$ and $\boldsymbol{t_j}$ freely. In the limit, competition authority can impose cost-based interconnection price to TR i, i.e., $\boldsymbol{t_i^r} = \boldsymbol{c_i}$.

Proposition 4. *Decrease in access fee $\boldsymbol{t_i^r}$ has no impact in intra-network price charged by TR i but implies a decrease in the intra-network price charged by the competitor, with* $\boldsymbol{\frac{dp_i}{dt_i^r} = 0}$, *and* $\boldsymbol{\frac{dp_j}{dt_i^r} > 0}$.

Proof. See Subsection 9.9 in Appendix.

As t_i^r decreases, TR j faces a lower burden for outbound inter-network access, allowing it to decrease p_j. No impact in p_i is observed for any level of $\boldsymbol{\beta_i}$, as TR i's inter-network connection revenues remain unaffected.

Equilibrium prices under asymmetrically regulated access fees $\boldsymbol{t_i^r}$ and $\boldsymbol{t_j}$

Considering regulated $\boldsymbol{t_i^r}$ and freely chosen $\boldsymbol{t_j}$, equilibrium prices are:

$$\boldsymbol{p_i^3} = \frac{\boldsymbol{a_0 - a_1\beta_i t_j + a_1 t_j + a_1 x_i}}{\boldsymbol{2a_1}} \tag{19}$$

$$\boldsymbol{p_j^3} = \frac{\boldsymbol{b_0 + b_1\beta_i t_i^r + b_1 x_i}}{\boldsymbol{2b_1}} \tag{20}$$

Proof. See Subsection 9.10 and Figure 12, Figure 13, and Figure 14 in Appendix 9.13.

As expected, with lower incumbency of TR i, TR j is more sensitive to reductions in regulated $\boldsymbol{t_i^r}$. For moderate $\boldsymbol{\beta_i}$, p_j is lower than p_i, however, under higher values for $\boldsymbol{\beta_i}$, TR j charges higher prices to compensate the burden of inter-network outbound access.

6.3.4. Minimum access fee as reciprocal charge

The TRs choose their prices subject to $\boldsymbol{p_i = t_i}$ and $\boldsymbol{p_j = t_j}$. The regulator imposes the reciprocal access fee ($\boldsymbol{t^m}$) as the lower between $\boldsymbol{t_i}$ and $\boldsymbol{t_j}$, according to:

i) $\boldsymbol{t^m = t_j}$ if $\boldsymbol{t_j < t_i}$, implying $\boldsymbol{t^m = p_j}$;

ii) $\boldsymbol{t^m = t_i}$ if $\boldsymbol{t_i < t_j}$, implying $\boldsymbol{t^m = p_i}$.

Proposition 5. *If TR j sets a lower access fee than TR i, as condition (i) above describes, an increase in $\boldsymbol{\beta_i}$ implies a decrease in $\mathbf{p_i}$ and an increase in $\mathbf{p_j}$, with $\boldsymbol{\frac{dp_i}{d\beta_i} < 0}$, and $\boldsymbol{\frac{dp_j}{d\beta_i} > 0}$.*

Proof. See Subsection 9.11 in Appendix.

As $\boldsymbol{\beta_i}$ increases, the burden for TR *j* in the inter-network also increases, which implies higher prices charged. For moderate $\boldsymbol{\beta_i}$, $\boldsymbol{p_i}$ and $\boldsymbol{p_j}$ are close in absolute value, with p_j facing small increments, but it soars for higher $\boldsymbol{\beta_i}$. The increment in market share for TR *i*, allows it to keep low values for $\boldsymbol{p_i}$.

Equilibrium prices under minimum access fee as reciprocal

Extending from Subsection 6.3.4, we consider that condition (i) prevails, which yields the equilibrium prices:

$$\boldsymbol{p_i^4 = \frac{a_0a_1\beta_i - a_0a_1 - 2a_0b_1 + a_1b_0\beta_i - a_1b_0 - 2a_1b_1x_i - a_1b_1x_j}{a_1^2\beta_i - a_1^2 - 4a_1b_1}} \tag{21}$$

$$\boldsymbol{p_j^4 = \frac{-a_0\beta_i + a_0 + a_1\beta_ix_i - a_1x_i - 2b_0\beta_i + 2b_0 + 2b_1x_j}{a_1\beta_i^2 - 2a_1\beta_i + a_1 - 4b_1\beta_i + 4b_1}} \tag{22}$$

Proof. See Subsection 9.12 and Figure 15 in Appendix 9.13.

As $\boldsymbol{\beta_i}$ increases to the point TR *j*'s permanence in the market is threatened, TR *j*'s pricing behavior indicates an intention to profit from its remaining market share of RECAP, leading to a significant increase in its price.

7. Conclusions

The regulatory changes implemented by the Central Bank of Brazil (BCB) in the Trade Repository (TR) market have had a significant impact on market dynamics, pricing, and competition. This study applied a modified version of the Laffont et al. (1998) model to analyze these effects, focusing on the implications of enforced interoperability and transparency.

The analysis demonstrates that regulatory constraints on prices effectively lower both intra-network and inter-network prices for incumbents and entrants. When prices are freely and individually set, market distortions can occur, leading to inefficiencies. In that scenario, both firms exert market power over their respective local monopolies either by subsidizing access or by charging significantly higher prices. Regulatory interventions help mitigate these distortions by enforcing more competitive pricing structures. The adoption of a reciprocal access fee demonstrates to be especially effective for lower levels of incumbency advantage, where firms' price sensitivity to reductions in the access fee is more pronounced. This suggests that a competitive market, with more than two rivals, combined with access fee regulation would be beneficial for decreasing equilibrium prices. The price equilibria found for each regulatory constraint against free prices demonstrated a relevant relationship between incumbency and changes in pricing behavior in interoperability networks.

The results align with Laffont et al.'s (1998) conclusion that freely negotiated access charges may hinder competition, questioning if competition alone can discipline access and retail prices. Similarly, it corroborates Armstrong's (1998) suggestion that in a market with a two-way interconnection, the role of regulator as arbitrator is not enough. Moreover, our observations corroborates Carter and Wright's (1999) conclusion that, when acting independently, competitors would be inclined to set higher prices, exploiting their monopoly power.

The findings underscore the importance of regulatory policies in fostering competition and reducing costs in the TR market. By enforcing interoperability and transparent pricing, the BCB might set regulatory constraints to lower prices and enhance competition. Currently, TRs operating in the RECAP market adhere to a market agreement outlined in subsection 6.3.1, which mandates that the access fee charged to other TRs must be equal to its own intra-network price. Our proposal for Brazilian competition policy in the TR market suggests that the reciprocal access fee regulation, developed in subsection 6.3.2, would be more effective in promoting a balance between competition and allocative efficiency.

These strategies can serve as a model for other sectors where network effects and market power are significant. The insights gained from this study are not only relevant to the TR market but also applicable to other financial market infrastructures and industries with similar characteristics. The regulatory approach can be extended to the broader asset market, such as the virtual duplicata or "entry-book trade bills," to achieve similar competitive outcomes.

While this study provides valuable insights, it is not without limitations. The relative novelty of the TR market in Brazil and the limited availability of firm-level data constrain the depth of the analysis. Future research should focus on collecting more comprehensive data and exploring the long-term effects of regulatory changes on market dynamics.

In conclusion, this research highlights the critical role of regulation in ensuring competitive and efficient market outcomes. By examining the specific case of the Brazilian TR market, it provides a blueprint for regulatory bodies worldwide to foster competition, reduce costs, and enhance market transparency.

8. References

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9. Appendix

9.1. Tables, figures, and graphs

Table 1 – Trade repositories in Brazil

TR name	Legal entity name
Balcão B3	B3 S.A. - Brasil, Bolsa, Balcão (B3)
Registradora BM&F BOVESPA	B3 S.A. - Brasil, Bolsa, Balcão (B3)
Câmara B3	B3 S.A. - Brasil, Bolsa, Balcão (B3)
Sistema UFIN	B3 S.A. - Brasil, Bolsa, Balcão (B3)
C3	CIP S.A. (CIP)
Sistema CERC	CERC S.A. (CERC)
Sistema CRDC	Central de Registro de Direitos Creditórios S.A.
Sistema CSD BR	CSD Central de Serviços de Registro e Depósito aos Mercados Financeiros e de Capitais S.A.
Sistema CRT4	CRT4 Central de Registro de Títulos e Ativos S.A. (acquired by CIP in early 2023)
Sistema de Registro da TAG IMF	TAG Tecnologia para o Sistema Financeiro S.A. (TAG)
Sistema de Registro SPC Grafeno	SPC Grafeno Infraestrutura e Tecnologia para o Sistema Financeiro S.A.

Table 2 - Financial assets recorded by authorized trade repositories

Financial Asset Description	Code	Legal entity (TR operator)						
		CIP	**B3**	**CERC**	**CRDC**	**CRT4**	**CSDBR**	**TAG**
Adiantamento sobre cambiais entregues	ACE							
Adiantamento sobre Contrato de Câmbio	ACC		X					
Cartão Consignado	CCONS			X				
Cédula de Crédito à Exportação	CCE		X	X				
Cédula de Crédito Bancário	CCB		X	X				
Cédula de Crédito Comercial	CCC		X	X				
Cédula de Crédito Imobiliário	CCI		X	X				
Cédula de Crédito Industrial	CCIN		X	X				
Cédula de Crédito Rural	CCR		X	X				
Cédula de Produto Rural	CPR		X	X	X			
Cédula Imobiliária Rural	CIR			X				
Certificado de Cédula de Crédito Bancário	CCCB		X					
Certificado de Depósito Agropecuário	CDA		X					
Certificado de Depósito Bancário	CDB		X	X		X	X	
Certificado de Depósito Bancário Vinculado	CDBV		X	X			X	
Certificado de Direitos Creditórios do Agronegócio	CDCA		X					
Certificado de Operações Estruturadas	COE		X					
Cheque Pós-Datado	CH			X				
Contrato de Crédito Contra Terceiros	CCT	X	X	X				
Contrato de Financiamento	CFIN	X	X	X				
Contrato de Mútuo	CMUT	X	X	X				
Contratos de CDC	CDC	X		X				
Contratos de Crédito Pessoal	CP	X		X				
Crédito Securitizado	CSEC		X					
Depósito a Prazo com Garantia Especial	DPGE		X					
Depósito Interfinanceiro	DI		X					
Duplicata	DUPLICATA	X	X	X	X			
Duplicata Rural	DR		X	X				
Empréstimo Consignado	EC	X		X				
Fundo de Desenvolvimento Social	FDS		X					
Instrumento de Confissão de Dívida	CDIV		X	X				
Letra de Arrendamento Mercantil	LAM		X				X	
Letra de Câmbio	LC		X			X		
Letra de Crédito do Agronegócio	LCA		X					
Letra de Crédito Imobiliário	LCI		X			X	X	
Letra Hipotecária	LH		X				X	
Nota de Crédito à Exportação	NCE		X	X				
Nota de Crédito Comercial	NCC		X	X				
Nota de Crédito Industrial	NCI		X	X				
Nota Promissória	NP	X	X	X				
Nota Promissória Rural	NPR		X	X				
Operações de Arrendamento Mercantil	AM			X				
Operações de crédito com garantias constituídas sobre imóveis	OCGI		X	X				
Operações de crédito com garantias constituídas sobre veículos automotores	OCGV		X	X				
Ouro	DIS OZ3		X					
Recebíveis Subcredenciadora	RECSUBC							X
Recebível de Arranjo de Pagamento	RECAP	X	X	X				X
Recebível Imobiliário	RECIM							
Recibo de Depósito Bancário	RDB		X	X		X	X	
Warrant Agropecuário	WA		X					

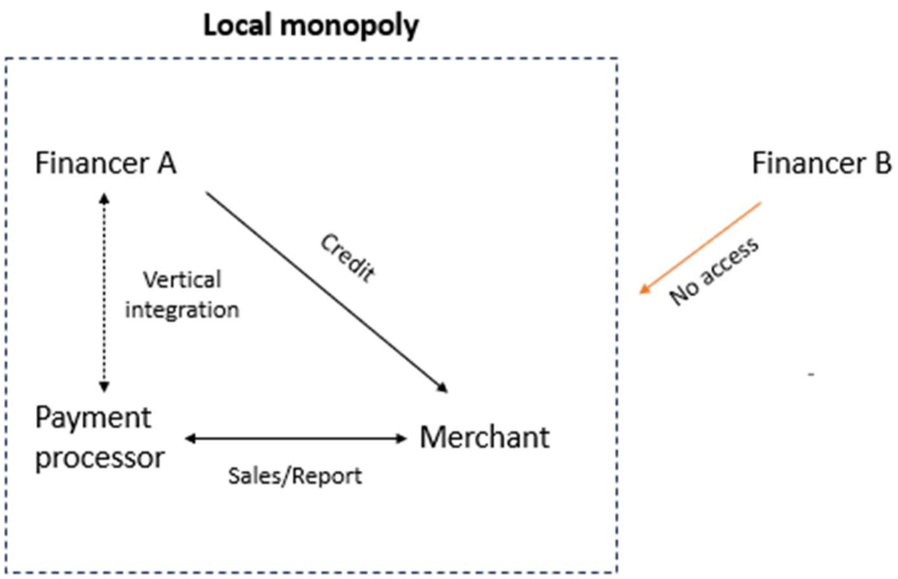

Figure 3 – Monopolistic dynamics before the reform

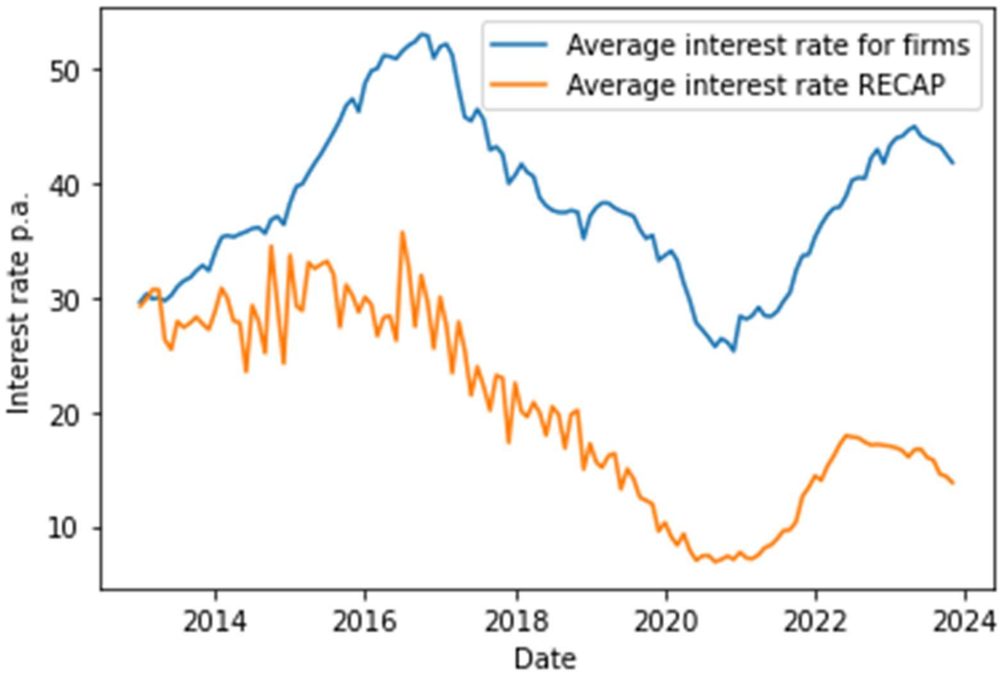

Figure 4 – Average interest rate

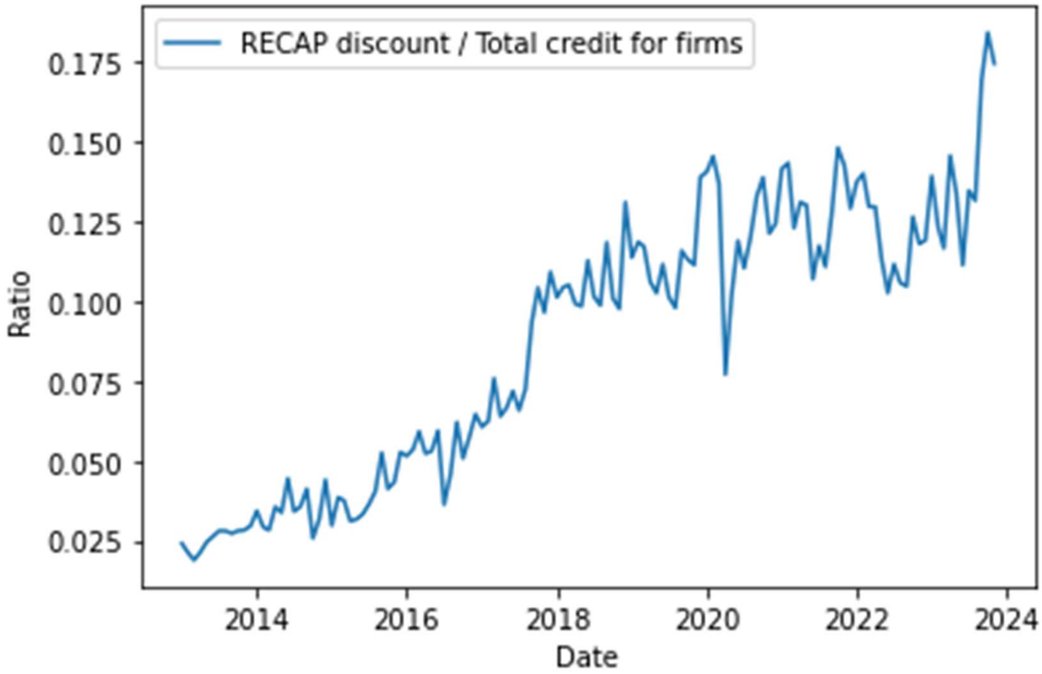

Figure 5 – Ratio of RECAP discount over total credit for firms

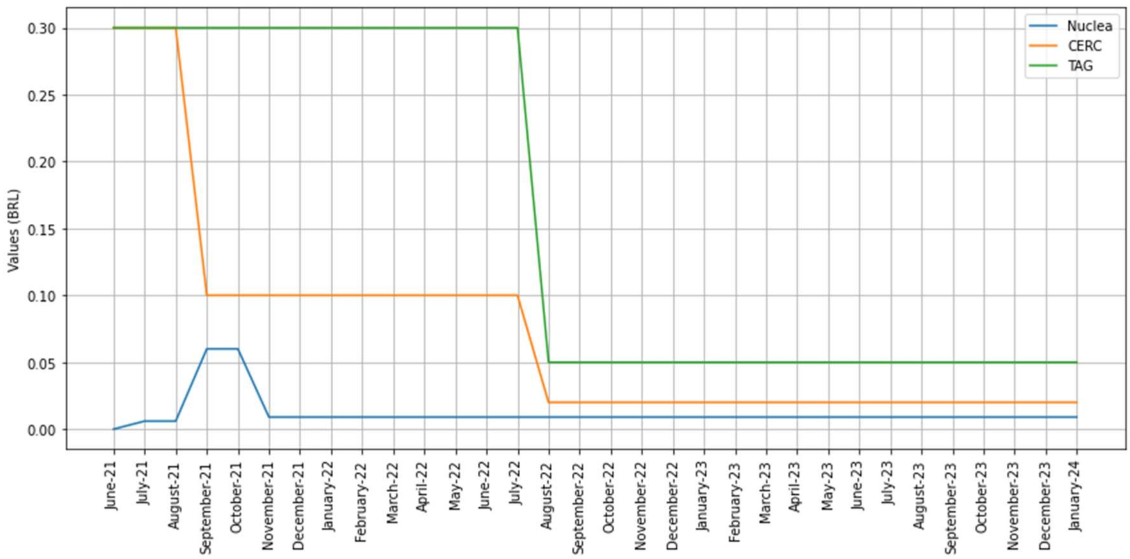

Figure 6 – Interconnection prices

9.2. Additional features of the model setup

Financers derive utility $u(D_i)$ from accessing D_i RECAPs. The individual demand of a financer to a TR i is obtained by:

$$\boldsymbol{D_i(p_i) = \max_{D_i}\{u(D_i) - D_i p_i\}}, \tag{23}$$

derived by first order condition for utility maximization $u'(D_i) = p_i$. Financers derive an additional fixed utility level $u_i^0 \geq 0$ from participating in TR i.[28] However, payment processors present no additional utility, i.e., $u_i^0 = 0$, since they are indifferent to being connected to one TR or another, neither paying charges or earing income in the arrangement. Indirect net utility of a customer of TR i is given by:

$$\boldsymbol{v_i(p_i) = u_i^0 + u[D_i(p_i)] - p_i D_i(p_i)}, \tag{24}$$

where u_i^0 is not access-related but $u[D_i(p_i)] - p_i D_i(p_i)$ is. Consumer's net utility is decreasing in prices, as expected:

$$\boldsymbol{\frac{\partial v_i(p)}{\partial p} = - D_i(p_i)}. \tag{25}$$

Financers choose a TR based on the highest utility level among $v_i(p_i)$ and $v_j(p_j)$.

I assume that financers linked to TR i are uniformly distributed on the interval $[0, Ms_i^F]$, where $M > 0$ is a given constant, which measures competitiveness.[29] *Ex ante*, we may consider $M \rightarrow 0$, i.e., TRs provide complete substitutable goods. *Ex post* $M \rightarrow \infty$, where switching costs are high enough to prevent changes from one TR to another, creating monopoly market power.

[28] From quality of service, brand loyalty, complementary services.
[29] Also related to switching costs, extent of product differentiation and lock-in effect.

Given the switching costs faced by financers, changing from incumbent (TR i) to entrant (TR j) will take place if and only if:

$$\boldsymbol{v_j(p_j) - m > v_i(p_i)}, \tag{26}$$

where m represents the switching cost and is extracted from the interval [0, $M\alpha$].[30]

For a total mass of n financers,[31] the following is obtained:

$$\int_0^{M\alpha_i} \frac{n}{M}\, dm + \int_0^{M\alpha_j} \frac{n}{M}\, dm = \frac{n}{M}\,(M\alpha_i + M\alpha_j) = \mathbf{n}. \tag{27}$$

One possible interpretation is that financers with m close to zero are inclined to switch TRs, but those with m close to $M\alpha$ would only be willing to change if the challenger considerably decreases its price. To observe the effect of switching cost and competition depicted by M, for given prices, suppose $v_j(p_j) > v_i(p_i)$. A financer will be indifferent between changing or not from TR i to TR j when:

$$\boldsymbol{v_j(p_j) - m_0 = v_i(p_i)}, \tag{28}$$

where m_0 represents the switching cost. As $m_0 = v_j(p_j) - v_i(p_i)$, a ratio of:

$$\frac{\boldsymbol{v_j(p_j) - v_i(p_i)}}{\boldsymbol{M\alpha_i}} \tag{29}$$

changes from TR i to TR j. Complementarily a share of:

$$\frac{\boldsymbol{M\alpha_i - (v_j(p_j) - v_i(p_i))}}{\boldsymbol{M\alpha_i}} \tag{30}$$

is maintained by TR i. Thus, market share of financers for TR i is given by:

[30] Consider switching costs uniformly distributed [0, M].
[31] Total number of financers is also denoted as n.

$$\alpha_{i,t+1}(p_i, p_j) = \alpha_{i,t} + \frac{v_i(p_i) - v_j(p_j)}{M\alpha_i}, \tag{31}$$

where $\alpha_{i,t+1}$ represents the market share of financers of TR *i* in period t + 1.

Initial market share increases with TR offering larger net utility, but changing between TRs diminishes in less competitive markets, forcing entrants to set lower prices to increase market share.

Producer's surplus (PS) is given by the sum of TR's profits, PS = $\sum_{i=1,2} \pi_i$. Consumer surplus (CS), can be obtained from the indirect utilities as:

$$\boldsymbol{CS = s_1 v_1[p_1] + s_2 v_2[p_2] - \frac{(s_1 v_1[p_1] - s_2 v_2[p_2])^2}{2M}}. \tag{32}$$

The term $\frac{(s_1 v_1[p_1] - s_2 v_2[p_2])^2}{2M}$ represents the average switching cost for financers to change between TRs. Finally, total welfare (W) is given by W = PS + CS.

9.3. Proof of Proposition 1

From profit equation (5) for TR *i*, respectively for TR *j*, we obtain FOCs with respect to $\boldsymbol{p_i}$, $\boldsymbol{\hat{p}_i}$, $\boldsymbol{p_j}$, and $\boldsymbol{\hat{p}_j}$:

$$\boldsymbol{V = \partial\pi_i/\partial p_i = 0} \tag{33}$$

$$\boldsymbol{V = D_i'(p_i, \hat{p}_i)\left[(p_i - x_i)\beta_i + (\hat{p}_i - x_i - t_j)\,(1 - \beta_i)\right] + D_i(p_i, \hat{p}_i)\beta_i;} \tag{34}$$

$$\boldsymbol{W = \partial\pi_i/\partial\hat{p}_i = 0} \tag{35}$$

$$\boldsymbol{W = D_i'(p_i, \hat{p}_i)[(p_i - x_i)\beta_i + (\hat{p}_i - x_i - t_j)(1 - \beta_i)] + D_i(p_i, \hat{p}_i)(1 - \beta_i);} \tag{36}$$

$$\boldsymbol{Y = \partial\pi_j/\partial p_j = 0} \tag{37}$$

$$\boldsymbol{Y = D_j'(p_j, \hat{p}_j)[(p_j - x_j)(1 - \beta_i) + (\hat{p}_j - x_j - t_i)\beta_i] + D_j(p_j, \hat{p}_j)(1 - \beta_i);} \tag{38}$$

$$\boldsymbol{Z = \partial\pi_j/\partial\hat{p}_j = 0} \tag{39}$$

$$\boldsymbol{Z = D_j'(p_j,\hat{p}_j)\big[(p_j - x_j)(1-\beta_i) + (\hat{p}_j - x_j - t_i)\beta_i\big] + D_j(p_j,\hat{p}_j)\beta_i.} \tag{40}$$

For the comparative statistics, I apply the chain rule, for V and W:

$$\frac{\partial V}{\partial p_i}\cdot\frac{dp_i}{d\beta_i} + \frac{\partial V}{\partial \hat{p}_i}\cdot\frac{d\hat{p}_i}{d\beta_i} + \frac{\partial V}{\partial \beta_i} = 0,$$

$$\frac{\partial W}{\partial p_i}\cdot\frac{dp_i}{d\beta_i} + \frac{\partial W}{\partial \hat{p}_i}\cdot\frac{d\hat{p}_i}{d\beta_i} + \frac{\partial W}{\partial \beta_i} = 0.$$

And for Y and Z:

$$\frac{\partial Y}{\partial p_j}\cdot\frac{dp_j}{d\beta_i} + \frac{\partial Y}{\partial \hat{p}_j}\cdot\frac{d\hat{p}_j}{d\beta_i} + \frac{\partial Y}{\partial \beta_i} = 0,$$

$$\frac{\partial Z}{\partial p_j}\cdot\frac{dp_j}{d\beta_i} + \frac{\partial Z}{\partial \hat{p}_j}\cdot\frac{d\hat{p}_j}{d\beta_i} + \frac{\partial Z}{\partial \beta_i} = 0,$$

and the Cramer's rule to compute the derivatives for $\boldsymbol{p_i}$, $\boldsymbol{p_j}$, $\boldsymbol{\hat{p}_i}$, and $\boldsymbol{\hat{p}_j}$ with respect to $\boldsymbol{\beta_i}$, the exogenous parameter:

$\frac{dp_i}{d\beta_i} = \frac{-\frac{\partial V}{\partial \beta_i}\cdot\frac{\partial W}{\partial \hat{p}_i} + \frac{\partial V}{\partial \hat{p}_i}\cdot\frac{\partial W}{\partial \beta_i}}{\frac{\partial V}{\partial p_i}\cdot\frac{\partial W}{\partial \hat{p}_i} - \frac{\partial V}{\partial \hat{p}_i}\cdot\frac{\partial W}{\partial p_i}} > 0$, $\frac{d\hat{p}_i}{d\beta_i} = \frac{-\frac{\partial V}{\partial p_i}\cdot\frac{\partial W}{\partial \beta_i} + \frac{\partial V}{\partial \beta_i}\cdot\frac{\partial W}{\partial p_i}}{\frac{\partial V}{\partial p_i}\cdot\frac{\partial W}{\partial \hat{p}_i} - \frac{\partial V}{\partial \hat{p}_i}\cdot\frac{\partial W}{\partial p_i}} < 0$, where $\frac{\partial V}{\partial \beta_i} > 0$, $\frac{\partial W}{\partial \hat{p}_i} < 0$, $\frac{\partial V}{\partial \hat{p}_i} < 0$, $\frac{\partial W}{\partial \beta_i} > 0$, $\frac{\partial V}{\partial p_i} < 0$, and $\frac{\partial W}{\partial p_i} < 0$.

Also,

$\frac{dp_j}{d\beta_i} = \frac{-\frac{\partial Y}{\partial \beta_i}\cdot\frac{\partial Z}{\partial \hat{p}_j} + \frac{\partial Y}{\partial \hat{p}_j}\cdot\frac{\partial Z}{\partial \beta_i}}{\frac{\partial Y}{\partial p_j}\cdot\frac{\partial Z}{\partial \hat{p}_j} - \frac{\partial Y}{\partial \hat{p}_j}\cdot\frac{\partial Z}{\partial p_j}} < 0$ and $\frac{d\hat{p}_j}{d\beta_i} = \frac{-\frac{\partial Y}{\partial p_j}\cdot\frac{\partial Z}{\partial \beta_i} + \frac{\partial Y}{\partial \beta_i}\cdot\frac{\partial Z}{\partial p_j}}{\frac{\partial Y}{\partial p_j}\cdot\frac{\partial Z}{\partial \hat{p}_j} - \frac{\partial Y}{\partial \hat{p}_j}\cdot\frac{\partial Z}{\partial p_j}} > 0$, where $\frac{\partial Z}{\partial \beta_i} > 0$, $\frac{\partial Y}{\partial \hat{p}_j} < 0$, $\frac{\partial Z}{\partial \hat{p}_j} < 0$, $\frac{\partial Y}{\partial \beta_i} > 0$, $\frac{\partial Z}{\partial p_j} < 0$, and $\frac{\partial Y}{\partial p_j} < 0$.

9.4. Computation of equilibrium under free prices

The new profit functions become:

$$\boldsymbol{\pi_i(p_i, \widehat{p}_i, t_i, t_j) = (a_0 - \frac{a_1 p_i}{2} - \frac{a_1 \widehat{p}_i}{2})[\beta_i(p_i - x_i) + (1-\beta_i)(\widehat{p}_i - x_i - t_j)] + \beta_i(b_0 - \frac{b_1 p_j}{2} - \frac{b_1 \widehat{p}_j}{2})[t_i - c_i]} \quad (41)$$

$$\boldsymbol{\pi_j(p_j, \widehat{p}_j, t_j, t_i) = (b_0 - \frac{b_1 p_j}{2} - \frac{b_1 \widehat{p}_j}{2})[(1-\beta_i)(p_j - x_j) + (1-\beta_i)(\widehat{p}_j - x_j - t_i)] + (1-\beta_i)(a_0 - \frac{a_1 p_i}{2} - \frac{a_1 \widehat{p}_i}{2})[t_j - c_j]} \quad (42)$$

FOCs for the new profit functions, with respect to $\boldsymbol{p_i, \widehat{p}_i, p_j}$, and $\boldsymbol{\widehat{p}_j}$ result in:

$$\boldsymbol{V = \partial\pi_i/\partial p_i = -\frac{a_1}{2}[\beta_i(p_i - x_i) + (1-\beta_i)(\widehat{p}_i - x_i - t_j)] + \beta_i\left(a_0 - \frac{a_1 p_i}{2} - \frac{a_1 \widehat{p}_i}{2}\right) = 0} \quad (43)$$

$$\boldsymbol{W = \partial\pi_i/\partial \widehat{p}_i = -\frac{a_1}{2}[\beta_i(p_i - x_i) + (1-\beta_i)(\widehat{p}_i - x_i - t_j)] + (1-\beta_i)\left(a_0 - \frac{a_1 p_i}{2} - \frac{a_1 \widehat{p}_i}{2}\right) = 0} \quad (44)$$

$$\boldsymbol{Y = \partial\pi_j/\partial p_j = -\frac{b_1}{2}[(1-\beta_i)(p_j - x_j) + (1-\beta_i)(\widehat{p}_j - x_j - t_i)] + (1-\beta_i)\left(b_0 - \frac{b_1 p_j}{2} - \frac{b_1 \widehat{p}_j}{2}\right) = 0} \quad (45)$$

$$\boldsymbol{Z = \partial\pi_j/\partial \widehat{p}_j = -\frac{b_1}{2}\left[(1-\beta_i)\left(p_j - x_j\right) + (1-\beta_i)\left(\widehat{p}_j - x_j - t_i\right)\right] + (1-\beta_i)\left(b_0 - \frac{b_1 p_j}{2} - \frac{b_1 \widehat{p}_j}{2}\right) = 0} \quad (46)$$

For the comparative statistics, the Cramer's rule is applied to compute the equilibrium prices in Subsection 6.2.1.

9.5. Proof of Proposition 2

For $\boldsymbol{t_i = p_i}$ and $\boldsymbol{t_j = p_j}$, profit equation for TR *i* (5) (respectively for TR *j*) can be rewritten as:

$$\boldsymbol{\pi_i(p_i, p_j) = D_i(p_i)[p_i - x_i - (1-\beta_i)p_j] + \beta_i D_j(p_j)[p_i - c_i]} \quad (47)$$

$$\boldsymbol{\pi_j(p_i, p_j) = D_j(p_j)[p_j - x_j - \beta_i p_i] + (1-\beta_i)D_i(p_i)[p_j - c_j].} \quad (48)$$

A system of First Order Conditions (FOC)s) is solved as:

$$\boldsymbol{V = \partial\pi_i/\partial p_i = D_i'(p_i)[p_i - x_i - (1-\beta_i)p_j] + D_i(p_i) + \beta_i D_j(p_j) = 0} \quad (49)$$

$$\boldsymbol{W = \partial\pi_j/\partial p_j = D_j'(p_j)[p_j - x_j - \beta_i p_i] + D_j(p_j) + (1-\beta_i)D_i(p_i) = 0.} \quad (50)$$

For the comparative statistics, I apply the chain rule,

$$\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{p_i}} \cdot \frac{\boldsymbol{dp_i}}{\boldsymbol{d\beta_i}} + \frac{\partial \boldsymbol{V}}{\partial \boldsymbol{p_j}} \cdot \frac{\boldsymbol{dp_j}}{\boldsymbol{d\beta_i}} + \frac{\partial \boldsymbol{V}}{\partial \boldsymbol{\beta_i}} = \mathbf{0}$$

$$\frac{\partial \boldsymbol{W}}{\partial \boldsymbol{p_i}} \cdot \frac{\boldsymbol{dp_i}}{\boldsymbol{d\beta_i}} + \frac{\partial \boldsymbol{W}}{\partial \boldsymbol{p_j}} \cdot \frac{\boldsymbol{dp_j}}{\boldsymbol{d\beta_i}} + \frac{\partial \boldsymbol{W}}{\partial \boldsymbol{\beta_i}} = \mathbf{0},$$

and the Cramer's rule to compute the derivatives for $\boldsymbol{p_i}$ and $\boldsymbol{p_j}$ with respect to $\boldsymbol{\beta_i}$, the exogenous parameter:

$$\frac{\boldsymbol{dp_i}}{\boldsymbol{d\beta_i}} = \frac{-\frac{\partial V}{\partial \beta_i} \cdot \frac{\partial W}{\partial p_j} + \frac{\partial V}{\partial p_j} \cdot \frac{\partial W}{\partial \beta_i}}{\frac{\partial V}{\partial p_i} \cdot \frac{\partial W}{\partial p_j} - \frac{\partial V}{\partial p_j} \cdot \frac{\partial W}{\partial p_i}} < \mathbf{0}, \text{ and}$$

$$\frac{\boldsymbol{dp_j}}{\boldsymbol{d\beta_i}} = \frac{-\frac{\partial V}{\partial p_i} \cdot \frac{\partial W}{\partial \beta_i} + \frac{\partial V}{\partial \beta_i} \cdot \frac{\partial W}{\partial p_i}}{\frac{\partial V}{\partial p_i} \cdot \frac{\partial W}{\partial p_j} - \frac{\partial V}{\partial p_j} \cdot \frac{\partial W}{\partial p_i}} \begin{cases} > \mathbf{0}, \boldsymbol{\beta_i} = \mathbf{0.51} \text{ or } \boldsymbol{\beta_i} = \mathbf{0.75} \\ < \mathbf{0}, \boldsymbol{\beta_i} = \mathbf{0.99} \end{cases}$$

Where $\frac{\partial V}{\partial \beta_i} < \mathbf{0}$; $\frac{\partial W}{\partial p_j} < \mathbf{0}$; $\frac{\partial V}{\partial p_j} < \mathbf{0}$; $\frac{\partial W}{\partial \beta_i} > \mathbf{0}$; $\frac{\partial V}{\partial p_i} < \mathbf{0}$; and $\frac{\partial W}{\partial p_i} > \mathbf{0}$.

9.6. Computation of equilibrium under equality for intra-network prices and access fee

Assuming price equality for intra-network and access fee, the resulting profit equations after substituting the linear demands (13) and (14) in profit equation (12) for TR *i*, and for TR *j*, respectively, are:

$$\boldsymbol{\pi_i} = (\boldsymbol{a_0} - \boldsymbol{a_1 p_i})[\boldsymbol{p_i} - \boldsymbol{x_i} - (\mathbf{1} - \boldsymbol{\beta_i})\boldsymbol{p_j}] + \boldsymbol{\beta_i}(\boldsymbol{b_0} - \boldsymbol{b_1 p_j})[\boldsymbol{p_i} - \boldsymbol{c_i}] \quad (51)$$

$$\boldsymbol{\pi_j} = (\boldsymbol{b_0} - \boldsymbol{b_1 p_j})[\boldsymbol{p_j} - \boldsymbol{x_j} - \boldsymbol{\beta_i p_i}] + (\mathbf{1} - \boldsymbol{\beta_i})(\boldsymbol{a_0} - \boldsymbol{a_1 p_i})[\boldsymbol{p_j} - \boldsymbol{c_j}]. \quad (52)$$

FOCs for the new profit functions result in:

$$\boldsymbol{V} = \partial \boldsymbol{\pi_i} / \partial \boldsymbol{p_i} = \boldsymbol{a_0} - \mathbf{2}\boldsymbol{a_1 p_i} + \boldsymbol{a_1 x_i} + (\mathbf{1} - \boldsymbol{\beta_i})\boldsymbol{a_1 p_j} + \boldsymbol{\beta_i b_0} - \boldsymbol{\beta_i b_1 p_j} = \mathbf{0} \quad (53)$$

$$\boldsymbol{W = \partial\pi_j/\partial p_j = b_0 - 2b_1p_j + b_1x_j + \beta_i b_1 p_i + (1-\beta_i)a_0 - (1-\beta_i)a_1 p_i = 0.} \quad (54)$$

For the comparative statistics, the Cramer's rule is applied to compute the equilibrium prices in Subsection 6.3.1.

9.7. Proof of Proposition 3

The profit functions with reciprocal access fee become:

$$\boldsymbol{\pi_i = D_i(p_i)[p_i - x_i - (1-\beta_i)t] + \beta_i D_j(p_j)[\mathrm{t} - c_i]}. \quad (55)$$

$$\boldsymbol{\pi_j = D_j(p_j)[p_j - x_j - \beta_i t] + (1-\beta_i)D_i(p_i)[\mathrm{t} - c_j]}. \quad (56)$$

FOCs for the equations result in:

$$\boldsymbol{V = \partial\pi_i/\partial p_i = D_i'(p_i)[p_i - x_i - (1-\beta_i)t] + D_i(p_i) = 0} \quad (57)$$

$$\boldsymbol{W = \partial\pi_j/\partial p_j = D_j'(p_j)[p_j - x_j - \beta_i t] + D_j(p_j) = 0.} \quad (58)$$

For the comparative statistics, I apply the chain rule and the Cramer's rule to compute the derivatives for $\boldsymbol{p_i}$ and $\boldsymbol{p_j}$ with respect to $\boldsymbol{t}$.[32]

$\boldsymbol{\frac{dp_i}{dt} = \frac{-\frac{\partial V}{\partial t}\cdot\frac{\partial W}{\partial p_j} + \frac{\partial V}{\partial p_j}\cdot\frac{\partial W}{\partial t}}{\frac{\partial V}{\partial p_i}\cdot\frac{\partial W}{\partial p_j} - \frac{\partial V}{\partial p_j}\cdot\frac{\partial W}{\partial p_i}} > 0}$ and $\boldsymbol{\frac{dp_i}{dt} = \frac{-\frac{\partial V}{\partial p_i}\cdot\frac{\partial W}{\partial t} + \frac{\partial V}{\partial t}\cdot\frac{\partial W}{\partial p_i}}{\frac{\partial V}{\partial p_i}\cdot\frac{\partial W}{\partial p_j} - \frac{\partial V}{\partial p_j}\cdot\frac{\partial W}{\partial p_i}} > 0}$, where $\boldsymbol{\frac{\partial V}{\partial t} > 0}$, $\boldsymbol{\frac{\partial W}{\partial p_j} < 0}$, $\boldsymbol{\frac{\partial V}{\partial p_j} = 0}$, $\boldsymbol{\frac{\partial W}{\partial t} > 0}$, $\boldsymbol{\frac{\partial V}{\partial p_i} < 0}$, and $\boldsymbol{\frac{\partial W}{\partial p_i} = 0}$.

9.8. Computation of equilibrium under reciprocal access fee

Assuming reciprocal access fee, the resulting profit equations after substituting the linear demands (13) and (14) in profit equation (12) for TR *i*, and for TR *j*, respectively, are:

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[32] I consider the following values for $\boldsymbol{t}$: 0.40, 0.45, and 0.50.

$$\boldsymbol{\pi_i = (a_0 - a_1 p_i)[p_i - x_i - (1-\beta_i)t] + \beta_i(b_0 - b_1 p_j)[t - c_i]} \quad (59)$$

$$\boldsymbol{\pi_j = (b_0 - b_1 p_j)[p_j - x_j - \beta_i t] + (1-\beta_i)(a_0 - a_1 p_i)[t - c_j].} \quad (60)$$

FOCs for the new profit functions result in:

$$\boldsymbol{V = \partial\pi_i/\partial p_i = a_0 - 2a_1 p_i + a_1 x_i + a_1(1-\beta_i)t = 0} \quad (61)$$

$$\boldsymbol{W = \partial\pi_j/\partial p_j = b_0 - 2b_1 p_j + b_1 x_j + b_1\beta_i t = 0} \quad (62)$$

For the comparative statistics, the Cramer's rule is applied to compute the equilibrium prices in Subsection 6.3.2.

9.9. Proof of Proposition 4

The profit functions with regulated access fee for the incumbent and free for the entrant are:

$$\boldsymbol{\pi_i = D_i(p_i)[p_i - x_i - (1-\beta_i)t_j] + \beta_i D_j(p_j)[t_i^r - c_i]}. \quad (63)$$

$$\boldsymbol{\pi_j = D_j(p_j)[p_j - x_j - \beta_i t_i^r] + (1-\beta_i) D_i(p_i)[t_j - c_j]}. \quad (64)$$

FOCs for the equations result in:

$$\boldsymbol{V = \partial\pi_i/\partial p_i = D_i'(p_i)[p_i - x_i - (1-\beta_i)t_j] + D_i(p_i) = 0} \quad (65)$$

$$\boldsymbol{W = \partial\pi_j/\partial p_j = D_j'(p_j)[p_j - x_j - \beta_i t_i^r] + D_j(p_j) = 0.} \quad (66)$$

For the comparative statistics, I apply the chain rule and the Cramer's rule to compute the derivatives for $\boldsymbol{p_i}$ and $\boldsymbol{p_j}$ with respect to $\boldsymbol{t_i^r}$.

$\boldsymbol{\frac{dp_i}{dt_i^r} = \frac{-\frac{\partial V}{\partial t_i^r}\cdot\frac{\partial W}{\partial p_j} + \frac{\partial V}{\partial p_j}\cdot\frac{\partial W}{\partial t_i^r}}{\frac{\partial V}{\partial p_i}\cdot\frac{\partial W}{\partial p_j} - \frac{\partial V}{\partial p_j}\cdot\frac{\partial W}{\partial p_i}} = 0}$ and $\boldsymbol{\frac{dp_j}{dt_i^r} = \frac{-\frac{\partial V}{\partial p_i}\cdot\frac{\partial W}{\partial t_i^r} + \frac{\partial V}{\partial t_i^r}\cdot\frac{\partial W}{\partial p_i}}{\frac{\partial V}{\partial p_i}\cdot\frac{\partial W}{\partial p_j} - \frac{\partial V}{\partial p_j}\cdot\frac{\partial W}{\partial p_i}} > 0}$, where $\boldsymbol{\frac{\partial V}{\partial t_i^r} = 0}$, $\boldsymbol{\frac{\partial W}{\partial p_j} < 0}$,

$\boldsymbol{\frac{\partial V}{\partial p_j} = 0}$, $\boldsymbol{\frac{\partial W}{\partial t_i^r} > 0}$, $\boldsymbol{\frac{\partial V}{\partial p_i} < 0}$, and $\boldsymbol{\frac{\partial W}{\partial p_i} = 0}$.

9.10. Computation of equilibrium prices under asymmetric regulation

Assuming asymmetric regulation of prices, the resulting profit equations after substituting the linear demands (13) and (14) in profit equation (12) for TR *i*, and for TR *j*, respectively, are:

$$\boldsymbol{\pi_i = (a_0 - a_1 p_i)\left[p_i - x_i - (1-\beta_i)t_j\right] + \beta_i(b_0 - b_1 p_j)[t_i^r - c_i]} \quad (67)$$

$$\boldsymbol{\pi_j = \left(b_0 - b_1 p_j\right)\left[p_j - x_j - \beta_i t_i^r\right] + (1-\beta_i)(a_0 - a_1 p_i)[t_j - c_j].} \quad (68)$$

FOCs for the new profit functions result in:

$$\boldsymbol{V = \partial\pi_i/\partial p_i = -a_1(p_i - x_i + (1-\beta_i)t_j + (a_0 - a_1 p_i) = 0} \quad (69)$$

$$\boldsymbol{W = \partial\pi_j/\partial p_j = -b_1\left(p_j - x_j - \beta_i t_i^r\right) + (b_0 - b_1 p_j) = 0} \quad (70)$$

For the comparative statistics, the Cramer's rule is applied to compute the equilibrium prices in Subsection 6.3.3.

9.11. Proof of Proposition 5

The profit functions with the access fee as the minimum value chosen by the TRs are:

$$\boldsymbol{\pi_i = D_i(p_i)\left[p_i - x_i - (1-\beta_i)p_j\right] + \beta_i D_j(p_j)[p_j - c_i],} \quad (71)$$

$$\boldsymbol{\pi_j = D_j(p_j)\left[p_j - x_j - \beta_i p_j\right] + (1-\beta_i)D_i(p_i)[p_j - c_j].} \quad (72)$$

FOCs for the equations result in:

$$\boldsymbol{V = \partial\pi_i/\partial p_i = D_i'(p_i)\left[p_i - x_i - (1-\beta_i)p_j\right] + D_i(p_i) = 0,} \quad (73)$$

$$\boldsymbol{W = \partial\pi_j/\partial p_j = D_j'(p_j)\left[p_j - x_j - \beta_i p_j\right] + D_j(p_j) - \beta_i D_j(p_j) + (1-\beta_i)D_i(p_i) = 0.} \quad (74)$$

For the comparative statistics, I apply the chain rule and the Cramer's rule to compute the derivatives for $\boldsymbol{p_i}$ and $\boldsymbol{p_j}$ with respect to $\boldsymbol{\beta_i}$.

$$\frac{\boldsymbol{dp_i}}{\boldsymbol{d\beta_i}} = \frac{-\frac{\partial V}{\partial \beta_i}\cdot\frac{\partial W}{\partial p_j}+\frac{\partial V}{\partial p_j}\cdot\frac{\partial W}{\partial \beta_i}}{\frac{\partial V}{\partial p_i}\cdot\frac{\partial W}{\partial p_j}-\frac{\partial V}{\partial p_j}\cdot\frac{\partial W}{\partial p_i}} < \mathbf{0}$$ and $$\frac{\boldsymbol{dp_j}}{\boldsymbol{d\beta_i}} = \frac{-\frac{\partial V}{\partial p_i}\cdot\frac{\partial W}{\partial \beta_i}+\frac{\partial V}{\partial \beta_i}\cdot\frac{\partial W}{\partial p_i}}{\frac{\partial V}{\partial p_i}\cdot\frac{\partial W}{\partial p_j}-\frac{\partial V}{\partial p_j}\cdot\frac{\partial W}{\partial p_i}} > \mathbf{0}$$, where $\frac{\partial V}{\partial \beta_i} < \mathbf{0}$, $\frac{\partial W}{\partial p_j} < \mathbf{0}$,

$\frac{\partial V}{\partial p_j} > \mathbf{0}$, $\frac{\partial W}{\partial \beta_i} > \mathbf{0}$, $\frac{\partial V}{\partial p_i} < \mathbf{0}$, and $\frac{\partial W}{\partial p_i} < \mathbf{0}$.

9.12. Computation of equilibrium under minimum access fee

Assuming the minimum access price chosen by TRs as reciprocal, the resulting profit equations after substituting the linear demands (13) and (14) in profit equation (12) for TR *i*, and for TR *j*, respectively, are:

$$\boldsymbol{\pi_i = (a_0 - a_1 p_i)\left[p_i - x_i - (1-\beta_i)p_j\right] + \beta_i(b_0 - b_1 p_j)[p_j - c_i]} \qquad (75)$$

$$\boldsymbol{\pi_j = \left(b_0 - b_1 p_j\right)\left[p_j - x_j - \beta_i p_j\right] + (1-\beta_i)(a_0 - a_1 p_i)[p_j - c_j].} \qquad (76)$$

FOCs for the new profit functions result in:

$$\boldsymbol{V = \partial\pi_i/\partial p_i = a_0 - 2a_1 p_i + a_1 x_i + (1-\beta_i)a_1 p_j = 0} \qquad (77)$$

$$\boldsymbol{W = \partial\pi_j/\partial p_j = \beta_i b_0 - 2b_1 p_j + b_1 x_j + 2\beta_i b_1 p_j + (1-\beta_i)(a_0 - a_1 p_i) = 0.} \qquad (78)$$

For the comparative statistics, the Cramer's rule is applied to compute the equilibrium prices in Subsection 6.3.4.

9.13. Graphical representation of equilibrium prices

This Subsection illustrates the relationship between the equilibrium prices computed for TR *i* and TR *j* in the scenarios of free prices, as well as under some market agreement or regulatory constraint.

I consider discrete values 0.51, 0.75 and 0.99 for $\boldsymbol{\beta_i}$, [33] and $\boldsymbol{x_i = 0.1}$, $\boldsymbol{x_j = 0.11}$, $\boldsymbol{a_0 = 0.5}$, $\boldsymbol{a_1 = 1}$, $\boldsymbol{b_0 = 0.5}$, $\boldsymbol{b_1 = 1}$.[34]

Price behavior under free prices

As values for $\boldsymbol{t_i}$ and $\boldsymbol{t_j}$, for each $\boldsymbol{\beta_i}$, I assume the prices computed in equilibrium under equality for intra-network price and access fee (i.e., the values for $\boldsymbol{p_i^1}$ and $\boldsymbol{p_j^1}$):

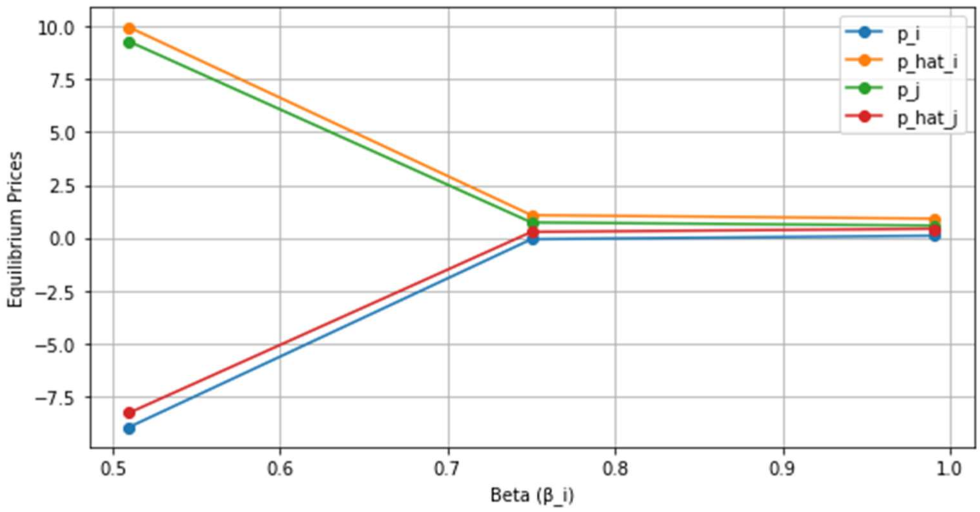
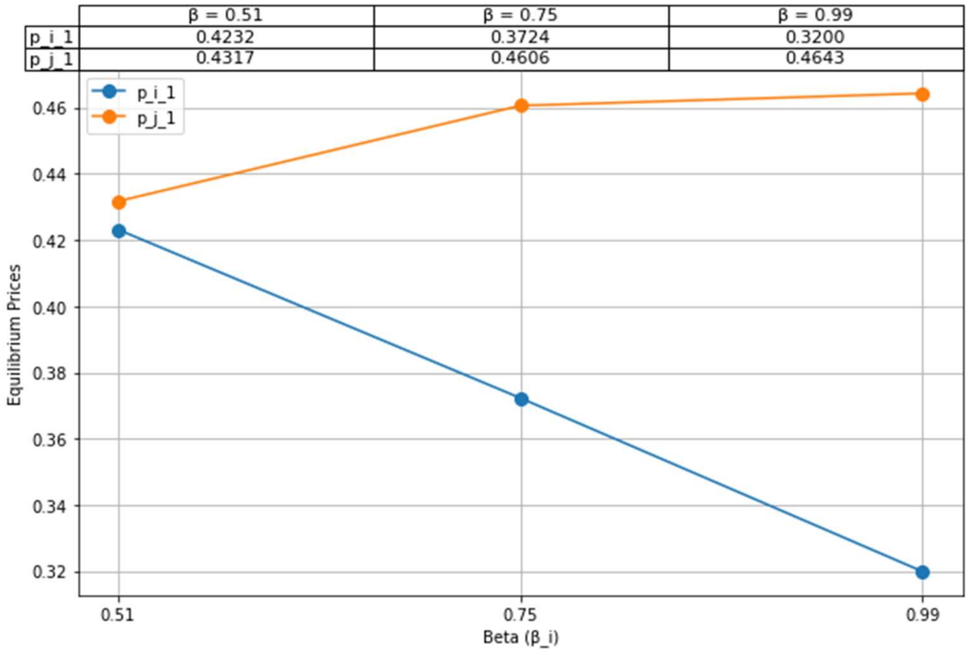


Figure 7 – Equilibrium prices

Price behavior under equality for intra-network prices and access fee

[33] $\boldsymbol{\beta_i}$ must be greater than 0.5 and lower than 1, which represents the incumbency and guarantees two TRs in the market, respectively.
[34] I assume higher cost for the entrant, which is reasonable given lower economies of scale compared to the incumbent.



	β = 0.51	β = 0.75	β = 0.99
p_i_1	0.4232	0.3724	0.3200
p_j_1	0.4317	0.4606	0.4643

Figure 8 – Equilibrium prices

Price behavior under reciprocal access fee

Assuming $\boldsymbol{t} = [\mathbf{0.30}, \mathbf{0.40}, \mathbf{0.45}]$:

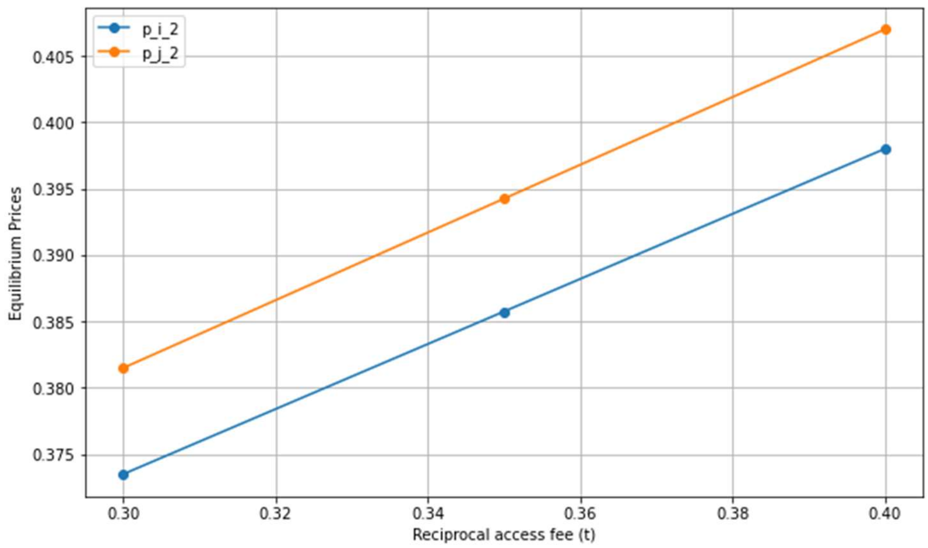


Figure 9 – Equilibrium prices for $\boldsymbol{\beta_i} = 0.51$

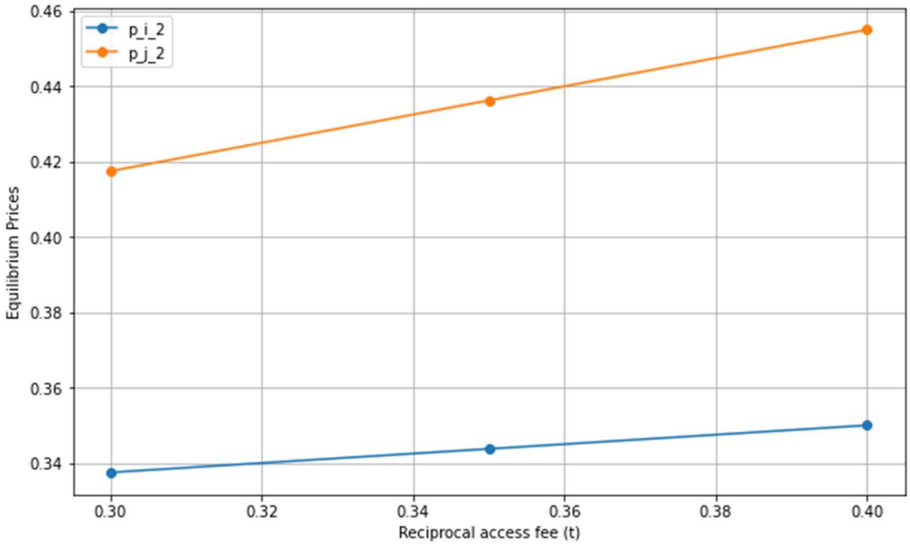


Figure 10 – Equilibrium prices for $\boldsymbol{\beta_i} = 0.75$

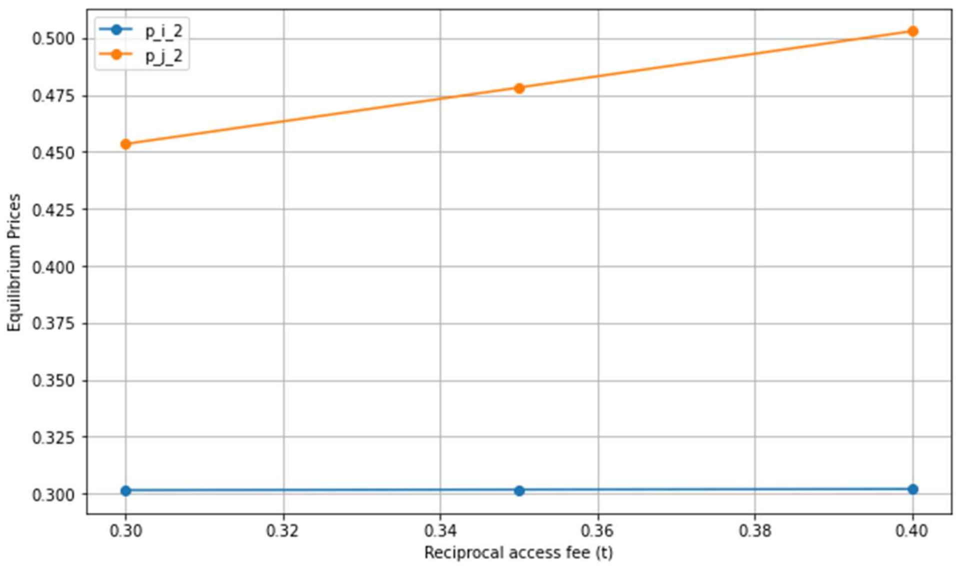


Figure 11 – Equilibrium prices for $\boldsymbol{\beta_i} = 0.99$

Price behavior under asymmetric regulation

Assuming $\boldsymbol{t_j = 0.45}$:[35]

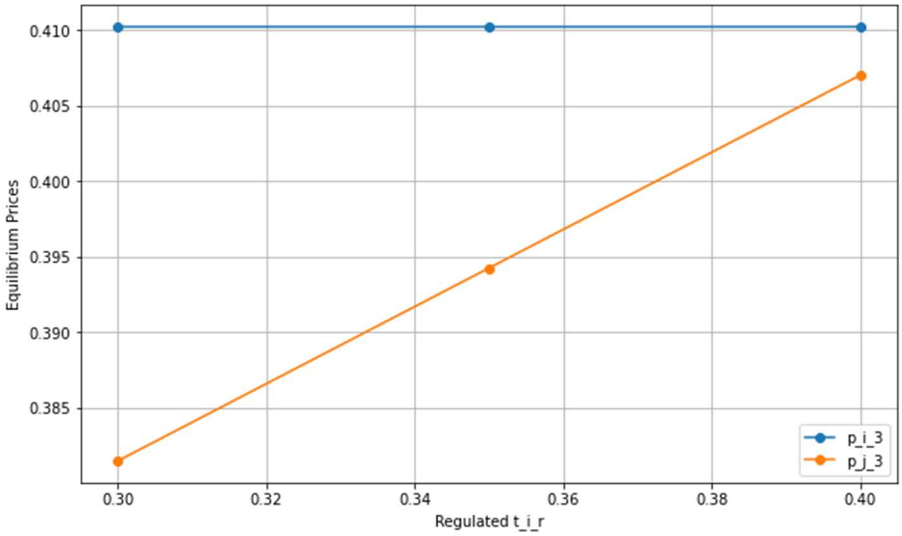


Figure 12 – Equilibrium prices for β_i=0.51

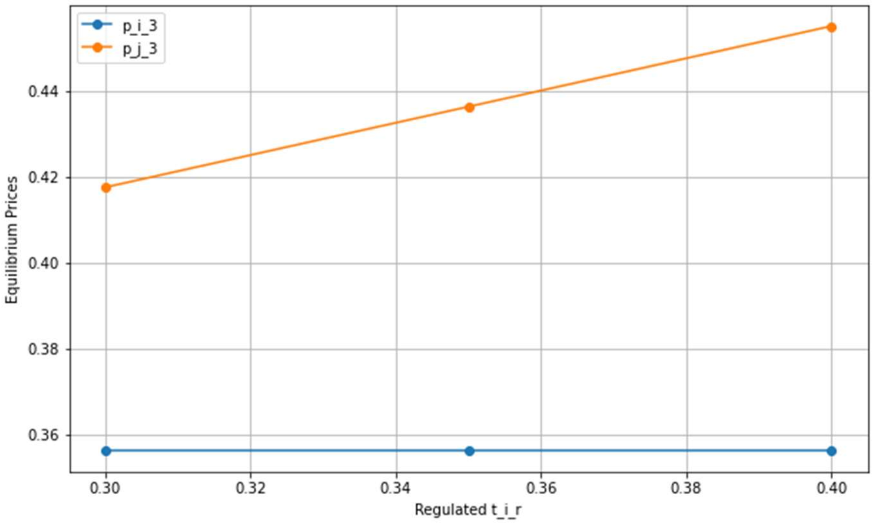


Figure 13 – Equilibrium prices for β_i=0.75

[35] *p_j* is always increasing and becomes higher than *p_i* as β_i increases.

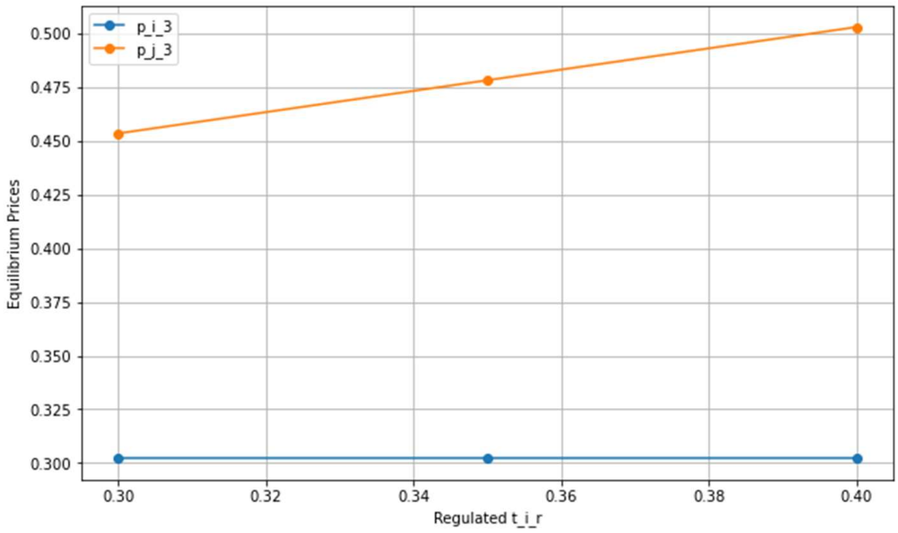


Figure 14 – Equilibrium prices for β_i=0.99

Price behavior under minimum access fee as reciprocal

beta_i	p_i	p_j
0.51	0.400890868596882	0.411799463660743
0.75	0.367058823529412	0.536470588235294
0.99	0.329177057356609	5.83541147132160

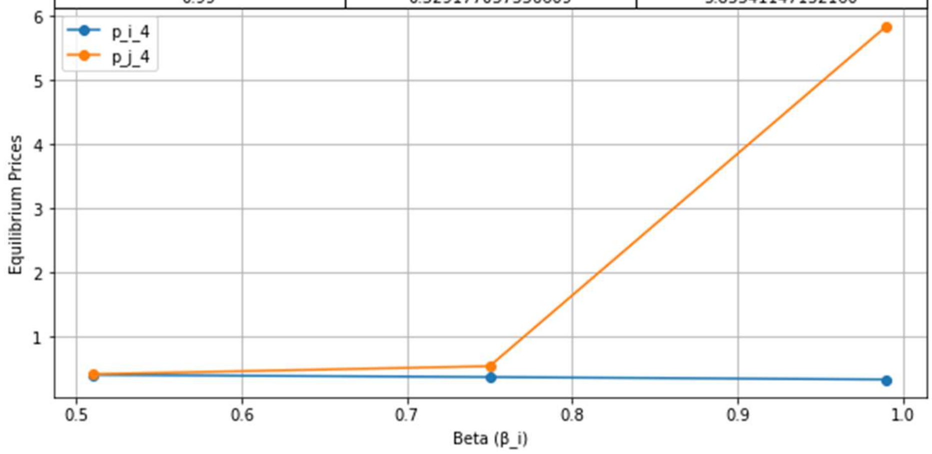


Figure 15 – Equilibrium prices for given β_i