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Abstract

Variations in repo haircuts play a crucial role in leveraging (or deleveraging) in security markets, as observed in the two major economic events that happened so far in this century, the US housing bubble that burst into the great recession and the European sovereign debts episode. Repo trades are secured but recourse loans. Default triggers insolvency. Collateral may be temporarily exempt from automatic stay but creditors' final reimbursement depends on the bankruptcy outcome. We show examples of bankruptcy equilibria. We infer how haircuts are related to asset or counterparty risks whenever a bankruptcy equilibrium exists.

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1 Introduction

In a repo trade, a security is pledged as collateral for a cash loan and can then by reused by the cash lender, that is, pledged in a another contract or short-sold. The reuse of the collateral makes repo trades quite different from mortgage loans where the durable good collateral stays put. The resulting leverage was studied in detail by Bottazzi, Luque and Páscoa (2012), under the assumption that agents always fulfilled their financial obligations.

Leverage played a major role in the recent financial crisis of 2008. Leading to the crisis, it was not only households that were highly indebted but also large financial institutions. These large institutions turned to the *shadow banking* system to finance themselves (see e.g. Gorton and Metrick (2010)). The repo market is a crucial part of this system. The haircut applied to the loan given in a repo trade is inversely related to how much agents can build up their positions in a security by using the repo market as a means of financing security positions. Security and repo trades can be combined in way that allows security positions to be increased, as the security gets pledged as collateral in repo, then repledged or short sold by the creditor (then again pledged by the counterparty of the short seller and so on). How do collateral reuse and haircuts determine what leverage is? The former may be an ingredient but does not determine by itself what leverage is (and limitations on reuse do not automatically translate into targeted reductions in leverage). For long agents to lever up to haircut potential, the reuse of the collateral only becomes necessary when the wealth of these agents is high enough and the haircut is low enough that the resulting aggregate leveraged long positions exceed aggregate initial holdings of the security. It is ultimately the haircut that determines what leverage is (and it could be the shorts being leveraged instead). More recently, in the sovereign debt crisis of 2010-12, there was substantial deleverage (also of those short selling) caused by the consecutive hikes in repo margins on bonds issued by several European governments.

Given that leverage and the haircut are inversely related, it is crucial to understand how the latter is determined. The haircut is the difference between the values of the collateral and the respective cash loan, at the time when the repo trade starts. It is usually expressed as a percentage (less than or equal to 1) of the collateral value. Equivalently, the initial margin captures that difference by expressing the collateral value as a percentage (greater than or equal to 1) of the cash loan. A repo trade has a purchase leg and then a repurchase leg at a repurchase price that is locked in at the first leg. The difference between the purchase price (the cash loan) and the repurchase price is the repo interest rate, agreed upon in advance. Hence, in the absence of default, there would be no reason to charge a haircut. The haircut reflects the cash lender's perceived risk of loss in the event of the cash borrower's default.

In this article we model the limited commitment involved in repo trades. In this respect, also, there is a key difference by comparison with what happens in many (but not all) mortgages, as captured in the GE collateral literature. Repo trades are recourse loans, whereas many (but not all) mortgages are non-recourse. If an household that has signed a non-recourse mortgage decides to default, it would just surrender the house and walk away without suffering any other penalties. That is not the case in recourse loans: in the event of default, creditors can be repaid above the collateral liquidation value by forcing the bankruptcy of the faulty borrower and then becoming claimants in the partition of the borrower's estate. It may also happen that creditors end up recovering less than the collateral liquidation value, when that is the outcome from the partition of the estate among all creditors. Repo collateral is exempted from certain provisions of the US Bankruptcy Code that normally apply to pledges, in particular, the automatic stay on enforcement of collateral in the event of insolvency. That is, creditors can keep the collateral that had been pledged to them (and can sell it) but, when the bankruptcy court takes the final decisions, they may get more or less than what their claim was (the promised repayment) and this may be different from the liquidation value of the collateral.

It should be noted that when an agent goes bankrupt, it is not just the repayment of the cash borrowed in repo that is at stake. If a security happened to be pledged to this agent in repo, then this collateral will not be given back to the cash borrowers - a *"fail"* occurs as a result of bankruptcy - and the respective manufactured dividends due to the beneficial owner will not be paid also.

Default is a very serious event and needs to be modeled by taking into consideration the whole bankruptcy process. It is not a decision that can be taken asset by asset, comparing promised payments and collateral values. Debtors can't be assumed to be repaying the minimum of these two, contrary to what happens in non-recourse loans, as shown in a long standing literature emerging from the work by Geanakoplos and Zame in the nineties (see Geanakoplos (1997), Geanakoplos and Zame (1997) and Geanakoplos and Zame (2014)). For the same reason, default can't be avoided by designing contracts so that collateral values never fall below promised payments, as was the case in a contemporaneous literature dating back to Kiyotaki and Moore (1997). Garnishable estates must now be set against total debts (net of credits that the defaulter may be entitled to). This creates a non-convexity in the borrower's budget set that seems to have put off previous research efforts.

There are however interesting results that can be established, in spite of the intrinsic non-convexity of individual decision problems. We consider binomial economies, where just two states of nature, U or D, may occur after the initial node (and each of these states may be followed without uncertainty into a third date). Our paper focus on over-the-counter (OTC) repo, that is, trades that are not centrally cleared through an exchange (or central clearing counterparty, CCP), and bilateral (as opposed to tri-party where collateral selection, payment, custody and settlement are outsourced to a third-party agent). Our finite-agent model does not let us explore the convexifying effect of large numbers that has been used in continuum of agents models in several contexts, including in consumer bankruptcy problems with unsecured loans (see Araujo and Pascoa (2002) and Sabarwal (2003)). However, modeling the agents set as a continuum is not appropriate in a context of OTC repo where each trader should anticipate counterparty bankruptcy risk and choose repo haircuts accordingly[2].

For equilibrium to exist, leverage should be bounded. Here there is another important distinction between credit backed by securities and credit backed by houses or productive resources.

[2] A bankruptcy analysis might be doable also for centrally cleared repo, but aggregate default risk should take the place of counterparty risk. The OTC case seems to be more informative and easier to relate to the applied literature on the determinants of repo haircuts.

In the latter, the aggregate supply of collateral is fixed and, therefore, under exogenous collateral margins, borrowing becomes bounded. In the former, the collateral supply is endogenous since it includes short-sales and, therefore, there are no a priori bounds on secured borrowing, even under exogenous margins. However, repo and security positions must be related in another way: the net security title balance held by each agent must be non negative. This is known as the *box constraint* and says that in to order to pledge the agent must be long in the security and in order to short-sell the agent must be long in repo (the security being pledged to him). In the one-security case, by combining the box and budget constraints we can bound secured borrowing. This would be enough to bound all sorts of leverage (long or short) in convex full commitment economies. However, in non-convex economies allowing for bankruptcy, we need to bound secured lending as well, since an equilibrium for a truncated economy (whose portfolios are assumed to be market feasible) may fail to be an equilibrium. In the multi-security case, it was already known that, even in the convex full commitment setting, other constraints should be added with the purpose of bounding repo and security trades[3].

In order to gain intuition and allow for a full characterization of equilibria, we start by examining a one-security and two-agent case. In this simple case, the optimist is long in the security (short in repo) and the pessimist is short in the security (long in repo). We find equilibria where both or just the former go bankrupt (the former in the state where the security has lower returns and the latter in the other state). Then, we contemplate the multi-security and multi-agent case to see what are the determinants of haircuts. On this issue, there are different views in the applied literature. Gorton and Metrick (2012) argue that haircuts depend both on the underlying asset and on who is the counterparty in a repo transaction but that, particularly in times of crisis, the latter gains importance. In contrast, Krishnamurty et al. (2014) report little variation of haircuts across counterparties and place much more weight on the underlying asset. Infante (2015) argues that these observed differences on haircuts arise because two different

[3]See Bottazzi, Luque and Páscoa (2012) on bounds that result from the segregation of haircuts or the distinction between dealers and non-dealers and Bottazzi, Luque and Páscoa (2017) on bounds that follow from equity requirements in the spirit of the Basel regulation of banks.

markets are studied: the bilateral and tri-party repo.

The loss that a lender may suffer from counterparties' default may be related to the collateral falling in value (or being sold in a fire sale) but, since the loan is recourse, the loss cannot be associated to that asset risk in such a simple way. It may happen that there is no *asset risk* but the *counterparty risk* will nevertheless govern what the lender gets back, which does not have to be equal to the collateral liquidation value. What a secured creditor recovers in the bankruptcy process depends on what is the liquidation value of the whole estate of the defaulter and how it will be partitioned among all creditors, even though the exemption from automatic stay allows the creditor to sell the collateral while waiting for the final outcome of the bankruptcy process.

We characterize how haircuts respond to asset and counterparty risks. Suppose there are many traders in the repo market of each security, repo rates are security-specific but haircuts are specific to each pair of traders. In such competitive setting, we should expect counterparty bankruptcy risk to affect pair-specific haircuts but not the repo rate, as opposed to what happens in the two-agent example. Say state D is the state where bankruptcy may occur. Suppose an agent i is solvent in state D and is, in terms of the whole portfolio, a net creditor to a counterparty j (in state D) and the expected repayment rate of this counterparty decreases (an increased counterparty risk). Then, agent i would like to raise (lower) the haircut charged to counterparty j when accepting collateral from j, for securities whose repo repayment exceeds (falls below) the collateral value. That is, when the asset is risky from the creditor's perspective, haircuts tend to move in the same direction as the counterparty risk. But for the other securities (risky from the debtors' point of view, wary of a repo fail), haircuts move in the opposite direction.

Quite differently, in the two-agent and one-security example, counterparty risk affects the repo rate and his effect is strong enough to make bankruptcy rates decrease as the haircut increases. In a small numbers context, it is now the other direction that may become more relevant: how are overall solvency rates affected when the haircut charged in one security changes? That is why in such extreme non-competitive case, haircuts and expected repayment rates may move together, contrary to our results for the competitive case. To summarize, the way counterparty risk may impact haircuts depends on how competitive the repo market is and to understand how that impact works in the competitive case we need to couple this risk with asset risk. When faced with a rise in counterparty risk, competitive creditors tend to ask for higher haircuts for securities that exhibit an asset risk from the creditors' perspective, but lower haircuts may arise if the security involves the opposite risk (a fail rather than a default risk).

2 The Model

2.1 Fundamentals

We consider a binomial economy with three dates. At an initial date (date 0) there is only one node in the event tree, followed by nodes U and D at the second date. Each second date note has a unique successor at the third date: U^+ and D^+ are the successors of U and D, respectively. As we will see, the third date just serves to guarantee that securities retain value at the second date, when borrowing and lending transactions are settled (and we may want to dispense with the third date in some cases, as discussed below).

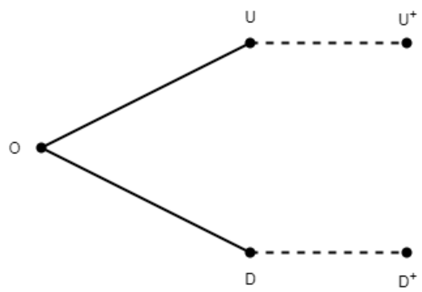


Figure 1: Events tree of the binomial economy.

Binomial models have been used to study the leverage cycle in economies with default on non-recourse loans (see e.g., Fostel and Geanakoplos (2012) and Fostel and Geanakoplos (2014)). Given that repo trades constitute recourse loans, we will model default as a bankruptcy process.

There is only one consumption good. Markets for this commodity open at each event. We denote the price of this good at event e by p_e. There is a finite set of $I \geq 2$ agents, indexed by i. A bundle of commodities consumed by agent i is denoted by $x^i = (x^i_0, x^i_U, x^i_D, x^i_{U+}, x^i_{D+})$. There are also F real securities indexed by f, each one being characterized by a vector of non-negative real returns $R_f = (R_{fU}, R_{fD}, R_{fU+}, R_{fD+})$. Given spot prices p_e, the nominal return of security f is $p_e R_{fe}$[4].

Trading of securities occurs at the first and second dates. Each agent chooses a securities portfolio $\phi^i \in \mathbb{R}^{3F}$ consisting of positions in the F securities at the initial nodes and nodes U and D. Security prices are denoted by $q \equiv (q^f_e) \in \mathbb{R}^{3F}$. Agents' endowments of commodities are $\omega^i \in \mathbb{R}^5_+$, with $\omega^i_s > 0$ in both states. Agents have initial holdings, at date 0, of each security f, $o^i_f > 0$. Preferences are described by utility functions $U^i : \mathbb{R}^5_+ \to \mathbb{R}$. For each security f, we normalize its positive net supply to be one: $\sum_i o^i_f = 1$.

2.2 Repo markets

Agents can have negative positions in securities, short-sales are permitted. Short-selling, however, is not the same as issuing (which we take as given this model, having occurred prior to date 0). In order to short-sell a security, an agent must go first in the repo market and *borrow* the desired amount of securities. This is the way short-selling is actually done in reality.

Borrowing of securities actually consists in buying the security and promising to resell it to the lender, at a future date and at a predetermined price. There is a difference between the price at which a security is bought, in the first leg of the transaction, and the price at which it is resold to its original owner, in the second leg of the transaction, at a future date. This difference is captured by the *repo rate.* The highest repo rate within its class of securities is referred to as the *general collateral rate* (GC).

The borrower of a security acquires possession rights associated with the security. However, any coupon or dividend paid to the borrower during the term of the transaction is passed through to the original owner; this is called a *manufactured payment* or a *manufactured dividend.*

[4]We could have considered nominal securities instead.

A repo transaction is actually a collateralized loan. If agent i buys security f from j in the first leg of the transaction, i is at the same time the borrower of the security and a lender of cash, and j is the lender of the security and borrower of cash. A haircut $(1-h_f^{ij})$ is applied to the market value of the security to compensate the lender of funds for the risk associated to the transaction, so that the cash loan may be lower than the value of the collateral. Haircut is such that $0 < h_f^{ij} \leq 1$.

For simplicity, repo trading takes place only at date 0. We denote by z_f^{ij} agent i's repo position, with counterparty j, on security f. If $z_f^{ij} > 0$, it means that i is the lender of cash and borrower of the security and we say that he is *long* in repo. At the same time, since markets must clear, we should expect that $z_f^{ji} < 0$, which means that agent j is a borrower of cash, a lender of the security, and we say that he is *short* in repo.

Given that a haircut is applied to every repo transaction, the amount of funds that can be borrowed by pledging one unit of security f in the repo market, in a transaction with counterparties i and j, is given by $h_f^{ij} q_{f0}$. As a matter of notation, we use both h_f^{ij} and h_f^{ji} to denote the haircut applied to transactions in the repo market involving security f and counterparties i and j, regardless of which one of the agents is long in repo for security f and which one is short. Denote by ρ_f the repo rate of a loan backed by security f and let $r_f = 1 + \rho_f$.

An important feature of a repo transaction is that the borrower of the security has the right to lend it in the repo market or short-sell it in the security market. That is, the collateral can be *rehypothecated* directly or indirectly, and this process may occur many times over for the same settlement period. This is an important feature of repo markets as it it the *reuse* of the collateral that allows agents to leverage their portfolio positions or their cash loans beyond what would be possible if collateral was used only once.

There is a constraint that captures both the need to pledge collateral when borrowing cash in repo and the need to borrow a security (accept it as collateral) when short-selling it. This is the *box constraint.* This restriction states that the agent must hold a nonnegative amount of the security in his possession. That is, the sum of security position and repo trades must be nonnegative. At the initial date (the only node where repo markets are open) the box constraint for security f is:

$$\phi^i_{fe} + \sum_{j \neq i} z^{ij}_f \geq 0 \tag{1}$$

At second date nodes, repo markets are not open and the box constraints reduce to plain no-short-sales constraints:

$$\phi^i_{fU} \geq 0, \ \ \phi^i_{fD} \geq 0 \tag{2}$$

When there is more than one security, without any further assumptions on portfolio or repo positions, leverage can be unbounded (see Bottazzi, Luque and Páscoa (2012) on some institutional arrangements that bound leverage). In this paper we take the simple approach of imposing some bounds on repo positions whenever there is a need to bound leverage[5].

2.3 Bankruptcy and feasible market plans

Given market prices and repo rates, (p, q, r), agents decide on a plan (x^i, ϕ^i, z^i), consisting of consumption and portfolios in the securities and repo markets. Let us define the budget constraints that these plans must satisfy. At date 0, the repo market opens and agents have initial endowments of goods and securities. Agent i's budget constraint at this date is:

$$p_0(x^i_0 - \omega^i_0) + \sum_f q_{f0}(\phi^i_{f0} - o^i_f) + \sum_f \sum_{j \neq i} q_{f0} h^{ij}_f z^{ij}_f \leq 0 \tag{3}$$

Repo markets do not open at the second date but agents can still trade in securities, using real payments from their previous securities positions. We allow for the possibility of agents not fulfilling their obligations in the second date. This only happens if agents become insolvent. For every agent, we need to see how do assets set against liabilities, in each state of the second date. On the assets' side we have a first component which is the new market value plus returns associated with the actual amount of each security that the agent has in his possession when he

[5]In the one-security model in Section 3, leverage will be bounded and no further constraints will be imposed on financial positions.

enters that node. This (non-negative) amount is what the agent had in his date 0 box for the corresponding security. By adding up the current market value and returns, across all securities, we get assets' component pertaining to *the value of the past box positions*:

$$\Xi_s^i = \sum_f \left(\phi_{f0}^i + \sum_{j \neq i} z_f^{ij} \right) (q_{fs} + p_s R_{fs})$$

To evaluate assets or liabilities resulting from repo positions taken at the previous node, we need to remember that repo positions consist of securities that the agent has received (borrowed) or pledged (lent) as collateral backing a cash loan. If agent i borrowed security f from agent j at the initial node ($z_f^{ij} > 0$), he must return the security together with the respective returns to its original owner, that is, pass on the non-negative value $z_f^{ij}(q_{fs}+p_s R_{fs})$ to agent j but at the same time agent j must repay (at the gross rate r_f) to i the cash loan he obtained before. Conversely, a repo short agent repays the cash loan and gets back the value of the security he pledged as collateral. The settling of all of agent i's repo transactions with agent j as counterparty is captured by the following term:

$$I_s^{ij} = \sum_f z_f^{ij} \left[q_{f0} h_f^{ij} r_f - (q_{fs} + p_s R_{fs}) \right]$$

If all repo transactions are settled as was agreed upon at the initial date, that is, if every agent is solvent in state s, agent i's corresponding budget constraint is:

$$p_s(x_s^i - \omega_s^i) + \sum_f q_{fs} \phi_{fs}^i \leq \Xi_s^i + \sum_{j \neq i} I_s^{ij} = \sum_f \left(\phi_{f0}^i \, (q_{fs} + p_s R_{fs}) + \sum_{j \neq i} z_f^{ij} q_{f0} h_f^{ij} r_f \right)$$

Since repos are recourse loans, insolvency will occur in state s only if the agent's assets are insufficient to cover his liabilities. The assets include the value of the past box positions, plus the positive settlements of his repo transactions, plus the garnishable portion (according to some coefficient β) of his commodity endowment in that state. Liabilities consist in the negative settlements of his repo trades. That is, insolvency will occur if, and only if, the following condition is satisfied:

$$\beta p_s \omega_s^i + \Xi_s^i + \sum_{j \neq i} \eta_s^j I_s^{ij+} \geq \sum_{j \neq i} I_s^{ij-}$$

Here, I_s^{ij+} and I_s^{ij-} denote the positive and the negative parts[6], respectively, of I_s^{ij}, while η_s^j denotes the portion of all of agent j's financial obligations that he *effectively* repays given his income in that state. That is, $\eta_s^j = 1$ if j is solvent in state s and $\eta_s^j < 1$ when he declares bankruptcy. In words, agent i declares bankruptcy in state s if, and only if, the garnishable portion of his commodity endowment ($\beta p_s \omega_s^i$), plus the value of the securities in his possession at the beginning of the second date (Ξ_s^i), plus the positive repo repayments the agent gets from all his counterparties ($\sum_{j \neq i} \eta_s^j I_s^{ij+}$) is not sufficient to repay all of agent i's repo obligations (I_s^{ij-}).

Once we allow bankruptcy, agent i's budget constraint in state s of the second date is:

$$(4) \qquad p_s(x_s^i - \omega_s^i) + \sum_f q_{fs}\phi_{fs}^i \leq \max\left\{ -\beta p_s \omega_s^i \ , \ \Xi_s^i + \sum_{j \neq i} I_s^{ij+}\eta_s^j - I_s^{ij-} \right\},$$

where

$$(5) \qquad \eta_s^i = \begin{cases} 1 \ , \ \beta p_s \omega_s^i + \Xi_s^i + \sum_{j \neq i} \eta_s^j I_s^{ij+} \geq \sum_{j \neq i} I_s^{ij-} \\ \frac{\beta p_s \omega_s^i + \Xi_s^i + \sum_{j \neq i} \eta_s^j I_s^{ij+}}{\sum_{j \neq i} I_s^{ij-}} \ , \quad \text{otherwise} \end{cases}$$

At the last date there is no trading of securities. Agents consume from their commodity endowments and security payments according to their security positions constituted at the previous nodes

$$(6) \qquad p_{s^+}(x_{s^+}^i - \omega_{s^+}^i) \leq \sum_f \phi_{fs}^i p_{s^+} R_{fs^+}$$

Given parameters (p, q, r), an agent's plan of consumption, of securities, and of repo positions (x^i, ϕ^i, z^i), will be called *feasible* if $x^i \geq 0$ and conditions (1), (2), (3), (4) and (6) hold. We denote by $B^i(p, q, r)$ the set of all feasible plans for agent i, and by $B_\star^i(p, q, r)$ the subset of utility maximizing plans in $B^i(p, q, r)$.

2.4 Equilibrium

For this economy, equilibrium is defined as follows:

[6] $I_s^{ij+} = \max\{0, I_s^{ij}\}$ and $I_s^{ij-} = -\min\{0, I_s^{ij}\}$.

Definition 1 (**Equilibrium**). *An equilibrium is an allocation of bundles, securities and repo positions* (x, ϕ, z) *together with prices* (p, q, r) *and haircuts implied by* $h = (h_f^{ij})$*, such that:*

(a) for each agent i*,* $(x^i, \phi^i, z^i) \in B_\star^i(p, q, r, h)$*,*

(b) commodity markets clear: $\sum_i \left(x_0^i - \omega_0^i\right) = 0$*,* $\sum_i \left(x_s^i - \omega_s^i\right) = \sum_f R_{fs}$ *for* $s = U, D$*, and* $\sum_i \left(x_{s^+}^i - \omega_{s^+}^i\right) = \sum_f R_{fs^+}$ *for* $s^+ = U^+, D^+$*,*

(c) security markets clear: $\sum_i \phi_{fe}^i = 1$ *at each event* $e = 0, U, D, U^+, D^+$*,*

(d) repo markets clear: $z_f^{ij} + z_f^{ji} = 0$ *for all* i, j *and* f*.*

3 A one-security and two-agent model

We can now take our base model and consider the simplest of economies. Suppose there are only two dates, only one security ($F = 1$ and we dispense with the index f for the security) and only two agents indexed i and j with $\omega_0^i = \omega_0^j = 0$ and $o^i = o^j = 1$. Agents utilities are simply the expected consumption at the second date: $U^i(x_U^i, x_D^i) = a^i x_U^i + (1 - a^i) x_D^i$ and $U^j(x_U^j, x_D^j) = a^j x_U^j + (1 - a^j) x_D^j$. To simplify we have dispensed with the third date and assumed repo to maturity[7], that is, both repo trades are settled at the maturity date of the security (even though the security does not have a price at the second date it can still serve as collateral, since its second date value consists in its returns given by R_s).

As there is only one security and one pair of agents, we simplify notation by letting $h = h^{ij} = h^{ji}$, $z^i = z^{ij}$ and $z^j = z^{ji}$. Suppose $R_U > R_D$ and that agents' subjective probabilities are different enough as to guarantee trade in the repo market: $E^i R > E^j R$, where $E^i R \equiv a^i R_U + (1 - a^i) R_D$. This is the same as assuming that $a^i > a^j$.

[7]This type of repo trades occurs in reality, although some agents (in particular, central banks) are not willing to engage in it.

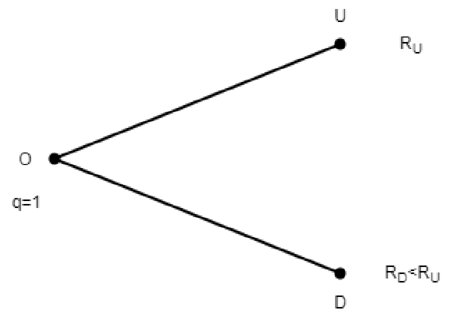


Figure 2: Economy with 2 dates. Security's price and payments.

Normalizing prices so that the security price is one, we can write agent i's constraints as:

$$
\begin{aligned}
\phi^i + hz^i &= o^i, && \text{(First date)} \\
\phi^i + z^i \geq 0 \Leftrightarrow o^i + (1-h)z^i &\geq 0, && \text{(Box constraint)} \\
x_s^i = \omega_s^i + \max\big\{ -\beta\omega_s^i, (o^i + (1-h)z^i)R_s & \\
+\eta_s^j[(hr - R_s)z^i]^+ - [(hr - R_s)z^i]^-\big\}, & && \text{(Second date, state } s\text{)}
\end{aligned}
$$

The box constraint determines what that largest short repo position an agent can take is:

$$z^i = -\frac{o^i}{1-h} \tag{7}$$

Note that the magnitude of the position in equation (7) can be many times higher than the total initial supply of the security in the economy $(o^i + o^j)$. Building such a large short repo position is possible because of the re-usability of collateral in repo markets. At the same time, the position is not unbounded because of the haircut[8]: how much an agent can leverage his initial endowment of the security is related inversely to the haircut applied in the repo market. This is one good reason to define the *asset specific leverage* as the inverse of the haircut applied to the security when used as collateral in the repo market: $\frac{1}{1-h}$.

At the first date, we will require that long repo positions are also limited by the asset specific

[8]See Bottazzi, Luque and Páscoa (2012) for a complete example of how these positions are built in repo markets with one security.

leverage:

$$z^i \le \frac{o^i}{1-h}$$

If we substitute x^i_U and x^i_D into agent i's utility function, we can write his problem as:

Maximize

$$E^i\omega^i + a^i \max\left\{ -\beta\omega^i_U, (o^i + (1-h)z^i)R_U + \eta^j_U[(hr - R_U)z^i]^+ - [(hr - R_U)z^i]^- \right\}$$
$$+ (1-a^i)\max\left\{ -\beta\omega^i_D, (o^i + (1-h)z^i)R_D + \eta^j_D[(hr - R_D)z^i]^+ - [(hr - R_D)z^i]^- \right\}$$

s. t.

$$o^i + (1-h)z^i \ge 0$$
$$z^i \le \frac{o^i}{1-h}$$

The only decision variable in the problem is z^i and the agent only needs to decide whether to be long ($z^i > 0$) or short ($z^i < 0$) in repo. Given our assumption on security payments and utilities, it is reasonable[9] to search for equilibria in which $hr \in (E^jR, E^iR)$. Given the relative weights of each state in agents i and j utility function it is also reasonable to start for equilibria by assuming agent i to be repo short ($z^i < 0$), and j to be repo long ($z^j > 0$).

Being short in repo, agent i can potentially transfer consumption from state D to state U, which gives him comparatively more utility. In other words, agent i is an optimist with regard to this security (as he puts more weight in the state where the security pays more) and this suggests that he should be long in the security and leverage his long position by being short in repo. However, taking a short repo position is not guaranteed to increase his consumption in state U, or to yield an increase in overall utility, since this depends on the agent's counterparty effective repayment rate in state U (η^j_U).

We cannot completely rule out agent i taking a long repo position even though this would transfer consumption from a high utility state to a state with low utility. The reason for this is

[9] In fact, it is easy to see that if $hr > E^iR$ or $hr < E^jR$ both agents will want to take either long or short repo positions, and there cannot be market clearing.

that bankruptcy limits the utility loss in state U and, depending on how large his long repo position is allowed to be, the increase of utility experienced in state D could more than compensate this loss.

Ultimately, whether agent i takes a long or short repo position will depend on his endowments on each state and in how much he can leverage his position. If leverage is low enough as to rule out bankruptcy, then i necessarily takes a short position ($z^i < 0$).

Starting with the assumption that i is short in repo, his position will be determined by the box constraint. In fact, as the utility function is linear, agent i will pick the largest possible short repo position. Since we know that $x^i_s \geq (1-\beta)\omega^i_s > 0$ no matter what the portfolio might be, agent i is not constrained in his choice by non negativity of x^i_s in any state. It is just the box that determines what that largest short repo position is. This is $z^i = -\frac{o^i}{1-h} = -\frac{1}{1-h}$.

Market clearing requires that if agent i is short in repo, agent j must be long. Again, the linearity of utilities requires that j takes the maximum position that he can in the repo market. We know this position to be $z^j = \frac{o^j}{1-h} = \frac{1}{1-h}$.

With these repo positions, agents i and j are solvent in states U and D, respectively. In fact, in state U agent i has a non-negative financial income: $(o^i + (1-h)z^i)R_U + \eta^j_U[(hr - R_U)z^i]^+ - [(hr - R_U)z^i]^- = (o^i + (1-h)z^i)R_U + \eta^j_U(R_U - hr)|z^i| \geq 0 > -\beta\omega^i_U$. Therefore, agent i does not become insolvent, actually makes $x^i_U \geq \omega^i_U$ (and analogously for agent j in state D).

However, agent i is decreasing consumption in state D and we cannot be sure of his solvency in that state. The same applies for agent j in state U. If we let $\alpha^i_s = 1$ iff agent i is solvent in state s and $\alpha^i_s = 0$ when he declares bankruptcy, we have four possible cases to consider:

Case	α^i_U	α^i_D	α^j_U	α^j_D
1	1	0	0	1
2	1	0	1	1
3	1	1	0	1
4	1	1	1	1

It will be useful to denote by z^s the position of the short repo agent and z^l the position of the long repo agent. We have argued that the short agent (whoever he is) will be solvent in state U while the long agent will be solvent in state D.

Next we note that whether an agent goes bankrupt or not in a certain state depends entirely on how the agent's obligation in that state compares with the garnishable portion of his income. For a given β and h we can compute the (gross) repo rate that equalizes the two and that we denote r^s for the short agent and r^l for the long agent. In the case of the short agent we have:

$$r^s = \frac{R_D}{h} + \frac{1-h}{h} \cdot \frac{\beta \omega_D^s}{o^s}$$

If $r < r^s$ we have $\alpha_D^s = 1$ and, if $r > r^s$, $\alpha_D^s = 0$.

For the long agent we have that:

$$r^l = \frac{R_U}{h} - \frac{1-h}{h} \cdot \frac{\beta \omega_U^l + 2o^l R_U}{o^l}$$

If $r < r^l$ we have $\alpha_U^l = 0$ and, if $r > r^l$, $\alpha_U^l = 1$.

Suppose that the parameters of the model are such that $R_D/h < r^s < r^l < R_U/h$. If we consider a given haircut and for a fixed β we have that depending on the repo rate, bankruptcy coefficients are necessarily as follow:

Case	α_U^s	α_D^s	α_U^l	α_D^l
$r < r^s$	1	1	0	1
$r^s < r < r^l$	1	0	0	1
$r^l < r$	1	0	1	1

Now, for each r, we can compute the consumption of both short and long agents. For the short agent we have:

$$x_U^s = \omega_U^s + \alpha_U^l (hr - R_U) z^s + (1 - \alpha_U^l)[\beta \omega_U^l + 2o^l R_U] \tag{8}$$

$$x_D^s = \omega_D^s + \max\{-\beta \omega_D^s, (hr - R_D) z^s\} \tag{9}$$

In (8) we have written $\alpha_U^l (hr - R_U) z^s + (1 - \alpha_U^l)[\beta \omega_U^l + 2o^l R_U]$ instead of $\eta_U^l [(hr - R_U) z^s]^+$. The two terms coincide because $\eta_U^l = \alpha_U^l$ when the long agent is solvent in state U and, when the long agent is insolvent, we have that $\beta\omega_U^l + 2o^l R_U = -\eta_U^l(hr - R_U)z_U^l$. From market clearing we have that $z^l = -z^s$, so that:

$$\eta_U^l[(hr - R_U)z^s]^+ = \eta_U^l(hr - R_U)z^s = -\frac{\beta\omega_U^l + 2o^l R_U}{(hr - R_U)z_U^l}(hr - R_U)z^s$$
$$= \frac{[\beta\omega_U^l + 2o^l R_U]z^s}{z^s} = \beta\omega_U^l + 2o^l R_U$$

Analogously, we can write the consumption of the long agent as:

$$x_U^l = \omega_U^l + \max\{-\beta\omega_U^l, 2o^l R^U + (hr - R_U)z^l\} \tag{10}$$

$$x_D^l = \omega_D^l + 2o^l R_D + \alpha_D^s(hr - R_D)z^l + (1 - \alpha_D^s)\beta\omega_D^s \tag{11}$$

From this consumption for the short and long agent, we can compute their respective utilities for a given value of r. The final step to confirm that consumption plans and portfolios correspond to an equilibrium is to check for optimality. This is done by comparing agents' utilities with the levels of utility they would attain by taking the opposite action (e.g. a short agent deciding to take a long position instead) *while considering the choice of the other agent as given.* That is, the short agent must compare his utility with the utility he would get if he chose the portfolio $z^{sl} > 0$ instead. The consumption implied by this portfolio would be given by:

$$x_U^{sl} = \omega_U^s + \max\{-\beta\omega_U^s, 2o^s R_U + (hr - R_U)z^{sl}\} \tag{12}$$

$$x_D^{sl} = \omega_D^s + 2o^s R_D + (hr - R_D)z^{sl} \tag{13}$$

Note that in (12), even though the (long) counterparty might be insolvent in state U, this does not affect consumption of the short agent because now, when he is also taking a long position, the term $(hr - R_U)z^{sl}$ constitutes an obligation for the agent and the repayment rate of his counterparty is irrelevant.

The long agent must also compare his utility with what he would get if he chose the short position $z^{ls} < 0$ instead. In this case his consumption would be given by:

$$x_U^{ls} = \omega_U^l + (hr - R_U)z^{ls} \tag{14}$$

$$x_D^{ls} = \omega_D^l + \max\{-\beta\omega_D^l, (hr - R_D)z^{ls}\} \tag{15}$$

We have that the original consumption plans are optimal (and we have an equilibrium) if it is true that $U^s(x_U^s, x_D^s) \geq U^s(x_U^{sl}, x_D^{sl})$ **and** $U^l(x_U^l, x_D^l) \geq U^l(x_U^{ls}, x_D^{ls})$.

We can for example, study an economy with initial parameters:

$$\begin{array}{lllll} \beta = 0.35 & R_U = 1.4 & \omega_U^i = 4 & \omega_U^j = 6 & a^i = 0.9 \\ h = 0.9 & R_D = 0.1 & \omega_D^i = 2 & \omega_D^j = 4 & a^j = 0.2 \end{array}$$

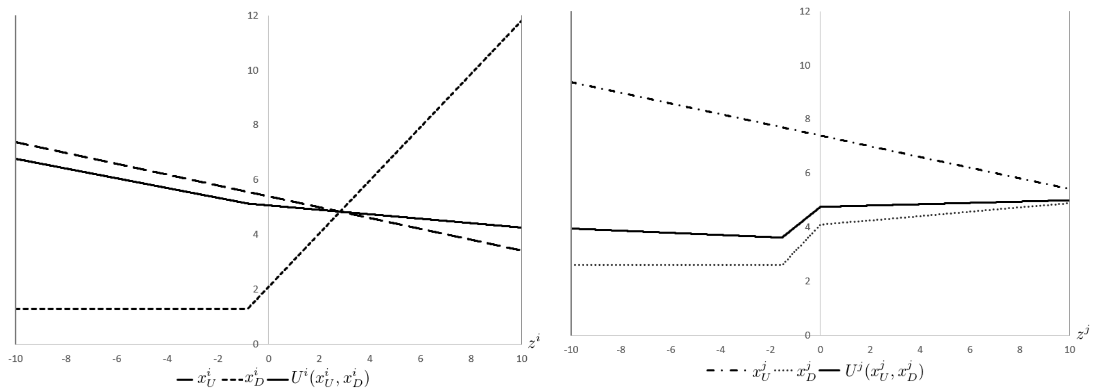
The following are equilibria in which i is repo short and j is repo long for this economy:

r	α_D^i	α_U^j	x_U^i	x_D^i	x_U^j	x_D^j	U^i	U^j
0.7755	0	0	8.9	1.3	3.9	4.9	8.14	4.7
0.8044	0	0	8.9	1.3	3.9	4.9	8.14	4.7
0.8333	0	0	8.9	1.3	3.9	4.9	8.14	4.7
0.8622	0	0	8.9	1.3	3.9	4.9	8.14	4.7
0.8911	0	0	8.9	1.3	3.9	4.9	8.14	4.7
0.9200	0	0	8.9	1.3	3.9	4.9	8.14	4.7
0.9488	0	0	8.9	1.3	3.9	4.9	8.14	4.7
0.9777	0	0	8.9	1.3	3.9	4.9	8.14	4.7
1.0066	0	0	8.9	1.3	3.9	4.9	8.14	4.7
1.0355	0	1	8.68	1.3	4.12	4.9	7.942	4.744
1.0644	0	1	8.42	1.3	4.38	4.9	7.708	4.796
1.0933	0	1	8.16	1.3	4.64	4.9	7.474	4.848
1.1222	0	1	7.9	1.3	4.9	4.9	7.24	4.9
1.1511	0	1	7.64	1.3	5.16	4.9	7.006	4.952
1.1800	0	1	7.38	1.3	5.42	4.9	6.772	5.004
1.2088	0	1	7.12	1.3	5.68	4.9	6.538	5.056
1.2377	0	1	6.86	1.3	5.94	4.9	6.304	5.108
1.2666	0	1	6.6	1.3	6.2	4.9	6.07	5.16
1.2955	0	1	6.34	1.3	6.46	4.9	5.836	5.212
1.3244	0	1	6.08	1.3	6.72	4.9	5.602	5.264

Notably, there are no equilibria corresponding to cases 3 or 4 in this economy. We can compute the exact values of r^s and r^l:

$$r^s = \frac{R_D}{h} + \frac{1-h}{h} \cdot \frac{\beta \omega_D^s}{o^s} = \frac{0.1}{0.9} + \frac{0.1}{0.9} \cdot \frac{0.35 \cdot 2}{1} = 0.1888$$
$$r^l = \frac{R_U}{h} - \frac{1-h}{h} \cdot \frac{\beta \omega_U^l + 2o^l R_U}{o^l} = \frac{1.4}{0.9} - \frac{0.1}{0.9} \cdot \frac{0.35 \cdot 6 + 2 \cdot 1.4}{1} = 1.0111$$

Figure 3 shows agents i and j's problems when the (gross) repo rate is 1.18 and clearly show that it is optimal for i to be short in repo (as much as the box constraint allows him) and for agent j it is optimal to take the highest long position that he can. The Figure shows consumption in states U and D for each agent. Sometimes, as with x^i_D, a kink occurs in the agents consumption at the point where z^i is such that the agents obligations equal his garnishable income and the agent is indifferent between being solvent or declaring bankruptcy. In other cases, as for x^i_U, no kink is observed. This is because the repo position that equalizes obligations and garnishable income occurs outside the interval that constraints repo positions. In this case, z^i at which the kink would occur is $z^i = 14.14$, which is the portfolio that satisfies the condition $-\beta\omega^i_U = (o^i + (1-h)z^i)R_U + (hr - R_U)z^i$. Similarly, for x^j_U the kink where j is marginally solvent occurs for $z^j = 15.9$, also beyond the upper bound on z^j. In the case of x^j_D, two kinks are observed. The one to the left corresponds to the repo position that makes the agent indifferent between being solvent or not. The one at $z^j = 0$ occurs because when $z^j < 0$, the agent is a debtor in state D (meaning that $(hr - R_D)z^j < 0$) and so he is not affected by i's repayment rate $\eta^i_D < 1$. When $z^j > 0$, he is a net creditor, is affected by i's repayment rate (meaning that his income is $\eta^i_D(hr - R_D)z^j$ instead of $(hr - R_D)z^j$) and this reduces the slope of x^j_D as a funtion of z^j.



(a) Agent i's problem.

(b) Agent j's problem.

Figure 3: How consumption and utility at the second date relate to repo positions, when $r = 1.18$.

As is clear from the previous discussion, for a given set of parameters, there are multiple equilibria. Regardless of this indeterminacy, we have two conditions that must be satisfied in any equilibrium:

$$\eta_D^i = \frac{\beta\omega_D^i(1-h)}{(rh-R_D)}, \quad \eta_U^j = -\frac{[\beta\omega_U^j + 2R_U](1-h)}{(rh-R_U)} \tag{16}$$

These equations suggest that $\frac{\partial \eta_D^i}{\partial h} = \frac{\beta\omega_D^i(R_D-r)}{(rh-R_D)^2}$. For the equilibria we have presented here we have $\frac{\partial h}{\partial \eta_D^i} < 0$, and $\frac{\partial h}{\partial \eta_U^j} < 0$.

Figure 4 shows how h is related to the equilibrium values of η_D^i and η_U^j, for h ranging from 0.85 to 0.99.

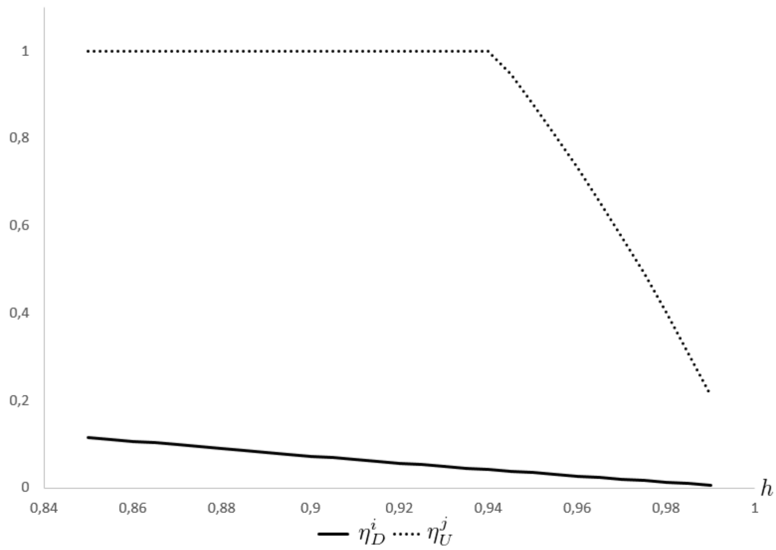


Figure 4: How η_D^i and η_U^j relate to h when $r = 1.18$.

Observe that haicuts move together with the counterparty's repayment rate (this must always happen in this 2-agent and 1-security economy. As we will see in section 4, in a competitive setting, where many agents trade many securities, the impact of the haircut in one security on the insolvency of an agent becomes less noticeable. It is the other direction that becomes more relevant: haircuts rise in response to lower repayment rates of the counterparty, for securities that involve a risk from the creditor's point of view (have a collateral liquidation value below the promised repo loan settlement). That is, in a competitive setting, creditors tend to focus on how to protect their individual credits rather than trying to influence the solvency of the counterparty.

4 Haircuts

Haircuts in pairwise repo trades are endogenously determined in the equilibrium that we defined. Existence was established and characterized for the 2-agent and 1-security case for a set of given parameters. We discuss now what may govern haircuts, that is, how should we expect haircuts to be set in equilibrium, depending on what are the parameters and other equilibrium variables for the relevant pair of repo traders.

Suppose agent i has a possession value for security f at the initial node, that is, a binding box constraint for security f at the initial node - more precisely, the shadow value μ^i_{f0} of this constraint is positive. Denoting by λ^i_e agent i's multiplier for the budget constraint at each node e and ν^{ij}_f the multiplier for the lower bound on repo positions value, from the first order conditions for agent i's problem, we get the following expression for h^{ij}_f

$$h^{ij}_f = \frac{\tau^{ij}_f(p,q) + \frac{\nu^{ij}_f}{\lambda^i_0}}{1 - r_f \sum_s \frac{\lambda^i_s}{\lambda^i_0}\alpha^i_s \kappa^{ij}_s} \tag{17}$$

where

$$\tau^{ij}_f(p,q) = \begin{cases} \sum_s \frac{\lambda^i_s}{\lambda^i_0}\alpha^i_s(1-\kappa^{ij}_s)\frac{(q_{fs}+p_s R_{fs})}{q_{f0}}, & \text{if } \mu^i_{f0} = 0 \\ 1 - \sum_s \frac{\lambda^i_s}{\lambda^i_0}\alpha^i_s \kappa^{ij}_s \frac{(q_{fs}+p_s R_{fs})}{q_{f0}}, & \text{if } \mu^i_{f0} > 0 \end{cases}$$

and $\kappa^{ij}_s = \gamma^{ij}_s \eta^j_s + (1-\gamma^{ij}_s)$, $\gamma^{ij}_s = 1$ if $I^{ij}_s > 0$, $\gamma^{ij}_s = 0$ if $I^{ij}_s < 0$, $\alpha^i_s = 1$ if agent i is solvent in state s, $\alpha^i_s = 0$ if i goes bankrupt in state s, and η^i_s satisfies (5).

It is worth recalling that η^j_s is the effective percentage of his debt that agent j pays to all of his counterparties, so it can be used as a measure of counterparty risk: the lower η^j_s is, the riskier (or less solvent) agent j is in state s, and this must be taking into account by agents deciding having j and counterparty and, in particular, in setting the terms of repo contracts (h^{ij}_f).

Equation (17) is true in any equilibrium and can be used to study *the incentives* that counterparties i and j have to either increase or decrease the haircut $(1 - h^{ij}_f)$ associated to their repo transactions in response to an increase in the risk of one of the counterparties. Let's look at the derivative of h^{ij}_f with respect to η^j_D, under the assumption that agents' marginal rates of income substitution remain unchanged. To be more precise, *Assumption* (Λ)*:* agent i's marginal rates of substitution of income across the first two dates, λ_s^i/λ_0^i, are not affected by a change in the counterparty j's effective repayment rate η_D^j.

Although we might not want to take this assumption literally, it is useful to get a sense of how haircuts move with counterparty risk in a context where agent i is trading in many securities and has many counterparties, so that a small variation in the default rate of one of them in some state won't affect the optimal inter-nodes deflators of agent i.

This assumption holds for linear utilities (recall that the bankruptcy structure ensures the positivity of consumption in each state, which implies that $DU_s^i(x^i) = \lambda_s^i p_s$) in the case of repo to maturity (dispensing with the third date and allowing for $p_s = 1$) and provided that agent i is consuming at the initial date (so that $DU_0^i(x^i) = \lambda_0^i p_0$) and that the equilibrium commodity price p_0 is not affected by a small change in the effective repayment rate η_D^j of counterparty j in state D.

$$\text{(18)} \qquad \frac{\partial h_f^{ij}}{\partial \eta_D^j} = \frac{-\alpha_D^i \gamma_D^{ij} \cdot \frac{1}{q_{f0}} \cdot \frac{\lambda_D^i}{\lambda_0^i} \cdot [(q_{fD} + p_D R_{fD}) - h_f^{ij} q_{f0} r_f] + h_f^{ij} \frac{\partial r_f}{\partial \eta_D^j} \sum_s \frac{\lambda_s^i}{\lambda_0^i} \alpha_s^i \kappa_s^{ij}}{1 - r_f \sum_s \frac{\lambda_s^i}{\lambda_0^i} \alpha_s^i \kappa_s^{ij}}$$

When most of the response to a variation in counterparty risk is channeled into a change in haircuts, rather than a change in the repo rate, we can be more specific about the direction of change. We say that *repo rates are competitive* if actions by a pair of agents i and j, in particular actions that change their solvency rates (η_s^i and η_s^j) do not affect equilibrium repo rates. This is a reasonable assumption if there are many agents (and therefore, many pairs of counterparties) in the economy, but not to be expected in an economy with only two (or very few) agents, as in the example of section 3. We have,

Proposition 1. *Suppose repo rates are competitive. Let us evaluate the impact of η_D^j on h_f^{ij}, under a scenario where agents' marginal rates of substitution are not affected. Say $I_D^{ij} > 0$ (i is a net creditor in the repo market with respect to j) and i is solvent in state D ($\alpha_D^i = \eta_2^i = 1$). If agent j's expected repayment rate η_D^j decreases, agent i will want to:*

- *Increase the haircut $(1 - h_f^{ij})$ he charges (pays) in his repo long (short) positions with agent j, of securities f such that $h_f^{ij} q_{f0} r_f > q_{fD} + p_D R_{fD}$.*

- *Decrease haircuts paid to (charged to) agent j for his short (long) repo positions in securities g such that $h_g^{ij} q_{g0} r_g < q_{gD} + p_D R_{gD}$.*

If $I_D^{ij} < 0$ (i is a net debtor in the repo market with respect to j), or if i is insolvent in state D, he has no incentives to increase or decrease the haircut $(1 - h_f^{ij})$ in response to expected changes in η_D^j.

Remark 1. The last part of the proposition reflects the fact that if i were a net debtor to agent j instead, he would not be entitled to any share in the liquidation of agent j's estate in the event of agent j's bankruptcy. Note that as long as agents i and j trade in the repo market, one of them must be a net creditor and proposition 1 applies to either i or j, as long as the agent is solvent in state D.

Proof. See the appendix. □

Suppose that i is a net creditor with counterparty j, and that $z_f^{ij} > 0$, and that $h_f^{ij} q_{f0} r_f > q_{fD} + p_D R_{fD}$. If agent i anticipates a decrease in j's expected repayment rate, η_D^j, then i would like to charge j a higher haircut (by lowering h_f^{ij}). To understand why this is so, note that the magnitude of j's net debt to i, is given by $z_f^{ij}[q_{f0} r_f h_f^{ij} - (q_{fD} + p_D R_{fD})]$ and lowering h_f^{ij} would reduce this debt and, therefore, the loss resulting from agent j's bankruptcy.

Now suppose $h_f^{ij} q_{f0} r_f < q_{fD} + p_D R_{fD}$. If everything else is as in the previous paragraph, a decrease in η_D^j will be an incentive for i to collect a lower haircut from j (by raising h_f^{ij}). Even though agent i is a net creditor to agent j when adding up all of his repo transactions with j, he has now a debt to j associate to his position on security f with absolute value $z_f^{ij}[(q_{fD} + p_D R_{fD}) - q_{f0} r_f h_f^{ij}]$. That is, the collateral kept by i when lending cash to j has now a higher market value than what j owes to i. If h_f^{ij} increases, he gets to keep more of the collateral in the event of j's bankruptcy.

In both cases, i has incentives to respond in a way that counteracts the loss in income when j becomes more insolvent in state D. The appropriate response depends on the relationship of the value of j's debt $(q_{f0} r_f h_f^{ij})$ with the market value of the collateral $(q_{fD} + p_D R_{fD})$. This relationship is what is usually understood as *asset risk*, the risk is precisely that the value of the collateral might decrease in some future date and become insufficient to cover the value of the debt that it is backing. In practice, haircuts are set so that it is expected that[10] $h_f^{ij} q_{f0} r_f < (q_{fD} + p_D R_{fD})$. This should be considered the most relevant case in the context the proposition.

Proposition 1 is useful to understand agents' incentives when a perceived increase in counterparty risk occurs. It should not be interpreted as a comparative statics analysis. One should expect to observe different haircuts but also different repo positions in equilibria with different repayment rates. One could expect that even the roles of net creditor and net debtor might get reversed when repayment rates change.

5 Concluding remarks

Repo markets have attracted a lot of attention in the applied macro and finance literatures, particularly since the 2008 financial crisis. These literatures have focused on how leverage in these markets has impacted on the whole economy, how this leverage depends on repo haircuts and what determines these margins. However, at the theoretical level, these important issues had not been addressed yet, possibly due to the complexity and intrinsic non-convexities involved in a bankruptcy model where counterparty risk could be understood and margins could be explained as endogenous variables.

We contribute in that direction, building a general model, exhibiting concrete examples of bankruptcy equilibrium for a simple economy, and characterizing how counterparty and asset risks interact to determine repo haircuts. Full recognition of the recourse nature of repo loans is crucial. This implies also that default must be modelled in terms of insolvency and according to bankruptcy rules. The model can be made richer, incorporating more detailed institutional aspects, or allowing also for centrally cleared repo, but the main drivers of margins seem to identifiable already in a simple model. Competitiveness versus small numbers of traders or asset

[10]Or if the value of the collateral decreases and doesn't cover the value of the debt, a margin call is issued by the lender of funds.

risk in a creditor's perspective (suggesting default tensions) versus a debtor's risk perspective (concerned with repo fails instead) are some of the key issues in assessing how counterparty risk impacts on repo haircuts and the resulting leverage.

Appendix

Proof of Proposition 1. From (18), if r_f is competitive, the derivative of h_f^{ij} with respect to η_s^j reduces to:

$$\frac{\partial h_f^{ij}}{\partial \eta_D^j} = -\alpha_D^i \gamma_D^{ij} \cdot \frac{1}{q_{f0}} \cdot \frac{\lambda_D^i}{\lambda_0^i} \cdot \frac{(q_{fD} + p_D R_{fD}) - h_f^{ij} q_{f0} r_f}{1 - r_f \left[\frac{\lambda_U^i}{\lambda_0^i}\alpha_U^i \kappa_U^{ij} + \frac{\lambda_D^i}{\lambda_0^i}\alpha_D^i \kappa_D^{ij}\right]}$$

Moreover, we have that $\frac{dh_f^{ij}}{d\eta_s^j}$ has the same sign as $[h_f^{ij} q_{f0} r_f - (q_{fs} + p_s R_{fs})]$ and $\frac{d(1-h_f^{ij})}{d\eta_s^j}$ has the same sign as $[(q_{fs} + p_s R_{fs}) - h_f^{ij} q_{f0} r_f]$. To see that this is the case, look at the first order condition of agent i's problem with respect to z_f^{ij}:

$$r_f = \frac{\lambda_0^i}{\sum_s \lambda_s^i \alpha_s^i \kappa_s^{ij}} - \frac{1}{q_{f0} h_f^{ij}} \cdot \frac{\sum_s \lambda_s^i \alpha_s^i [(1-\kappa_s^{ij})(q_{fs} + p_s R_{fs})]}{\sum_s \lambda_s^i \alpha_s^i \kappa_s^{ij}} - \frac{1}{h_f^{ij}} \cdot \frac{(\mu_{f0}^i / q_{f0}) + \nu_f^{ij}}{\sum_s \lambda_s^i \alpha_s^i \kappa_s^{ij}} \tag{19}$$

Lets focus on the term in the middle of the right hand side of (19):

$$\frac{\lambda_U^i \alpha_U^i [(1-\kappa_U^{ij})(q_{fU} + p_U R_{fU})] + \lambda_D^i \alpha_D^i [(1-\kappa_D^{ij})(q_{fD} + p_D R_{fD})]}{\lambda_U^i \alpha_U^i \kappa_U^{ij} + \lambda_D^i \alpha_D^i \kappa_D^{ij}} \tag{20}$$

In an equilibrium as the one which existence we have proven, we have $\alpha_U^i = 1$. The theorem assumes that i is a net creditor so we have $\gamma_D^{ij} = 1$. We have that agent j is solvent in state U. We must have $\kappa_U^{ij} = 1$. We have assumed that i is solvent in state D, so that $\alpha_D^i = 1$, we can suppose that $\eta_D^j \in (0,1)$ (so that j was risky to begin with and so that it makes sense to take the derivative with respect to η_D^j which belongs in the set $[0,1]$). From all these considerations, we have that (20) is actually positive:

$$\frac{\lambda_D^i [(1-\eta_D^j)(q_{fD} + p_D R_{fD})]}{\lambda_U^i + \lambda_D^i \eta_D^j} > 0 \tag{21}$$

We can conclude from equation (19) that $r_f < \lambda_0^i / \sum_s \lambda_s^i \alpha_s^i \kappa_s^{ij}$ as long as at least one of the following conditions holds:

(a) Agent i values the possession of security f ($\mu_{f0}^i > 0$).

(b) Agent i's position z_f^{ij} has a market value at the lower bound.

(c) Counterparty j is *somewhat* risky in state D ($\eta_D^j \in (0,1)$).

When at least one of the conditions is satisfied we have $r_f \sum_s \frac{\lambda_s^i}{\lambda_0^i} \alpha_s^i \kappa_s^{ij} < 1$. □

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