DEPARTMENT OF MATHMATICS

IBERIAN ENERGY MARKET: SPOT PRICE FORECAST BY MODELLING MARKET OFFERS

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Master/BSc in Physics Engineering

DOCTORATE IN PHD DEGREE IN STATISTICS AND RISK MANAGEMENT

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Para a Vanessa, Afonso e Alexandre.

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"Go as far as you can see; when you get there, you'll be able to see further." (Thomas Carlyle)

Abstract

Electricity is a very special commodity since it is economically non-storable, and thus requiring a constant balance between production and consumption. At the corporate level, electricity price forecasts have become a fundamental input to energy companies' decision making mechanisms [22, 45]. Electric utilities are higly vulnerable to economical crisis, since they generally cannot pass their excess costs on the wholesale market to the retail consumers [77] and, since the price depends on variables like weather (temperature, wind speed, precipitation, etc.) and the intensity of business and everyday activities (on-peak vs. off-peak hours, weekdays vs. weekends, holidays and near-holidays, etc.) it shows specific dynamics not observed in any other market, exhibiting seasonality at the daily, weekly and annual levels, and abrupt, short-lived and generally unanticipated price spikes.

These extreme price volatility make price forecasts from a few hours to a few months ahead to become of particular interest to power portfolio managers. An utility company or large industrial consumer who is able to accurately forecast the wholesale prices and it's volatility, can adjust its bidding strategy and its own production/consumption schedule in order to reduce the risk or maximize the profits in day-ahead trading.

In this work I discuss the dynamics of the Iberian electricity day-ahead market (OMIE), review the state-of-the-art forecasting techniques and introduce a new approach to Electricity Price Forecasting, by forecasting the underlying dynamics, the market demand/supply curves. With this method it is possible to predict not only the electricity prices for the next hours, but also the market curves, which can then be used for risk management and a more accurate schedule of generation units. I analyze the model results and benchmark them against other models in the industry.

Keywords: Electricity Price Forecasting, Market Curves, Electricity Price, OMIE, Vector Autoregression

Resumo

A eletricidade é uma *commodity* muito especial, uma vez que não é possível armazená-la, e por isso, requer um constante equilíbrio entre a produção e consumo. ao nível empresarial, a previsão de preços de eletricidade tornou-se um *input* fundamental para os mecanismos de tomada de decisão das companhias [22, 45]. As empresas de eletricidade são altamente vulneráveis a crises económicas, uma vez que, em geral, não conseguem passar os seus custos excessivos para o mercado retalhista [77] e, uma vez que o preço depende de variáveis como meteorologia (temperatura, velocidade do vento, precipitação, etc.) e da intensidade de negócio e das atividades do dia-a-dia (pico vs vazio, dias da semana vs fim-de-semana, feriados e pontes, etc.) apresenta uma dinâmica que não é observada em mais nenhum mercado, com sazonalidade diária, semanal e anual, e com picos de preço abruptos de pouca duração e, em termos gerais, impossíveis de antecipar.

Esta volatilidade de preços torna a previsão de preços particularmente interessante para gestores de portfólio, seja a curto ou a longo prazo. Uma companhia de eletricidade ou grande consumidor industrial que seja capaz de prever corretamente os preços do mercado grossista e a sua volatilidade, pode ajustar a estratégia de oferta da sua produção/ seu consumo de maneira a reduzir o risco ou maximizar os ganhos no mercado à vista.

Neste trabalho abordo a dinâmica do mercado de eletricidade ibérico (Operador de Mercado Iberico - Polo Español (OMIE)), revendo o estado da arte dos métodos de previsão de preços de eletricidade, e introduzo uma nova técnica de previsão de preços de eletricidade, através da previsão da sua dinâmica subjacente, as curvas de mercado da procura e oferta. Com este método é possível prever, não só o preço de eletricidade para as próxima shoras, mas também as próprias curvas de oferta, o que pode ser utilizado na gestão de risco ao melhor a capacidade de programar as suas unidades de geração.Os resultados do modelo são analisados e comparados com outros modelos já utilizados na industria.

Palavras-chave: Previsão de Preço de Eletricidade, curvas de mercado, preço de eletricidade, OMIE, Vector Autoregression

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Acronyms

ACF Autocorrelation Factors *(p. 50)*

ADF Augmented Dickey-Fuller Test *(p. 48)*

AIC Akaike Information Criterion *(pp. 45, 48)*

ANFIS Adaptive-Network-Based Fuzzy Inference System *(p. 23)*

ANN Artificial Neural Networks *(pp. 20–25)*

ARIMA Autoregressive Integrated Moving Average *(pp. 15–18, 21–23, 25, 26)*

ARMA Autoregressive Moving Average *(pp. 16, 17, 25, 39)*

ARMAX Autoregressive Moving Average Exogenous *(p. 17)*

BIC Bayesian Information Criterion *(pp. 45, 48)*

CNN Convolutional Neural Network *(pp. 23, 24)*

DL Deep Learning *(pp. 23, 24)*

DNN Deep Neural Network *(pp. 20, 23)*

EEX European Energy Exchange *(p. 17)*

ELM Extreme Learning Machines *(pp. 50, 52, 53)*

EPF Electricity Price Forecast *(pp. 8, 10, 12–15, 17, 18, 20–22, 24–26, 28, 29, 58, 59)*

ERCOT Electric Reliability Council of Texas *(p. 19)*

FNN Fuzzy Neural Networks *(pp. 22, 23)*

GARCH Generalized Autoregressive conditional hesteroskedasticity *(pp. 15, 18)*

ISO Independent System Operator *(p. 7)*

LASSO Least Absolute Shrinkage and Selection Operator *(pp. 15, 24)*

LSTM Long Short-Term Memory *(pp. 23–25)*

MAE Mean Absolute Error *(pp. xiv, 11, 35, 48, 51, 54, 55)*

MAPE Mean Absolute Percentage Error *(pp. 11, 12)*

MCP Market Clear Price *(pp. 6, 7, 9, 37, 52)*

MIBEL Mercado Ibérico de Eletricidade *(pp. 2, 5)*

ML Maximum Likelihood *(p. 44)*

MLP Multi-Layer Perceptron *(pp. 21, 23)*

NORDPOOL Nordic POOL *(p. 18)*

NYPOOL New York Pool *(p. 19)*

OLS Ordinary Least Squares *(p. 44)*

OMIE Operador de Mercado Iberico - Polo Español *(pp. ix, 3, 32)*

OMIP Operador de Mercado Iberico - Polo Português *(p. 3)*

PABA Pay-As-Bid Auctions *(p. 5)*

PACF Partial Autocorrelation Factors *(p. 50)*

PI Prediction Interval *(p. 10)*

PJM Pennsylvania-New Jersey-Maryland Interconnection *(p. 19)*

RBF Radial Basis Function *(pp. 18, 21, 23, 25)*

RCGA Real-Coded Genetic Algorithm *(p. 21)*

RMSE Root Mean Square Error *(p. 12)*

RNN Recursive Neural Network *(p. 23)*

RSS Residual Sum of Squares *(p. 15)*

SARIMA Seasonal Autoregressive Integrated Moving Average *(pp. 16–18, 24, 39)*

SVARIMA Support Vector Autoregressive Integrated Moving Average *(p. 22)*

SVM Support Vector Machine *(pp. 20, 22)*

UPA Uniform Price Auctions *(pp. 5–7)*

VAR Vector Autoregression *(pp. 43, 44, 48–50)*

WMAE Weighted Mean Absolute Error *(pp. 12, 48, 51)*

XGBOOST Extreme Gradient Boosting *(pp. 27, 50, 52–55)*

Symbols

B	backward shift operator *(p. 16)*
$b_{i,h}$	block offer i at time h *(p. 34)*
$\mathcal{C}$	set of curves *(p. 30)*
$d(x)$	demand market curve *(p. 29)*
ϵ_h	error term *(p. 14)*
γ_k	autocovariance function of a time process Z_t *(p. 40)*
h	instant of time *(p. 11)*
$\hat{P}_h$	predicted price at instant h *(p. 11)*
$\mathbf{l}$	latent variables of a given curve *(p. 29)*
MWh	unit of energy *(p. 6)*
ω	variable of the frequency domain *(p. 40)*
P_h	actual price at instant h *(p. 11)*
ϕ	autoregressive parameters *(p. 16)*

$s(x)$	supply market curve *(p. 29)*
T	period of time *(p. 11)*
θ	moving average parameters *(p. 16)*
$V_{i,h}$	energy of the block $b_{i,h}$ *(p. 34)*
$\mathbf{X}_h$	exogenous variable measures at instant h *(p. 14)*
Z_t	time series process *(p. 40)*

1

The Iberian Electricity Market

The energy markets are commodity markets that deal exclusively with the supply and trade of energy, which can refer to electricity or any other source of energy. Although each commodity is managed and dealt in its own market, they are highly connected and have many similarities, leading the industry to believe that we are in the presence of a bigger and much complex unique system [79].

This becomes clear when thinking that all commodities are used to meet the same fundamental needs: heat, light and transport. Electricity, in addition to meeting these needs, is also used to maintain and support the information infrastructure in which the modern society is built on. The energy commodities compete as substitutes for each other, while at the same time, are important sources of production of other energy commodities. The proportions in which each energy commodity is used vary from location to location and over time, depending on their availability and production costs. The final energy mix is the result of interactions between market forces and political processes, and one has to understand the entire dynamic system.

Electricity stands out from other commodities because it has specific physical and economical characteristics. You can't see, smell or touch it, and you can't even guarantee it's motion from one point to another in the same way as other commodities all because of the complex physics laws that govern it. It is economically non-storable and the power system requires a constant balance between production and consumption. At the same time, its demand and production is dependent on many changeable variables, like temperature, wind-speed or solar radiation, or even the hours of the day or the days of the week where business and everyday activities are more intense. These unique characteristics lead to a very volatile market of high importance for governments, industry and consumers, with price dynamics that are not observable in any other market.

1.1 Liberalisation of Energy Markets

A strong drive for liberalisation of the energy markets, specifically electricity markets, started at the end of the millennium, with purpose of reducing prices and increasing competitiveness between parties [74]. In addition to this assumed advantages, this market paradigm had other strengths: it lowered the risk of over-investment and possible price disparities; but, on the other hand, one can find shortcomings for this approach: the domestic consumer can be disadvantaged, as the costs of supply can be easily passed onto them; and absence of incentives for technological innovation or new investments on generation capacity. Due to this limitations on the liberalised markets, there are still four prime justification for regulatory intervention [130]:

1. Significant economies of scale and scope that favor large integrated utilities
2. The natural monopoly character of electricity transport and distribution
3. Segments of the electricity value chain that are regarded as indivisible
4. The belief that private investment will not occur without long-term hedging.

In the European Union, the liberalisation of the many electricity markets over Europe has been directed by the European Comission directives promulgated in 1996, 2003, 2009 [44, 42, 43]. They aimed at a gradual market opening, supported by the increasing interconnection capacity between the different electricity markets, that would eventually lead to a common electricity market in Europe. For the countries of the Iberian Peninsula it was no different, there was a phased and meticulously prepared plan for the liberalization of the market that led to the signing of a collaboration protocol in 2001 between the Republic of Portugal and the Kingdom of Spain for the creation for the Iberian electricity market (Mercado Ibérico de Eletricidade (MIBEL)), with its effective start on 2007.

1.2 The Iberian Case

As already stated, the creation of the MIBEL, on the 1st of july 2007, was the product of the cooperation between the administrations of Portugal and Spain with the integration of the electrical systems of both countries as the main objective[105].

In the process of liberalisation the activities related to network activities, transport and distribution, were considered natural monopolies, meaning that, because of the high fixed costs involved, the existence of only one supplier can prove more efficient than two or more, leading to less costs [78, 120]. Because of this they are object of economic regulation, leaving only the production and commercialization open to competition, leading to the creation of organized markets where this activities could relate.

The production of electricity is associated with the existence of a wholesale market, where every producer can place its energy and every supplier can supply itself with the energy needed to provide to final client. On the other hand, the commercialization is associated with the existence of a retail market where every supplier compete to secure a customer base.

Electricity trading in the wholesale market can take many forms, from trading for the day-ahead, or closer to real time with intraday trading, to longer term trading, through the forward market or through bilateral contracts. All these legal mechanisms for trading on the organized market led to the creation of two distinct divisions, one for managing the derivate market (Operador de Mercado Iberico - Polo Português (OMIP), Portuguese division) and the other for managing the day-ahead and intraday market (Operador de Mercado Iberico - Polo Español (OMIE), Spanish division). The markets are managed with respect to transmission constraints between both countries and in case of congestions, the single market is split into submarkets [23].

1.2.1 The day-ahead market

The liberalisation and reorganization of the electric sector changed the way generation companies and supplier companies interact, bringing about the need for regulated markets. As already stated, one of the mechanism that was created are spot markets, know as pool markets (because it includes all the producers and suppliers of the system), with the primary objective of achieving the balance between production and demand. The day-ahead markets are the main object of the spot-markets, named due to take place on the day before the scheduled delivery. They help optimizing the system on the short term by facilitating generation companies to correctly schedule their units, specifically their slower-start units [131, 31].

Startup is costly, and the value of committing (starting) a unit of generation depends on the cost of power produced by many other producers. The theory of perfect competition assumes that there is a price that clears the market when suppliers take that price as given [59, 101]. In this case a competitive market is efficient, but if production costs are nonconvex, there may be no market clearing price, and when there is none, economic theory does not guarantee efficiency. Startup costs and no-load costs are nonconvex and some constraints on generation output cause similar problems.

A market clearing price is a single price that causes supply to equal demand. On electricity markets this might not happen, since start-up costs are high and mostly fixed, there might not be one price at which the demand is equalled, if only a small amount of energy, in volume or in time, is needed, it might not be possible for generators to sell energy without incurring in losses. In fact, this costs also depend on the other generation companies, since it's cheaper for only one producer to meet the demand than it is for two producers, along these lines, it is cheaper for a producer to produce for two hours than it is to produce for one hour, since it can spread the start-up costs for a longer time, and this is where the nonconvexity arises [131].

The design of the day-ahead market aims to minimize the inefficiency that comes with nonconvex costs as it can provide a reliable dispatch, increasing reliability and minimizing the risk for producers. Since this is a pool market where every producer is included, the percentage of slower-start units is small, reducing the nonconvexity of the problem and allows for small corrections in the closer to real-time markets, it also helps minimizing the risk since the market firms the operation for the next day, reducing the uncertainty around the operation between consequent time intervals.

In addition to the presented arguments, the day-ahead market also has the benefits of providing market signals to demand-side management providers (encouraging greater participation), reducing price volatility when compared to a real-time market and increasing liquidity by creating an arbitrage opportunity with other markets.

1.2.2 Why auctions?

As practical economic approaches, the auctions are used in electricity markets trading. There has been a growing interest in the use of auctions in the electricity industry as a way to promote efficient procurement and foster competition in all sectors: generation, transmission, and distribution. Throughout the world, auctions have been employed in diverse settings ranging from the hourly dispatch of generators in day-ahead markets to long-term contracts for the concession rights to build and operate hydroelectric, wind or solar plants plants or transmission assets. They were introduced in electric systems during the liberalisation of the industry as they are very efficient and have low costs and could easily be implemented by independent system operators as a dispatch process for the day-ahead markets[102, 114].

The auction leads to awarding some participants attend a financial program and compete according to market rules. Any auction is based on three fundamentals, (1) bidding, (2) clearing, and (3) pricing rules. The bidding rules determine the quality and quantity of submitted bids. The clearing rules determine how to settle the market, choose the awarded participants and determine the amount of traded product. The pricing rules determine at what price the production will be traded[102, 114].

Auctions have been used extensively outside of the power sector as a procurement mechanism for many years. While the existence of auctions dates back to 500 B.C. in Greece, its use in network industries has been pioneered by telecommunications companies to assign and trade bandwidth. The gas and oil industries rely on auctions to define prices and allocate exploration rights. In the electricity industry, auctions have been used in different parts of the world as the basis for trading energy and capacity, transmission congestion rights, ancillary services, and other products.

An auction is a transparent mechanism that should achieve a fair, open, and timely procurement process, reducing opportunities for corruption. These are widely sought after features that minimize the likelihood of future challenges to the selection process and its outcome, avoiding post-auction delays.

There are many kinds of auction designs used in electricity markets around the world. The first group is named sealed-bid auctions, including firstprice sealed bid auctions, Uniform Price Auctions (UPA), and Pay-As-Bid Auctions (PABA). The second group is dynamic pricing auctions, in particular, descending clock auctions for electricity markets. The third group is the hybrid auctions containing a combination of the mentioned auctions, e.g., descending clock auction followed by PABA, or firstprice sealed-bid stage followed by an iterative descending auction, and sequential auctions[102].

1.2.3 Uniform price auction

The MIBEL Day-ahead market is based in a UPA. The UPA is a kind of sealed-bid auction that doesn't discriminate the price over the different players and incentives the use of marginal price as the base for the competitive pricing. In UPA, all of the winners would be paid at a single pre-determined price, Market Clear Price (MCP), irrespective of what they had bidded, and the last winner bidder, which is called marginal unit, will get exactly the bidded price.

This auction is a generalization from the Vickrey auction [140] and was introduced by Friedman in the 1960s [49]. The pricing rule and the whole process of clearing the market is simple and straightforward, it is based on the equilibrium between production and demand. The independent system operator aggregates the supply offers and demand bids and arranges them in the form of ascending and descending curves, as shown in Fig. 1.1. The intersection of these two curves will determine the market price and the cleared quantity, which satisfies the aggregated cost of winner sellers and demand of winner buyers, respectively [129].

All sales occur at market price, meaning that every producer and every supplier will be paid the same per MWh. The shaded area in Fig. 1.1 shows the total revenue of winner producers. The upper area of the shaded area represents the surplus of buyers, i. e. the extra income generated because the MCP is higher than their bids.

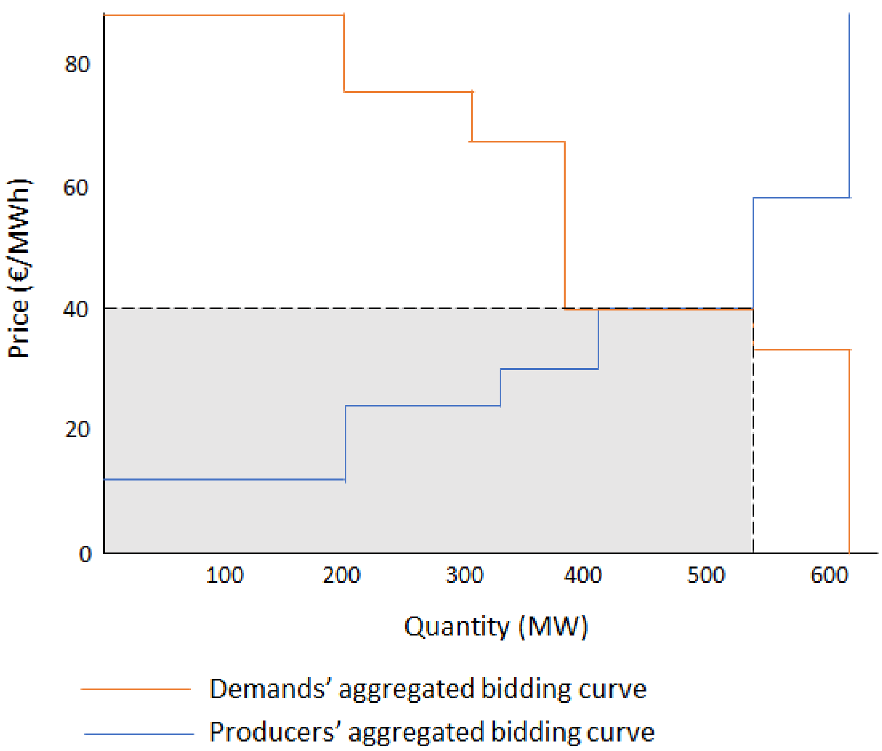

Figure 1.1: Ilustration of a Uniform Price Auction

Although in the UPA, there is only one MCP, the fact that the market consists of two countries, Portugal and Spain, means that there are physical constraints regarding transmission, and, in case of congestions, the market is split into two submarkets. This will lead to two distinct MCP. The zonal pricing scheme divides an electrical network into several zones, where in each zone, the probability of congestion is ignored and the price is unique.

Considering the concept of UPA, the winner sellers will receive the highest accepted price automatically, so there is not a loss for none of the sellers if they had bided their marginal cost [116]. On the other hand, the buyers will pay the least possible value for their traded good. Since the electricity price is the same for all winners whose offers are less than the MCP, the producers tend to be more honest regarding price bidding. If a producer bids at a higher price than its actual marginal price, it might not be cleared and miss out the opportunity of participation in the market; on the other hand, if it bids at a lower price than its actual marginal cost, it may incur in loss. Consequently, bidding their marginal cost is the best strategy for the power plants to assure that they will not suffer economically. It also help guarantee that the Independent System Operator (ISO) will find the least price in which the demand is satisfied. This is the reason why the UPA is well-known as an efficient pricing mechanism. It is stated that the UPA is a transparent pricing mechanism which leads to the efficient selection of the least-cost producers and social welfare maximization. Furthermore, the UPA creates opportunity and incentivizes small producers to take part and take advantage of the higher prices determined by large-scale producers.

However, the probability of gaming still exists, which can reduce the overall efficiency [41]. It is stated by Nazemi and Mashayekhi [112] that only producers that their cost is near to the MCP will exercise market power and have the incentive to bid dishonestly, yet others will not risk losing the market participation opportunity by offering at above of their actual cost. In addition, the existence of subsequent markets, like the intraday-market, creates a chance for arbitrage which also reduce efficiency.

1.3 The importance of Electricity Price Forecasting

In the current deregulated scenario, the forecasting of electricity demand and price has emerged as one of the major research fields [21]. The liberalisation was intended to encourage competition among companies in order to decrease the cost of electricity. Unfortunately, occurrences that were of no concern or rarely happened in the regulated market, such as outages, possible blackouts and price peaks are now subject of increasing concern. Moreover, deregulation increases electricity prices uncertainty, giving an increased role on forecasting.

Precision in forecasting these electricity prices is very important, since a higher accuracy reduces the risk of under (or over) estimating the revenue from the production units for the companies and provides better risk management [15]. Forecast errors have significant implications for profits, market shares and ultimately shareholder value[21]. In actual electricity markets, price curve exhibits considerably richer structure than load curve and has the following characteristics: high frequency, nonconstant mean and variance, multiple seasonality, calendar effect, high level of volatility and high percentage of unusual price movements. These characteristics arise from the properties of the electricity and also by the fact that the market equilibrium is influenced by both load and generation side uncertainties [21]. Therefore, electricity price forecasting is essential for all market participants for their survival. Even accurate load forecasts cannot guarantee profits and the market risk due to trading is considerable because of extreme volatility of electricity prices.

In the short-term, a producer needs to forecast electricity prices to be able to derive its bidding strategy and to optimally schedule its electric energy resources [37]. In a regulated environment, traditional generation scheduling of energy resources was based on cost minimization, satisfying the electricity demand and all operating constraints [47] and therefore, the key issue was how to accurately forecast electricity demand. In a deregulated environment, since generation scheduling of energy resources, such as hydro [26] and thermal resources[104], is now based on profit maximization[48], it is an accurate price forecasting that embodies crucial information for any decision making. Suppliers end up by needing short-term price forecasts exactly for the same reasons as producers.

In the next chapter, I make a thorough analysis of the state of the art of Electricity Price Forecast (EPF), analysing the most recent evolution in the methods and models presented in the literature. This will be the starting line for the work presented in this thesis, where I embrace the challenge of EPF and try new approaches for forecasting while giving a special attention to the implication on the risk management and scheduling of production units.

2

Electricity Price Forecast

2.1 Introduction

Over the last years different methods and approaches have been tried for electricity price forecasts with varying degrees of success. The increasing number of studies published has been highly motivated not only by industry needs, but also by the challenges that the electricity markets present, making it a challenging field to work on.

Despite the increasing interest, the large amount of electricity markets and their different complexity make it difficult to compile all the existing models in literature and compare them with each other. Market configuration has a high influence in the price formation, price caps/floors or the existence of capacity or balancing markets and ancillary services can highly influence the price formation and the way that all market players act. In addition, the geographical characteristics also play a major role, the generation mix or grid design of each market is enough to completely alter the performance of the models when applied to different markets.

Nonetheless, electricity price forecasting has been of the most importance for the industry as it plays a major role in minimizing the risks that each player is exposed to.

In this chapter I make the main focus the forecasting of the day-ahead electricity price and make a review of the literature, more specifically, I approach the different methods used and the most recent trends in the field.

2.2 Predictive framework for EPF

Recalling the previous chapter, the MCP is obtained from all demand and supply offers on an organized auction. The main reason behind this auction is the system operators requiring advance notice in order to verify that the schedule is feasible and falls within transmission constraints. This auction happens once a day and results in prices for all periods of the following day (typically 24 hours).

This means that, when forecasting or modelling the day-ahead market one has to take into account that all contracts of the next day are determined at the same time with exactly the same market information. There are two interesting effects that result from this: first, the dynamics of hourly price does not behave as a pure time series process, instead, these prices may be seen as daily arrays of 24 hours [70]. They only approximate a time series because most of the underlying dynamics in the bids are in fact time series (demand, wind production, solar production, etc.). Second, the prices have no relation to the real values observed in the electrical system - demand or renewable energy sources production - but instead are highly dependent on what the market foresaw as expected for the next day. This mean that forecasts of these variables will work better as exogenous variables for the models than the actual observed values.

Regarding predictive models, most of the literature refers to point forecasts, where the target is the actual day-ahead price. The point forecast can be obtained using different techniques, by modelling the hourly price in the past and forecasting for the next hours or by simulating the supply and demand curve for the next hours and obtaining the price by intercepting both curves. Nevertheless, models where the demand and supply curves are the object of modelling are still to be found among the main EPF authors and journals. In this work, I model the supply and demand curve and forecast this curves for the next hours.

A more recent line of work around EPF has been probabilistic forecasting, where the result is not a single value (point) but a Prediction Interval (PI), that predicts the distribution of individual future and bears the same relationship that a frequentist confidence interval or a Bayesian credible interval bears to an unobservable population parameter. The increased interest in this branch of EPF is a direct consequence of the renewable integration, existance of smart grids, and distributed generation, where uncertainty and probabilistic forecasting play a big role. Probabilistic forecasting can offer an improved assessment of future uncertainty, a broader ability to plan different strategies for the range of possible outcomes and an increased effectiveness of submitted bids.

2.2.1 Forecasting horizons

The literature separates the forecasting horizons in 3 groups: short, mid and long-term, but there is no actual agreement as to what actually the thresholds should be. Short-term often refers to forecasts that range from the next-day to a few days ahead and is the main forecasting tool in day-to-day market operations, being used to assure an optimized bidding strategy for all assets. Mid-term time horizons suggest from a few days or weeks to a few months ahead and are mainly used for budget and balance sheet control, elaboration of business plans, risk management and derivative trading. Long-term forecast can range from a few months to dozens of years ahead and its main objective is investment profitability analysis and planning.

The forecasting method used can change significantly for the different horizons. Longer periods often require forecasts to be anchored in structural variables, like commodity prices, installed capacity, energy consumption growth, network design changes, regulatory changes and so on, while short periods ahead are mostly dependent on high volatility variables, like meteorological data or electricity demand changes from business work day.

2.2.2 Forecasting evaluation

Most literature agrees in the forecasting evaluation methods and metrics and sometimes even provide different metrics for their forecasts. Measuring point forecasting accuracy is based on absolute errors $AE_h = \left|P_h - \hat{P}_h\right|$, where P_h is the actual and $\hat{P}_h$ is the predicted price for the period h. Obviously, the accuracy will increase with lower AE_h. For hourly forecasts, it's quite commons to see a Mean Absolute Error (MAE) as the mean of the absolute errors over the forecast period T:

$$MAE = \frac{1}{T}\sum_{h=1}^{T}\left|P_h - \hat{P}_h\right| \tag{2.1}$$

Since electricity prices can change significantly from market to market, absolute errors are quite hard to compare between different datasets. Many authors use measures based on absolute percentage errors: $APEh = AEh/Ph$. By far, the most popular is the Mean Absolute Percentage Error (MAPE), which is computed as the mean of absolute percentage errors over T:

$$MAPE = \frac{1}{T}\sum_{h=1}^{T}\left|\frac{P_h - \hat{P}_h}{P_h}\right| \tag{2.2}$$

The MAPE measure is suitable for markets where prices are significantly higher than zero, but it can be misleading when applied to most electricity prices. In particular, when electricity prices are close to zero, MAPE values become very large, regardless of the actual absolute errors. On the other hand, when electricity prices spike, the resulting MAPE values are small, irrespective of the absolute differences. In a balance sheet impact point of view, this is inaccurate, as a badly forecasted price spike might have a much bigger finantial impact than a poorly forecasted price close to zero.

The most common approach to tackle the comparison issues between different models and different price scales is to normalize the absolute error by the average price obtained in the evaluation interval T. This yields the Weighted Mean Absolute Error (WMAE):

$$WMAE = \frac{1}{T}\sum_{h=1}^{T}\frac{\left|P_h - \hat{P}_h\right|}{\bar{P}_T} \tag{2.3}$$

where $\bar{P}_T = \frac{1}{T}\sum_{h=1}^{T} P_h$ is the mean price in the time interval T.

In more statistical and econometric papers it is also common to see l^2-type norms, with the most common one being the Root Mean Square Error (RMSE):

$$RMSE = \sqrt{\frac{1}{T}\sum_{h=1}^{T}\left(P_h - \hat{P}_h\right)^2} \tag{2.4}$$

Like in all the above absolute error based measures, the squared differences can be normalized by the square of the actual price or by the square of the mean price over the period T.

2.3 Forecasting methods overview

The field of EPF aims at predicting the spot prices in markets, however, given the diversity of trading regulations available across the globe, EPF always has to be tailored to the specific market, which renders the field of EPF very diverse. In this section I will make an overview of the most common methods and aim at reviewing the state-of-art models.

Electricity price forecasts models can be separated in four distinct families: *Statistical*, *Fundamental*, *Machine Learning* and *Hybrid*. Statistical models are quite common in the literature and come from direct application of statistical techniques, they include regression type models, autoregressive type time series models and reduced-form models. While the first two types are still famous among authors. mostly in hybrid models, the latter has decreased in popularity.

Fundamental models try to describe the price dynamics by modelling important physical and economic factors and simulate the operation of a system of heterogeneous agents interacting with each-other, although favoured in the industry, this models are often used for mid or long-term forecasts, as they perform poorly on the short-term.

Machine Learning are models which combine elements of learning, evolution and fuzziness to create approaches that are capable of adapting to complex dynamic systems. There has been an increased interest in machine learning models because of their capability to capture non-linear behaviour.

Finally, hybrid models try to combine two or more techniques from the other families for EPF, they have increased in popularity because of their ability to capture the pros of different models, minimizing the cons. Since their classification is almost impossible they have a family of their own.

2.3.1 Statistical models

Historically, the first statistical EPF techniques consisted on replicating statistical methods of load forecasting, by a simple substitution of loads or temperatures for electricity prices, the researchers were able to obtain EPF models. As time passed, more and more contemporary statistical, econometric or signal processing techniques were introduced to this area.

Statistical methods forecast the current price by using a mathematical combination of the previous prices and exogenous factors, typically consumption and production figures, or weather variables. The two most important categories are additive, where the predicted price is the sum of the components, and multiplicative models, where the predicted price is the product of a number of factors. Note, however, that the two are closely related: a multiplicative model for prices can be transformed into an additive model for log-prices.

Most models in this family rely on linear regression and represent the target variable , i.e. the price p_h for time (hour) h, by a linear combination of independent variables (the predictors):

$$p_h = \theta_h \mathbf{X}_h + \epsilon_h \tag{2.5}$$

where $\theta_h = [\theta_{h,0}, \theta_{h,1}, \ldots, \theta_{h,n}]$ is a vector of coefficients specific to hour h, $\mathbf{X}_h = [1, X_{h,1}, X_{h,2}, \ldots, X_{h,n}]$ is a vector of inputs and ϵ_h is an error term.

Statistical models are attractive because some physical interpretation may be taken from their results, thus allowing the user to understand their behavior. They are often criticized for their limited ability to model the nonlinear behavior of electricity prices and related fundamental variables, nonetheless, in practical applications, their performances are comparable to those of their non-linear alternatives.

Similar-day methods

In most EPF papers, the author refers to a benchmark model, this model is called a similar-day method. It is based on searching historical data for a day with similar characteristics to the predicted day and use this historical values as forecasts for the future. We may include day of the week, day of the year, holidays, weather conditions or consumption levels as similar characteristics.

One of the more common applications of the similar-day approach was introduced by [1] and is called the naïve method. The procedure is so basic that it became a must have benchmark for all methods in literature, if your method can't outperform the naïve then it's just simply not good enough. The procedures consists in taking the day of the week from the previous week and apply it to the next one, this way a Monday will be equal to last Monday, a Tuesday to last Tuesday, and so on.

Regression models

Regression is one of the most widely used statistical techniques. The general purpose of multiple regression is to learn more about the relationships between several independent or predictor variables and a dependent or criterion variable. Multiple regression is based on least squares: the model is fitted such that the sum-of-squares of the residues is minimized.

Regression models are so common and popular in EPF that it is almost impossible to make a full review on all of them. Besides that, most papers use regression models combined with some other methods, normally more sophisticated. It is also hard to separate regression and auto-regressive models since most of them use some kind of lagged electricity price as regressors. Nonetheless, there are papers that introduced some kind of innovation to the field and deserve to be mentioned.

One of the first application of regression models in EPF was proposed by [85]. In this approach, a regression model is used in combination with wavelet decomposition, i.e., the coefficients are estimated using wavelet decomposition detail series. The day-ahead price forecast is then given by previous day's low frequency and the predicted high frequency components. Similar techniques are also found in [36] and [126].

Most of the innovations that followed involved some kind of model combination, for instance, the authors of [82] consider regression models with Fractional Autoregressive Integrated Moving Average (ARIMA) models and Generalized Autoregressive conditional hesteroskedasticity (GARCH) disturbances. The regressors capture seasonal cycles, holiday effects and possible interventions on the mean and variance. An algorithm which switches between neural networks, fuzzy regression and a standard regression based on some rules can be found in [13] . A two-step method for EPF, where a time varying regression is used to explain nonstationary influences from load and wind power generation and in a second step, an autoregressive model and exponential smoothing is applied to account for residual autocorrelation and seasonal dynamics was used by the authors of [76] .

In the last few years the most relevant contribution to EPF has been the appearance of linear regression with a large number of input features that utilize some kind of regularization technique. If the number of regressors is to large, using Least Absolute Shrinkage and Selection Operator (LASSO) or the elastic net. By using a penalty factor jointly with minimizing the Residual Sum of Squares (RSS) one can turn some of the coefficients to zero and effectively eliminate redundant regressors. These regularized regression models exhibit superior performance [134, 162, 135, 91, 161, 160].

Another innovation in this family is the use of different calibration windows on the same model and combining them afterwards, it has been proven that this approach outperforms the predictions obtained for the best ex-post selected calibration window [69, 100] .

Autoregressive-type models

Autoregressive models take into account the time correlations of the data we are studying, the X_t is expressed linearly in terms of it's p past values (autoregressive parameter), and in terms of q values of the noise (moving average part):

$$\phi(B)X_t = \theta(B)\epsilon_t \tag{2.6}$$

Where B is the backward shift operator $B^h X_t = X_{t-h}$, $\phi(B)$ is the notation for $\phi(B) = 1-\phi_1 B - \cdots - \phi_p B^p$, and $\theta(B)$ is a notation for $\theta(B) = 1+\theta_1 B + \cdots + \theta_1 B^q$, where $\phi_1, \ldots, \phi_p$ and $\theta_1, \ldots, \theta_q$ are the coefficients of the model. This is called the Autoregressive Moving Average (ARMA) model and its stylized ARMA(p,q).

The ARMA model assumes that the time series is stationary. If it is not, a transformation of the time series is required to assure stationarity. One of the simplest way to ensure stationarity is differencing. [20] introduced a new model where differencing was explicitly included and named it AutoRegressive Integrated Moving Average ARIMA. The ARIMA has three types of parameter: the $(\phi_1, \ldots, \phi_p)$, the number of differencing passes d and the moving average parameter $\theta_1, \ldots, \theta_q$):

$$\phi(B)\nabla^d X_t = \theta(B)\epsilon_t \tag{2.7}$$

where $\nabla_h x_t = (1-B^h)x_t = x_t - x_{t-h}$ is the lag-h differencing operator.

For time series where seasonality has high periods, differencing for a small number of times might not be enough to ensure stationarity. In particular, for electricity prices or load one can find seasonality with 24 hour and 168 hours, representing a daily and a weekly cycle. This requires to difference at very long lags. This process is known as a Seasonal Autoregressive Integrated Moving Average (SARIMA) and the general notation is ARIMA$(p,d,q) \times (P,D,Q)_s$. The term (p,d,q) represents the order of the non-seasonal part, while $(P,D,Q)_s$ represents the order of the seasonal part. s represents the period for the seasonal pattern. The SARIMA model can be written as:

$$\phi(B)\Phi(B^s)\nabla^d\nabla_s^D X_t = \theta(B)\Theta(B^s)\epsilon_t \tag{2.8}$$

Looking to the notation one can note that it's quite easy, although extensive, to transform a model ARIMA or SARIMA in a ARMA model in the variable $\tilde{X}_t = \nabla^d\nabla^D X_t$.

This time-series models relate the variable in study into its own past, and do not explicitly receive information contained in other exogenous time series. However, electricity prices are highly influenced by other factors like load profiles and weather conditions. To capture the behaviour between prices, past values and exogenous fundamental variables, time series model with input variables can be used. These models are generalization of the ARMA models discussed and are referred to as Autoregressive Moving Average Exogenous (ARMAX) models. One can write the expression for this model taking (2.6):

$$\phi(B)X_t = \theta(B)\epsilon_t + \sum_{i=1}^{k} \psi^i(B)V_t^{(i)} \tag{2.9}$$

where $V^{(1)}, \dots, V^{(k)}$ are the exogenous variables, $\psi^i(B) = \psi_0^i + \psi_1^i B + \cdots + \psi_{r_i}^i B^{r_i}$, with r_i being the order of the exogenous variables and ψ_j^i the corresponding coefficients.

Autoregressive-type models represent the backbone of all time series models of electricity prices. The authors of [39] apply variants of AR(1) model and general ARMA processes to forecast the german European Energy Exchange (EEX) market. They conclude that modelling each hour of the day separately provides slightly better forecasts that modelling the entire time series. They also included a probabilistic process for the detection of extreme price events that leads to an improvement in the forecasting of univariate models.

In a related study, [144] use various autoregression schemes for modelling and forecasting prices in the California market. They observe that an autoregressive model with lags of 24, 48 and 168 hours where each hour of the day is modelled separately, performs better than the single large SARIMA specification for all hours proposed in [38]. A similar simple Autoregressive model (with lags of 24, 48 and 168 hours) to account for residual autocorrelation and seasonal dynamics, with the inclusion of load forecasts, was successfully used for short-term EPF [76].

A wavelet-ARIMA technique was proposed in [36], which consists of decomposing the price series using a discrete wavelet transform, modelling the resulting detail and approximation series using ARIMA processes to obtain 24 hourly predicted values, and then applying the inverse wavelet transform, to yield the predicted prices for the next twenty-four hours. The performance of the wavelet-ARIMA technique is generally better than that of a standard ARIMA process.

In the same spirit, [128] propose a hybrid method for forecasting day-ahead electricity prices, in which a wavelet transform provides a set of 'better-behaved' time series, an

ARIMA model is used to generate a linear forecast, and then a Radial Basis Function (RBF) network is used to correct the estimation error of the wavelet-ARIMA forecast.

A set of 24 hourly ARIMA models for weekdays (which are calibrated only to weekday prices) and a set of 24 hourly ARIMA models for weekends (which are calibrated to weekday and weekend prices) was proposed by [53]. This model may be treated as a generalization of the approach advocated by [39] and [106], where each hour of the day is modelled by a separate AR-type model.

An interesting methodology was decribed in [90] which combines elements of time series and multi-agent modelling. They forecast 24 hourly prices for the next day using an ARIMA model applied to the conjectural variations of the firms participating in the Spanish power market. They find that this model performs slightly better than a pure ARIMA model. Further applications of SARIMA models in EPF include the studies by [10], [32], [40], and [132]. In these studies, the SARIMA are used as benchmarks for more complicated models involving neural networks, support vector machines or GARCH components.

2.3.2 Fundamental models

The next class of models, known as fundamental or structural models, tries to capture the basic physical and economic relationships which are present in the day-ahead market. The functional associations between fundamental drivers (loads, weather conditions, system and power plant parameters, etc.) are theorized, and the fundamental inputs are modelled and predicted independently, often via statistical or machine learning techniques. Moreover, many of the EPF approaches considered in the literature are hybrid solutions with time series, regression and neural network models using fundamental factors, like loads, fuel prices, wind power or temperature, as input variables [58, 81, 94, 145].

Many of these models are often developed as proprietary, and therefore, their details are not disclosed publicly. In fact, one can find several providers of software for long-term EPF with parameter-rich fundamental models. At the academic level, most of the results published relate to hydro-dominant power markets, since the functional behaviour of hydro power plants bidding it's easier to emulate in this models than with statistical ones. In particular, [75] presents a supply–demand model for the Norwegian power market from a time before the common Nordic market Nordic POOL (NORDPOOL) had started. He uses hydro inflow, snow and temperature conditions to explain spot price formation. The authors of [45] develop a hybrid fundamental model and calibrate it to data from Electric Reliability Council of Texas (ERCOT), New York Pool (NYPOOL) and Pennsylvania-New Jersey-Maryland Interconnection (PJM). They start with the processes for the primary drivers (such as fuels, outages and temperature/demand), then construct the bid stack transformation and obtain electricity prices. The simulated price processes exhibit spikes, mean reversion, fat tails of the price distributions, and a correct forward price volatility structure.

An even more parameter-rich fundamental model for the Nordic market was built [139], considering stochastic climate factors like temperature and precipitation, they model the hydrological inflow and snow-pack development that affect hydro power generation, the major source of electricity in Scandinavia. They derive the electricity price from 27 parameters and 29 formulas defining relationships between the fundamental variables. Their model is able to capture the observed fundamentally motivated market price movements on a monthly scale.

Under the fundamental models, there is a subclass of simpler structural models that can be built from empirical analysis of market supply and demand curves. The first approach was made by [14], he builds the day-ahead spot price process by applying the inverse of the Box-Cox transformation to an Ornstein-Uhlenbeck process. As a result, Barlow obtains a jumpless spot price model that can exhibit spikes.

The authors of [80], with a similar approach, define a hockey-stick shaped supply curve that matches the empirically observed curves better than the inverse of the Box-Cox transformation. They combine this with an inelastic vertical demand curve with horizontal stochastic variations driven from and Ornstein-Uhlbeck process.

A fundamental model for spot electricity prices, based on stochastic processes for the underlying factors like fuel prices, demand and generation capacity availability, as well as a parametric form for the bid stack function that maps these price drivers to the power price can be found in [65]. Using observed bid data, they find high correlations between the movements of bids and the corresponding fuel prices. [24] use a stochastic model of the bid stack that translates the demand and the prices of fuel commodities into electricity spot prices. The stack structure allows for a range of generator parameters. The derived spot price process captures important behaviour of historical electricity prices, including both spikes and the complex dependence upon its underlying supply and demand drivers. Furthermore. In a similar context, [6] develop a structural risk-neutral model in which a scarcity function is introduced to allow for deviations of the spot price from the marginal fuel price, thus leading to price spikes. They focus on pricing and hedging electricity derivatives, and show that, when far from delivery, electricity futures behave like a basket of futures on fuels.

2.3.3 Machine Learning models

Machine learning is the study of computer algorithms and techniques to make descisions or predictions by the use of data. Is a very diverse group that has been developed to solve problems which traditional methods (e.g., statistical) cannot handle efficiently. It combines elements of learning, evolution and fuzziness to create approaches that are capable of adapting to complex non-linear dynamic systems, and because of that are regarded as 'intelligent' .

From all the Machine Learning, Artificial Neural Networks (ANN), fuzzy systems, Support Vector Machine (SVM) and evolutionary computation (genetic algorithms, evolutionary programming, swarm intelligence) are definitely the main classes. This models are sophisticated and flexible enough to handle complex systems and non-linearity. This makes them very promising for short-term EPF, specially in complex markets, with effects from varied sources. Although artificial neural networks have probably received the most attention [3, 143], other non-parametric techniques, such as fuzzy logic, genetic algorithms, evolutionary programming and swarm intelligence, have also been applied, but typically in hybrid constructions.

With the increase of computational power available to the common user, in the last six years there has been a growing interest in Deep Neural Network (DNN), a neural network with a more complex structure, with several hidden layers. However, despite this trend, most of the published studies are quite limited, with poor benchmarks and usually avoid state-of-the-art statistical methods.

Artificial Neural Networks

Every ANN model can be classified in terms of its architecture and learning algorithm. The architecture describes the neural connections, and the learning algorithm provides information on how the ANN adapts its weights for every training vector, however, the analysis and explanation of all the different topologies is beyond the scope of this thesis.

ANN models has been widely employed in EPF. One output node has been used to forecast the next hour's price (see e.g. [57, 99]), the price h hours ahead (see e.g[8, 67, 123], the next day's peak price [12], the next day's average on-peak price [61], or the next day's average baseload price[118]. Using models with several output nodes it's possible to forecast a vector of prices, typically 24 nodes for forecasting the next day's complete price profile [150].

Multi-Layer Perceptron (MLP) have also been used on EPF, as they proved to be good at capturing global data trends. [4] use back-propagation as a training algorithm, while [27, 119, 123] use the Levenberg–Marquardt algorithm. [9] argues that it trains a network 10–100 times faster than back-propagation. [10] propose a hybrid system in which a Real-Coded Genetic Algorithm (RCGA) with an enhanced stochastic search capability is used to train a MLP and show that this method can provide more accurate results for the Spanish market than a standard ARIMA model, a wavelet-ARIMA model or a fuzzy ANN.

[54] use a combination of univariate MLP networks, in which three auxiliary networks forecast maximum, minimum and medium values of the price, and then this information is fed to five principal MLP networks in order to forecast the electricity price. [67] use a market concentration index, a measure of the oligopolistic structure of the power market, as an input variable for a MLP, and show that it has a considerable impact on the forecasts.

A hybrid ANN model that involves wavelet decomposition, a mixed data model that includes time and wavelet-domain features, a relief algorithm for feature selection, and a multilayer perceptron for forecasting and cross-validation can be found in [83]. They propose a model in which the numbers of hidden and input neurons are adjusted based on an iterative procedure, after which an evolutionary algorithm is used to make further adjustments to the weights of the network in the neighbourhood of the weights found initially. The auhtors of [28] model the residuals of an ARIMA model using a ANN with past prices as inputs. The auhtors of [128] construct a hybrid wavelet-ARIMA-Radial Basis Function network, in which a RBF network corrects the estimation error of the wavelet-ARIMA forecast. Like in [68], a particle swarm optimization is used to optimize the network structure.

Other examples of articial neural networks use on EPF are [33, 40, 52, 54, 99, 119, 150, 60, 95, 152].

2.3.3.1 Support vector machines

The SVM is a classification and regression tool that has been initially proposed on Vapnik's statistical learning theory [137]. Unlike Artificial neural netwroks, which try to explain complex functions of the input space, SVM performs a non-linear mapping of the data into a high dimensional space, then uses simple linear functions to create linear decision boundaries in the new space. An attractive feature of SVM is that it gives a single solution that is characterized by the global minimum of the optimized function, rather than multiple solutions associated with local minima, as do ANNs.

In electricity price forecasting, SVM are normally seen in hybrid models, however, in one of the first papers on this topic, [125] compares a multi-layer Perceptron and a SVM with exactly the same inputs, and conclude that the SVM has a more consistent behaviour while requiring less time for optimal training. Also, [156] employ a SVM to forecast the value of the spot price. The use self-organizing map classifiers to cluster hourly electricity price data according to their similarity, then employing SVM to predict the prices within each subset were also used in EPF [46, 113].

The authors of [32] propose a hybrid model called Support Vector Autoregressive Integrated Moving Average (SVARIMA). this model combines support vector regression to capture the nonlinear patterns and ARIMA models. The results demonstrate that the SVARIMA model outperforms some of the existing ANN approaches and traditional ARIMA models.

Fuzzy neural networks

Fuzzy logic is a generalization of the usual Boolean logic where the input has certain qualitative ranges associated with it, for example, a temperature may be low, medium or high instead of the traditional Boolean 0 or 1. Fuzzy logic allows outputs to be deduced from fuzzy or noisy inputs with no need to specify a precise mapping of inputs to outputs. Following the logical processing of fuzzy inputs, a defuzzification process may be used in order to produce precise outputs (e.g., prices for particular hours). Fuzzy Neural Networks (FNN) combine the learning and computational power of traditional ANNs with fuzzy logic [88, 124].

One of the first applications of fuzzy logic to EPF can be found in [66], which utilize fuzzy-c-means for classifying historical data into three distinct clusters representing peak, medium and off-peak, and then employ a ANN for forecasting. The tecnhique has been used by more authors [136]. The authors of [123] build an Adaptive-Network-Based Fuzzy Inference System (ANFIS), which combines an adaptive mechanism with Sugenotype rules and uses a combination of the least squares method and back-propagation for training the membership function and the linear combination parameters. They show that the ANFIS performs better than a multi-layer perceptron. A FNN with an inter-layer and a feed-forward architecture was proposed by [8] and uses a new hypercubic training mechanism. The method is shown to predict Spanish hourly day-ahead electricity prices better than ARIMA, wavelet-ARIMA, MLP or a RBF network. A hybrid approach was proposed in [25], which combines a wavelet transform, particle swarm optimization and an adaptive-network-based fuzzy inference system.

Deep learning methods

Deep learning is a family of machine learning methods closely related to artificial neural networks, in which several layers of neurons are used, hence the name "deep". The existence of many layers allows for the search of more features in the input data, since each layer can identify different signals.

The first published Deep Learning (DL) paper[142] proposes a DNN using stacked de-noising autoencoders. The new method is compared against machine learning techniques but also against two statistical methods. In the second published deep learning study [92], a DNN for modeling market integration is proposed. Afterwards, four DL models (a DNN, two Recursive Neural Network (RNN)s, and a Convolutional Neural Network (CNN)) was built the results compared between them using a whole year of data against a benchmark of 23 different models, including 7 machine learning models, 15 statistical methods and a commercial software [91]. Moreover, among the statistical methods, the comparison includes the fARX-Lasso and fARX-EN which are among the state-of-the-art statistical methods. The study shows that the deep learning algorithms perform slightly better than the others.

The studies that followed focused on one of three topics: evaluating the performance of different deep recurrent networks [133, 35, 159, 108]; proposing new hybrid methods based on RNNs and Long Short-Term Memory (LSTM) Recurrent Neural Networks [155, 89, 148]; or employing regular DNN models [35]. In detail, [133] studies the use of RNNs for forecasting electricity prices and compares the result against a simple statistical methods (a

SARIMA model, a Markov regime-switching model, and a self exciting threshold model). A similar approach where a hybrid deep learning method composed of a CNN and a LSTM is used for forecasting electricity prices[89, 155]. [87] proposes a series of DL models and compares them for a year of data against several advanced statistical methods such as LASSO and a simpler ANN method. Although the great approach and the thorough comparison of state-of-art models, the study focuses on intraday electricity prices, while most of the literature considers forecasting day-ahead electricity prices.

More recently the application of deep learning models can be found in [34, 30, 158, 109, 107, 149, 103, 29, 73, 5, 7, 127].

Deep Learning became popular buzzword, being the techniques seen as the state-of-the-art. However they impose a severe computational burden, their relative performance is not well tested, the training process is described by many hyper-parameters which the optimal values are unknown, thus requiring a lot of effort to tune and apply.

2.3.4 Hybrid

Within the field of EPF, the research area that had the biggest contribution has been hybrid forecasting methods, in fact, most of the approaches already discussed here are actually hybrid in some sense.

Hybrid models are very complex forecasting frameworks that are composed of several algorithms and techniques. Usually, they comprise different models for different components or modules of a forecasting technique, which can be data decomposition, feature selection, data clustering, some heuristic optimization of hyperparameters or prediction ensemble.

In terms of decomposition methods, the most widely used technique is the wavelet transform [30, 111, 151, 117, 16, 11, 50, 153]. Alternatives methods include empirical mode decomposition [64, 154], variational mode decomposition [141, 93], and singular spectrum analysis [138, 147].

For feature selection, the most commonly used algorithms are correlation analysis [16, 64, 84, 17, 86] and the mutual information technique [50, 84, 50, 121, 2]. Other algorithms include classification and regression trees with recursive feature elimination or Relief-F [110].

For clustering data, the algorithms are usually based on k-means [55, 72], self-organizing maps [111, 55, 56], enhanced game theoretic clustering [55] or fuzzy clustering

[50, 71].

For the optimization of hyperparameters, the most used algorithm is particle swarm optimization [151, 11, 93, 121, 71], but other approaches can also be found like differential evolution [141], genetic algorithm [121], backtracking search [121], deterministic annealing [71], bat algorithm [16], vaporization precipitation-based water cycle algorithm [17], cuckoo search [147, 86] or honey bee mating optimization [117].

As for forecast ensemble, its use became more popular has combining forecasts is less risky in practice than to select an individual forecasting method. Although used in other fields, they have only been employed in EPF quite recently by [18, 98, 115, 122] by providing empirical support for the benefits of combining forecasts in obtaining better predictions of electricity spot prices.

I have to mention here that combining forecasts is related to the concept of committee machines [62], which is also referred to as ensemble averaging. A committee machine is composed of multiple networks. The individual ANNs are trained, perform predictions and then are updated in such a way as if they were stand-alone. Then, a 'weight calculator' generates weighting coefficients by which individual predictions are combined linearly in a 'combiner' neuron. [61] were the first authors to use committee machines for EPF. They combine a RBF network, which uses 23 inputs and six clusters, and a multi-layer predictor, which uses 55 inputs and eight hidden neurons, to compute daily average on-peak electricity prices for New England. They consider three committee machines: one with simple arithmetic averaging, one where the correlation matrix used to determine weighting coefficients is re-calculated whenever new prediction errors become available, and one newly developed combiner.

Only recently have we seen new approaches on forecast ensembling by using multi-layer predictors [111, 16, 11, 64, 138, 147, 84, 17, 2, 157], followed by the adaptive network-based fuzzy inference system [111, 154, 121], radial basis function network [117, 71, 157], and autoregressive models like ARMA or ARIMA [117, 154, 157, 151]. Other models include LSTM [30], linear regression [110], extreme learning machine [151, 110], CNN [110], Bayesian neural network [55, 56], exponential GARCH [154], echo state neural network [141], Elman neural networks [51], and support vector regressors [157].

2.4 Industry Applications

There is a significant discrepancy between the recent advances in EPF at an academic level and at an industrial level. In fact, despite all the models that have been developed and published by the academy, most models used for industry application are much simpler. This is understandable as the industry has different objectives from the academy.

Most of the models discussed are complex, hard to optimize and computation heavy. If the industry requires fast inputs for decision making it becomes hard to use these models on a daily basis. It also requires a specific level of expertise to be able to correctly tune and maintain most models, specially artificial neural networks and deep learning models, which may be hard (and expensive) to find. Besides, because of the lack of a common framework for studying and benchmarking EPF models in the literature, it becomes unclear for decision makers if the time and money needed will have the expected return. Nonetheless, Electricity Price Forecasting is of major importance in the industry and requires constant investment.

The most used models in industry are statistical models, in particular autoregressive dynamic models, which are an extension of the ARIMA models where the exogenous variable is also a time series forecast. In this case, the use of lagged values for the exogenous variable improves the performance of the overall model. Using the notation introduced in 2.3.1 we can write the model has:

$$\phi(B)X_t = \phi(B)V_t + \epsilon_t \tag{2.10}$$

A curious fact about this models (and most statistical ones) is that the number of exogenous or explanatory variables V used can affect the model performance. It has been prooven that a model with fewer exogenous variable performs better than one that uses all possible explanatory variables. These has been the motivation for the development of new hybrid methods that focus on regressor selection already discussed. These leads to a large number of models available to the industry.

The difficulty found in choosing the right model for the right situation raised the need for methods that combined forecasts with the most used approach being the ensemble averaging. Although ensemble averaging had the advantages of minimizing the risk of having a poor forecast (it's never as bad as the worst available single forecast) it also makes it impossible to have the best forecast possible (it's always worse that the best single forecast available). The pursue for a method that could lead to the best average possible while minimizing the risk of taking a poor forecast lead to new techniques, in particular the use of random forests and Extreme Gradient Boosting (XGBOOST) algorithms to "correct" the forecasts errors from statistical methods. In these approach, the statistical models predictions are used as inputs for the random forests or gradient boosting models which in turn outputs new forecasts that are slightly better than the simple average.

Besides these methods, is also common to see simple artificial neural network machine learning models in the industry, like single-layer perceptrons or support vector machines. However, the performance of this models when compared to statistical ones is not clear. Statistical models are usually preferred over machine learning because they are easier to interpret and help in fundamental market analysis.

3

New Approach - Curve Forecast

3.1 Introduction

As the literature review from the last chapter as shown, apart from a small family of models (fundamental or structural) most of the EPF approaches focus on studying the electricity price as a vector of single hourly observations. These observations can then be studied as a time series and all the known techniques are available for analysis and forecasting. The prediction methods tend to disregard that there is an actual physical structure behind the price, i.e., the price is a consequent construction of a series of offers that the agents do in an organized market. This means that the electricity price is not the unknown, the real unknowns are the bids/asks (or the market curves) by themselves, if you knew how the market curves would look for the future we could accurately infer the electricity prices.

From an industry point of view, the forecast of the supply and demand curves brings many advantages when compared to simple price forecasts. You can derive different measures by looking at the curves, obtaining information impossible to get otherwise, such as: i) price sensitivity - how will the price change when small changes occur in the curves; ii) asset optimization - with the market curves predictions we can optimize our bidding; iii) risk management - how risky it is to operate my power plant in a given time.

For the first point, price sensitivity is important to understand how easily can a price drop (or rise) if small changes happen in the curve, e.g. the uncertain of the demand or renewable energy sources production. For the second point, knowing the curve for the next hours or even days let generation companies optimize their assets in terms of start-up costs, available power and correct bidding. If we combine the first two points, one can even correctly manage technologies that suffer from cannibalization, such as hydro power plant, by knowing exactly how to bid them and the impact that new generation might have in the price, helping in the computation of the water value. For the third and last point, you can easily derive possible losses from the operation and potential gains from day-ahead trading by comparing the curves to future prices.

With this in mind, in this thesis I formulate a framework for supply and demand curve forecast, always with the assumption that knowing the exact curves will lead to the correct price for the future. I start by introducing a Theory for Market Curve Spaces, how exogenous variables affect the elements of the space and its intrinsic relationship to electricity prices. Afterwards I design a new method for modulation of the demand and supply curves and try to forecast this curves for the future. I then calculate the prices from this curves and compare this method to other methods used in industry for EPF.

3.2 Theory

Taking the electricity price p at each time period h we can write that:

$$p_h = g(s(x), d(x)) \tag{3.1}$$

where $g : \mathcal{C} \to \mathcal{P}$ is a function that assigns elements from the set of Market Curves to elements of the set of Electricity prices, meaning that the price is a function of (interaction between) the supply curve s and the demand curve d. In fact this function is well known, and it's given by the intersection of both curves:

$$p_h = \{s(x) : s(x) = d(x)\} \tag{3.2}$$

meaning that only $s(x)$ and $d(x)$ are needed to find the price.

Let $\mathbf{l} = (l_1, l_2, \ldots, l_n)$ be a vector of n variables that I shall call latent variables. $\mathbf{l}$ is composed from many variables that might or might not be observable and from which, we do not have the whole information. The number of elements of $\mathbf{l}$ might actually be infinite.

The elements of this vector are all the variables that somehow affect the shape of the market curves, like forecasted wind or solar production, forecasted demand, fuel prices, hour of the day, day of the week, day of month, available capacity, which are easily observable, but also the actions of the market agents, bidding strategy, external markets inluence, etc, which are much more difficult or even impossible to observe.

Let C be a set to which all the Market Curves belong, called Market Curves Set. For each $\mathbf{l}$ there is a subset C' of C, such that

$$C' = \{s(x) \in C|\mathbf{l}\} \tag{3.3}$$

meaning that the elements of C' are all the curves that have in common the elements of $\mathbf{l}$. For simplification, let me say that C' are the curves generated by $\mathbf{l}$.

Axiom 3.1 (Bijection). *For a given s, there is one and only one $\mathbf{l} = (l_1, l_2, \dots, l_n, sl_{n+1}, \dots, l_{n+k})$ such that $s(x) \in (C)|\mathbf{l}$, i.e. for any element s of C it's possible to find a subset $C' = C|\mathbf{l}$ of order one whose only element is s. Likewise, if a set C'' is of order one, there are only one vector of variables $(l_1, l_2, \dots, l_k)$, with k large enough, that conditions C such that $C'' = C|(l_1, l_2, \dots, l_k)$.*

This axiom is a clear conclusion from the observation of the market curves. Every curve is unique and its shape results from a set of factors that somehow conditioned the offers, if anything changed in these factors, then the curve wouldn't be the same, you would have, even if only slightly, a different curve.

Theorem 3.1 (Continuity). *Let C_n be a subset of C and $\mathbf{l} = (l_1, l_2, \dots, l_n)$ a vector of latent variables, such that $C_n = \{s(x) : s(x) \in C|\mathbf{l}\}$. If we have $\mathbf{l}' = (l_1, l_2, \dots, l_n, l_{n+1})$ and $C_{n+1} = \{s(x) : s(x) \in C|\mathbf{l}'\}$ then $C_{n+1} \subseteq C_n$.*

Proof. Let's consider the subset $C_1 = \{s(x) : s(x) \in C|l_1\}$ and the subset $C_2 = \{s(x) : s(x) \in C|l_2\}$, we know from conditional set theory that, for $C_{1,2} = \{s(x) : s(x) \in C|l_1, l_2\}$, $C_{1,2} \subseteq C_1$ and $C_{1,2} \subseteq C_2$. It easily follow that for a given subset $C_{1,2,\dots,n-1,n} = \{s(x) : s(x) \in C|l_1, l_2, \dots, l_{n-1}, {}_n\}$ and a subset $C_{1,2,\dots,n-1,n+1} = \{s(x) : s(x) \in C|l_1, l_2, \dots, l_{n-1}, l_{n+1}\}$, the subset $C_{1,2,\dots,n-1,n,n+1} = \{s(x) : s(x) \in C|l_1, l_2, \dots, l_{n-1}, l_{n+1}\}$ is such that $C_{1,2,\dots,n-1,n,n+1} \subseteq C_{1,2,\dots,n-1,n,n+1}$ and $C_{1,2,\dots,n-1,n} \subseteq C_{1,2,\dots,n-1,n+1}$, thus prooving the theorem. □

Theorem 3.2 (Convergence). *Let $c(x)$ be an element of the set C and $\mathbf{l} = (l_1, l_2, \dots, l_n)$ a vector of latent variables, such that $c(x) \in C' = \{s(x) : s(x) \in C|\mathbf{l}\}$, then exists, for k large enough, a vector $\mathbf{l}' = (l_1, l_2, \dots, l_n, l_{n+1}, \dots, l_k)$ such that $c(x) \in C'' = \{s(x) : s(x) \in C|\mathbf{l}'\}$, with C'' or order one.*

Proof. It easily follows from the continuity theorem 3.1 and the bijection axiom 3.1. Take a vector of latent variables $\mathbf{l}_n = (l_1, l_2, \ldots, l_n)$ and another vector $\mathbf{l}_{n+1} = (l_1, \ldots, {}_n, l_{n+1})$, we know from theorem 3.1 that $C_{n+1} \subseteq C_n$ and an element $c \in C_n$ will also belong to C_{n+1}. By induction, if we keep conditioning C to new elements $l_{n+2}, \ldots, l_k$ we will find the set $C_k = C|(l_1, l_2, \ldots, l_k)$ stated in axiom 3.1 such that C is the only element in it. □

The theory just stated guarantees that, in order to find a particular curve, you only need to know all the factors that condition it. It also guarantees that, if you have a set of curves and want to identify a particular one, adding the right conditions on the set will reduce the number of curves available, converging to the desired one.

These points will be central to the forecasting of market curves discussed below, as I predict the market curves taking into account my knowledge of the latent variables. The real challenge is to find out the correct latent variables as most of them, like the agents strategy, is not observable and thus probably impossible to incorporate in a model.

It is important to note that everything said in this chapter for the supply curve $s(x)$ also applies for curves demand curve $d(x)$ as all theory can easily be rewritten replacing $s(x)$ by $d(x)$.

4

Curve Forecast Model

4.1 The Parametric Curve Forecast

My main purpose is to identify market curves and forecast them to the future. The theory explained in the last chapter gives the support needed by guaranteeing that there is a relationship between the market curves and a set of exogenous variables. If such relationship exists, then you can use any known pattern recognition model, statistical or neuronal, to infer it.

To use any mathematical model we face the first obstacle: the definition of the object "market curves". The easiest way to approach this problem is by imagining that the market curve is in fact a function defined in the space (Energy, Price) and, although we don't know this function explicitly, we can approximate it to a simpler known function or family of functions. the goal of this approach is to define:

$$s_h(x) = f(\theta_h, x) \tag{4.1}$$

with f is a known function and θ_h a vector of parameters of f for time period h. What defines the curves for each time period is the parameter θ making it the main target to predict.

4.1.1 The market data

For the construction and evaluation of this model I used market curves data from 2017 to 2020 from the Iberian Market OMIE. This data includes all the offers (energy, price) from market agents for each hour of the period.

I also include in the analysis, different market data available, like wind production forecasts, solar production forecasts, load forecasts, availability forecasts for different technologies, interconnection capacity and forecasts, French and German market data. This data will be used as latent variables in the model and will help explain the curve shape for the future.

4.1.2 Evenly spaced point interpolation

Since the market curves are offers that consist in pairings (energy, price), the actual spatial representation of them does not define a continuous function, but a discrete function defined by different blocks, as presented in Fig. 4.1:

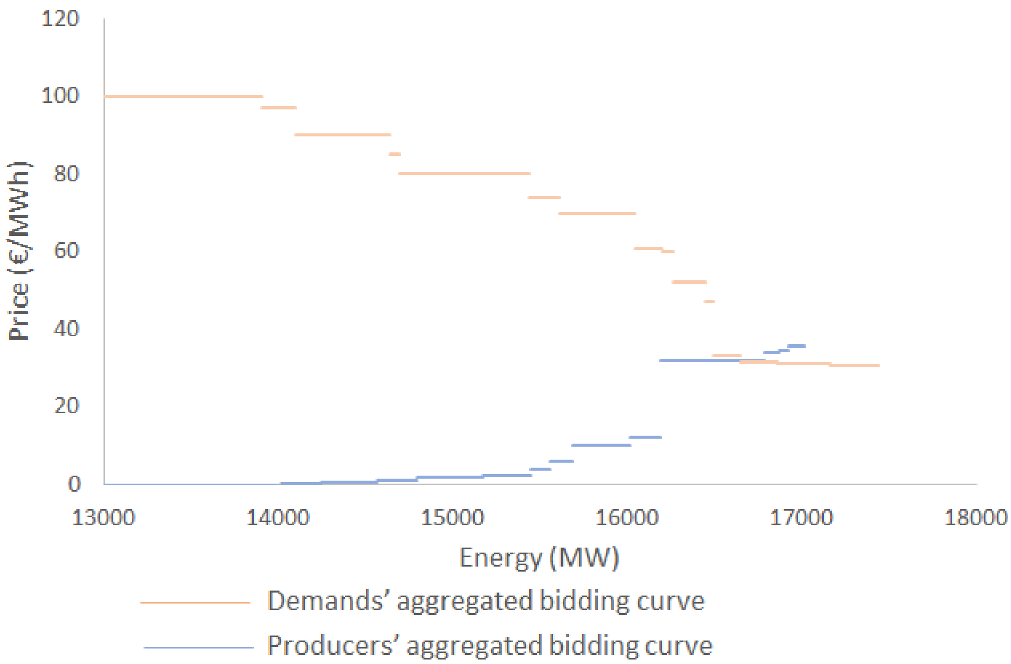

Figure 4.1: Example of block offers for a specific hour in the Day-Ahead Market

This block representation make it hard for curve fitting, the existence of very small (low amount of energy offered) with very big (high amount of energy offered) blocks makes the space non-uniform and the points might be irregularly spaced. Besides, the blocks form a monotonic function but not entirely increasing/decreasing, having many uniform *plateaus* over the curve, that can vary in size, and the information they contain might be important for future needs.

The analysis of the blocks and the evaluation of the market curves must be carefully done and depends both on the market in study and the purpose of the analysis. If the market is composed by a large amount of *plateaus* of a significant length, then shape of the blocks can't be ignored - is the case of French electricity market, where the non-flexibility of nuclear installed capacity creates large *plateaus* where energy is offered at the same price; on the other hand, if the curves are mainly composed of small ascending offers, then the blocks can be ignored and the curve can be studied as explicitly ascending - like in the Iberian market, where the generation mix is composed mainly of renewable energy with no costs and gas plants to give the needed flexibility to the system.

In this case, I shall ignore the blocks over 0€/MWh and take only it's mid-point, the curve fitting will then be made over this mid-points. Since this market hasn't seen negative prices regularly, I consider every block below 0€/MWh has a single large offer of price 0€/MWh.

Consider any offer block $b_{i,h}$ in the market curve as:

$$b_{i,h} = (V_{i,h}, p_{i,h}) \tag{4.2}$$

where $V_{i,h}$ is the volume offered and p its price at time period h. The coordinates of the block are:

$$b_{i,h} = ((v_{1,i,h}, p_{i,h}), (v_{2,i,h}, p_{i,h})) \tag{4.3}$$

where:

$$\begin{aligned} v_{1,i,h} &= \sum_{j=1}^{i-1} V_{j,h} \;\; \text{with} \;\; p_{j+1,h} >= p_{j,h} & (4.4) \\ v_{2,i,h} &= v_{1,i,h} + V_{i,h} & (4.5) \end{aligned}$$

The interpolation points are then

$$\begin{aligned} x_i &= \begin{cases} \max(v_{2,i,h}) \;\; if \;\; p_{i_h} <= 0 \\ v_{1,i,h} + \frac{v_{2,i,h} - v_{1,i,h}}{2} \;\; otherwise \end{cases} & (4.6) \\ y_i &= p_{i,h} & (4.7) \end{aligned}$$

For the demand market curve I do exactly the same but invert the direction and set a maximum price instead of the minimum 0€/MWh.

As already pointed out, you also have to take into account that this points are not evenly spaced and since curve fitting is done with the least squares method, the concentration of point in a given part of the curve can affect the results. A simple way to avoid the problem is to linearly interpolate the points in (4.6) and then take evenly spaced points $(x_{i,h}, y_{i,h})$ from the interpolation results, as illustrated in Fig. 4.2.

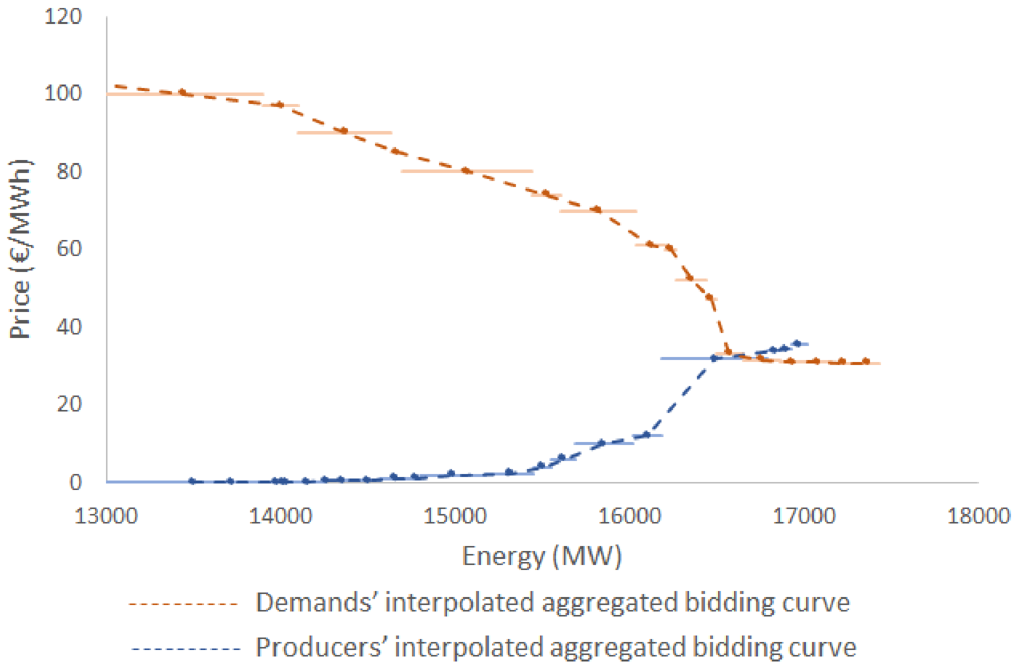

Figure 4.2: Interpolation of the block offers

Since I am ignoring the block structure by linearly interpolating points over blocks midpoint, one concern to take into account is the error in the price forecast that the interpolation can bring to the model. To evaluate the error of this method I calculate the price with the interpolated curves and evaluate the error. I obtained a MAE of 0.18€/MWh for the period 2017-2020, meaning that the interpolation has little to no affect in the price estimation.

4.1.3 Price Sensitivity Curve

Now that I have estimated the points that represent the market curves, it's possible to start the curve fitting.

Since there are two distinct market curves, supply and demand, I have in fact two sets of coordinate pairs (x, y) and will need two functions $f(\theta_s, x)$ and $g(\theta_d, x)$ to correctly simulate the market, which implies two distinct parameter vector θ_s and θ_d, bringing some complexity to the system in study. However, as a turnaround to this complexity and recalling the advantages of the curves - price sensitivity to demand volume changes, confidence intervals over price forecasts and portfolio optimization - the same results can be achieved looking at the difference:

$$\eta(x) = d(x) - s(x) \tag{4.8}$$

where $d(x)$ is the demand curve and $s(x)$ is the supply curve.

This leads to only one single function and only one set of parameters. Let us call this function the "price sensitivity curve", since it is a direct measure of the price changes over volume. The electricity price can be found in the root of this function, more precisely:

$$p_h = \{s(x) : \eta = 0 \iff d(x) = s(x)\} \tag{4.9}$$

On a pratical point a view, since the price is the image of the supply (or demand) curve on the root η, I can simplify the price electricity calculation if I actualy take its inverse:

$$\zeta = \eta^{-1}(x) \tag{4.10}$$

this way the market clearing price will be the root of ζ.

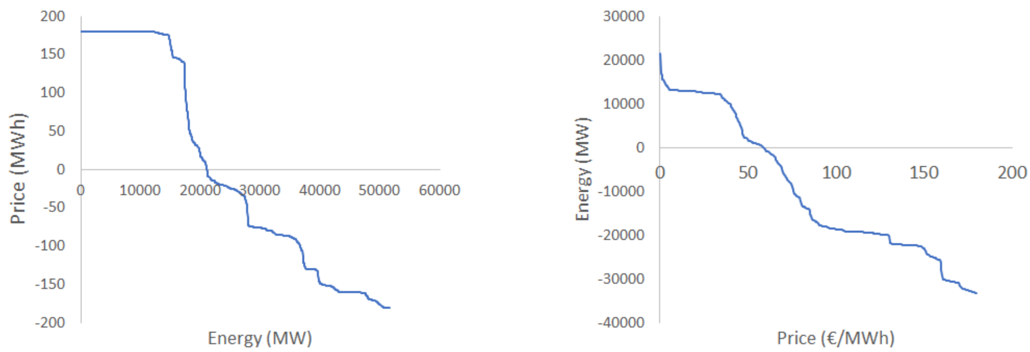

Figure 4.3: On the right, the η function. On the left, it's inverse $\zeta = \eta^{-1}(x)$

4.1.4 Curve fitting

With all data from the offers processed as mentioned in this chapter, I am in conditions to start the curve fitting and to model historic market curves. You have to bring to the curve fitting your own knowledge of the market, i. e., anything that you know from the curve shape or size can and should be used as an assumption for the curve fitting and function family selection, e.g:

- The function ζ is monotonic;
- The markets usually have a negative or zero price floor and a cap of a few hundreds of euros;

Looking at the shape of the points and taking into account the number of *plateaus* expected in the ζ function there are several function families that can be tried. For this market in particular I evaluated curve fitting using polynomials of different orders, sigmoid functions and stepwise linear functions. The purpose of the method is to have the curve shape around the MCP, meaning that the measures used to compare the different function have to give weight to points near the root of ζ and I also performed a weighted fit on the points, using the euclidean distance to the price as the weight.

It's usual to measure the goodness of fit using measures of the it's residuals. However, to understand how the curve behaves near its root, I calculated the MCP as the root of the fitted function and calculated the error to the real price observed.

Function	Weight	MAE	RMSE	RSE
poly3	1	6.18	6.98	2347.1
poly4	1	5.63	6.50	1875.4
poly5	1	5.39	6.23	1413.2
poly6	1	4.44	6.32	743.8
poly3	1/y	1.22	1.50	23.0
poly4	1/y	1.01	1.28	18.8
poly5	1/y	0.82	1.04	15.4
poly6	1/y	0.70	0.88	12.0
poly3	Erf	1.13	1.72	2169.1
poly3	Sigmoid	1.11	1.7	2278.5
poly2	Gaussian	6.88	6.17	22.4
poly3	Gaussian	5.83	5.79	22.1
Segmented		1.28	1.56	747.5

Table 4.1: Parameterization results, error measures for each function family for different weights.

Taking into account all the measures presented in tab. 4.1, the best function to use is a weighted 6th degree polynomial $ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g$, where x is the price and the parameters $\theta = (a, b, c, d, e, f, g)$ are estimated using the weighted least squares method.

4.1.5 Parameter analysis

The parameters θ can be seen as the representation of the market curve ζ, every curve has their own, which make them the main target for our predictions and analysis. Based in the theory exposed in 3.2, there will be a set of latent variables that characterises each θ, i. e., each curve. Our main objective is to find patterns and relationships between this variables so that we can, with a given set of latent variables, find the respective market curve.

By observing figure 4.4, where every parameter over the period between April and May of 2020 is plotted, it's possible to perceive their time series behaviour.

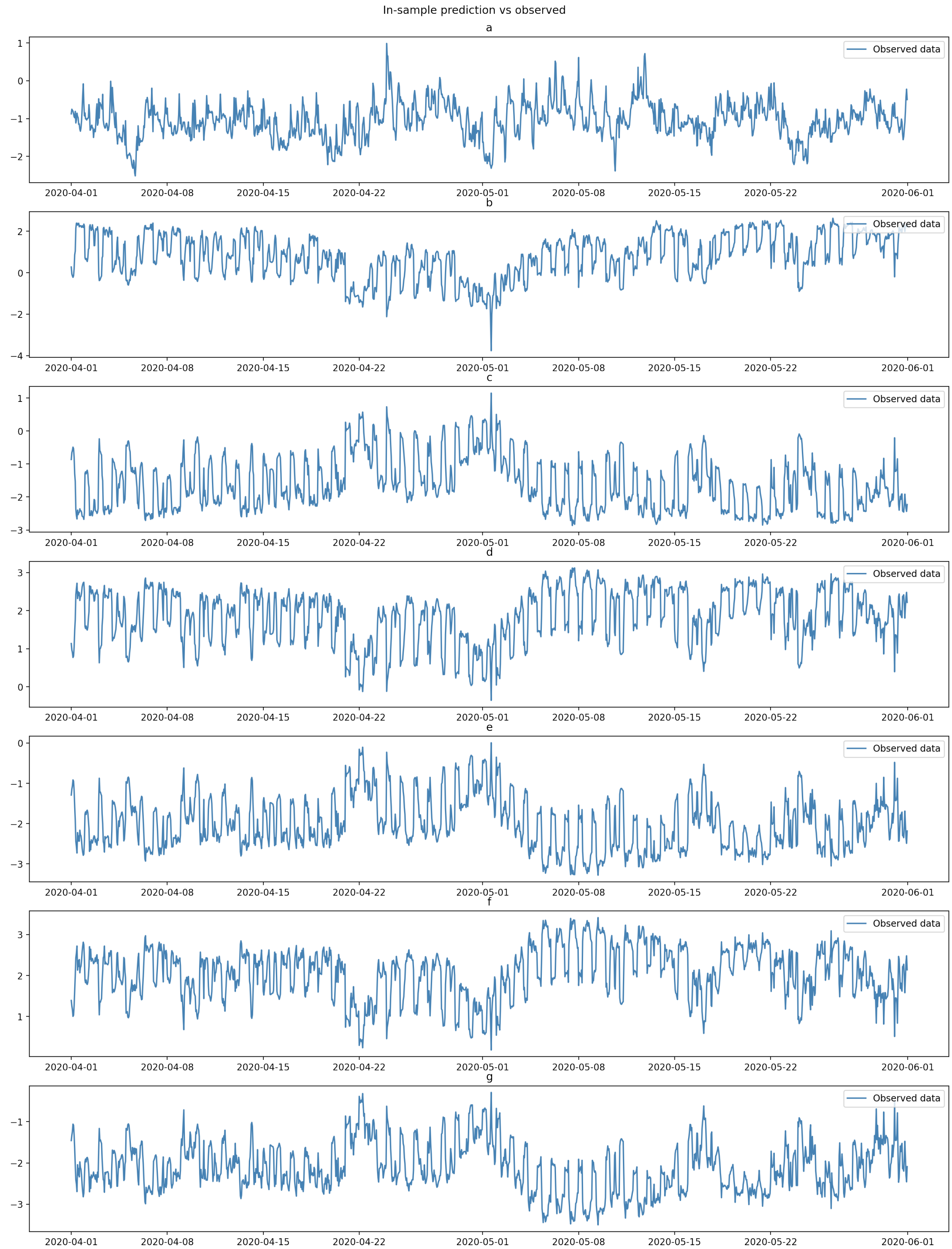

Figure 4.4: Observed standardized parameters for the period range Apr-20 to May-20.

In fact, if you zoom in the time series and observe any random two weeks (Fig. 4.5) a seasonal behaviour is clear. In electricity day-ahead markets seasonal behaviour is very common and arises mainly from demand. The workday and work week cycle dictates the shape of the demand, which in turn highly influence the electricity prices. This means that in the data we will most likely find two seasonal periods: 24 hours (daily seasonality) and 168 hours (weekly seasonality).

4.1.5.1 Seasonality of the parameters

In time series analysis, the presence of various seasonal patterns has always been a challenge, as most models do not naturally incorporate them. Among all the forecasting techniques, the seasonal ARIMA (SARIMA) model [19] and the exponential smoothing [146, 63] are the most common and classic approaches, however the basic forms of this methods are only suited in modelling single seasonality, and unable to account for multiple seasonal patterns.

The first successful approach for complex multi-seasonality was built under state space formulations in [96] and is composed of a Trigonometric seasonal effect, a Box-Cox transformation, an ARMA model, a Trend and Seasonal components, with the acronym TBATS. This model is still considered the state-of-the-art for multiple seasonality models and accounts for multiple, non-integer seasonality.

In order to include seasonality in the model, I first studied the periodicity of the data series by computing its power spectrum and analysing the periodogram. The periodogram is an estimate of the power spectrum of a time series, i. e., it shows the strength of the variations of the energy as a function of the frequency. It refers to the spectral energy distribution that would be found per unit time, since the total energy of such a signal over all time would generally be infinite. Summation or integration of the spectral components yields the total power (for a physical process) or variance (in a statistical process).

The spectrum of a physical process $x(t)$ often contains essential information about the nature of x. For instance, the pitch and timbre of a musical instrument are immediately determined from a spectral analysis. The colour of a light source is determined by the spectrum of the electromagnetic waves electric field as it fluctuates at an extremely high frequency.

The power spectrum is important in statistical signal processing and in the statistical study of stochastic processes, as well as in many other branches of physics and engineering. Typically the process is a function of time, but one can similarly discuss data in the spatial domain being decomposed in terms of spatial frequency. The mathematical process used for this decomposition is the Fourier transform.

For a given time series process Z_t, the spectral representation is given by

$$Z_t = \int_{-\pi}^{\pi} e^{i\omega t} dU(\omega), \; -\pi \leq \omega \leq \pi \tag{4.11}$$

where $dU(\omega)$ is a complex-valued orthogonal stochastic process for each ω such that

$$E\left[dU(\omega)\right] = 0, \tag{4.12}$$

$$E\left[dU(\omega)dU^*(\lambda)\right] = 0, \text{ for all } \omega \neq \lambda \tag{4.13}$$

$$\text{and } E\left[|dU(\omega)|^2\right] = E\left[dU(\omega)dU^*(\omega)\right] = dF(\omega) \tag{4.14}$$

where $U^*(\omega)$ is the complex conjugate of $U(\omega)$.

Let γ_k be the autocovariance function of Z_t. Its sprectral represetation is

$$\gamma_k = \int_{-\pi}^{\pi} e^{i\omega k} dF(\omega), \; -\pi \leq \omega \leq \pi \tag{4.15}$$

where $F(\omega)$ is the spectral distribution function. When the autocovariance is absolutely summable, the spectral density exists and is equal to

$$f(\omega)d\omega = dF(\omega) \tag{4.16}$$

In this case, we have

$$f(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma_k e^{-i\omega k}, \; -\pi \leq \omega \leq \pi \tag{4.17}$$

which means that the power spectral density is the Fourier transform of the autocovariance function γ_k.

Recalling that the Fourier transform of the time series Z_t at the Fourier frequencies $\omega_j = \frac{2\pi j}{n}, -\frac{n-1}{2} \leq j \leq \frac{n}{2}$, is

$$y(\omega_j) = \frac{1}{\sqrt{2\pi n}} \sum_{t=1}^{n} Z_t e^{-i\omega_j t} \tag{4.18}$$

the sample spectrum is given by

$$\tilde{f}(\omega_j) = y(\omega_j)y^*(\omega_j) \tag{4.19}$$

$$= \frac{1}{2\pi n} \sum_{t=1}^{n} \sum_{r=1}^{n} Z_t Z_r e^{-i\omega_j (t-r)} \tag{4.20}$$

After some algebraic manipulation, we obtain that

$$\tilde{f}(\omega_j) = \frac{1}{2\pi}\sum_{k=-(n-1)}^{n-1}\hat{\gamma}_k e^{-i\omega_j k} \tag{4.21}$$

$$= \frac{1}{2\pi}\left[\hat{\gamma}_0 + 2\sum_{k=1}^{n-1}\hat{\gamma}_k e^{-i\omega_j k}\right] \tag{4.22}$$

where $\hat{\gamma}_k$ is the sample autocovariance function and is given by

$$\hat{\gamma}_k = \frac{1}{n}\sum_{t=1}^{n-k} Z_t Z_{t+k} \tag{4.23}$$

Although $\tilde{f}(\omega_j)$ is an unbiased estimate, since the sample spectrum is only defined at the Fourier frequencies and its variance is independent of the sample size n, it is a rather poor estimate of the spectrum. To solve this problem, we introduce a suitable kernel or spectral window to smooth the sample spectrum, i.e.,

$$\tilde{f}(\omega_p) = \sum_{j=-M_n}^{M_n} W_n(\omega_j)\tilde{f}(\omega_p - \omega_j) \tag{4.24}$$

where ω_j are the Fourier frequencies, M_n is a function of n such that $M_n \to \infty$ but $\frac{M_n}{n \to 0}$ as $n \to \infty$ and $W_n(\omega_j)$is the kernel or spectral window that is a weighting function with the following properties

$$\sum_{j=-M_n}^{M_n} W_n(\omega_j) = 1, \tag{4.25}$$

$$W_n(\omega_j) = W_n(\omega_{-j}), \tag{4.26}$$

$$\lim_{n\to\infty}\sum_{j=-M_n}^{M_n} W_n^2(\omega_j) = 0 \tag{4.27}$$

Applying the spectral window to the sample autocovariance in 4.24

$$\hat{f}(\omega) = \frac{1}{2\pi}\sum_{k=-M}^{M} W(k)\hat{\gamma}_k e^{-i\omega k} \tag{4.28}$$

the result is a smoothed power spectrum which is a more accurate representation for the true spectrum.

The computation of the power spectrum is a useful tool for analysing seasonal and cyclic components in the time series. It allows for the calculation of harmonic components present in the parameters of the market curves, i.e., identifies cyclic patters and its frequency. A frequency with a higher power is also a frequency where the autocovariance is higher, representing a significant seasonality.

The spectrum obtained from the calculated parameters can be observed in the fig. 4.6.

The analysis of the spectrum is easier when split in two: high frequencies and low frequencies. For low frequencies the spectrum shows a peak at frequency $f = 0.005952$, representig a cycle of 168 hours, i.e., weekly seasonality. For low frequencies, the highest peak is shown at frequency $f = 0.04167$, representing a cycle of 24 hours, i.e., daily seasonality. In both cases the spectrum shows lower peaks at frequencies that are multiples of the ones already identified.

4.1.5.2 Correlations of the parameters

The main purpose of the model is to find the market curve from a set of latent variables. The statistical linear relationship between two random variables can be measured by calculating the correlation between them. In most forecasting methods, the study of the correlations is the starting point for variable selection.

The correlation between the parameters and a set of important market variables can be seen in fig. 4.7.

It is clear that there is a high, positive and negative, correlation between the different parameters, we can also observe high correlation with the load forecast, thermal gap and gas prices. This correlations give some hint on how we should approach the model construction.

The exogenous variables used for this analysis were the demand forecast for Spain, Portugal, France and Germany, wind production forecast for Spain, Portugal, France and Germany, thermal gap forecast for Iberia, the price of gas and the price of CO_2.

4.1.6 Curve forecast model

From the analysis of the parameters there are important remarks:

1. Complex seasonality patterns, with two distinct cycles: 24hours and 168hours,
2. High correlations between the different parameters to estimate, meaning that a multivariate model capable of capturing these correlations is needed,
3. High linear correlation to known market variables.

The model proposed consists of a combination a Vector Autoregression (VAR) model with a seasonal harmonic component of periods 24 and 168 and dynamic exogenous variables. This model captures the autoregressive behaviour of the time series as well as the autocorrelations between the parameters. The harmonic component helps to correctly model the seasonality of the parameters and, since the electricity prices are highly dependent on external variables like electricity demand or wind and solar production, which are time series *per se*, the dynamic external variables are essential to capture the dynamics of the market curves.

The autoregressive models were lightly described in 2.3.1, recalling the notation, we can write our model:

$$\boldsymbol{\phi}(B)_p \mathbf{Y}_t = \sum_{i=0}^{l} \boldsymbol{\beta}_i \mathbf{X}_{t-i} + \sum_{k=0}^{K} \left(\mathbf{a_1} \cos\left(\frac{2\pi}{P} kt\right) + \mathbf{a_2} \sin\left(\frac{2\pi}{P} kt\right)\right) + \mathbf{c} + \epsilon_t \tag{4.29}$$

Where $\mathbf{Y}_t = (y_{1,t}, y_{2,t}, \ldots, y_{m,t})$ are the m endogenous market curve parameters, B is the backward shift operator $B^h X_t = X_{t-h}$, $\phi(B)$ is the notation for $\boldsymbol{\phi}(B) = 1 - \boldsymbol{\phi}_1 B - \cdots - \boldsymbol{\phi}_p B^p$, $\boldsymbol{\phi}$ are coefficient $m \times m$ matrices for the autoregressive terms, $\boldsymbol{\beta}_i$ are l coefficient $(m \times n)$ matrices for the n dynamic exogenous variables for each lag l, $\mathbf{a_1}$ and $\mathbf{a_2}$ the coefficient vectors $(m \times 1)$ for the harmonics and ϵ_t a white noise process with mean 0 and covariance matrix $\boldsymbol{\Sigma}$.

The parameters can be estimated with ordinary least squares method by easily transforming the model into a simple regression:

$$\mathbf{Y} = \mathbf{LU} + \epsilon \tag{4.30}$$

where

$$\mathbf{Y} = \begin{pmatrix} \mathbf{Y}_t \\ \mathbf{Y}_{t-1} \\ \vdots \\ \mathbf{Y}_{t-p+1} \\ \mathbf{X}_t \\ \mathbf{X}_{t-1} \\ \vdots \\ \mathbf{X}_{t-l} \\ a_1 \\ a_2 \\ 1 \end{pmatrix}, \quad \mathbf{U} = \begin{pmatrix} \mathbf{Y}_{t-1} \\ \mathbf{Y}_{t-2} \\ \vdots \\ \mathbf{Y}_{t-p} \\ \mathbf{X}_t \\ \mathbf{X}_{t-1} \\ \vdots \\ \mathbf{X}_{t-l} \\ a_1 \\ a_2 \\ 1 \end{pmatrix} \tag{4.31}$$

and

$$\mathbf{L} = \begin{pmatrix} \phi_1 & \phi_2 & \dots & \phi_{p-1} & \phi_p & \beta_1 & \beta_2 & \dots & \beta_l & c \\ I_m & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & I_m & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I_m & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & I_m & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & I_m & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & I_m & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & I \end{pmatrix} \tag{4.32}$$

The ordinary least squares estimate of **L** can be obtained by

$$\hat{\mathbf{L}} = \mathbf{Y}U(UU')^{-1} \tag{4.33}$$

This estimates assume that the time series **Y** a normally distributed Gaussian stationary process, and, if no parameter restrictions are employed on the VAR system, the Ordinary Least Squares (OLS) estimator is identical to the Maximum Likelihood (ML) estimator.

The hyperparameters p, l and K can be obtained by performing a search on the hyperparameter space. First we evaluate of the model results with different hyperparameters and then we compare the results and choose the best combination possible. For autoregressive models, it's common to evaluate the hyperparameters by using the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC). In this case, since the main purpose is to find correct market curves I will evaluate the hyperparameters by compairing the residuals of the different models.

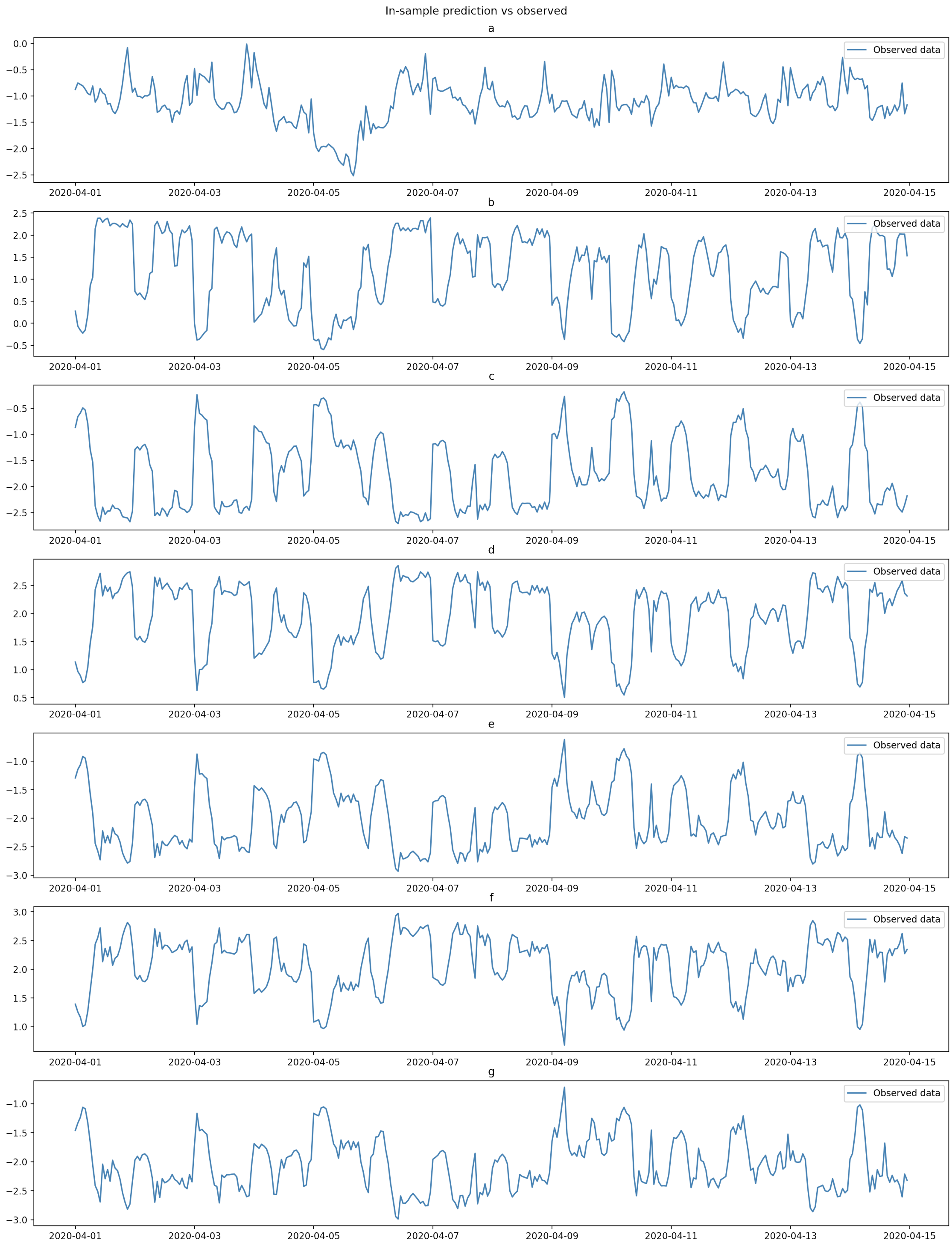

Figure 4.5: Observed standardized parameters for 2 weeks. It is possible to see some degree of daily and weekly seasonality.

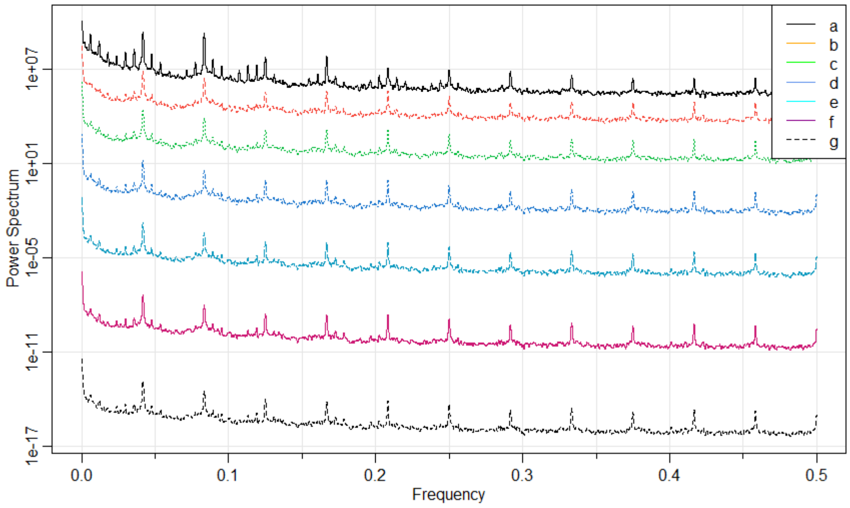

Figure 4.6: Calculated power spectrum for the market curve parameters.

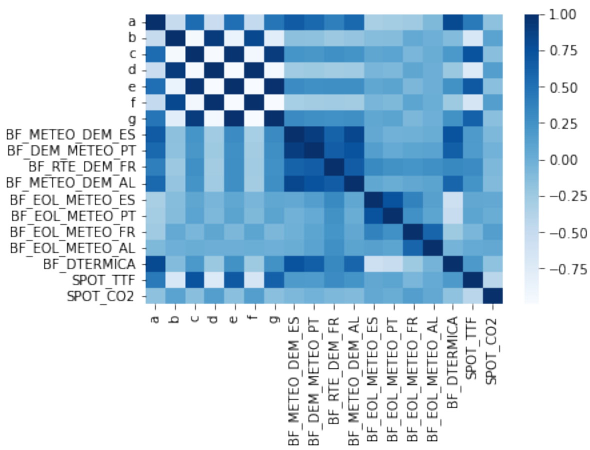

Figure 4.7: Correlation matrix for the parameters and a selection of latent variables

5

Empirical Results

In this chapter I present the results obtained from the model implementation, as well as a benchmark with other models from the industry.

In a VAR model, it's important to guarantee that all variable time series are stationary. To check the stationarity of the parmeters of the market curves, I used the Augmented Dickey-Fuller Test (ADF), which tests the null hypothesis that a unit root is present in the sample. The p-value obtained for all the parameters was $p < 0.005$ rejecting the null hypothesis and assuming that the time series are indeed stationary.

The model proposed has a large number of hyperparameters: the number of lags p for the VAR model, the number of lags l for the exogenous variables and the max iteration K for Fourier Series. The hyperparameters were estimated by performing a grid search. I tried a range of values for each parameter: p between 1 and 168, l between 1 and 24 and K between 1 and 12; and evaluated the model performance in three distinct ways: computing information criterion AIC, BIC, by calculating the MAE and WMAE of each parameter and by calculating the MAE of the resulting electricity price. Since the main target of the model is to produce forecasts for the the 24 hours ahead the main metric used to evaluate its performance is the MAE and WMAE, objectively, the combination with a lower combined MAE for the 7 parameters was chosen.

The criterions were evaluated for one year of insample data and the errors for the parameters and price were calculated for 1 month of outsample predictions and the results can be summarized in:

1. The AIC shows a descending trend until $p = 72$ and an ascending trend for higher values. This shows that, for this criterion, the model should be complex enough, having a high number of lags in the autoregressive term;

2. On the other hand, the BIC and HQIC seem to be lower with lower parameters (low p, l and K) and start rising rapidly, suggesting a more simple model. However, the outsample residuals of parameters and price for the models with lower BIC/HQIC, are higher than what is expected in the industry.

3. For the parameters error, the higher order parameters MAE was lower with $p = 24$, but for the lower order parameters, the lowest MAE was found for $p = 72$. Since we are in the presence of 7 distinct parameters, it is not possible to find a scenario where all the parameters MAE are lower, however, when comparing the two scenarios just mentioned, the average of the MAE is lower for $p = 72$.

4. The price MAE is lower when $p = 48$ or $p = 72$, showing some relationship with the parameters MAE.

5. For every measure, the best models for the remaining hyperparameter were found for $l = 24$ and $K = (K_1, K_2) = (10, 1)$

After observing this results, the model chosen consisted on the hyperparameters $p = 72$, $l = 24$ and $K = (10, 1)$. I ran this model for 2020 and calculated the outsample predictions for each hour and then compared to some of the best models used in the industry, both statistical and machine learning.

5.1 Residual analysis

The analysis of the residuals of the model is essential in forecasting models. From (4.29) we expect the residuals of the model to be a white noise process with mean 0, if not, there are still some factors that the model does not take into account, leading to some kind of bias in the results.

In the VAR model, one of the most recurrent problem is the autocorrelation of the residuals. Autoregressive models tend to propagate errors along the forecasts, meaning that the residuals will have high correlations with each other for different lags. For this study in particular, I divided the analysis in two: the residuals of the parameters, which are the direct results of the model, and the residuals of the calculated electricity price, that is obtained by calculating the root of a 6^{th} degree polynomial with the parameters obtained from the model. This is done because the electricity price is also an important figure that can be taken from the predicted curves, probably the most important, and we have to ensure that there is no bias in the prices obtained.

I tested for normality by plotting the histogram and Q-Q Plot of the residuals from both parameters (Fig. 5.1 and 5.3) and electricity prices (Fig. 5.2 and 5.4). The histogram look symmetric for all parameters and the electricity prices, but the Q-Q plots show that the residuals have a fat-tail when compared to the normal distribution, although not the direct scope of this work, this may imply that calculation of prediction intervals has to be taken with caution.

The autocorrelation was tested with the Durbin-Watson test and validated by observing the Autocorrelation Factors (ACF) and Partial Autocorrelation Factors (PACF). The result of the test was close to 2 for all parameters (2.0 for parameter a and 1.99 for the remaining parameters) and for the price, showing that there are no significant autocorrelations in the residuals (Fig. 5.5 and 5.6).

The same conclusion can be taken by looking at the PACF of the parameters, where the values obtained are within confidence intervals for all lags. However, when observing the ACF, it's possible to see significant autocorrelations for lags until around 24 hours. One of the possible reasons for this autocorrelation is the fact that we observe new data in blocks of 24 hours, meaning that everyday we are forecasting 24 new parameters for the day-ahead, and everyday the new 24 market curves are known at the same time. This can lead to some bias in the data, because when we over (or under) estimate the parameters the effect may propagate for 24 hours (Fig. 5.7 and 5.8).

5.2 Model benchmark

The model has been benchmarked against other models used in the industry, both statistical and machine learning, and the results obtained can be seen in table 5.1.

The statistical models are dynamic regression models with different combination of the exogenous variables already discussed in this work, and used in the market curves VAR model. The Extreme Learning Machines (ELM) models have a 500 sample random runs and their average is considered as the final forecast, they use different exogenous variabes between them, but the same hyperparameters. The XGBOOST have a significant difference to the other models: they use the results from the other models, statistical and machine learning, as inputs.

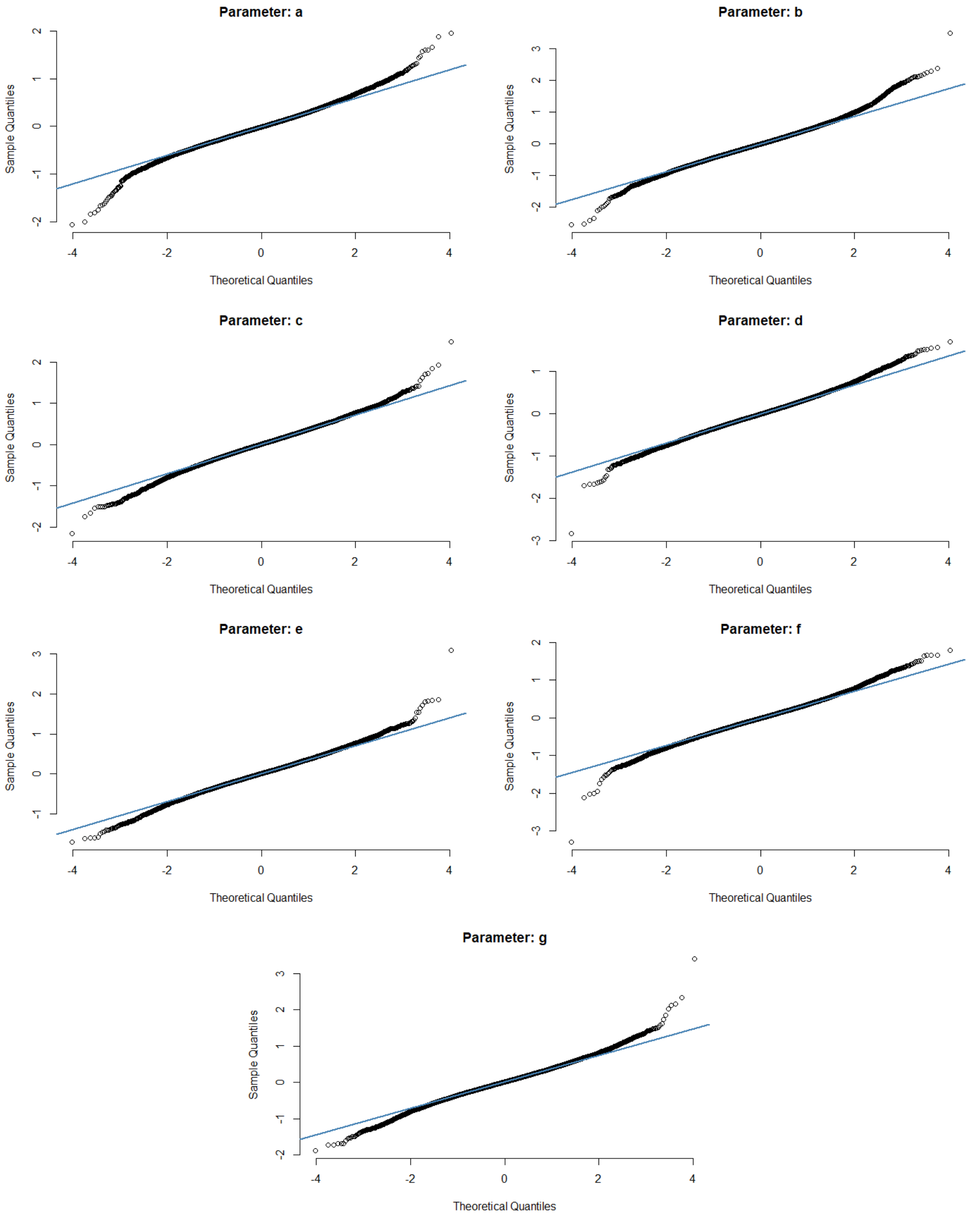

Figure 5.1: QQ-Plot of the residuals for each of the parameters of the market curve

This models forecast the electricity hourly price itself, so, in order to benchmark the model proposed in this thesis, the metric used for comparison is the electricity price MAE and WMAE, although, the model actually forecast the market curves for each hour of the day-ahead. The results can be seen in Table 5.1.

It is shown that the market curve model outperforms the different statistical models

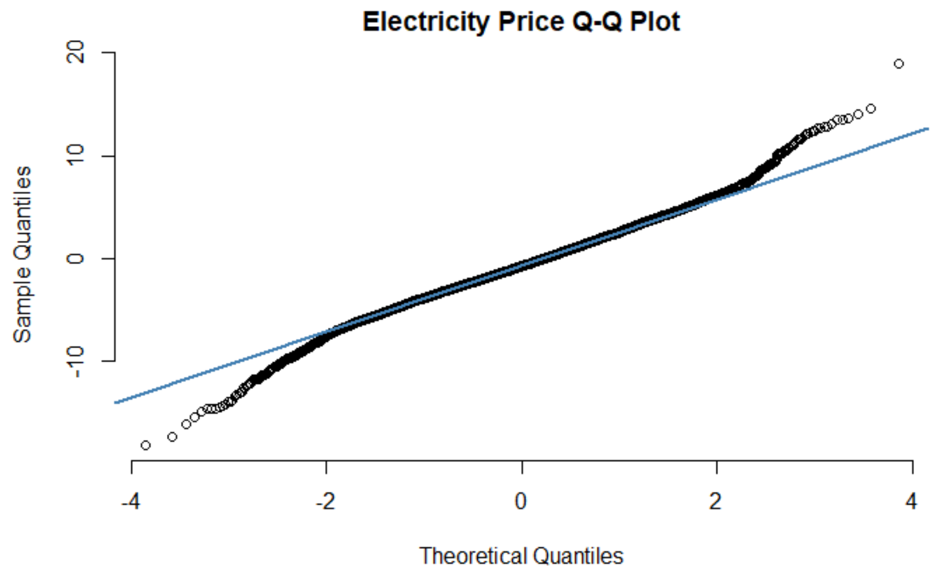

Figure 5.2: Q-Q plot for the residuals of the electricity prices

Model	MAE (€/MWh)	WMAE
Market Curves	2.7	8.0%
Statistical1	3.6	10.6%
Statistical2	3.7	10.9%
Statistical3	3.4	10.0%
Statistical3	3.7	10.8%
ELM1	3.5	10.2%
ELM2	3.5	10.3%
XGBOOST1	2.8	8.3%
XGBOOST2	2.7	7.9%

Table 5.1: Model results and benchmark against industry models

already employed in the industry and some machine learning models, like the ELM, just falling short of the XGBOOST models. However, as already stated, this models work as an assemble for the forecasts obtained with the other methods, using them as an input. All the known market variables exogenous to the other models are also used in XGBOOST as dependent variables.

In the evaluation of the model, I found that in 6 hours (0.000683% of the cases) the forecasted 6^{th} degree polynomial did not have a root in an acceptable range of prices, and therefore, it was not possible to find a MCP. For this reason, these hours were removed of the analysis. The method for the electricity price calculation still needs some work so it can account for ths cases.

Since the proposed model outperforms the pure electricity price forecasting ones -

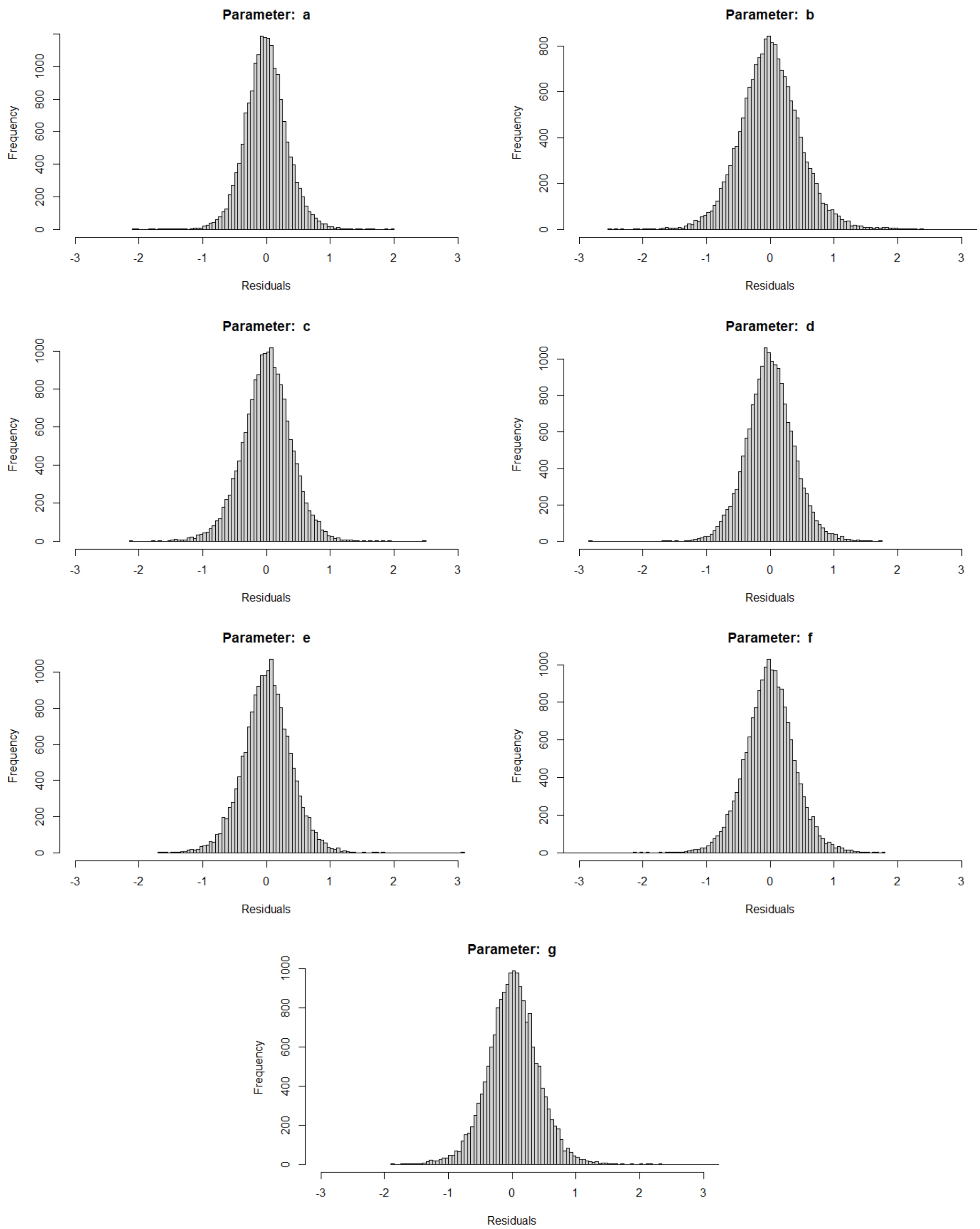

Figure 5.3: Histogram of the residuals of the parameters

statiscal and ELM - it is reasonable to conclude that it captures some information that the others cannot find, leading to lower errors. On the other hand, the comparison with XGBOOST is proof of what is seen in the literature, ensemble models have better performance than individual forecasting models, being able to capture the advantages of each models while evading the disadvantages. Being this an assemble model, it is possible

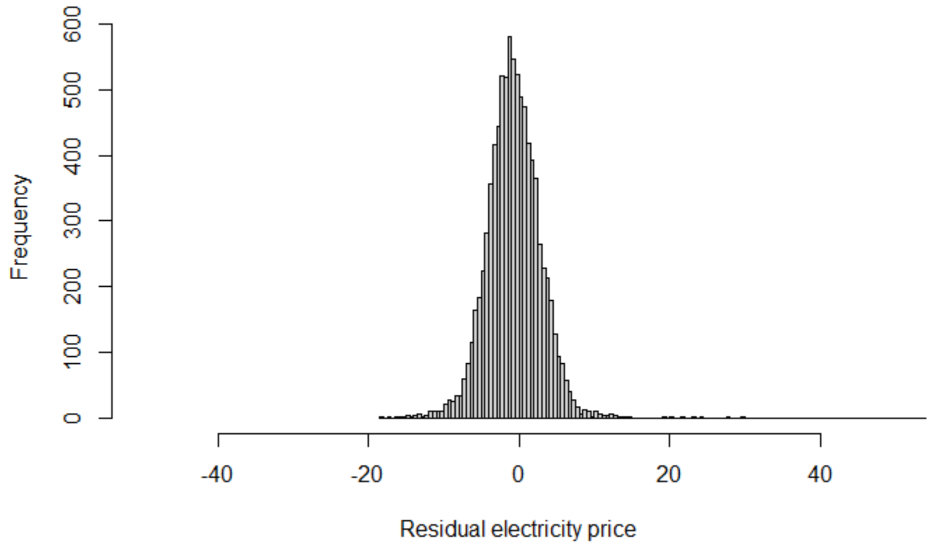

Figure 5.4: Histogram of the residuals of the electricity prices

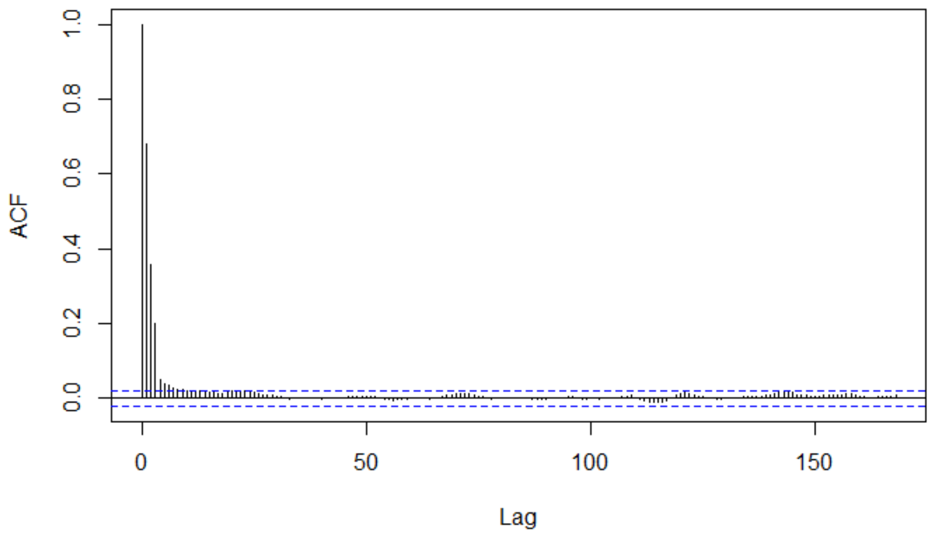

Figure 5.5: Autocorrelation factors for the electricity prices

to include the new market curve results in the pool of forecasts already used. By doing this, the MAE of the XGBOOST can decrease to 2.24€/MWh as seen in Table 5.2.

This means that, not only the market curve model outperforms all the other pure price forecasting models, but also adds relevant information to the ensemble, significantly improving its results. This shows that the proposed model is useful on capturing market dynamics that the other models cannot, dynamics that are only obtainable with the ask/bid offers information.

Besides that, the model has shown a good adaptability when there are significant price

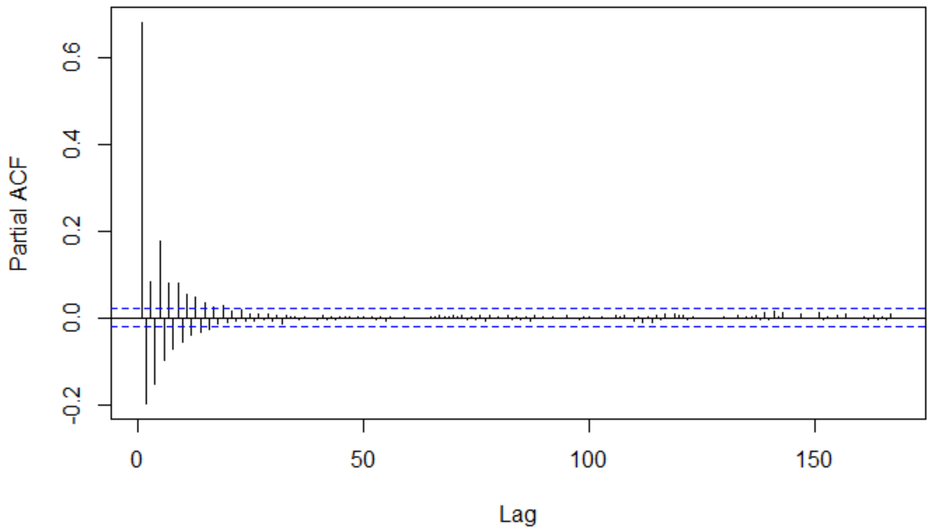

Figure 5.6: Autocorrelation factors for the electricity prices

Model	MAE (€/MWh)	WMAE
Market Curves	2.7	8.0%
XGBOOST2	2.7	7.9%
XGBOOST with Market Curves	2.2	6.6%

Table 5.2: Using the market curve forecasts in the XGBOOST clearly improves the forecasts

Model	MAE (€/MWh)	WMAE
Market Curves	3.5	9.8%
XGBOOST2	4.1	11.5%
XGBOOST with Market Curves	3.1	8.7%

Table 5.3: MAE of the models when evaluated on the days where the average daily price changes by more than 3 standard deviations

changes, being able to capture regime changes better than any other model in the industry. by calculating the MAE of the models in the days, where the average daily price changes by more than three standard deviations, we can see a lower error than in the XGBOOST that can be improved even further by including the market curve price forecasts in the XGBOOST, as shown in Table 5.3.

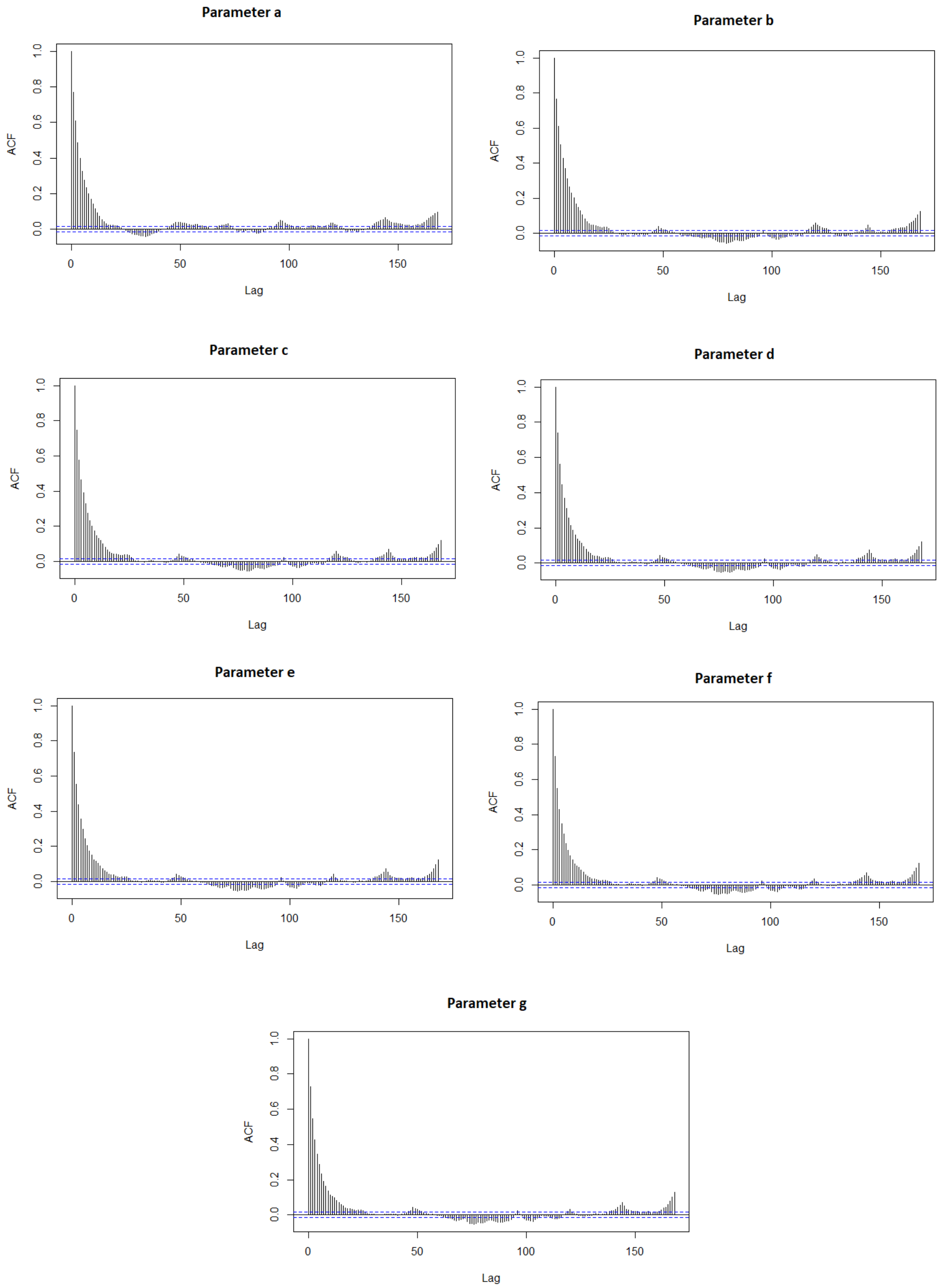

Figure 5.7: Autocorrelation factors for the parameters residuals

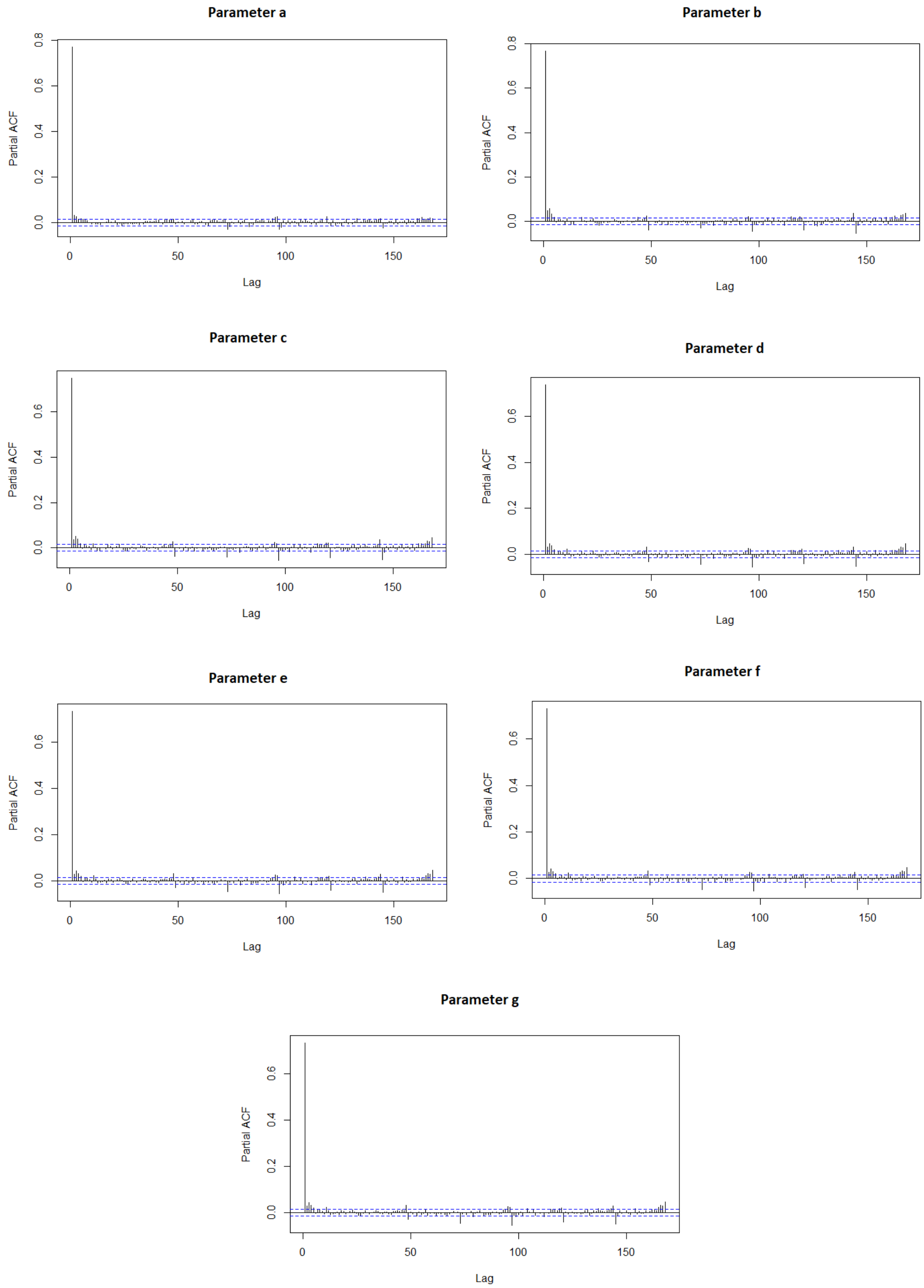

Figure 5.8: Partial autocorrelation factors for the parameters residuals

6

Conclusions and Future Work

The Electricity Price Forecasting field is a very varied field that is attactive for many reasons: the dynamics of the market are unique, with high volatilities, big complexity and a chance for arbitrage because of the many organized markets available; the importance of energy in the actual contest, with direct impact in the world infrastructure and global climate changes; the challenge of predict electricity prices together with the possibility to employ a large amount of mathematical techniques. In the industry point of view, it is a must have for energy management, since it is impossible to correctly dispatch energy or manage risk without accurate electricity price predictions.

In the actual contest, accurate predictions become more than just a management tool, they are a differentiating factor from the competition, researching and investing in new EPF models is needed to keep the competitiveness. This thesis presents not only a new method for EPF, but a new framework for reaserchers to work with. Looking at the market curves instead of the electricity price has prooven more efficient than the traditional point forecasts for electricity price.

A new market curve model was developed for forecasting electricity prices. This new approach is not only useful for electricity price forecasting, but also brings clear advantegous for risk management in the industry. To the best of my knowledge, there is no other model that makes this approach to electricity price forecasting, as models and methods proposed often disregard the market curves structures that is behind the market clearing price.

The electrictricity price forecasts made by the market curve model outperformed traditional statistical models and some machine learning models and also showed a capacity to adapt to extreme price changes. Regime changes in time series are alway hard to forecast without a complete knowledge of the underlying processes, in the electricity price forecast, the underlying process are the market curves, incorporating them into the forecasts increases the adaptability of the model in scenarios where there are abrupt changes to the price.

It has been shown, through the model results, that the offers of the agents in the market are important and should be included in the forecasting methods available.

Althought the model proposed in this thesis in classic in the sense of the techniques used - Vector Autoresgression with exogenous variables and an harmonic factor for seasonality - it was able to obtain better results than the models that are actually used by the industry in its everyday operation. This shows that the method of using and forecasting the market curves/offers, which is a novel method built for this nodel, is essential for EPF and must be studied.

This approach is inovative in the sense that the model described in the literature do not incorporate the information about the bids/asks in their price forecasts, and this is, most probably, the first model with a pratical application to be designed with the market curves and offers in the market at its core.

There are two important remarks to take from this work: First, the job of modelling, identifying and forecasting market curves is complex and there is still much room to improve. The second remark is that incorporating this knowledge to the electricity price forecasting gives better results than traditional models and gives additional information that are not usually captured by the remaining models.

As for future work, the first remark has to be approached, this work suggested a parametrization of the curves by fitting them to a known easier to work function and then analyzing this parameters, however, more complex methods, with a higher number of parameters, can be used and will probably lead to better overall forecasts. Besides that, the comparison of curves can be done using different techniques, by defining a distance betweem them it is possible to apply other techniques, like clustering, that could give additional information about the market curves. The second remark suggests that more complex models can be used, like applying feature selection techniques to the exogenous variables, like the state-of-art statistical models, or use machine learning techniques for curve forecasting.

Bibliography

[1] F. J. .Nogales et al. “Forecasting next-day electricity prices by time series models”. In: *IEEE Transactions on Power Systems* 17.2 (2002), pp. 342–348. DOI: 10.1109/TPWRS.2002.1007902 (cit. on p. 14).

[2] O. Abedinia et al. “Electricity price forecast using Combinatorial Neural Network trained by a new stochastic search method”. In: *Energy Conversion and Management* 105 (2015), pp. 642–654. ISSN: 0196-8904. DOI: https://doi.org/10.1016/j.enconman.2015.08.025. URL: https://www.sciencedirect.com/science/article/pii/S0196890415007712 (cit. on pp. 24, 25).

[3] S. Aggarwal, L. M. Saini, and A. Kumar. “Short term price forecasting in deregulated electricity markets: A review of statistical models and key issues”. In: *International Journal of Energy Sector Management* 3 (2009-11), pp. 333–358. DOI: 10.1108/17506220911005731 (cit. on p. 20).

[4] S. K. Aggarwal, L. M. Saini, and A. Kumar. “Electricity price forecasting in deregulated markets: A review and evaluation”. In: *International Journal of Electrical Power & Energy Systems* 31.1 (2009), pp. 13–22. ISSN: 0142-0615. DOI: https://doi.org/10.1016/j.ijepes.2008.09.003. URL: https://www.sciencedirect.com/science/article/pii/S0142061508000884 (cit. on p. 21).

[5] W. Ahmad et al. “Electricity Price Forecasting in Smart Grid: A Novel E-CNN Model”. In: 2019-03, pp. 1132–1144. ISBN: 978-3-319-98284-7. DOI: 10.1007/978-3-030-15035-8_109 (cit. on p. 24).

[6] R. Aïd, L. Campi, and N. Langrené. “A STRUCTURAL RISK-NEUTRAL MODEL FOR PRICING AND HEDGING POWER DERIVATIVES”. In: *Mathematical Finance*

23.3 (2013), pp. 387–438. DOI: https://doi.org/10.1111/j.1467-9965.2011.00507.x. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1467-9965.2011.00507.x. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-9965.2011.00507.x (cit. on p. 19).

[7] D. Aineto et al. "On the Influence of Renewable Energy Sources in Electricity Price Forecasting in the Iberian Market". In: *Energies* 12.11 (2019). ISSN: 1996-1073. DOI: 10.3390/en12112082. URL: https://www.mdpi.com/1996-1073/12/11/2082 (cit. on p. 24).

[8] N. Amjady. "Day-ahead price forecasting of electricity markets by a new fuzzy neural network". In: *IEEE Transactions on Power Systems* 21.2 (2006), pp. 887–896. DOI: 10.1109/TPWRS.2006.873409 (cit. on pp. 21, 23).

[9] N. Amjady. "Short-Term Bus Load Forecasting of Power Systems by a New Hybrid Method". In: *IEEE Transactions on Power Systems* 22.1 (2007), pp. 333–341. DOI: 10.1109/TPWRS.2006.889130 (cit. on p. 21).

[10] N. Amjady and M. Hemmati. "Day-ahead price forecasting of electricity markets by a hybrid intelligent system". In: *European Transactions on Electrical Power* 19 (2009-01), pp. 89–102. DOI: 10.1002/etep.242 (cit. on pp. 18, 21).

[11] Anamika, R. Peesapati, and N. Kumar. "Electricity Price Forecasting and Classification Through Wavelet–Dynamic Weighted PSO–FFNN Approach". In: *IEEE Systems Journal* 12.4 (2018), pp. 3075–3084. DOI: 10.1109/JSYST.2017.2717446 (cit. on pp. 24, 25).

[12] P. Areekul et al. "A New Method for Next-Day Price Forecasting for PJM Electricity Market". In: *International Journal of Emerging Electric Power Systems* 11.2 (2010). DOI: doi:10.2202/1553-779X.2266. URL: https://doi.org/10.2202/1553-779X.2266 (cit. on p. 21).

[13] A. Azadeh et al. "Optimum Long-Term Electricity Price Forecasting in Noisy and Complex Environments". In: *Energy Sources, Part B: Economics, Planning, and Policy* 8.3 (2013), pp. 235–244. DOI: 10.1080/15567249.2012.678559. eprint: https://doi.org/10.1080/15567249.2012.678559. URL: https://doi.org/10.1080/15567249.2012.678559 (cit. on p. 15).

[14] M. Barlow. "A Diffusion Model for Electricity Prices". In: *Mathematical Finance* 12 (2002-10), pp. 287–298. DOI: 10.1111/j.1467-9965.2002.tb00125.x (cit. on p. 19).

[15] J. Bastian et al. "Forecasting energy prices in a competitive market". In: *IEEE Computer Applications in Power* 12.3 (1999), pp. 40–45. DOI: 10.1109/67.773811 (cit. on p. 8).

[16] P. Bento et al. "A bat optimized neural network and wavelet transform approach for short-term price forecasting". In: *Applied Energy* 210 (2018), pp. 88–97. ISSN: 0306-2619. DOI: https://doi.org/10.1016/j.apenergy.2017.10.058. URL: https://www.sciencedirect.com/science/article/pii/S0306261917314782 (cit. on pp. 24, 25).

[17] R. Bisoi, P. K. Dash, and P. P. Das. "Short-Term Electricity Price Forecasting and Classification in Smart Grids Using Optimized Multikernel Extreme Learning Machine". In: *Neural Comput. Appl.* 32.5 (2020-03), pp. 1457–1480. ISSN: 0941-0643. DOI: 10.1007/s00521-018-3652-5. URL: https://doi.org/10.1007/s00521-018-3652-5 (cit. on pp. 24, 25).

[18] S. Bordignon et al. "Combining day-ahead forecasts for British electricity prices". In: *Energy Economics* 35.C (2013), pp. 88–103. DOI: 10.1016/j.eneco.2011.12.0. URL: https://ideas.repec.org/a/eee/eneeco/v35y2013icp88-103.html (cit. on p. 25).

[19] G. E. P. Box and G. M. Jenkins. "Seasonal models: Time Series Analysis, Forecasting and Control". In: San Francisco, CA: USA: Holden-Day, Inc., 1990. Chap. 9, pp. 327–366. ISBN: 978-1-118-67502-1. DOI: 10.1007/978-3-030-13929-2_13 (cit. on p. 39).

[20] G. E. P. Box and G. M. Jenkins. *Time Series Analysis: Forecasting and Control*. SF: Holde-Day, 1976. ISBN: 9780816211043 (cit. on p. 16).

[21] D. W. Bunn. "Forecasting loads and prices in competitive power markets". In: *Proceedings of the IEEE* 88.2 (2000), pp. 163–169. DOI: 10.1109/5.823996 (cit. on pp. 7, 8).

[22] D. W. Bunn. *Modelling Prices in Competitive Electricity Markets*. Chichester: Wiley, 2004. ISBN: 978-0-470-84860-9 (cit. on pp. vii, ix).

[23] J. Capelo et al. “ERGEG South-West Electricity REM: First achievements and market coupling”. In: *2008 5th International Conference on the European Electricity Market*. 2008, pp. 1–7 (cit. on p. 3).

[24] R. Carmona, M. Coulon, and D. Schwarz. “Electricity price modeling and asset valuation: A multi-fuel structural approach”. In: *Mathematics and Financial Economics* 7 (2012-05), pp. 167–202. DOI: 10.1007/s11579-012-0091-4 (cit. on p. 19).

[25] J. P. S. Catalão, H. M. I. Pousinho, and V. M. F. Mendes. “Hybrid wavelet-PSO-ANFIS approach for short-term electricity prices forecasting”. In: *2011 IEEE Power and Energy Society General Meeting*. 2011, pp. 1–1. DOI: 10.1109/PES.2011.6038968 (cit. on p. 23).

[26] J. Catalão et al. “Parameterisation effect on the behaviour of a head-dependent hydro chain using a nonlinear model”. In: *Electric Power Systems Research* 76.6 (2006), pp. 404–412. ISSN: 0378-7796. DOI: https://doi.org/10.1016/j.epsr.2005.09.002. URL: https://www.sciencedirect.com/science/article/pii/S0378779605002026 (cit. on p. 8).

[27] J. Catalão et al. “Short-term electricity prices forecasting in a competitive market: A neural network approach”. In: *Electric Power Systems Research* 77.10 (2007), pp. 1297–1304. ISSN: 0378-7796. DOI: https://doi.org/10.1016/j.epsr.2006.09.022. URL: https://www.sciencedirect.com/science/article/pii/S0378779606002422 (cit. on p. 21).

[28] N. Chaâbane. “A hybrid ARFIMA and neural network model for electricity price prediction”. In: *International Journal of Electrical Power & Energy Systems* 55 (2014), pp. 187–194. ISSN: 0142-0615. DOI: https://doi.org/10.1016/j.ijepes.2013.09.004. URL: https://www.sciencedirect.com/science/article/pii/S0142061513003773 (cit. on p. 21).

[29] Z. Chang, Y. Zhang, and W. Chen. “Effective Adam-Optimized LSTM Neural Network for Electricity Price Forecasting”. In: *2018 IEEE 9th International Conference on Software Engineering and Service Science (ICSESS)*. 2018, pp. 245–248. DOI: 10.1109/ICSESS.2018.8663710 (cit. on p. 24).

[30] Z. Chang, Y. Zhang, and W. Chen. “Electricity price prediction based on hybrid model of adam optimized LSTM neural network and wavelet transform”. In: *Energy* 187 (2019), p. 115804. ISSN: 0360-5442. DOI: https://doi.org/10.101

6/j.energy.2019.07.134. URL: https://www.sciencedirect.com/science/article/pii/S0360544219314768 (cit. on pp. 24, 25).

[31] H.-p. Chao and H. G. Huntington. *Designing Competitive Electricity Markets*. Boston, MA: Springer, 1998. ISBN: 978-0-7923-8282-9 (cit. on p. 3).

[32] J. Che and J. Wang. "Short-term electricity prices forecasting based on support vector regression and Auto-regressive integrated moving average modeling". In: *Energy Conversion and Management* 51.10 (2010), pp. 1911–1917. ISSN: 0196-8904. DOI: https://doi.org/10.1016/j.enconman.2010.02.023. URL: https://www.sciencedirect.com/science/article/pii/S0196890410000890 (cit. on pp. 18, 22).

[33] X. Chen et al. "Electricity Price Forecasting With Extreme Learning Machine and Bootstrapping". In: *IEEE Transactions on Power Systems* 27.4 (2012), pp. 2055–2062. DOI: 10.1109/TPWRS.2012.2190627 (cit. on p. 21).

[34] Y. Chen et al. "BRIM: An Accurate Electricity Spot Price Prediction Scheme-Based Bidirectional Recurrent Neural Network and Integrated Market". In: *Energies* 12.12 (2019). ISSN: 1996-1073. DOI: 10.3390/en12122241. URL: https://www.mdpi.com/1996-1073/12/12/2241 (cit. on p. 24).

[35] R. A. Chinnathambi et al. "Deep Neural Networks (DNN) for Day-Ahead Electricity Price Markets". In: *2018 IEEE Electrical Power and Energy Conference (EPEC)*. 2018, pp. 1–6. DOI: 10.1109/EPEC.2018.8598327 (cit. on p. 23).

[36] A. J. Conejo et al. "Day-ahead electricity price forecasting using the wavelet transform and ARIMA models". In: *IEEE Transactions on Power Systems* 20.2 (2005), pp. 1035–1042. DOI: 10.1109/TPWRS.2005.846054 (cit. on pp. 15, 17).

[37] A. J. Conejo et al. "Forecasting electricity prices for a day-ahead pool-based electric energy market". In: *International Journal of Forecasting* 21.3 (2005), pp. 435–462. ISSN: 0169-2070. DOI: https://doi.org/10.1016/j.ijforecast.2004.12.005. URL: https://www.sciencedirect.com/science/article/pii/S0169207004001311 (cit. on p. 8).

[38] J. Contreras et al. "ARIMA models to predict next-day electricity prices". In: *IEEE Transactions on Power Systems* 18.3 (2003), pp. 1014–1020. DOI: 10.1109/TPWRS.2002.804943 (cit. on p. 17).

[39] J. Crespo Cuaresma et al. "Forecasting electricity spot-prices using linear univariate time-series models". In: *Applied Energy* 77.1 (2004), pp. 87–106. ISSN: 0306-2619. DOI: https://doi.org/10.1016/S0306-2619(03)00096-5. URL: https://www.sciencedirect.com/science/article/pii/S0306261903000965 (cit. on pp. 17, 18).

[40] A. Cruz et al. "The effect of wind generation and weekday on Spanish electricity spot price forecasting". In: *Electric Power Systems Research* 81.10 (2011), pp. 1924–1935. ISSN: 0378-7796. DOI: https://doi.org/10.1016/j.epsr.2011.06.002. URL: https://www.sciencedirect.com/science/article/pii/S0378779611001313 (cit. on pp. 18, 21).

[41] R. Ethier et al. "A uniform price auction with locational price adjustments for competitive electricity markets". In: *International Journal of Electrical Power & Energy Systems* 21.2 (1999), pp. 103–110. ISSN: 0142-0615. DOI: https://doi.org/10.1016/S0142-0615(98)00034-9. URL: https://www.sciencedirect.com/science/article/pii/S0142061598000349 (cit. on p. 7).

[42] C. of the European Union. *Directive 2003/54/EC. Common rules for internal market in electricity*. https://eur-lex.europa.eu/resource.html?uri=cellar:caeb5f68-61fd-4ea8-b3b5-00e692b1013c.0004.02/DOC$_1&$format=PDF. 2003 (cit. on p. 2).

[43] C. of the European Union. *Directive 2009/72/EC. Common rules for internal market in electricity*. https://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2009:211:0055:0093:EN:PDF. 2009 (cit. on p. 2).

[44] C. of the European Union. *Directive 96/92/EC. Common rules for internal market in electricity*. https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:31996L0092$&$from=EN. 1996 (cit. on p. 2).

[45] A. Eydeland and K. Wolyniec. *Energy and Power Risk Management: New Developments in Modeling, Pricing, and Hedging*. Hoboken, NJ: Wiley, 2003. ISBN: 978-0-471-10400-1 (cit. on pp. vii, ix, 19).

[46] S. Fan, C. Mao, and L. Chen. “Next-day electricity-price forecasting using a hybrid network”. In: *Generation, Transmission & Distribution, IET* 1 (2007-02), pp. 176–182. DOI: 10.1049/iet-gtd:20060006 (cit. on p. 22).

[47] L. A. F. M. Ferreira et al. “Short-term resource scheduling in multi-area hydrothermal power systems”. In: *Electrical Power Energy Systems* 11.3 (1989), pp. 200–212 (cit. on p. 8).

[48] O. Fosso et al. “Generation scheduling in a deregulated system. The Norwegian case”. In: *IEEE Transactions on Power Systems* 14.1 (1999), pp. 75–81. DOI: 10.1109/59.744487 (cit. on p. 8).

[49] M. Friedman. *A program for monetary stability*. New York, NY: Fordham University Press, 1960. ISBN: 978-0823203710 (cit. on p. 6).

[50] W. Gao et al. “Combination of fuzzy based on a meta-heuristic algorithm to predict electricity price in an electricity markets”. In: *Chemical Engineering Research and Design* 131 (2018). Energy Systems Engineering, pp. 333–345. ISSN: 0263-8762. DOI: https://doi.org/10.1016/j.cherd.2017.09.021. URL: https://www.sciencedirect.com/science/article/pii/S0263876217304744 (cit. on pp. 24, 25).

[51] W. Gao et al. “Different states of multi-block based forecast engine for price and load prediction”. In: *International Journal of Electrical Power & Energy Systems* 104 (2019), pp. 423–435. ISSN: 0142-0615. DOI: https://doi.org/10.1016/j.ijepes.2018.07.014. URL: https://www.sciencedirect.com/science/article/pii/S0142061517319865 (cit. on p. 25).

[52] C. García-Ascanio and C. Maté. “Electric power demand forecasting using interval time series: A comparison between VAR and iMLP”. In: *Energy Policy* 38.2 (2010), pp. 715–725. ISSN: 0301-4215. DOI: https://doi.org/10.1016/j.enpol.2009.10.007. URL: https://www.sciencedirect.com/science/article/pii/S0301421509007344 (cit. on p. 21).

[53] C. Garcia-Martos, J. Rodriguez, and M. J. Sanchez. “Mixed Models for Short-Run Forecasting of Electricity Prices: Application for the Spanish Market”. In: *IEEE Transactions on Power Systems* 22.2 (2007), pp. 544–552. DOI: 10.1109/TPWRS.2007.894857 (cit. on p. 18).

[54] R. Gareta, L. M. Romeo, and A. Gil. "Forecasting of electricity prices with neural networks". In: *Energy Conversion and Management* 47.13 (2006), pp. 1770–1778. ISSN: 0196-8904. DOI: https://doi.org/10.1016/j.enconman.2005.10.010. URL: https://www.sciencedirect.com/science/article/pii/S0196890405002566 (cit. on p. 21).

[55] M. Ghayekhloo et al. "A combination approach based on a novel data clustering method and Bayesian recurrent neural network for day-ahead price forecasting of electricity markets". In: *Electric Power Systems Research* 168 (2019), pp. 184–199. ISSN: 0378-7796. DOI: https://doi.org/10.1016/j.epsr.2018.11.021. URL: https://www.sciencedirect.com/science/article/pii/S0378779618303961 (cit. on pp. 24, 25).

[56] M. Ghofrani et al. "A new day-ahead hourly electricity price forecasting framework". In: *2017 North American Power Symposium (NAPS)*. 2017, pp. 1–6. DOI: 10.1109/NAPS.2017.8107269 (cit. on pp. 24, 25).

[57] A. M. Gonzalez, A. M. S. Roque, and J. Garcia-Gonzalez. "Modeling and forecasting electricity prices with input/output hidden Markov models". In: *IEEE Transactions on Power Systems* 20.1 (2005), pp. 13–24. DOI: 10.1109/TPWRS.2004.840412 (cit. on p. 21).

[58] V. Gonzalez, J. Contreras, and D. W. Bunn. "Forecasting Power Prices Using a Hybrid Fundamental-Econometric Model". In: *IEEE Transactions on Power Systems* 27.1 (2012), pp. 363–372. DOI: 10.1109/TPWRS.2011.2167689 (cit. on p. 18).

[59] R. J. Green and D. M. Newbery. "Competition in the British Electricity Spot Market". In: *Journal of Political Economy* 100.5 (1992), pp. 929–953. ISSN: 00223808, 1537534X. URL: http://www.jstor.org/stable/2138629 (visited on 2022-05-08) (cit. on p. 4).

[60] J.-J. Guo and P. B. Luh. "Selecting input factors for clusters of Gaussian radial basis function networks to improve market clearing price prediction". In: *IEEE Transactions on Power Systems* 18.2 (2003), pp. 665–672. DOI: 10.1109/TPWRS.2003.811012 (cit. on p. 21).

[61] J.-J. Guo and P. Luh. "Improving market clearing price prediction by using a committee machine of neural networks". In: *IEEE Transactions on Power Systems* 19.4 (2004), pp. 1867–1876. DOI: 10.1109/TPWRS.2004.837759 (cit. on pp. 21, 25).

[62] S. Haykin. *Neural Networks: A Comprehensive Foundation*. Prentice-Hall, 1998. ISBN: 9780132733502 (cit. on p. 25).

[63] C. C. Holt. "Forecasting seasonals and trends by exponentially weighted moving averages". In: *International Journal of Forecasting* 20.1 (2004), pp. 5–10. ISSN: 0169-2070. DOI: https://doi.org/10.1016/j.ijforecast.2003.09.015. URL: https://www.sciencedirect.com/science/article/pii/S0169207003001134 (cit. on p. 39).

[64] Y.-Y. Hong et al. "Short-term LMP forecasting using an artificial neural network incorporating empirical mode decomposition". In: *International Transactions on Electrical Energy Systems* 25.9 (2015), pp. 1952–1964. DOI: https://doi.org/10.1002/etep.1949. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/etep.1949. URL: https://onlinelibrary.wiley.com/doi/abs/10.1002/etep.1949 (cit. on pp. 24, 25).

[65] S. D. Howison and M. Coulon. "Stochastic Behaviour of the Electricity Bid Stack: From Fundamental Drivers to Power Prices". In: *J. Energy Markets* 2 (2009-02). DOI: 10.21314/JEM.2009.032 (cit. on p. 19).

[66] C.-Y. Hsiao. "Locational marginal price forecasting in deregulated electricity markets using artificial intelligence". English. In: *IEE Proceedings - Generation, Transmission and Distribution* 149 (5 2002-09), 621–626(5). ISSN: 1350-2360. URL: https://digital-library.theiet.org/content/journals/10.1049/ip-gtd_20020371 (cit. on p. 22).

[67] Z. Hu et al. "A game-theoretic model for electricity markets with tight capacity constraints". In: *International Journal of Electrical Power & Energy Systems* 30 (2008-03), pp. 207–215. DOI: 10.1016/j.ijepes.2007.07.008 (cit. on p. 21).

[68] C.-M. Huang, C.-J. Huang, and M.-L. Wang. "A Particle Swarm Optimization to Identifying the ARMAX Model for Short-Term Load Forecasting". In: *Power Systems, IEEE Transactions on* 20 (2005-06), pp. 1126–1133. DOI: 10.1109/TPWRS.2005.846106 (cit. on p. 21).

[69] K. Hubicka, G. Marcjasz, and R. Weron. "A Note on Averaging Day-Ahead Electricity Price Forecasts Across Calibration Windows". In: *IEEE Transactions on Sustainable Energy* 10.1 (2019), pp. 321–323. DOI: 10.1109/TSTE.2018.2869557 (cit. on p. 15).

[70] R. Huisman, C. Huurman, and R. Mahieu. "Hourly electricity prices in day-ahead markets". In: *Energy Economics* 29.2 (2007), pp. 240–248. ISSN: 0140-9883. DOI: https://doi.org/10.1016/j.eneco.2006.08.005. URL: https://www.sciencedirect.com/science/article/pii/S0140988306000995 (cit. on p. 10).

[71] S. Itaba and H. Mori. "A Fuzzy-Preconditioned GRBFN Model for Electricity Price Forecasting". In: *Procedia Computer Science* 114 (2017), pp. 441–448 (cit. on p. 25).

[72] S. Itaba and H. Mori. "An Electricity Price Forecasting Model with Fuzzy Clustering Preconditioned ANN". In: *Electrical Engineering in Japan* 204.3 (2018), pp. 10–20. DOI: https://doi.org/10.1002/eej.23094. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/eej.23094. URL: https://onlinelibrary.wiley.com/doi/abs/10.1002/eej.23094 (cit. on p. 24).

[73] H. Jahangir et al. "A Novel Electricity Price Forecasting Approach Based on Dimension Reduction Strategy and Rough Artificial Neural Networks". In: *IEEE Transactions on Industrial Informatics* 16.4 (2020), pp. 2369–2381. DOI: 10.1109/TII.2019.2933009 (cit. on p. 24).

[74] T. Jamasb and M. Pollitt. "Electricity Market Reform in the European Union: Review of Progress toward Liberalization & Integration". In: *The Energy Journal* 26 (2005), pp. 11–41. ISSN: 01956574, 19449089. URL: http://www.jstor.org/stable/23297005 (visited on 2022-05-08) (cit. on p. 2).

[75] T. A. Johnsen. "Demand, generation and price in the Norwegian market for electric power". In: *Energy Economics* 23.3 (2001), pp. 227–251. ISSN: 0140-9883. DOI: https://doi.org/10.1016/S0140-9883(00)00052-9. URL: https://www.sciencedirect.com/science/article/pii/S0140988300000529 (cit. on p. 18).

[76] T. Jonsson et al. "Forecasting Electricity Spot Prices Accounting for Wind Power Predictions". In: *IEEE Transactions on Sustainable Energy* 4.1 (2013), pp. 210–218. DOI: 10.1109/TSTE.2012.2212731 (cit. on pp. 15, 17).

[77] P. L. Joskow. "California's Electricity Crisis". In: *Oxford Review of Economic Policy* 17.3 (2001-09), pp. 365–388. ISSN: 0266-903X. DOI: 10.1093/oxrep/17.3.365. eprint: https://academic.oup.com/oxrep/article-pdf/17/3/365/1203692/170365.pdf. URL: https://doi.org/10.1093/oxrep/17.3.365 (cit. on pp. vii, ix).

[78] A. E. Kahn. *The Economics of Regulation: Principles and Institutions*. New York: Wiley, 1970. ISBN: 9780262111294 (cit. on p. 3).

[79] V. Kaminski. *Energy Markets*. London: Risk Books, 2013. ISBN: 9781906348793 (cit. on p. 1).

[80] T. Kanamura and K. Ōhashi. "A structural model for electricity prices with spikes: Measurement of spike risk and optimal policies for hydropower plant operation". In: *Energy Economics* 29.5 (2007), pp. 1010–1032. ISSN: 0140-9883. DOI: https://doi.org/10.1016/j.eneco.2006.05.012. URL: https://www.sciencedirect.com/science/article/pii/S0140988306000600 (cit. on p. 19).

[81] N. V. Karakatsani and D. W. Bunn. "Forecasting electricity prices: The impact of fundamentals and time-varying coefficients". In: *International Journal of Forecasting* 24.4 (2008). Energy Forecasting, pp. 764–785. ISSN: 0169-2070. DOI: https://doi.org/10.1016/j.ijforecast.2008.09.008. URL: https://www.sciencedirect.com/science/article/pii/S0169207008001076 (cit. on p. 18).

[82] N. V. Karakatsani and D. W. Bunn. "Fundamental and Behavioural Drivers of Electricity Price Volatility". In: *Studies in Nonlinear Dynamics & Econometrics* 14.4 (2010). DOI: doi:10.2202/1558-3708.1657. URL: https://doi.org/10.2202/1558-3708.1657 (cit. on p. 15).

[83] F. Keynia and N. Amjady. "Electricity price forecasting with a new feature selection algorithm". In: *Journal of Energy Markets* 1 (2008-12), pp. 47–63. DOI: 10.21314/JEM.2008.015 (cit. on p. 21).

[84] M. G. Khajeh et al. "Electricity price forecasting using neural networks with an improved iterative training algorithm". In: *International Journal of Ambient Energy* 39.2 (2018), pp. 147–158. DOI: 10.1080/01430750.2016.1269674. eprint: https://doi.org/10.1080/01430750.2016.1269674. URL: https://doi.org/10.1080/01430750.2016.1269674 (cit. on pp. 24, 25).

[85] C.-i. Kim, I.-K. Yu, and Y. Song. "Prediction of system marginal price of electricity using wavelet transform analysis". In: *Energy Conversion and Management* 43.14 (2002), pp. 1839–1851. ISSN: 0196-8904. DOI: https://doi.org/10.1016/S0196-8904(01)00127-3. URL: https://www.sciencedirect.com/science/article/pii/S0196890401001273 (cit. on p. 15).

[86] M. Kim. “Short-term price forecasting of Nordic power market by combination Levenberg–Marquardt and Cuckoo search algorithms”. In: *IET Generation, Transmission & Distribution* 9 (2015-05). DOI: 10.1049/iet-gtd.2014.0957 (cit. on pp. 24, 25).

[87] J. K. Kolberg and K. Waage. “Artificial intelligence and Nord Pool’s intraday electricity market Elbas: a demonstration and pragmatic evaluation of employing deep learning for price prediction: using extensive market data and spatio-temporal weather forecasts”. MA thesis. Norwegian School of Economics, 2018 (cit. on p. 24).

[88] A. Konar. *Computational Intelligence: Principles, Techniques and Applications*. Springer, 2005. ISBN: 978-3-540-27335-6 (cit. on p. 22).

[89] P.-H. Kuo and C.-J. Huang. “An Electricity Price Forecasting Model by Hybrid Structured Deep Neural Networks”. In: *Sustainability* 10.4 (2018). ISSN: 2071-1050. DOI: 10.3390/su10041280. URL: https://www.mdpi.com/2071-1050/10/4/1280 (cit. on pp. 23, 24).

[90] J. Lagarto et al. “Price forecasting in the day-ahead Iberian electricity market using a conjectural variations ARIMA model”. In: *2012 9th International Conference on the European Energy Market*. 2012, pp. 1–7. DOI: 10.1109/EEM.2012.6254734 (cit. on p. 18).

[91] J. Lago, F. De Ridder, and B. De Schutter. “Forecasting spot electricity prices: Deep learning approaches and empirical comparison of traditional algorithms”. In: *Applied Energy* 221 (2018), pp. 386–405. ISSN: 0306-2619. DOI: https://doi.org/10.1016/j.apenergy.2018.02.069. URL: https://www.sciencedirect.com/science/article/pii/S030626191830196X (cit. on pp. 15, 23).

[92] J. Lago et al. “Forecasting day-ahead electricity prices in Europe: The importance of considering market integration”. In: *Applied Energy* 211 (2018), pp. 890–903. ISSN: 0306-2619. DOI: https://doi.org/10.1016/j.apenergy.2017.11.098. URL: https://www.sciencedirect.com/science/article/pii/S0306261917316999 (cit. on p. 23).

[93] S. Lahmiri. “Comparing Variational and Empirical Mode Decomposition in Forecasting Day-Ahead Energy Prices”. In: *IEEE Systems Journal* 11.3 (2017), pp. 1907–1910. DOI: 10.1109/JSYST.2015.2487339 (cit. on pp. 24, 25).

[94] D. Liebl. “Modeling and forecasting electricity spot prices: A functional data perspective”. In: *The Annals of Applied Statistics* 7.3 (2013), pp. 1562–1592. DOI: 10.1214/13-AOAS652. URL: https://doi.org/10.1214/13-AOAS652 (cit. on p. 18).

[95] W.-M. Lin, H.-J. Gow, and M.-T. Tsai. “An enhanced radial basis function network for short-term electricity price forecasting”. In: *Applied Energy* 87 (2010-10), pp. 3226–3234. DOI: 10.1016/j.apenergy.2010.04.006 (cit. on p. 21).

[96] A. M. D. Livera, R. J. Hyndman, and R. D. Snyder. “Forecasting Time Series With Complex Seasonal Patterns Using Exponential Smoothing”. In: *Journal of the American Statistical Association* 106.496 (2011), pp. 1513–1527. DOI: 10.1198/jasa.2011.tm09771. eprint: https://doi.org/10.1198/jasa.2011.tm09771. URL: https://doi.org/10.1198/jasa.2011.tm09771 (cit. on p. 39).

[97] J. M. Lourenço. *The NOVAthesis LaTeX Template User's Manual*. NOVA University Lisbon. 2021. URL: https://github.com/joaomlourenco/novathesis/raw/master/template.pdf (cit. on p. ii).

[98] K. Maciejowska. “Fundamental and speculative shocks, what drives electricity prices?” In: *11th International Conference on the European Energy Market (EEM14)*. 2014, pp. 1–5. DOI: 10.1109/EEM.2014.6861289 (cit. on p. 25).

[99] P. Mandal, T. Senjyu, and T. Funabashi. “Neural networks approach to forecast several hour ahead electricity prices and loads in deregulated market”. In: *Energy Conversion and Management* 47 (2006-09), pp. 2128–2142. DOI: 10.1016/j.enconman.2005.12.008 (cit. on p. 21).

[100] G. Marcjasz, T. Serafin, and R. Weron. “Selection of Calibration Windows for Day-Ahead Electricity Price Forecasting”. In: *Energies* 11.9 (2018). ISSN: 1996-1073. DOI: 10.3390/en11092364. URL: https://www.mdpi.com/1996-1073/11/9/2364 (cit. on p. 15).

[101] A. Mas-Colell, M. D. Whinston, and J. R. Green. *Microeconomic Theory*. New York, NY: Oxford University Press, 1995. ISBN: 9780195073409 (cit. on p. 4).

[102] L. T. A. Maurer and L. A. Barroso. “Electricity Auctions : An Overview of Efficient Practices.” In: *World Bank Study. World Bank.* (2011). URL: https://openknowledge.worldbank.org/handle/10986/2346 (cit. on p. 5).

[103] J.-H. Meier et al. "ANN-Based Electricity Price Forecasting Under Special Consideration of Time Series Properties: 14th International Conference, ICTERI 2018, Kyiv, Ukraine, May 14-17, 2018, Revised Selected Papers". In: 2019-01, pp. 262–275. ISBN: 978-3-030-13928-5. DOI: 10.1007/978-3-030-13929-2_13 (cit. on p. 24).

[104] V. Mendes et al. "Emission constraints on short-term schedule of thermal units". In: *39th International Universities Power Engineering Conference, 2004. UPEC 2004.* Vol. 3. 2004, 1068–1072 vol. 2 (cit. on p. 8).

[105] *Mercado Ibérico de Eletricidade (MIBLE)*. URL: www.erse.pt (visited on 2021-01-12) (cit. on p. 2).

[106] A. Misiorek, S. Trueck, and R. Weron. "Point and Interval Forecasting of Spot Electricity Prices: Linear vs. Non-Linear Time Series Models". In: *Studies in Nonlinear Dynamics & Econometrics* 10 (2007-02), pp. 1362–1362. DOI: 10.2202/1558-3708.1362 (cit. on p. 18).

[107] S. Mujeeb and N. Javaid. "ESAENARX and DE-RELM: Novel schemes for big data predictive analytics of electricity load and price". In: *Sustainable Cities and Society* 51 (2019-05). DOI: 10.1016/j.scs.2019.101642 (cit. on p. 24).

[108] S. Mujeeb et al. "Big Data Analytics for Price and Load Forecasting in Smart Grids". In: *Advances on Broadband and Wireless Computing, Communication and Applications*. Ed. by L. Barolli et al. Cham: Springer International Publishing, 2019, pp. 77–87. ISBN: 978-3-030-02613-4 (cit. on p. 23).

[109] S. Mujeeb et al. "Deep Long Short-Term Memory: A New Price and Load Forecasting Scheme for Big Data in Smart Cities". In: *Sustainability* 11.4 (2019). ISSN: 2071-1050. DOI: 10.3390/su11040987. URL: https://www.mdpi.com/2071-1050/11/4/987 (cit. on p. 24).

[110] A. Naz et al. "Short-Term Electric Load and Price Forecasting Using Enhanced Extreme Learning Machine Optimization in Smart Grids". In: *Energies* 12.5 (2019). ISSN: 1996-1073. DOI: 10.3390/en12050866. URL: https://www.mdpi.com/1996-1073/12/5/866 (cit. on pp. 24, 25).

[111] M. S. Nazar et al. "Hybrid model using three-stage algorithm for simultaneous load and price forecasting". In: *Electric Power Systems Research* 165 (2018), pp. 214–228. ISSN: 0378-7796. DOI: https://doi.org/10.1016/j.epsr.2018.09.004. URL:

https://www.sciencedirect.com/science/article/pii/S0378779618302979 (cit. on pp. 24, 25).

[112] A. Nazemi and M. Mashayekhi. "Competitiveness assessment of Iran's restructured electricity market". In: *Energy Economics* 49.C (2015), pp. 308–316. DOI: 10.1016/j.eneco.2015.02.0. URL: https://ideas.repec.org/a/eee/eneeco/v49y2015icp308-316.html (cit. on p. 7).

[113] D. Niu, D. Liu, and D. D. Wu. "A soft computing system for day-ahead electricity price forecasting". In: *Appl. Soft Comput.* 10 (2010), pp. 868–875 (cit. on p. 22).

[114] S. Nojavan and K. Zare. *Electricity Markets. New Players and Pricing Uncertainties*. Switzerland: Springer, 2020. ISBN: 978-3-030-36979-8 (cit. on p. 5).

[115] J. Nowotarski and R. Weron. "Computing electricity spot price prediction intervals using quantile regression and forecast averaging". In: *Computational Statistics* 30 (2015), pp. 791–803. DOI: 10.1007/s00180-014-0523-0 (cit. on p. 25).

[116] C. A. Ockenfels, V. Grimm, and G. Zoettl. "The Pricing Mechanism of the Day Ahead Electricity Spot Market Auction on the EEX". In: (2008) (cit. on p. 7).

[117] J. Olamaei et al. "Day-ahead price forecasting based on hybrid prediction model". In: *Complexity* 21 (2016-05). DOI: 10.1002/cplx.21792 (cit. on pp. 24, 25).

[118] H.-T. Pao. "A Neural Network Approach to m-Daily-Ahead Electricity Price Prediction". In: *Lecture Notes in Computer Science*. 2006-05, pp. 1284–1289. ISBN: 978-3-540-34437-7. DOI: 10.1007/11760023_186 (cit. on p. 21).

[119] N. M. Pindoriya, S. N. Singh, and S. K. Singh. "An Adaptive Wavelet Neural Network-Based Energy Price Forecasting in Electricity Markets". In: *IEEE Transactions on Power Systems* 23.3 (2008), pp. 1423–1432. DOI: 10.1109/TPWRS.2008.922251 (cit. on p. 21).

[120] R. Pindyck and D. Rubinfeld. *Microeconomics*. Pearson, 2018. ISBN: 9780134184241 (cit. on p. 3).

[121] A. Pourdaryaei et al. "Short-Term Electricity Price Forecasting via Hybrid Backtracking Search Algorithm and ANFIS Approach". In: *IEEE Access* 7 (2019), pp. 77674–77691. DOI: 10.1109/ACCESS.2019.2922420 (cit. on pp. 24, 25).

[122] E. Raviv, K. E. Bouwman, and D. van Dijk. "Forecasting day-ahead electricity prices: Utilizing hourly prices". In: *Energy Economics* 50.C (2015), pp. 227–239. DOI: 10.1016/j.eneco.2015.05.0. URL: https://ideas.repec.org/a/eee/eneeco/v50y2015icp227-239.html (cit. on p. 25).

[123] C. Rodriguez and G. Anders. "Energy Price Forecasting in the Ontario Competitive Power System Market". In: *Power Systems, IEEE Transactions on* 19 (2004-03), pp. 366–374. DOI: 10.1109/TPWRS.2003.821470 (cit. on pp. 21, 23).

[124] L. Rutkowski. *Computational Intelligence: Methods and Techniques*. Springer, 2008. ISBN: 978-3-540-76288-1 (cit. on p. 22).

[125] D. Sansom, T. Downs, and tapan Saha. "Evaluation of support vector machine based forecasting tool in electricity price forecasting for Australian national electricity market participants". In: *Journal of electrical and electronics engineering, Australia* 22 (2003-01) (cit. on p. 22).

[126] A. Schmutz and P. Elkuch. "Electricity price forecasting: application and experience in the European power markets". In: *6th IAEE European Conference*. Zurich, 2004 (cit. on p. 15).

[127] S. Schnürch and A. Wagner. "Electricity Price Forecasting with Neural Networks on EPEX Order Books". In: *Applied Mathematical Finance* 27.3 (2020-05), pp. 189–206. DOI: 10.1080/1350486x.2020.1805337. URL: https://doi.org/10.1080%2F1350486x.2020.1805337 (cit. on p. 24).

[128] M. Shafie-khah, M. Moghaddam, and M. Sheikholeslami. "Price forecasting of day-ahead electricity markets using a hybrid forecast method". In: *Energy Conversion and Management* 52 (2011-05), pp. 2165–2169. DOI: 10.1016/j.enconman.2010.10.047 (cit. on pp. 17, 21).

[129] M. Shahidehpour and Z. L. Hatim Yamin. *Market Operations in Electric Power Systems: Forecasting, Scheduling, and Risk Management*. New York, NY: Wiley-IEEE Press, 2002. ISBN: 9780471443377 (cit. on p. 6).

[130] F. P. Sioshansi. "Electricity Market Reform: What Have We Learned? What Have We Gained?" In: *The Electricity Journal* 19.9 (2006), pp. 70–83. ISSN: 1040-6190. DOI: https://doi.org/10.1016/j.tej.2006.10.003. URL: https:

//www.sciencedirect.com/science/article/pii/S1040619006001199 (cit. on p. 2).

[131] S. Stoft. *Power System Economics: Designing Markets for Electricity*. Piscataway, NJ: Wiley-IEEE Press, 2002. ISBN: 9780471150404 (cit. on pp. 3, 4).

[132] Z. Tan et al. "Day-ahead electricity price forecasting using wavelet transform combined with ARIMA and GARCH models". In: *Applied Energy* 87.11 (2010), pp. 3606–3610. ISSN: 0306-2619. DOI: https://doi.org/10.1016/j.apenergy.2010.05.012. URL: https://www.sciencedirect.com/science/article/pii/S0306261910001807 (cit. on p. 18).

[133] U. Ugurlu, I. Oksuzand, and O. Tas. "Electricity Price Forecasting Using Recurrent Neural Networks". In: *Energies* 11.5 (2018). ISSN: 1996-1073. DOI: 10.3390/en11051255. URL: https://www.mdpi.com/1996-1073/11/5/1255 (cit. on p. 23).

[134] B. Uniejewski, J. Nowotarski, and R. Weron. "Automated Variable Selection and Shrinkage for Day-Ahead Electricity Price Forecasting". In: *Energies* 9.8 (2016). ISSN: 1996-1073. DOI: 10.3390/en9080621. URL: https://www.mdpi.com/1996-1073/9/8/621 (cit. on p. 15).

[135] B. Uniejewski and R. Weron. "Efficient Forecasting of Electricity Spot Prices with Expert and LASSO Models". In: *Energies* 11.8 (2018). ISSN: 1996-1073. DOI: 10.3390/en11082039. URL: https://www.mdpi.com/1996-1073/11/8/2039 (cit. on p. 15).

[136] V. Vahidinasab, S. Jadid, and A. Kazemi. "Day-ahead price forecasting in restructured power systems using artificial neural networks". In: *Electric Power Systems Research* 78.8 (2008), pp. 1332–1342. ISSN: 0378-7796. DOI: https://doi.org/10.1016/j.epsr.2007.12.001. URL: https://www.sciencedirect.com/science/article/pii/S0378779607002362 (cit. on p. 23).

[137] V. N. Vapnik. *The Nature of Statistical Learning Theory*. Springer, 1995. ISBN: 978-1-4757-3264-1 (cit. on p. 22).

[138] H. Varshney, A. Sharma, and R. Kumar. "A hybrid approach to price forecasting incorporating exogenous variables for a day ahead electricity Market". In: *2016 IEEE 1st International Conference on Power Electronics, Intelligent Control and Energy*

Systems (ICPEICES). 2016, pp. 1–6. DOI: 10.1109/ICPEICES.2016.7853355 (cit. on pp. 24, 25).

[139] I. Vehviläinen and T. Pyykkönen. "Stochastic factor model for electricity spot price—the case of the Nordic market". In: *Energy Economics* 27.2 (2005). Special Issue on Electricity Markets, pp. 351–367. ISSN: 0140-9883. DOI: https://doi.org/10.1016/j.eneco.2005.01.002. URL: https://www.sciencedirect.com/science/article/pii/S0140988305000034 (cit. on p. 19).

[140] W. Vickrey. "Counterspeculation, Auctions, and Competitive Sealed Tenders". In: *The Journal of Finance* 16.1 (1961), pp. 8–37. ISSN: 00221082, 15406261. URL: http://www.jstor.org/stable/2977633 (visited on 2022-05-08) (cit. on p. 6).

[141] A. A. Victoire et al. "Two-Stage Machine Learning Framework for Simultaneous Forecasting of Price-Load in the Smart Grid". In: *2018 17th IEEE International Conference on Machine Learning and Applications (ICMLA)*. 2018, pp. 1081–1086. DOI: 10.1109/ICMLA.2018.00176 (cit. on pp. 24, 25).

[142] L. Wang, Z. Zhang, and J. Chen. "Short-Term Electricity Price Forecasting With Stacked Denoising Autoencoders". In: *IEEE Transactions on Power Systems* 32.4 (2017), pp. 2673–2681. DOI: 10.1109/TPWRS.2016.2628873 (cit. on p. 23).

[143] R. Weron. *Modeling and forecasting electricity loads and prices: astatistical approach*. Chichester: Willey, 2006. ISBN: 978-0-470-05753-7 (cit. on p. 20).

[144] R. Weron and A. Misiorek. "Forecasting spot electricity prices with time series models". In: *Proceedings of the European Electricity Market EEM-05 Conference*. 2005-05 (cit. on p. 17).

[145] R. Weron and A. Misiorek. "Forecasting spot electricity prices: A comparison of parametric and semiparametric time series models". In: *International Journal of Forecasting* 24.4 (2008). Energy Forecasting, pp. 744–763. ISSN: 0169-2070. DOI: https://doi.org/10.1016/j.ijforecast.2008.08.004. URL: https://www.sciencedirect.com/science/article/pii/S0169207008000952 (cit. on p. 18).

[146] P. R. Winters. "Forecasting Sales by Exponentially Weighted Moving Averages". In: *Management Science* 6.3 (1960), pp. 324–342. ISSN: 00251909, 15265501. URL: http://www.jstor.org/stable/2627346 (visited on 2022-05-08) (cit. on p. 39).

[147] L. Xiao et al. "Research and application of a hybrid wavelet neural network model with the improved cuckoo search algorithm for electrical power system forecasting". In: *Applied Energy* 198 (2017-07), pp. 203–222. DOI: 10.1016/j.apenergy.2017.04.039 (cit. on pp. 24, 25).

[148] X. Xie, W. Xu, and H. Tan. "The Day-Ahead Electricity Price Forecasting Based on Stacked CNN and LSTM". In: *Intelligence Science and Big Data Engineering*. Ed. by Y. Peng et al. Cham: Springer International Publishing, 2018, pp. 216–230. ISBN: 978-3-030-02698-1 (cit. on p. 23).

[149] J. Xu and R. Baldick. "Day-Ahead Price Forecasting in ERCOT Market Using Neural Network Approaches". In: *Proceedings of the Tenth ACM International Conference on Future Energy Systems*. e-Energy '19. Phoenix, AZ, USA: Association for Computing Machinery, 2019, pp. 486–491. ISBN: 9781450366717. DOI: 10.1145/3307772.3331024. URL: https://doi.org/10.1145/3307772.3331024 (cit. on p. 24).

[150] H. Yamin, S. Shahidehpour, and Z. Li. "Adaptive short-term electricity price forecasting using artificial neural networks in the restructured power markets". In: *International Journal of Electrical Power & Energy Systems* 26.8 (2004), pp. 571–581. ISSN: 0142-0615. DOI: https://doi.org/10.1016/j.ijepes.2004.04.005. URL: https://www.sciencedirect.com/science/article/pii/S0142061504000420 (cit. on p. 21).

[151] Z. Yang, L. Ce, and L. Lian. "Electricity price forecasting by a hybrid model, combining wavelet transform, ARMA and kernel-based extreme learning machine methods". In: *Applied Energy* 190 (2017), pp. 291–305. ISSN: 0306-2619. DOI: https://doi.org/10.1016/j.apenergy.2016.12.130. URL: https://www.sciencedirect.com/science/article/pii/S0306261916319134 (cit. on pp. 24, 25).

[152] S. J. Yao et al. "Prediction of system marginal price by wavelet transform and neural network". In: *Electric Machines and Power Systems* 28 (2000), pp. 537–549 (cit. on p. 21).

[153] J. Zhang, Z. Tan, and C. Li. "A Novel Hybrid Forecasting Method Using GRNN Combined With Wavelet Transform and a GARCH Model". In: *Energy Sources, Part B: Economics, Planning, and Policy* 10.4 (2015), pp. 418–426. DOI: 10.1080/15567

249.2011.557685. eprint: https://doi.org/10.1080/15567249.2011.557685. URL: https://doi.org/10.1080/15567249.2011.557685 (cit. on p. 24).

[154] J.-L. Zhang et al. "Forecasting day-ahead electricity prices using a new integrated model". In: *International Journal of Electrical Power & Energy Systems* 105 (2019), pp. 541–548. ISSN: 0142-0615. DOI: https://doi.org/10.1016/j.ijepes.2018.08.025. URL: https://www.sciencedirect.com/science/article/pii/S0142061518314194 (cit. on pp. 24, 25).

[155] W. Zhang, F. Cheema, and D. iSrinivasan. "Forecasting of Electricity Prices Using Deep Learning Networks". In: *2018 IEEE PES Asia-Pacific Power and Energy Engineering Conference (APPEEC)*. 2018, pp. 451–456. DOI: 10.1109/APPEEC.2018.8566313 (cit. on pp. 23, 24).

[156] J. H. Zhao et al. "A Statistical Approach for Interval Forecasting of the Electricity Price". In: *IEEE Transactions on Power Systems* 23.2 (2008), pp. 267–276. DOI: 10.1109/TPWRS.2008.919309 (cit. on p. 22).

[157] L. Zhou et al. "Seasonal classification and RBF adaptive weight based parallel combined method for day-ahead electricity price forecasting". In: *2018 IEEE Power Energy Society Innovative Smart Grid Technologies Conference (ISGT)*. 2018, pp. 1–5. DOI: 10.1109/ISGT.2018.8403372 (cit. on p. 25).

[158] S. Zhou et al. "An Optimized Heterogeneous Structure LSTM Network for Electricity Price Forecasting". In: *IEEE Access* 7 (2019), pp. 108161–108173. DOI: 10.1109/ACCESS.2019.2932999 (cit. on p. 24).

[159] Y. Zhu et al. "Power Market Price Forecasting via Deep Learning". In: *IECON 2018 - 44th Annual Conference of the IEEE Industrial Electronics Society*. 2018, pp. 4935–4939. DOI: 10.1109/IECON.2018.8591581 (cit. on p. 23).

[160] F. Ziel. "Forecasting Electricity Spot Prices Using Lasso: On Capturing the Autoregressive Intraday Structure". In: *IEEE Transactions on Power Systems* 31.6 (2016), pp. 4977–4987. DOI: 10.1109/TPWRS.2016.2521545 (cit. on p. 15).

[161] F. Ziel, R. Steinert, and S. Husmann. "Forecasting day ahead electricity spot prices: The impact of the EXAA to other European electricity markets". In: *Energy Economics* 51 (2015-10), pp. 430–444. DOI: 10.1016/j.eneco.2015.08.005 (cit. on p. 15).

[162] F. Ziel and R. Weron. "Day-ahead electricity price forecasting with high-dimensional structures: Univariate vs. multivariate modeling frameworks". In: *Energy Economics* 70 (2018), pp. 396–420. ISSN: 0140-9883. DOI: https://doi.org/10.1016/j.eneco.2017.12.016. URL: https://www.sciencedirect.com/science/article/pii/S014098831730436X (cit. on p. 15).

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