

An Autopilot to Make Circular Loops: from Vertical to Horizontal Circular Loops

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Abstract—In this paper we define an autopilot to make vertical circular loops, making the centripetal acceleration, G_f , to vary through the loop and maintaining a constant engine thrust, although it varies a little bit with altitude. We prove that this autopilot could be implemented easily in the flight computer of an F16, F15, F18, or in any other aircraft with a high $G_{onSetRate}$, the positive derivative in order to the time of G_f , greater than 0.62 g/s . Then we define an autopilot to make horizontal circular loops varying only the centripetal acceleration. We found that this autopilot can be deployed on any aircraft with the negative derivative in order to the time of G_f , $G_{offSetRate}$, such that $|G_{offSetRate}| > 1\text{ g/s}$, which includes almost all aircrafts. Finally we revise and correct our previous work where we define the G-LOC risk in a *flyup*, and we obtain the exact G-LOC risk in a loop. In the near future we plan to design a system of planning aerobatic flights based on our model of G-LOC risk.

Index Terms—penetrating vertical loops; autopilot to make vertical circular loops; spiral like horizontal loops; autopilot to make horizontal circular loops; G-LOC; G-LOC risk.

I. INTRODUCTION

In a preliminary study with vertical loops with constant centripetal acceleration, $G_f=3\text{g}$, through the loop [1], generated with a simplified model of the F16 [2], we obtained a result apparently paradoxal: for an initial speed greater than 450 Knots the final altitude is less than the initial altitude, i.e., we got penetrating vertical loops that may end in a crash, if we continue to increase the initial speed. In Figure 1 we show how penetrating vertical loops arise, if we use a constant centripetal acceleration through the loop and continue to increase the initial speed, maintaining a constant engine thrust through the loop, and in Figure 2 we show the evolution of speed through these loops. In Table I we present the main parameters of our simplified model of the F-16 aircraft.

TABLE I
F16 MAIN PARAMETERS

$C_x=0.2-1.2$
 $S_x=10\text{m}^2$
 $C_L=0.5-2$
 $(L/D)_{max}=7$
 $G_{f_max}=9\text{g}$
 $m=9280\text{Kg}$

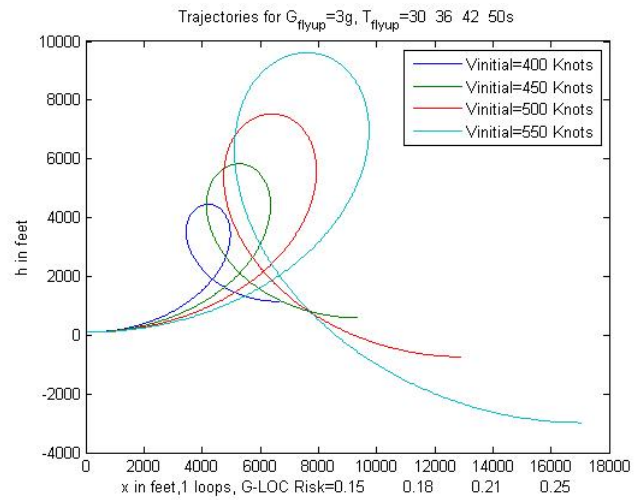


Fig. 1. Loops with constant $G_f=3\text{g}$ and initial speeds 400, 450, 500 and 550 knots. For 500 and 550 knots we have penetrating vertical loops, with $h(-360^\circ) < h(0^\circ)$.

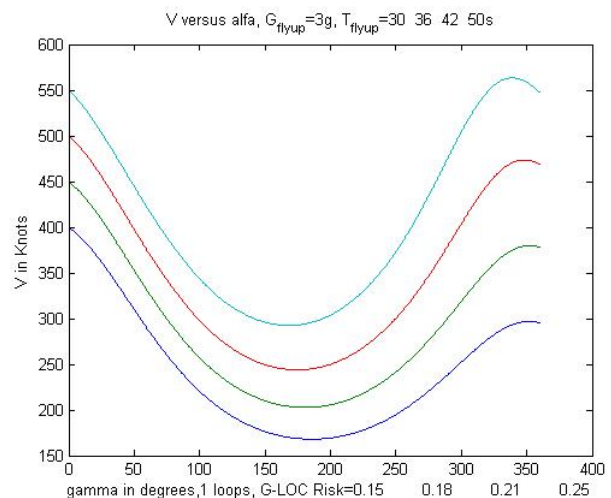


Fig. 2. Evolution of speed through the loops. For the initial speed of 400 knots the loop is clearly unfeasible since $V(-180^\circ) < 200$ knots, the stall speed of the F-16 aircraft.

In practice the centripetal acceleration is not constant through the loop, and we will see that a circular loop corresponds to the usual practice of fighter and aerobatic pilots, because it prevents penetrating vertical loops for higher initial speeds. In this paper we define an autopilot to make vertical circular loops varying only the centripetal acceleration and then we generalise this result to make horizontal circular loops varying only the centripetal acceleration. To our knowledge there exists no previous similar work to this paper. In addition to being a starting point to an autopilot to make circular loops, our work could be used as a training tool of fighter and aerobatic pilots through the use of a flight simulator.

II. AN AUTOPILOT TO MAKE VERTICAL CIRCULAR LOOPS

Next we will define an autopilot to make circular vertical loops varying only the centripetal acceleration, G_f , through the loop and maintaining a constant engine thrust, although it varies a little bit with altitude [2]. Taking into account the geometry of the problem, the radius, R , of the loop will be given by (1).

$$R = \frac{V_{initial}^2}{G_{f_{initial}} - g} \quad (1)$$

Then, after some algebraic manipulations, for a trajectory angle γ , the centripetal acceleration, in g's, that corresponds to a vertical circular loop of radius R will be given by (2).

$$G_f(\gamma) = \frac{V(\gamma)^2}{g R} + \cos(\gamma) \quad (2)$$

Equation (2) can be interpreted as the control equation of an autopilot that would command the centripetal acceleration through the loop, resulting a circumference of radius R given by (1). Since in the F-16 and other aircrafts we can control directly the centripetal acceleration through the pilot *stick*, the implementation of this autopilot seems feasible and easily implementable.

III. SIMULATION OF THE AUTOPILOT TO MAKE VERTICAL CIRCULAR LOOPS

Next we show some simulation results using the referred simplified model of the F-16 implemented in Matlab. We will consider $G_{f_{initial}}=5g$, $V_{initial}=\{500, 600, 650, 700\}$ knots, 1000 points for each simulation run and initial altitude, $h_{initial}=5000$ feet. The evolution of speed through the loop for each of the initial speeds is shown in Figure 3. Note that for an initial speed of 500 knots at $\gamma=-180^\circ$ the F-16 attains a speed less than the stall speed of 200 Knots. This means that for an initial centripetal acceleration of 5g, the initial speed must be greater than 500 knots. In Figure 4 we show the evolution of centripetal acceleration, $G_f(\gamma)$, for each of the four loops. In Figure 5 we confirm that the trajectories of the four loops are circumferences, and in Figure 6 we show the four curves of $G_f(\gamma)V_f(\gamma)/K_{pilot}$. We will show in section VI that the area under these curves correspond to the exact G-LOC risk of the respective four loops.

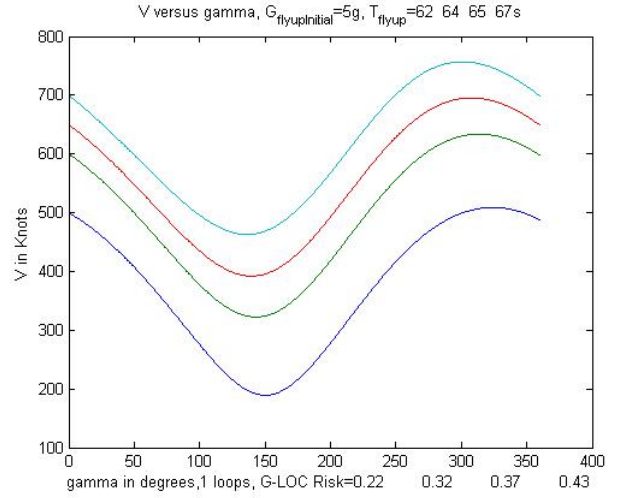


Fig. 3. Evolution of speed through the loops. For the initial speed of 500 knots the loop is clearly unfeasible since $V(-180^\circ) < 200$ knots, the stall speed of the F-16 aircraft.

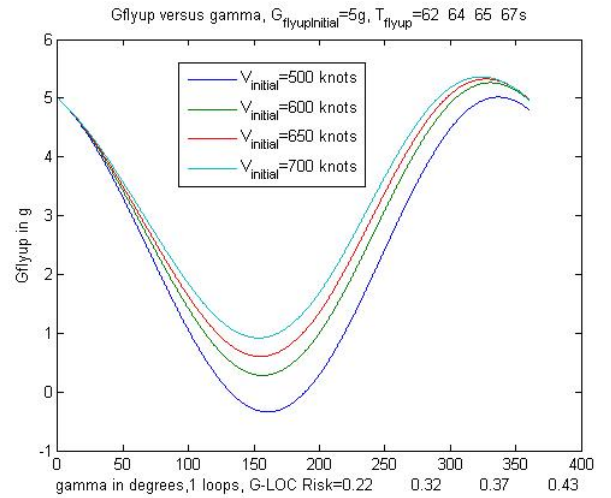


Fig. 4. Evolution of $G_f(\gamma)$ through the loop for $G_f(0^\circ) = 5g$ and various initial speeds. Note that since $G_f(-180^\circ) < 0$ for $V_{initial} = 500$ knots this loop would be unfeasible.

IV. ANALYSIS OF VIABILITY OF THE AUTOPILOT TO MAKE CIRCULAR VERTICAL LOOPS

Since the F-16 passes from $G_f = 1g$ to $9g$ in about 1s, i.e., it has a $G_{onSetRate} = 8g/s$, it seems that will be possible to implement this autopilot in the F-16 or in any other aircraft with a high $G_{onSetRate}$. The critical parts of the loop will be those where G_f increases, i.e. between -180° and -360° . In mathematical terms, the viability of our autopilot for vertical circular loops will be given by (3).

$$\left(\frac{dG_f}{dt} \right)_{max} < G_{onSetRate} \quad (3)$$

We will see that the positive dG_f/dt , the instantaneous $G_{onSetRate}$, has a maximum for $\gamma \approx -263^\circ$. Next, we

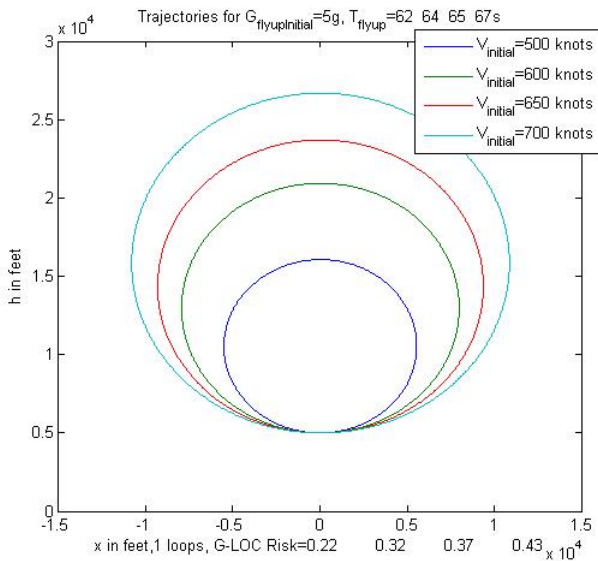


Fig. 5. Trajectories of the four loops. As expected we obtain circumferences with increasing radius and tangent for $h(0^\circ) = h(-360^\circ) = 5000$ feet.

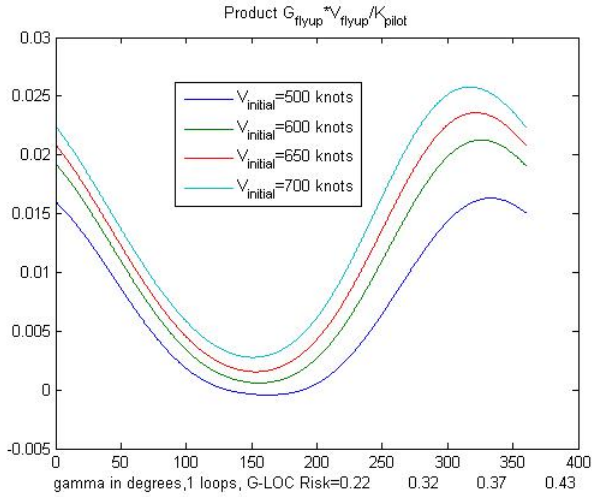


Fig. 6. Product $G_f(\gamma)V_f(\gamma)/K_{pilot}$ through the various loops. We will see in section VI that the area under these curves correspond to the exact G-LOC risk of the respective loops.

will verify if (3) is true for the F-16, simulating the more unfavourable situation, a minimum initial speed that would imply a minimum loop time and a maximum initial centripetal acceleration, G_f , say (450 Knots, 9g) maximizing this way the first term of (3). Equation (3) being true translates in condition (4) be always true during the descendent part of the loop, $\gamma \in [-180^\circ - 360^\circ]$, with the referred parameters of the loop.

$$\frac{G_f(i) - G_f(i-1)}{\Delta t(i-1)} < G_{onSetRate} \quad (4)$$

In Figure 7 we show the evolution of $G_{onSetRate}(\gamma)$ through a loop with $G_f(0^\circ) = 9g$ and $V(0^\circ)=450$ knots. The maximum is about 0.62 g/s at $\gamma \approx -260^\circ$. This means that

this autopilot can be implemented in the F-16 and in any aircraft with $G_{onSetRate} > 0.62g/s$. In Figure 8 we show the evolution of speed through the loop, which confirms that this circular loop is feasible, since the minimum speed is greater than the stall speed of the F-16, about 200 knots. In Figure 9, we show the evolution of centripetal acceleration through the loop, where we can see that its instantaneous positive derivative in order to the time attains a maximum around $\gamma = -260^\circ$. In Figure 10 we show the trajectory of this loop, where we confirm that it is a circular loop.

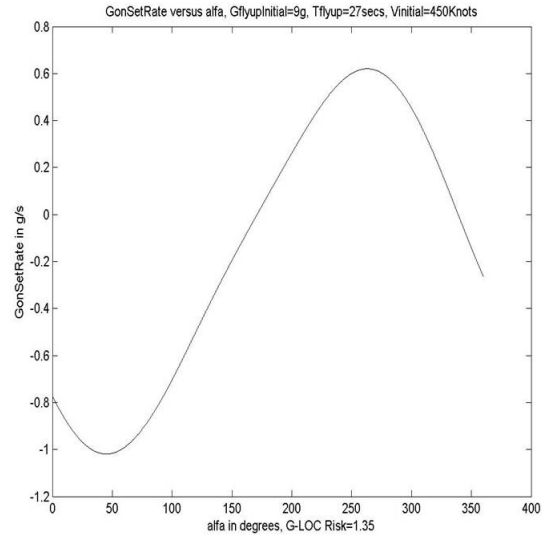


Fig. 7. Evolution of $G_{onSetRate}(\gamma)$ through a circular loop with initial centripetal acceleration of 9g and initial speed of 450 knots.

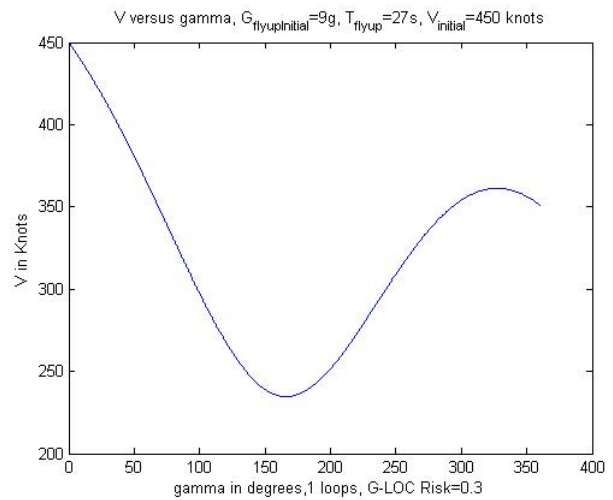


Fig. 8. Evolution of speed for a circular loop with initial centripetal acceleration of 9g and initial speed of 450 knots.

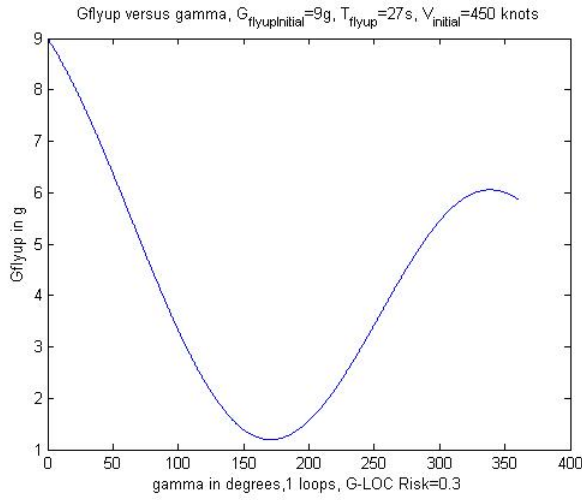


Fig. 9. Evolution of centripetal acceleration, $G_f(\gamma)$, for a circular loop with initial centripetal acceleration of 9g and initial speed of 450 knots.

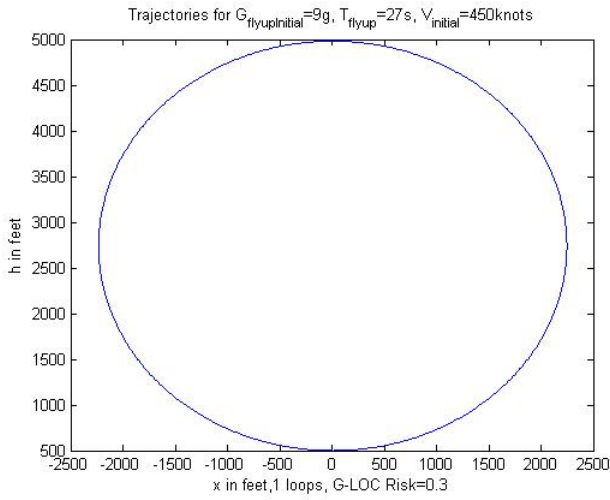


Fig. 10. Confirmation that the trajectory, for an initial centripetal acceleration of 9g and initial speed of 450 knots, it is a circumference.

V. DESIGN OF AN AUTOPILOT TO MAKE CIRCULAR HORIZONTAL LOOPS

Now we will show how to design an autopilot to make horizontal circular loops varying only the centripetal acceleration through the loop. Although a spiral like horizontal loop, which results from a constant G_f , it is not dangerous if the final speed is not less than the stall speed, we will show that in a circular horizontal loop the final speed is greater than the final speed associated to a horizontal spiral like loop, and we will have a lower risk of G-LOC. In this sense, we can say that horizontal circular loops are safer than horizontal spiral like loops. In Figure 11, we show the evolution of speed through spiral like horizontal loops, with constant centripetal acceleration of 9g, for initial speeds between 400 and 650 knots. For an initial speed of 400 knots the final speed reaches a value less than 200 knots, so this spiral like horizontal loop is not feasible.

In Figure 12, we confirm that these horizontal loops with constant centripetal acceleration, are spiral like loops, since their radius of curvature reduces through the loop, because the speed reduces through the loop, as a consequence of increased induced drag.

The radius of curvature, R , of a circular horizontal loop with initial speed, $V_{initial}$ and initial centripetal acceleration, $G_{f_{initial}}$, will be given by (5).

$$R = \frac{V_{initial}^2}{G_{f_{initial}}} \quad (5)$$

Then the control equation of centripetal acceleration through the loop, $G_f(\alpha)$, that guarantees a circular horizontal loop will be given by (6).

$$G_f(\alpha) = \frac{V(\alpha)^2}{R} \quad (6)$$

We can say that (5) and (6) are the mathematical control model of our autopilot to make circular horizontal loops. In Figure 13, we show the evolution of speed through various loops, with initial speeds between 400 and 650 knots and initial centripetal acceleration, $G_{f_{initial}} = 9g$. Now the loop with initial speed of 400 knots is feasible, since the final speed is greater than 200 knots, and we have lower risks of G-LOC associated to the various loops, since the centripetal acceleration reduces through the various loops, as is shown in Figure 14. In Figure 15 we confirm that the resultant trajectories of the various loops are circumferences. In Figure 16, we show the evolution of the instantaneous $G_{offsetRate}$ through the loops, $G_{offsetRate}(\alpha)$, where we show that $|G_{offsetRate}|$ must be greater than 1 g/s. In Figure 17, we show the evolution of the product $V(\alpha)G_f(\alpha)/K_{pilot}$ for the various loops. We will see in next section, that the areas under these curves correspond to the risk of G-LOC of the respective circular horizontal loops.

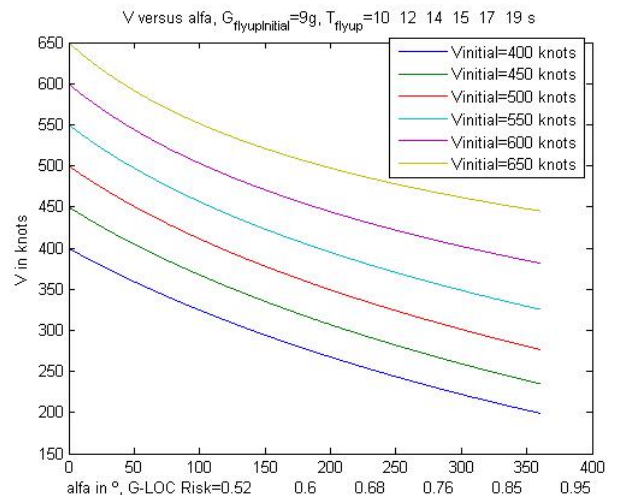


Fig. 11. Evolution of speed versus α for a constant centripetal acceleration of 9g through the loops and initial speeds between 400 and 650 knots.

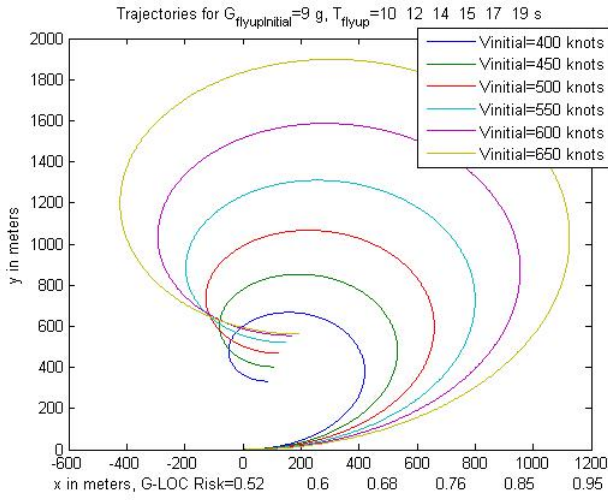


Fig. 12. Spiral like trajectories for a constant centripetal acceleration of $9g$ and initial speeds between 400 and 650 knots.

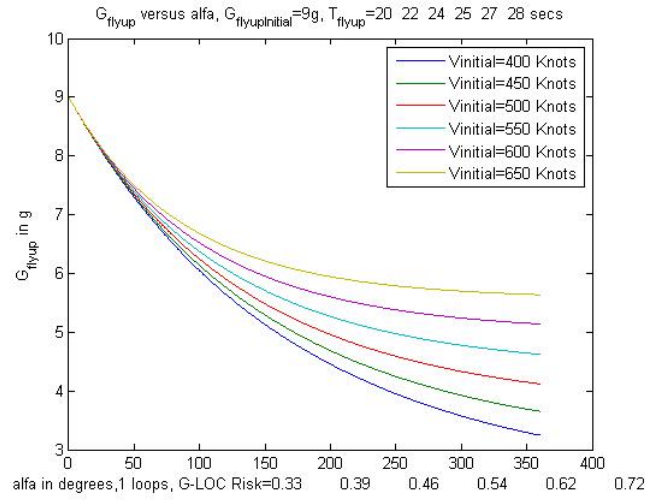


Fig. 14. Evolution of centripetal acceleration versus α for $G_{flyup}(0^\circ) = 9g$ and initial speeds between 400 and 650 knots.

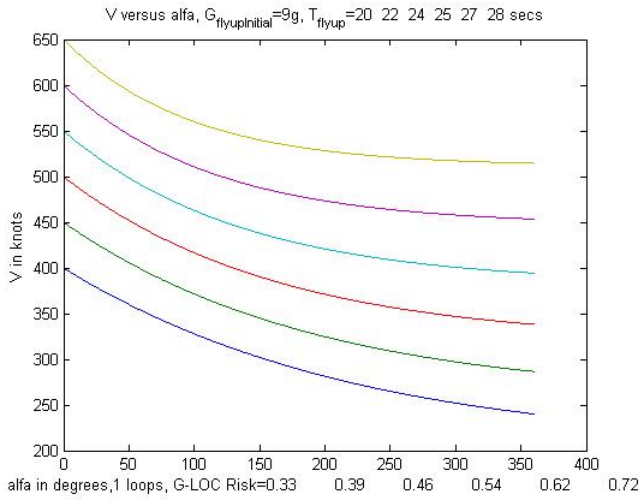


Fig. 13. Evolution of speed versus α for an initial centripetal acceleration of $9g$ and initial speeds between 400 and 650 knots.

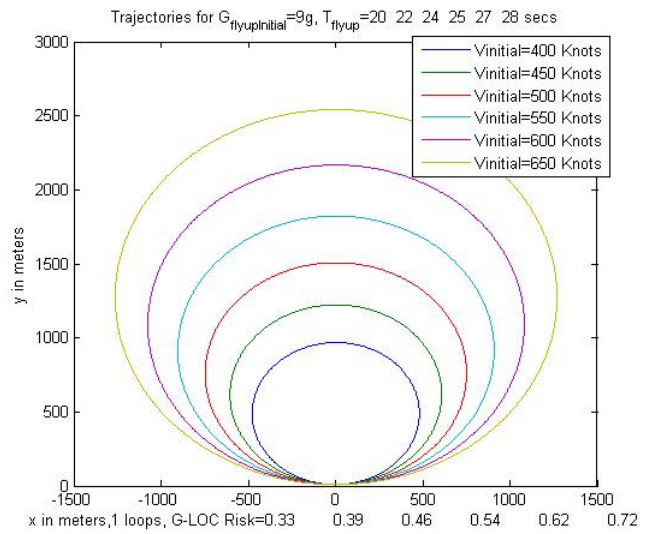


Fig. 15. Confirmation that the trajectories, for an initial centripetal acceleration of $9g$ and initial speeds between 400 and 650 knots, are circumferences tangent in the point $(x, y) = (0, 0)$.

VI. REVISION OF THE MATHEMATICAL MODEL OF G-LOC RISK

Next we will revise and correct the model of G-LOC risk presented in our previous work [3]. The first result of the G-LOC risk model is the paradox of the flyup. This result tells us that the flyup time of a flyup made with constant centripetal acceleration is proportional to the flyup speed. The centripetal acceleration, G_f , is given by (7).

$$G_f = \omega V_f \quad (7)$$

In a flyup of $\Delta\alpha$ radians, the flyup time, Δt , will be given by (8).

$$\Delta t = \frac{\Delta\alpha}{\omega} \quad (8)$$

Substituting (7) in (8) we get (9), that expresses the referred result, since G_f and $\Delta\alpha$ are constants.

$$\Delta t = \frac{\Delta\alpha}{G_f} V_f \quad (9)$$

The G-LOC risk will be given by (10), where $\Delta t_{G-LOC}(G_f)$ is a model of the pilot +Gz tolerance [4].

$$Risk_{G-LOC} = \frac{\Delta t}{\Delta t_{G-LOC}(G_f)} \quad (10)$$

After some simplifications the pilot +Gz tolerance from [4] can be written as (11).

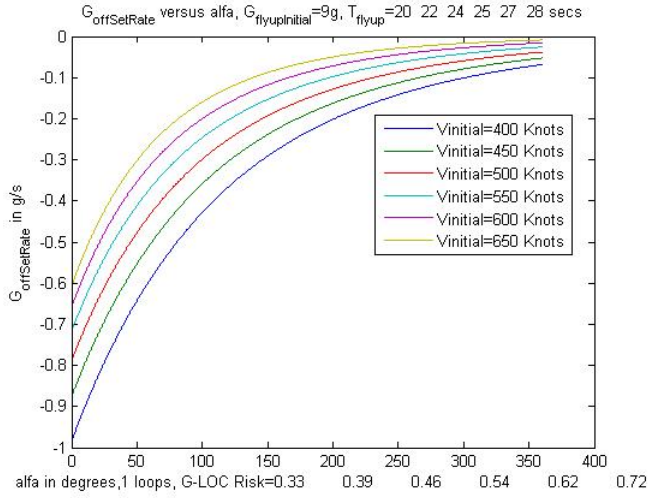


Fig. 16. Evolution of the instantaneous negative variation of centripetal acceleration, $G_{offSetRate}(\alpha)$, for an initial centripetal acceleration of 9g and initial speeds between 400 and 650 knots.

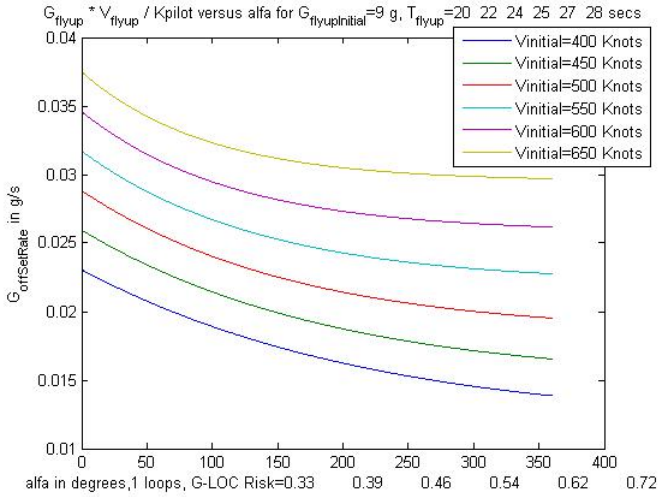


Fig. 17. Evolution of $V(\alpha)G_f(\alpha)/K_{pilot}$ for an initial centripetal acceleration of 9g and initial speeds between 400 and 650 knots. We will see in the next section that the area under these curves correspond to the G-LOC risk of the respective circular horizontal loops.

$$\Delta t_{G-LOC}(G_f) = \frac{K_{pilot}}{G_f^2} \quad (11)$$

In (11) the pilot +Gz tolerance is characterized by K_{pilot} . A typical value is given by $\Delta t_{G-LOC}(9g) = 20s$ for fighter pilots, since it is required that F-16 pilots must be submitted to 9g of centripetal acceleration during 20s in a human centrifuge, without suffering a G-LOC. Substituting (9) and (11) in (10) we obtain the final expression of the G-LOC risk in a flyup given by (12).

$$Risk_{G-LOC} = \frac{\Delta\alpha V_f G_f}{K_{pilot}} \quad (12)$$

Dividing the loop in N flyups of $\Delta\alpha$ radians, considering constants the centripetal acceleration G_{f_i} and speed V_{f_i} through each flyup, the G-LOC risk in a horizontal loop can be written as (13).

$$Risk_{G-LOC} = \frac{\Delta\alpha}{K_{pilot}} \sum_{i=1}^N V_{f_i} G_{f_i} \quad (13)$$

In the limit, when $N \rightarrow \infty$ and $\Delta\alpha \rightarrow 0$, we get the exact value of G-LOC risk in a horizontal loop given by (14).

$$Risk_{G-LOC} = \frac{1}{K_{pilot}} \int_0^{-2\pi} G_f(\alpha) V_f(\alpha) d\alpha \quad (14)$$

Equation (14) tells us that the exact G-LOC risk in a horizontal loop is given by the area under the curve of $V_f(\alpha)G_f(\alpha)$ divided by K_{pilot} . Equation (13) can be a good approximation of the exact value of G-LOC risk given by (14), if we decrease $\Delta\alpha$ and increase N . By trial and error we found that it is enough to use $N=1000$. To estimate the risk of G-LOC in a vertical loop, we just substitute, in (13), $\Delta\alpha$ by $\Delta\gamma$ and to obtain the exact G-LOC risk we must substitute, in (14), α by γ .

VII. CONCLUSIONS AND FUTURE WORK

We revised our previous work of an autopilot to make circular vertical loops, correcting the estimation of the risk of G-LOC in each loop, and we showed that this autopilot can be deployed on any aircraft with $G_{onSetRate} > 0.62g/s$. Then we designed an autopilot to make circular horizontal loops, and we showed that it can be deployed on any aircraft with $|G_{offSetRate}| > 1g/s$. We also showed that a circular horizontal loop has a lower G-LOC risk than a spiral like loop with the same constant centripetal acceleration and the same initial speed. Finally we made a revision and correction of our model of G-LOC risk and obtained the mathematical expression of the exact G-LOC risk in a loop. In a near future we plan to design an acrobatic flight planning system based on this model of the exact G-LOC risk.

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