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On the determinants of Tail Risk

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Abstract:

This paper aims to describe tail risk dynamics in the U.S equity market and put it in context of ESG practices. By estimating a firm-specific conditional tail index, the paper looks at the different dynamics that firm-specific characteristics play in the cross-section of firms with different loadings of Reputational Risk. The graphical evidence presented suggests that firms tail risk with lower Reputational Risk, indeed are less likely to be affected by market wide uneasiness.

Keywords Time Varying Risk, Pareto-distribution, ESG, Hill-estimator

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1 Introduction

The distribution of stock returns is a statistical representation of the possible outcomes or variations in the returns that can be expected from investing in a stock. Its distribution is typically characterized by a normal distribution, which means that most of the returns are centered around the average return, with a few extreme positive or negative returns on either side. This type of distribution is also known as a Gaussian distribution and is often used in financial modeling and analysis to estimate the probabilities of various outcomes. Considering that financial returns follow a normal distribution, the standard deviation arises as an appropriate risk measure. Mean-variance optimizing theory utilizes the idea of balancing expected return and risk, represented by standard deviation. However, as Mandelbrot (1963) showed, asset returns seem to have fatter tails than what the normal distribution suggests. This is a profound finding, as it could greatly impact asset pricing models as well as risk management techniques. For the mean-variance optimizing investor, the measure of risk could potentially be grossly misleading. The hypothesis of Kelly and Jiang (2014) is that investors' marginal utility is increasing in tail risk, and subsequently that tail risk will have implications on asset pricing.

Building on the works of Nicolau, Rodrigues, and Stoykov (2023), this work project will look at the dynamic time varying tail risk of individual firms and the role firm characteristics play in this. This paper expands their idea to include one more firm-specific variable (leverage), and to view results through an ESG-perspective. It will include the method as discussed by Beirlant and Goegebeur (2003) which estimates tail risk as a function of covariate information by utilizing a Maximum Likelihood estimator to obtain the appropriate parameter estimates.

The returns of stocks are considered to follow a Pareto-type distribution conditional on exceeding a threshold u on which we define the "tail" to start.

2 Literature Review

Bachelier (1900), made groundbreaking advancements in the application of probabilities and statistics for forecasting stock returns by employing the Central Limit Theorem (CLT). He found that price fluctuations were entirely random, statistically independent, and identically distributed, commonly abbreviated to i.i.d. As a result, his article "Theory of Speculation" proposed a Gaussian model with a zero mean, which he named the Random Walk hypothesis. Since these price fluctuations were i.i.d., the returns of these stocks would approach a bell curve, as returns of a stock over time are a simple sum of the price movements (Fama 1963). Assuming Bachelier's premises hold true, a Gaussian model can be constructed for any tradeable security returns. If one were to depict a Gaussian distribution, its characteristics would be that it is symmetrical, continuous and resembles a bell, thus earning it the moniker "bell-curve."

Bachelier's model, which features i.i.d. price movements and foreseeable risk, laid the foundation for the work of future theorists and practitioners. In accordance with this Gaussian-based random walk model, volatility and risk is best measured using variance and standard deviations. Motivated by the practical applications of this model, Markowitz (1952) introduced an equation to quantify risk in relationship to its reward. To estimate this, he utilized the mean and variance of the returns. Markowitz's equations serve as the basis for creating "efficient portfolios," which aim to maximize the ratio between profit and risk. When constructing a portfolio, following Markowitz's approach, an investor needs to estimate the variance, covariance, and mean return of each (tradeable) security. As a result, the investor can estimate "an efficient portfolio" – one that is supposedly resilient to extreme price fluctuations and optimized for the highest possible ratio between return and risk, as measured by variance.

The assumptions of normality and finite variance for financial asset returns are fundamental to classical financial models such as the Capital Asset Pricing Model (CAPM) (developed by Sharpe (1964), Lintner (1965), and Mossin (1966)), and the Black-Scholes formula.

This *Gaussian hypothesis* remained unchallenged until the groundbreaking studies by Mandelbrot (1963) and Fama (1963; 1965) emerged. Mandelbrot argued that, despite Kendall and Hill (1953) showing evidence of normality in asset prices, they overemphasized the agreement between the *Gaussian hypothesis* and their respective empirical distributions, and that the departures from normality had been neglected. Subsequently, numerous studies discovered that the empirical unconditional distribution of financial returns presents fatter tails and higher peaks around the center compared to Gaussian distributions (Fama 1965; Officer 1972; Fama and Roll 1968). This finding indicates the indisputable presence of leptokurtosis in empirical distributions of asset returns and the nonstationary nature of return variability (Fama 1965; So 1987).

In response to these observations, researchers have suggested alternative distributions that exhibit characteristics suitable for modeling asset returns. Fama (1963) and Mandelbrot (1963) aimed to account for excess kurtosis by modeling the distribution of continuously compounded returns as a member of stable-Lévy or stable Paretian distributions, where the normal distribution is a special case with a characteristic exponent of $\alpha=2$. Subsequent studies by Fama (1965), Fama and Roll (1968) and So (1987) have provided evidence supporting stable Paretian data-generating processes across a wide variety of financial time series.

The literature offers mixed empirical evidence in support of the stable Paretian hypothesis. In his influential paper, Fama (1965) found that daily returns of large US mature companies, as well as the DJIA stock index, follow stable Paretian distributions with a characteristic exponent $\alpha<2$. This result implies that Mandelbrot (1963) Paretian hypothesis aligns more closely with the data than the Gaussian hypothesis. It is important to note that when $\alpha<2$, the second moment does not exist, indicating infinite variance. This infinite population variance suggests that even as the sample size increases, the sample variance continues to display erratic behavior and does not appear to converge to a specific value.

Kelly and Jiang (2014) proposed a new measure for time varying tail risk, as estimated by the cross-section of returns. The main assumption being that each stock is loaded with a constant multiplicative factor towards the dynamic time varying market tail risk. To find this common time varying tail risk, they utilize the tail index estimator developed by Hill (1975). They show that their new measure has strong predictive power of aggregate stock returns. The measure shows persistence across a timespan from 1963 and until 2010. Their finding being that an increase in tail risk predicts an increase in market excess return over the next year. This indicates that tail risk could be a determinant of returns. However, Nicolau, Rodrigues, and Stoykov (2023) show evidence that the assumption of the common dynamic with a constant multiplicative parameter is not as simple as proposed by Kelly and Jiang (2014).

Beirlant and Goegebeur (2003) developed a method for modelling a tail index parameter in the presence of covariate information. They model the tail index as a function of the covariates, however assuming that the data is i.i.d. Nicolau, Rodrigues, and Stoykov (2023) relax this assumption to allow for weak dependence, thus making the method appropriate in a time-series context.

Nicolau, Rodrigues, and Stoykov (2023) apply their methodology to find a time-varying firm-specific tail index. The specifics of this method are going to be discussed in the methodology-part of this paper. An interesting possibility when having the firm-specific time-varying tail index, is that one can check if they in fact follow a common process as suggested by Kelly and Jiang (2014). Their results imply that there could be some common process governing the tail behavior, but likely more complex than what was suggested by the latter authors.

Aboura and Arisoy (2019) looks to explain if the performance of small, value, momentum and idiosyncratic volatility sorted portfolios could be explained by time-varying tail index as suggested by Kelly and Jiang (2014). Amongst other, they construct a six-factor model to include a tail-risk measure which performs better than the Fama and French (2015) five factor model over their sample, further showing the importance of tail-risk determining returns. They conclude that this tail-risk dynamics in fact seems to explain the four other pricing anomalies, as delineated in Fama and French (2015). One of the arguments made is that *value* firms are more prone to macroeconomic shocks, since they have more assets in place which could decline sharply in value.

According to Amihud and Mendelson (1986), Reinganum (1981) and Banz (1981), small stocks seem to be more prone to risk than stocks with high market capitalization. Possible explanations for this might be liquidity, higher transaction costs and other behavioral reasons. It could also be that small companies do not have the firepower to cope with business downturns (Lettau and Ludvigson 2001; Petkova and Zhang 2005). The latter shows that *value* stocks (stocks with low market-to-book ratio) indeed display lower sensitivity to tail risk than growth stocks and that it goes in the right direction to explain the value-premium, albeit with an underwhelming magnitude.

Choi (2013) empirically demonstrates the interaction between asset risk and financial leverage in explaining the equity risk dynamics of value and growth stocks. It reveals that during economic downturns, *value* firms experience an increase in asset betas and leverage, leading to a spike in equity betas. In contrast, *growth* firms exhibit less sensitivity to economic conditions, and their lower leverage contributes to the stability of their equity betas.

Bollerslev, Todorov, and Xu (2015) discuss how the variance risk premium, which according to their results, predicts future market returns, can be attributed to time variation in the part of the premium related to investor compensation for bearing jump tail risk.

Andersen, Fusari, and Todorov (2020) discovered that the risk premium linked to negative tail events, as evidenced in index options across global equity markets, serves as a powerful predictor of future returns for all indices. This tail risk premium is distinct from volatility, with compensation for negative jump risk being the main contributor to the equity premium.

Bloom (2009) provides an explanation for tail risk at the firm level, attributing it to tail uncertainty. This economic uncertainty adversely influences companies' investment choices. The author asserts that firm-level tail uncertainty serves as a conduit for tail risk's effect on the equity premium. Moreover, the tail exponent, which gauges tail heaviness, plays a crucial role in determining asset prices. Consequently, stocks sensitive to crashes demand a risk premium.

Ashwin Kumar et al. (2016) showed that firms scoring high on ESG indices, provided better returns and lower risk. They also show this effect to be different across industries. Both the Sharpe and Treynor ratio turned out better for the ESG-portfolios than their reference portfolios in 9 out of 12 industries. The annualized volatility was lower in all 12 industries. Meanwhile Hörter and Anderson (2019) argue that ESG factors mostly impact "downside" risk, and acknowledge the hard reality that judging firms' ESG behavior is not a black or white judgment, but often falls into the grey area.

3 Methodology

This section outlines the methodology used. The paper will analyze various measures of tail risk. First, the methodology proposed by Kelly and Jiang (2014) to calculate the cross-sectional tail risk is introduced. Then, the firm-specific tail risk estimation procedure proposed by Nicolau, Rodrigues, and Stoykov (2023) is presented.

3.1 Kelly and Jiang (2014) setup

Kelly and Jiang (2014) modelled the set of returns that fell below a threshold, as:

$$P\left(r_{i,t+1} < r \mid r_{i,t+1} < w_t \text{ and } F_t\right) = \left(\frac{r}{w_t}\right)^{\frac{-\alpha_i}{\lambda_t}}. \tag{1}$$

They refer to λ_t as a common time-varying component of stock i's left tail shape parameter. According to this setup each stock can have their own unique tail-shape, however the only time-varying component is one that is common to all stocks. To calculate this common time varying component, they utilize the well-established Maximum Likelihood estimator proposed by Hill (1975), i.e.,

$$\lambda_t^{Hill} = \frac{1}{K_t} \sum_{k=1}^{K_t} \ln \frac{R_{k,t}}{u_t}$$

(2)

where $R_{k,t}$ is the k-th daily return that falls below some threshold u_t during month t. In this paper, a month corresponds to 20 trading days, and the threshold u_t is set at 5% as suggested by Kelly and Jiang (2014). This threshold is to be determined by the econometrician conducting the analysis. For the rest of this work project consider

$$\alpha_{Mt} = \frac{1}{\lambda_t^{Hill}}.$$

(3)

Thus, the tail shape parameter in (1) can be rewritten as $\alpha_i \alpha_{Mt}$. This work project will refer to α_{Mt} as market tail risk.

3.2 Pareto-type Setup

In light of results and arguments presented by Nicolau, Rodrigues, and Stoykov (2023), we consider the following survival function for stock i's returns,

$$\bar{F}(r_{it}|\boldsymbol{X}_{it}, w_{in}) = 1 - F(r_{it}|\boldsymbol{X}_{it}, w_{in}) = \left(\frac{r}{w_{in}}\right)^{-\alpha(\boldsymbol{X}_{it}, \boldsymbol{\beta})} \mathcal{L}(r_{it}|\boldsymbol{X}_{it})$$

(4)

where $\boldsymbol{X}_{it}$ is a vector of explanatory variables $\boldsymbol{X}_{it} = (mtb_{it}, leverage_{it}, size_{it}, ES_t, vix_t)$, $\boldsymbol{\beta}$ is a vector of parameters and $\mathcal{L}(r_{it}|\boldsymbol{X}_{it})$ is a slowly varying function at infinity. With this setup, we assume that $r_{i,t}$ follows a Pareto-type distribution, with conditional tail index,

$$\alpha_{it}(\boldsymbol{X}_{it}, \boldsymbol{\beta}) = \exp(\beta_0 + \beta_1 mtb_{it} + \beta_2\ leverage_{it} + \beta_3 size_{it} + \beta_4 ES_t + \beta_5 Vix_t). \tag{5}$$

The regressors will be explained in a later section. w_{in} is the threshold based on which we define where the "tail" of an assets' returns' distribution begins. It has been chosen in line with procedures shown in Wang and Tsai (2009) and Nicolau, Rodrigues, and Stoykov (2023) to ensure the smallest discrepancy possible between the empirical distribution of $\{\hat{\mathrm{U}}(\boldsymbol{X}_{it}): r_{it} < w_{in}\} = \exp\left(-\exp(\boldsymbol{X}_t\hat{\boldsymbol{\beta}})\ln\left(\frac{r_{it}}{w_{in}}\right)\right)$ and $U = [0,1]$.

The discrepancy measure is:

$$\hat{D}(w_{in}, \boldsymbol{X}_{it}) = \frac{1}{n_0}\sum_{t=1}^{n}\left\{\hat{\mathrm{U}}_{it} - \hat{F}_n(\hat{\mathrm{U}}_{in})\right\}^2 I(r_{it} < w_{in}) \tag{6}$$

and w_{in} is selected as the minimum of the discrepancy measure, i.e.,

$$w*_{in} = \arg\min \hat{D}(w_{in}, \boldsymbol{X}_{it}). \tag{7}$$

For further details regarding this procedure, please refer to Nicolau, Rodrigues, and Stoykov (2023).

3.3 Dataset Creation

The data analyzed was downloaded from the CRSP/Compustat-merged database accessed through the WRDS platform. It is filtered to only consist of Common Ordinary Shares (Share code 10 and 11 on the CRSP database) traded on New York Stock Exchange, American Stock Exchange, or NASDAQ. Another requirement is that the variables needed to calculate the explanatory variables need to be available. Only stocks with at least 500 observations are included to ensure enough observations to estimate the tail behavior. The data analyzed has a span from 2000-01-01 until 2022-12-31. The final sample consisted of n = 8980 securities over 5787 trading days.

The CRSP-stock file includes nominal returns; however, they do not include the potential returns when delisting occurs. For some stocks, this delisting return is not available and is assumed to be 0, however when available the return of the asset is,

$$return_t = (1 + delisting\ return_t) * (1 + trading\ return_t) - 1 \tag{8}$$

as suggested by the CRSP manual.

The market capitalization of a firm is calculated by aggregating the market capitalization of all listed stocks the firm has. As an illustration, Berkshire Hathaway has had two stocks listed the whole period, and thus the market capitalization for Berkshire Hathaway follows the firm and not the individual stock. As the other firm-specific variables are dependent on this one, this is also the case with $leverage_{it}$ and mtb_{it}.

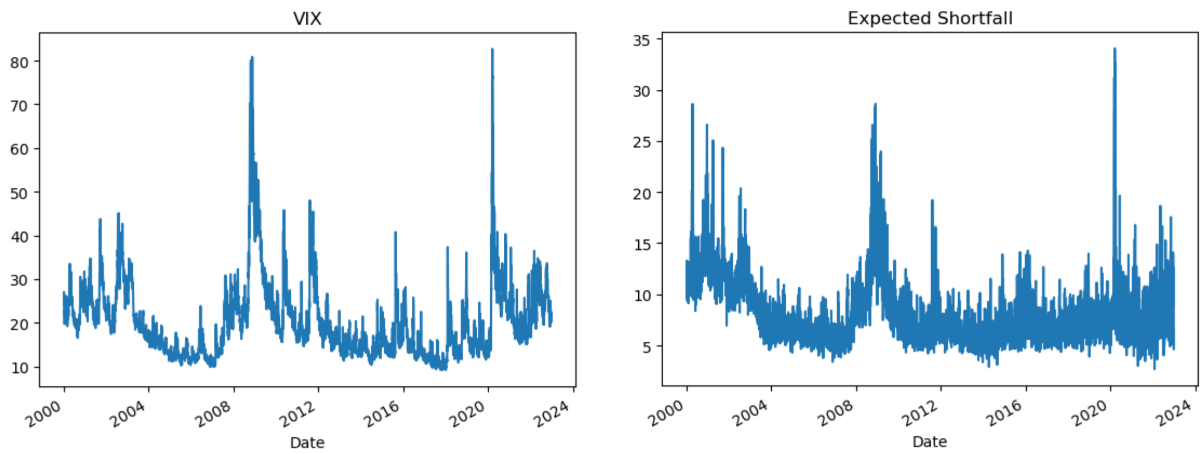
Leverage is the ratio between long term debt and market capitalization while mtb is the ratio between market capitalization and the book value of common equity.

The variables $market\ cap_{it}$, $leverage_{it}$ and mtb_{it} are all cross-sectionally ordered percentiles of the specific firms' market capitalization, leverage, and market to book ratio, respectively.

Expected shortfall is calculated as the average of returns in each day that is lower than the 5% cross sectional quantile times -100. This means that the variable ES_t is the loss of an equal weighted portfolio with initial value $100 if the portfolio was equally invested in the 5% worst performing stocks that day.

The CBOE Volatility Index (from now on referred to as VIX) is a measure of expected volatility in the stock market over the next 30 days, based on S&P 500 index options prices and is given by the Chicago Board Options Exchange. It is often called the "fear index" because it tends to increase during times of uncertainty or market turmoil. It is calculated by looking at the implied volatility in the index options. A higher value of the VIX, means a higher implied volatility. In Figure 1, we see the VIX and *ES* plotted.

Figure 1:Market indicators VIX and Expected Shortfall



To present the analysis through an ESG-perspective, we utilize the RepRisk database and sort firms according to their most recent RepRisk Rating (from here on, RepRisk Rating is referred to as ESG-rating). The scale from best to worst: AAA, AA, A, BBB, BB, B, CCC, CC, C and D. This scale can be compared to a credit rating. The RepRisk database is accessed through the WRDS platform. It leverages machine learning to categorize sentiment in social media, news, and other publicly available sources towards different risk factors that companies are exposed to. Therefore, this ESG-rating is only related to what is known to the public, and not something done by an independent external auditor with privileged access. This means that it is not a measure of the effectiveness of real ESG-practices, but rather a measure of the perception of a firms ESG-practices. Not all firms have an ESG-rating. The number of firms covered by the database in our sample is n = 2885. Results will therefore have to be interpreted with this in mind. When looking at the distribution of ESG-ratings in our sample, we decide to define our own categories for ESG-behavior. Rating AAA and AA as Best, A and BBB as Good, BB is Medium and the rest is Low or unknown. From here on, this is referred to as ESG-category. This is done to ensure that each category has at least 100 assets.

Table 1: Number of firms in each Sector ESG-rating.

Sector	n
Communication Services	315
Consumer Discretionary	1240
Consumer Staples	334
Energy	441
Financials	1614
Health Care	1704
Industrials	1052
Information Technology	1710
Materials	344
Real Estate	64
Unknown	16
Utilities	146
Total	8980

ESG-rating	n
AAA	4
AA	1495
A	814
BBB	328
BB	128
B	59
CCC	50
CC	6
C	1
D	0
Unknown	6095
Total	8980

ESG	n
Best	1499
Good	1142
Medium	128
Low	116
Unknown	6095
Total	8980

4 Results

In this section, we first analyze the relationship between market tail risk and implied volatility as measured by the VIX. It is found that they show coherence between each other. Second, the firm specific tail risk themselves are analyzed and presented across ESG-rating. A clear conclusion does not seem to emerge. Lastly, the significance of the firm specific determinants of tail risk are presented.

4.1 Relationship between implied volatility and market tail risk

As disclosed, the VIX is often referred to as the "investor fear gauge". One could assume that in times of investor fear, one would also observe higher tail risk. To test for this, we run an OLS regression where:

$$\text{Model 1: } \Delta Vix_t = \beta_0 + \beta_1 \Delta\alpha_{Mt} + u_t.$$

The hypothesis being that when the market tail risk α_{Mt}, as calculated by the methodology suggested by Kelly and Jiang (2014) outlined in section 3.1, increases (recall that lower values of α_{Mt} means higher tail risk) the implied volatility of options on the S&P also rises. If this were to be true, then $\beta_1 < 0$. In fact, we find this relationship to be true as seen in Table 2 with statistical significance.

Table 2: OLS results on the change in VIX and change in market tail risk

	Δvix_t
Intercept	-0.0005
	(-0.021)
$\Delta\alpha_{Mt}$	-1.9549***
	(-3.422)
Observations	5786
R^2	0.002
Note:	*p<0.1; **p<0.05; ***p<0.01

This shows that there is coherence between the methodology suggested by Kelly and Jiang (2014) and how investors behave in the market. One possible explanation for this phenomenon is that when investors witness deviations from normality, such as more extreme events than what would be expected based on a normal distribution, they tend to seek protection through the derivative market.

4.2 Firm specific tail risk

In Figure 2, we see the estimated conditional tail index $\hat{\alpha}_{it}(\boldsymbol{X}_{it}\beta_i)$ for selected companies (companies that remain public for most of the period, one for each ESG-rating). A lower (higher) $\hat{\alpha}_{it}(\boldsymbol{X}_{it}\beta_i)$ corresponds to high (low) probability mass in the tail. For example, generally all firms show their smallest values in 2008, the midst of the financial crisis and at the outbreak of the pandemic in 2020. It is interesting that these times of prominent economic crisis seem to affect all the selected firms in the figure. According to Figure 2, this further suggests that there indeed is some common process governing the tail risk of the market and of the individual firms that deserve further inspection. Conversely, Figure 1 also indicates that these were times of economic distress, as measured by the VIX and ES.

For instance, CITIGROUP's high tail risk in 2008 is explained by the financial crisis, where they for a period were on the brink of bankruptcy but received a government bailout. Another analogous evidence is Boeing's tail risk through the pandemic years of 2020-2021, where the commercial airline industry was effectively halted for extended periods of time. BOEING CO has of course other lines of business, but the capital-intensive industry that airplane manufacturing is saw them struggle in this period.

In Figure 3**,** the average $\hat{\alpha}_{it}$ for each ESG-category is plotted. As in Figure 2, the commonality across time seems evident. Interestingly, the movements seem more prominent the lower the ESG-category. This suggests that firms that have high Reputational Risk, as measured by RepRisk database, indeed are more prone to tail risk.

Figure 2: Tail Index Estimates ($\hat{\alpha}_{it}$) for selected companies and their ESG-rating

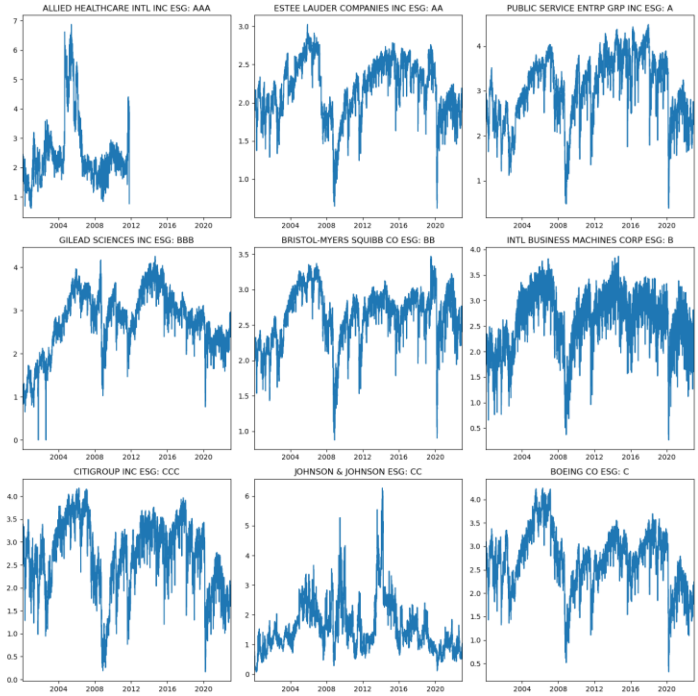
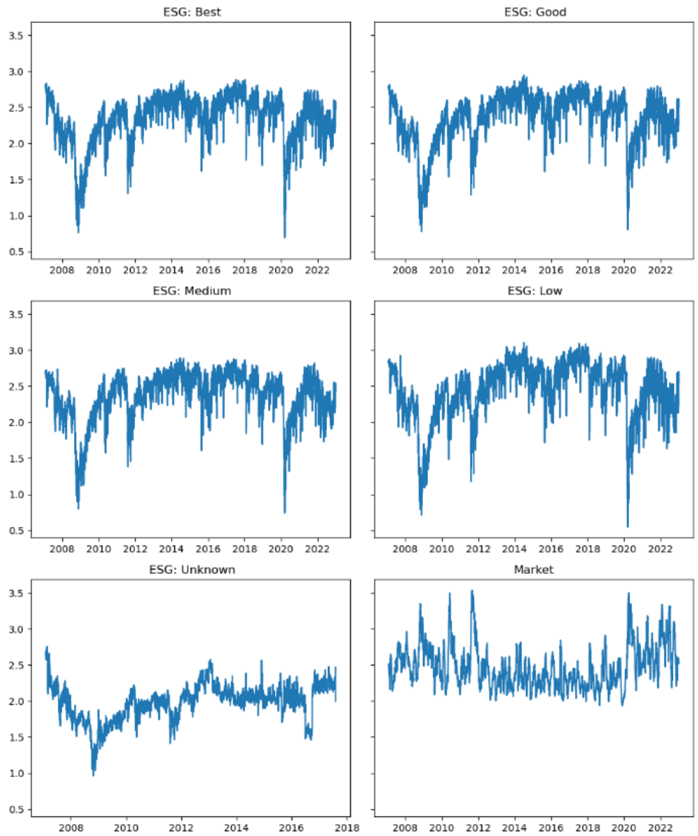


Figure 3: Average Tail Index Estimates $\hat{\alpha}_{it}$ for each ESG and α_{Mt}



4.3 Determinants of Tail Risk

and estimated by the procedure in outlined in section 3.2.

Considering the literature review we would assume that *large*, *growth* and *solid* stocks (meaning firms with high market capitalization, high ratio between market capitalization and book value of equity and low ratio between market capitalization and debt) would be considered less risky. We capture these levels with the variables $size$, mtb and $leverage$, as discussed in section 3. One could summarize the impact of these firm specific variables on $\alpha_{it}(\boldsymbol{X}_{it}, \boldsymbol{\beta})$ as: $\beta_1 > 0,\ \beta_2 < 0\ and\ \beta_3 > 0$.

Table 3, we show some selected stocks (same as in Figure 2) and their coefficients as specified in (5) and estimated by the procedure in outlined in section 3.2.

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Table 3: Results of $\hat{\beta}_{i,k}$ as specified in Section 3.2

	β_1 *mtb*	β_2 *lev*	β_3 *size*	β_4 *ES*	β_5 *Vix*	*n*
ALLIED HEALTHCARE INTL INC	-1.229*	0.173	5.121***	-0.064***	0.009	2971
	(-1.835)	(0.755)	(2.994)	(-3.123)	(0.953)	
ESTEE LAUDER COMPANIES INC	-1.356	-1.338	-1.022	-0.010	-0.015***	5787
	(-0.640)	(-1.272)	(-0.322)	(-0.723)	(-3.641)	
PUBLIC SERVICE ENTRP GRP INC	-0.642	-1.425	5.456	-0.007	-0.030***	5787
	(-0.748)	(-1.506)	(0.666)	(-0.343)	(-5.448)	
GILEAD SCIENCES INC	-0.142	-0.798	11.078***	-0.022	-0.013**	5787
	(-0.149)	(-1.006)	(6.618)	(-1.401)	(-2.173)	
BRISTOL-MYERS SQUIBB C	0.251	0.616	-6.742	-0.011	-0.014***	5787
	(0.245)	(0.719)	(-0.331)	(-0.596)	(-2.878)	
INTL BUSINESS MACHINES CORP	-0.150	0.609	17.799	-0.058***	-0.012**	5787
	(-0.099)	(0.254)	(0.380)	(-3.437)	(-2.220)	
CITIGROUP INC	-0.080	-0.772	17.924**	-0.023	-0.033***	5787
	(-0.174)	(-0.675)	(2.038)	(-1.221)	(-5.312)	
JOHNSON & JOHNSON	-5.261***	4.096**	1103.035***	0.047***	-0.031***	5787
	(-4.470)	(2.142)	(7.038)	(3.145)	(-4.229)	
BOEING CO	-0.809	0.209	37.998	-0.022	-0.019***	5787
	(-1.177)	(0.308)	(1.450)	(-1.083)	(-3.850)	

Note: *p<0.1; **p<0.05; ***p<0.01

Our market specific measures impacts would be $\beta_4 < 0$ *and* $\beta_5 < 0$. As ES is a measure of what is happening cross-sectionally at time t, and a higher value of ES indicates times of uneasiness in the market, one would be rational to assume that this would increase the tail risk (which translate in a decline in α_{it}). The same applies to the Vix.

In Figure 4, we report the percentage of statistically significant βs with the predicted sign within each sector. In Figure 5, we do the same, but sort them according to their ESG-category. When interpreting these figures, it is important to keep in mind the distribution of firms within each bucket (as seen in Table 1).

In Figure 4, we see that the Vix shows clear persistence as a determinant of tail risk, across industry sectors. It is the most consistent determinant, in the expected direction. 60-80% of coefficients are statistically significant in the expected direction, in most sectors. In *Utilities* over 80% of coefficient are statistically significant in the expected direction. ES also shows impact. Across sectors, it seems that the market specific risk measures are statistically significant in the expected direction. Vix ranges from 40% in *Health Care* to above 80% in *Utilities*. The finding that Vix is only statistically significant in 40% in the *Health Care* sector can be explained by its not so cyclical nature. Meaning that stocks in this sector are less prone to what happens in the financial markets than others. It seems that capital intensive sectors, (*utilities*, *energy*, and *real estate*) are more likely to be exposed to the Vix. The reason may be the increased cost of capital because of the unrest in the financial markets, thus making firms in these industries riskier in times of market unrest.

The most promising firm-specific variable seems to be *size,* which even outscores *ES* in some sectors. It ranges from 25% to 45% across industries. The other suggested firm-specific variables show little consistency, $leverage$ seem to have a greater significance than mtb, ranging between 10-15% and below 10% respectively.

Figure 4: Statistically significant coefficients with expected signs grouped by sector

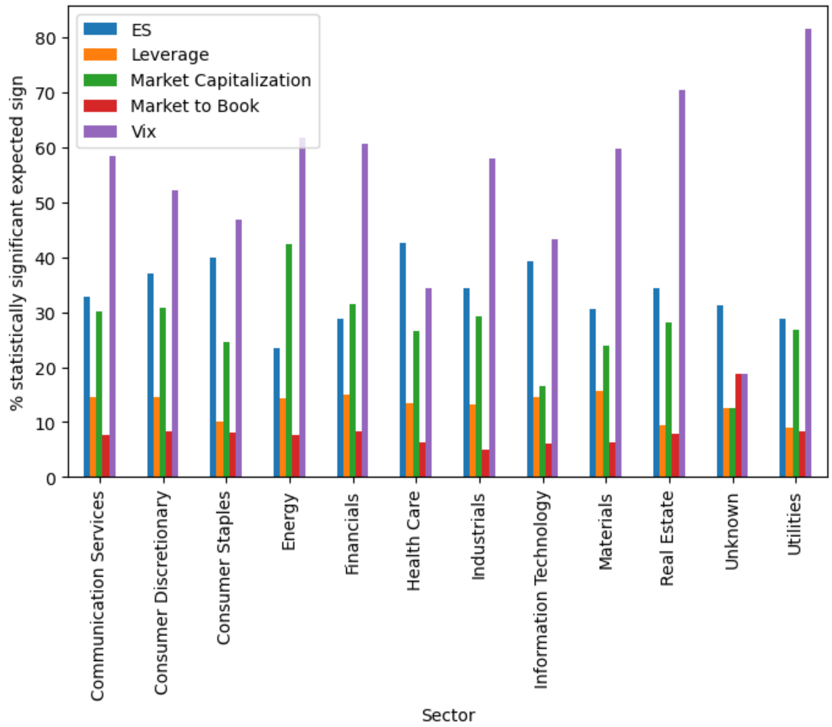
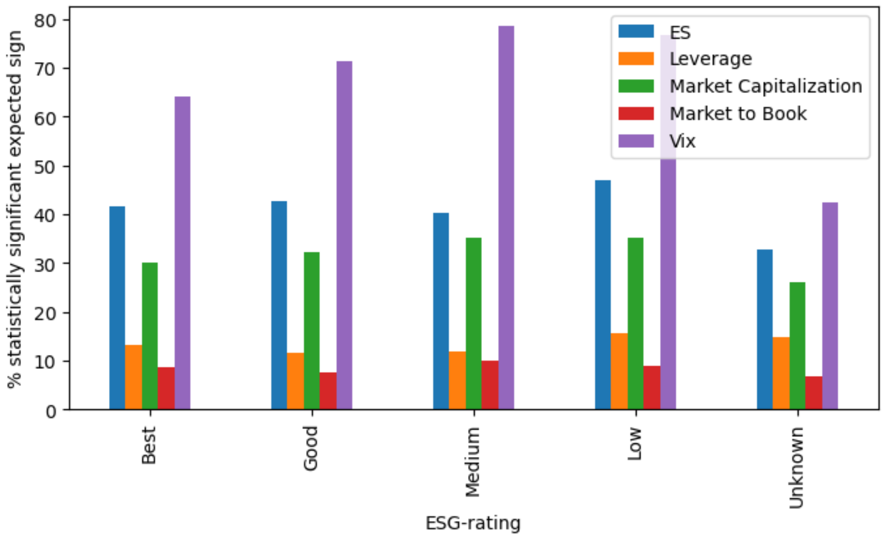


Figure 5: Statistically significant coefficients with expected signs, grouped by ESG-Category



It does seem like better ESG-rating, relates to a weakening of the significance of the Vix while the others are not that fluctuating. This implies that, as Ashwin Kumar et al. (2016) argued, firms scoring better on ESG indices show less tail risk in times of rising implied volatility. The percentage of significant coefficients decreases from 80% to 65% when going from ESG-category Low to Best. The other variables show similar significance when compared across different ESG-categories.

5 Discussion and Conclusion

Throughout this paper, various dynamics of firms' tail risk have been studied. The results in 4.1 indicate that there is coherence between implied volatility as indicated by the VIX and the market tail risk estimated as suggested by Kelly and Jiang (2014). Thus, when the market tail risk increases, the implied volatility in the market also increases.

Section 4.2 shows the conditional tail index estimated in nine randomly selected companies, across the ESG-ratings where we identify a pattern consistent with the idea of a common process determining the tail index. This is also evident when calculating the average tail risk across ESG-categories, where a similar pattern emerges. Arguably, this implies that there exists a common process determining firm specific tail risk.

Furthermore, section 4.3 inspects the coefficient estimates, $\hat{\beta}_i$, of $\alpha_{it}(\boldsymbol{X}_{it}, \boldsymbol{\beta})$, and analyzes their statistical significance across sectors and ESG-category. It is shown that the most consistent coefficients are the market specific variables; ES and Vix. Interestingly, it is shown that the significance decreases when the ESG category gets better. Implying that firms with a good ESG score react less aggressively to tail risk than those with a bad rating. The additional firm specific variable $leverage$ showed more statistical significance than the mtb, with $size$ performing the best.

Concluding this paper, the VIX seems to be the most important determinant of tail risk supporting the notion of a common process governing the time varying tail risk of firms. Moreover, it seems that the tail risk of firms with low reputational risk behaves less aggressive to this common process than those with high reputational risk.

6 Future research

As the availability of ESG-ratings increases and the timespan of which they have existed, it would be interesting to see something like the RepRisk Index linked to the conditional tail index. Exploring other firm-specific characteristics might also be of interest. One interesting approach would be something like the setup of this paper but viewing the results through the lens of managerial competence, compensation, or engagement in M&A activity. Applying this firm specific conditional tail index in an asset pricing setting would also be of great interest, and to my knowledge has not yet been done.

Considering that leverage more consistency than market-to-book in determining tail risk suggests that it deserves to be explored further as a determinant of tail risk in the future.

Another area of research could be to look at the relationship between the firm specific tail index suggested by Nicolau, Rodrigues, and Stoykov (2023) and the cross sectional tail index suggested by Kelly and Jiang (2014) quantitatively.

Even though the methodology used in this paper is heavily based on the one suggested by Nicolau, Rodrigues, and Stoykov (2023), the results are not coherent with theirs. This is a cause for concern and suggests that nuances in the approach and sample selection play an important role for the results. Therefore, it is suggested to do further research, to further make the approach more robust to such nuances.

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