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LEARNING TO COMPETE AND VICE VERSA

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ABSTRACT

We consider a dynamic price-setting duopoly selling to a sequence of buyers with uncertain demand. We assume that each firm's unit cost is a decreasing function of its cumulative past sales, and characterize equilibrium. We provide sufficient conditions for the leader's dominance to be self-reinforcing, in the sense that the probability of winning the next sale increases with past success. Next, we compare private and social benefits from learning-by-doing. Although learning economies are beneficial from a social standpoint, industry profits are strictly lower in a world of learning than in a world with no learning. Furthermore, in equilibrium, learning occurs too slowly from society's standpoint. Finally, we develop a theory of predatory pricing based on learning economies, showing that entry and subsequent exit can be an equilibrium outcome, that the possibility of a rival's exit induces more aggressive pricing, and that predatory pricing might be socially beneficial.

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1. INTRODUCTION

There is a sparse and rather dormant industrial organization literature on the importance of the learning curve for strategic oligopoly incentives.[1] The learning curve hypothesis is that unit costs decline with cumulative production. The learning curve gives firms an obvious incentive to produce more early to lower production costs later. Spence (1981) showed numerically that firm-specific learning might be a barrier to entry, although Ghemawat and Spence (1985) demonstrated that this effect was attenuated by industry spillovers. Fudenberg and Tirole (1983) showed for special cases that firms' outputs might actually decrease with learning, that learning is socially beneficial, and that a balanced-budget tax-subsidy scheme can improve welfare. Ross (1986) showed that learning enhanced a Stackelberg advantage. All of these results assumed Cournot quantity setting behavior by firms, excepting Ross's Stackelberg leader.

Our approach to the learning curve is very different, is in some respects more general, and yields new insights. We model a duopoly selling to a sequence of buyers with uncertain demand. In each period, prices are set simultaneously and one unit is sold. Each firm's unit cost decreases with its cumulative past sales. Section 2, which introduces the model in more detail,

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[1] A business strategy literature, pioneered by the Boston Consulting Group (1972), discusses a firm's optimal pricing strategy with a learning curve. There is also an empirical literature, e.g. Lieberman (1984) and references therein. Another strand of literature, including articles by Arrow (1962) and Romer (1986), evaluates the implications of learning-by-doing for economic growth.

provides a general characterization of equilibrium prices.[2] Section 3 shows how market dominance might be self-reinforcing, in the sense that a leader's probability of winning a sale increases with the length of the lead, although this is not always so.[3] Section 4 shows that learning is privately disadvantageous, in the sense that industry profits are strictly lower in a world of learning compared to a world with no learning. Section 5 shows that learning is always socially beneficial, although specialization of production might come too slowly from society's point of view. Finally, Section 6 shows how our model leads to a theory of predatory pricing. While prices below marginal cost are not necessarily predatory, predatory pricing can occur in equilibrium if firms have avoidable fixed costs, and may be socially beneficial. Section 7 concludes with a brief discussion of some open questions.

2. THE MODEL

Consider a price setting differentiated duopoly selling to an infinite sequence of heterogeneous buyers. In each period, a buyer demands at most one

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[2] Mookherjee and Rey (forthcoming) analyze a dynamic price-setting oligopoly with deterministic demand and learning by doing. They focus on "folk theorem" results, thus indicating the potential scope for cooperative behavior. In contrast, our restriction to Markov Perfect equilibria focuses on noncooperative strategic interaction.

[3] This suggests that Arthur's (1989) model of lock-in by small historical events is robust to the introduction of strategic behavior. Independent work by Salant (1991) shows market dominance is self-reinforcing in a model with deterministic demand and stochastic learning.

unit of the good from either firm i or firm j. The buyer's willingness to pay for firm 1's and firm 2's products is given by $b + y$ and $b + z$, with $y \geq 0$ and $x \geq 0$. Thus, $x \equiv z - y$ is the price premium the buyer is willing to pay for firm 2's product. x varies across buyers, possibly taking any real value. We refer to x as a buyer's "preference parameter". Throughout, we assume b is sufficiently large that in equilibrium a sale always occurs.[4]

Firms do not observe buyers' preferences. The preference parameter x is i.i.d. according to a twice differentiable c.d.f. $F(x)$ with full support. $f(x)$ denotes the corresponding density function, and is symmetric around zero.[5]

Assumption 1: $H(x) \equiv F(x)/f(x)$ is increasing.

Assumption 1 has the following interpretation. If $x = p_2 - p_1$ is the price differential between firms 2 and 1, then $F(x)$ is the probability that firm 1 makes a sale, and $p_1/H(x)$ is the price elasticity of expected demand for firm 1's product. Thus, expected marginal revenue equals $p_1 - H(x)$. The

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[4] The significance of this assumption will be clear in the proof of Theorem 2.1 below. See note 8.

[5] The assumption that $f(\cdot)$ is differentiable only matters for Lemmas 3.2 and 5.2. It is not entirely innocuous. Suppose that y and z are i.i.d. with a density $\phi(\cdot)$ on the positive real line. Then

$f(x) = \int_0^\infty \phi(|x|+z)\phi(z)\,dz$. $f(x)$ is differentiable everywhere except zero if

$\phi(\cdot)$ is differentiable, and $f'(0)$ exists if and only if $\phi(0) = 0$.

assumption implies that, given a rival's price, expected marginal revenue decreases as a lower price increases the probability of a sale.

The key assumption of our model is that a firm's unit cost is a decreasing function of cumulative past sales, $c(q)$. We further assume that learning is finite, in that a firm reaches the bottom of its learning curve upon making m sales.

Assumption 2: $c(q) > c(q+1)$ for $q < m$ and $c(q) = c(m)$ for $q \geq m$.

We maintain Assumptions 1 and 2 throughout.

Our solution concept is Symmetric Markov Perfect Equilibrium (SMPE). This is a subgame perfect equilibrium with the property that each firm's strategy depends only on the state of the game, and not on the firm's identity. The state of the game is defined by a pair (i,j), where i and j are the cumulative sales of each firm. Since learning is bounded, state $(i,m+k)$ is equivalent to state (i,m) for $k > 0$. Therefore, a strategy for each firm is a mapping $p(i,j)$ which gives its price for each possible state of the game and has the property that $p(i,m+k) = p(i,m)$ for $k \geq 0$.

Given a symmetric strategy, we can define recursively the value functions $v(i,j)$, giving the value at the beginning of state (i,j) for the firm with cumulative sales i. Our first result relates these value functions to the equilibrium strategies.

Before stating this result, we need more notation. Let $C(i,j) = c(i) - c(j)$ denote the cost difference between firms in state (i,j). Also, define the function $\Pi(x) = F(x)H(x)$. $\Pi(x)$ is equal to the one-shot expected profit of firm 1 if x is the Bertrand equilibrium price difference. Finally, let $G(x) = \Pi(x) - \Pi(-x) = H(x) - H(-x)$. Note that $\Pi'(0) = 1$, $\Pi(x)$ is increasing, $G(0) = 0$, $G'(0) = 2$, and $G(x)$ is increasing.[6]

Theorem 2.1. A SMPE has the following properties:

$$p(i,j) = c(i) + H(-P(i,j)) - \delta w(i,j), \tag{2.1}$$

where $w(i,j) = v(i+1,j) - v(i,j+1)$ is firm i's value of winning a sale, and $P(i,j) = p(i,j) - p(j,i)$ is the equilibrium price difference;

$$v(i,j) = \Pi(-P(i,j)) + \delta v(i,j+1); \tag{2.2}$$

$$P(i,j) = C(i,j) - G(P(i,j)) - \delta W(i,j), \tag{2.3}$$

where $W(i,j) = w(i,j) - w(j,i)$; and

$$v(i,m+1) = v(i,m). \tag{2.4}$$

Proof.[7] At each stage (i,j), the firm with cumulative sales i (referred to as firm i) solves the following maximization problem:

$$\max F(q-p)[p-c(i) + \delta v(i+1,j)] + [1 - F(q-p)]\delta v(i,j+1) \tag{2.5}$$

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[6] These facts follow from the symmetry and differentiability of $f(x)$ and Assumption 1.

[7] The proof of Theorem 2.1 does not require Assumption 1.

p

where q is the price set by the rival firm.[8] The first-order conditions are given by

$$F(q-p) - f(q-p)[p - c(i) + \delta v(i+1,j)] + f(q-p)\delta v(i,j+1) = 0, \qquad (2.6)$$

which simplifies to $p = c(i) + H(q-p) - \delta w(i,j)$. Substituting $p(i,j)$ and $p(j,i)$ for p and q, we obtain (2.1). Furthermore, substituting $p(i,j)$ and $p(j,i)$ for p and q in the maximand of problem (2.5), and simplifying, we obtain (2.2). Taking the difference between (2.1) and its symmetric counterpart for firm j, and simplifying, we obtain (2.3). Finally, the boundary conditions (2.4) follow from the fact that (i,m) and $(i,m+1)$ are equivalent states by Assumption 2.

Q.E.D.

<u>Theorem 2.2</u>. A SMPE exists uniquely.

<u>Proof</u>. We first show that the necessary conditions of Theorem 2.1 have a unique solution. First, consider state (m,m). By Assumption 2, $C(m,m) = w(m,m) = W(m,m) = 0$. Since $G(x)$ is increasing and $G(0) = 0$, equations (2.3), (2.2) and (2.1) imply $P(m,m) = 0$, $v(m,m) = \Pi(0)/(1-\delta)$ and $p(m,m) = c(m) + H(0)$. Next, consider state (i,m), $i < m$, and adopt the induction hypothesis that a unique SMPE exists for the subgame beginning in state $(i+1,m)$. Equation (2.2) implies that

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[8] This specification implicitly assumes that $p \le b$ over the relevant range, which is true if b is sufficiently large. A price above b would choke off some demand.

$$v(i,m) = \frac{\Pi(-P(i,m))}{(1-\delta)} \quad . \tag{2.7}$$

and

$$v(m,i) = \Pi(P(i,m)) + \delta v(m,i+1). \tag{2.8}$$

Therefore,

$$w(i,m) = v(i+1,m) - \frac{\Pi(-P(i,m))}{(1-\delta)}, \tag{2.9}$$

$$w(m,i) = \Pi(P(i,m)) - (1-\delta)v(m,i+1), \tag{2.10}$$

and

$$W(i,m) = v(i+1,m) + (1-\delta)v(m,i+1) - \frac{\Pi(-P(i,m))}{1-\delta} - \Pi(P(i,m)). \tag{2.11}$$

Thus, it follows from equation (2.3) and the definition of $G(x)$ that $P(i,m)$ equals the value of x solving

$$x + (1+\delta)\Pi(x) - \frac{\Pi(-x)}{(1-\delta)} = C(i,m) - \delta v(i+1,m) - \delta(1-\delta)v(m,i+1). \tag{2.12}$$

The right hand side is bounded and independent of x, while the left hand side is continuously increasing, ranging between $-\infty$ and $+\infty$. It follows that equation (2.12) yields a unique solution for $P(i,m)$. Given $P(i,m)$, both $p(i,m)$ and $p(m,i)$ are determined by equation (2.1). A similar, but even simpler, induction argument establishes a unique solution to (2.1)-(2.3) for other subgames. Thus, equations (2.1)-(2.4) characterize unique necessary conditions for a SMPE. Sufficiency follows from the fact that Assumption 1 implies the maximand of (2.5) is quasi-concave.

Q.E.D.

3. MARKET DOMINANCE

It is intuitively plausible that as cost differences between firms widen, so should price differences. This is generally true in static models, and is true in our model also, if the future is sufficiently unimportant. However, in our model, this result is not a comparative static one, but has dynamic implications for the evolution of market structure. With learning, winning a sale increases the probability of winning the next sale, and so, once ahead, a firm tends to stay ahead.

Theorem 3.1. For sufficiently small δ, and $i \le j \le m$,

(i) $P(i,j) > 0$ if $i < j$,

(ii) $P(i,j) < P(i,j+1)$ if $j < m$,

(iii) $P(i,j) > P(i+1,j)$ if $i < j$.

Proof. Since Assumption 1 implies $G(\cdot)$ is increasing, the results follow from Assumption 2 and equation (2.3) as $\delta \to 0$.

Q.E.D.

We can also prove that market dominance is self-reinforcing if one firm is at the bottom of the learning curve and the other firm has one step to go. By implication, self-reinforcing market dominance holds for the special case of "one-step learning" with $m = 1$.

Theorem 3.2. $P(m-1,m) > 0$.

Proof. From Theorem 2.1:

$$v(m,m) = \frac{\Pi(0)}{(1-\delta)} ;$$

$$v(m-1,m) = \frac{\Pi(-P(m-1,m))}{(1-\delta)} ;$$

$$v(m,m-1) = \Pi(P(m-1,m)) + \frac{\delta\Pi(0)}{(1-\delta)} .$$

Thus,

$$\begin{aligned} W(m-1,m) &= 2v(m,m) - v(m-1,m) - v(m,m-1) \\ &= \frac{2\Pi(0)}{(1-\delta)} - \frac{\Pi(-P(m-1,m))}{(1-\delta)} - \Pi(P(m-1,m) - \frac{\delta\Pi(0)}{(1-\delta)} \\ &= \frac{(2-\delta)}{(1-\delta)} [\Pi(0) - \Pi(-P(m-1,m))] - G(P(m-1,m)). \end{aligned}$$

It follows from equation (2.1) that $P(m-1,m)$ is the unique value of x that solves

$$\psi(x) \equiv x + (1-\delta)G(x) + \frac{\delta(2-\delta)}{(1-\delta)} [\Pi(0)-\Pi(-x)] = C(m-1,m),$$

and the result follows from the facts that $C(m-1,m) > 0$ by Assumption 2, $\psi(x)$ is increasing by Assumption 1, and $\psi(0) = 0$.

Q.E.D.

We can also show that self-reinforcing market dominance holds if the future matters sufficiently. This result builds on two important lemmas.

Lemma 3.1.

$$\lim_{\delta \to 1} P(i,j) = 0. \tag{3.1}$$

Proof. See Appendix.

Lemma 3.2.

$$\lim_{\delta \to 1} \frac{dP(i,j)}{d\delta} = -\sum_{k=i}^{j-1} C(k,m) \tag{3.2}$$

for $i < j$.

Proof. See Appendix.

Theorem 3.3. For sufficiently large δ, and for $i \le j \le m$,

(i) $P(i,j) > 0$ if $i < j$,

(ii) $P(i,j) < P(i,j+1)$, if $j < m$,

(iii) $P(i,j) > P(i+1,j)$. if $i < j$.

Proof. Since $P(i,j) \to 0$ by Lemma 3.1, for values of δ in the left neighborhood of 1, we have $P(i,j) > P(k,l)$ if and only if $\frac{dP(i,j)}{d\delta} < \frac{dP(k,l)}{d\delta}$. The results then follow from Lemma 3.2.

Q.E.D.

The details of the proof of Lemma 3.2, from which Theorem 3.3 follows, are somewhat complicated. However, the underlying effects can be found in equation (2.3), which we rewrite as

$$P(i,j) + G(P(i,j)) = C(i,j) - \delta W(i,j), \tag{3.3}$$

where the left-hand-side is an increasing function of $P(i,j)$ which equals zero when $P(i,j) = 0$. It follows from this equation that $P(i,j) > 0$ if and only if the right-hand-side of (3.3) is positive.

We thus have two effects to consider. First there is the cost-difference effect, which explains Theorem 3.1. Second, there is the effect given by $W(i,j)$. Recall that, by definition, $W(i,j) = w(i,j) - w(j,i)$, i.e., $W(i,j)$ is the difference between the values of winning for firm i and for firm j. Therefore, a sufficient condition for monotonicity is that $W(i,j)$ increases with i and decreases with j; i.e. winning a sale increases the leader's relative value of winning the next sale, and conversely. The proof of Lemma 3.2 shows this is so for δ close to 1, in which case the second effect reinforces the cost-difference effect.

There is an alternative way of interpreting the effect of $W(i,j)$. Recall that $w(i,j) = v(i+1,j) - v(i,j+1)$, and thus

$$\begin{aligned} W(i,j) &= w(i,j) - w(j,i) \\ &= [v(i+1,j) - v(i,j+1)] - [v(j+1,i) - v(j,i+1)] \end{aligned}$$

$$= [v(i+1,j) + v(j,i+1)] - [v(i,j+1) + v(j+1,i)]$$

$$= U(i+1,j) - U(i,j+1)$$

where $U(i,j) = v(i,j) + v(j,i)$ is the industry value function at state (i,j). This equality means that the condition of the value of winning a sale being greater for the leader than for the follower is equivalent to the condition that industry profits become greater if the leader wins than if the follower wins. This is a generalization of the well-known condition that industry profits be greater under monopoly than under duopoly (cf. Gilbert and Newbery, 1982).[9]

We have not found a general result for intermediate values of δ when $m > 1$. In fact, there are numerical examples violating the monotonicity of prices. Figure 1 plots the equilibrium values of $P(0,2)$ and $P(1,2)$ for a special case as a function of the discount factor δ. Notice first that as $\delta \rightarrow 1$, both $P(0,2)$ and $P(1,2) \rightarrow 0$ (Lemma 3.1). Furthermore, the derivative of $P(0,2)$ with respect to δ is twice the derivative of $P(1,2)$ with respect to δ at $\delta = 1$ (Lemma 3.2). As a result, for values of δ close to 1, $P(0,2) > P(1,2)$ (Theorem 3.3). However, for lower values of δ, but not too low (Theorem 3.1), it may be that $P(0,2) < P(1,2)$, as the figure shows.

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[9] Budd, Harris and Vickers (1990) also conclude that "competition tends to evolve in the direction in which joint profits (i.e. the sum of the two firms' payoffs in the continuation game) are higher." (p.3) However, the model they use is different form ours in two important ways. They consider a one-dimensional state space and an exogenously given effort cost function, where effort determines the expected motion along the state space.

4. PRIVATE BENEFITS OF LEARNING

We next demonstrate how learning is privately disadvantageous, in the sense that symmetric firms would be better off if no further learning were possible. All of the benefits of learning, and more, are competed away.

Theorem 4.1. $v(i,i) > v(i,i+1)$ for $i < m$, implies that $v(i,i)$ is lower than it would be in a world with no learning beyond (i,i).

Proof. If $v(i,i) > v(i,i+1)$, then from equation (2.2) and symmetry

$$\begin{aligned} v(i,i) &= \Pi(0) + \delta v(i,i+1) \qquad (4.1) \\ &< \Pi(0) + \delta v(i,i) \\ &< \frac{\Pi(0)}{1-\delta}. \end{aligned}$$

which equals the present value of repeated Bertrand profits for symmetric firms.

Q.E.D.

Equation (4.1) has an interesting interpretation. Letting $\delta = 0$, $\Pi(0)$ is the short run Bertrand profit of a firm facing a symmetric rival. However, if $\delta > 0$, the ability to reduce costs with cumulative sales creates a "prize" for winning the next sale. Symmetric firms value this prize the same, and so completely bid it away competing for the next sale. Thus, short run expected profits are reduced below $\Pi(0)$ by the expected value of the prize. It follows that the present value of current and future expected profits of each firm equals the short run Bertrand profit plus the value of losing. But why does learning make symmetric firms strictly worse off? The reason is that a firm does not capitalize the losses that low prices impose on its rival.

In conjunction with Theorem 4.1, the next result points out that learning is privately disadvantageous whenever market dominance is self-reinforcing. Recall that section 3 showed this was the case if the future was either sufficiently unimportant or sufficiently important, and for one-step learning ($m = 1$).

Theorem 4.2. If $P(i,k)$ is increasing in k, then $v(i,j) > v(i,j+1)$.

Proof. From equation (2.2),

$$v(i,j) - v(i,j+1) = \Pi(-P(i,j)) - (1-\delta)v(i,j+1).$$

$$= \Pi(-P(i,j)) - (1-\delta) \sum_{k=1}^{\infty} \delta^{k-1}\Pi(-P(i,j+k)) < 0.$$

Since $\Pi(x)$ is increasing, the last inequality follows from $P(i,j+k) > P(i,j)$ because

$$(1-\delta) \sum_{k+1}^{\infty} \delta^{k-1}\Pi(-P(i,j+k))$$

is a weighted average of terms each of which is greater than $\Pi(-P(i,j))$.

Q.E.D.

It should be clear from the proof of Theorem 4.2 that monotonicity of P(i,k) is sufficient but not necessary for v(i,j) > v(i,j+1).

Anecdotal evidence suggests that learning does indeed have a negative impact on industry profits. A first example is given by commercial aircraft construction, which is known to be subject to significant learning economies. Seitz et al (1985) state that out of 22 commercial aircraft developed, only three - the Boeing 707,727 and the DC-8 - have been profitable. Returns on assets and on sales have been significantly below that for all manufacturing.

A second example comes from the semiconductor industry. "The chip business is a matter of yields, learning from experience and forward pricing. The greater the investment a manufacturer makes in a semiconductor plant, the more chips it can turn out. The higher its output, the lower its unit costs and the greater its operating experience. That translates, in turn, into higher yields and still lower unit costs" (The Economist, July 14, 1990). Competition for sales in the previous generation of memory chips (1-Megabit DRAMs) was such that chip makers "almost bankrupted themselves." Now that a new generation of memory chips (4-Megabit DRAMs) is overtaking the previous one, "Japan's giant chipmakers are rushing into what looks like a suicidal expansion of 4-megabit chip production" (The Economist, October 13, 1990).

5. SOCIAL BENEFITS OF LEARNING

In this section, we contrast equilibrium outcomes with those corresponding to maximized social welfare. First, we derive the socially optimal solution and show that there are always social benefits from learning, contrary to the absence of private benefits. Second, we show that the equilibrium rate of learning by the leader is too low from a social point of view if the future matters sufficiently.

We define social welfare to be the sum of expected producer and consumer surpluses. We view the social planner as choosing a symmetric function $P^*(i,j)$ such that, in state (i,j), the buyer receives firm 1's product if his preference parameter (x) exceeds $P^*(i,j)$, and firm 2's product otherwise. Again, we implicitly assume b is sufficiently high that a transaction always takes place.

In the following theorem, $b + \bar{y}$ is a buyer's expected value for firm 2's product.

<u>Theorem 5.1</u>. The socially optimal solution is characterized by the following properties:

$$P^*(i,j) = C(i,j) - \delta W^*(i,j); \tag{5.1}$$

$$U^*(i,j) = b + \bar{y} - c(j) + \delta U^*(i,j+1) + \Gamma(P^*(i,j)); \tag{5.2}$$

where $W^*(i,j) = U^*(i+1,j) - U^*(i,j+1)$, and $\Gamma(x) = \int_x^\infty (1-F(a))\,da$; and

$$U^*(i,m+1) = U^*(i,m). \tag{5.3}$$

Proof: By Bellman's principle, $U^*(i,j)$ obeys the following equation:

$$U^*(i,j) = b + \bar{y} - c(j) + \delta U^*(i,j+1) + \max_P \int_P^\infty [x - C(i,j) + \delta W^*(i,j)]\,dF(x)$$

The first-order condition for optimality can be simplified to get (5.1). Substituting this first-order condition back into the Bellman equation yields (5.2). Finally, the boundary conditions (5.3) follow from the fact that (i,m) and (i,m+1) are equivalent states.

Q.E.D.

Theorem 5.2. The maximized social benefits from learning are greater than in a world with no learning beyond (i,i).

Proof: Trivial.

Next we turn to the results which characterize optimum values for $P^*(i,j)$.

Lemma 5.1. $\lim_{\delta\to 1} P^*(i,j) = 0.$

Proof: Analogous to proof of Lemma 3.1. Q.E.D.

Lemma 5.2. $\lim_{\delta\to 1} \frac{d\ P^*(i,j)}{d\delta} = 2 \lim_{\delta\to 1} \frac{d\ P(i,j)}{d\delta}$.

Proof: From (5.2), we have

$$U^*(i,m) = \frac{b + \bar{y} - c(m) + \Gamma(P^*(i,m))}{1 - \delta}$$

Therefore,

$$W^*(i,m) = \frac{\Gamma(0) - \Gamma(P^*(i,m))}{1 - \delta} - \frac{\Gamma(0) - \Gamma(P^*(i+1,m))}{1 - \delta} \tag{5.4}$$

By Lemma 5.1 and equation (5.1), we have $\lim_{\delta\to 1} W^*(i,j) = C(i,j)$. Therefore, (5.4) can be solved recursively to get

$$\lim_{\delta\to 1} \frac{\Gamma(0) - \Gamma(P^*(i,m))}{1 - \delta} = S(i).$$

Finally, applying L'Hôpital's rule and $\Gamma'(0) = -1/2$ we get

$$\lim_{\delta\to 1} \frac{d\ P^*(i,m)}{d\delta} = -2S(i) = 2\lim_{\delta\to 1} \frac{d\ P(i,m)}{d\delta}.$$

The remainder of the proof is similar to that of Lemma 2.2.

Q.E.D.

Lemmas 3.1, 5.1 and 5.2 lead immediately to the following result.

Theorem 5.3. For sufficiently large δ, the equilibrium probability of a sale by the leader is too low from a social point of view.

6. PREDATORY PRICING

Predation refers to actions which are unprofitable but for their possible contribution to a rival's exit (Ordover and Willig, 1981). By introducing an avoidable fixed cost in our model, we demonstrate how a learning curve can create equilibrium incentives for predatory pricing.

Specifically, we amend our model by assuming that firms incur a positive fixed cost each period they remain in the market. At the beginning of each period, firms decide simultaneously whether to remain or exit. Exiting, a firm gets a payoff of zero.[10] Staying in, the firm pays a fixed cost in the amount $(1-\delta)A$.

If both firms stay in the market, then competition for that period proceeds as before, except that the possible future exit of a rival can affect equilibrium pricing, as we shall demonstrate. If its rival exits, a firm with experience i gets a monopoly payoff of $v^{\#}(i) - A$ for the subgame beginning at that date. We assume b is sufficiently large that $v^{\#}(i) \geq v(i,j) + v(j,i)$ for all j; monopoly profit is greater than the sum of duopoly profits. It is easy enough to characterize $v^{\#}(i)$ explicitly, but we won't need to do so.

In what follows, we adopt the following notational convention. For $A = 0$, $v(i,j)$ denotes the equilibrium value function for the subgame beginning

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[10] For expositional simplicity we assume firms cannot re-enter after exiting.

in state (i,j), as characterized in Section 2. For $A > 0$, $v'(i,j)$ denotes the value function for the subgame beginning in state (i,j) and before firms have decided whether to exit, and $v''(i,j)$ denotes the corresponding value function given that both firms have decided not to exit. $P(\cdot)$, $P'(\cdot)$ and $P''(\cdot)$ are distinguished similarly, as are $p(\cdot)$, $p'(\cdot)$, and $p''(\cdot)$.

As our purpose is to establish "possibility" results, we proceed by analyzing special cases and examples. We begin with the case of one-step learning, $m = 1$. In this case we know from Section 3 that

$$v(1,0) > v(1,1) > v(0,0) > v(0,1).$$

If $v(0,1) = \Pi(-P(0,1))/(1 - \delta) \geq A$, there obviously exists a SMPE with no exit, but we show there might exist also a SMPE with exit in state $(0,1)$. Obviously, if $v(0,1) < A$, exit must occur in state $(0,1)$.

<u>Theorem 6.1</u>. If $m = 1$ and $(1 - \delta)A = \Pi(-P(0,1)) - \epsilon$, then, for $\epsilon > 0$ sufficiently small, there exists a SMPE in which

(a) both firms enter initially,

(b) the firm losing the first sale exits, and

(c) $p'(0,0) < p(0,0)$.

<u>Proof</u>. Adopt the equilibrium hypothesis that exit only occurs in state $(0,1)$. Thus, $v'(0,1) = 0$ and $v'(1,1) = v(1,1) - A$. Suppose that the lagging firm were to deviate and stay in. Under the hypothesis of equilibrium play in future periods, and letting q denote the rival's price, the deviant chooses p

to maximize $\{p - c(0) + \delta v'(1,1)\}F(q-p)$. The first-order condition is $p + H(p-q) = c(0) - \delta[v(1,1) - A]$, and the corresponding payoff is $[\Pi(q-p) - (1 - \delta)A]$. On the other hand, the first-order condition for the rival is

$$q - H(p-q) = c(1) - \delta[v^{\#}(1) - v(1,1)].$$

Thus, the equilibrium price difference, $P = p - q$, satisfies

$$\begin{aligned} P + G(P) &= C(0,1) - \delta[v(1,1) - A] + \delta[v^{\#}(1) - v(1,1)] \\ &= C(0,1) - \delta W(0,1) - \delta[v(0,1) - A] + \delta[v^{\#}(1) - v(1,0)] \\ &> C(0,1) - \delta W(0,1). \end{aligned}$$

Since $v^{\#}(1) > v(1,0) + v(0,1)$ it follows that $P > P(0,1)$, and the firm's payoff is less than $[\Pi(-P(0,1)) - (1 - \delta)A]$. Thus, it is an equilibrium for the lagging firm to exit in state (0,1) if

$$\Pi(-P(0,1)) < (1 - \delta)A + \epsilon,$$

for $\epsilon > 0$ sufficiently small. Furthermore, initial entry will be profitable if $v'(0,0) > 0$. By an analogue of equation (2.2), we have

$$v'(0,0) = \Pi(0) - (1 - \delta)A + \delta v'(0,1).$$

Since $v'(0,1) = 0$ because of exit, it follows that initial entry occurs if $\Pi(0) > (1 - \delta)A$, which follows from Theorem 3.2. It remains to show that exit in state (0,1) influences pricing in state (0,0). An analogue of equation (2.1) yields

$$p'(0,0) = c(0) + H(0) - \delta[v^{\#}(1) - A].$$

Since $v^{\#}(1) > v(1,0)$ and $v(0,1)$ is approximately equal to A, $v^{\#}(1) - A > w(0,0)$, and we conclude that $p'(0,0) < p(0,0)$.

Q.E.D.

Thus, if $A = \Pi(-P(0,1)) + \epsilon$ where ϵ is a small positive number, there exist multiple equilibria, one involving predatory pricing and exit. The intuition behind the predation result is clear. The possibility of the rival's exit increases the "prize" from winning the first sale, inducing more aggressive pricing. Moreover, it is clear from the proof that parts (a) and (b) of the theorem hold if $\Pi(0) \geq (1-\delta)\ A \geq \Pi(-P(0,1))$. Thus, multiple equilibria exist in a wide range of circumstances.

In cases of multiple equilibria, the predatory one might be considered a "bootstrap" equilibrium. The leading firm sets a predatory price expecting that winning the next sale will induce exit, and this pricing behavior makes that expectation self-fulfilling. It is perhaps surprising that Markov perfection does not eliminate bootstrap equilibria of this sort; the reason is that the learning curve makes history "payoff relevant" (Maskin and Tirole, 1989). Moreover, it is tempting to think that an appropriate forward induction argument would eliminate such equilibria, but that is not clear. Staying in rather than exiting might "signal" that the lagging firm expects to play the non predatory equilibrium, but a predatory response might in turn signal that the leader expects to play the predatory one. Which signal is more convincing?

A possible policy drawback with the $m = 1$ predation result is that there is no distinction between predator and prey. Both firms are predating against each other. However, for more complicated learning curves, asymmetric market positions can emerge in which the lower cost firm predates. We show this for the case of two step learning.

Theorem 6.2. Let $m = 2$, there exist parameters supporting an equilibrium with

(a) exit occurring only in state $(0,2)$;

(b) $P'(0,1) > P(0,1)$

(c) $p'(1,0) < p(1,0)$

Proof. Define P° and A° to satisfy

$$P^{\circ} + G(P^{\circ}) = C(0,2) - \delta[v(1,2) + v(2,1) - v^{\#}(2) - A^{\circ}]$$

and

$$A^{\circ} = \Pi(-P^{\circ})/(1 - \delta).$$

If $A = A^{\circ} + \epsilon$ for $\epsilon > 0$, then $P''(0,2) = P^{\circ}$. A° is the lowest value of A consistent equilibrium exit in state $(0,2)$. Moreover, for ϵ sufficiently small and δ sufficiently close to one, an evaluation of first-order conditions establishes that this is the only state in which exit occurs. By an analogue of equation (2.3),

$$P'(0,1) + G(P'(0,1)) = C(0,1) - \delta W'(0,1)$$

$$= C(0,1) - \delta[v(1,1) - A] + \delta[v^{\#}(2) - v(1,1)]$$

$$= C(0,1) - \delta W(0,1) + \delta A + \delta[v^{\#}(2) - v(2,0) - v(0,2)]$$

$$> C(0,1) - \delta W(0,1).$$

Therefore, $P'(0,1) > P(0,1)$. By an analogue of equation (2.1), we have

$$p'(1,0) = c(0) + H(P'(0,1)) - \delta[v^{\#}(2) - v(1,1)]$$

$$= c(0) + H(P'(0,1)) - \delta w(0,1) - \delta[v^{\#}(2) - v(2,0)]$$

$$< c(0) + H(P(0,1)) - \delta w(0,1) = p(1,0).$$

Q.E.D.

These results provide a very satisfactory theory of predatory pricing, with the following features. First, an incumbent firm perceives that a lower price increases the probability that a rival will exit the market. Second, such exit is rational for the rival. Third, it was rational for the rival to have entered in the first place. Fourth, the possibility of the rival's exit leads the firm to price lower than it would were the rival committed not to exit. Fifth, such pricing increases the probability of exit.

We have shown that, for specific parameter values, there may exist two equilibria, one where predation and exit occurs, and one where it does not. A natural question to ask is then which of the equilibria is socially better. It is well known that free entry into an industry with scale economies may lead to excessive entry (e.g. Mankiw and Whinston, 1986). Therefore, it is not surprising that there may exist parameter values such that the predation equilibrium is socially better than the equilibrium with no predation. In fact, when the two equilibria co-exist, a comparison involves a trade-off between several effects. The predation equilibrium features (1) less total production because of monopoly power; (2) less product variety; and (3) lower production costs due to quicker learning. It is not hard to construct examples where this last effect dominates, making the predatory equilibrium better from society's standpoint.

We next show that prices below marginal cost are not necessarily predatory (cf. Areeda and Turner, 1975). Below cost pricing can happen even when A = 0 precludes the possibility of exit. However, once a firm reaches the bottom of its learning curve, there is no explanation for below marginal cost pricing except to induce exit. This suggests that prices below c(m) indicate a predatory intent.

<u>Theorem 6.3</u>. $A = 0$ implies $\lim_{\delta \to 1} p(i,j) = c(m) + H(0)$.

<u>Proof</u>. From equation (2.1) and by Lemma 3.1, we have

$$\lim_{\delta \to 1} p(i,j) = c(i) + H(0) - \lim_{\delta \to 1} w(i,j).$$

From the proof of Lemma 3.1 (see Appendix), we have $\lim_{\delta \to 1} w(i,j) = C(i,m)$.

Substituting and simplifying we get the desired result.

Q.E.D.

Our assumptions certainly allow $c(i) > c(m) + H(0)$, in which case $p(i,j) < c(i)$ for δ sufficiently large. Thus, competition to move down the learning curve can itself induce equilibrium prices below short run marginal cost.

<u>Theorem 6.4</u>. $A = 0$ implies $p(m,i) > c(m)$.

<u>Proof</u>. From equation (2.2), $w(m,i) = v(m,i) - v(m,i+1) = \Pi(P(i,m)) - (1-\delta)v(m,i+1)$. Therefore, from equation (2.1)

$$\begin{aligned} p(m,i) &= c(m) + H(P(i,m)) - \delta w(m,i) \\ &= c(m) + H(p(i,m)) = \delta H(P(i,m))F(P(i,m)) + \delta(1-\delta)v(m,i+1) \\ &\geq c(m) + (1-\delta)\Pi(i,m)) + \delta(1-\delta v(m,i+1) > c(m). \end{aligned}$$

Q.E.D.

It is also possible to explain predatory pricing by a mature incumbent (at the bottom of the learning curve), by introducing declining demand into the model. Specifically, suppose that beyond a certain date, say t', buyers begin to arrive more slowly. This is captured by assuming that δ is lower after t'. Now suppose that the market is in state (i,m) at some date t prior to date t', and let $v(i,m,t)$ denote the value function for the lagging firm. Suppose that $v(i,m,t+1) < v(i,m,t)$ since, other things being equal, the future looks less favorable as time goes on. The firm might stay in at date t, hoping to move down the learning curve, but exit at date $t+1$ if it fails to do so. This gives the mature firm an incentive for predatory pricing at date t.

7. CONCLUSION

We modeled dynamic price competition with learning by doing for a duopoly facing a sequence of buyers with uncertain demands. We showed that market dominance can be self-reinforcing, that learning can be socially desirable but privately disadvantageous, and that equilibrium learning can be too slow from society's standpoint. Finally, we developed a theory of predatory pricing arising from learning economies, and argued that predation might speed learning in a socially desirable way.

However, this agenda only scratches the surface. Learning economies give rise to many other interesting issues for strategic oligopoly interaction. For example, important issues surround the timing of production.[11] In our model, the timing of production was determined by the exogenous arrival of buyers. Certainly, this is very artificial. In industries such as airframe manufacturing, where it has been long known that the learning curve matters (Wright, 1936), delivery dates are determined endogenously by negotiation between buyers and sellers, and actual delivery lags are very significant. The optimal timing of production must trade off between short run economies of

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[11] Majd and Pindyck (1989) analyze the optimal production plan of a price-taking firm with a learning curve, short run decreasing returns to scale, and facing uncertain future prices. Fershtman and Spiegel (1983) compare optimal production under monopoly and competition. See also Spence (1981), Fudenberg and Tirole (1983), and Gulledge and Womer (1986) on optimal production plans for a monopolist.

scale and long run economies of learning. A very interesting issue is how strategic interaction affects this tradeoff.

There are also various policy issues surrounding learning economies. We addressed predatory pricing, but only briefly. Obviously, it would be desirable to have a much clearer delineation of when predatory pricing is likely to be harmful. Also, it would be interesting to address how the enforcement rules against predatory conduct, such as the Areeda-Turner standard, might effect dynamic oligopoly competition and the evolution of market structure. Still another area of policy application is the so-called infant industry argument for tariff protection in the presence of learning economies, briefly analyzed by Krugman (1984). Finally, oligopoly competition with learning economies might have implications for the optimal design of patent policy.

In short, there is much work left to be done.

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APPENDIX

Proof of Lemma 3.1. We begin by defining

$$S(i) = \lim_{\delta\to 1} \frac{\Pi(0) - \Pi(-P(i,m))}{1-\delta}$$

We will prove by induction that $P(i,j)\to 0$ and that $S(i) = \sum_{k=i}^{m-1} C(k,m)$. Using equation (2.2), we can write

$$w(i,m) = v(i+1,m) - v(i,m)$$

$$= \frac{\Pi(0) - \Pi(-P(i,m))}{1-\delta} - \frac{\Pi(0) - \Pi(-P(i+1,m))}{1-\delta} ,$$

and

$$w(m,i) = v(m,i) - v(m,i+1)$$

$$= \Pi(P(i,m)) + \ldots + \delta^{m-i-1}\Pi(P(m-1,m)) + \delta^{m-i}\Pi(0)/(1-\delta)$$

$$- \Pi(P(i+1,m)) - \ldots - \delta^{m-i-2}\Pi(P(m-1,m)) - \delta^{m-i-1}\Pi(0)/(1-\delta).$$

$$= \Pi(P(i,m) - (1-\delta)\Pi(P(i+1,m)) - \ldots - (1-\delta)\delta^{m-i-2}\Pi(P(m-1,m))$$

$$- \delta^{m-i-1}\Pi(0)$$

If our induction hypothesis holds true for every i' > i, then

$$\lim_{\delta\to 1} w(i,m) = \lim_{\delta\to 1} \frac{\Pi(0) - \Pi(-P(i,m))}{1-\delta} - S(i+1)$$

and

$$\lim_{\delta\to 1} w(m,i) = \lim_{\delta\to 1} \Pi(P(i,m)) - \Pi(0).$$

Substituting $w(i,m) - w(m,i)$ for $W(i,m)$ in (2.3) and taking limits as $\delta \to 1$, we get

$$\lim_{\delta\to 1} [P(i,m) + \Pi(0) - \Pi(-P(i,m))] + \lim_{\delta\to 1} \frac{\Pi(0) - \Pi(-P(i,m))}{1-\delta}$$

$$= C(i,m) + S(i+1).$$

Since the right-hand side is finite under the induction hypothesis, it must be that $\lim_{\delta\to 1} P(i,m) = 0$. Therefore, the first term in the left-hand side is zero, and we get

$$S(i) = S(i+1) + C(i,m). \tag{A.1}$$

Since $S(m) = 0$ by symmetry, we get

$$S(i) = \sum_{k=i}^{m-1} C(k,m).$$

Now suppose $\lim_{\delta\to 1} P(i',j') = 0$ is true for all (i',j'). It can be shown that

$$\lim_{\delta\to 1} w(i,j) = S(i) - S(i+1) = C(i,m).$$

Substituting $w(i,j) - w(j,i)$ for $W(i,j)$ in equation (2.3), we get

$$\lim_{\delta\to 1} [P(i,m) + G(P(i,m))] + C(i,m) - C(j,m) = C(i,j),$$

which, canceling terms, is equivalent to

$$\lim_{\delta \to 1} [P(i,m) + G(P(i,m))] = 0$$

which, since G(x) is increasing and G(0) = 0, implies

$$\lim_{\delta \to 1} P(i,j) = 0.$$

Q.E.D.

<u>Proof of Lemma 3.2</u>. Differentiating equation (2.3) with respect to δ, we get

$$\frac{dP(i,j)}{d\delta}[1 + G'(P(i,j))] + W(i,j) + \delta\frac{dW(i,j)}{d\delta} = 0. \tag{A.2}$$

Since $G'(0) = 2$, from Lemma 3.1 we get $\lim_{\delta \to 1} W(i,j) = C(i,j)$ and

$$R(i,j) = -\lim_{\delta \to 1}\frac{dP(i,j)}{d\delta} = \frac{1}{3}\left(C(i,j) + \lim_{\delta \to 1}\frac{dW(i,j)}{d\delta} \right). \tag{A.3}$$

From the proof of Lemma 3.1, we know that

$$\begin{aligned} W(i,m) = {} & \frac{\Pi(0) - \Pi(-P(i,m))}{1-\delta} - \frac{\Pi(0) - \Pi(-P(i+1,m))}{1-\delta} \\ & - \Pi(P(i,m)) + (1-\delta)\Pi(P(i+1,m)) + \ldots + (1-\delta)\delta^{m-i-2}\Pi(P(m-1)) \\ & + \delta^{m-i-1}\Pi(0) \end{aligned} \tag{A.4}$$

Define

$$T(i) = \lim_{\delta \to 1}\frac{d}{d\delta}\left(\frac{\Pi(0) - \Pi(-P(i,m))}{1-\delta} \right).$$

Taking the limits of the derivatives with respect to δ in (A.4), and simplifying, we get

$$\lim_{\delta \to 1}\frac{dW(i,m)}{d\delta} = T(i) - T(i+1) + R(i,m). \tag{A.5}$$

Applying L'Hôpital's rule to

$$\lim_{\delta\to 1} \frac{\Pi(0) - \Pi(-P(i,m))}{1-\delta} = S(i),$$

we get R(i,m) = S(i). Using this fact and combining (A.3) and (A.5) yields

$$2S(i) = C(i,m) + T(i) - T(i+1)$$

Using (A.1) this simplifies to

$$T(i) - T(i+1) = S(i) + S(i+1).$$

We now consider the general (i,j) case, with i,j<m.

$$\begin{aligned} W(i,j) \equiv\ & \Pi(-P(i+1,j)) + \delta\Pi(-P(i+1,j+1)) + \ldots + \delta^{m-j}\Pi(-P(i+1,m))/(1-\delta) - \\ & - \Pi(-P(i,j+1)) - \delta\Pi(-P(i,j+2)) - \ldots - \delta^{m-j-1}\Pi(-P(i,m))/(1-\delta) \\ & = \Pi(-P(i+1,j)) + \delta\Pi(-P(i+1,j+1)) + \ldots + \delta^{m-j-1}\Pi(-P(i+1,m-1)) - \\ & - \Pi(-P(i,j+1)) - \delta\Pi(-P(i,j+2)) - \ldots - \delta^{m-j-2}\Pi(-P(i,m-1)) \\ & - \delta^{m-j-1}\Pi(0) + \delta^{m-j-1} \frac{\Pi(0) - \Pi(-P(i,m))}{1-\delta} - \delta^{m-j} \frac{\Pi(0) - \Pi(-P(i+1,m))}{1-\delta} . \end{aligned}$$

Taking the limit of the derivative with respect to δ, and simplifying, we get

$$\begin{aligned} \lim_{\delta\to 1} \frac{dW(i,j)}{d\delta} &= R(i+1,j) + R(i+1,j+1) + \ldots + R(i+1,m-1) \\ & - R(i,j+1) - R(i,j+2) - \ldots - R(i,m-1) \\ & + (m-j-1)S(i) - (m-j)S(i+1) + T(i) - T(i+1). \\ & = R(i+1,j) + R(i+1,j+1) + \ldots + R(i+1,m-1) \end{aligned}$$

$$- R(i,j+1) - R(i,j+2) - \dots - R(i,m-1).$$

$$+ (m-j)S(i) - (m-j-1)S(i+1)$$

Together with equation (A.2) and using $S(i) = R(i,m)$, this defines a recursive system in $R(i,j)$. Since

$$R(i,m) = \sum_{k+i}^{m-1} C(k,m),$$

and the system is linear, a unique solution exists. It can be easily checked that $R(i,j) = S(i) - S(j)$ solves the system. Therefore, for $i<j$,

$$\lim_{\delta \to 1} \frac{dP(i,j)}{d\delta} = -R(i,j) = -[S(i) - S(j)] =$$

$$= - \left(\sum_{k=i}^{m-1} C(k,m) - \sum_{k=j}^{m-1} C(k,m) \right)$$

$$= - \sum_{k=i}^{j-1} C(k,m).$$

Q.E.D

FIGURE 1

Equilibrium prices for the special case:
$m = 2$;
$c(2) = 0$, $c(1) = c(0) = 1$;
x distributed $N(0,1)$.