A Work Project presented as part of the requirements for the Award of a Master's degree in Finance from the Nova School of Business and Economics.

ANALYSIS OF QUANTITATIVE INVESTMENT STRATEGIES

Trend Following – A Multiple Moving Average Approach

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17/12/2025

Abstract

This thesis evaluates a Trend Following strategy based on Multiple Moving Averages. Two novel metrics are introduced to capture trend strength based on the relative ordering of the averages and aim to adjust position sizing beyond the directional signal from a moving-average crossover. Applied to a broad set of macro futures indices, the strategy is profitable. However, most returns originate from the moving-average crossover, while the additional metrics provide limited contribution. Compared with a breakout-based Trend Following strategy, the approach underperforms.

Keywords:

Trend Following, Moving Averages, Time Series Momentum, Trading, Quantitative Investment Strategies, Hedge Funds

1. Introduction

This thesis examines the use of moving averages to capture long-term trend direction and extends the traditional moving-average crossover framework with two additional metrics designed to assess trend strength and adjust position sizing. The resulting strategy is constructed to be directly comparable to the breakout-based trend-following approach of Moskowitz, Ooi, and Pedersen (2012). The first novel metric, Ratio, captures the relative ordering of the moving averages, while the second novel metric, Dispersion, measures their spread and directional alignment. When applied to futures indices, the strategy generates positive overall performance. However, this performance is only weakly attributable to the two proposed metrics. The Ratio metric provides limited informational value, as its historical distribution is highly skewed and fails to meaningfully distinguish between market conditions. Similarly, the Dispersion metric does not exhibit a consistently positive relationship with future returns, restricting its effectiveness in adjusting position sizes. Factor regressions based on Fama–French risk factors show that statistically significant alpha emerges only at the portfolio level. This outcome appears to be driven by diversification across asset classes and by excluding the cross-sectional momentum factor. At the trade level, the moving-average crossover signals display more favorable performance characteristics than the breakout strategy, albeit with higher volatility of absolute returns. These differences arise entirely from the distinct weighting of historical returns inherent in the two strategy constructions.

2. Literature Review

Time-series momentum refers to the predictability of an asset's future returns based on its own past return history. Moskowitz, Ooi, and Pedersen (2012) document the existence of time-series momentum and establish it as a distinct asset-pricing factor in their paper "Time Series Momentum". They construct portfolios of index futures whose historical returns generate significant excess performance even after controlling for market, value, size, and cross-sectional momentum exposures. Their results show that time-series momentum persists for return lookback horizons of up to one year, while longer horizons exhibit reversal behavior. Importantly, they demonstrate that time-series momentum and cross-sectional momentum are distinct factors despite their conceptual similarity. In this thesis, the resulting breakout-based strategy serves as a benchmark against which the multiple moving-average strategy is evaluated, enabling a direct comparison of the respective signal-generation mechanisms. Valeyre (2025) addresses the risk of overfitting in trend-following systems, which can arise when multiple indicators are calibrated to historical data. To mitigate this issue, Valeyre models asset returns using stochastic processes whose analytical expressions are directly linked to the smoothing parameter of the exponential moving average (EMA). Within this framework, a theoretical Sharpe ratio is derived as a function of the smoothing parameter, allowing for the identification of an optimal EMA indicator. By contrast, this thesis employs multiple indicators to determine final asset positioning, where the two additional metrics serve as leverage-adjustment mechanisms, while the core signal is generated by a moving-average crossover of two EMAs and is therefore less tightly optimized to an individual asset's stochastic process. For additional theoretical context, Zakamulin and Giner (2018) provide a comprehensive comparison of moving-average and momentum-based strategies, highlighting their close conceptual relationship and similar forecasting properties. They show that both approaches perform similarly during strong trend regimes, while moving-average rules tend to produce more robust signals by effectively varying the historical window length used for signal construction.

3. Data

The strategy is applied to 50 index futures of all four major asset classes (Equities, Bonds, Commodities and Currencies; see Table 13), downloaded from Refinitiv. The analysis covers the period from January 1986 to July 2025. Equity index futures are extended with MSCI country equity indices back to 1986. While the equity futures data begins uniformly in 1986, the bond futures start dates range mostly from the early 1980s to the mid-1990s. Commodity futures generally begin between the early 1970s and the late 1980s, depending on the contract. Currency data is available from the early 1980s. For regression purposes, factor data is collected using the Fama-French Data Library as well as the AQR Data Library for specific time-series momentum factor data.

4. Trend Following Background & Motivation

Trend Following is an investment style that fascinates through its simplicity as well as its historical performance over decades and across asset classes. Instead of forecasting future prices, it identifies the prevailing directional movement in a price series and takes positions accordingly, implicitly relying on the existence of autocorrelation in asset returns. Autocorrelation measures whether past and future values of a time series are related or independent. For a Trend Following strategy to perform positively, assets must exhibit positive return autocorrelation. Figure 1 illustrates the autocorrelation structure for each asset class by computing the correlation between a k-month return block and the subsequent one-month return. The figure reports average autocorrelations across all assets within each of the four asset classes. The results show that Equities, Bonds, and to a lesser extent Currencies exhibit positive autocorrelation for return histories of up to 12 months, while Commodities display predominantly negative autocorrelation. Despite their unfavorable autocorrelation properties, Commodities are retained in the analysis to ensure full comparability across the four major asset classes. My strategy is motivated by the Guppy Multiple Moving Average (GMMA) strategy, originally developed as a Trend Following approach for stock trading. GMMA relies on discretionary investor judgment, as it evaluates 12 exponential moving averages (EMAs) of varying lengths, divided into two groups: six short-term averages representing short-term oriented traders, and six long-term averages representing long-term oriented investors. The strategy's trading edge arises from extracting additional information about trend strength through the consolidation and expansion of these two groups. An illustrative example is shown in Figure 5, which applies GMMA to Tesla stock in 2015.

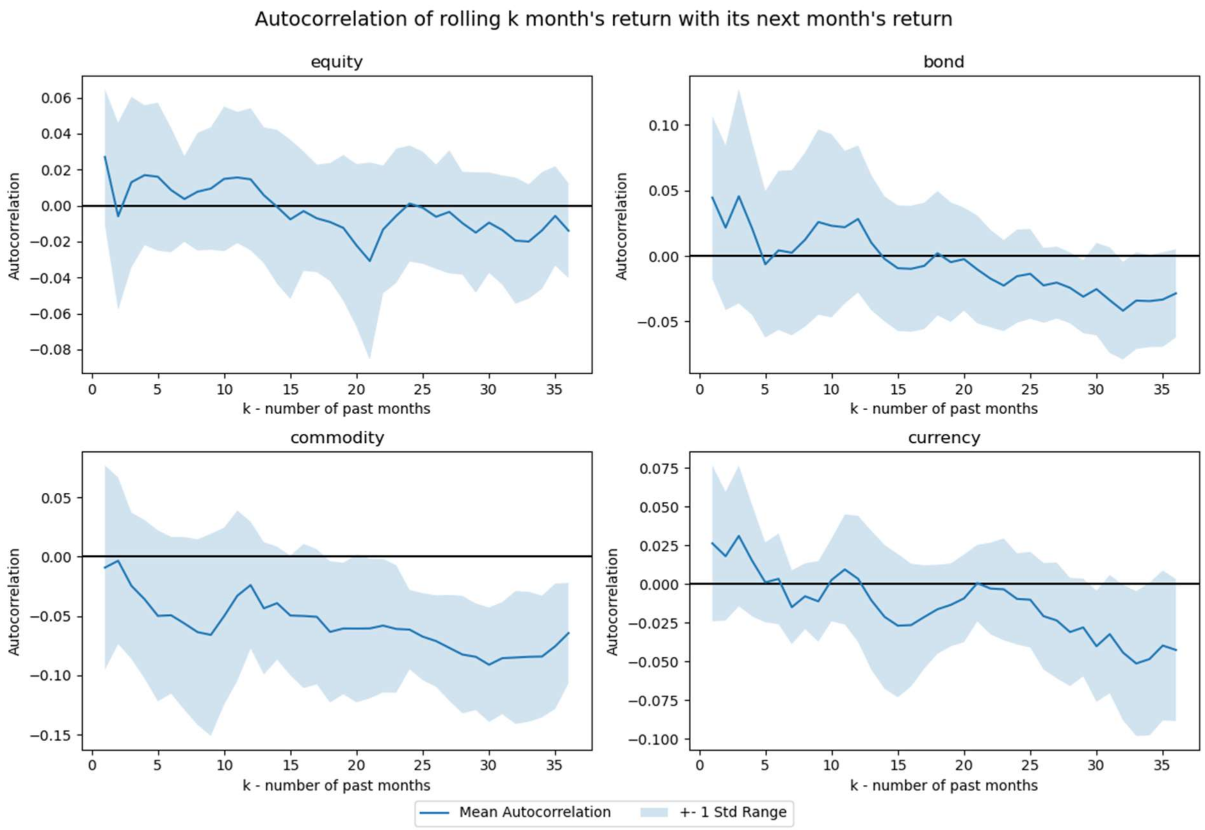

Figure 1: Autocorrelational structure exhibited within the four asset classes

As a strategy designed for equities, GMMA focuses on recent months of price history, reflecting the fact that stock prices can change rapidly and substantially. The concept of using multiple moving averages to infer trend strength forms the basis of the present strategy and is extended to index futures. Index futures are used because the benchmark strategy, the Time-Series Momentum Strategy (TSMOM) from Moskowitz/Ooi/Pedersen, is also using them. Consequently, the GMMA framework is adapted to a more long-term orientation. TSMOM is a breakout strategy that generates signals based on an asset's past one-year returns. This design allows it to capture long-term trends in well-diversified macro index futures, which, relative to individual stocks, are less prone to rapid trend reversals, making short-term price changes less informative about true trend shifts. Accordingly, I construct a strategy that preserves the relative structure of GMMA while scaling it to a longer investment horizon to ensure direct comparability with TSMOM. Specifically, the effective long-term center average of GMMA (which is equivalent to an EMA with a lag of 43) is scaled to an EMA with a lag of 251, corresponding to a one-year span parameter. The resulting strategy, referred to as MMA, follows a moving-average crossover framework augmented by position size adjustments. Table 7 in the Appendix documents the original EMA lengths, the scaling methodology, and the resulting EMA lengths.

5. Methodology

The Multiple Moving Average (MMA) strategy derives its final position from three metrics and provides a systematic representation of the discretionary GMMA framework. Metric 1, the moving-average crossover (MA-Crossover), serves as the primary trend indicator and is based on the center averages of the short-term and long-term EMA groups. A long position is taken when the short-term center average exceeds the long-term center average, while a short position is defined by the reverse condition. Metric 2, referred to as the Ratio, measures the internal ordering of EMAs within each group and captures early signals of internal crossovers. It can be interpreted as each group's assessment of the prevailing trend direction. For each group, the metric is computed by evaluating pairwise differences between adjacent EMAs and counting positive and negative differences. The Ratio is defined as the difference between positive and negative counts relative to the total number of comparisons. The final Ratio value is obtained as a center-weighted average of the short-term and long-term group measures. Metric 3, termed Dispersion, quantifies the spread of EMAs within each group. It is calculated as the daily range between the highest and lowest EMA in the group, normalized by the group's center average to ensure comparability across price levels. Directional alignment is incorporated using the 10-day change in the center average. If the two groups move in opposing directions, the metric is reduced to reflect weaker trend conviction, whereas a wide and directionally consistent spread increases position size. The final position is defined as the product of the three metrics. While the directional sign is determined exclusively by the MA-Crossover, Metric 2 scales exposure downward when internal ordering is inconsistent, and Metric 3 adjusts exposure upward or downward depending on the magnitude and directional consistency of dispersion. Consequently, the strategy allows for both leveraged and reduced exposure based on the combined strength of the metrics. Appendix AX 2–AX 5 provide detailed commentary on the formulas used to compute each metric, which in turn are listed below in equation 1–9. To construct the return series for the MMA strategy applied to an asset, the final signal is multiplied by the asset's subsequent monthly return and by a leverage factor calibrated to target an annualized volatility of 40%. This portfolio construction mirrors the approach employed by the benchmark TSMOM strategy.

$$(Moving - Average\ Crossover)\ Signal = sign(st_{center} - lt_{center}) \tag{1}$$

$$Ratio_{lt} = \frac{\sum_1^5 sign(EMA_{lt}(i) - EMA_{lt}(i+1))}{\sum_1^5 abs(sign(EMA_{lt}(i) - EMA_{lt}(i+1)))} \tag{2}$$

$$Ratio_{st} = \frac{\sum_1^5 sign(EMA_{st}(i) - EMA_{st}(i+1))}{\sum_1^5 abs(sign(EMA_{st}(i) - EMA_{st}(i+1)))} \tag{3}$$

$$Ratio_{final} = \frac{abs(N_{st-applied} * Ratio_{st} + N_{lt-applied} * Ratio_{lt})}{N_{st-applied} + N_{lt-applied}} \tag{4}$$

$$Dispersion_{st} = sign(st_{center}.diff(10)) * \frac{Price\ Span}{st_{center}} \tag{5}$$

$$Dispersion_{lt} = sign(lt_{center}.diff(10)) * \frac{Price\ Span}{lt_{center}} \tag{6}$$

$$Dispersion_{final} = \frac{abs(N_{st-applied} * Dispersion_{st} + N_{lt-applied} * Dispersion_{lt})}{N_{st-applied} + N_{lt-applied}} + 1 \tag{7}$$

$$Signal_{final} = Signal * Ratio_{final} * Dispersion_{final} \tag{8}$$

$$MMA\ return\ series = Signal_{final} * \frac{40\%}{vol_{estimated}} * next\ month\ return \tag{9}$$

6. Performance Overview

6.1 Portfolio and Asset Class Performance

The MMA strategy is evaluated against the TSMOM benchmark and a Long-Only strategy. Portfolio performance, measured as cumulative monthly returns over the sample period, shows that TSMOM outperforms MMA, which in turn outperforms the Long-Only strategy (left panel of Figure 2). This ranking is already established by the mid-1990s. The close co-movement of TSMOM and MMA indicates broadly similar positioning, while the gradual widening of the performance gap (shown in the right panel of Figure 2) suggests that MMA underperforms TSMOM at the portfolio level, likely due to frequent but short-lived differences in positioning. Disaggregating performance by asset class (Figure 3) clarifies the source of this divergence. In Equities, TSMOM and MMA perform almost identically. In Bonds, the initial performance gap is larger but stabilizes over time. Commodities exhibit a steadily increasing divergence, while Currencies show little initial separation but begin to drift apart persistently after the financial crisis.

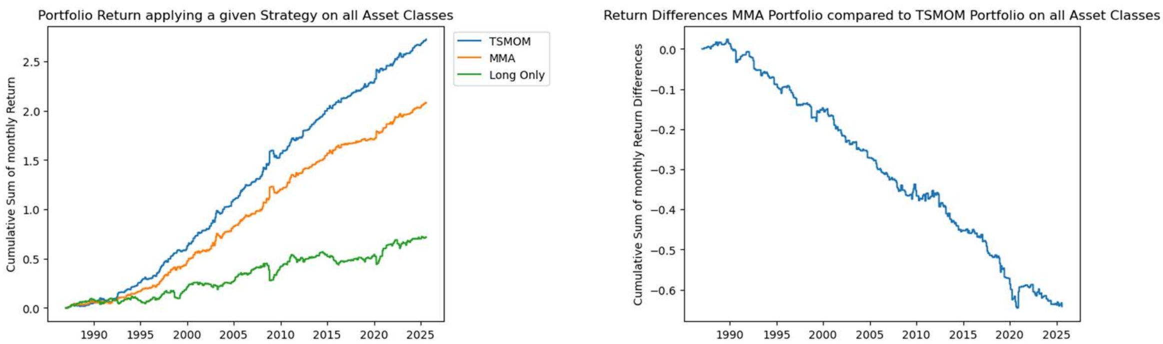

Figure 2: Comparison of Portfolio returns of the strategies TSMOM, MMA and Long-Only and visualization of the portfolio return difference between TSMOM and MMA over time

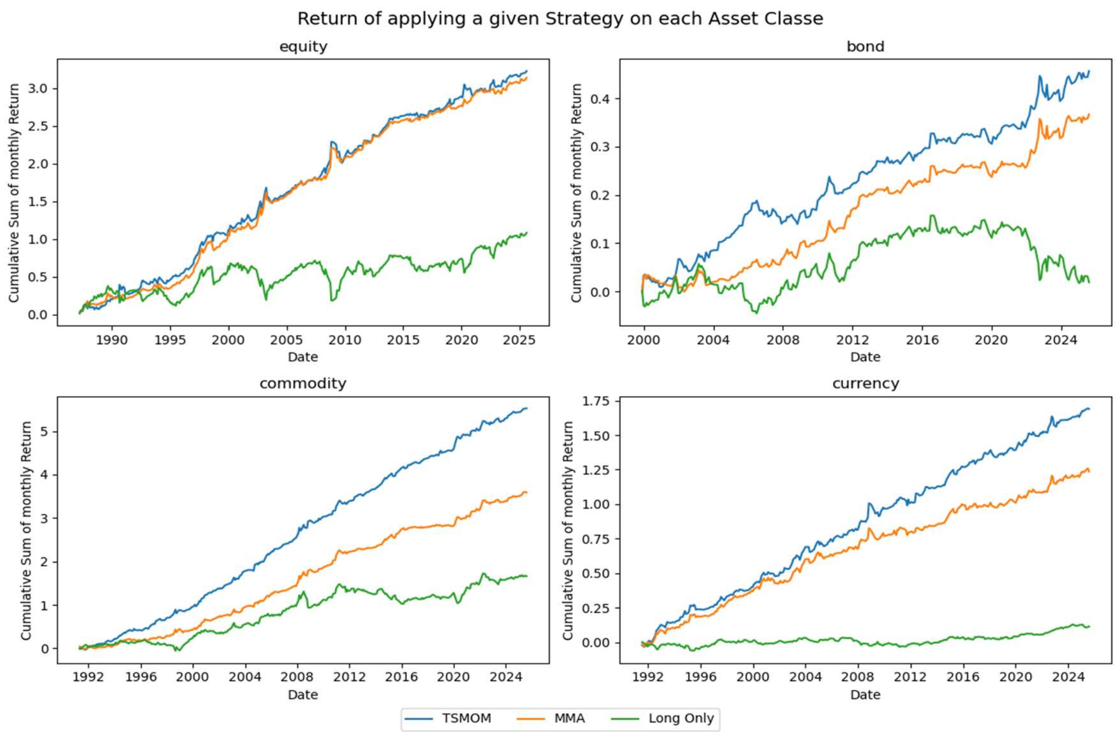

Figure 3: Visualization of asset class specific Portfolio returns per strategy

This pattern is mirrored in the performance statistics reported in Table 1. TSMOM achieves a Sharpe Ratio of 1.92, exceeding MMA's Sharpe Ratio of 1.62, primarily due to higher annualized returns (11.68% versus 9.01%, compared to 2.92% for the Long-Only strategy). All strategies display similar annualized volatility of approximately 6%, while maximum drawdowns are substantially higher for the Long-Only strategy (−16.57%) than for TSMOM (−7.51%) and MMA (−6.68%). This underscores a defining feature of Trend Following strategies, termed as crisis alpha, which represents their ability to generate positive returns during prolonged market downturns. Within the MMA strategy, Commodities emerge as the strongest-performing asset class, delivering an annualized return of 16.84% and a Sharpe Ratio of 1.71. This performance is nearly twice that of the second-best asset class, Equities, which achieve an annualized return of 8.92% and a Sharpe Ratio of 1.22. Bonds and Currencies also exhibit solid performance, with Sharpe Ratios of 0.71 and 1.10, supported by relatively low annualized volatilities of 2% and 3.8%, respectively. The strong performance of trend following in Commodities is particularly noteworthy, as their autocorrelation structure would have suggested more limited return potential.

Table 1: *Performance Metric Comparison Table*

in %	TSMOM	MMA	Long Only	MMA-Equity	MMA-Bond	MMA-Commodity	MMA-Currency
Ann. Ret	11.68	9.01	2.92	8.92	1.53	16.84	4.18
Ann. Vol	6.09	5.55	6.1	7.3	2.16	9.85	3.8
SR	1.92	1.62	0.48	1.22	0.71	1.71	1.1
CAGR	8.4	6.42	1.95	6.27	1.07	12.15	2.93
Max DD	-7.51	-6.68	-16.57	-6.72	-3.55	-11	-5.27
Best Month	12.05	11.21	5.86	8.45	3.13	13.36	5.57
Worst Month	-3.4	-3.22	-10.74	-5.06	-1.64	-6.99	-3.13

6.2 Regression Analysis

An additional perspective on MMA returns is provided by regression analysis using factor models such as the Fama–French 3-Factor and 4-Factor models, which assess whether MMA performance can be attributed to systematic exposures to established risk factors. Estimation of the 3-Factor Model (Table 2) yields a small but statistically significant monthly alpha of

0.4% for the MMA portfolio, indicating that the included factors do not fully explain MMA returns at the portfolio level. The Market factor loading is significantly negative, while the Value and Size factor loadings are negative but not statistically significant, implying a short exposure to broad market excess returns. At the asset class level, this negative market exposure is driven primarily by Fixed Income and Currencies, both of which display strongly negative and statistically significant Market-factor loadings, whereas Equity returns are significantly positively related to the Market factor. No asset class exhibits significant exposure to the Size factor, and only Bonds show a significantly negative loading on the Value factor. None of the individual asset classes generate statistically significant alpha, suggesting that the significant portfolio-level alpha arises partly from diversification across asset classes. Extending the analysis to the 4-Factor Model (Table 3) by including the cross-sectional Momentum factor (UMD) allows for a clearer assessment of MMA's factor dependencies. Under this specification, the MMA portfolio no longer exhibits significant alpha and shows no significant exposure to any factor other than UMD. The large t-value of the UMD loading indicates strong co-movement between MMA returns and the Momentum factor. At the asset class level, Equities, Bonds, and Commodities exhibit highly positive and statistically significant UMD coefficients, while Currencies show no significant UMD exposure and retain a significantly negative Market-factor loading. As in the 3-Factor Model, Equities remain significantly positively related to the Market factor, Bonds significantly negatively related, and neither the Value nor the Size factor provides significant explanatory power. Again, no asset class generates significant alpha under the 4-Factor Model.

A direct comparison between MMA and TSMOM is conducted by regressing TSMOM returns on MMA returns to assess whether MMA generates alpha relative to TSMOM. Table 4 reports results for both portfolio-level and asset-class-specific regressions. In all cases, MMA is strongly and significantly related to TSMOM, with beta t-values exceeding 19. The alpha estimates align with the cumulative return analysis, showing Equity alpha is effectively zero, consistent with the near-identical performance of the two strategies, while Bonds and Commodities exhibit significant negative alphas of −1.5% and −0.3%, respectively. Currencies do not generate significant alpha. At the portfolio level, MMA exhibits a significant negative alpha of −0.6%, which is smaller in magnitude than the sum of the individual asset-class alphas, again indicating diversification benefits across asset classes.

Table 2: *Regression of the MMA strategy using Fama-French 3-Factor-Model*

MMA ~ FF 3FM	Equity	Bond	Commodity	Currency	Portfolio
Alpha	0.005	0.007	0.001	0.005	0.004
t-value Alpha	1.297	1.724	0.496	1.331	2.211
MIW-RF	0.165	-0.354	-0.015	-0.265	-0.117
t- value MIW-RF	1.98	-4.072	-0.283	-3.352	-2.663
SMB	0.178	-0.16	0.001	-0.137	-0.03
t- value SMB	1.52	-1.311	0.015	-1.234	-0.478
HML	-0.005	-0.253	-0.042	-0.122	-0.106
t- value HML	-0.045	-2.243	-0.596	-1.187	-1.842

Table 3: *Regression of the MMA strategy using Fama-French 4-Factor-Model*

MMA ~ FF 4FM	Equity	Bond	Commodity	Currency	Portfolio
Alpha	0.002	0.006	0	0.004	0.003
t- value Alpha	0.709	1.498	0.059	1.162	1.678
MIW-RF	0.362	-0.279	0.075	-0.215	-0.014
t- value MIW-RF	4.282	-3.001	1.323	-2.534	-0.317
SMB	0.152	-0.17	-0.011	-0.144	-0.043
t- value SMB	1.372	-1.4	-0.146	-1.296	-0.738
HML	0.15	-0.194	0.029	-0.083	-0.024
t- value HML	1.431	-1.684	0.413	-0.785	-0.439
UMD	0.473	0.179	0.218	0.12	0.248
t- value UMD	6.305	2.173	4.303	1.587	6.21

Table 4: *Regression of the MMA strategy with TSMOM as Factor*

MMA ~ TSMOM	Equity	Bond	Commodity	Currency	Portfolio
Alpha	0	-0.015	-0.003	-0.002	-0.006
t- value Alpha	0.146	-5.011	-3.285	-1.108	-5.096
Beta	0.811	0.633	0.734	0.748	0.716
t- value Beta	26.758	19.405	33.416	27.813	24.895

6.3 Summary Performance Overview

Overall, the performance analysis indicates that the MMA strategy is structurally similar to TSMOM. However, the persistent pattern of small but recurring underperformance over the full sample suggests that the chosen parameterization results in slower reactions to trend reversals. Factor-model regressions show that the MMA portfolio exhibits statistically significant alpha only when the cross-sectional momentum factor is excluded, while no alpha is observed at the asset-class level. Instead, return variation is primarily explained by exposures to the Market and UMD factors. At the asset-class level, Commodities generate notably strong trend-following performance despite their unfavorable autocorrelation properties, whereas Currencies remain comparatively weak. Diversification across asset classes enhances portfolio-level outcomes, resulting in more favorable performance than suggested by the asset-class results in isolation.

7. MMA Analysis

This section aims to look behind the technical construction of the MMA strategy by analyzing each component of the final MMA signal individually.

7.1 Return Signature Plot

The first component of the MMA strategy is the moving-average crossover between the short-term and long-term center averages, which determines the general long or short position. As this component defines the directional stance, it is the primary driver of MMA returns and the direct counterpart to the breakout rule used in TSMOM. Accordingly, understanding how both strategies generate their signals is essential for identifying the conditions under which they diverge. Since Trend Following strategies rely exclusively on historical prices or returns, their key distinction lies in how past information is weighted. The Return Signature Plot, introduced by Levine and Pedersen (2016) in their paper “Which Trend Is Your Friend?”, visualizes this weighting by displaying the contribution of daily historical returns to each strategy’s signal. Figure 6 presents the return weights for both the MA-Crossover and the breakout strategy. TSMOM applies a one-year lookback window, assigning equal weight to the past 251 daily returns. In contrast, the MA-Crossover effectively incorporates the entire price history, although the weight assigned to observations beyond approximately 400 days is close to zero. The MA-Crossover therefore places substantially more emphasis on recent weeks than TSMOM, implying greater sensitivity to short-term price dynamics. An additional insight is provided by the 50% point, which partitions historical returns into two segments that each contribute half of the signal. A signal change occurs when these segments differ in sign and one dominates in absolute magnitude. For TSMOM, the 50% point lies at a lag of 125, comparing the most recent six months of returns with the preceding six months. For the MA-Crossover, the 50% point occurs at a lag of 49, indicating that the most recent two to three months of returns are weighed against the entire earlier return history. Overall, the Return Signature Plot demonstrates that any divergence in signals between MMA and TSMOM arises exclusively from differences in how the two strategies weight past returns.

7.2 Trade Performance Metrics and Absolute Returns

As the two strategies frequently generate different signals, it is informative to analyze their realized trades. Accordingly, all trades for each asset class are collected for both strategies and their performance metrics are compared in Table 5.

Table 5: *Performance Metrics of realized trades per strategy*

	Equity		Bond		Commodity		Currency	
	TSMOM	MA Crossover	TSMOM	MA Crossover	TSMOM	MA Crossover	TSMOM	MA Crossover
Win Rate in %	36.12	39.38	31.73	34.14	39.46	28.2	38.23	35.18
Avg Win in %	18.35	25.37	3.87	4.99	17.51	21.78	6.87	10.21
Avg Loss in %	-6.48	-9.07	-2.27	-2	-9.17	-11.69	-3.88	-3.8
Payoff Ratio	2.88	2.81	1.63	2.62	2	1.98	1.79	2.88
Expectancy in %	2.67	4.57	-0.27	0.32	1.26	-1.94	0.28	1.26
Mean Return in %	2.78	4.57	-0.27	0.33	1.27	-1.94	0.28	1.26
Volatility in %	20.03	25.43	4.54	4.47	22.16	22.33	8.08	9.29
Sharpe Ratio	0.1	0.16	-0.1	0.08	0.04	-0.16	0.01	0.11
Sample Size	515	320	561	536	1490	1149	497	361
Avg Turnover per Asset	57	36	44	41	79	61	56	40

In Equities, a comparison of realized absolute returns shows that the MA-Crossover achieves higher average gains on winning trades but also larger losses on losing trades. This results in an identical Payoff Ratio for both strategies, while producing a substantially higher

Expectancy and mean return per trade for the MA-Crossover. The wider dispersion between gains and losses, however, leads to higher volatility of absolute returns. A key advantage of the MA-Crossover is its significantly lower turnover per asset, implying reduced transaction costs. Together with the nearly identical equity portfolio return paths of both strategies, this suggests that the MA-Crossover is the more favorable approach within the Equity asset class. In Bonds, the MA-Crossover again outperforms TSMOM, achieving a higher Payoff Ratio driven by both larger average gains and smaller average losses. This results in a slightly positive Expectancy, compared to the slightly negative Expectancy of TSMOM, while volatility and turnover per asset remain similar across strategies. In Commodities, TSMOM clearly outperforms the MA-Crossover due to its higher Win Rate and, consequently, higher Expectancy. Although both strategies exhibit a comparable Payoff Ratio of approximately two, implying that winning trades yield roughly twice the losses of losing trades, the low Win Rate of the MA-Crossover is insufficient to offset its losses, resulting in a negative Expectancy. Volatility is similar for both strategies, while turnover per asset is lower for the MA-Crossover. Notably, despite Commodities being the strongest asset class in terms of annualized MMA returns, the MA-Crossover component alone generates negative mean returns per trade. Since annualized MMA returns reflect the full MMA signal, whereas mean trade returns capture only the MA-Crossover component, this discrepancy suggests that Metric 2 and Metric 3 contribute materially to performance beyond the MA-Crossover. In Currencies, the MA-Crossover is preferable on a per-trade basis, exhibiting a higher Payoff Ratio, substantially higher Expectancy, and lower turnover. The higher Payoff Ratio is driven by larger average gains per winning trade, accompanied by slightly higher volatility. Overall, the trade-level metrics indicate that the MA-Crossover generally produces more favorable individual trade characteristics than TSMOM. Nevertheless, as shown previously, TSMOM outperforms MMA across the full sample in all asset classes. This outcome is likely driven by

TSMOM's larger number of trades, implying that despite less attractive individual trades, higher trade frequency and cumulative compounding lead to superior aggregate performance.

7.3 Portfolio Return Contribution from Metric 2 and Metric 3 combined

To assess the impact of Metric 2 and Metric 3 on overall MMA performance, Figure 4 reports the difference between MMA portfolio returns and MA-Crossover portfolio returns. This isolates the contribution of the position-size adjustments introduced by the final two metrics.

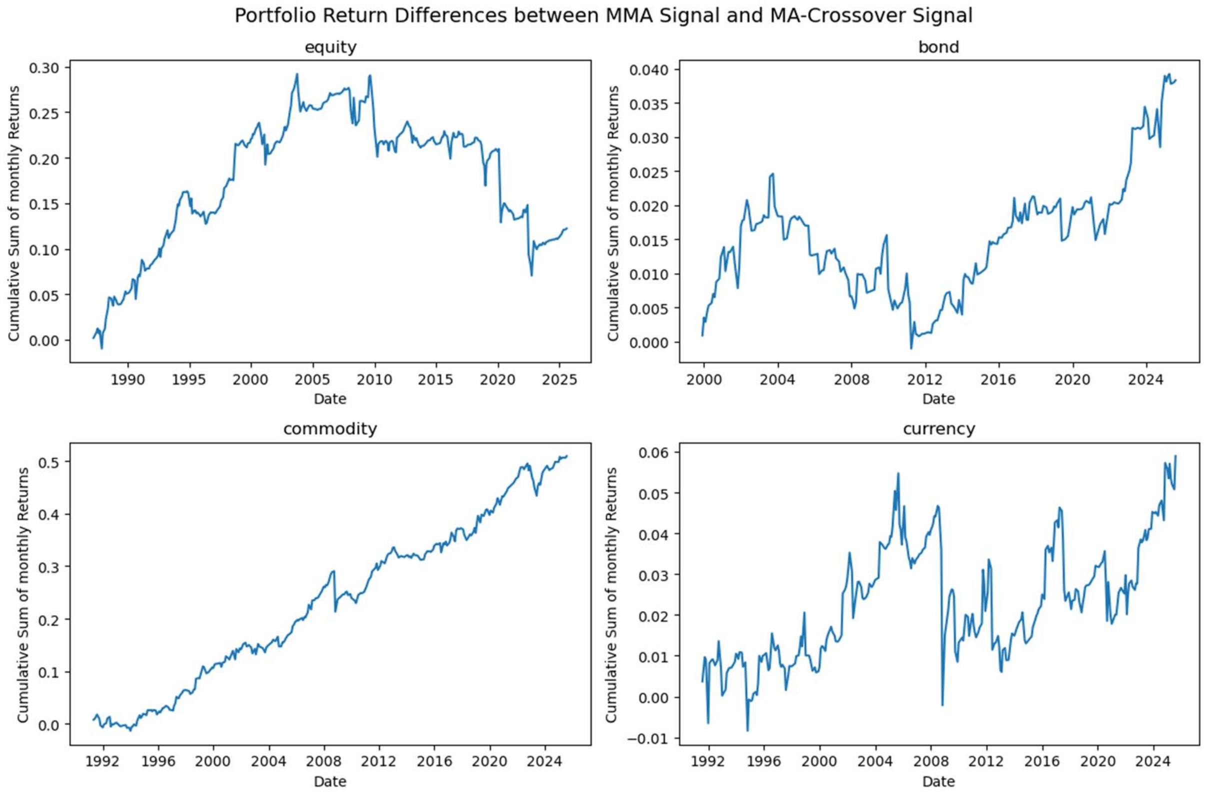

Figure 4: Portfolio Return Differences between MMA and MA-Crossover per asset class

A key initial observation is that, over the long run, the combined effect of Metric 2 and Metric 3 does not detract from portfolio performance. However, their contribution differs markedly across asset classes. For Bonds and Currencies, the impact is minimal, adding only 4 and 6 percentage points, respectively, to cumulative returns over the full sample period. In contrast, the contribution in Commodities is substantial, amounting to approximately 50 percentage points. Given that MMA achieves roughly a 300% cumulative return in Commodities, around one-sixth of this performance can be attributed to Metric 2 and Metric 3. This finding helps reconcile the discrepancy between the strong annualized MMA returns and the negative average per-trade returns of the MA-Crossover component alone. In Equities, the two metrics contribute positively from the beginning of the sample until the financial crisis, adding approximately 30 percentage points to cumulative returns. These gains are partially reversed thereafter, primarily during isolated periods. Overall, this analysis indicates that the combined effect of Metric 2 and Metric 3 is highly beneficial in Commodities, largely negligible in Bonds and Currencies, and mixed in Equities.

7.4 Analysis Metric 2 – "Ratio"

As defined previously, the Ratio metric is given by:

$$Ratio_{lt} = \frac{\sum_1^5 sign(EMA_{lt}(i) - EMA_{lt}(i+1))}{\sum_1^5 abs(sign(EMA_{lt}(i) - EMA_{lt}(i+1)))} \quad (2)$$

$$Ratio_{st} = \frac{\sum_1^5 sign(EMA_{st}(i) - EMA_{st}(i+1))}{\sum_1^5 abs(sign(EMA_{st}(i) - EMA_{st}(i+1)))} \quad (3)$$

$$Ratio_{final} = \frac{abs(N_{st-applied} * Ratio_{st} + N_{lt-applied} * Ratio_{lt})}{N_{st-applied} + N_{lt-applied}} \quad (4)$$

Further insight into Metric 2 is obtained by examining the distribution of its possible configurations. As each group can take six distinct Ratio values, there are 36 possible combinations. Figure 7 shows, for each asset class, how the sample period is distributed across these configurations, illustrating how frequently each final Ratio value is used for position-size adjustments. Across all asset classes, the most common configuration occurs when both the short-term and long-term groups exhibit a perfectly ordered upward structure, characteristic of sustained uptrends. This situation accounts for approximately 20% of the sample for Commodities and Currencies, 27% for Bonds, and 38% for Equities, indicating that Equities spend nearly one-third of the sample period in this upward-ordered state. The second most frequent configuration is the opposite extreme, where both groups are perfectly ordered downward, typical of extended drawdowns. This occurs in roughly 20% of the sample for Commodities and Currencies, 15% for Bonds, and 10% for Equities. Overall, Commodities and Currencies spend similar amounts of time in both extremes, while Equities are clearly biased toward upward movements. An additional insight from the heatmaps is that intermediate configurations occur only rarely. This suggests that once the internal ordering of a group begins to change, it tends to transition rapidly toward one of the two extremes rather than remaining in intermediate states. This observation raises the question of whether all 36 configurations are necessary, or whether a simplified classification based solely on the four most frequent cases would capture essentially the same information. To investigate this, only the four extreme situations are considered, together with their average next-month returns, as shown in Figure 8. Since the final Ratio is computed as a center-weighted average that assigns greater weight to the long-term group, performance improves when the sign of next-month returns aligns with the sign of the long-term Ratio, particularly when both group-specific Ratios coincide. For Equities, Metric 2 is beneficial in three of the four extreme cases. Average next-month returns are positive when both Ratios are positive (long positioning) and negative when both are negative (short positioning). The highest average next-month return, approximately 1%, occurs when the long-term group remains downward-oriented while the short-term group has already turned upward. In this case, Metric 2 reduces the position to a modest short exposure, resulting in a small loss. For Bonds, all four extreme situations yield positive average next-month returns, including cases where both groups are downward-oriented. Since return signs do not consistently align with the Ratio classifications, Metric 2 appears unable to capture return direction meaningfully and is therefore of questionable value for Bonds. A similar pattern emerges for Commodities. While Metric 2 correctly identifies positive returns when both groups are upward-oriented, its best-performing situation, with an average next-month return of 1.1%, occurs when both groups are downward-oriented, leading to losses for the MMA strategy. Combined with earlier evidence that Metric 2 and Metric 3 jointly improve Commodity performance, this suggests that the positive contribution stems primarily from Metric 3 rather than Metric 2. For Currencies, Metric 2 appears largely neutral. It contributes marginally in clear long and short cases, but the highest average next-month return occurs when the final Ratio remains slightly downward-oriented, implying small losses attributable to Metric 2. In summary, the highly uneven distribution of observations across the 36 possible configurations casts doubt on the necessity of Metric 2. Moreover, focusing on the most relevant extreme cases reveals a neutral to slightly negative contribution across all asset classes. Taken together, these findings suggest that Metric 2 is a weak component of the MMA strategy and could potentially be excluded due to its limited contribution to performance.

7.5 Analysis Metric 3 – Dispersion

As defined previously, the Dispersion metric is given by:

$$Disperison_{st} = sign(st_{center}.diff(10)) * \frac{Price\ Span}{st_{center}} \quad (5)$$

$$Disperison_{lt} = sign(lt_{center}.diff(10)) * \frac{Price\ Span}{lt_{center}} \quad (6)$$

$$Dispersion_{final} = \frac{abs(N_{st-applied} * Dispersion_{st} + N_{lt-applied} * Dispersion_{lt})}{N_{st-applied} + N_{lt-applied}} + 1 \quad (7)$$

To assess the impact of Metric 3, its predictive relationship with next-month asset returns is examined. As Dispersion is strictly positive and is intended to identify both strong uptrends and strong downtrends, the analysis is conducted on split samples. Specifically, the data are partitioned according to the long or short signal generated by Metric 1. This setup tests whether high Dispersion values are associated with higher next-month returns in long positions and lower next-month returns in short positions, thereby evaluating whether the leverage applied by Metric 3 is activated at favorable times. As reported in Table 6 for a subsample of assets as well as in Table 9-12, most assets across Equities, Bonds, and Currencies, as well as many Commodities, show no statistically significant relationship between Dispersion and next-month returns. Moreover, in cases where a significant relationship exists (Table 6), the estimated coefficient often has the opposite sign from what would be desired. For example, Natural Gas exhibits a significantly positive alpha in long positions, indicating positive expected next-month returns following a long signal. However, its Dispersion coefficient is significantly negative, implying that higher Dispersion values are associated with lower subsequent returns. While this does not imply negative returns per se, it indicates that leverage tends to be applied during periods of below-average future returns. A similar pattern is observed in short positions. Although the alpha correctly reflects negative next-month returns, the Dispersion coefficient is significantly positive, suggesting higher subsequent returns when Dispersion is high. In general, leverage remains beneficial whenever next-month returns align with the MA-Crossover signal. However, the intended effect of increasing exposure during periods of strong trend intensity can also be detrimental, as such periods are sometimes followed by sharp trend reversals. Given earlier evidence that Metric 2 and Metric 3 jointly contribute positively to Commodity performance, while Metric 2 alone performs poorly, the results indicate that, in this asset class, favorable returns are levered more frequently than adverse ones. In summary, the predictive-power analysis indicates that when Dispersion exhibits significant explanatory power, the leverage effect of Metric 3 is inherently risky and can amplify adverse outcomes. Nonetheless, the empirical evidence shows that in Commodities, leverage is applied predominantly during periods of favorable subsequent returns, making Metric 3 beneficial to MMA performance in this asset class.

Table 6: *Regression of the next month return using Dispersion as regressor (subsample)*

	Long Signals		Short Signals	
t-value	**const**	**Dispersion**	**const**	**Dispersion**
IBEX 35 (Spain)	-0.668	0.730	-2.844	2.850
Canada 10Y Bond	2.441	-2.434	-0.658	0.660
UK Long Gilt	2.507	-2.509	0.394	-0.389
Copper (COMEX)	-1.095	1.137	-2.811	2.840
WTI Crude Oil	1.033	-1.005	-3.813	3.878
Brent Crude Oil	0.864	-0.825	-2.072	2.134
Heating Oil	0.837	-0.799	-2.195	2.249
Gas Oil	0.803	-0.763	-2.431	2.475
Natural Gas	2.891	-2.865	-1.873	1.966

8. Limitations

The analysis focuses on signal construction within an MA-Crossover framework augmented by two position-sizing metrics and does not account for market impact, liquidity constraints, or transaction costs, all of which may affect real-world feasibility and profitability. The MA-

Crossover parameters are fixed to ensure comparability with TSMOM and alignment with long-term trends in futures indices. However, calibrating the short- and long-term EMAs to asset-specific autocorrelation structures could better capture heterogeneous trend dynamics and improve signal accuracy. Moreover, portfolio-level performance differences between MMA and TSMOM are also influenced by volatility targeting. As a time-varying leverage mechanism, it scales exposure down during volatile periods and up during calmer ones, thereby affecting the weighting of subsequent returns. When the two strategies differ in their signals, this mechanism may amplify adverse returns for MMA. While signal generation remains the primary performance driver, volatility targeting materially influences the magnitude of return differentials.

9. Conclusion

The MMA strategy proposed in this thesis is profitable when applied to a diversified set of macro futures indices. Within a factor-regression framework, MMA exhibits statistically significant alpha only at the portfolio level and only when the cross-sectional momentum factor is excluded. Performance differences between MMA and TSMOM arise mainly from their distinct signal-generation mechanisms, which differ solely in how historical returns are weighted. At the trade level, the MA-Crossover component achieves higher expectancy and payoff ratios but also entails greater risk, reflected in higher volatility of absolute returns. The two additional metrics, Ratio and Dispersion, contribute positively to performance only within the Commodity asset class, as indicated by their cumulative return contribution. The Ratio metric appears weak, exhibiting a neutral to slightly negative standalone effect. For Dispersion, regression results show that, when evaluated separately in long and short regimes, higher EMA-group dispersion is often associated with below-average next-month returns. While Dispersion contributes positively to Commodity performance within the MMA framework, the results also highlight the risk that leverage may amplify adverse outcomes.

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11. Appendix

Figure 5: Example of the GMMA strategy

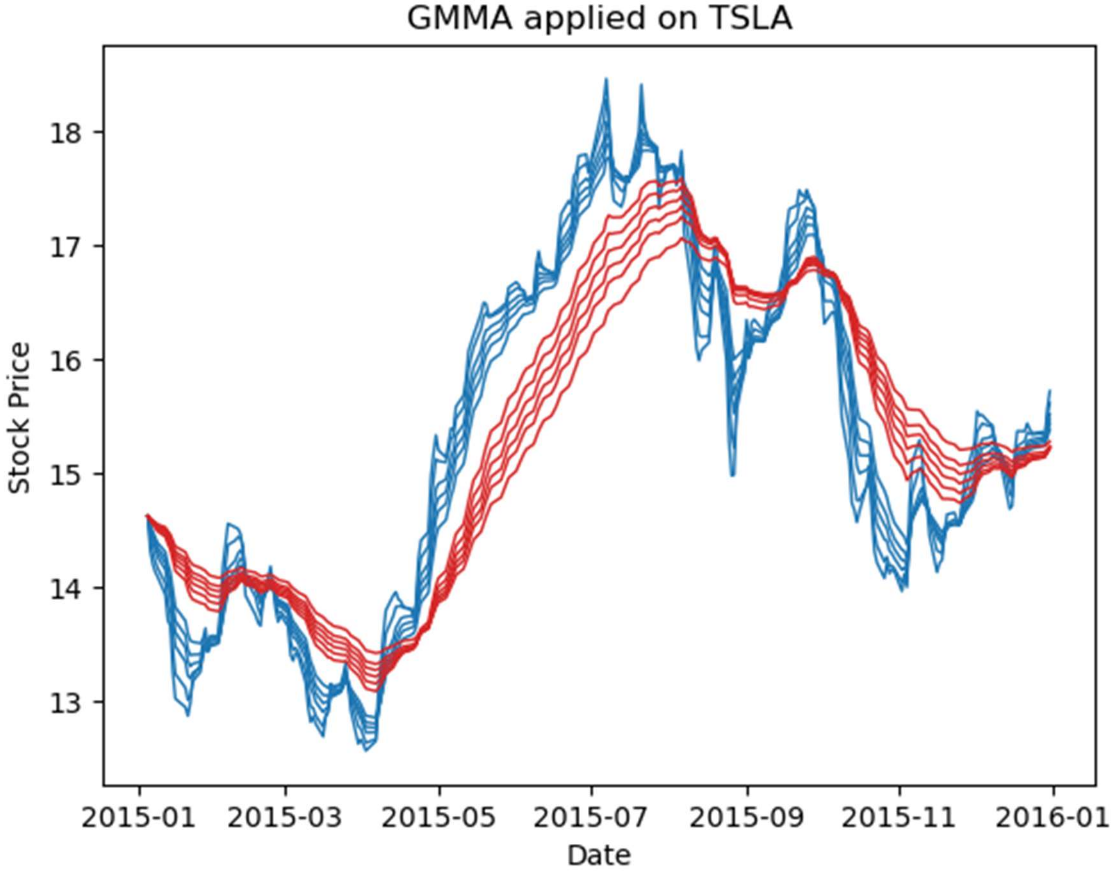

Figure 5: Visualized example of applying the GMMA strategy's Moving-Averages on the Tesla stock during the year 2015

AX 1: Comments about converting GMMA into MMA:

The following table presents how the 12 original MA lengths with their center average are converted into the actually applied MA lengths of my MMA strategy.

The span parameter (N) can also be expressed as smoothing constant (sc) or center of mass (com). The short-term center average as well as the long-term center average is the value-weighted average of the "original N" values. The "Ratio to Center" is calculated by dividing the "original N" by the respective center-average N. The "applied N" of the long-term center average is set exogenously at 251 to match the lookback period used in the TSMOM strategy (251 days).

By considering the ratio between the short-term center average and the long-term center average of the "original N", the resulting "applied N" of the short-term center average is 52. With these two applied N values of the center averages, and with the Ratio-to-Center values, the conversion of all applied N is done. After this, it is also possible to calculate the respective applied sc and applied com.

Table 7: *MA-lengths conversion from GMMA to MMA*

Original MA	original N	original sc	original com	Ratio to Center	applied N	applied sc	applied com
3	3	0.500	1.00	0.34	17	0.109	8.18
5	5	0.333	2.00	0.56	29	0.067	13.96
7	7	0.250	3.00	0.78	40	0.048	19.75
10	10	0.182	4.50	1.12	58	0.034	28.43
12	12	0.154	5.50	1.34	69	0.028	34.21
15	15	0.125	7.00	1.68	87	0.023	42.89
ST Center Avg	9	0.201	3.97		52	0.038	25.38
30	30	0.065	14.50	0.69	174	0.011	86.28
35	35	0.056	17.00	0.81	202	0.010	100.74
40	40	0.049	19.50	0.92	231	0.009	115.21
45	45	0.043	22.00	1.04	260	0.008	129.67
50	50	0.039	24.50	1.15	289	0.007	144.14
60	60	0.033	29.50	1.38	347	0.006	173.06
LT Center Avg	43	0.045	21.19		251	0.008	125.00

AX 2: Comments about Metric 1 – Moving-Average-Crossover:

- Has a value of either +1 or -1
- Formula for calculating the MA-Crossover signal:

$$N_{st-applied} = applied\ Span\ of\ the\ short - term\ center\ Average \tag{10}$$

$$N_{lt-applied} = applied\ Span\ of\ the\ long - term\ center\ Average \tag{11}$$

$$st_{center} = EMA(N_{st-applied}) \tag{12}$$

$$lt_{center} = EMA(N_{lt-applied}) \tag{13}$$

AX 3: Comments about Metric 2 – "Ratio":

- The Ratio-value for the short-term group and for the long-term group can be one of the following, respectively: +1, +0.6, +0.2, -0.2, -0.6, -1
- The final Ratio value is non-negative between 0 and +1

AX 4: Comments about Metric 3 – "Dispersion":

- The "Price Span" is the difference between the daily highest and lowest value of a group's EMAs.
- The Price Span is set relative to the day's center-average to compare the result across time and different price levels.
- The sign of the center-average difference is supposed to reduce the final Dispersion value in case the two groups move in opposite directions.
- The center-average difference uses for its calculation a difference-period of 10 days to smooth the sign factor

AX 5: Comments about Final Signal:

- For each month, a final Signal and Volatility estimate is calculated and applied on the asset's next month return to derive the MMA return series
- The annualized Volatility estimate is based on a GARCH model with a center of mass of 60 days.

Figure 6: Return Signature Plot

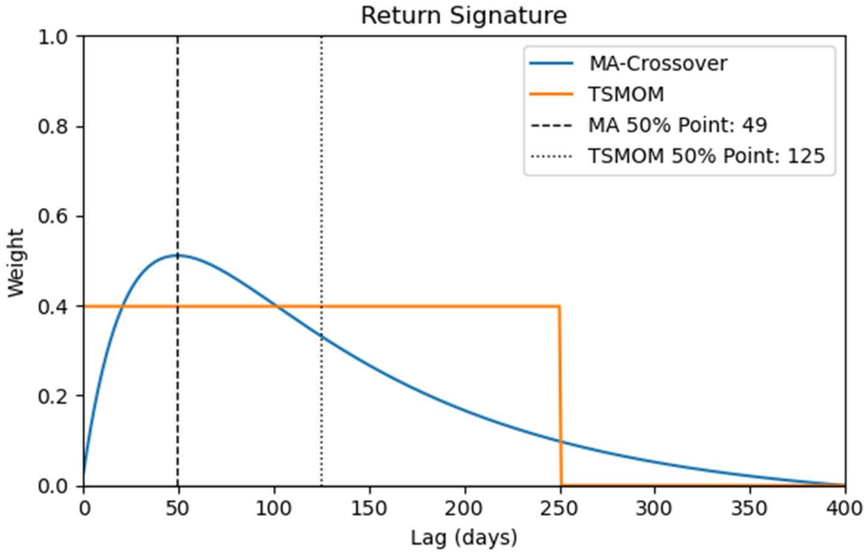

Figure 6: Comparison of the weighting of past returns in TSMOM and MA-Crossover

AX 6: Explanation of the Performance metrics used for evaluating the realized trades

Table 8: *Explanation of Performance Metrics used to evaluate the realized Trades*

Win Rate in %	Number of positively returning Trades to total number of Trades
Avg Win in %	Average absolut return of positively returning Trades
Avg Loss in %	Average absolut return of negatively returning Trades
Payoff Ratio	Ratio of Average Win to Average Loss in absolut values
Expectancy in %	Weighted average of Avg Win and Avg Loss using Win Rate as weight for Avg Win and using 1-Win Rate as weight for Avg Loss
Mean Return in %	Average absolut return of all Trades
Volatility in %	Volatility of the absolut returns of all Trades
Sharpe Ratio	Mean Return of all Trades / Volatility of all Trades
Sample Size	Total number of Trades
Avg Turnover per Asset	Average number of Trades per Asset

Figure 7: Percentage Distribution of the 36 Ratio Situations per asset class

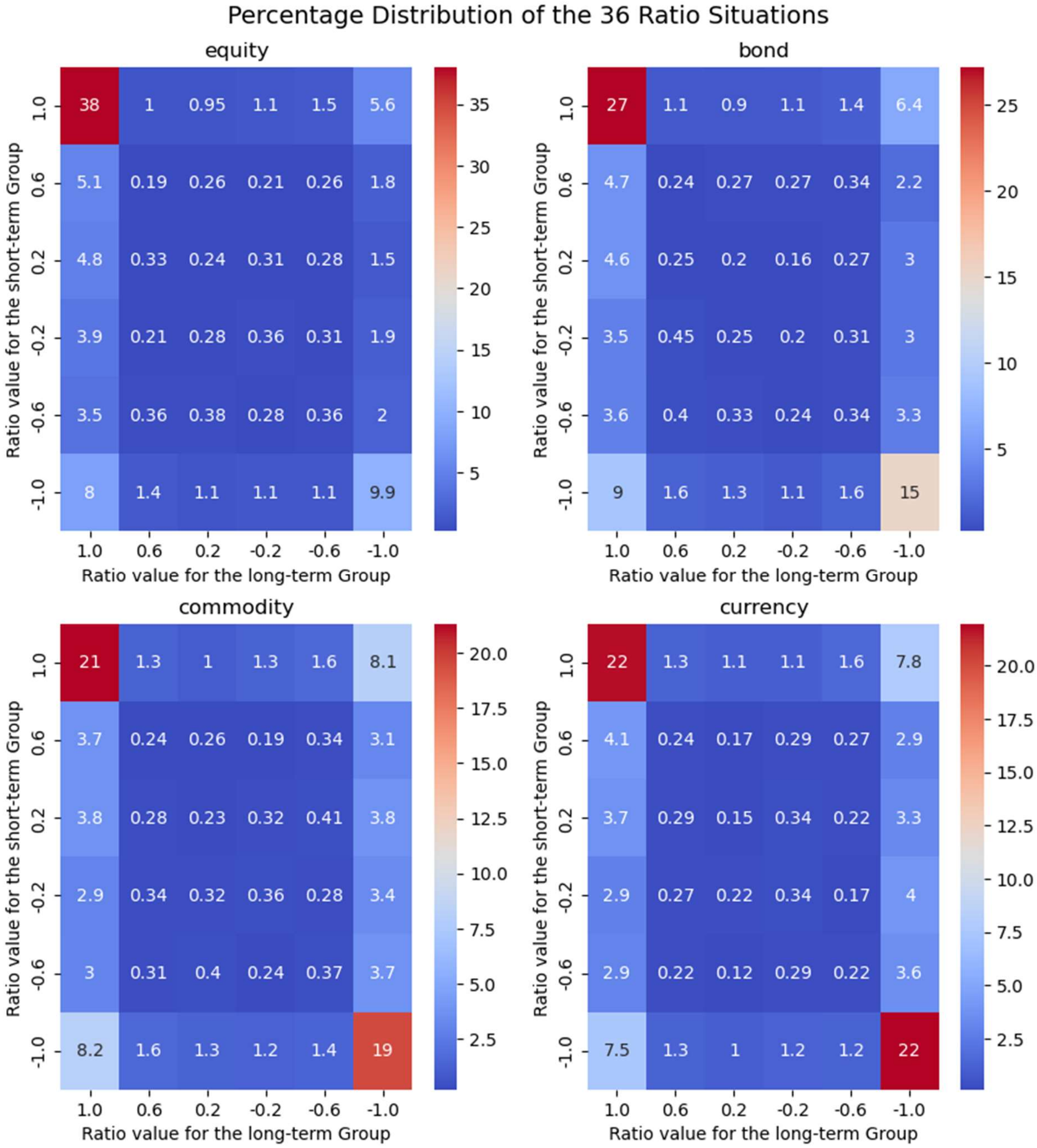

Figure 7: Percentage Distribution of the 36 possible situations for "Ratio" per asset class

Figure 8: Average next month return in the 4 extreme Ratio situations per asset class

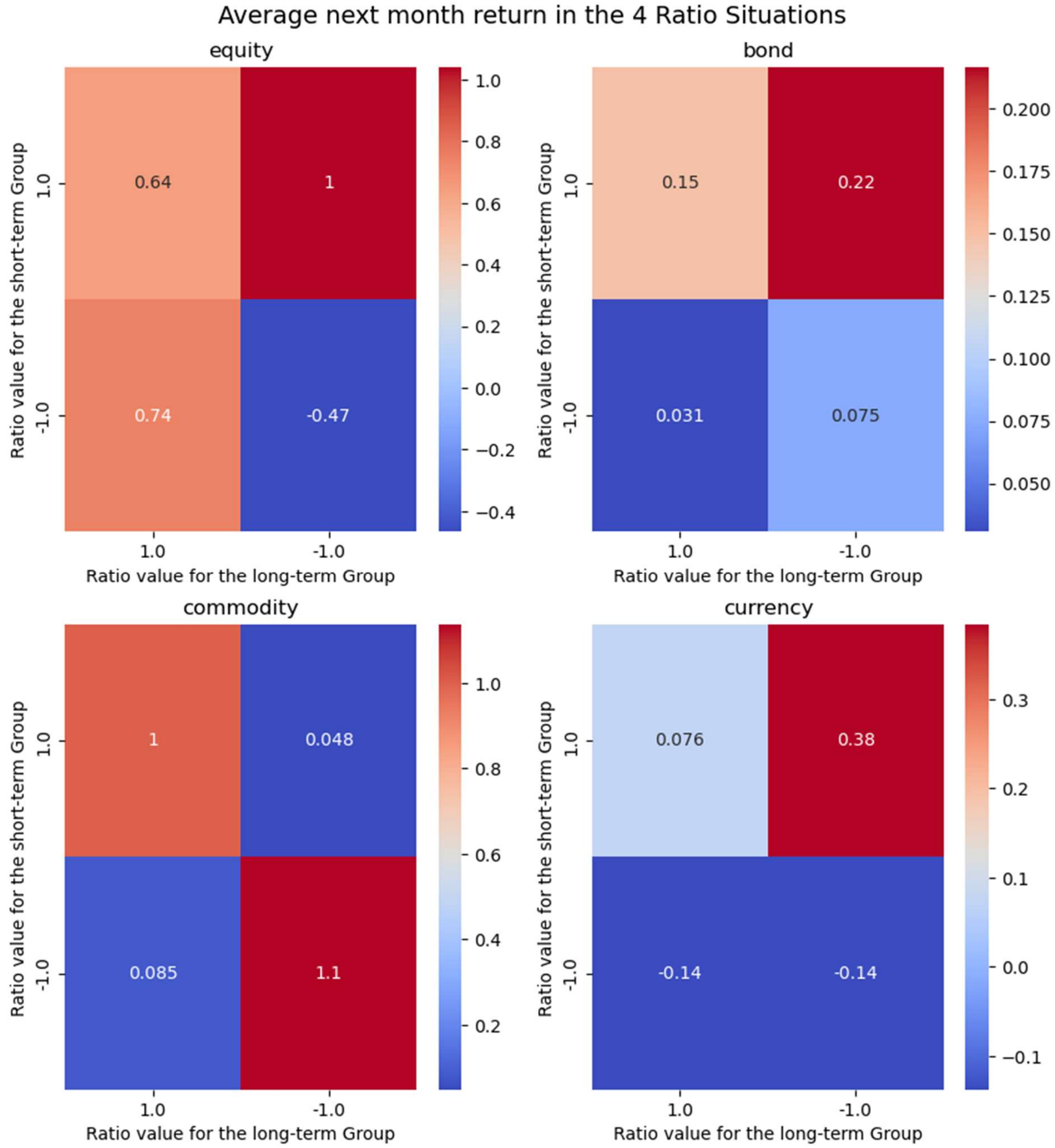

Figure 8: Average next month return separated in the 4 extreme situations for "Ratio" per asset class

AX 7: Regression results – Dispersion on next-month-return

Table 9: *Regression of the next month return using Equity Dispersion as regressor*

	Long Signals		Short Signals	
t-value	**const**	**Dispersion**	**const**	**Dispersion**
S&P 500	1.295	-1.233	-0.154	0.185
SPI 200 (Australia)	1.501	-1.480	0.085	-0.025
CAC 40 (France)	0.435	-0.391	-0.589	0.611
DAX (Germany)	0.457	-0.394	-0.821	0.868
FTSE/MIB (Italy)	1.037	-0.989	-2.014	2.013
TOPIX (Japan)	0.466	-0.428	-1.611	1.606
AEX (Netherlands)	0.002	0.058	-0.260	0.287
IBEX 35 (Spain)	-0.668	0.730	-2.844	2.850
FTSE 100 (UK)	0.961	-0.931	-0.152	0.197

Table 10: *Regression of the next month return using Bond Dispersion as regressor*

	Long Signals		Short Signals	
t-value	**const**	**Dispersion**	**const**	**Dispersion**
Australia 3Y Bond	1.831	-1.830	-0.739	0.739
Australia 10Y Bond	1.417	-1.416	-0.446	0.446
Euro Schatz (2Y)	0.428	-0.426	1.006	-1.006
Euro Bobl (5Y)	1.280	-1.277	1.051	-1.047
Euro Bund (10Y)	1.830	-1.821	0.939	-0.938
Euro Buxl (30Y)	1.448	-1.429	0.324	-0.328
Canada 10Y Bond	2.441	-2.434	-0.658	0.660
Japan 10Y Bond	1.061	-1.055	-1.570	1.573
UK Long Gilt	2.507	-2.509	0.394	-0.389
US 2Y Note	-0.009	0.014	-0.658	0.652
US 5Y Note	1.245	-1.242	-0.120	0.119
US 10Y Note	0.674	-0.669	-0.736	0.742
US Long Bond (30Y)	2.679	-2.668	-0.974	0.973

Table 11: *Regression of the next month return using Commodity Dispersion as regressor*

	Long Signals		Short Signals	
t-value	**const**	**Dispersion**	**const**	**Dispersion**
Corn	0.878	-0.870	-1.408	1.443
Wheat	1.063	-1.071	-1.208	1.240
Soybeans	1.183	-1.182	-0.346	0.377
Soy Meal	2.320	-2.325	-1.326	1.377
Soy Oil	0.840	-0.810	-1.084	1.111
Cotton	0.180	-0.183	-3.186	3.222
Coffee	0.909	-0.871	-1.140	1.181
Cocoa	-0.534	0.600	-2.355	2.390
Sugar	0.434	-0.394	-0.760	0.819
Live Cattle	-0.768	0.762	-2.851	2.886
Lean Hogs	1.621	-1.656	-3.759	3.848

Gold	-0.686	0.776	-1.960	1.963
Silver	0.043	0.023	-0.254	0.299
Copper (COMEX)	-1.095	1.137	-2.811	2.840
WTI Crude Oil	1.033	-1.005	-3.813	3.878
Brent Crude Oil	0.864	-0.825	-2.072	2.134
Heating Oil	0.837	-0.799	-2.195	2.249
Gas Oil	0.803	-0.763	-2.431	2.475
Natural Gas	2.891	-2.865	-1.873	1.966

Table 12: *Regression of the next month return using Currency Dispersion as regressor*

	Long Signals		**Short Signals**	
t-value	**const**	**Dispersion**	**const**	**Dispersion**
AUD	0.006	0.004	-2.484	2.477
CAD	1.055	-1.044	0.119	-0.125
EUR	0.758	-0.751	-1.779	1.776
JPY	0.806	-0.789	-0.184	0.154
NZD	-0.953	0.972	-2.145	2.136
NOK	0.676	-0.663	-1.189	1.192
SEK	0.582	-0.569	-0.781	0.786
CHF	0.136	-0.145	0.000	-0.007
GBP	0.964	-0.966	-0.926	0.930

AX 8: Assets and their Tickers

Table 13: *Assets and Tickers per asset class for Refinitiv*

Equity Index Futures		**Bond Futures**		**Commodity Futures**		**Currency Spot and Forward**	
Asset	**Ticker**	**Asset**	**Ticker**	**Asset**	**Ticker**	**Asset**	**Ticker**
S&P 500	ESc1	Australia 3Y Bond	YTTc1	Corn	Cc1	AUD Spot	AUD=
SPI 200 (Australia)	YAPc1	Australia 10Y Bond	YTCc1	Wheat	Wc1	AUD 1M FWD	AUD1M=
CAC 40 (France)	FCEc1	Euro Schatz (2Y)	FGBSc1	Soybeans	Sc1	CAD Spot	CAD=
DAX (Germany)	FDXc1	Euro Bobl (5Y)	FGBMc1	Soy Meal	SMc1	CAD 1M FWD	CAD1M=
FTSE/MIB (Italy)	IFSc1	Euro Bund (10Y)	FGBLc1	Soy Oil	BOc1	EUR Spot	EUR=
TOPIX (Japan)	JTIc1	Euro Buxl (30Y)	FGBXc1	Cotton	CTc1	EUR 1M FWD	EUR1M=
AEX (Netherlands)	AEXc1	Canada 10Y Bond	CGBc1	Coffee	KCc1	JPY Spot	JPY=
IBEX 35 (Spain)	MFXIc1	Japan 10Y Bond	JGBc1	Cocoa	CCc1	JPY 1M FWD	JPY1M=
FTSE 100 (UK)	FFIc1	UK Long Gilt	FLGc1	Sugar	SBc1	NZD Spot	NZD=
		US 2Y Note	TUc1	Live Cattle	LCc1	NZD 1M FWD	NZD1M=
		US 5Y Note	FVc1	Lean Hogs	LHc1	NOK Spot	NOK=
		US 10Y Note	TYc1	Gold	GCc1	NOK 1M FWD	NOK1M=
		US Long Bond (30Y)	USc1	Silver	SIc1	SEK Spot	SEK=
				Copper (COMEX)	HGc1	SEK 1M FWD	SEK1M=
				WTI Crude Oil	CLc1	CHF Spot	CHF=
				Brent Crude Oil	LCOc1	CHF 1M FWD	CHF1M=
				Heating Oil	HOc1	GBP Spot	GBP=
				Gas Oil	LGOc1	GBP 1M FWD	GBP1M=
				Natural Gas	NGc1		

A Work Project presented as part of the requirements for the Award of a Master's degree in Finance from the Nova School of Business and Economics.

ANALYSIS OF QUANTITATIVE INVESTMENT STRATEGIES

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Work Project carried out under the supervision of:

Professor Nicholas Hirschey

17/12/2025

1. Introduction

Modern portfolio theory began with Markowitz's (1952) paper, which argued that investors should evaluate portfolios not only by expected returns but also by the risks of the assets. The main finding of this paper is that it is possible to diversify across imperfectly correlated assets to reduce risk and achieve an optimal risk-return trade-off. Over the following decades, financial markets became more competitive, causing many conventional sources of excess returns to diminish as investors exploited and arbitraged them. More recently, technology has been advancing rapidly, and more sophisticated techniques have been developed to keep up with increased competition in financial markets. This gave rise to quantitative investment strategies that rely on rule-based frameworks, statistical and mathematical models, and algorithms to identify consistent investment opportunities while managing risk. Some recent methods, such as machine learning and AI, aim to identify patterns in data that may generate consistent abnormal returns beyond those of traditional methods.

In this context, our paper examines whether combining three independent investment strategies into a single strategy can build a more robust portfolio. The three strategies are as follows: Trend Following (MMA); Attention and Sentiment in Industries in the U.S. equity market (Cycle); and Median of Ranks (MoR) in the Norwegian equity market. These strategies differ across regions, asset classes (via MMA), and methods, suggesting that there is potential for diversification benefits when combined. We explore combining these strategies using equal weighting and mean-variance optimization to determine whether they enhance diversification, reduce risk, and increase returns relative to the individual strategies and selected benchmarks, the 60/40 portfolio, and the SPY.

2. Individual Strategies

2.1. Trend Following (MMA)

Trend following is an investment approach that seeks to exploit the tendency of asset prices to move in persistent trends. Rather than predicting future price levels, the strategy identifies directional movement based purely on historical price data, allowing the investor to systematically follow the trend. This style of investing has demonstrated robustness across decades and asset classes, delivering attractive returns as well as exhibiting the ability to generate crisis-alpha during extended market downturns.

The Trend Following strategy (MMA) developed in this thesis builds upon a moving-average crossover framework and enhances it by adjusting position size through two uniquely defined metrics (the Ratio metric and the Dispersion metric) designed to measure trend strength. Applied to a broad set of macro futures indices, the MMA strategy proves to be profitable over the full sample period from 1985 to 2025, achieving an annualized return of 9% with an annualized volatility of 5.55%, resulting in a Sharpe Ratio of 1.62. The analysis shows that most of the performance is driven by the moving-average crossover itself, while the additional trend-strength metrics contribute meaningfully only in the case of commodities, with limited effects in other asset classes.

2.2. Attention and Sentiment in industries (Cycle)

This strategy combines two variables examined in prior research but not usually paired together: attention and consumer sentiment. Attention is measured using the Abnormal Search Volume Index (ASVI) from Google Trends, which captures higher-frequency information on stocks and industry-specific factors. Consumer sentiment, sourced from the University of Michigan, is a slower-moving indicator designed to reflect economic cycles rather than predict returns. This analysis is conducted monthly at the industry level, using the Fama-French 12 industry classification, derived from stocks listed on the NYSE, NASDAQ, and AMEX. The combination of these variables can reveal when investors are focused on certain industries or industry-specific factors, and whether the overall economic environment can provide support or weaken those industries. The main idea is that different industries may respond differently under various economic conditions. This strategy tests whether high-ASVI industries benefit more from improved environments, and whether lower-ASVI industries perform better when these conditions reverse.

During the in-sample period (March 2004 to December 2015), the Cycle strategy achieved an annualized return of 11.23%, a volatility of 16.88%, resulting in a Sharpe ratio of 0.59. Out-of-sample performance improved, with a return of 13.21%, slightly lower volatility at 16.71%, and a Sharpe ratio of 0.68. Overall, the strategy performs moderately, suggesting that attention and sentiment can help explain some of the variation in industry behavior. Still, no evidence was found that either variable helps predict monthly industry returns.

2.3. Median of ranks (MoR)

The Median of Ranks (MoR) strategy is a cross-sectional multi-factor approach applied to Norwegian equities. Each month, all eligible stocks are ranked within their industry on four signals: profitability, momentum (6-1), asset growth, and low beta. For each stock, the median of these four industry-neutral ranks is computed and used as its overall score. At each quarterly rebalance, the 20 stocks with the lowest median rank are selected and held in an equal-weighted portfolio, with a baseline weight of 5 per cent per stock and a 10 per cent cap. The strategy is implemented with strict T+1 execution and the same transaction-cost assumptions as the other portfolios, so that all decisions are based on information that would have been available at the time.

Over the full sample period from July 2008 to August 2025, the MoR portfolio achieves an annualized return of 12.3% with annualized volatility of 29.5%, corresponding to a Sharpe ratio of 0.55 after costs. Average quarterly one-way turnover is about 11.9%, which is noticeably lower than for the more aggressive robust z-score composite. Compared with the OSEAX benchmark, which delivers 9.5% annualized return, 17.0% volatility and a Sharpe ratio of 0.62, the MoR strategy offers a higher absolute return at the cost of higher risk, while remaining relatively stable and trading-efficient given its multi-factor tilt.

3. Correlation Analysis

The correlation structure in *Table 1* shows that the in-sample correlations are comparable to the out-of-sample correlations among the strategies. The Cycle and MoR strategies are positively correlated at around 0.65. Both strategies exhibit a negative correlation with MMA, approximately -0.20 for Cycle and -0.15 for MoR. These relationships provide a solid foundation for diversification when combining strategies. In particular, the negative correlation implies that MMA tends to move in the opposite direction relative to Cycle and MoR, enhancing the potential for risk reduction through portfolio construction.

Table 1: *Correlation In-Sample and Out-Of-Sample among the three strategies*

In-Sample	**MMA**	**Cycle**	**MoR**
MMA	1.00	-0.23	-0.17
Cycle	-0.23	1.00	0.66
MoR	-0.17	0.66	1.00

Out-Of-Sample	**MMA**	**Cycle**	**MoR**
MMA	1.00	-0.16	-0.13
Cycle	-0.16	1.00	0.65
MoR	-0.13	0.65	1.00

The rolling correlations of the pairwise strategy combinations (*Appendix* F*igure A1*) show that Cycle and MoR reach a stable correlation already by around 2010, at a level of approximately 0.75, indicating a relatively high co-movement over the full sample period. For MMA, its correlation with both Cycle and MoR stabilizes only after 2012. While MMA remains negatively correlated with both other strategies, its initially strongly negative correlation rises toward approximately -0.25, thereby reducing part of its potential diversification benefit. The rolling correlations suggest that frequent rebalancing of the portfolio to maintain diversification alone is not necessary, as correlations remain largely stable, at least after 2012. However, beyond diversification effects, the individual performance dynamics of each strategy must also be considered when determining rebalancing frequency to achieve an optimal risk-adjusted portfolio return.

4. Combined Strategy

4.1 Portfolio Construction

a. Mean-Variance Frontier and Performance - In-sample and Out-of-sample

Following Markowitz's (1952) mean-variance framework, the first approach to construct a meaningful portfolio applies portfolio theory by using the correlation structure between the strategies to determine optimal weights that improve the risk-return profile. This approach exploits diversification effects to locate a portfolio on the mean-variance frontier that achieves the lowest annualized volatility for a given annualized return. The portfolio with the highest Sharpe ratio is obtained by additionally considering the risk-free rate, by drawing a tangency line from the risk-free rate to the mean-variance frontier. The tangency portfolio delivers the highest Sharpe ratio, given the historical covariance and return structure of the strategies.

Appendix Figures A2 and A3 present the mean-variance plots for the in-sample and out-of-sample periods.

Table 2: *Performance Metrics In-Sample and Out-Of-Sample per Strategy*

IS	MM	Cycle	MoR
Ann. Ret	-0.48%	14.78%	10.28%
Ann. Vol	11.62%	18.21%	20.54%
SR	-0.06	0.80	0.49
CAGR	-1.21%	14.85%	8.98%
Max DD	-31.41%	-40.99%	-36.58%
Best Month	9.57%	29.94%	17.18%
Worst Month	-7.6%	-16.31%	-16.15%

OOS	MMA	Cycle	MoR
Ann. Ret	3.63%	11.67%	27.87%
Ann. Vol	12.33%	18.04%	42.74%
SR	0.11	0.52	0.60
CAGR	3.42%	12.42%	23.38%
Max DD	-23.89%	-20.18%	-58.41%
Best Month	10.29%	15.07%	35.07%
Worst Month	-8.09%	-11.62%	-50.25%

In the in-sample setting, the tangency portfolio realizes approximately 12.5% annualized return with 15% annualized volatility. *Table 2* shows the exact performance metrics for the in-sample period and out-of-sample period, respectively. Cycle delivers the highest annualized return of about 15% with a Sharpe ratio of 0.80, whereas MMA achieves the lowest, slightly negative return with a Sharpe ratio of -0.06. MoR achieves a Sharpe ratio of 0.49 with approximately 10% annualized return and 20% annualized volatility. The optimal weighting for the in-sample period is 20% MMA, 80% Cycle, and 0% MoR. This allocation reflects the strong weighting toward Cycle due to its superior Sharpe ratio, and a modest allocation to MMA due to its negative correlation with the other strategies. MoR is excluded, as it does not provide meaningful benefits in terms of return enhancement or risk reduction.

In the out-of-sample period, the situation changes. MoR achieves the highest annualized return of approximately 28%, accompanied by a high annualized volatility of 43%, resulting in a Sharpe ratio of 0.60. Cycle achieves a Sharpe Ratio of 0.52, with an annualized return of 11.6%. In comparison, MMA performs modestly with a 3.6% annualized return and 12% annualized volatility, resulting in a Sharpe ratio of only 0.11. The tangency portfolio in this period is composed of 40% MMA, 35% Cycle, and 25% MoR. Compared to the in-sample allocation, this results in a more balanced weighting between Cycle and MoR, as they now perform equally in terms of Sharpe ratio and drive the tangency portfolio's return. MMA again plays primarily a diversification role and is included with a relatively high allocation of 40%. The tangency portfolio yields about 12% annualized return with 17% annualized volatility.

b. Mean-Variance Portfolio

For the mean-variance optimized portfolio (MVarP), the weights of the tangency portfolio are recalculated every month, ensuring that the portfolio is rebalanced monthly to reflect the most recent covariance structure among the strategies.

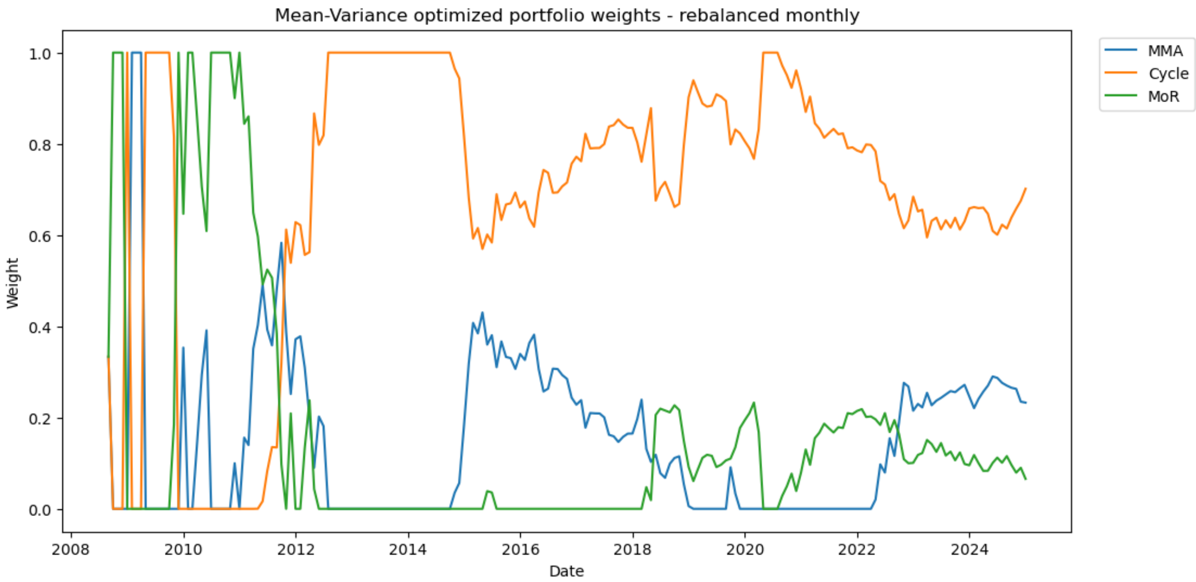
Figure 1 illustrates the evolution of strategy weights over time. In the early phase, the weights fluctuate substantially due to the limited historical data available for estimating stable correlations between the strategies. As more monthly observations accumulate, the correlation structure becomes more reliable, and weight fluctuations diminish.

A particularly notable observation is the extended period in 2013-2014 during which Cycle becomes almost the exclusive component of the portfolio. During this time, neither MMA nor MoR provides additional performance or diversification benefits. *Appendix Figure A4,* which displays the cumulative returns of the strategies, shows that this period corresponds to a phase where Cycle delivers low volatility while outperforming MMA and MoR. MoR is excluded due to its higher volatility, while MMA is excluded due to its negative performance in that period.

The subsequent rise in MMA's allocation after 2015 is mainly driven by MMA's strong performance in 2015, while Cycle shows comparatively neutral performance.

The years after 2015 reflect the mechanism of mean-variance optimization. Cycle is used as the primary return contributor due to its attractive risk-adjusted performance, while MMA or MoR are selectively added either for diversification or for incremental return enhancement. However, neither MoR nor MMA is consistently strong enough to become the dominant portfolio component.

Figure 1: *Mean-Variance Optimized Weights per Strategy from 2008 to 2024*



c. Comparison between the Mean-Variance and Equal-Weighted Portfolios

The second approach for constructing a portfolio is a simple equal-weighted allocation (Equal-Weighted Portfolio, EWP), assigning one-third of the portfolio to each strategy. The EWP and MVarP are compared in terms of their performance metrics and cumulative return over time to evaluate whether the more frequent rebalancing of the MVarP leads to an outperformance relative to the EWP. *Table 3* presents the performance metrics for both portfolio constructions and shows similar results with respect to annualized return, volatility, and Sharpe ratio.

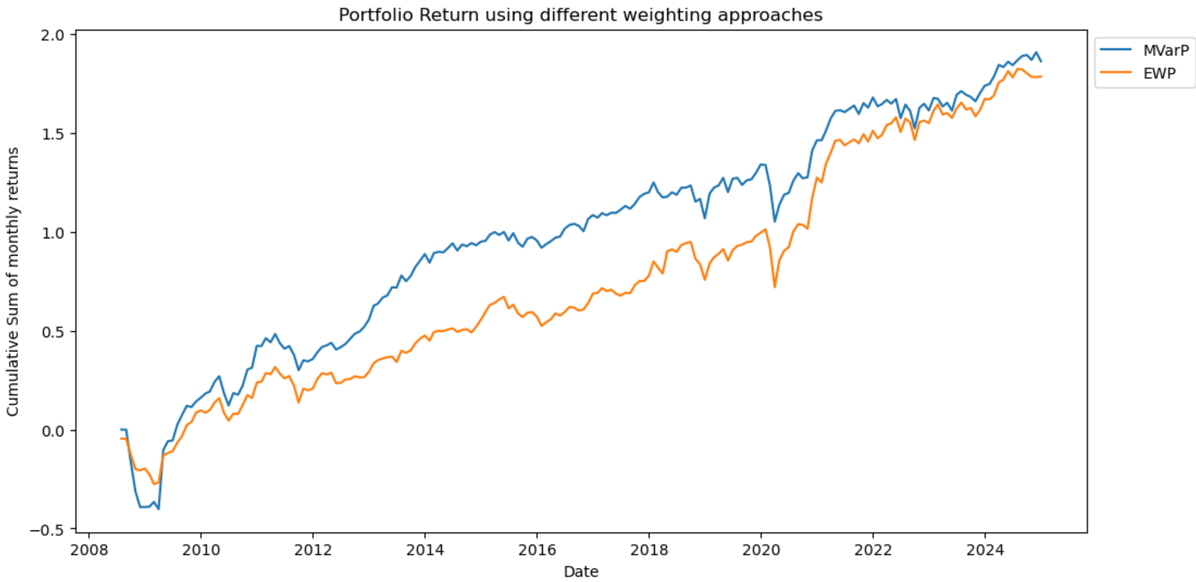
Notably, over the full sample period, the MVarP achieves a lower Sharpe ratio than the EWP, despite explicitly optimizing for the highest Sharpe ratio. With respect to maximum drawdown, the EWP exhibits less risk, and the timing of this maximum drawdown can be observed in *Figure 2,* occurring shortly after the financial crisis. Both portfolios recover by 2010, after which the MVarP clearly outperforms the EWP until around 2021, when the EWP begins to converge toward the performance of the MVarP. Over the full sample, the MVarP accumulates slightly higher total returns, but loses much of its mid-2010s outperformance.

Overall, the MVarP is recommended for implementation, as it dynamically incorporates the evolving correlation structure and performance characteristics of the strategies to determine a weighting that aims at maximizing risk-adjusted returns, which proved effective throughout the 2010s, even though it delivered lower absolute returns than the EWP in the 2020s.

Table 3: *Performance Metrics MVarP and EWP*

	MVarP	**EWP**
Ann. Ret	11.29%	10.82%
Ann. Vol	17.07%	14.92%
SR	0.6	0.65
CAGR	10.65%	10.49%
Max DD	-35.24%	-27.43%
Best Month	29.94%	15.57%
Worst Month	-18.12%	-19.93%

Figure 2: *Cumulative returns of each portfolio for the full sample period*



4.2 Comparison with Benchmarks

Two benchmarks were selected for comparison with the combined strategy allocations. The first benchmark selected is the traditional 60/40 portfolio, where 60% is allocated to equities and 40% to bonds. This allocation is widely used because equities and bonds typically exhibit low or negative correlation. This portfolio is beneficial as it enhances diversification, reduces overall volatility, and provides protection during economic downturns. We constructed this portfolio by allocating 60% to the MSCI World Equity Index (MSCIW) and 40% to the FTSE World Government Bond Index (WGBI). We use global indexes because our individual strategies focus on the U.S. and Norwegian equity markets and futures, so this is intended to reflect a global market index for comparison with the combined strategy.

The second benchmark selected is the SPDR S&P 500 ETF (SPY) because the combined strategy has more equity exposure, with all strategies including equities, and two of them being entirely equity-based, making this benchmark useful for comparison. Although the strategies span different regions, the Cycle strategy is entirely in the U.S. equity market, whereas the MoR is from the Norwegian equity market and is strongly positively correlated with the Cycle strategy. Also, the Cycle strategy has notably higher weights across the sample under the mean-variance weighting scheme. This implies that the U.S. equities play a significant role in the dynamics of the combined strategy. The SPY is a simpler, widely used benchmark that allows us to compare whether a more complex multi-strategy with a high equity weighting can provide meaningful value relative to investing in a simpler U.S. equity fund.

Table 4: *Performance metrics for the combined strategy and benchmark*

	EWP		**MVarP**		**60/40**		**SPY**	
	IS	OOS	IS	OOS	IS	OOS	IS	OOS
Ann. Exc. Ret	8.00%	12.14%	12.44%	7.23%	4.16%	2.96%	10.8%	12.13%
Ann. Vol	11.59%	18.63%	16.68%	17.73%	10.91%	11.75%	14.89%	17.14%
SR	0.69	0.65	0.75	0.41	0.38	0.25	0.73	0.71
Kurt	1.95	2.49	12.71	1.86	2.05	0.13	2.19	0.07
Max DD	-21.21%	-27.43%	-35.24%	-26.93%	-30.2%	-24.25%	-41.79%	-23.92%

The table above summarizes the performance metrics for both weighting schemes of the combined strategy and the selected benchmarks (*Table 4*). The results show notable differences between the in-sample (IS) and out-of-sample (OOS) periods, showcasing how the combined strategy performs under different market conditions.

In the in-sample period, both EWP and MVarP outperform the 60/40 portfolio on a risk-adjusted basis. Both have higher volatility than the 60/40 portfolio, but this is compensated for by higher excess returns, resulting in higher Sharpe ratios. However, the MVar has a higher maximum drawdown than the 60/40 portfolio, indicating that this strategy emphasizes riskier components of the individual strategies, which makes the combined strategy more vulnerable to market reversals. Compared with the SPY, the EWP and MVarP exhibit different behaviors. The EWP underperforms the SPY on a risk-adjusted basis and earns lower excess returns, as expected, because this method applies the same weights to the individual strategies, two of which are purely equity, with one in the U.S., making the division less effective than U.S. equity's performance in this period. Despite this, the EWP has lower volatility than the SPY, making it a more defensive, stable allocation at the cost of lower returns. On the other hand, the MVarP portfolio exhibits stronger risk-adjusted performance and higher excess returns than the SPY. The volatility of the MVar is higher than that of the SPY. Still, its maximum drawdown is lower, indicating that diversification through MMA's strategy offers some protection against market downturns, when equities tend to perform worse, as in 2009 (*Appendix Figure A5*). Additionally, the MVarP has the highest kurtosis (12.71) among all strategies in the table, indicating greater tail risk and a higher likelihood of extreme return outcomes than the others.

In the out-of-sample period, the 60/40 portfolio continues to be surpassed by the EWP and MVarP, in terms of Sharpe ratios and excess returns. This shows that the combination of individual strategies continues to add value in the out-of-sample period. However, compared to the SPY, the EWP and MVarP underperform in terms of the Sharpe ratio. Despite having a slightly higher excess return than the SPY, the EWP has higher volatility, a lower Sharpe ratio, and a higher max drawdown. The MVarP also performs poorly, with lower excess returns, higher volatility, and a higher max drawdown than the SPY. This deterioration in performance is evident in the MVarP's weight dynamics. Since 2018, the weights have been heavily and persistently allocated to the Cycle strategy, and as such have become increasingly exposed to equities and have lost diversification value. As a result, the MVarP portfolio becomes more correlated with equity markets, more volatile, and less resilient during market downturns. This portfolio showed higher cumulative returns than all strategies until around 2020, when it was overtaken by the SPY (*Appendix Figure A6*). Since 2019, the SPY has performed well (except in 2022) due to the dominance of large-cap companies, especially in tech, and periods of lower interest rates following the pandemic, making this a tough benchmark to beat.

It is important to note that the MVarP declined notably from in-sample to out-of-sample. The mean-variance optimization relies on historical data for returns, risks, and correlations, but these figures can change over time. This method might assign weights that seem optimal based on past data, but it can perform poorly when market conditions change suddenly, especially if the strategies differ from historical values. This might explain why the MVarP performs worse out-of-sample, whereas EWP remains more stable.

4.3 Fama-French 3-Factor Model

We conducted regressions using the CAPM and Fama-French 3- and 5-factor models. Our primary focus is on the Fama-French 3-factor model (FF3), which provides a more comprehensive view of portfolio performance. The CAPM offers no additional insights, as market excess returns dominate both the FF3 model and the CAPM (*Appendix Table A1*). Likewise, the profitability (RMW) and investment (CMA) factors from the FF5 model do not significantly add much value to the results, as they only cause minor alterations to the model (*Appendix Table A2*). The results for the FF3 regression are in *Table 5*, with t-statistics in brackets. The ***, **, and * represent significance levels of 1%, 5% and 10%, respectively.

Table 5: *FF3 regression results for all sample periods*

	EWP			MVarP		
	IS	OOS	Full	IS	OOS	Full
Alpha	0.102%	0.346%	0.188%	0.287%	-0.179%	0.109%
	(0.543)	(1.321)	(1.034)	(0.959)	(-1.131)	(0.533)
Mkt-Rf	0.6052	0.7794	0.6707	0.7528	0.8718	0.7915
	(12.19)***	(9.07) ***	(9.71) ***	(7.69) ***	(18.79)***	(13.26)***
SMB	−0.0315	0.1723	0.0821	0.2717	0.0608	0.1637
	(−0.28)	(1.85) *	(0.99)	(1.18)	(0.91)	(1.38)
HML	0.0067	0.3167	0.1890	−0.0485	0.3215	0.1767
	(0.07)	(5.33) ***	(2.48) **	(−0.24)	(8.08) ***	(1.78) *
R^2	0.63	0.71	0.64	0.56	0.88	0.69

It can be observed that R^2 remains high across all sample periods for both EWP and MVarP, indicating that the FF3 model accounts for a significant portion of the strategies' variation in returns. The explanatory power reaches the maximum value for the MVarP strategy out-of-sample (0.88). The alphas are statistically insignificant across all strategies and sample periods. This indicates that, after accounting for the model's factors, there is no evidence that either strategy earns abnormal returns. This means that most of the excess returns can be explained by the FF3 factors.

The market factor is highly statistically significant for both strategies in all periods. The coefficients are large and positive, which signals that the strategies are sensitive to market fluctuations. This heavy dependence is consistent with the construction of the combined strategy, in which the MVarP allocates substantial weight to the Cycle strategy, which is linked to U.S. market dynamics. The SMB factor is mostly statistically insignificant, except out-of-sample for the EWP, indicating no consistent inclination for either small-cap or large-cap firms. The HML factor is statistically significant and positive in both the out-of-sample and full samples. This implies that the strategies' returns tend to move with value-oriented stocks.

However, this factor is statistically insignificant in-sample for both strategies, indicating that this relationship emerged only in later years.

5. Limitations

The reported results are based on an idealized backtest and should not be interpreted as directly achievable net performance. The most important limitation is that transaction costs and fees are not modelled. All three underlying strategies - MMA, Cycle, and MoR - rebalance regularly, and the mean-variance portfolio (MVarP) adjusts the allocations over time. In practice, commissions, bid-ask spreads, market impact, and any fund or ETF management fees would reduce returns and Sharpe ratios, and could alter the relative attractiveness of MVarP compared with the equal-weight and 60/40 benchmarks, especially in periods with frequent reallocations.

A second limitation is that the entire analysis is based on historical data. All inputs to the model, returns, volatilities, correlations, and factor signals are estimated from past observations. Financial markets evolve continuously, and relationships that were useful for explaining returns in the past may become weaker or even irrelevant in the future. For example, investor behavior can change over time, regulation and market structure can shift, and new sources of risk can emerge. There is also a risk that a model that fits historical data very well, including noise and random patterns, may lose its ability to generalize to new data. The results should therefore be interpreted as evidence of how the strategies would have behaved in the sample period, not as a guarantee that the same patterns will persist.

Finally, the analysis abstracts from several practical implementation frictions. The three-component strategies are treated as perfectly investable “black boxes”, and the combined portfolio does not account for overlapping holdings, capacity constraints, liquidity shocks, margin requirements, or short-selling and leverage constraints in the underlying instruments. Taxes and operational considerations are also ignored. Taken together, these simplifications mean that the findings are best viewed as an indication that a diversified combination of the three strategies can be attractive in a frictionless setting, rather than as a fully engineered product ready for implementation without further adjustments.

6. Conclusion

This thesis examined whether combining three rule-based strategies - a macro trend-following futures strategy (MMA), a U.S. industry-level equity strategy based on attention and sentiment (Cycle), and a Norwegian multi-factor equity strategy (MoR) - improves portfolio performance from July 2008 onwards. The results show that each component adds a distinct return and risk profile: Cycle and MoR behave like higher-risk equity strategies, while MMA delivers lower returns but offers meaningful diversification through low, often negative, correlation with equities.

When the strategies are combined, the equal-weight portfolio provides the most robust outcome. It delivers higher long-run returns and Sharpe ratios than a 60/40 stock-bond benchmark, and smoother performance with shallower drawdowns than any single component strategy. In contrast, the mean-variance tangency portfolio performs very well in-sample but its advantage largely disappears out-of-sample. The optimizer tends to concentrate in the most recent winner, which makes the portfolio more sensitive to regime shifts and increases volatility and maximum drawdown relative to the simple equal-weight rule.

Factor-model regressions indicate that excess returns of the individual and combined strategies can largely be explained by exposure to standard equity risk premia rather than by unexplained alpha. The main benefit of the combination is therefore better diversification and a more attractive risk-return trade-off, not the discovery of a new independent source of return. Overall, the findings suggest that investors who already use systematic strategies may gain more from combining a small number of transparent, complementary models with simple, stable weights than from relying on aggressive mean-variance optimization based on noisy estimates.

7. References

Markowitz, Harry. “Portfolio Selection.” *The Journal of Finance* 7, no. 1 (1952): 77-91. https://doi.org/10.2307/2975974.

8. Appendix

Figure A1*: Rolling Correlation of the pairwise combinations of the strategies over the sample period*

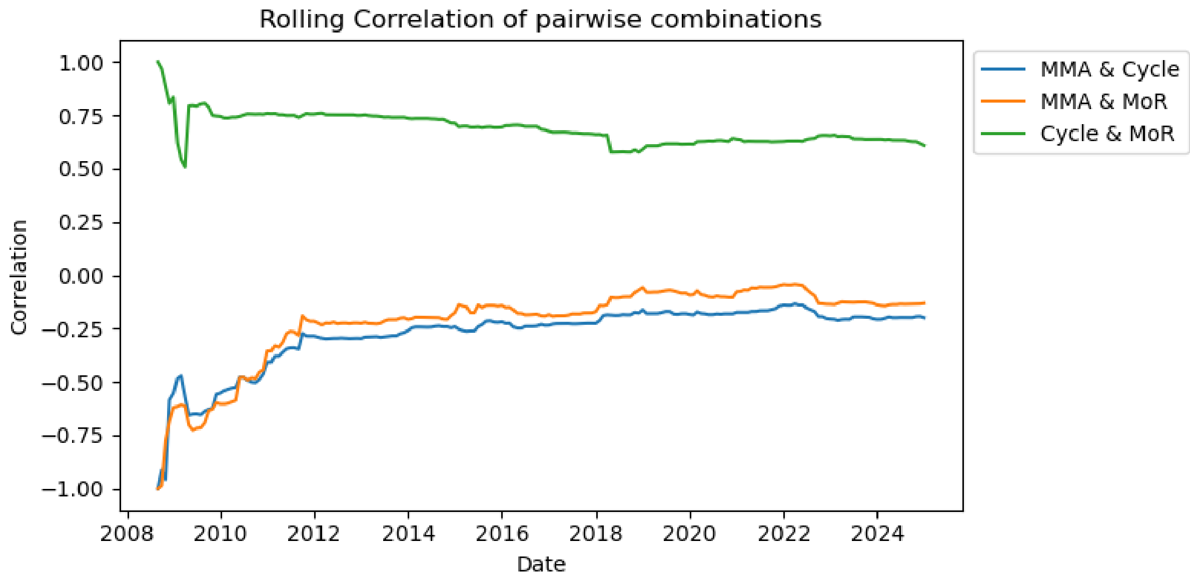


Figure A2: *Mean-Variance Frontier and the optimal Tangency Portfolio using the three strategies over the In-Sample period from 2008-2017*

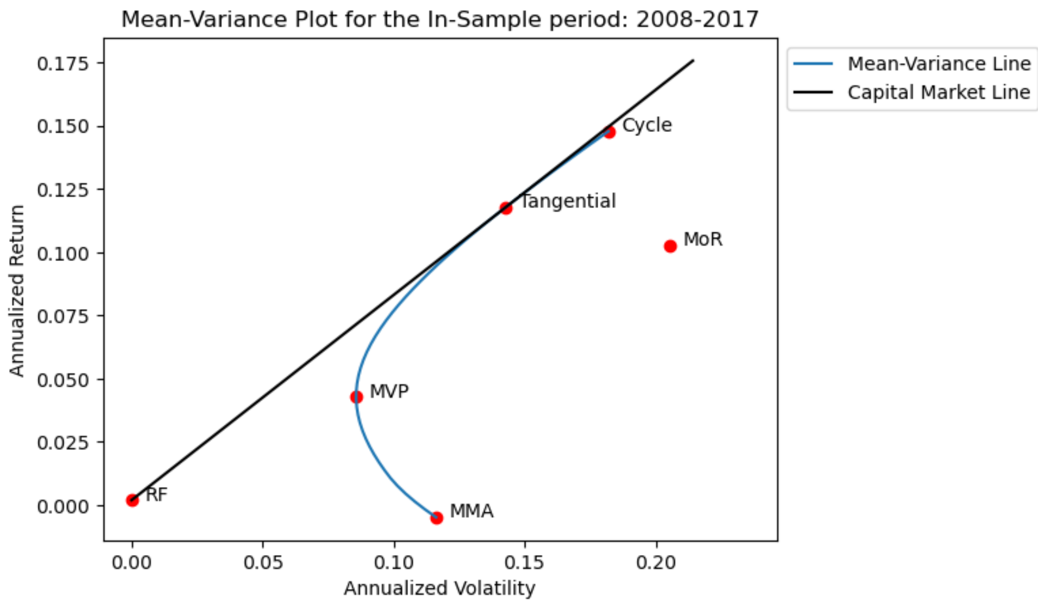


Figure A3: *Mean-Variance Frontier and the optimal Tangency Portfolio using the three strategies over the Out-Of-Sample period from 2018-2024*

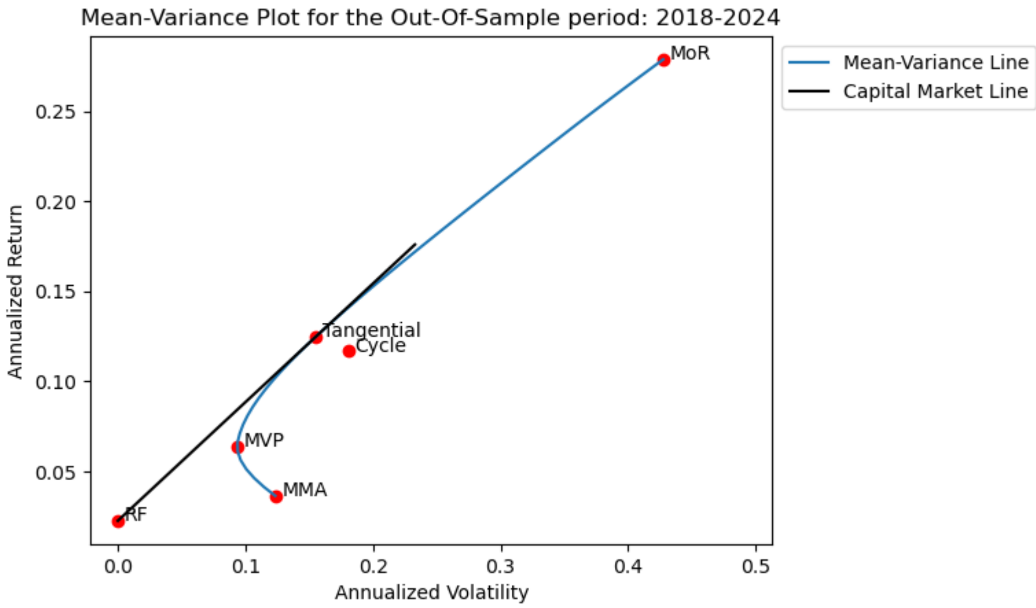


Figure A4: *Cumulative returns of each individual Strategy Return over the full sample period*

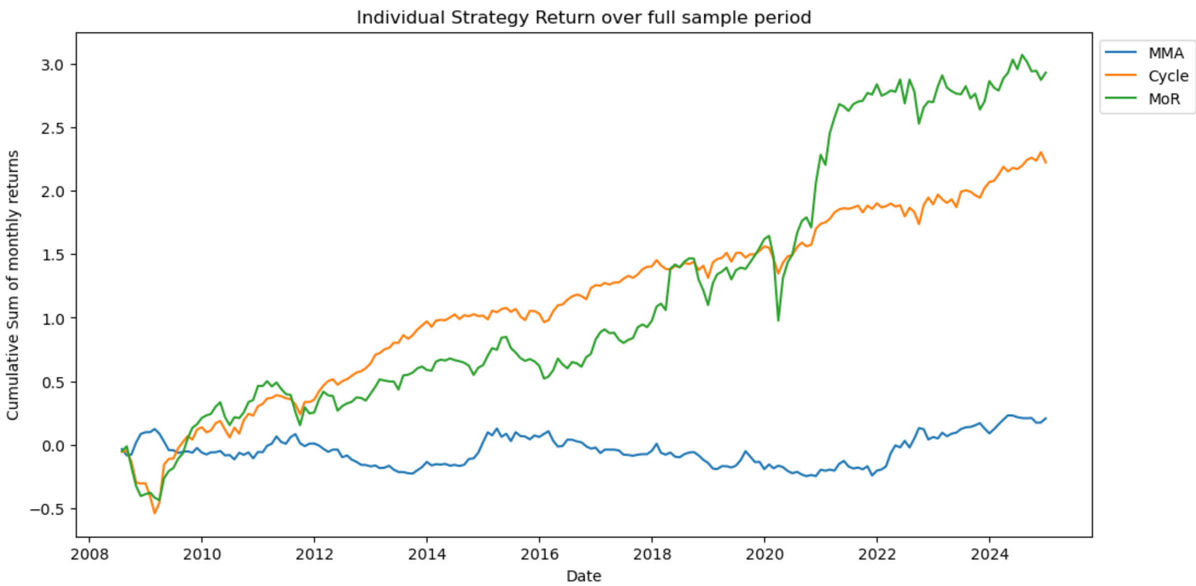


Figure A5: *Drawdowns of the Combined Strategies (EW and MVar) and the benchmarks, for the whole sample period*

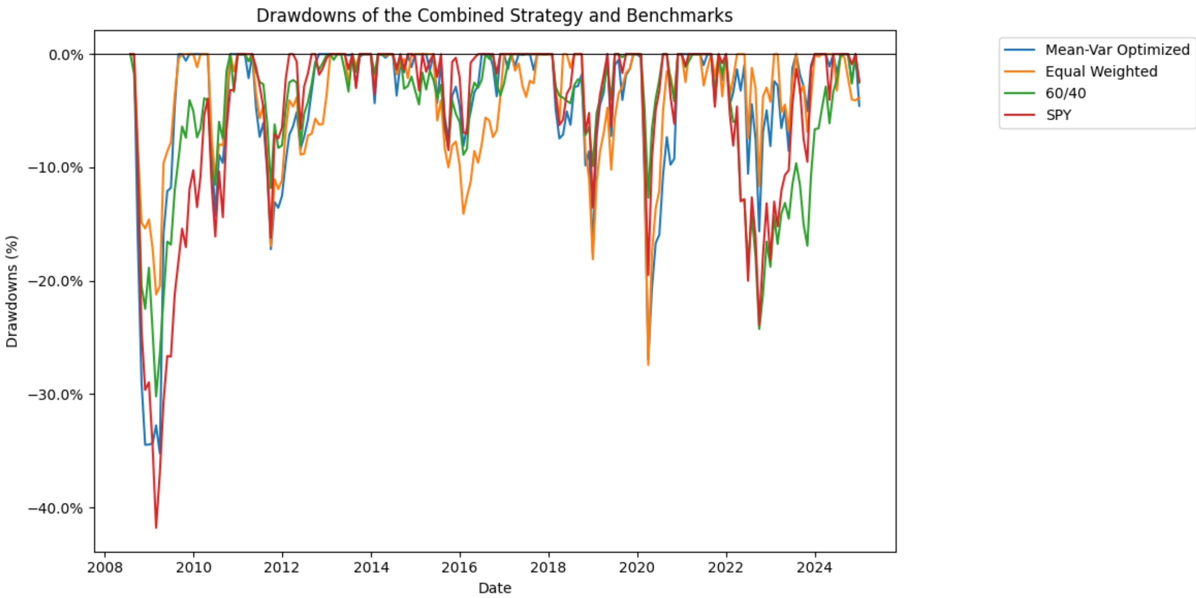


Figure A6: *Cumulative Returns of the Combined Strategies (EW and MVar) and the benchmarks, for the whole sample period*

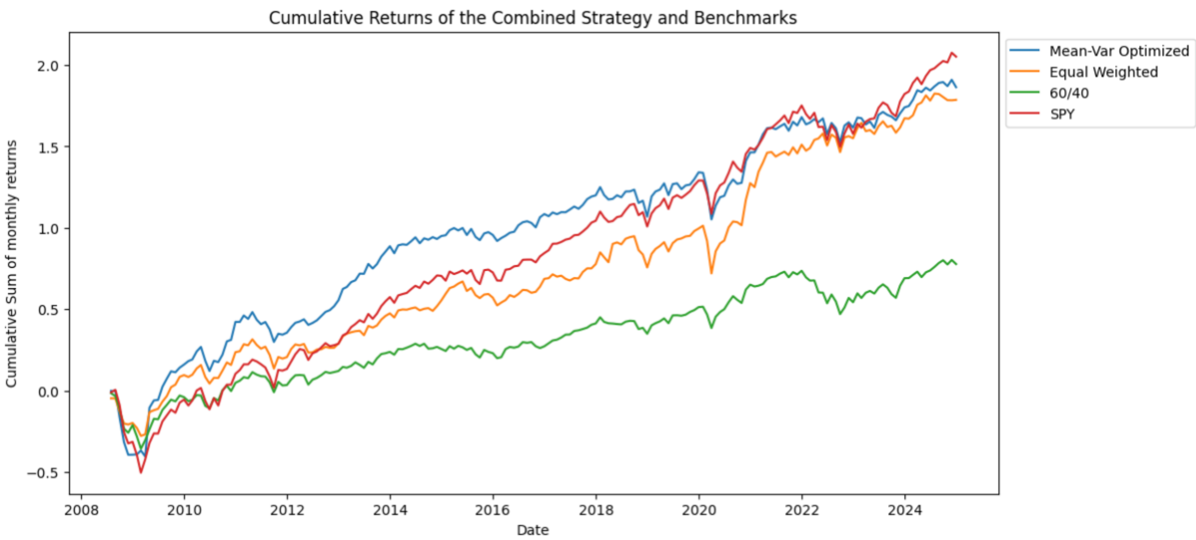


Table A1: *CAPM regression, for both strategies (EWP and MVarP) and all sample periods*

	EWP			MVarP		
	IS	OOS	Full	IS	OOS	Full
Alpha	0.103%	0.187%	0.125%	0.279%	-0.292%	0.032%
	(0.550)	(0.462)	(0.599)	(0.990)	(-0.998)	(0.153)
Mkt-Rf	0.5993	0.8247	0.7120	0.8052	0.8940	0.8492
	(12.19)***	(7.61) ***	(9.77) ***	(10.36) ***	(13.59)***	(16.06) ***
R^2	0.63	0.62	0.61	0.55	0.81	0.66

Table A2: *Fama-French 5-factor regression, for both strategies (EWP and MVarP) and all sample periods*

	EWP			MVarP		
	IS	OOS	Full	IS	OOS	Full
Alpha	0.027%	0.32%	0.103%	0.194%	-0.236%	-0.007%
	(0.155)	(1.137)	(0.591)	(0.832)	(-1.499)	(-0.041)
Mkt-Rf	0.6334	0.7696	0.6777	0.7896	0.8645	0.8138
	(14.79)***	(8.88) ***	(11.97) ***	(9.06) ***	(17.71)***	(15.57)***
SMB	−0.0181	0.2053	0.1536	0.2764	0.1301	0.2492
	(−0.14)	(1.68) *	(1.5)	(1.01)	(1.68) *	(1.58)
HML	-0.0571	0.3054	0.1493	−0.1609	0.2621	0.0691
	(-0.46)	(2.75) ***	(1.22) **	(−0.71)	(4.3) ***	(0.49)
RMW	0.116	0.0857	0.2229	0.0927	0.1399	0.2238
	(0.67)	(0.47)	(1.69) *	(0.26)	(1.17)	(1.64)
CMA	0.2337	-0.0119	0.0509	0.3933	0.0703	0.2199
	(1.34)	(-0.07)	0.3	(2.02) **	(0.93)	(1.64)
R^2	0.64	0.71	0.65	0.57	0.89	0.7