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Testing the Performance of the Risk Budgeting Asset Allocation Strategy in the Brazilian Market Context

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Abstract:

Risk-based strategies to portfolio selection have become popular among researchers and practitioners in developed markets, as they provide reasonable returns without requiring the estimation of expected returns for its calculations. However, few studies verify these strategies' performance in an emerging market context. This article tests the performance of the risk budgeting allocation strategy in the Brazilian market. The results of the strategy are then compared against the weight budgeting and minimum variance approaches. The findings suggest that the strategy's result is consistent with existing literature that the risk budgeting strategy provides an intermediary solution between its weight budgeting equivalent and the minimum variance portfolio.

Keywords: Financial Markets, Risk Management, Risk Budgeting, Asset Allocation,

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1. Introduction

The origins of the risk budgeting allocation strategy can be traced back to Bridgewater Associates in 1996, with the inception of the All Weather Fund. First introduced for Ray Dalio's trust assets, the goal of the fund was to create a portfolio that would achieve reasonable returns regardless of the economic environment, by allocating uncorrelated asset classes equally based on their risk. This is now known as the Risk Parity strategy.

Since then, risk-based strategies became widely used by practitioners. This was not only due to Bridgewater's success, which can be partly attributed to favorable bond performance over the period, but because it is an alternative to the arduous task of estimating expected returns, leading to a more balanced portfolio.

Despite the strategy's success, there are not many studies that verify its performance exclusively in emerging markets. Results may differ from existing literature, given the differences in risk behavior when compared to developed markets' assets. For instance, inflation can be more of a consistent threat in emerging markets than in Europe and the US and given the high levels of political and economic uncertainty, these markets lack a definitive safe haven asset, which can hinder the risk budgeting portfolio's performance.

The goal of this article is to evaluate the risk budgeting strategy's performance in a multi-asset portfolio predominantly composed by Brazilian assets. This is accomplished by constructing a portfolio comprised of four indexes that serve as benchmarks for passive investment funds and reflect the major asset classes normally analyzed by Brazilian investors. Then, multiple risk budget allocation techniques are examined by running an optimization algorithm that yields the amount of wealth that would have to be invested in each index to distribute the proper amount of risk into each asset-class, given pre-defined risk budgets. The risk-based results are put up against the minimum variance portfolio and their respective weight budgeting equivalents. This allows us to verify if the results found in developed countries during the preceding decades hold true to the Brazilian market.

Analyzing the risk-based strategies' performance in the Brazilian market can be a great advantage to native and international investors alike, given Brazil's long relationship with risk due to historic high levels of inflation, political instability and the lack of literature on the subject. Brazil remains as the largest market in Latin America as of 2020, with over one trillion dollars in assets under management[1].

The rest of the report is structured as follows: Section 2 briefly goes through the most relevant findings in the field of asset allocation, including notable literature discussing the risk budgeting strategy in developed markets and Brazil. In Section 3 the risk budgeting problem and the methodology used to calculate the risk budgeting portfolios are formally described. The data collected is discussed in Section 4, while the results and its limitations are analyzed in Section 5. Finally, the final comments alongside directions for later research can be found in Section 6.

2. Literature Review

The study of capital allocation has its roots in the landmark article Markowitz (1952), in which the author develops the mean-variance approach to portfolio selection. The paper remains as the most notable contribution to the field since it provides an elegant solution to the asset allocation problem,

[1] Data taken from Boston Consulting Group - https://www.bcg.com/publications/2020/global-asset-management-protect-adapt-innovate -Accessed in 07/12/2021

formalizing the concept that investors make decisions with the goal of maximizing their returns given a certain level of risk.

Nonetheless, the mean-variance approach has come under scrutiny from practitioners and researchers alike over the decades. The strategy's key drawbacks can be summarized by the two following points. First, the portfolios created by this method are heavily concentrated in few securities, resulting in an unpractical strategy. Second, the mean-variance approach generates results that are highly sensitive to the input parameters, the expected returns and covariance matrix, all of which can be extremely difficult to estimate. Merton (1980) suggests that the main source of sensibility comes from the estimation of expected returns, rather than the covariance matrix input.

These issues have been extensively addressed by academics. For instance, Black and Litterman (1992) tackles the problem of estimating expected returns by creating a model which incorporates investor's own views on asset returns to the results. Tütüncü and Koenig (2004) develop a more complex approach, creating an algorithm which replaces the conventional estimation process by describing the input parameters in the form of uncertainty sets.

Investors, on the other hand, prefer solutions that are less computationally intensive, allowing room for the minimum variance and equally weighted portfolios. The former is a unique portfolio that is located on Markowitz's efficient frontier. Despite being regarded as more robust because it does not require the estimation of expected returns for its calculations, it still fails to overcome the problem of excessive concentration. The latter addresses this issue by distributing the same amount of wealth to each security under evaluation. It is frequently used in practice, and its naive approach has been proven to be efficient out-of-sample, as shown by DeMiguel et al. (2009). The downside of this strategy is that it can lack diversification in risk in cases in which assets' volatilities significantly diverge from each other. Therefore, the risk budgeting strategy was developed.

Even though risk budgeting portfolios were being implemented by practitioners since the mid-nineties, the concept of risk contribution was first notably formalized by Qian (2006), where the author shows that the contribution of an asset in the overall portfolio's risk is closely related to their expected contributions to the portfolio's losses.

Maillard et. al (2010) takes a step further, and defines the specific problem of an equal risk contribution portfolio, i.e. the risk parity portfolio. They show that the risk parity strategy results in an intermediary solution between the equally weighted and the minimum variance portfolio, and it can be defined as a variance-minimizing portfolio subject to a diversification constraint. Bruder and Roncalli (2012) expand the article's solution by deriving the theoretical properties of a general risk budgeting portfolio, instead of restricting the analysis to the risk parity case. They confirm Maillard et. al (2010) solution stating that the volatility derived by the risk budgeting strategy lies between the minimum variance and their respective weight budgeting portfolios. More recently, Kapsos et. al (2018) have looked to improve the methods previously established by the authors, by incorporating uncertainty sets to the volatility estimation to create more robust results to the risk budgeting portfolio.

Many articles have since tried to explain the risk budgeting strategy's returns. Asness et. al (2012) proposes the theory that the risk parity portfolio's performance is a result of leverage aversion. In many cases, the risk parity portfolio solution requires the investor to leverage their allocations in the least risky assets to enable a higher risk exposure. If investors are unwilling or unable to take leverage, they will be obligated to hold a suboptimal portfolio with higher concentration in riskier assets, allowing leveraged investors to take advantage of this market imbalance.

Souza et. al (2016) were the first to verify the risk budgeting allocation strategy with Brazilian assets, by implementing a risk parity portfolio composed by Brazilian equity sector indexes. They confirm the results previously found in developed markets, and further state that the method to estimate the covariance matrix has little influence on the final portfolio results. Bortoluzzo et. al (2018) expands this result by incorporating all liquid stocks in the Brazilian stock exchange to the securities analyzed.

Despite being the most relevant literature regarding risk-based allocation strategies in Brazil, these studies display an important drawback by confining their analyses to a single asset class. To take full advantage of risk diversification is crucial to consider cross-asset correlation, which will be the focus of this report.

3. Defining the Risk budgeting Problem

This section describes the approach developed by Maillard et. Al (2010) and Bruder and Roncalli (2012) to mathematically solve for the risk budgeting portfolio, which will be the method utilized in this paper.

3.1 Defining Risk Contribution

Consider a portfolio of n assets and let x_i be the nominal exposure of asset i in the total portfolio, i.e., the amount of wealth invested in asset i. Following the same approach as most relevant literature on the subject, volatility is used as the risk measure for the remainder of the paper. Therefore, we can define the risk of the portfolio $\sigma_{(x)}$ as:

$$\sigma_{(x)} = \sqrt{x^T \Sigma \, x} \tag{1}$$

In which Σ is the covariance matrix, and *x* is the vector of weights. We can define the marginal risk contribution (*MRC*) of asset *i* by taking the vectorial derivative of $\sigma_{(x)}$ according to the chain rule, as follows:

$$MRC_i = \frac{\partial \sigma_{(x)}}{\partial x_i} = \frac{\partial \sqrt{x^{\mathrm{T}} \Sigma\, x}}{\partial x^{\mathrm{T}} \Sigma\, x} * \frac{\partial x^{\mathrm{T}} \Sigma\, x}{\partial x_i}$$

$$MRC_i = \frac{1}{2\sqrt{x^{\mathrm{T}} \Sigma\, x}} * 2(\Sigma x)_i$$

$$MRC_i = \frac{(\Sigma x)_i}{\sqrt{x^{\mathrm{T}} \Sigma\, x}} \tag{2}$$

Therefore, for every small increase *g* of exposure in asset *i,* the portfolio's risk will increase by g*MRC_i. Following this approach, we find that the total contribution (*RC*) of asset *i* in the portfolio's risk can be defined as the marginal risk contribution of asset *i* multiplied by its total weight in the portfolio (x_i).

$$RC_i = x_i * MRC_i = x_i * \frac{(\Sigma x)_i}{\sqrt{x^{\mathrm{T}} \Sigma\, x}} \tag{3}$$

If we add all the risk contributions, we find that:

$$\sum_{i=1}^{n} RC_i = \sum_{i=1}^{n} x_i * \frac{(\Sigma x)_i}{\sqrt{x^{\mathrm{T}} \Sigma\, x}}$$

$$\sum_{i=1}^{n} RC_i = x^{\mathrm{T}} * \frac{\Sigma x}{\sqrt{x^{\mathrm{T}} \Sigma\, x}}$$

$$\sum_{i=1}^{n} RC_i = \sqrt{x^{\mathrm{T}} \Sigma\, x} = \sigma_{(x)} \tag{4}$$

Thus, we conclude that the total risk of the portfolio is equal to the sum of total risk contributions.

3.2 Defining Risk Budgets

The next step is to define the constraints that will be applied to calculate the risk budgeting portfolio.

If we divide the RC of asset *i* by $\sigma_{(x)}$, we arrive at the relative contribution of asset *i* in the total risk of the portfolio, i.e., we can analyze what is the weight of asset *i* in overall portfolio's volatility.

$$\frac{RC_i}{\sigma_{(x)}} = x_i * \frac{(\Sigma x)_i}{\sqrt{x^{\mathrm{T}}\Sigma\, x}} * \frac{1}{\sqrt{x^{\mathrm{T}}\Sigma\, x}} = x_i * \frac{(\Sigma x)_i}{x^{\mathrm{T}}\Sigma\, x} \tag{5}$$

We can then assume a set of risk budgets $\{b_1, b_2, \dots, b_n\}$ that represent the amount of risk the investor is willing to take in any given asset in relation to the overall portfolio. For instance, if b_i is equal to 0.3, the investor is willing to allocate 30% of the portfolio's risk in asset *i*. Therefore, the risk budgeting portfolio will be the one that satisfies the following constraints:

$$\begin{cases} \dfrac{RC_1}{\sigma_{(x)}} = b_1 \\ \dfrac{RC_2}{\sigma_{(x)}} = b_2 \\ \quad (\dots) \\ \dfrac{RC_n}{\sigma_{(x)}} = b_n \end{cases}$$

For the special case of the risk parity portfolio, we can assume that all risk budgets are equal ($b_1 = b_2 = b_n$), and, therefore, the portfolio solution will follow the constraint that $\frac{RC_i}{\sigma_{(x)}} = \frac{RC_j}{\sigma_{(x)}}$ for every *i* and *j*.

3.3 Finding the Solution

Finally, we can then solve for the risk budgeting portfolio by using a SLSQP (Sequential Least Squares Programming) algorithm, minimizing the sum of square differences between the relative risk contributions and their respective risk budgets, as follows:

$$x^{*} = \arg\min \sum_{i=1}^{n} \left(\frac{RC_i}{\sigma_{(x)}} - b_i \right)^2 \tag{6}$$

$$\textbf{u.c.} \sum_{i=1}^{n} \boldsymbol{x_i} = \mathbf{1} \ \textbf{ and } \ \mathbf{0} \leq \boldsymbol{x} \leq \mathbf{1}$$

The analysis is further constrained by the scenario in which the investor is fully invested at all times and short selling is not allowed. We follow this approach so the analysis can be impartial across strategies in terms of constraints, since the equally weighted strategy automatically assumes positive weights.

4. Data Description

The daily price data from four different asset classes were extracted from Bloomberg to build the diversified portfolio: Equities, Nominal Bonds, Inflation-linked Bonds and Commodities. After removing non-common trading days, the sample taken spans through 18 years and 4336 observations, going from September 2003 to September 2021. Log returns are utilized throughout the analysis. Summary Statistics can be found on **Table 1** and the annualized sample covariance matrix for the entirety of the sample can be found in **Appendix 1.**

To represent equities, the Ibovespa Index (IBOV) was chosen. Ibovespa is a market-value weighted theorical portfolio that captures the stocks with the highest volume traded in the Brazilian stock market, composing Brazil's main equity benchmark. Brazilian equities were the worst asset-class in terms of risk-adjusted returns over the period analyzed, displaying an Info Sharpe of 0.39.

IBOV clearly outperformed the other asset classes in the first quarter of the sample, form 2003 – 2007. However, the index suffered a significant drawdown in 2008, during the Global Financial Crisis, and even though it had a quick rebound until 2010, the index was stagnant from 2011 to 2016.

During this period, Brazil faced great political and economic uncertainty, including a corruption scandal that broke on mid 2014 and led to the President's impeachment in 2016. In addition to the impact of the Covid-19 crisis, these events can indicate why the index displayed such a high volatility across the sample. The index cumulative performance, along with the other three indexes can be found in **Figure 1.**

The IMA-B index tracks the performance of inflation-linked government bonds of different maturities, which is a particularly important asset-class for Brazilian investors, given the country's long-lasting history with inflation. From 2003-2020, Brazil's main inflation indicator averaged 5.75%[2], while the cumulative inflation rate for 2021 already reached 8.24%, as of November, 2021. It is the index with the highest performance across the sample, with an annualized return of 13.95% and an Info Sharpe of 2.03.

The IRF-M index tracks the performance of nominal government bonds across different maturities. It was the best performing index in terms of risk-adjusted returns over the period analyzed, with an

2 Author calculations, data taken from Brazilian Institute of Geography and Statistics (IBGE). Accessed in 10/11/2021. **https://www.ibge.gov.br/estatisticas/economicas/precos-e-custos/9256-indice-nacional-de-precos-ao-consumidor-amplo.html?=&t=series-historicas**

Info Sharpe of 3.90. This performance can be explained by high central bank rates, which are common in emerging markets and economies with high levels of inflation. Brazil's central bank rate averaged 10.16%[3] during the sample. IRF-M is also the asset with the lowest risk, with annualized volatility of 3.10%

Global commodities' performance is tracked by the Commodity Research Bureau Index (CRB), which is composed by a basket of energy contracts, agricultural commodities, and precious and industrial metals. It was the asset with the worst annualized return over the period, with only 4.40%, due to the low global inflation environment for most of the 21st century. However, the importance of this asset-class is evidenced by its performance during the aftermath of the Covid-19 pandemic. Given the high volume of liquidity injected by central banks around the world, and a sudden rise in demand combined with disrupted supply chains, global economies have experienced a sudden rise in inflation, which applies an upward pressure in commodities' prices. This has led CRB to outperform all indexes in 2021 as of September 16th, with a return of 22.59%.

Table 1 – Summary Sample Statistics

Index	Annualized Return	Annualized Volatility	VaR 1D 1%	VaR 1M 1%	Info Sharpe	Max Drawdown
IBOV	11.19%	28.22%	4.14%	18.95%	0.3964	-62.96%
IMAB	13.95%	6.87%	1.01%	4.62%	2.0301	-14.57%
IRFM	12.12%	3.10%	0.45%	2.08%	3.9064	-4.35%
CRB	4.40%	7.50%	1.10%	5.04%	0.5869	-40.91%

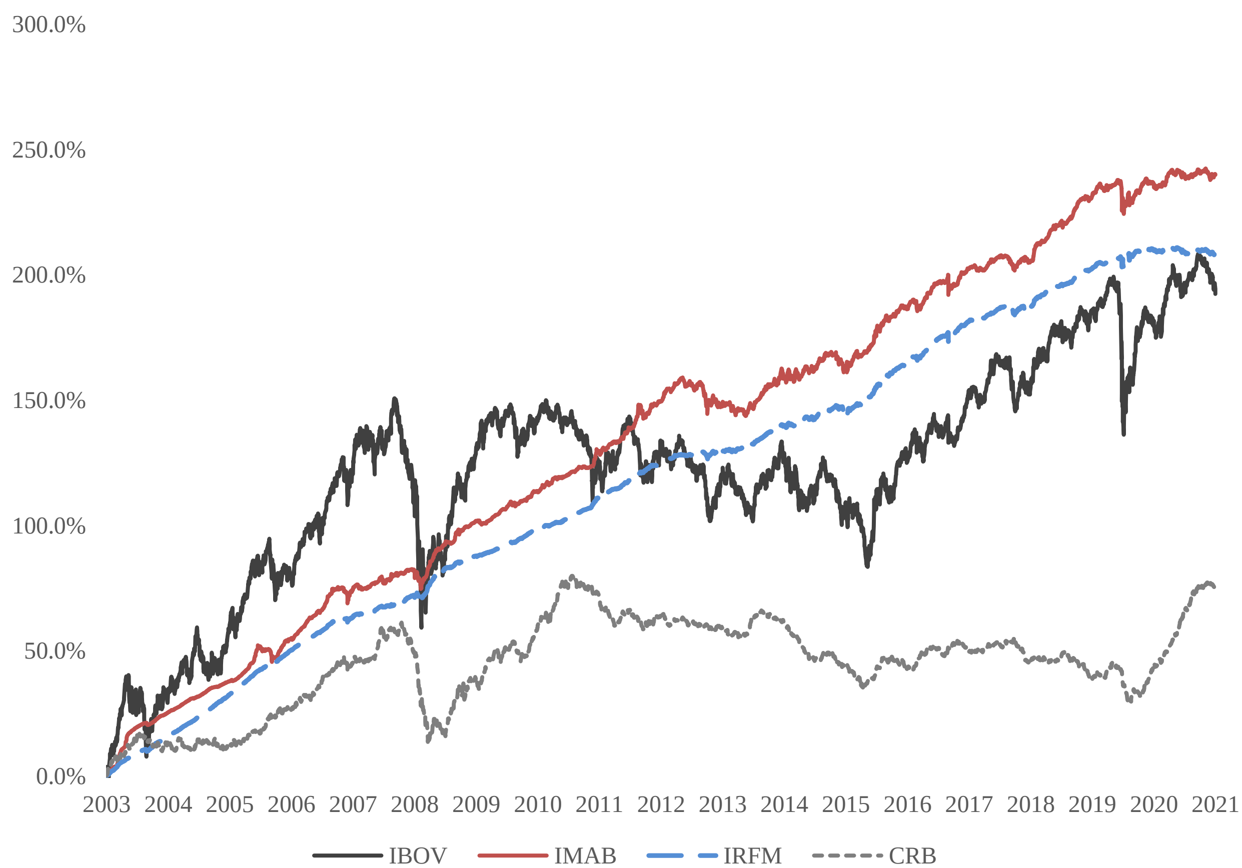
With only these few indexes we create a diversified portfolio with a seemingly low degree of correlation given different economic environments. IBOV is favored in periods of optimism

[3] Author calculations, data taken from Bloomberg.

regarding Brazil's economic growth while IRF-M plays the counterpart for when the economy slows down.

IMA-B provides protection against periods of high inflation, which has a negative impact on both assets previously mentioned. Finally, the CRB index is used as an exposure against global inflation and economic growth.

Figure 1 – Cumulative Return of The Four Indexes Analyzed



5. Empirical Results

The performances of five different risk budgeting strategies are tested against the minimum variance and weight budgeting portfolios. First, we test the standard risk parity strategy, the case in which the risk budgets for all asset-classes are equal (1/n). Then, to avoid biased results due to a single index performance, we analyze four scenarios in which the investor chooses to allocate a higher risk budget in a certain asset class relative to the rest. This is done for every asset class, by allocating 40% of the risk in the index chosen and 20% of the risk in each of the remaining three. The same weights are used for the weight budgeting portfolios to serve as comparison.

The rebalancing is done in a monthly basis and the performance is tested out-of-sample in a daily basis. A 252-day rolling window is used for the covariance matrix estimation to verify the portfolio's volatility. **Table 2** illustrates the risk budgets for each strategy tested.

Table 2 – Risk Budgets For Each Portfolio Tested

Strategy	IBOV	IRFM	IMAB	CRB
Risk Parity	25.0%	25.0%	25.0%	25.0%
Overweight Equities	**40.0%**	20.0%	20.0%	20.0%
Overweight Nominal Bonds	20.0%	**40.0%**	20.0%	20.0%
Overweight IL-Bonds	20.0%	20.0%	**40.0%**	20.0%
Overweight Commodities	20.0%	20.0%	20.0%	**40.0%**

To compare the strategies' performances, the annualized returns and volatilities are calculated, along with measures of Value at Risk and maximum Drawdowns to further evaluate the portfolios' risk. As in Maillard et. Al (2010), the Herfindahl Index is used to assess the concentration for each portfolio. Let x_i be the exposure of asset *i* in time *t*, the Herfindahl index is defined as follows:

$$h_t = \sum_{i=1}^{n} x_{t,i}^2 \qquad (7)$$

Since $x_{t,i} \in [0,1]$ and all the weights sum to 1, the higher the Herfindahl index, the higher the concentration of the portfolio. We can scale the index to fit onto [0,1] by:

$$H_t = \frac{h_t - {}^{1}\!/_{n}}{1 - {}^{1}\!/_{n}} \tag{8}$$

Therefore, the equally weighted portfolio and a perfectly concentrated one will have an H_t equal to 0 and 1, respectively. Finally, to compute the portfolio's turnover rate, we can verify the changes in exposures of the portfolio's assets in each rebalancing date, by:

$$T_t = \sum_{i=1}^{n} \frac{|x_{t,i} - x_{t-1,i}|}{2} \tag{9}$$

This definition implies that the turnover rates for all weight budgeting portfolios are equal to zero. In practice, this is not true, since rebalancing the portfolio to fit the pre-defined exposures will incur in transaction costs. However, for the purpose of this study, we will focus on the transaction costs led by active management decisions. The average of H_t and T_t across time for each strategy, along with the remaining resuts can be seen in **Table 3.**

The minimum variance portfolio had the best risk-adjusted performance among all strategies, displaying the best annualized return and lowest volatility across the inspected sample. This can be explained by the low perfoamnce of the two riskiet asset-classes, equities and commodities, in relation to bonds. Since interest rates were sustained at a high level throughout the whole period, nominal bonds provided reasonable returns at a low risk. Nevertheless, the minimum variance portfolio remains as the strategy with the highest concentration, with a Hefindahl Index of 65.14%, corroborating the drawbacks brought up by literature.

Table 3 - Summary Statistics of Empirical Results

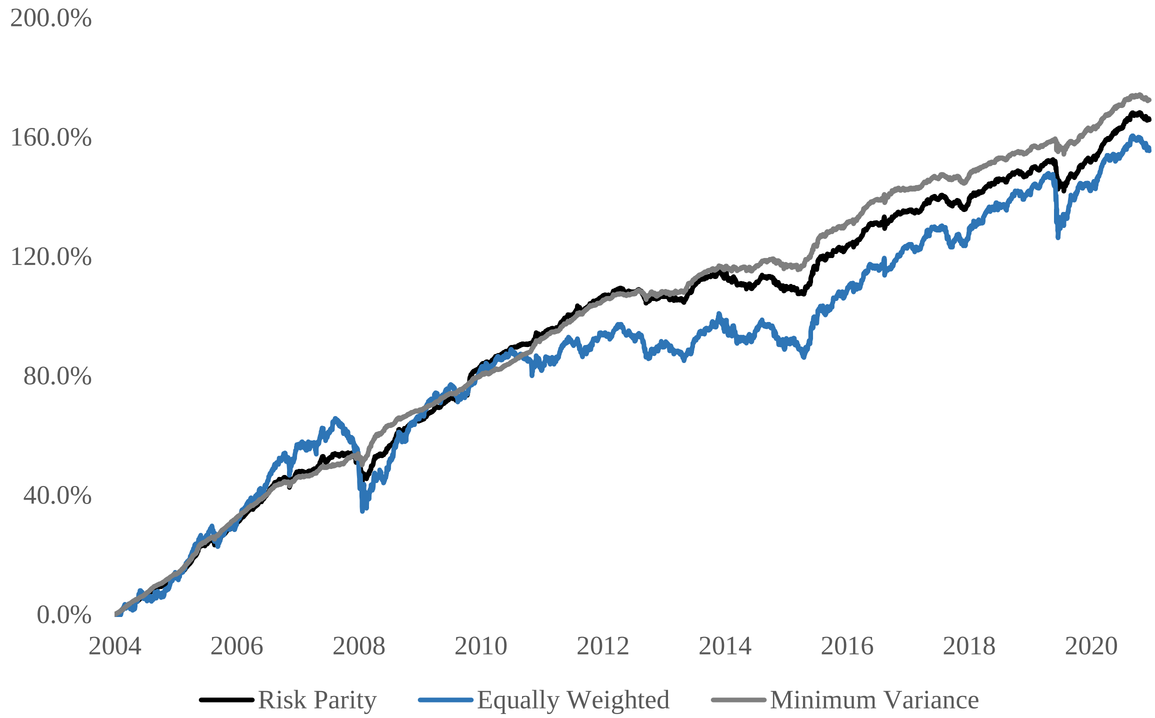
Strategy	Annual. Return	Annual. Volatility	VaR 1D 1%	VaR 1M 1%	Max Drawdown	Info Sharpe	Ht	Tt
Equally Weighted	9.5992%	8.9655%	1.3140%	6.0216%	-27.3367%	1.07	0.0000%	0.0000%
Risk Parity	10.2387%	4.2574%	0.6240%	2.8594%	-10.5786%	2.40	16.0347%	2.6079%
Minimum Variance	10.6524%	2.9357%	0.4303%	1.9718%	-5.2185%	3.63	65.1427%	2.5150%
Overweight Nominal Bonds								
Fixed Weighted	10.0450%	7.5639%	1.1086%	5.0802%	-22.2262%	1.33	4.0000%	0.0000%
Risk Budgeting	10.6047%	3.8154%	0.5592%	2.5626%	-8.2646%	2.78	26.6872%	2.3630%
Overweight Commodities								
Fixed Weighted	8.4657%	7.9051%	1.1586%	5.3094%	-27.9155%	1.07	4.0000%	0.0000%
Risk Budgeting	9.5874%	4.2616%	0.6246%	2.8623%	-11.5120%	2.25	16.5003%	3.9357%
Overweight Inflation-Linked Bonds								
Fixed Weighted	10.3187%	8.1080%	1.1883%	5.4456%	-23.3182%	1.27	4.0000%	0.0000%
Risk Budgeting	10.2680%	4.4399%	0.6507%	2.9820%	-11.0317%	2.31	12.9596%	1.7407%
Overweight Equities								
Fixed Weighted	9.5674%	12.6291%	1.8510%	8.4822%	-36.0847%	0.76	4.0000%	0.0000%
Risk Budgeting	9.9083%	4.7583%	0.6974%	3.1959%	-12.1881%	2.08	13.8648%	1.8117%

The risk parity strategy achieved a higher annualized return compared to the equally weighted portfolio, although transaction costs hinder this effect. However, the biggest advantage of the risk-based strategy is volatility mitigation, since the risk parity portfolio displayed less than half of the equally weighted portfolio's risk across all measures analyzed. Therefore, the results confirm the findings of Maillard et. Al (2010) that the volatility derived from the risk parity approach lies between the volatility of the minimum variance and equally weighted portfolios.

Figure 2 illustrates the strategies' cummulative returns over the period, in which it becomes clear how the equally weighted portfolio's performance is dependent on equities. The strategy clearly outperforms the minimum variance and risk parity portfolios during bull markets, mainly through the periods 2004-2007 and 2016-2019. However, during IBOV's fickle performance over the early 2010's, the strategy struggled to deliver returns. Furthermore, given the fact that IBOV's volatility is significantly higher than other indexes, the equally weighted portfolio experienced more severe drawdowns across the sample, specially during the Global Financial Crisis and the Covid-19 pandemic.

We can verify the strategy's risk exposure to IBOV by analyzing the historical risk contribution of each asset. **Figure 3** shows the distribution of risk contributions and weights of each asset across the sample, on the top and bottom graphs, respectively. IBOV was the biggest factor to the portfolio's risk by a far margin, with an average contrubition to total volatility of 74.96%. IMA-B, IRF-M and CRB had average contributions of 11.51%, 4.89% and 8.63%, respectively.

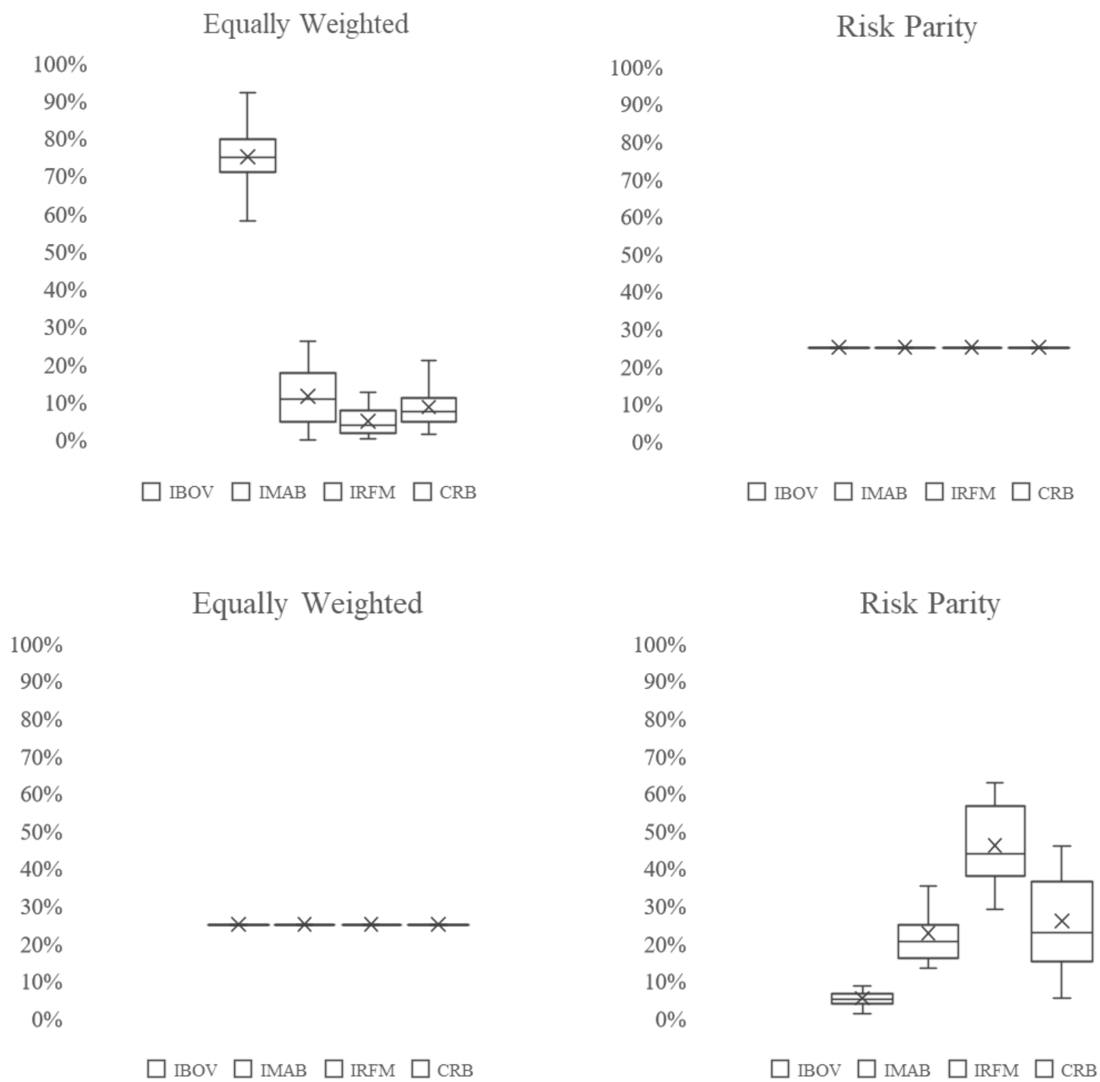
Figure 2 – Cumulative Return Comparison Between Risk Parity, Equally Weighted and Minimum Variance Strategies



In the risk parity case, by definition, all indexes contributed the same amount to the total volatility across the entire period. To achieve this result, the investor would have to give up almost all their investments in IBOV. The average nominal weight of the asset in the portfolio was 5.20%. Conversely, IRF-M displayed the highest nominal exposure over the period, with an average of 46.10%, which is expected for being the asset with the lowest volatility in the sample. IMA-B and CRB had average weights of 22.63% and 25.94%, respectively.

The outcomes from the remaining four risk budgeting strategies are consistent with these results. All strategies had slightly better annualized returns than their weight budgeting equivalents, except for the strategy of being overweight in IMA-B. Furthermore, the discrepancy between volatilities remains consistent with the outcomes from the risk parity case. More importantly, all results confirm the findings of Bruder and Roncali (2012), displaying volatilities that lie between the minimum variance and the weight budgeting portfolios.

Figure 3 – Historical Distribution of Risk Contributions (Top) and Weights (Bottom)



Given CRB's low returns across the sample, being overweight in Commodities would have been the worst perfoming decision regardless of asset allocation method, with the weight budgeting portfolio being the worst performing strategy overall, with an annualized return of 8.47%. Furthermore, the risk budgeting portfolio for this strategy had the highest turnover rate, which is due to the fact that the volatility for this asset flunctuated a lot over the period, resulting in greater rebalancing costs. Given the current scenario of rising global inflation, these results might change in the near future.

On the other hand, being overweight in bonds would have payed off. The risk budgeting portfolio overweighted in Nominal Bonds had the highest risk-adjusted return across risk-based strategies, with an Info Sharpe of 2.78. However it is also the most concentrated risk-based portfolio, with a Hefindahl Index of 26.69%. This is expected since this strategy allocates the biggest risk budget in the asset with the lowest volatility. Being overweight in Inflation-Linked Bonds also delivered high returns over the period, and the risk budgeting portfolio had the most reasonable turnover among the risk-based strategies implemented.

As previously mentioned, Brazilian equities had a poor performance across the sample, therefore the strategy of being overweight in IBOV had the lowest risk-adjusted returns among the strategies analyzed. The weight budgeting portfolio had the worst Info Sharpe, at 0.76, given the fact that it was the only portfolio with double-digit annualized volatility in the sample.

Evidently, past returns are not a fair indicator of future performance, and the results might change in the following decades, specially if Brazil finds itself in a bull market scenario with low interest rates. Nevertheless, the calculations show that the risk budgeting approach can be used as an alternative to the weight budgeting and minimum variance methods, resulting in a portfolio with reasonable volatility and capital distribution among securities.

6. Conclusion

In this article, we study the risk budgeting asset allocation strategy within a multi-asset portfolio majorly composed by Brazilian assets. We backtest the strategy’s performance over several scenarios in a seventeen-year span and compare the results against the equally weighted and minimum variance approaches.

We start by testing the risk parity strategy, in which the investor chooses to allocate the same amount of risk in each asset-class. We find that the risk parity approach creates an intermediary solution between the equally weighted and the minimum variance portfolio, corroborating the results from previous literature.

Furthermore, we discuss four other risk budgeting strategies that might incorporate investor's views on the market by going overweight in a specific asset. The results are consistent with the risk parity case, in which the risk budgeting portfolios display similar results to their weight budgeting equivalents with significantly lower risk.

There are still few articles that study the risk budgeting strategy in the Brazilian market, therefore, there are several ways that one might improve the findings of this paper. First, one could improve the analysis by creating a method to minimize transaction costs, something that was mentioned in this article but not analyzed thoroughly. For instance, the investor could use an algorithm that rebalances the portfolio only if the current weights stray too far away from the optimal ones.

Second, one could verify the impact of implementing different volatility estimation methods to find more realistic results. It might be interesting to test autoregressive methods to estimate volatility such as the GARCH and EWMA models, or the robust risk budgeting method described in Kapsos et. al (2018).

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Appendix

A.1 –Sample Covariance Matrix

Index	IBOV	IMAB	IRFM	CRB
IBOV	**0.079656**	0.007718	0.003551	0.004646
IMAB		**0.004723**	0.001811	0.000374
IRFM			**0.000962**	0.000175
CRB				**0.005633**